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Mathematical Optimization — Assignment 7

https://moodle-app2.let.ethz.ch/course/view.php?id=3610

Exercise 1: Interior Point Method
Consider the following dual problem, and solve it using the interior point method seen in the lecture.

$$\begin{array}{rrl} \min & y_1 + 2y_2 & \\ \text{s.t.} & y_1 + y_2 & \geq 4 \\ & y_2 & \geq 0 \end{array}$$

Exercise 2: Strict vs. Strong Convexity
A function is called **strictly convex** if $\forall \lambda \in (0,1)$ and $x, y \in \mathrm{dom}(f), x \neq y$:

$$f(\lambda x + (1-\lambda)y) < \lambda f(x) + (1-\lambda) f(y).$$

A function is called **strongly convex** with modulus $\sigma > 0$ if $\forall \lambda \in (0,1)$ and $x, y \in \mathrm{dom}(f), x \neq y$:

$$f(\lambda x + (1-\lambda)y) + \frac{\sigma}{2}\lambda(1-\lambda)||y-x||_2^2 \leq \lambda f(x) + (1-\lambda) f(y).$$

i) Give an example of a convex function, which is not continuous.
ii) Give an example of a convex function, which is not strictly convex.
iii) Give an example of a strictly convex function, which is not strongly convex.
iv) Give an example of a strongly convex function.

Prove your claims.

Exercise 3: Convergence of the Newton method
In the lecture's proof of quadratic convergence of the Newton method, you have seen that (using the notation from the lecture) $\|\nabla f(x_{t+1})\|_2 \leq \frac{L}{2\sigma^2}\|\nabla f(x_t)\|_2^2$. Use the assumption $\|\nabla f(x_0)\|_2 \leq \frac{\sigma^2}{L}$ and apply induction to conclude

$$\frac{L}{2\sigma^2}\|\nabla f(x_T)\|_2 \leq (1/2)^{2^T}.$$

Exercise 4: 'Invariance' of the Newton Step Under Linear Transformations
Show that the (pure) Newton method is 'invariant' under linear transformations: Let $f : \mathbb{R}^n \to \mathbb{R}$ and let $A \in \mathbb{R}^{n\times n}$ be an invertible matrix. Consider the Newton method for the following two cases:

(i) $\min_y f(y)$
i.e. $y^{k+1} := y^k - (\nabla^2 f(y^k))^{-1} \cdot \nabla f(y^k)$
(ii) $\min_x \tilde{f}(x) := f(Ax)$
i.e. $x^{k+1} := x^k - (\nabla^2 \tilde{f}(x^k))^{-1} \cdot \nabla \tilde{f}(x^k)$

Show that if $y^k = Ax^k$, then $y^{k+1} = Ax^{k+1}$.
Hint: Recall that

$$\begin{array}{rcl} \nabla f(Ax) & = & A^T \nabla f(Ax) \\ \nabla^2 f(Ax) & = & A^T \nabla^2 f(Ax) A \end{array}$$