

# Volatility Arbitrage

MATH 5010 Introduction to the Mathematics of Finance

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Technology Tool: R, Excel

Rehearsal Date: May 1, 2018

Rehearsal Time: 5:40 pm

Rehearsal Place: Uris Hall

Rehearsal Attendee: All group members

Guest Listener: Wanxia Chen, Yingting Yu

Group 15:

Chong Liang

Jiun Ying Chen

Mengxue Wang

Xiaojie Wei

Ziyue Wang

# Introduction

## What is “Volatility Arbitrage”

- Volatility arbitrage is a trading strategy that attempts to profit from the difference between the forecasted future price-volatility of an asset and the implied volatility of options based on that asset.

## Breaking Down “Volatility Arbitrage”

- Options pricing is affected by the volatility of the underlying asset, if the forecasted and implied volatilities differ, there will be a discrepancy between the expected price of the option and its actual market price.
- A volatility arbitrage strategy can be implemented through a delta-neutral portfolio consisting of an option and its underlying asset

# Implied Volatility vs. Forecasted Volatility

Implied Volatility:

- The Black-Scholes-Merton Formula

$$C(S_0, t) = S_0 N(d_1) - Ke^{-r(T-t)} N(d_2)$$

$$d_1 = \frac{\ln \frac{S_0}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \qquad d_2 = d_1 - \sigma\sqrt{(T-t)}$$

Forecasted Volatility (Historical Volatility):

- Realized volatility of the underlying asset over a previous time period
- Measuring the standard deviation from the mean during that time period

# Delta Neutral

## Delta Neutral Portfolio:

- Portfolio value remains unchanged when small changes occur in the value of the underlying security.

$$call\Delta = \frac{\partial c}{\partial S} = e^{-\delta\tau}N(d_1) > 0$$

## Delta Hedge:

- The process of setting or keeping the delta of a portfolio as close to zero as possible
- Buying or selling an amount of the underlying asset that corresponds to the delta of the portfolio
- Keeping the portfolio delta neutral

# Data Selection

Data source: Wharton Research Data Services (WRDS) & Bloomberg

Stock traded: SPY ETF

Option traded: SPY ETF Call Option (American option)

- Reduce firm-specific volatility

Training data: 2015.10.23 - 2016.12.31

Test data: 2017.01.01 - 2017.12.31

# Strategy Signal

When **Implied volatility** < **Historical volatility**, the option is underpriced and will rise in the future since the volatility will revert to the historical mean.

When **Implied volatility** > **Historical volatility**, the option is overpriced and will fall in the future.

Spread (difference between implied volatility and historical volatility) narrows

# Strategy Signal

Daily Spread = Historical volatility - Implied volatility

If Spread > Average(spread) + 0.5\*STDEV(spread) (spread > 0):

**Long 1 SPY ETF ATM call, short Delta units underlying SPY ETF**

If Spread < Average(spread) - 0.5\*STDEV(spread) (spread < 0):

**Short 1 SPY ETF ATM call, long Delta units underlying SPY ETF**

Otherwise: No Action

93 Trading signals (Long spread: 27 & Short spread: 66)

# Implementation

01/17/2017

- SPY implied volatility = 9.835 Historical Volatility = 5.3864
- Volatility Spread = -4.4486 < Signal = -3.54 Action: **Short**
- Strategy: short an ATM SPY Call option with strike \$226 at \$2.7
- Delta = 0.53 Long 53 shares of SPY at \$226.25

Duration

- Dynamic re-hedge

Expiration 02/15/2017

- Stock price = 234.92
- Settle option and sell the remaining stock
- P&L = payoff on day 1 - re-hedge cost + payoff of closing position at expiration = \$903.6
- Total cost is \$34269 Return =2.64%



# Trading Strategy Backtest

## Return

At time  $t$ , the strategy return

- Long call option, short underlying stock

- $(P\&L)_t = 100(-C_0 + (S_t - K)_+ + \Delta(S_0 - S_t))$

$C_0$  : call option price at trading day

$S_0$  : equity price at trading day

$S_t$  : equity price at expiration date

$K$  : strike price at trading day

$\Delta$  : stock units to long/short in the strategy

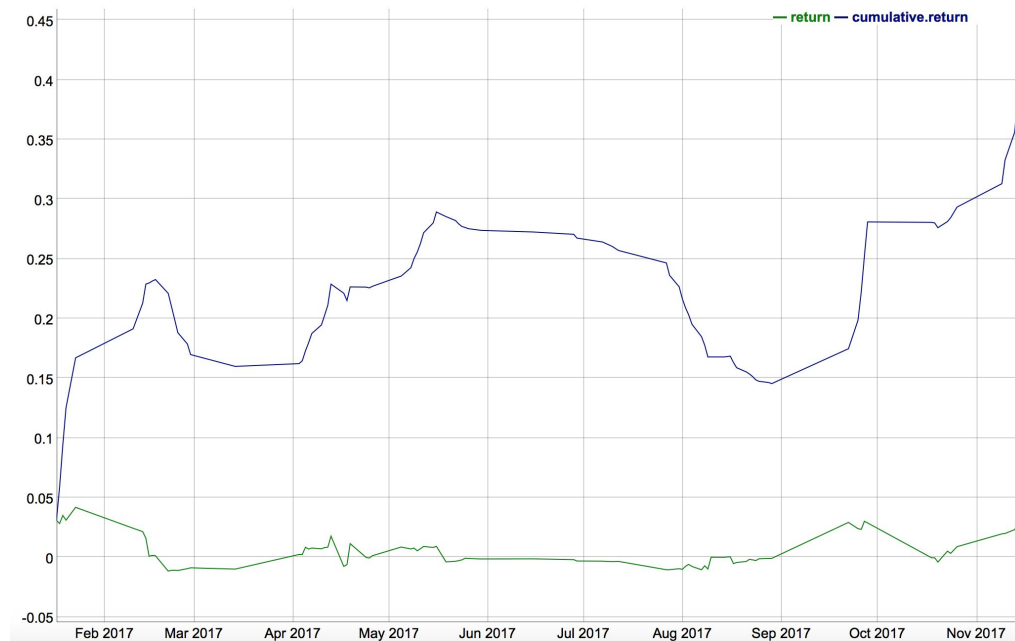
- Short call option, long underlying stock

- $(P\&L)_t = 100(C_0 - (S_t - K)_+ - \Delta(S_0 - S_t))$

# Trading Strategy Backtest

## Return

- Daily Return of the strategy on a delta-hedged option position
  - 39 days out of 93 trading days, the return is less than or equal to 0.
- Annualized Return:
  - 10.96%
- Cumulative Return until expiration (12.31.2017)
  - 41.67%
- Annualized Sharpe Ratio ( $R_f = 2.5\%$ )
  - 0.68



## Risk

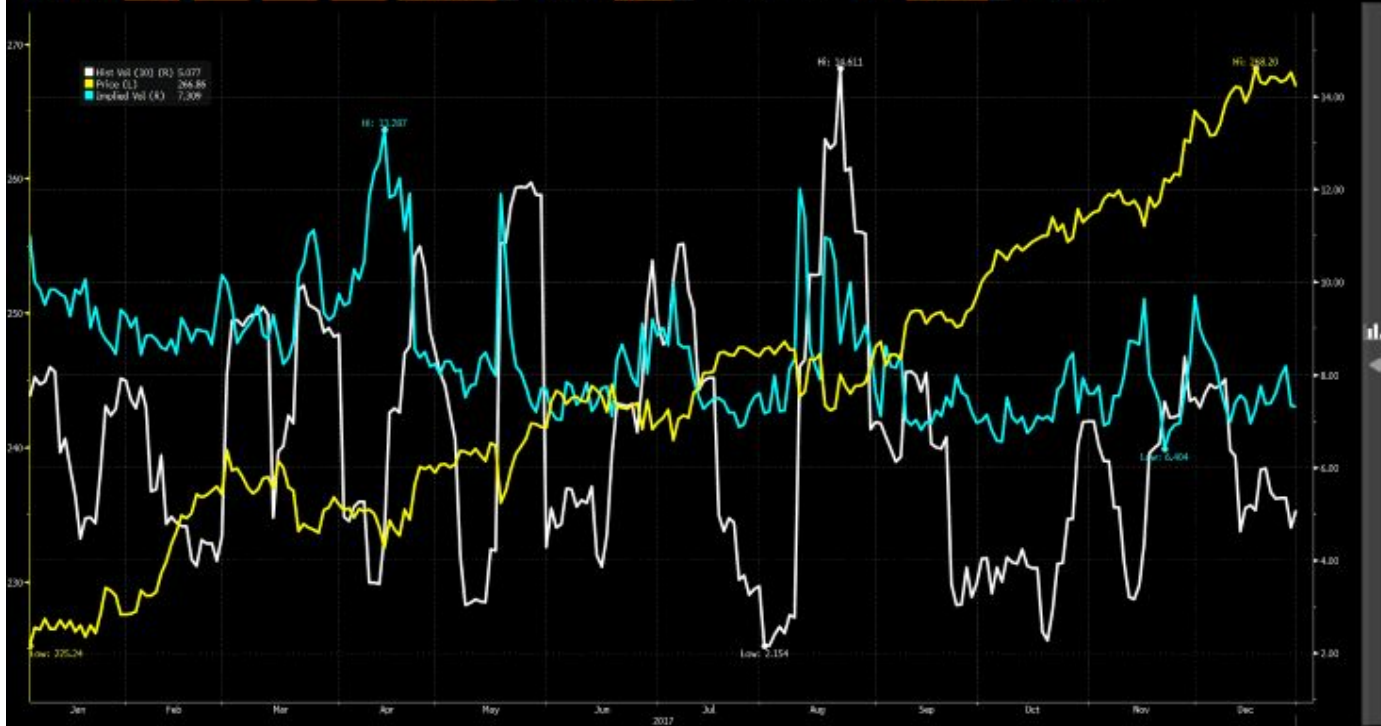
- Annualized standard deviation: 12.45%

GRAB

SPY US Equity	Actions ▾	Templates ▾	Table	Historical Implied Volatility Graph
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Period	Daily ▾	Range	01/03/17 - 12/31/17	Ann. Factor	260	Currency	LCL ▾
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Hist Vol	10	Trade ▾	Model	CLV ▾	Normal	Y/P	Price ▾	<input checked="" type="checkbox"/> IVOL
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Implied Volatility

Historical Volatility

Australia 61 2 9777 8600	Brazil 5511 2395 9000	Europe 44 20 7330 7500	Germany 49 69 9204 1210	Hong Kong 852 2977 6000
Japan 81 3 3201 8900	Singapore 65 6212 1000	U.S. 1 212 318 2000	Copyright 2018 Bloomberg Finance L.P.	

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# Limitations/Assumptions

There are several assumptions a trader must make, which will increase the complexity of a volatility arbitrage strategy:

- Transaction Cost
  - In our strategy, we assume that there is no transaction costs in buying or selling the stock or the option.
- Operation Cost
  - The investor must be right about whether implied volatility is really overpriced or underpriced
  - The investor must be correct about the amount of time it will take for the strategy to profit or time value erosion could outpace any potential gains.
  - The investors have to make the decision about re-hedge frequency, which will greatly impact P&L in volatility arbitrage.

# References

“Delta Neutral.” *Wikipedia*, Wikimedia Foundation, 10 Apr. 2018, [en.wikipedia.org/wiki/Delta\\_neutral](https://en.wikipedia.org/wiki/Delta_neutral).

“Volatility Arbitrage.” *Investopedia*, Investopedia, 19 Jan. 2018, [www.investopedia.com/terms/v/volatility-arbitrage.asp](https://www.investopedia.com/terms/v/volatility-arbitrage.asp).

“Options delta hedging with no options at all” *Faculty of Economic Sciences, University of Warsaw*, 2014.

Q&A

Thank You