168 - Homework 4. Due on Gradescope by February 9th.

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The penalty-free extended submission deadline for this homework is February 14th (i.e. there is an automatic 5 day extension).

Exercise 1. Exercise 11.1 from Newman. ($G(n, p)$ is notation for the E-R graph with parameters n and p).

Exercise 2. Exercise 11.9 from Newman.

Exercise 3. (Computer simulation problem).

(a) Calculate, and plot, the fraction, S of nodes within the largest connected component of a set of E-R graphs with 50 nodes as a function of the mean node degree, using values: $c = 0.5$, 1, 1.5, 2, 2.5, ... 5. (Hint: if you are using Matlab, then the function `conncomp` can be used to find all of the components in a graph).

(b) (* Optional) To make a nicer plot, make 30 E-R graphs for each value of c, and plot against c, the average value of S across the 30 replicates.

The last four problems in this homework can be treated as a practice midterm exam.

Exercise 4. A network with 7 nodes, labeled 1, 2, ... 7 has an adjacency matrix:

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

(a) From the adjacency matrix answer: which node in the network has the highest degree?

(b) Calculate the local clustering coefficient for the highest degree node you identified in part (a)

Exercise 5. In this exercise we will consider two versions of the same network: in (a) a directed version, and in (b) an undirected version. The two versions of the network are shown in Figure 1.

(a) Consider first the directed version of the graph. Calculate the Katz centrality of node i, labeled on the graph.

(b) Now consider the undirected version of the same graph. Calculate the betweenness centrality of node i.

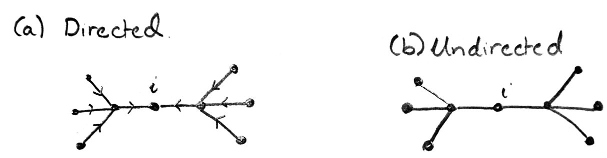

Figure 1: Two versions of a network, to be used in Exercise 5.

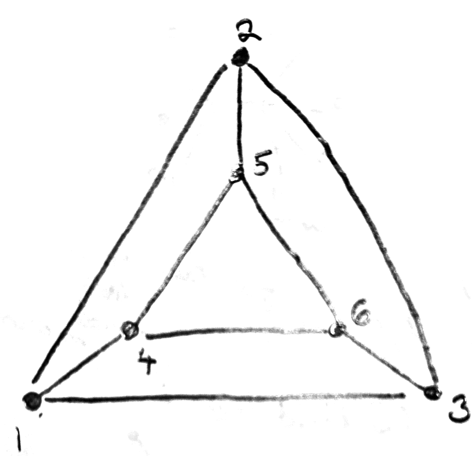

Figure 2: Network for Exercise 6.

Exercise 6. Consider the network shown in Figure 2. Find the largest positive eigenvalue of the network. (Hint: You do not need to write out the 6×6 adjacency matrix, instead you should start by explaining why each node has the same eigenvalue centrality).

Exercise 7. Tetradic closure is a measure of the number of 3 paths in the network that are closed (i.e. how many 3-paths start and end on vertices i and j that are also neighbors of each other). The mathematical definition of the tetradic closure coefficient is:

$$C = \frac{8 \times \ \text{\#squares in the network}}{\text{\#3-paths}}$$

Here a square is a closed loop that contains 4 nodes and the edges that connect them.
(a) Explain why the coefficient 8 is required in the numerator.
(b) Calculate the tetradic closure coefficient for the $G(n, p)$ Erdös-Rényi network.