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Mathematical Optimization — Assignment 4

https://moodle-app2.let.ethz.ch/course/view.php?id=3610

Exercise 1: The Dual program of an LP

(a) Formulate the dual program for the following LP:

$$\begin{array}{ll} \max & x_1 + 2x_2 - x_3 \\ \text{s.t.} & 2x_1 + 3x_2 + 5x_3 \le 1 \\ & -x_1 + 2x_2 - x_3 \ge 5 \\ & x_1 - 3x_3 = 2 \\ & x_1 \ge 0, x_2 \le 0, x_3 \in \mathbb{R}. \end{array}$$

(b) Determine the dual program for the following LP:

$$\begin{array}{ll} \max & c^T x + h^T y \\ \text{s.t.} & Ax + By \le b \\ & Cx + Dy = \gamma \\ & x \in \mathbb{R}^n, y \in \mathbb{R}^p, y \ge 0. \end{array}$$

Exercise 2: Complementary Slackness Conditions

Consider the following (primal) LP:

$$\begin{array}{lccccccccl} \max & 5x_1 & + & 3x_2 & + & x_3 & & & & \\ \text{s.t.} & 2x_1 & + & x_2 & + & x_3 & \le & 6 & & \\ & x_1 & + & 2x_2 & + & x_3 & \le & 7 & & \\ & & & & & x_i & \ge & 0 & & i = 1, 2, 3 \end{array}$$

Graphically solve the dual of this LP. Then use the complementary slackness conditions to solve the primal problem.

Exercise 3: Infeasibility and LP Duality

(a) Construct an infeasible LP whose dual is feasible. Is it possible to find one such that the dual optimal value is zero?

(b) Construct an infeasible LP whose dual is also infeasible. Explain how you constructed such an example.

Exercise 4: Recession Cone - Extreme Ray with Positive Cost

Let $C := \{x \in \mathbb{R}^n \mid Ax \le 0\}$ be a pointed cone. Prove the following theorem from the lecture:

Let $c \in \mathbb{R}^n$. Then, $\max\{c^{\mathrm{T}} x \mid x \in C\} = +\infty \Leftrightarrow$ there exists an extreme ray $r \in \mathbb{R}^n$ of C with $c^{\mathrm{T}} r > 0$.

(*) Can you find a solution which does not use the Theorem of Minkowski-Weyl?