# Computational Physics Homework 3

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## Problem 1

a) For the first year:

$$\langle t_1 \rangle = 23.3115, \quad \sigma_{t_1} = 3.4718$$
$$\langle h_1 \rangle = 41.5770, \quad \sigma_{h_1} = 2.7897$$

For the second year:

$$\langle t_2 \rangle = 23.4154, \quad \sigma_{t_2} = 3.3338$$
$$\langle h_2 \rangle = 40.6153, \quad \sigma_{h_2} = 2.9752$$

For the two years together:

$$\langle t \rangle = 23.3635, \quad \sigma_t = 3.4039$$

$$\langle h \rangle = 41.0962, \quad \sigma_h = 2.9238$$

From the results, It is obvious that average values and *rms* fluctuations of temperature and humidity are similar between the first year, second year and the two years together. This is reasonable, because they should be in the same range in every year if there is no significant climate change. Also, the two years' data is just the average of the first and the second year.

b) First, I subtract the average value from the data stream, so we can easily see the fluctuation without the peak at wavenumber equals to 0. Here, I use *periodogram*, which the power spectrum is defined at $N/2+1$ frequencies.

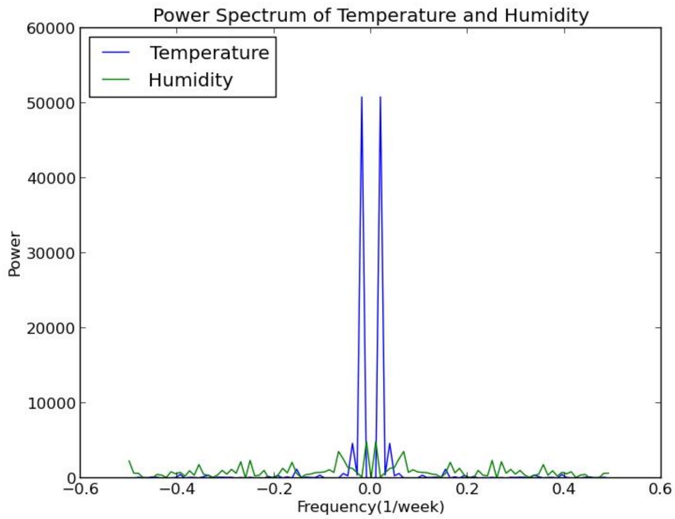


Figure 1: Power Spectrum of Temperature and Humidity

In the Figure 1, the power spectral density of temperature is very high at the frequency of $1/52\ week^{-1}$, which corresponds to the period of one year. Also, the power spectral density of humidity has no obvious high peak like that of temperature.

c) To eliminate the seasonal variations, I take away the frequency corresponding to the period of one year in frequency-domain space, and then transform back to time-domain space.

After taking away the seasonal variation under frequency-domain space,

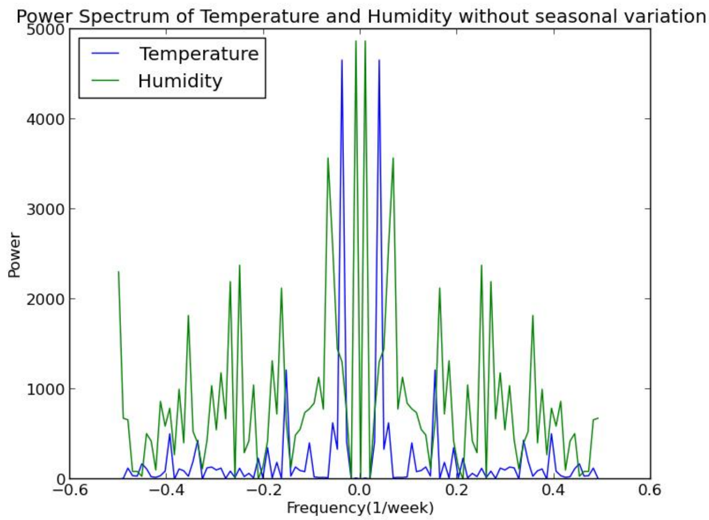


Figure 2: Power Spectrum of Temperature and Humidity without seasonal variation

After taking away the seasonal variation under Time-domain space,

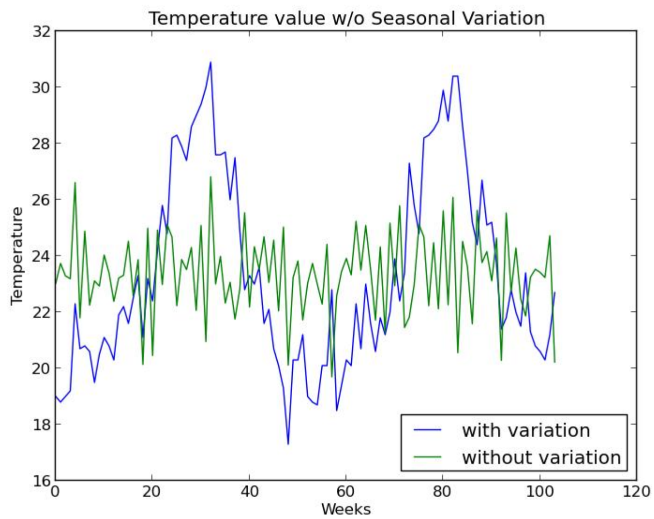


Figure 3: Temperature value w/o seasonal variation

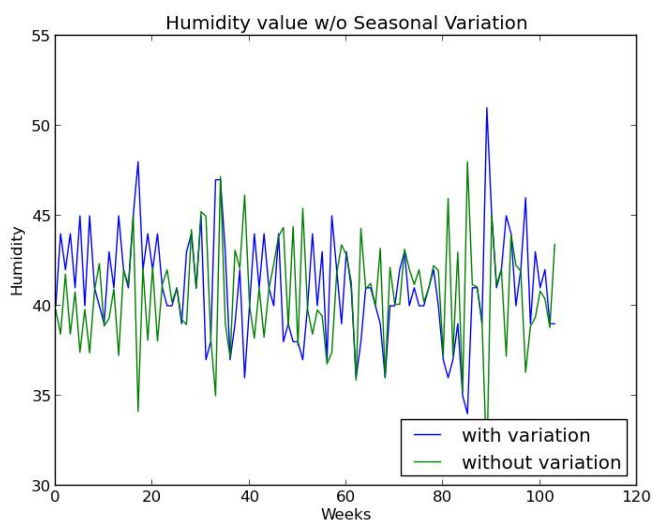


Figure 4: Humidity value w/o seasonal variation

After this computation, it is clear that under time-domain space the temperature fluctuation is much even with no periodic change. However, humidity fluctuation is no much difference just as what I mentioned before. There is no obvious high peak, which means that humidity does not change with season. To compare the fluctuation, I divide $rms$ by average to cancel units. After taking away the seasonal variation, hunidity has larger fluctuation.

$$C = \frac{rms}{average} \tag{1}$$

$$C(t) = 0.0633$$

$$C(h) = 0.0709$$

# Problem 2

a) The FFT code is in the attachment, hw3.py.

b) 2-dimension power spectra can be gotten by 2-dimension FFT.

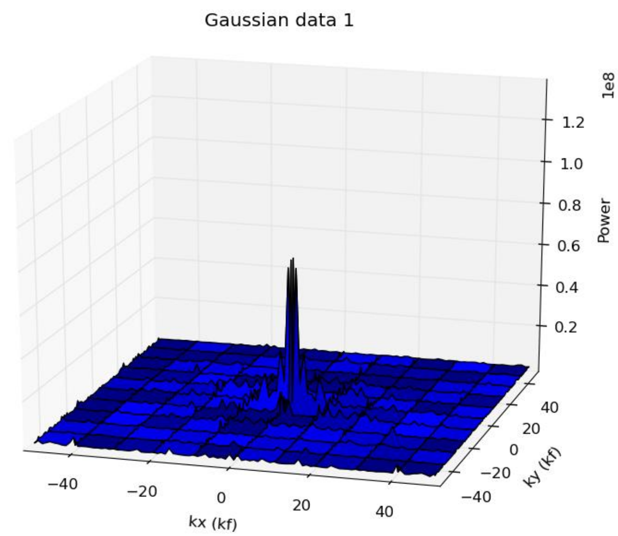


Figure 5: Power Spectrum of Gaussian data 1

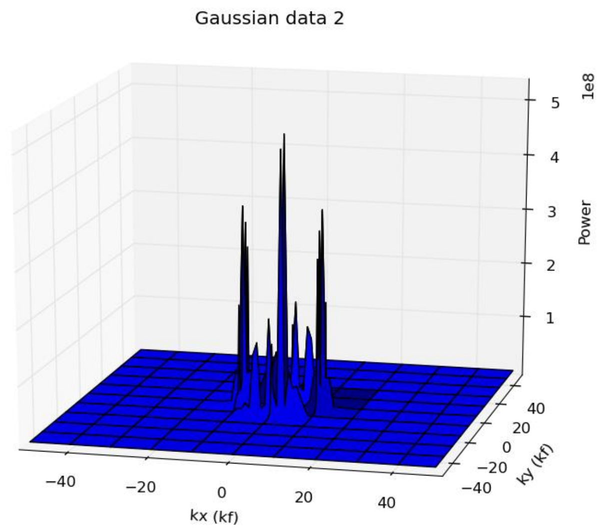


Figure 6: Power Spectrum of Gaussian data 2

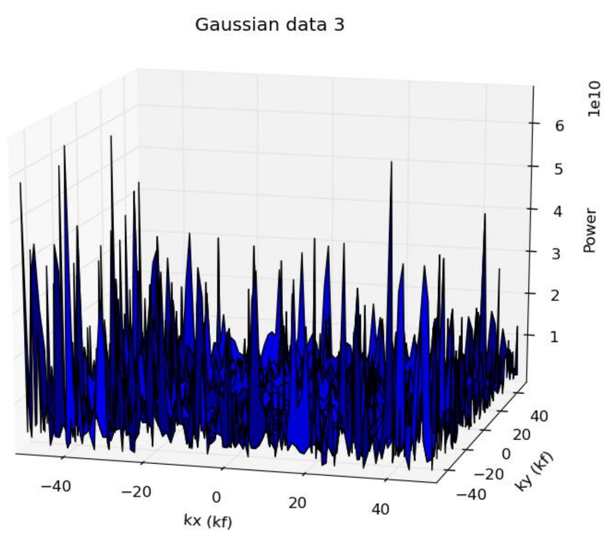


Figure 7: Power Spectrum of Gaussian data 3

Now the 2D power spectra is in the coordinate of $(k, \theta)$. To get the log-log plot of all of them. I integrate the $\theta$ of power spectrum field from 0 to $2\pi$ as a function of $k$.

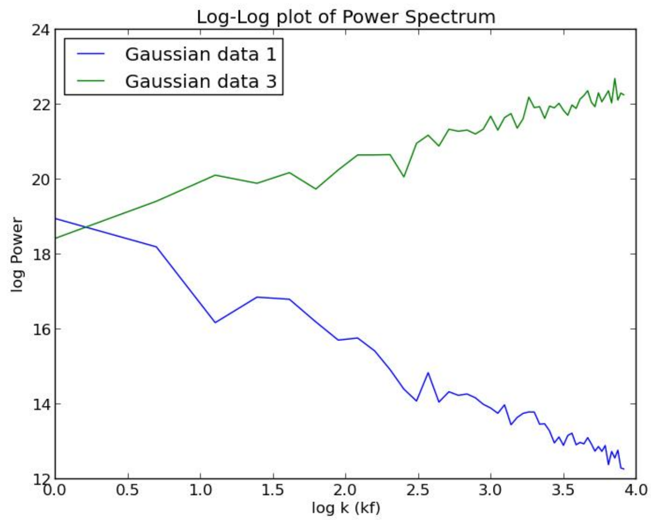


Figure 8: log-log plot of Power Spectrum for data 1 and 2

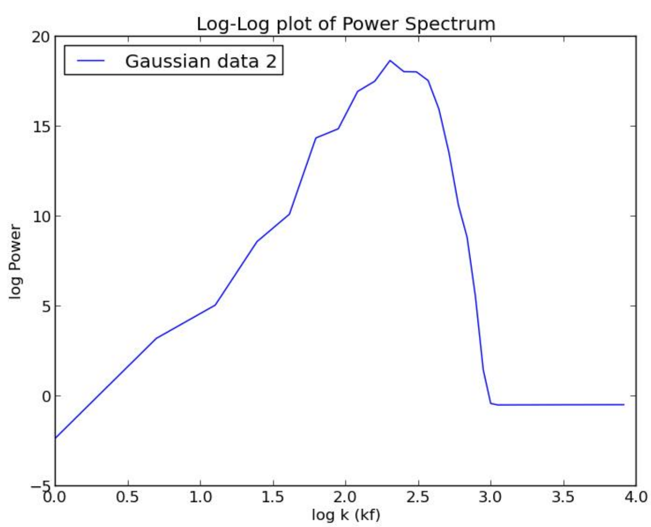


Figure 9: log-log plot of Power Spectrum for data 3

c) In data 1, the power spectrum density mainly clusters at small wavenumber, and the slope of log-log plot is around-5.Therefore, the power spectrum is generated by $P = k^-5$. In data 3, the power spectrum density mainly clusters at large wavenumber, and the slope of log-log plot is around 1. Therefore, the power spectrum is generated by $P = k^1$. In data 2, the power spectrum density mainly clusters at about $k^{2.5}(k_f)$. This power spectrum might be generated by a specific frequency signal.

d) After I resample the field in a grid two times coarser, Nyquist frequency becomes half of the original. This leads to aliasing. The power beyond the Nyquist will be aliased back to the frequency range $-\frac{\pi}{L} \leq k \leq \frac{\pi}{L}$, which means that the high frequency part leaks to low frequency in Fourier space.

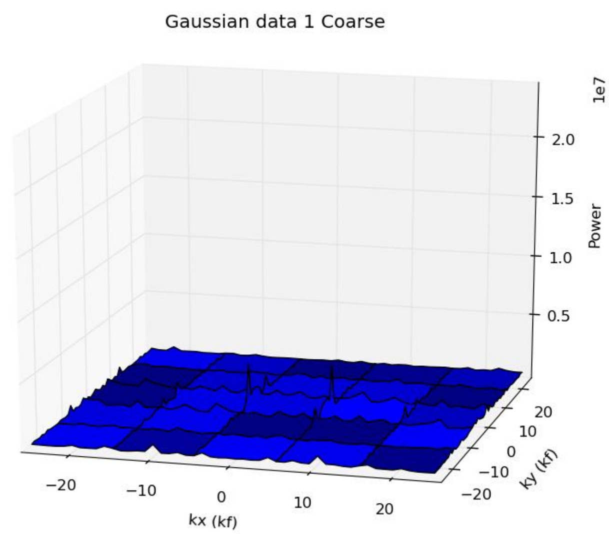


Figure 10: Power Spectrum of coarse Gaussian data 1

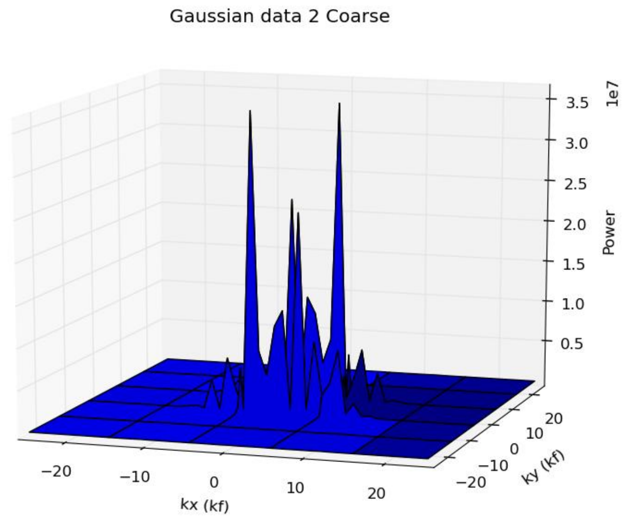


Figure 11: Power Spectrum of coarse Gaussian data 2

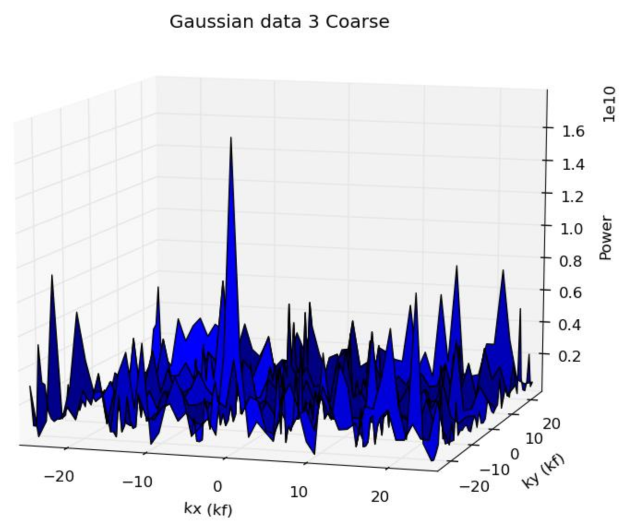


Figure 12: Power Spectrum of coarse Gaussian data 3

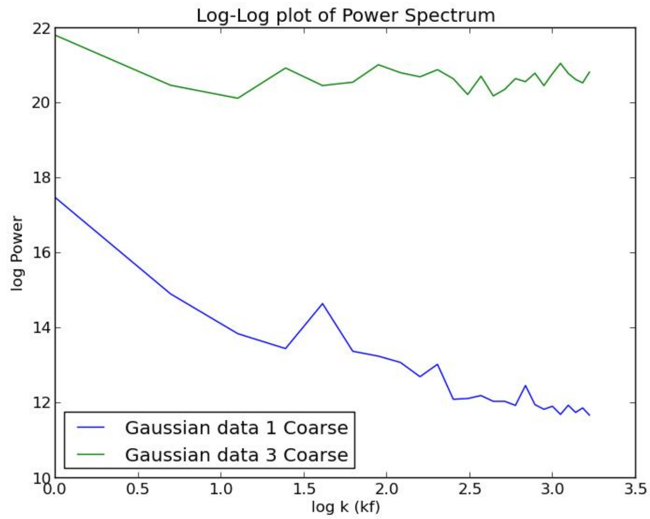


Figure 13: log-log plot of Power Spectrum for coarse data 1 and 2

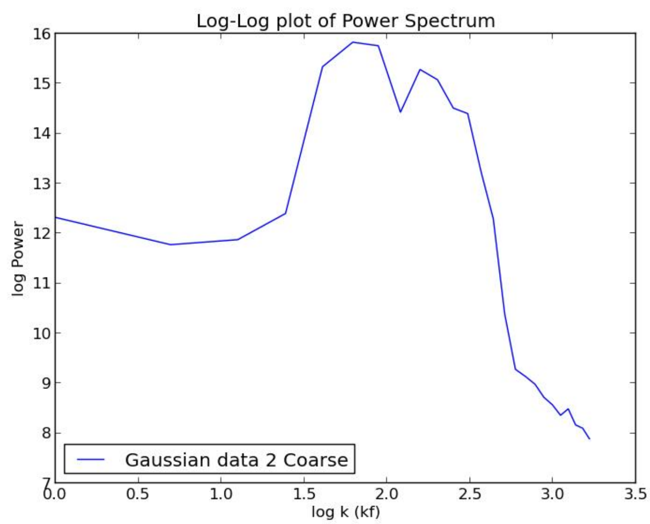


Figure 14: log-log plot of Power Spectrum for coarse data 3

# Problem 3

a) The code is in the attached file, hw3.py. Two clusters of $N_s$ particles are shown as below:

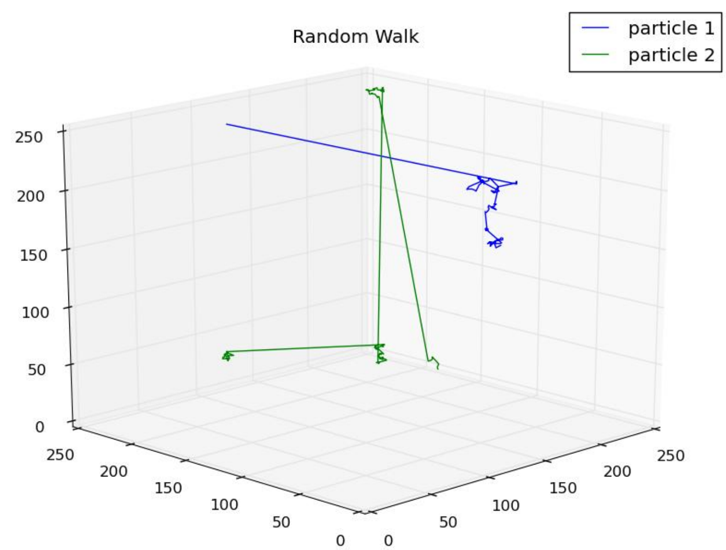


Figure 15: Rayleigh-Levy Random Walk

b) In the window corrected power spectrum, the power spectrum density increases slowly in the small wavenumber part. All the clusters share the same cumulative distribution. Thus, the distances of different clusters are related. As the distance increases, the distances of different clusters are less likely to be the same. So, the power spectrum density increase slowly.

We can also see that the power spectrum density increases rapidly in the large wavenumber part. In each cluster, certain particle and the next particle are related by the same cumulative distribution. And the distance between certain particle and the next particle is short. So, when it comes to the large wavenumber part, we should also take the correlation within clusters into consideration. So,the power spectrum density increases more rapidly.