168 - Homework 6. Homework is due on March 1st, with a penalty free late deadline of March 6th

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Exercises 4-7 may be regarded as a practice midterm exam

Exercise 1. Newman 13.4

Exercise 2. Newman 14.1

Exercise 3. (a) Calculate the entries in the modularity matrix, $\mathbf{B}$ for a bi-stellate graph (see the midterm for the definition of such a graph). You will need to consider 5 types of entries B_{ij}:

(i) i is a central node and j is a peripheral node on the same side

(ii) i is a central node and j is a peripheral node on the opposite side

(iii) i is a central node and j is the other central node

(iv) i, j are peripheral nodes on the same sides of the graph

(v) i, j are peripheral nodes on opposite sides of the graph.

(b) For the special case where $n = 3$, calculate the eigenvector corresponding to $\mathbf{B}$'s largest eigenvalue. You may use Matlab or some other software to do the calculation. What does your calculation say about the best way to group the network into two communities.

Exercise 4. A network has adjacency matrix:

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Given that:

$$A^2 = \begin{pmatrix} 4 & 2 & 2 & 2 & 4 & 2 \\ 2 & 4 & 2 & 2 & 2 & 4 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 2 & 2 & 4 & 4 & 2 & 2 \\ 4 & 2 & 2 & 2 & 4 & 2 \\ 2 & 4 & 2 & 2 & 2 & 4 \end{pmatrix} \quad \text{and} \quad A^3 = \begin{pmatrix} 8 & 12 & 12 & 12 & 8 & 12 \\ 12 & 8 & 12 & 12 & 12 & 8 \\ 12 & 12 & 8 & 8 & 12 & 12 \\ 12 & 12 & 8 & 8 & 12 & 12 \\ 8 & 12 & 12 & 12 & 8 & 12 \\ 12 & 8 & 12 & 12 & 12 & 8 \end{pmatrix}$$

(a) Calculate the number of triangles in the network.

(b) Calculate the global clustering coefficient for the network.

Exercise 5. The probability generating function for a configuration network in which the degree distribution is geometrically distributed with parameter p is $g_0(z) = \frac{p}{1-qz}$, where $q \equiv 1 - p$.

(a) Show that, in general: $\langle k \rangle = g_0'(1)$, $\langle k(k-1) \rangle = g_0''(1)$.

(b) Recall that the condition for the existence of a GC is that $\langle k^2 \rangle > 2\langle k \rangle$. Find the corresponding condition on p that leads to the existence of a GC in this network

Exercise 6. (a) Suppose p_k is the degree distribution in a configuration model. Show that the excess degree of a neighbor is $q_k = \frac{(k+1)p_{k+1}}{\langle k \rangle}$.

(b) A configuration network is built using two types of nodes. A fraction p of nodes have degree 1, and a fraction $1 - p$ have degree 3. Calculate the mean excess degree of a neighbor node in this network.

Exercise 7. A graph is constructed in the following way: 2 n-complete cliques are first created (each containing n-nodes, with every node connected to all $n - 1$ nodes in the same clique). We then consider all possible edges between a single node in clique 1, and a single node in clique 2. Each such edge is added to the graph with the same probability p.

(a) What is the average degree of a node within this network?

(b) Calculate the total expected number of edges, m, in the network.

(c) Calculate the expected modularity score for the network, if the two different cliques are assigned to two different communities. Recall the formula for modularity is:

$$Q = \frac{1}{2m} \sum_{i,j} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta_{g_i g_j} \ .$$