STAT GR5261   Statistical Methods in Finance

Final Project Report

Professor: Hammou EI Barmi

Group Members:

Cheng PENG (cp2990)

Jiaying ZHANG (jz2897)

Jiun-Ying CHEN (jc4878)

Yuhao KANG (yk2758)

Content

I Summary

There's a saying that "any monkey can beat the market", meaning portfolio constructed with randomly chosen assets sometimes outperform the market and many portfolio managers. This raised our interest. We randomly chose ten assets including various industries(***See in appendix1***), and analyze their returns. We also do research on the portfolio.

II Descriptive Analysis

2.1 Introduction to data

We chose 10 stocks randomly. We use their daily closing price during 2010-01-01 to 2018-12-01. Their symbols are 'CMCSA','FISV', 'MSFT','JNJ', 'CSCO','ADS', 'CL', 'BBT', 'AAPL', 'BFB'. Also, we use 'FVX' to denote the 5 years treasury yield and 'GSPC' for S&P500.

2.2 The sample descriptive statistics for 10 stocks

In this section, we report sample statistics. The result is as following.

	CMCSA	*FISV*	*MSFT*	*JNJ*	*CSCO*	*ADS*	*CL*	*BBT*	*AAPL*	*BFB*
Beta	1.198	0.940	1.114	0.540	1.274	1.357	0.557	1.024	1.005	0.718
Mean	0.018	0.018	0.016	0.011	0.011	0.013	0.008	0.009	0.023	0.016
s.d.	0.062	0.044	0.062	0.037	0.070	0.079	0.039	0.057	0.076	0.052
Skew.	-0.104	0.006	0.122	0.253	-0.107	-0.252	-0.240	-0.249	-0.070	0.077
Kurt.	-0.304	0.235	0.522	-0.032	0.418	0.737	0.660	0.308	-0.252	0.342

Table 1 Sample Statistics for 10 stocks

From the table above, we can see that beta for 'JNJ' and 'CL' are less than 1. This is because JNJ and CL are two worldwide companies who sell daily products like household and health care products. This two stocks are 'defensive' stocks. And 'FISV', 'BBT','AAPL' are not sensitive. Their beta(s) are very close to 1. And 'CMCSA','CSCO','ADS' tend to be more profitable(risky).

As for mean and standard deviation, mean price for 'CL' and 'BBT' are lower than others. The mean price for 'AAPL' is highest, accompanied by the highest deviation.

We also computed skewness and kurtosis coefficients, 'FISV','MSFT','JNJ','BFB' are positive skewness distributed, meaning there exists extreme positive returns while others are negative skewness distributed with extreme negative returns. And 'CMCSA', 'JNJ','AAPL' are platykurtic distributed, meaning the returns are flat-tailed while others are leptokurtic with thinner returns.

2.3 Monthly price for 10 stocks

Here are the plots for monthly price of 10 assets.

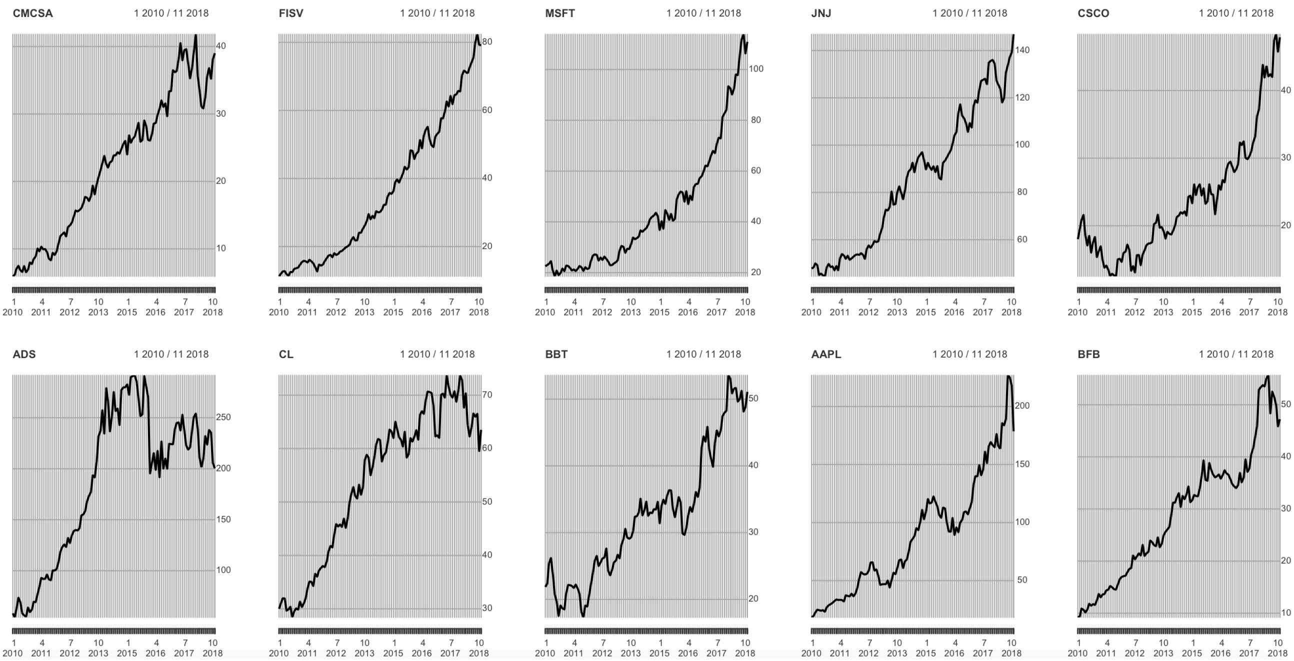


Fig. 1 monthly price for 10 assets

‘CMCSA’,’MSFT’,’CSCO’,’ADS’,’AAPL’ are 5 digital-related companies. For ‘CNCSA’, its price fluctuates in 2018, which caused by the acquisition of ‘SKY’. For ‘MSFT’, in 2014, their product Windows phone didn’t perform well, causing the fall of stock price. The price went up in 2015 because their investment in ‘cloud service’. Price of ‘CSCO’ and ‘ADS’ has stayed stable in latest few years. ‘AAPL’, refers to APPLE, has encountered a fall in 2018, which caused by the decreased sales of IPhone. In total, price for ‘MSFT’, ‘CSCO’,’AAPL’ has stayed stable. They are all biggest companies in the world.

Then we’ll talk about ‘JNJ’,’CL’ and ‘BFB’. ‘JNJ’ and ‘CL’ are 2 famous companies selling daily personal care products. Their price stayed stable since their prime business take a great portion of the market. As for ‘JNJ’, in 2011, they acquired a medical equipment company ‘Synthes’, which caused a great go-up for their stock price.

As for 2 financial service companies, ‘FISV’ went up in 2011 because it acquires ‘M-com’ and ‘CashEdge’.

2.4 Monthly returns for 10 stocks and S&P500

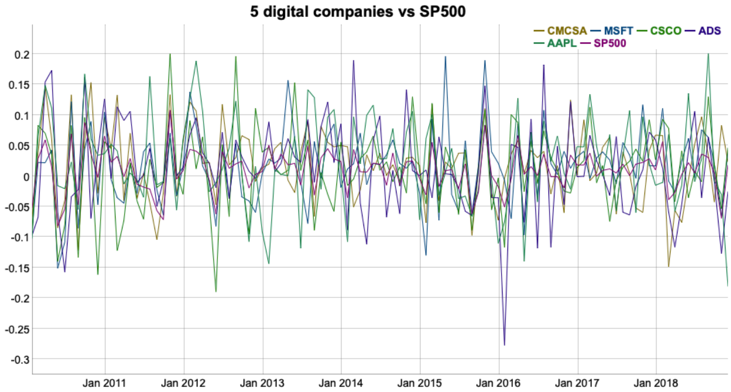


Fig. 2 Monthly returns for 5 digital-related companies vs SP500

Above is the plot for comparison between 5 digital companies and S&P500. We can see that 'ADS' fluctuates greatly. 'CMCSA' shows stability. Trend for 'MSFT' is consistent to S&P500.

As for 3 selling companies, from the plot above (***see in appendix 3***), return for 'BFB' fluctuates greatly. Since 'BFB' is a company selling wine, whose market is not as stable as that of daily personal care products that 'CL' and 'JNJ' sell. Monthly return for 'JNJ' and 'CL' are higher than S&P500.

As for 2 financial companies(***see in appendix 3***), we can see that loss for 'BBT' is greater than S&P500. Also, 'BBT' fluctuates acutely.

2.5 Equity curve for 10 stocks

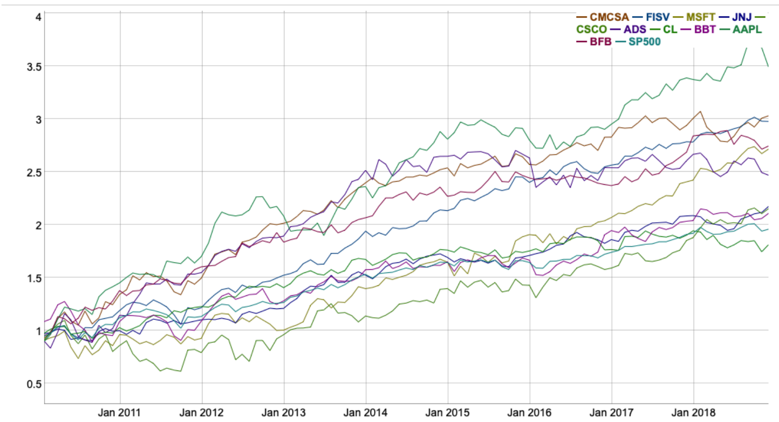


Fig. 3 Equity curve for 10 assets and S&P500

Here is the equity curve for 10 assets. We can see that 'AAPL' is the highest curve for most time. Positive slope shows its profitability. And equity curve for 'CSCO' is below other 9 assets. Overall speaking, all chosen stocks except for 'CSCO' have equity curves at least outperforming the benchmark S&P500 during year 2010 and 2018 during which was a strong bullish trend.

2.6 Test for stationarity

	CMCSA	FISV	MSFT	JNJ	CSCO	ADS	CL	BBT	AAPL	BFB	SP500
p-values	0.553	0.180	0.153	0.535	0.145	0.083	0.212	0.527	0.747	0.124	0.325
X-squared	0.353	1.795	2.037	0.386	2.129	2.999	1.559	0.401	0.104	2.362	0.968

Table 2 Stationary test

We use Ljung-Box test. We can see that p-values for 10 assets are all greater than 0.05, thus we can not conclude significant serial correlation.

2.7 Test for normality and outliers

	CMCSA	FISV	MSFT	JNJ	CSCO	ADS	CL	BBT	AAPL	BFB	SP500
p-values	0.782	0.789	0.431	0.584	0.550	0.083	0.207	0.337	0.866	0.681	0.431
X-squared	0.491	0.475	1.685	1.076	1.197	4.984	3.153	2.176	0.288	0.768	1.684

Table 3 Normality test

We use Jarque-Bera test. We can see that p-values for 10 assets are all greater than 0.05. Indicating all return series are skewed which can clearly support the fact that normality assumption of asset returns is generally unfavorable in a real market environment. It also can be demonstrated by the qq plot and histograms for residuals (***see in appendix2***).

As for outliers, from boxplots, we know there are only a few outliers in several assets. Consider the data size, it doesn't have much influence.

2.8 Fit different distributions to the data

	CMCSA	FISV	MSFT	JNJ	CSCO	ADS	CL	BBT	AAPL	BFB	SP500
Normal	146.71	182.27	145.64	200.23	132.28	121.37	194.58	155.53	124.55	164.50	208.15
T (df=5)	144.41	181.42	146.29	198.95	132.56	121.50	195.35	156.16	122.26	164.52	208.78
T(df=10)	145.80	182.34	146.48	199.88	132.88	122.21	195.58	156.27	123.69	164.95	208.84
T(df=15)	146.17	182.46	146.34	200.07	132.81	122.19	195.42	156.13	124.05	164.92	208.70

Table 4 fit different distributions

We tried to fit asset returns using normal and t-distribution with various degrees of freedoms. The correspondent log-likelihood is shown from the above table. Clearly, across all assets, it's not easy to distinguish which one fits better for the log-likelihood under all models are close, for stocks: 'MSFT', 'JNJ' and 'AAPL', all of which are very largely capitalized company and pioneering companies in the industries, it seems normal distribution is slightly better than t-distribution.

2.9 pair plot and sample covariance matrix between returns

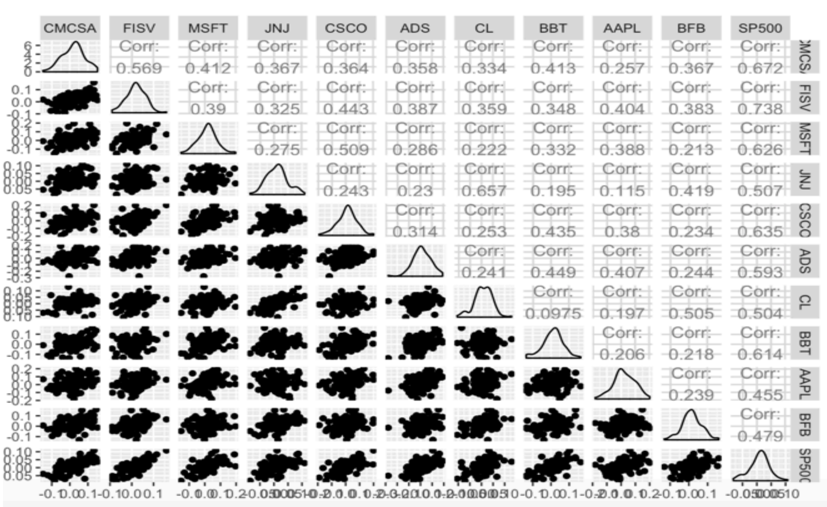


Fig. 4 covariance matrix between returns

The above pair plot and table show that the correlation and covariance matrix between asset returns and S&P500 index. While from the scatter plots, it can be seen that S&P500 index has linear relationship with all other assets with correlation range from 0.738 to 0.479, stock 'CMCSA', also shows similar patterns with other 9 assets. Comcast Corp. is a very large US telecommunication company with some well-known programs like CNBC financial news platform, reporting everyday business news across the world, and this may be one of the reasons their stock are correlated with ones across various industries. The highest correlation pair is 'JNJ' vs 'CL', Johnson and Johnson is a very large US pharmaceutical company and Colgate, a well-known company making toothpaste. Their correlation is 0.657 with a linear relationship seen from the scatter plot.

The covariance matrix is ***in appendix 4.***

III. Portfolio Theory

3.1 MVP

3.1.1 no short-sell

	CMCSA	FISV	MSFT	JNJ	CSCO	ADS	CL	BBT	AAPL	BFB
Weight	0	0.17618	0.03012	0.33266	0	0	0.24558	0.14994	0.03602	0.02951

	MONTHLY	ANNUALLY
RETURN	0.01182895	0.1419474
VOLATILITY	0.03027372	0.1048713

Table 6 MVP without short sells

The table shows the minimum variance portfolio when short sales are not allowed. We should invest most money to 'JNJ' and 'CL' with 33.266% and 24.558%, respectively. The total allocation is shown in table 1. The mean return, standard deviation for MVP are shown in the right table with annualized return and volatility of 14.2% and 10.5%, respectively.

Assume $100,000 to invest now. For the MVP without short-sell, the monthly Value at Risk (0.05) and monthly Expected Shortfall (0.05) computed with normality assumption are $3,768.21 and $5,033.23, respectively.

	NoShort.MVP	*NoShort.Tan*	*Short.MVP*	*Short.Tan*
Monthly VaR(0.05)	3768.213	4078.495	3692.671	4500.012
Monthly ES(0.05	5033.228	5549.486	4937.661	6166.3

Table 7 Monthly VaR

The correspondent VaR(0.05) and ES(0.05) of all 10 individual asset and S&P500 are as follows:

	CMCSA	*FISV*	*MSFT*	*JNJ*	*CSCO*	*ADS*	*CL*	*CSCO*	*ADS*	*CL*	*BBT*	*AAPL*	*BFB*	*SP500*
Monthly VaR(0.05)	8253.68	5433.83	8649.33	5059.16	10539.42	11490.55	5730.43	10539.42	11490.55	5730.43	8311.43	10159.06	6965.87	4820.69
Monthly ES(0.05	10832.09	7283.21	11253.63	6622.77	13490.24	14758.08	7378.89	13490.24	14758.08	7378.89	10685.80	13330.99	9149.32	6272.80

Table 8 VaR and ES

3.1.2 short-sell allowed

CMCSA	***FISV***	***MSFT***	***JNJ***	***CSCO***	***ADS***	***CL***	***BBT***	***AAPL***	***BFB***
-0.0794	0.23358	0.05805	0.33628	-0.04873	-0.04125	0.25881	0.1963	0.0498	0.03656

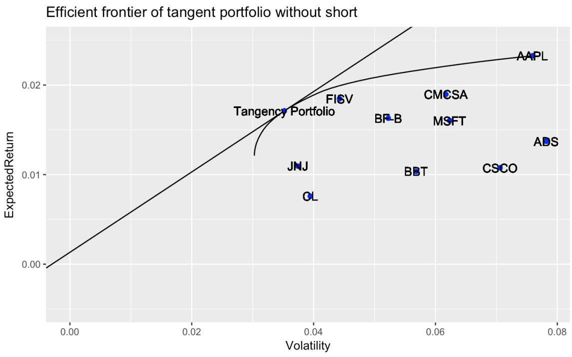
	MONTHLY	**ANNUALLY**
RETURN	0.0118212	0.1418544
VOLATILITY	0.02979447	0.1032111

Table9 MVP with short sells

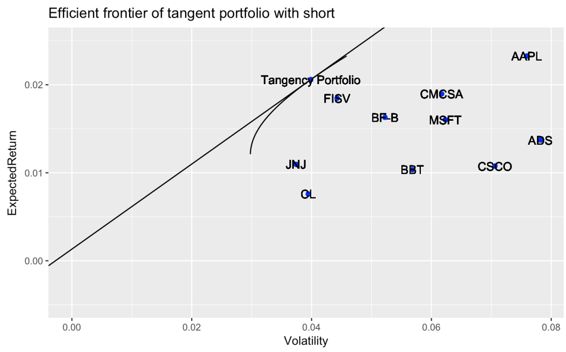
In this part we compute the MVP when short sales are allowed. And its mean return, standard deviation is shown as the right table. We annualize the monthly mean and risk by multiplying the mean by 12 and the risk by the square root of 12.

Assume $100,000 to invest now. For the MVP when short sell is allowed, the monthly Value at Risk (0.05) and monthly Expected Shortfall (0.05) computed with normality assumption are $3,692.67 and $4,937.66, respectively(see in table 7).

3.2 tangency portfolio, Sharpe ratio and efficient frontier



no short-sell



short-sell allowed

Fig. 5 efficient frontier

Here are the plots for the efficient portfolio frontier with and without short sell. Clearly, when we enable the short-sell condition, the efficient frontier expanded upward and the tangency line shift with higher slope (i.e. the Sharpe ratio), and thus we reach a more efficient portfolio combination.

	CMCSA	FISV	MSFT	JNJ	CSCO	ADS	CL	BBT	AAPL	BFB
SHARPERATIO	0.285	0.3871	0.23583	0.2574	0.133	0.1584	0.1587	0.1588	0.289	0.2864

	SHARPE RATIO
NO SHORT SALES TENGENCY PORTFOLIO	0.4372466
SHORT SALES ALLOWED TTENGENCY PORTFOLIO	0.4703565

Table 10 Sharpe ratio

From the tables above, we know that the 'FISV' has the highest Sharpe Ratio of 0.3872 among all individual asset, Fiserv, Inc. is a US provider of financial services technology. The company's clients include banks, thrifts, credit unions, securities broker dealers, leasing and finance companies, and retailers. Additionally, when short-sell are allowed, Sharpe Ratio increased from 0.4372 to 0.4703, again, reflect in the shifting of portfolio efficient frontier and its tangent line.

	CMCSA	FISV	MSFT	JNJ	CSCO	ADS	CL	BBT	AAPL	BFB
Weight	0.04068	0.40266	0.03559	0.21823	0	0	0	0	0.14608	0.15676

	MONTHLY	ANNUALLY
RETURN	0.01671608	0.2005929
VIOLATILITY	0.03520304	0.1219469

Table 11 Tangency portfolio without short sells

Tangency portfolio without short sell is shown as the left table. Its expected return and volatility are shown as the right table which are 20.06% and 12.2% annualized, respectively. And the Sharpe Ratio is 0.4372, which is lower than that of the circumstance that short sales are allowed.

	CMCSA	FISV	MSFT	JNJ	CSCO	ADS	CL	BBT	AAPL	BFB
Weight	0.0608	0.51731	0.09335	0.44999	-0.16631	-0.07312	-0.34957	0.02618	0.20716	0.2342

	MONTHLY	ANNUALLY
RETURN	0.02059135	0.2470962
VIOLATILITY	0.03987678	0.1381372

Table 12 Tangency portfolio wit short sells

Tangency portfolio with short sell is shown as the left table. Its expected return and volatility are 24.71% and 13.81% annualized, respectively. And the Sharpe Ratio is 0.4703, which is lower than that of the circumstance that short sales are allowed.

The correspondent normal monthly VaR(0.05) and ES(0.05) of tangency portfolio without shortsell are $4,078.5 and $5,549.49, respectively and that for tangency portfolio with short sell are $4,500.01 and $6,166.30, respectively(see in table 7).

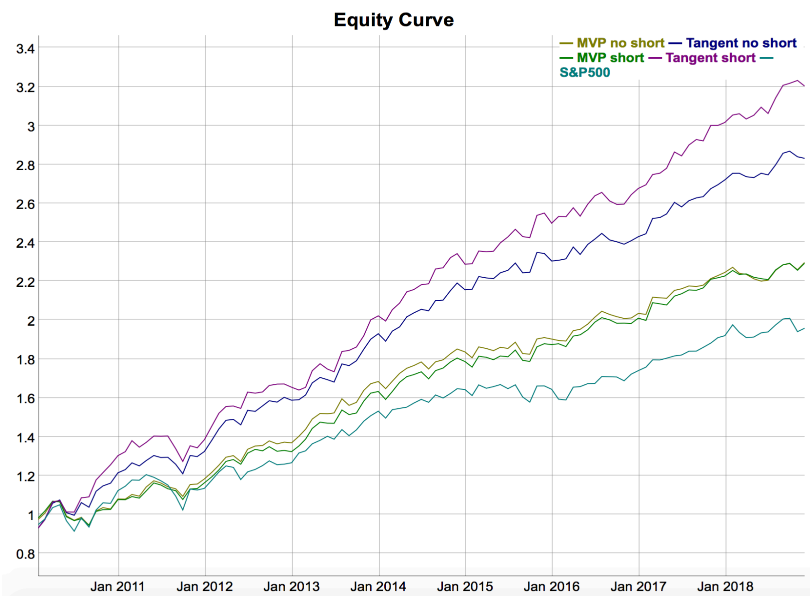


Fig. 6 Equity curve for 4 portfolio vs S&P500

Finally, the equity curve of four portfolios compared to S&P500 is shown as the above. Clearly, when the short-sell is allowed, the curve can be expanded with higher profitability. And it's obvious that our portfolio constructed with "randomly selected assets" outperforms S&P500 benchmark.

IV. Asset Allocation

4.1 Only risky assets

In this part, our goal return is 6% per year. However, as it showed in table2, all the assets selected has the average monthly return greater than 0.5%. Since no short sale is allowed, it's impossible to meet the goal. Thus, we set our goal to the return rate of SP500.

	CMCSA	FISV	MSFT	JNJ	CSCO	ADS	CL	BBT	AAPL	BFB
Weight	0	0	0	0.21246	0.00209	0	0.55229	0.23315	0	0

Expected Return	**0.846%**
Volatility	3.23%
VaR(0.05)	$4,415.51
ES(0.05)	$5,764.73

Table 13 Portfolio with only risky assets

The left table shows efficient portfolio. The right table shows monthly expected return, volatility and VaR and Expected shortfall which are 0.846%, 3.23%, $4,415.51 and $5,764.73, respectively.

4.2 Combination of T-Bills and the tangency portfolio

Same to the last part, no short sales are allowed. We have to set the goal return to monthly return of S&P500. Now we can use the combination of T-bill and risky assets.

	CMCSA	FISV	MSFT	JNJ	CSCO	ADS	CL	BBT	AAPL	BFB	Risky-free
Weight	0.0254	0.19454	0.01467	0.10731	0	0	0	0	0.07149	0.07453	0.51693

Expected Return	**0.846%**
Volatility	1.699%
VaR(0.05)	$1,899.37
ES(0.05)	$2,609.34

Table 14 Portfolio with risk-free assets

The left table shows the efficient portfolio with T-bill. And the right table shows relative expected return, volatility and VaR. Compared to part 4.1, we can see that-with the inclusion of riskless asset, we can reduce monthly volatility to 1.7%, VaR to $1,899.37 and Expected Shortfall to $2,609.34.

V. PCA

In this part, we did Principal Component Analysis (PCA) on our ten sample assets. Firstly we computed the sample correlation matrix of the returns.

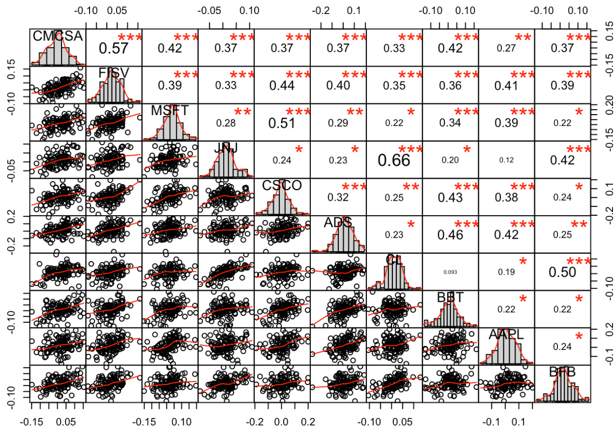


Fig.7 Correlation matrix

We found that JNJ and CL have the highest pairwise correlation. BBT and CL have the lowest pairwise correlation. Based on the estimated correlation values, we can see that there are definitely some certain correlations between these assets, and the correlations are not high. This implies that diversification might reduce risk with these assets, although reduction in risk might not be significant. Then we ran PCA. Below are the importance of components result, Scree plot of our assets, and plots of first three of our eigenvectors.

```
Importance of components:
                          PC1     PC2     PC3     PC4     PC5     PC6
Standard deviation     0.1225 0.06547 0.06424 0.05806 0.04808 0.04538
Proportion of Variance 0.4233 0.12097 0.11649 0.09513 0.06525 0.05813
Cumulative Proportion  0.4233 0.54426 0.66075 0.75588 0.82112 0.87926
                           PC7     PC8     PC9    PC10
Standard deviation     0.04120 0.03426 0.03061 0.02168
Proportion of Variance 0.04791 0.03312 0.02645 0.01326
Cumulative Proportion  0.92716 0.96029 0.98674 1.00000
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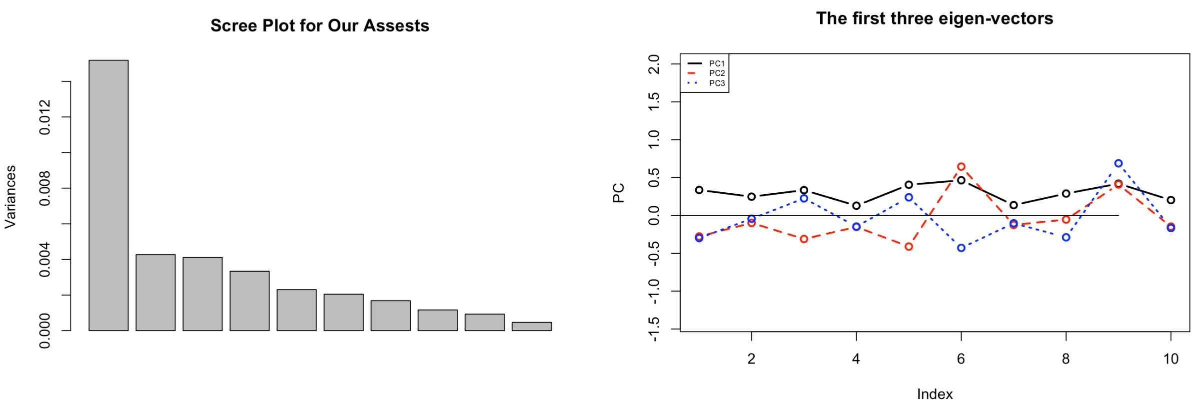


Fig. 8 PCA

According to the results, we can see that one needs eight principal components to get more than 95% of the variance. The first eigenvector has only positive values, and returns in this direction are either positive for all of the funds or negative for all of them. The second eigenvector is positive for funds 6 (Alliance Data Systems Corporations) and 9 (Apple Inc.), and negative for the other funds. The plot of third principal component is a little bit hard to interpret, but we can see that it loading on fund 9 (Apple Inc.) is higher than on the other funds, which might indicate something different about Apple's equities. Nevertheless, we do not need to pay much attention to second and third principal component, because the first principal component already took 42.33% of the total variance.

VI. Risk Management

5.1 Analysis for single asset

In this part, we compute VaR and expected shortfall. Assuming $100,000 to invest. For each asset and each portfolio, we estimated 5% VaR and expected shortfall over a one month investment horizon. We used both nonparametric and parametric method. Below are the results we got.

	CMCSA	*FISV*	*MSFT*	*JNJ*	*CSCO*	*ADS*	*CL*	*BBT*	*AAPL*	*BFB*	*SP500*
VAR_NONP	7640.92	5046.33	8478.84	4301.88	11071.07	11527.10	6927.35	9187.40	10766.77	7873.70	5591.85
ES_NONP	10562.55	7359.24	11035.69	5893.45	14417.45	15230.19	8236.80	12025.41	13357.11	9452.35	6752.73

	CMCSA	*FISV*	*MSFT*	*JNJ*	*CSCO*	*ADS*	*CL*	*BBT*	*AAPL*	*BFB*	*SP500*
VaR_par	8206.14	5399.73	8601.32	5030.33	10485.02	11430.30	5700.03	8267.65	10100.58	6925.61	4793.92
ES_par	10772.47	7240.45	11193.42	6586.62	13422.01	14682.53	7340.78	10630.90	13257.65	9098.84	6239.22

Table 15 VaR and ES for single asset with parametric and non-parametric method

As we can see from the charts, for single asset, under nonparametric method, ADS has the highest VaR and ES; JNJ has the lowest VaR and ES. Under parametric method, ADS has the highest VaR and ES; S&P 500 has the lowest VaR and ES.

5.2 Analysis for portfolio

	NoShort.MVP	*NoShort.Tan*	*Short.MVP*	*Short.Tan*	*Noshort(SP500)*	*Short (SP500)*	*Noshort (0.5%)*	*Short (0.5%)*
VaR_nonp	3526.88	3838.46	3526.55	4981.22	1795.89	1894.16	792.60	844.20
ES_nonp	4832.84	5197.17	4802.28	6099.46	2436.19	2331.49	1101.95	1051.15

	NoShort.MVP	NoShort.Tan	Short.MVP	Short.Tan	Noshort (SP500)	Short (SP500)	Noshort (0.5%)	Short (0.5%)
VaR_par	3744.921	4051.381	3669.724	4469.267	1886.318	1689.976	838.9153	746.8847
ES_par	5004.024	5515.484	4908.882	6127.741	2592.974	2346.747	1179.0553	1063.6364

Table 16 VaR and ES for portfolio with parametric and non-parametric method

Under nonparametric method, tangent portfolio with short sells has the highest VaR and also the highest expected shortfall. Target = 0.5% without short sells has the lowest VaR, and target = 0.5% with short sells has the lowest expected shortfall. Under parametric method, tangent portfolio with short sells has the highest VaR and also the highest expected shortfall. Target = 0.5% with short sell has the lowest VaR and also the lowest expected shortfall.

5.3 Bootstrap

Then we used bootstrap to compute 95% confidence intervals for 5% VaR and expected shortfall.

	NoShort.MVP	NoShort.Tan	Short.MVP	Short.Tan	No short (SP500)	Short (SP500)	No short (0.5%)	Short (0.5%)
VaR_nonp	3526.88	3838.46	3526.55	4981.22	1795.89	1894.16	792.60	844.20
ES_nonp	4832.84	5197.17	4802.28	6099.46	2436.19	2331.49	1101.95	1051.15
VaR_nonp_low	3368.87	3588.58	3350.35	4384.95	1779.61	1682.21	775.77	738.72
VaR_nonp_up	3666.70	3936.18	3610.37	5443.54	1817.29	2061.65	806.77	919.68
ES_nonp_low	4541.98	4912.51	4492.79	5949.54	2328.75	2250.55	1050.99	1019.12
ES_nonp_up	5769.69	6023.36	5809.24	6625.93	2853.47	2519.24	1296.57	1149.62

	NoShort.MVP	NoShort.Tan	Short.MVP	Short.Tan	No short (SP500)	Short (SP500)	No short (0.5%)	Short (0.5%)
VaR_par	3744.92	4051.38	3669.72	4469.27	1886.32	1689.98	838.92	746.88
ES_par	5004.02	5515.48	4908.88	6127.74	2592.97	2346.75	1179.06	1063.64
VaR_par_low	3480.58	3713.21	3401.03	4111.18	1759.72	1544.85	757.57	681.33
VaR_par_up	4018.28	4360.98	3948.25	4829.40	2028.04	1819.58	907.76	825.88
ES_par_low	4682.30	5141.86	4618.65	5738.50	2440.66	2174.38	1087.44	987.67
ES_par_up	5346.99	5888.01	5232.82	6532.23	2769.39	2506.62	1260.61	1151.44

Table 17 bootstrap

In every chart, first two lines are values of VaR and ES (same with what we got above). From third line to sixth line, values represent lower bound of 95% confidence intervals of VaR, upper bound of 95% confidence intervals of VaR, lower bound of 95% confidence intervals of ES, upper bound of 95% confidence intervals of ES, respectively.

Additionally, we used bootstrap to compute estimated standard errors of our portfolios.

SE	list [8]	List of length 8
[[1]]	double [4]	116 321 143 167
[[2]]	double [4]	161 305 164 187
[[3]]	double [4]	183 335 142 161
[[4]]	double [4]	409 176 182 207
[[5]]	double [4]	66.3 132.3 74.2 85.5
[[6]]	double [4]	154.7 74.5 76.6 88.2
[[7]]	double [4]	34.7 68.7 39.8 45.4
[[8]]	double [4]	57.8 35.1 36.3 41.5

Table 18 standard error for portfolio

For each of eight portfolios, we got four standard errors, each representing standard error of VaR under nonparametric method, standard error of ES under nonparametric method, standard error of VaR under parametric method, standard error of ES under parametric method, respectively.

VII. Copulas

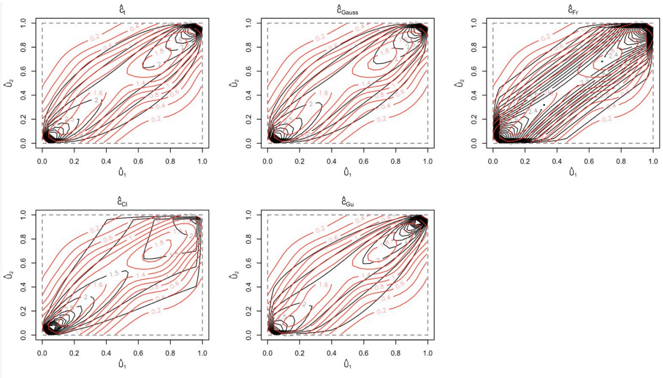
In this section we tried various copulas to model the joint distribution of the returns. First, we defined the t-copula using omega as the correlation parameter and 4 as the degrees-of-freedom (tail index) parameter. We fit the copula using different methods and got two estimates. The *fitCopula()* function in R has an input that should be the cumulative density of the marginal variables. Our first method was using '*pstd*' which assumes that the marginal is from a t-distribution. Second method was using the empirical cumulative distribution. The results for two estimates are similar.

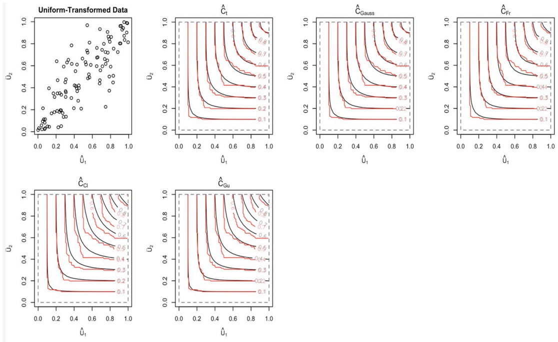
```
Call: fitCopula(copula, data = data, method = "ml", start = ..2)
Fit based on "maximum likelihood" and 107 2-dimensional observations.
t-copula, dim. d = 2
      Estimate Std. Error
rho.1   0.8402      0.023
df     29.2068     68.126
The maximized loglikelihood is 65.39
Optimization converged
Number of loglikelihood evaluations:
function gradient
     729      358
```

```
Call: fitCopula(copula, data = data, method = "ml", start = ..2)
Fit based on "maximum likelihood" and 107 2-dimensional observations.
t-copula, dim. d = 2
      Estimate Std. Error
rho.1   0.8509      0.023
df     12.1015     15.622
The maximized loglikelihood is 65.79
Optimization converged
Number of loglikelihood evaluations:
function gradient
     117       48
```

Fig 9　two methods for Copulas

We chose S&P 500 and our MVP portfolio without short sale allowed as our target to fit the copula. Then we defined and fit a meta-t-distribution by specifying its t-copula and its univariate marginal distributions. We also tried fitting normal (Gaussian), Frank, Clayton, Gumbel and Joe copulas to the data. We compared estimated copulas with the empirical copula using different methods including two-dimensional KDE of the copula's density and AIC comparison.





	est	max_loglik	AICs
t	0.8401993	64.87351	-125.74702
Gauss	0.8399273	64.78459	-127.56918
Frank	9.0975764	61.73932	-121.47864
Clayton	2.1387678	53.45814	-104.91628
Gumbel	2.5318592	56.89688	-111.79376
Joe	2.9307466	41.98779	-81.97557

Fig. 10 Results for copulas

Above are the contour plots for the density function (upper) and distribution function for various fitted copula models (middle), and also the AIC results (lower). Seeking the smallest value of AIC would point to the Gaussian model as the best one. Hence we can conclude that joint distribution between S&P 500 and our target portfolio is Gaussian distribution.

VIII. Conclusion

The Wall Street Journal says 'When S&P measured performance over a longer period, the results got worse. More than 90% of active managers underperformed their benchmark indexes over a 15-year period. Equity mutual funds do beat the market sometimes, but seldom can they do it consistently, year over year.'(see in https://www.rebalance360.com/news/index-funds-still-beat-active-portfolio-management/) Clearly, the portfolio we chose randomly defeated S&P500 and 90% active invest managers.

Of course, this good performance of the randomly chosen portfolio may happen by chance. Further explanation can be done in the future research.

Appendix

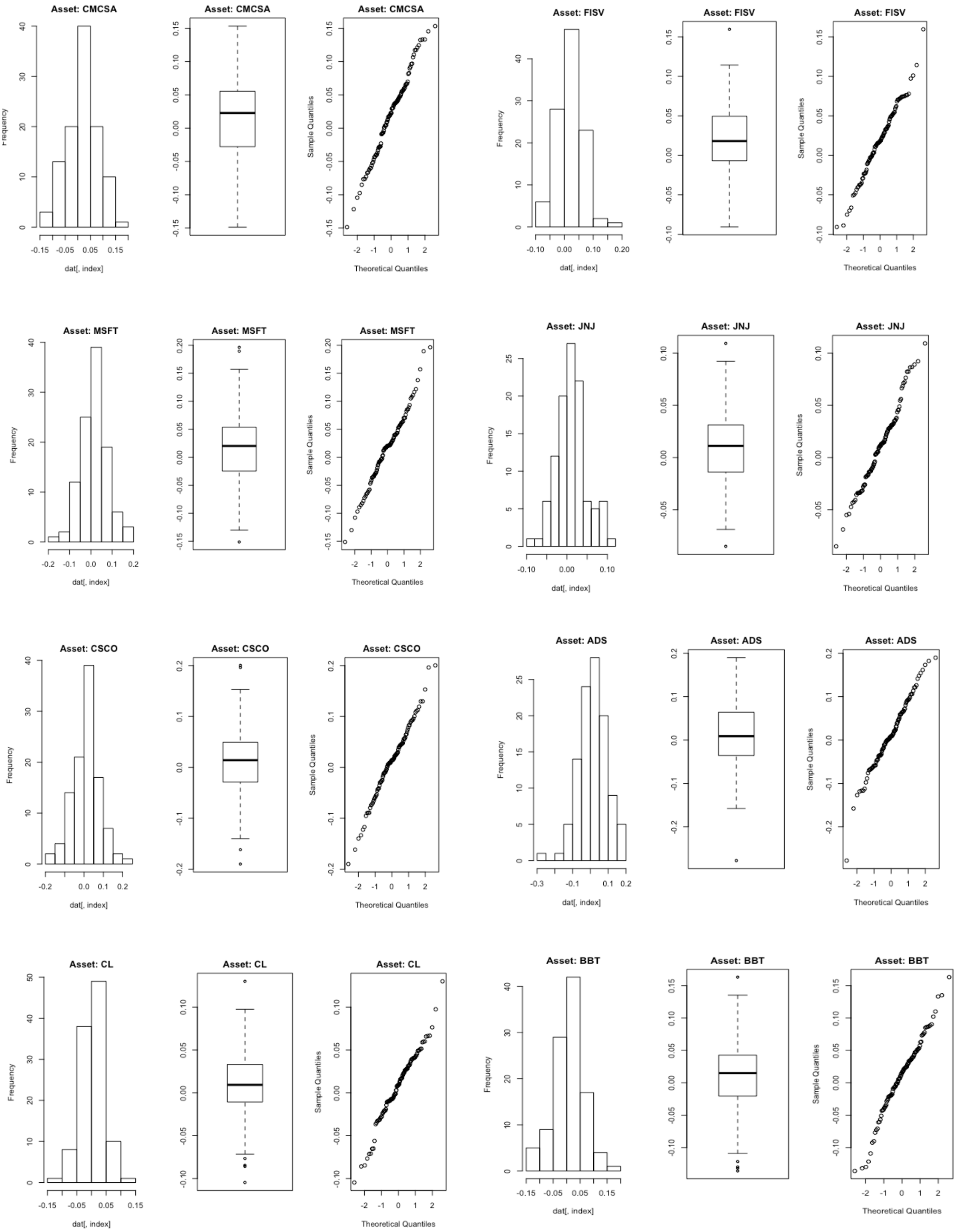
1. Introduction to 10 assets selected

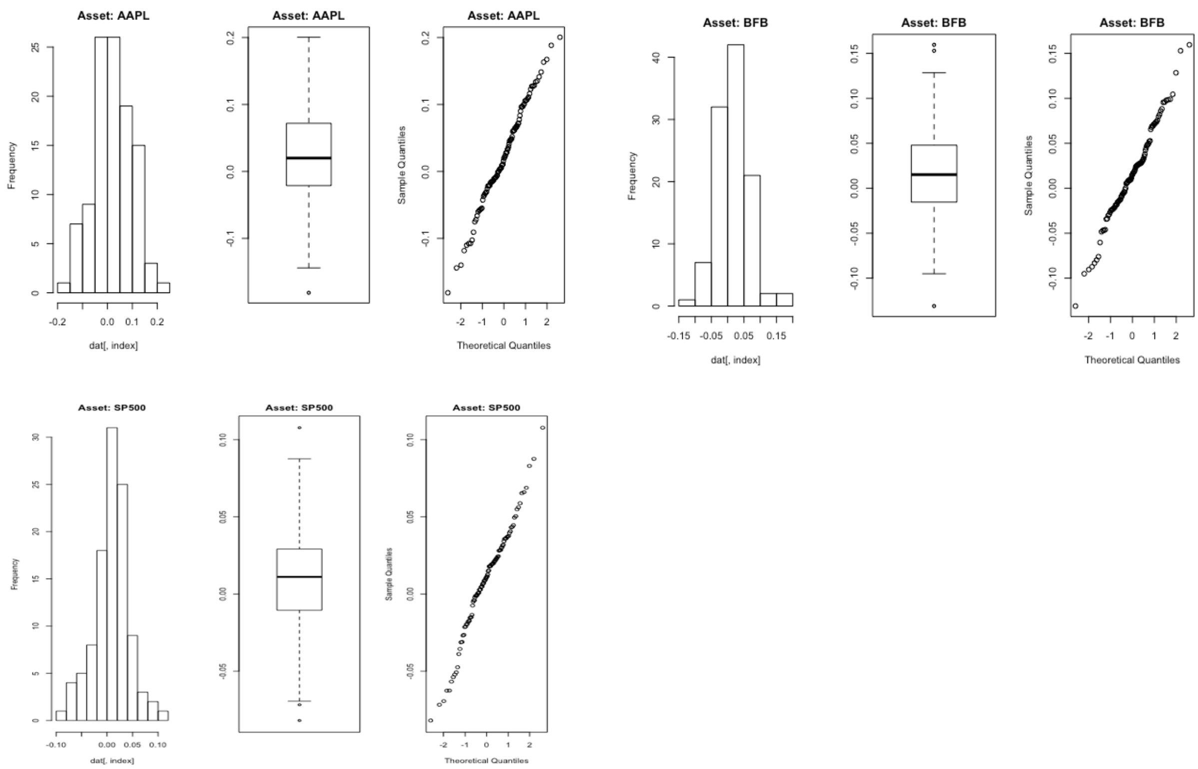
We chose 10 assets on "Finviz S&P500 map" (https://finviz.com/map.ashx?t=sec&st=peg). The basic information for them is as following.

	NAME	SYMBOL	INDUSTRY	BASIC INTRODUCTION
1	Comcast Corporation	CMCSA	Telecommunication	Comcast is an American company. It's the largest cable TV company and largest home internet service provider in the United States
2	Fiserv Inc.	FISV	Financial service	is a US provider of financial services technology. In October 2015, American banker and BAI ranked the company third by revenue among technology providers to U.S. banks.
3	Microsoft	MSFT	Technology	Microsoft is an American multinational technology company. It develops, manufactures, licenses, supports and sells computer software, consumer electronics, personal computers, and related services.
4	Johnson and Johnson	JNJ	Retailing	Johnson and Johnson is an American multinational medical devices, pharmaceutical and consumer packaged goods manufacturing company.
5	Cisco Systems Inc.	CSCO	Technology	Cisco develops, manufactures and sells networking hardware telecommunication and other high-technology services and products.

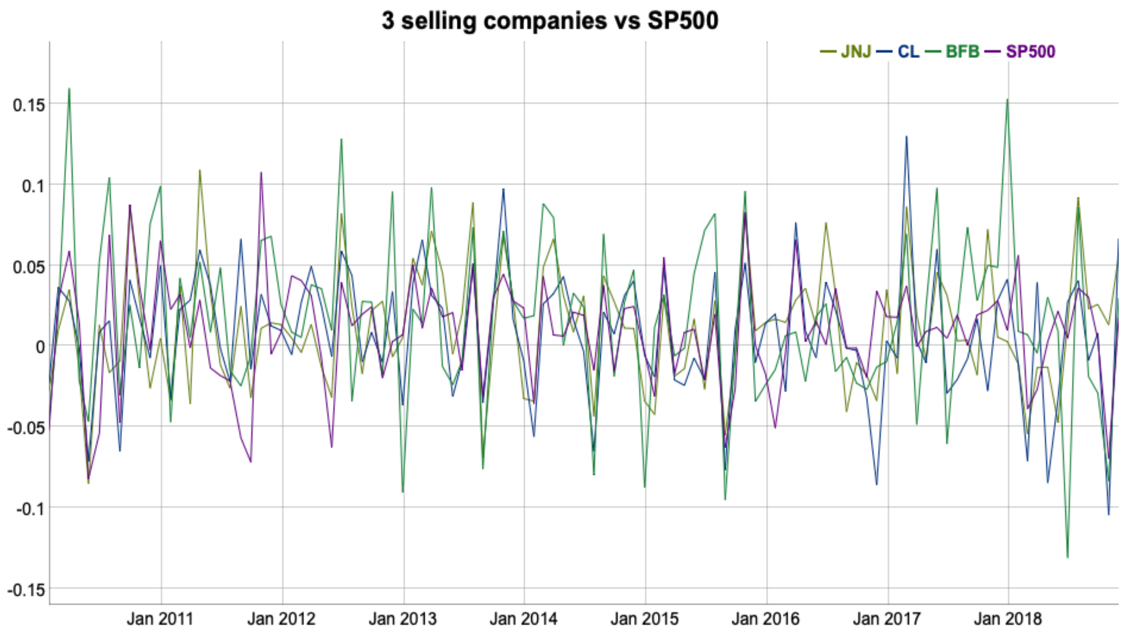
6	Alliance Data Systems Corporation	ADS	Public data	Alliance Data Systems Corporation is a publicly traded provider of loyalty and marketing services, such as private labelcredit cards, coalition loyalty programs, and direct marketing, derived from the capture and analysis of transaction-rich data.
7	Colgate-Palmolive Company	CL	Retailing	Colgate-Palmolive Company is an American worldwide consumer products company focused on the production, distribution and provision of household, health care and personal care products.
8	BB&T Corporation	BBT	Financial Service	BB&T Corporation (Branch Banking and Trust Company) is a bank holding company. It offers consumer and commercial banking, securities brokerage, asset management, mortgage, and insurance products and services.
9	Apple Inc.	AAPL	Technology	Apple Inc. is an American multinational technology company. It designs, develops, and sell consumer electronics, computer software, and online services.
10	Brown-Forman Corporation Class B	BFB	Retailing	The Brown–Forman Corporation is one of the largest American-owned companies in the spirits and wine business.

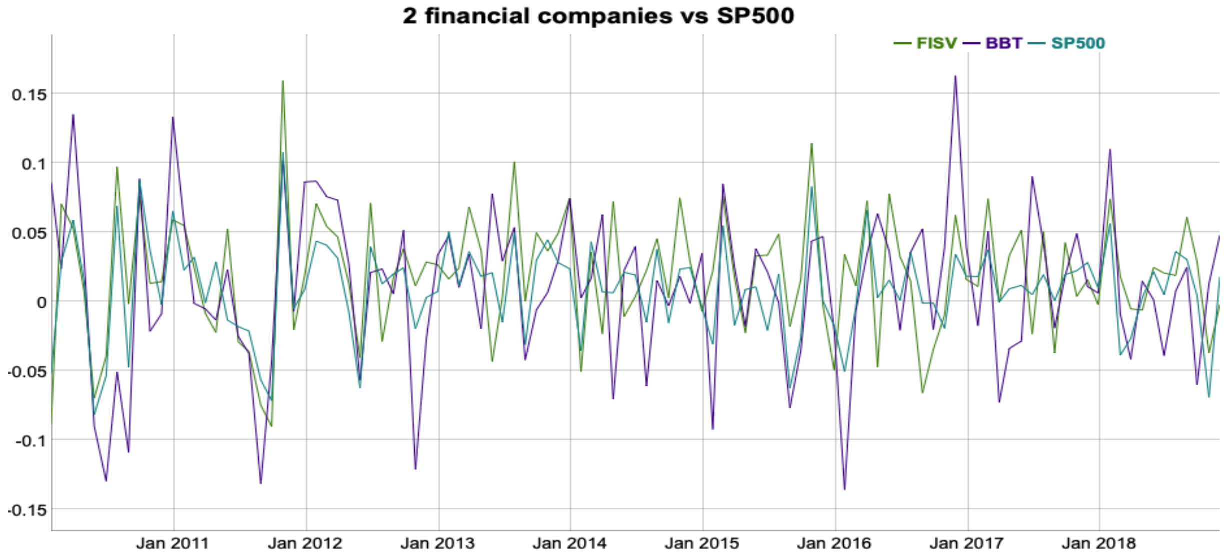
2. Histogram, boxplot and qq plots for returns


Asset: CMCSA
Asset: FISV
Asset: MSFT
Asset: JNJ
Asset: CSCO
Asset: ADS
Asset: CL
Asset: BBT
Frequency
dat[, index]
Sample Quantiles
Theoretical Quantiles



3. Monthly returns plot for 3 selling companies and 2 financial companies vs S&P500





4. Covariance matrix

	CMCSA	*FISV*	*MSFT*	*JNJ*	*CSCO*	*ADS*	*CL*	*BBT*	*AAPL*	*BFB*	*SP500*
CMCSA	0.0038	0.0016	0.0016	0.0008	0.0016	0.0017	0.0008	0.0014	0.0012	0.0012	0.0014
FISV	0.0016	0.0020	0.0011	0.0005	0.0014	0.0013	0.0006	0.0009	0.0014	0.0009	0.0011
MSFT	0.0016	0.0011	0.0039	0.0006	0.0022	0.0014	0.0005	0.0012	0.0018	0.0007	0.0014
JNJ	0.0008	0.0005	0.0006	0.0014	0.0006	0.0007	0.0010	0.0004	0.0003	0.0008	0.0007
CSCO	0.0016	0.0014	0.0022	0.0006	0.0050	0.0017	0.0007	0.0017	0.0020	0.0009	0.0016
ADS	0.0017	0.0013	0.0014	0.0007	0.0017	0.0061	0.0007	0.0020	0.0024	0.0010	0.0016
CL	0.0008	0.0006	0.0005	0.0010	0.0007	0.0007	0.0016	0.0002	0.0006	0.0010	0.0007
BBT	0.0014	0.0009	0.0012	0.0004	0.0017	0.0020	0.0002	0.0032	0.0009	0.0006	0.0012
AAPL	0.0012	0.0014	0.0018	0.0003	0.0020	0.0024	0.0006	0.0009	0.0058	0.0009	0.0012
BFB	0.0012	0.0009	0.0007	0.0008	0.0009	0.0010	0.0010	0.0006	0.0009	0.0027	0.0009
sp500	0.0014	0.0011	0.0014	0.0007	0.0016	0.0016	0.0007	0.0012	0.0012	0.0009	0.0012