168 - Homework 5. Exercises 2-7 are due on Gradescope by February 23rd.

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Exercise 1. Write a 1 page abstract for your final project. The abstract should include at least three references associated with your proposed project, which could be scientific papers or book chapters, websites including data. It should include a brief description of the tasks that each group member will be focusing on and how they will get started working on those tasks. You should write the abstract either on google docs, or, if you prefer, Overleaf, and post a link to it, when ready, as a PM on Piazza, for both the course instructor and TA. To give us time to give you feedback on your proposal, the deadline for this abstract is 11.59pm, Wednesday February 21st, with no extensions allowed.

Exercise 2. A directed graph version of the configuration model, assigns every node an in degree k_i^{in} and an out degree k_i^{out}. Edges are added by connecting in and out stubs.

(a) Explain why $2m = 2n\langle k^{in}\rangle$. (Hint: how are $\langle k^{in}\rangle$ and $\langle k^{out}\rangle$ related?

(b) Explain why the probability of an edge from node i to node j is $\frac{k_i^{in}k_i^{out}}{2m-1} \approx \frac{k_i^{in}k_i^{out}}{2m}$.

(c) Calculate a formula for the expected number of reciprocal pairs of nodes (in a directed network, a pair of nodes is reciprocal if there is an edge i to j and also and edge j to i).

Exercise 3. There are two important pgfs for the configuration network: the degree distribution pgf, $g_0(z)$ and the excess degree distribution $g_1(z)$.

(a) Show that:

$$g_1(z) = \frac{g_0'(z)}{g_0'(1)} ,$$

where $'$ denotes differentiation.

(b) The Poisson random graph is one in which the degree distributions are Poisson random variables. It therefore approximates the E-R graph. If $p_k = e^{-c}\frac{c^k}{k!}$, find $g_0(z)$. Then show that $g_1(z) = g_0(z)$. What does this mean for the E-R graph?

(c) Now, instead imagine that every node in your graph has the same degree k_0.

(i) Show that the formula from (a) predicts, as we expect, that every node has the same excess degree $k_0 - 1$.

(ii) Using the calculation for S given in class, show that $S = 1$ if $k_0 \geq 3$.

Exercise 4. Newman, 12.3.

Exercise 5. Newman, 12.11.

Exercise 6. (Computer simulation problem).

In class we briefly discussed the Strogatz-Wells model for a small world network. Initially, imagine n nodes arranged around a circle, with each node connected by an edge to its c nearest neighbors (for simplicity, you may assume that c is even).

(a) What is the diameter of this network?

To turn the network into a small world network, do the following. Take each edge in turn, and with probability p delete that edge and add a new edge to the network between any randomly chosen pair of edges. Networks so formed have two parameters: c and p. When $p = 0$, all networks are connected only locally. As $p \to 1$ the network approaches a Poisson-random network (a form of configuration network, with degree distribution given by the Poisson distribution, which is the same as the Erdös-Rényi graph).

(b) Simulate a set of small-world network with $n = 100$ nodes, $c = 4$ and (i) $p = 0$, (ii) $p = 0.001$, (iii) $p = 0.01$, (iv) $p = 0.1$. Make 30 networks for each value of $p > 0$. Calculate the average diameter of the networks made for each p value (if you are using Matlab for your simulations, look up the function **distances**), and plot it as a function of p. You will find that even a small number of random connections are enough to reduce the diameter of the graph to something similar to the Erdös-Rényi diameter.