168 - Homework 1. Due on Gradescope by January 19th

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January 12, 2024

Exercise 1. (A reflective exercise) Read Tom Körner's essay "In Praise of Lectures":

`https://www.dpmms.cam.ac.uk/~twk/Lecture.pdf`

Explain in one or two sentences the practices that Dr. Körner recommends students follow in order to get the most learning out of lectures.

Exercise 2. (From M.A. Porter)

Take a photo of something on the UCLA campus that you would model by a network (include the photo in your homework submission). Explain briefly what would be the nodes in your model and what would be its edges.

Exercise 3. In class we spent some time discussing properties that would enable us to classify a network. For each of the following give one example of a network with these properties (remember to specify what are the nodes in the network and what are its edges).

1. A static simple network.
2. A dynamic bipartite network
3. An observable transport network
4. A weighted network with directed edges

Exercise 4. A (simple) complete graph is one in which node is connected to every other node by exactly one edge.

(a) Write down the adjacency matrix $\mathbf{A}$ and Laplacian $\mathbf{L}$ for a complete graph with $|N| = 4$.
(b) How many edges does a complete graph with $|N|$ nodes have?
(c) Give the eigenvalues of $\mathbf{A}$ (Hint: $\lambda = -1$ is certainly an eigenvalue – what is its multiplicity?).
(d) Give the eigenvalues of $\mathbf{L}$ (see hint above).

Exercise 5. Given a complete graph with 3 nodes:

(a) Use the formula derived in class for counting walks to confirm that there are two paths of length 3 that start and end at node 1.
(b) How many paths of length 3 connect any pair of different nodes?

Exercise 6. Newman problem 6.6.

Exercise 7. A graph has adjacency matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

Use a **computer code** to answer the following questions

(a) Calculate the eigenvalues of **A**.

(b) Draw the graph of the network.

Exercise 8. (* Optional, challenge question). We showed in 4(d) that the Laplacian of a complete graph always has 0 has one of its eigenvalues. Using the analogy with resistor networks that was given in class, explain why any network will have at least one 0 eigenvalue.