Theory of Computation
Problem Set 2
Universidad Politecnica de San Luis Potosi

TPlease start solving these problems immediately, don't procrastinate, and work in study groups. Please do not simply copy answers that you do not fully understand;

Advice: Please try to solve the easier problems first (where the meta-problem here is to figure out which are the easier ones). Don't spend too long on any single problem without also attempting (in parallel) to solve other problems as well. This way, solutions to the easier problems (at least easier for you) will reveal themselves much sooner (think about this as a "hedging strategy" or "dovetailing strategy").

The remainder of these problems should be completed and returned by Friday, October 5 at the start of class.

Problem One: Elementary Set Theory

For the purposes of this problem, suppose that we are dealing with the following sets:

$A = \{ 1, 2, 3, 4 \}$

$B = \{ 2, 2, 2, 1, 4, 3 \}$

$C = \{ 1, 3 \}$

$D = \{ 2, 3, 4 \}$

$E = \{ x \mid x \in \mathbb{N} \text{ and } x \text{ is even} \}$

For each of the following, is the claim true or false? Explain why. You do not need to prove your assertions.

i. $A = B$.

ii. $C \Delta D = C$

iii. $|D| > |A|$

iv. $E \cap D = C \cap D$

v. $C \in A$.

vi. $C \subseteq A$.

Problem Two: Properties of Sets

Below are four claims about sets. For each statement, if it is always true, prove it. If it is always false, prove that it is always false. If it is sometimes true and sometimes false, provide an example for which it is true and an example for which it is false and briefly explain why your examples are correct.

To prove that two sets are equal, remember that you need to show that any element of the first set must also be an element of the second set and vice versa. Recall that this is equivalent to showing that the two sets are subsets of one another. It is not sufficient to use Venn diagrams or any other informal reasoning here. You need to formally prove each result.

i. If $A \in B$ and $B \in C$, then $A \in C$.

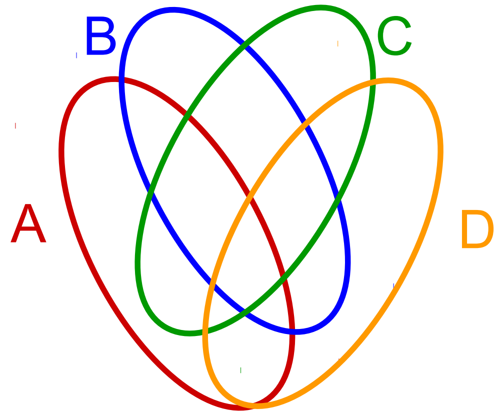
ii. If $\wp(A) = \wp(B)$, then $A = B$.

iii. $(A - B) \cup B = A$.

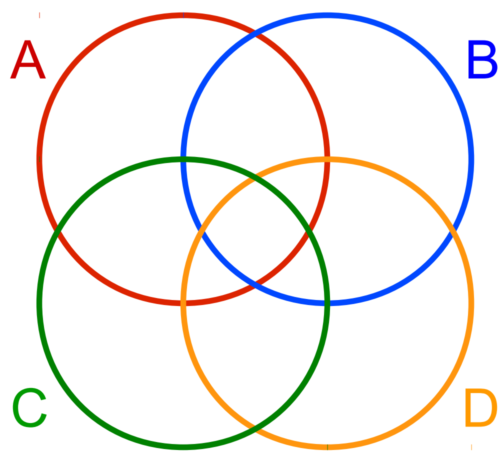
iv. $A \cap (B - A) \neq \emptyset$.

Problem Three: Venn Diagrams

In our first lecture, we saw the following picture, which represents a Venn diagram for four sets:



This picture is probably not what you would have initially expected. It might seem more reasonable to draw the Venn diagram this way:



However, the way that these circles overlap is not sufficient to show all possible ways that four different sets can overlap. Come up with four sets A, B, C, and D such that there is no way to accurately represent the overlap of those four sets with the second Venn diagram, and briefly explain why your sets have this property.

**Help -?

https://www.asu.edu/courses/mat142ej/readings/Sets_and_Counting_bars.pdf

https://core.ac.uk/download/pdf/29195171.pdf

https://tel.archives-ouvertes.fr/tel-00662789/file/Thesis.pdf page 45

Problem Four: Demostrate the conditional of next properties

$(A - B) \cup B = A$.

$A \cap (B - A) \neq \emptyset$.

a)$A-(B\cap C) = (A-B)\cup(A-C)$

b)$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

Problem Five: Grphs

Whay programming with this Primitives stated earlier in C ++:

```
template <class Tbase>

Grafo<Tbase>::Grafo()
   {
   inicio = new nodo;
   inicio->etiqueta = 0;
   inicio->nodo = 1;
   inicio->ady  = 0;
   inicio->sig  = 0;
   }
template <class Tbase>

void Grafo<Tbase>::InsertarNodo(const Tbase dato)
   {
   nodo aux,p;
   aux = new nodo;
   p=this.inicio;
   while(p->sig != 0)
       p = p->sig;
   aux->etiqueta = dato;
   aux->nodo = (p->nodo)+1;
   aux->ady = 0;
   aux->sig = 0;
   p->sig = aux;
   (this.nnodos)++;
   }
```