Written Problem

- $3n^2+10n+1$

Let $k=5$ and $n_0=10$ This will be true ~~for~~ for any $n \geq n_0$

$3(10)^2+10(10)+1 \leq 5(10)^2$ $3(20)^2+10(20)+1 \leq 5(20)^2$

$401 \leq 500$ $1401 \leq 2000$

As n increases, the difference between $3n^2+10n+1$ and kn^2 increases, showing the function to be $O(n^2)$.

- 2^n

There will be no witness for 2^n to be $O(n^2)$

Let $k=100$ and $n_0=10$ This, however, will not be true for any $n \geq n_0$

$2^{10} \leq 100(10)^2$ $2^{20} \leq 100(20)^2$

$1024 \leq 10000$ $1048576 \not\leq 40000$

There will never be a k and n_0 where kn^2 will always be greater than 2^n, so the function is not $O(n^2)$.

- $n\lg n$

Let $k=1$ and $n_0=10$ This will be true for any $n \geq n_0$

$10\lg(10) \leq 10^2$ $100\lg(100) \leq 100^2$

$10 \leq 100$ $200 \leq 10000$

Since the difference between the functions is increasing and staying true, the function is $O(n^2)$.

- $n^3/\lg n$

There will be no witness for $n^3/\lg n$ to be $O(n^2)$

Let $k = 100$ and $n_0 = 10$ This will not be true for any $n \geq n_0$

$$10^3/\lg 10 \leq 100(10)^2 \qquad 1000^3/\lg 1000 \leq 100(1000)^2$$

$$1000 \leq 10000 \qquad 333{,}333{,}333\tfrac{1}{3} \not\leq 100{,}000{,}000$$

There will never be a k and n_0 where kn^2 will always be greater than $n^3/\lg n$, so the function is not $O(n^2)$.

- $f(n) + h(n)$, where each of $f(n)$ and $h(n)$ are $O(n^2)$

Let k_f and n_{0f} represent the witnesses for $f(n)$.
Let k_h and n_{0h} represent the witnesses for $h(n)$.
$f(n) \leq k_f g(n)$ and $h(n) \leq k_h g(n)$ will be true for some $n \geq n_{0f}$ and $n \geq n_{0k}$.
Adding the two inequalities, we get
$f(n) + h(n) \leq (k_f + k_h) g(n)$, which will be true for any $n \geq n_{0f}$ and $n \geq n_{0k}$

Thus, the witness of $f(n)+h(n)$ will be $k = k_f + k_h$ and $n_0 =$ the greater of n_{0f} and n_{0k}.
The function is therefore $O(n^2)$.