

Holes bound to acceptors as qubits: Tunability, coherence and entanglement

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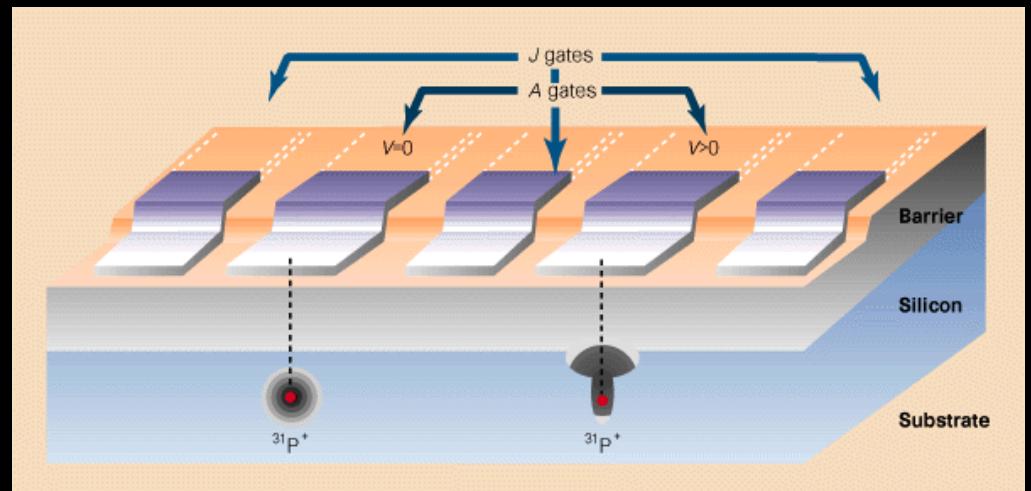
OUTLINE

- 1. BACKGROUND & MOTIVATION
- 2. ACCEPTOR DESCRIPTION
- 3. LH ACCEPTOR QUBIT 101
- 4. IN-PLANE B FIELD. NEW PHENOMENA
- 5. EXPERIMENTAL APPLICATIONS
- 6. CONCLUSIONS

DOPANT BASED QUANTUM COMPUTING

Proposed by Bruce Kane, Nature 393, 133 (1998).

- P donors in Si: nuclear spin $\frac{1}{2}$
- Electron spin coupled to nuclear spin by hyperfine interaction



- A gate controls the hyperfine interaction (for 1 qubit gates)
- J gate controls the overlap of electrons (for 2 qubit gates)
- Readout: spin-to-charge conversion

DONORS: CURRENT STATUS

- High fidelity single shot readout of electron and nuclear spin. Pla et al., Nature 489, 541 (2012). J. Pla et al., Nature 496, 334 (2013)
- Electron spin relaxation and coherence times $> T_1$ in the order of hundreds of seconds. $T_2 = 10\text{ s}$. Nature Materials 11, 143-147 (2012)
- Nuclear spin coherence times $> 30\text{ s}$. Nature Nanotechnology 9, 986 (2014)
- A-gate implementation. Laucht et al. Science Advances 1, e1500022 (2015)
- Fidelities above 99.95-99.99% for single qubit operations (electron and nuclear spin respectively). J. Phys: Condens. Matter 27, 154205 (2015)

DONORS: CHALLENGES

- Valley degeneracy: Exchange oscillations
- Small SO interaction: Slow interactions
- Oscillating Magnetic fields: Experimentally challenging and require too much power

MOTIVATION

- Electrical spin manipulation: **spin-orbit qubits**
 - Spin-orbit enhances the coupling to electric fields
 - Single-qubit operations: EDSR
 - Also scalability e.g. cQED, dipole-dipole coupling
 - Electric fields are easier to apply and localize than magnetic fields
- Current problems
 - Spin-orbit also enhances coupling to stray fields ~ noise and phonons
 - Scalability: exchange gates vulnerable to electrical noise

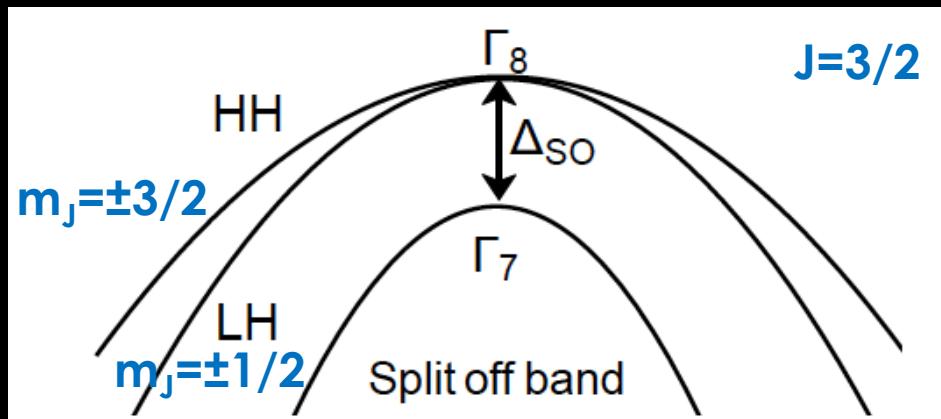
WHY ACCEPTORS?

Confinement potential is free and reproducible cf. donor
Holes have interesting properties

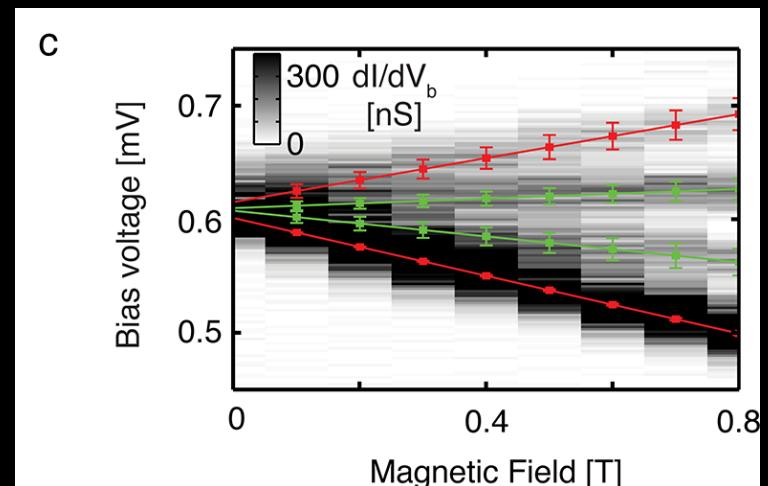
- Strong spin-orbit coupling in the valence band ($L=1$)
- Limited coupling to nuclear spins
- Effective spin-3/2 – completely different from electrons
- No valleys so no extra Hilbert space complications
- Enhanced dipole-dipole interaction
- Flexibility – can work in HH or LH manifolds

VALENCE BANDS

Strong spin-orbit interaction
Effective $J=3/2$ GS

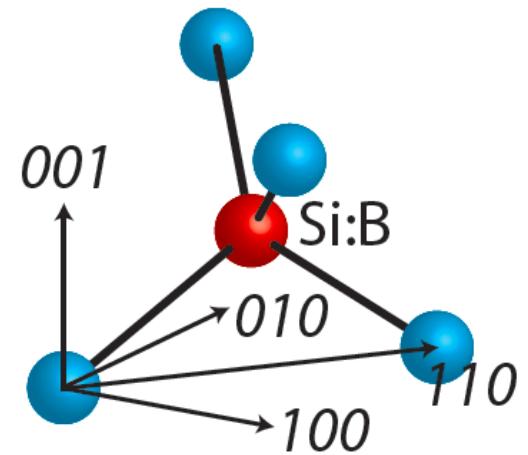
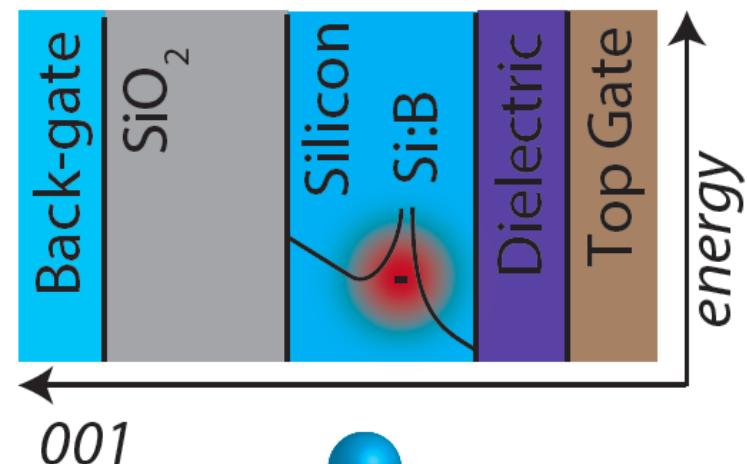
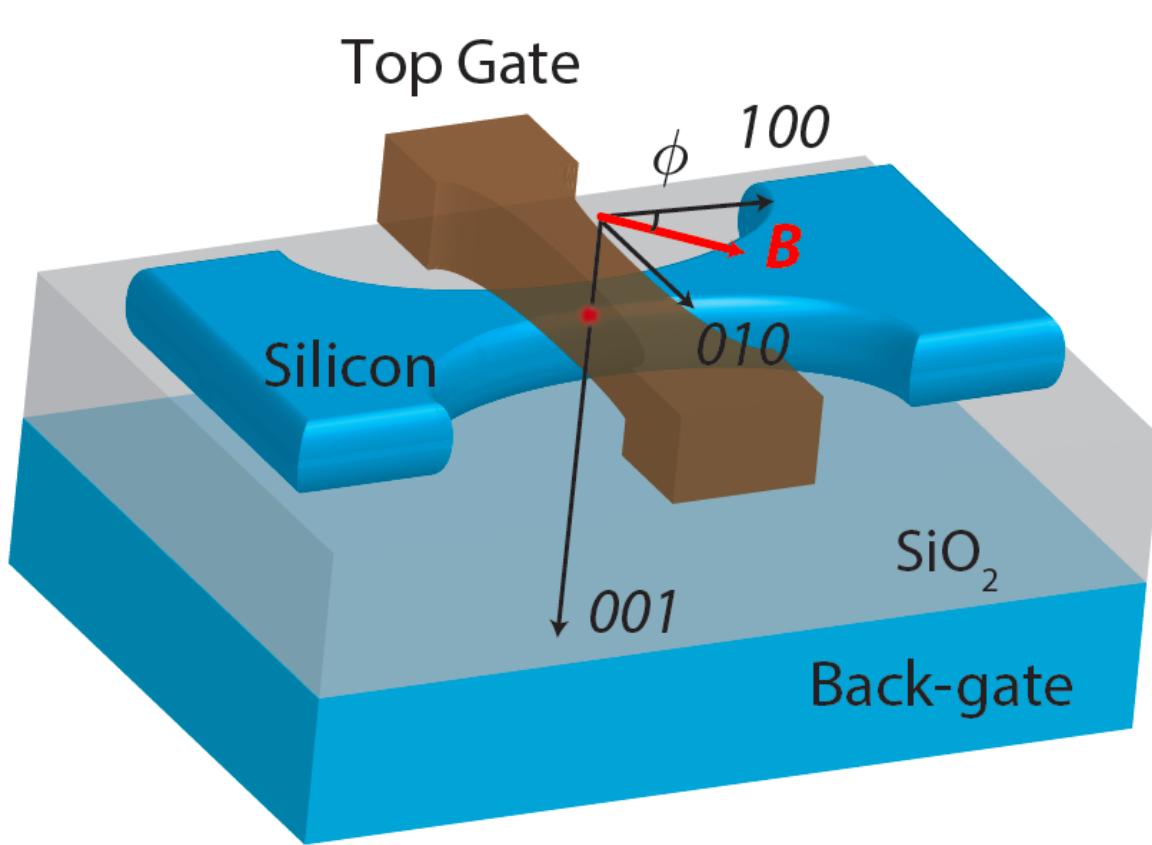


Luttinger and Kohn, Phys. Rev. 97, 869 (1955)
Si:B GS is 45 meV on top of VB
1st excited state is 21 meV below



Van de Heijden et al, Nano Letters 14, 1492 (2014)

FOCUS



Acceptor in Si close to a SiO₂ interface.

ACCEPTOR HAMILTONIAN

$$H = H_{KL} + H_{BP} + H_c + H_{\text{inter}} + H_F + H_B + H_{T_d}$$

-Kohn-Luttinger Hamiltonian for the VB

-Bir-Pikus Hamiltonian for the strain

-Coulomb impurity

-Si/SiO₂ Interface

-Electric and magnetic fields

-T_d symmetry of the ion

EMA+SW transformation

4x4 Effective low energy

Hamiltonian

For the 4-fold degenerate

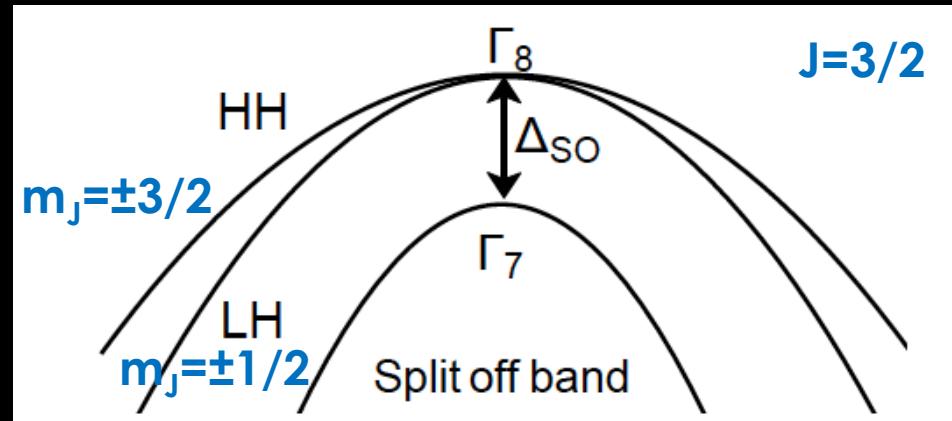
GS



INTUITIVE BUILDING THE EFFECTIVE GS HAMILTONIAN:

$$\{3/2, 1/2, -1/2, -3/2\}$$

$$H_{\text{eff}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



States with $|m_J|=3/2$ are predominantly HH like

$|m_J|=1/2$ are
predominantly LH like

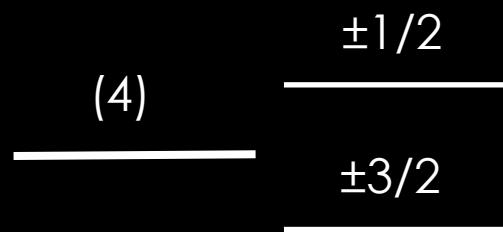
INTUITIVE BUILDING THE EFFECTIVE GS HAMILTONIAN: BREAKING DEGENERACIES

$$H_{\text{eff}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta_{HL} & 0 & 0 \\ 0 & 0 & \Delta_{HL} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

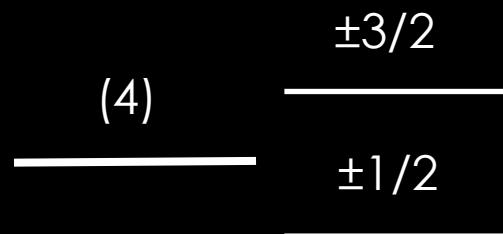
Bir-Pikus Hamiltonian

Uniaxial (001) strain

In-plane compressive
HH ground state



In-plane tensile
LH ground state



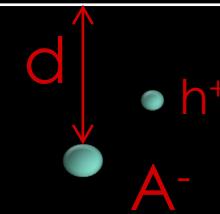
INTUITIVE BUILDING THE EFFECTIVE GS HAMILTONIAN: BREAKING DEGENERACIES

$$H_{\text{eff}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta_{HL} & 0 & 0 \\ 0 & 0 & \Delta_{HL} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Quantum confinement + dielectric mismatch

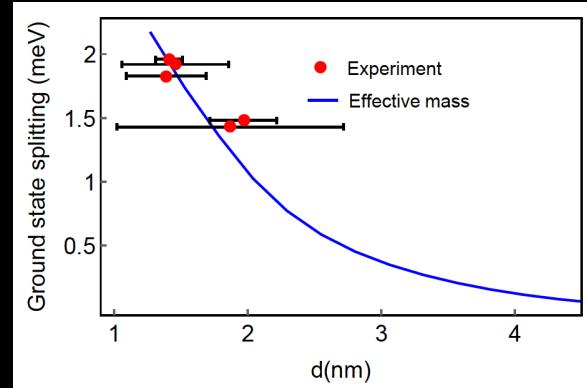
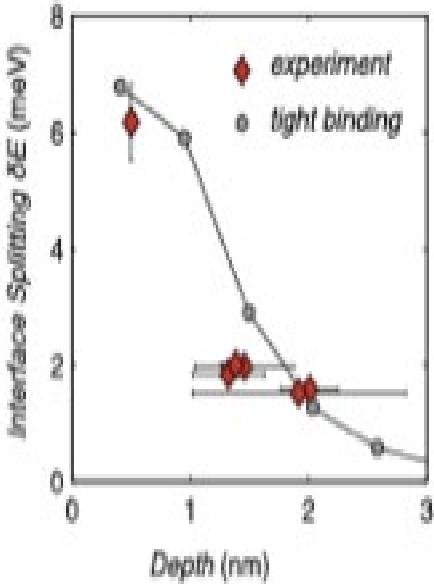
- A⁻ image
- h⁺ image

SiO₂



Si(001)

HH ground state



BUILDING THE EFFECTIVE GS HAMILTONIAN: MIXING LH-HH

$$\{3/2, 1/2, -1/2, -3/2\}$$

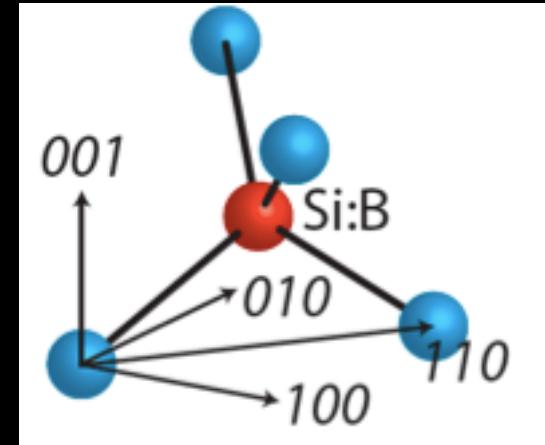
$$H_{\text{eff}} = \begin{pmatrix} 0 & 0 & -ipF_z & 0 \\ 0 & \Delta_{HL} & 0 & -ipF_z \\ ipF_z & 0 & \Delta_{HL} & 0 \\ 0 & ipF_z & 0 & 0 \end{pmatrix}$$

$$p = e \int_0^a f^*(r) r f(r)$$

This term mixes LH and HH.

p depends

- acceptor “depth”
- electric field
- distance to interface



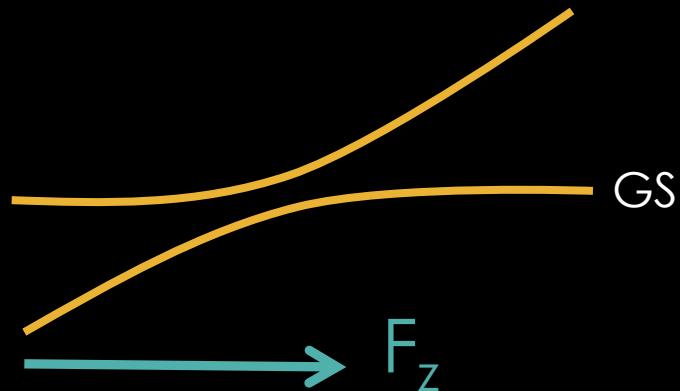
Bir, Butekov, Pikus J. Phys. Chem. Solids 24, 1467 (1963); 24, 1475 (1963)

BUILDING THE EFFECTIVE GS HAMILTONIAN: BROKEN DEGENERACY + MIXING LH-HH

2 branches (Kramers doublets):

$$E_u = \frac{1}{2}(\Delta_{HL} + \sqrt{\Delta_{HL}^2 + 4p^2 F_z^2})$$

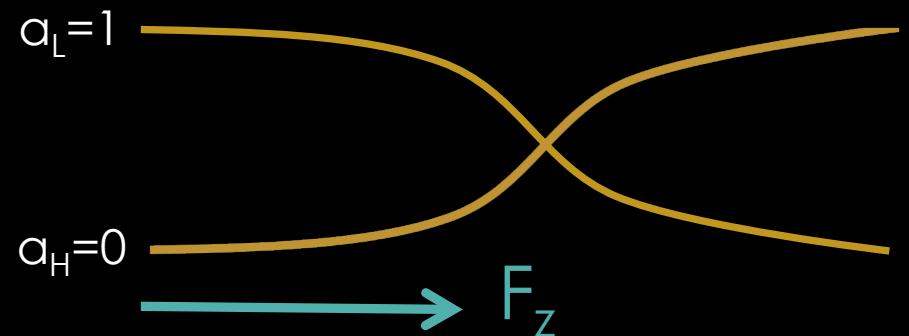
$$E_l = \frac{1}{2}(\Delta_{HL} - \sqrt{\Delta_{HL}^2 + 4p^2 F_z^2})$$



The qubit is defined in the lower branch with probability amplitudes on LH and HH:

$$a_L = E_l / \sqrt{E_l^2 + p^2 F_z^2}$$

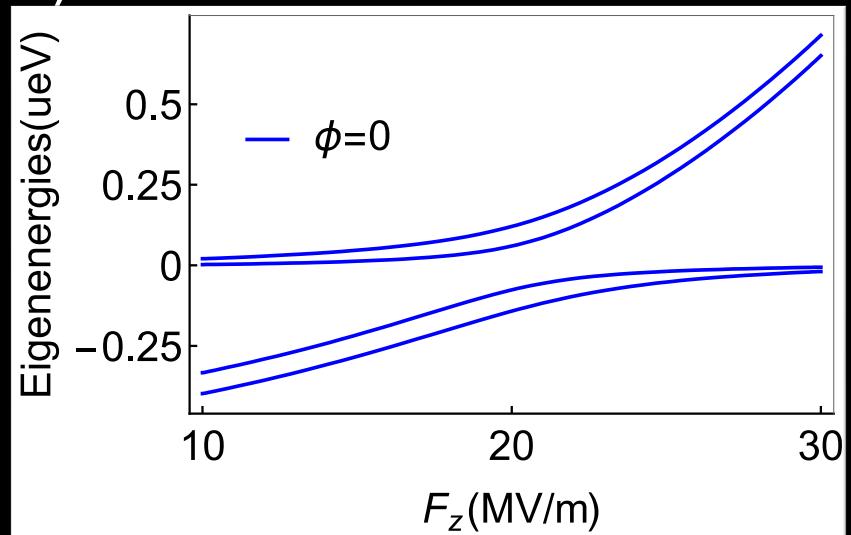
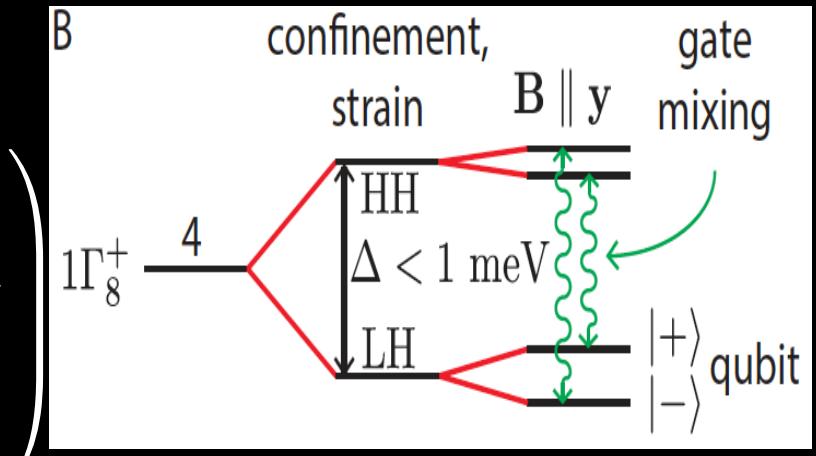
$$a_H = pF_z / \sqrt{E_l^2 + p^2 F_z^2}$$



BUILDING THE EFFECTIVE GS HAMILTONIAN: IN-PLANE MAGNETIC FIELD IN CRYSTAL AXIS

$$H_{\text{eff}} = \begin{pmatrix} \{3/2, 1/2, -1/2, -3/2\} & & & \\ 0 & \frac{\sqrt{3}}{2}\varepsilon_Z & -ipF_z & 0 \\ \frac{\sqrt{3}}{2}\varepsilon_Z & \Delta_{HL} & \varepsilon_Z & -ipF_z \\ ipF_z & \varepsilon_Z & \Delta_{HL} & \frac{\sqrt{3}}{2}\varepsilon_Z \\ 0 & ipF_z & \frac{\sqrt{3}}{2}\varepsilon_Z & 0 \end{pmatrix}$$

$$\varepsilon_Z = g_1 \mu_B B$$



HAMILTONIAN IN THE QUBIT BASIS

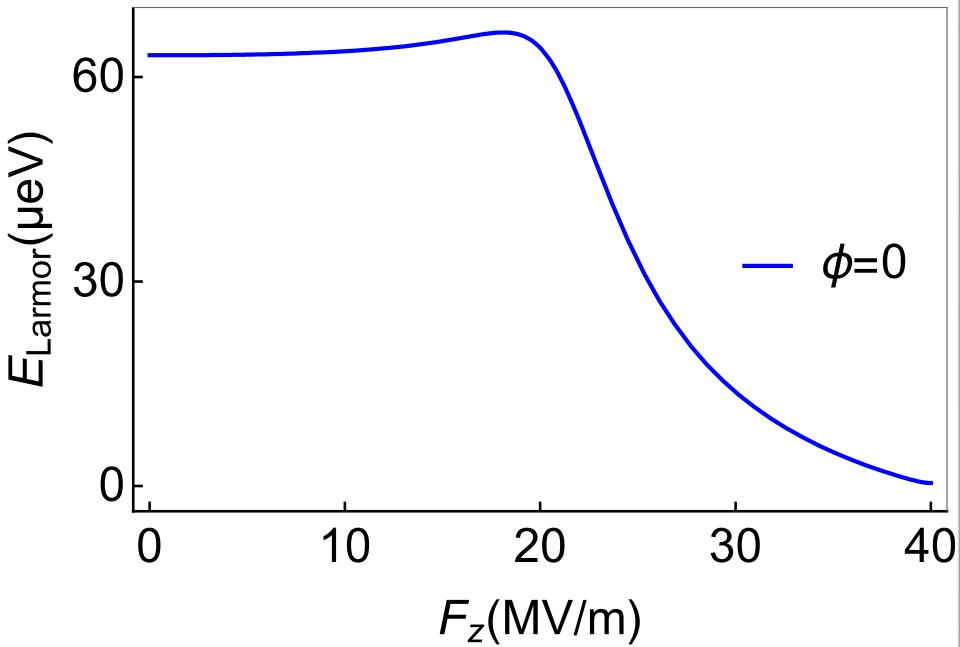
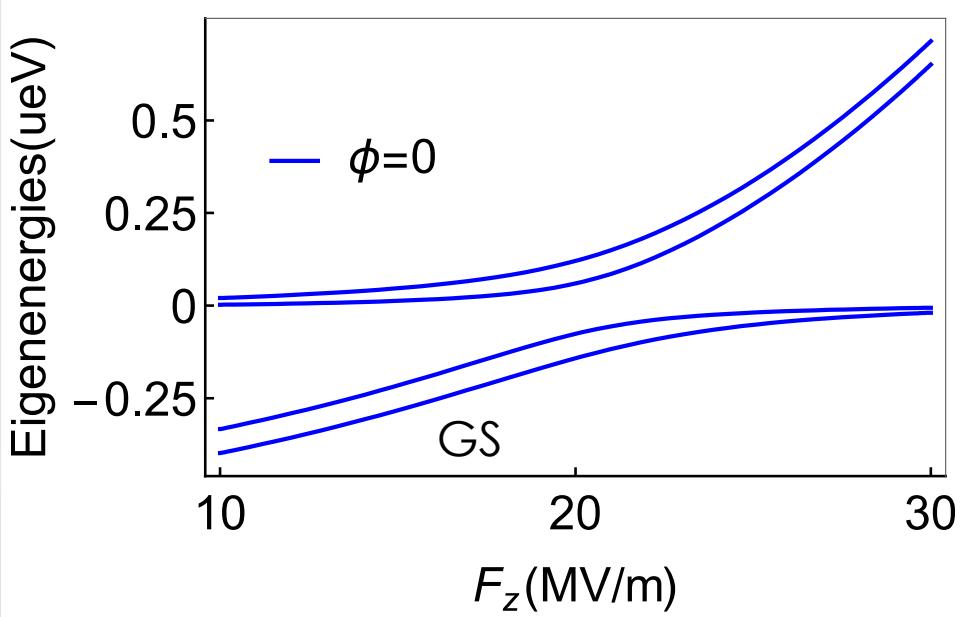
$$H_{\text{op}} = \begin{pmatrix} E_l - \frac{1}{2}\varepsilon_{Zl} & 0 & Z_1 & Z_2 \\ 0 & E_l + \frac{1}{2}\varepsilon_{Zl} & Z_2 & Z_1 \\ Z_1 & -Z_2 & E_u - \frac{1}{2}\varepsilon_{Zu} & 0 \\ -Z_2 & Z_1 & 0 & E_u + \frac{1}{2}\varepsilon_{Zu} \end{pmatrix}$$

$Z_1, Z_2 \propto \varepsilon_{Zo}$ Qubit branch-Excited branch interaction

Splittings and mixings come from ε_z .

Depend on the LH-HH mixing and the Zeeman interaction.

ENERGY LEVELS



Sweet spot at

$$F_z^* = -\frac{\sqrt{3}\Delta_{HL}}{2p}$$

(There is another sweet spot at $F_z=0$)

Qubit is insensitive to charge noise in these sweet spots

Salfi et al PRL (2016)

SINGLE-QUBIT MANIPULATION

Lack of inversion symmetry: Effective Rashba interaction

$$\hat{H}_E = \begin{pmatrix} 0 & 0 & E_1 & E_2 \\ 0 & 0 & E_2 & E_1 \\ -E_1 & E_2 & 0 & 0 \\ E_2 & -E_1 & 0 & 0 \end{pmatrix}$$

Mixes qubit-excited branches

$$E_1, E_2 \propto \alpha$$

$$H_{\text{EDSR}}^{(2)} = \alpha \frac{\varepsilon_{Zo}}{\Delta} F_{\parallel} \sigma_x$$

This interaction is maximized at sweet spot

Single gate times: 0.2 ns at the sweet spot for acceptor at 4.6 nm.

TWO-QUBIT MANIPULATION

Two-qubit interaction: Dipole-Dipole interaction

$$V_{dd} = (\mathbf{v}_a \cdot \mathbf{v}_b R^2 - 3(\mathbf{v}_a \cdot \mathbf{R})(\mathbf{v}_b \cdot \mathbf{R}))/4\pi\epsilon R^5$$

\mathbf{v} are spin dependent charge dipoles

$$H_{dd} \propto \alpha^a \alpha^b \varepsilon_{Zo}^a \varepsilon_{Zo}^b (\sigma_+^1 + \sigma_-^1)(\sigma_+^2 + \sigma_-^2)/R^3$$

For 20 nm : $\sqrt{\text{SWAP}} = 2\text{ns}$

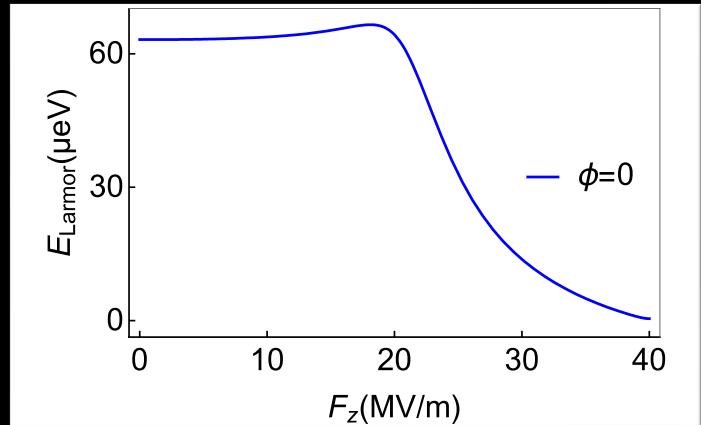
We can also use exchange for entanglement
No valley interference effects like in donor qubits
Circuit QED using microwave photons

DECOHERENCE LIMITATIONS

Strong spin-orbit coupling: electrical noises cause dephasing

Qubit is insensitive to charge noise fluctuations to first order at sweet spots

$$-\frac{\sqrt{3}}{2}|\pm 1/2\rangle \mp i\frac{1}{2}|\mp 3/2\rangle$$



Phonon-induced relaxation

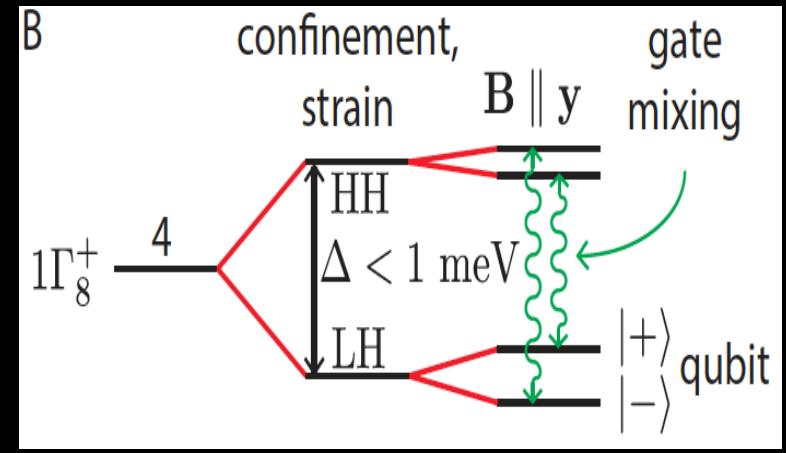
$$\frac{1}{T_1} = \frac{(\hbar\omega)^3}{20\hbar^4\pi\rho} C_d \left(\frac{\varepsilon_{Zo}}{\Delta} \right)^2$$

T₁=20μs for B=0.5T at sweet spot

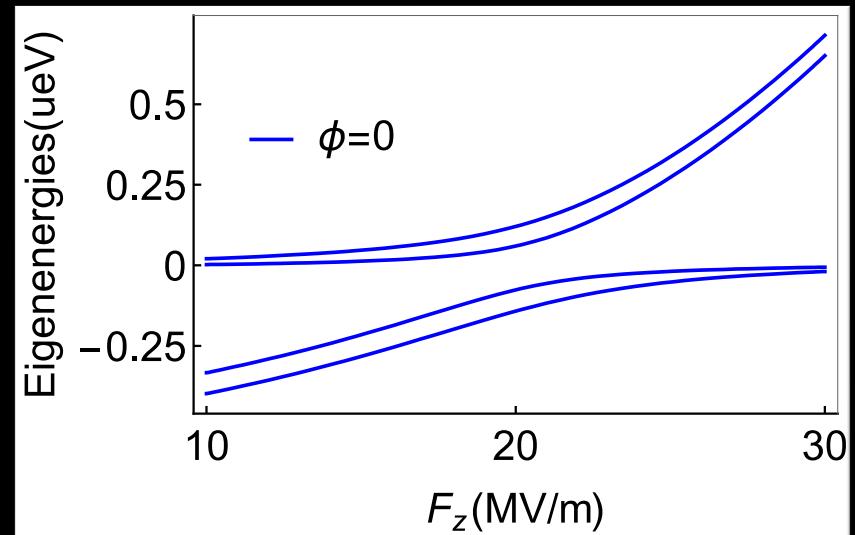
Still 10⁵ operations in this time can be performed

ARBITRARY IN-PLANE MAGNETIC FIELD

$$H_{\text{eff}} = \begin{pmatrix} \{3/2, 1/2, -1/2, -3/2\} & & & \\ 0 & \frac{\sqrt{3}}{2}\varepsilon_Z & -ipF_z & 0 \\ \frac{\sqrt{3}}{2}\varepsilon_Z^* & \Delta_{HL} & \varepsilon_Z & -ipF_z \\ ipF_z & \varepsilon_Z^* & \Delta_{HL} & \frac{\sqrt{3}}{2}\varepsilon_Z \\ 0 & ipF_z & \frac{\sqrt{3}}{2}\varepsilon_Z^* & 0 \end{pmatrix}$$



$$\varepsilon_Z = g_1 \mu_B B \rightarrow g_1 \mu_B B e^{i\phi}$$

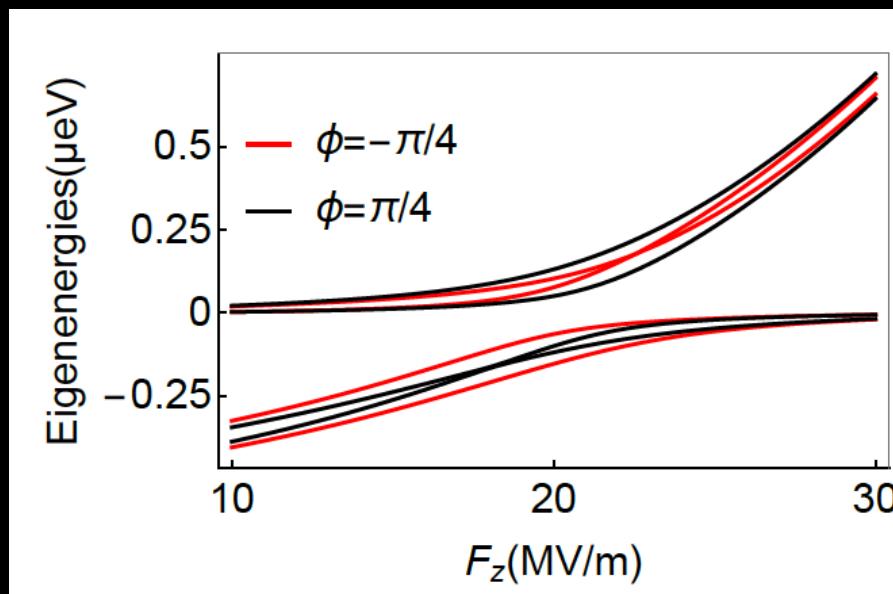


HAMILTONIAN IN THE QUBIT BASIS

$$H_{\text{op}} = \begin{pmatrix} E_l - \frac{1}{2}\varepsilon_{Zl} & 0 & Z_1 & Z_2 \\ 0 & E_l + \frac{1}{2}\varepsilon_{Zl} & Z_2 & Z_1 \\ Z_1 & -Z_2 & E_u - \frac{1}{2}\varepsilon_{Zu} & 0 \\ -Z_2 & Z_1 & 0 & E_u + \frac{1}{2}\varepsilon_{Zu} \end{pmatrix}$$

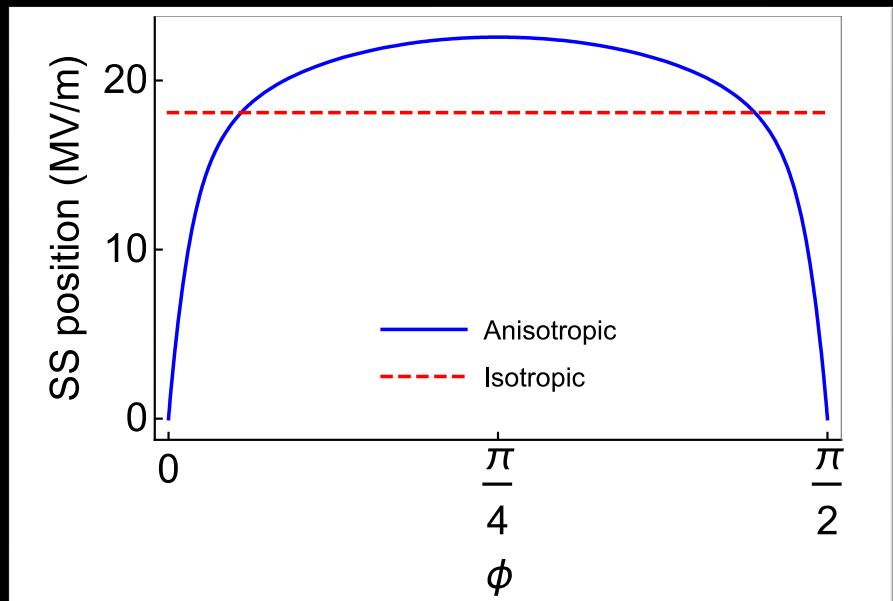
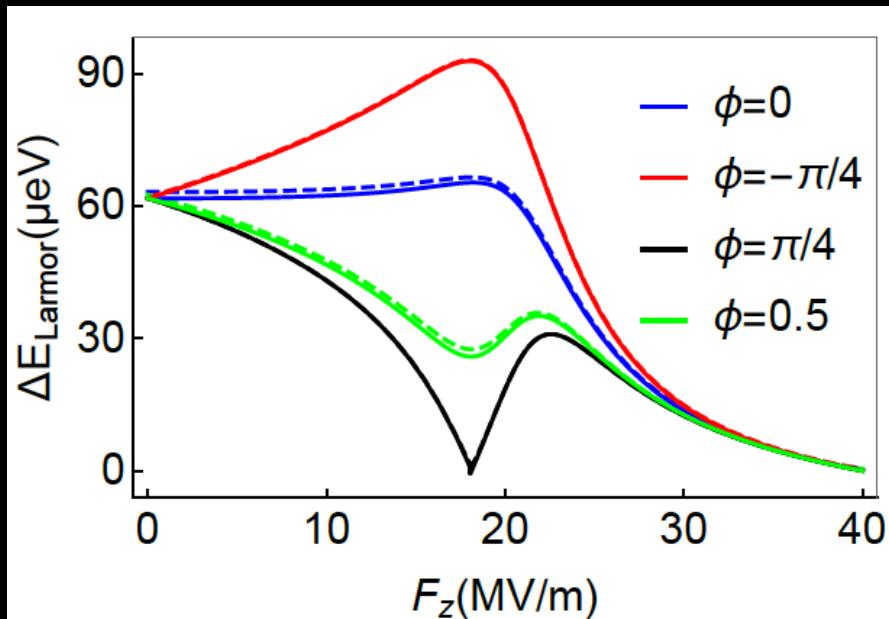
Qubit-upper states interaction terms

Splittings and mixings come from ε_z . They are complex functions of ϕ .



Strong magnetic field orientation dependence

MAGNETIC FIELD DEPENDENCE

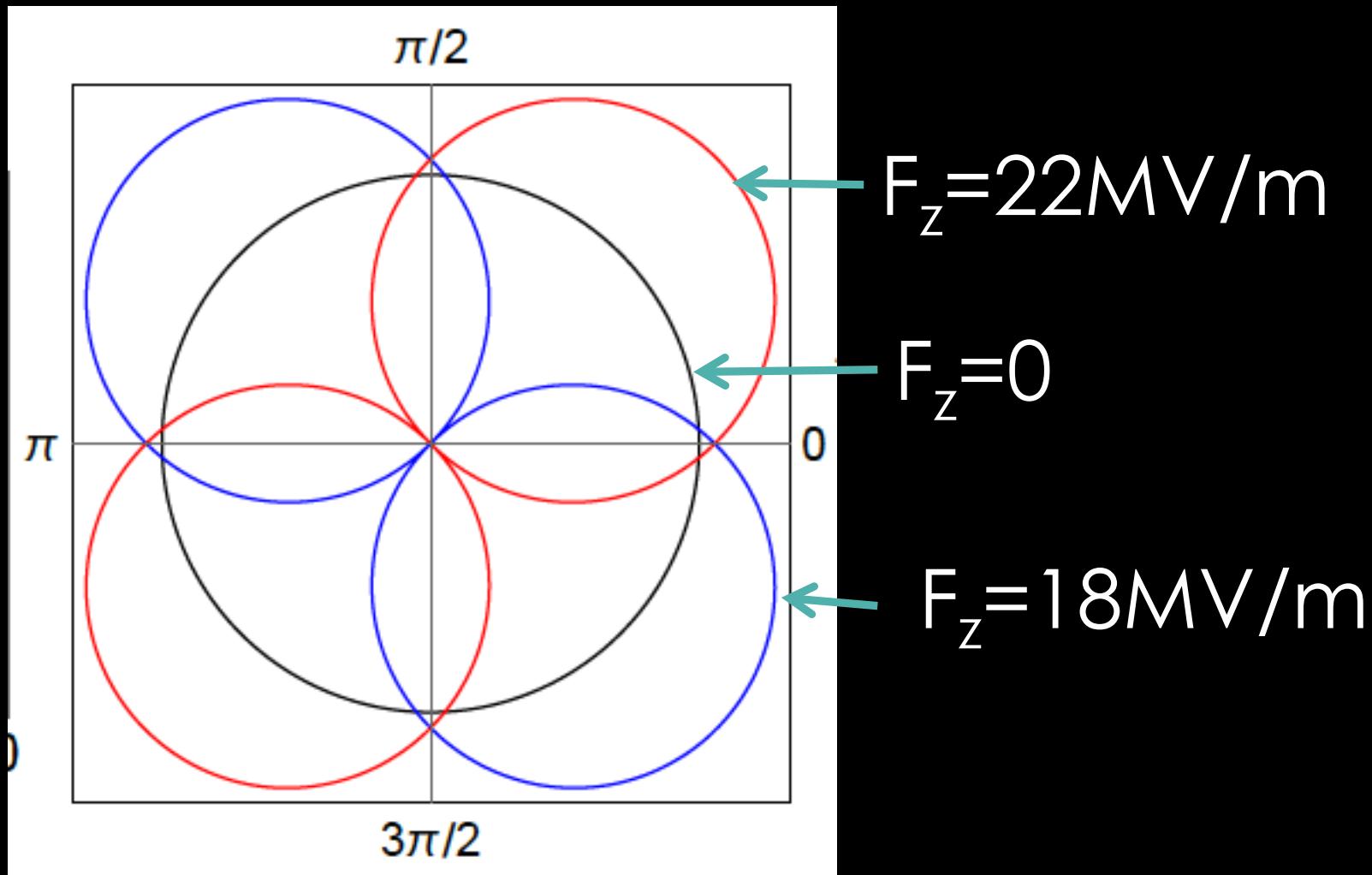


Two sweet spots:

- Isotropic sweet spot $F_z^* = -\frac{\sqrt{3}\Delta_{HL}}{2p}$
- $$-\frac{\sqrt{3}}{2}|\pm 1/2\rangle \mp i\frac{1}{2}|\mp 3/2\rangle$$

Anisotropic (dependent on ϕ) sweet spot

G-FACTOR IN-PLANE ANISOTROPY

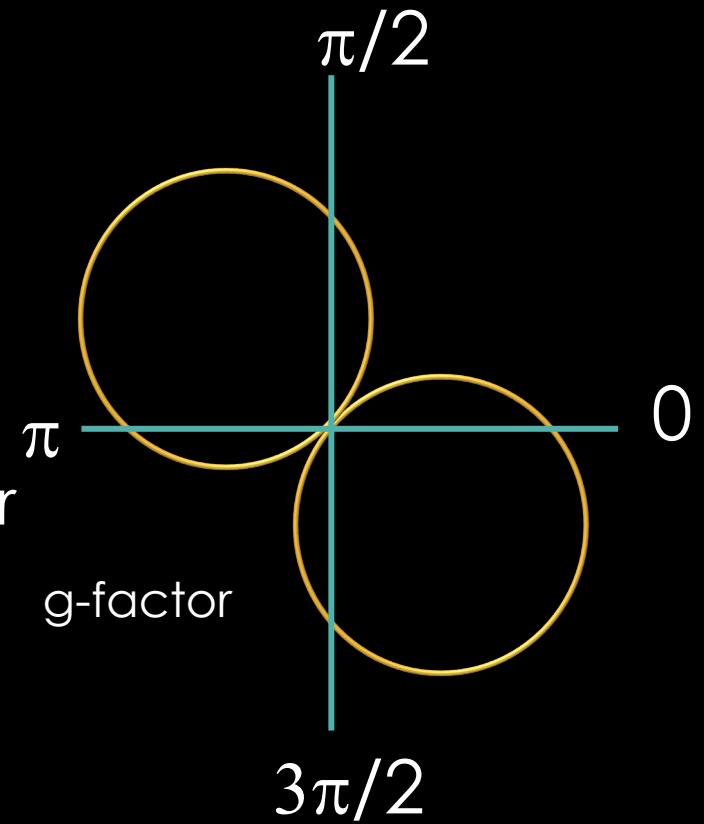


SPIN POLARIZATION AT SWEET SPOTS

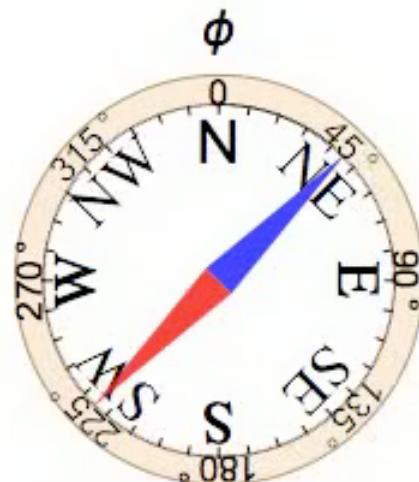
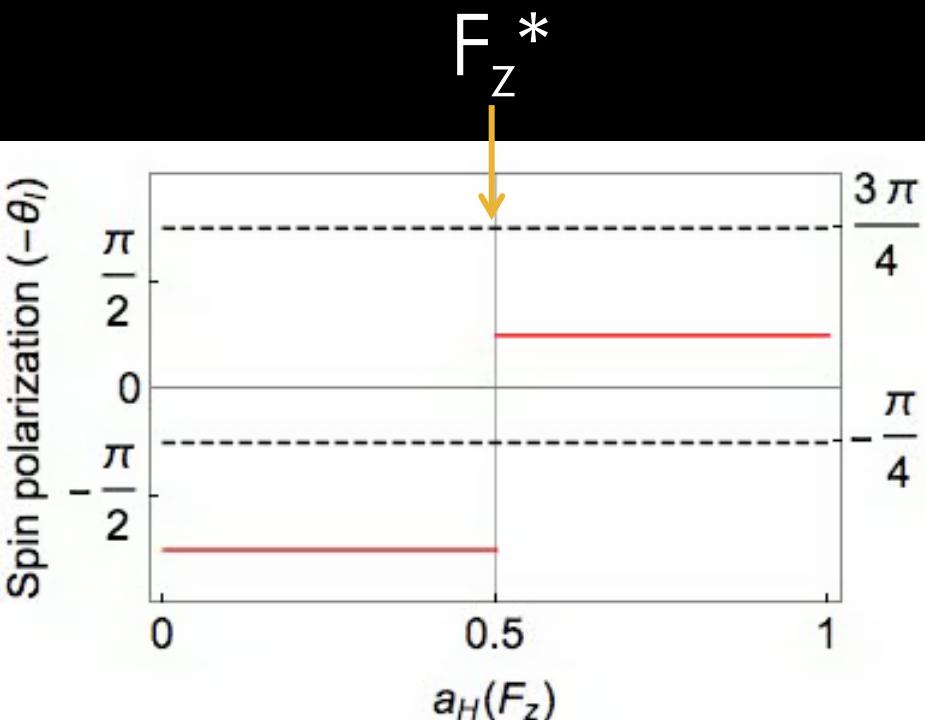
At F_z^* the spin polarization is in the $-\pi/4+n\pi$ direction

The g-factor is 0 in the perpendicular direction and maximum in the parallel direction.

The maximum corresponds to a complete decoupling of the lower and upper branches: decoherence free subspace.



SPIN POLARIZATION AT SWEET SPOTS



$$H_{\text{op}} = \begin{pmatrix} E_l - \frac{1}{2}\varepsilon_{Zl} & 0 & Z_1 & Z_2 \\ 0 & E_l + \frac{1}{2}\varepsilon_{Zl} & Z_2 & Z_1 \\ Z_1 & -Z_2 & E_u - \frac{1}{2}\varepsilon_{Zu} & 0 \\ -Z_2 & Z_1 & 0 & E_u + \frac{1}{2}\varepsilon_{Zu} \end{pmatrix}$$

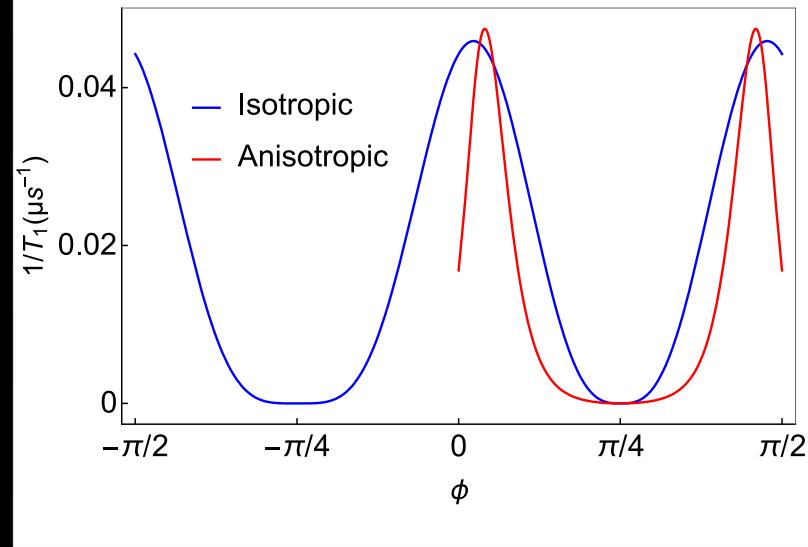
Decoherence free subspace $[H_{\text{inter}} + H_{T_d}, H_B] = 0$

$$H_{\text{op}} = \begin{pmatrix} E_l - \frac{1}{2}\varepsilon_{Zl} & 0 & 0 & 0 \\ 0 & E_l + \frac{1}{2}\varepsilon_{Zl} & 0 & 0 \\ 0 & 0 & E_u - \frac{1}{2}\varepsilon_{Zu} & 0 \\ 0 & 0 & 0 & E_u + \frac{1}{2}\varepsilon_{Zu} \end{pmatrix}$$

CONSEQUENCES: T_1

Phonon-induced relaxation

$$\frac{1}{T_1} = \frac{(\hbar\omega(\phi))^3}{20\hbar^4\pi\rho} C_d \left(\frac{\varepsilon_{Zo}(\phi)}{\Delta} \right)^2$$



This mechanism is canceled to 1st order:

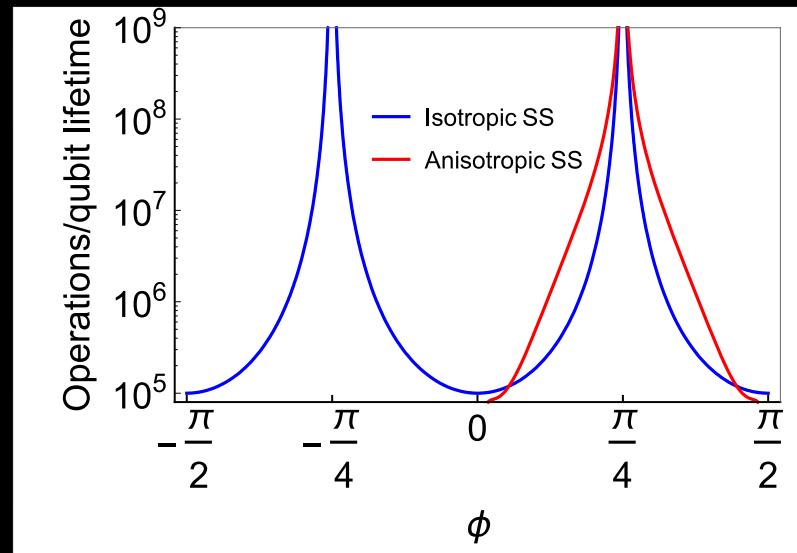
- When effective g-factor is 0
- In the DFS

CONSEQUENCES: SINGLE-QUBIT MANIPULATION

EDSR coupling depends on ϕ

$$D \propto \varepsilon_{Z_0}(\phi)$$

$$\frac{1}{T_1} = \frac{(\hbar\omega(\phi))^3}{20\hbar^4\pi\rho} C_d \left(\frac{\varepsilon_{Z_0}(\phi)}{\Delta} \right)^2$$



EDSR slows down near the DFS
T1 diverges faster than EDSR at DFS

The number of single-qubit operations in T1 diverges at DFS
-In real devices this will be limited by second order processes.
-Still carefully choosing ϕ should strongly reduce the T1 limitations

CONSEQUENCES: TWO-QUBIT MANIPULATION

Two-qubit interaction: Dipole-Dipole interaction

$$H_{dd} \propto \alpha^a \alpha^b \varepsilon_{Zo}^a \varepsilon_{Zo}^b G(F_z^a, F_z^b, \phi, \theta_E) (\sigma_+^1 + \sigma_-^1)(\sigma_+^2 + \sigma_-^2)/R^3$$

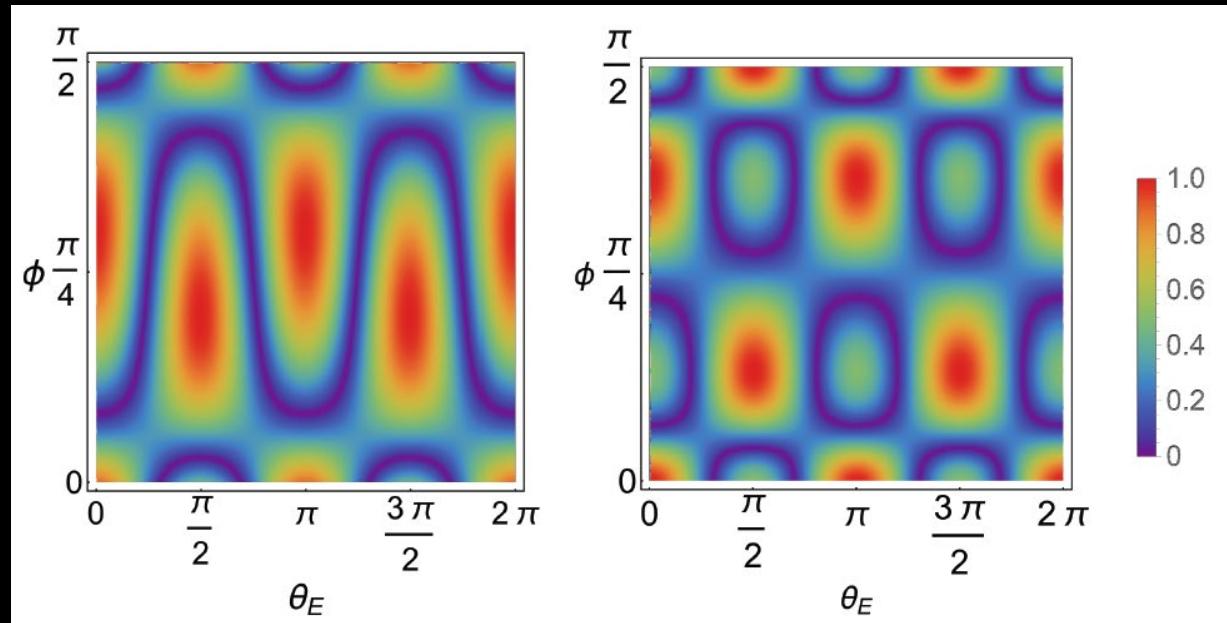
T1 diverges as fast as Hdd goes to zero at DFS: No direct improvement

But the modulating function depends on the gate fields: $G(F_z^a, F_z^b, \phi, \theta_E)$
-Two qubits at isotropic sweet spot: G is maximized and isotropic
-With at least one qubit at the Anisotropic sweet spot
this coupling becomes anisotropic

CONSEQUENCES: TWO-QUBIT MANIPULATION

Two-qubit interaction: Dipole-Dipole interaction

$$H_{dd} \propto \varepsilon_{Zo}^a \varepsilon_{Zo}^b G(F_z^a, F_z^b, \phi, \theta_E) (\sigma_+^1 + \sigma_-^1)(\sigma_+^2 + \sigma_-^2)$$



$$\mathbf{R} = R \cos(\theta_E) \hat{x} + R \sin(\theta_E) \hat{y}$$



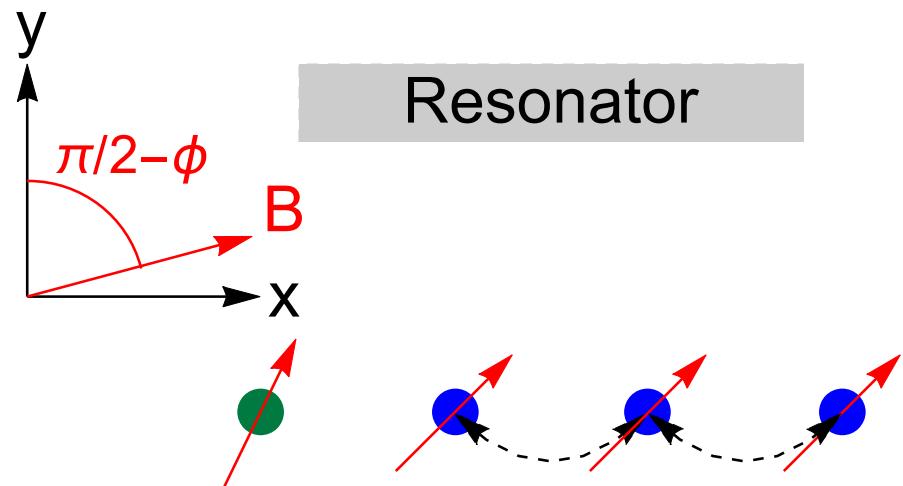
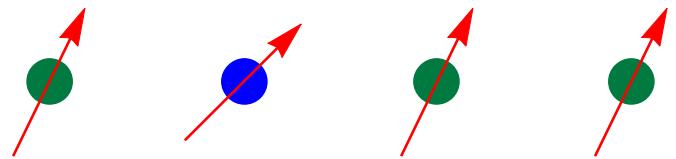
MAGIC ANGLES



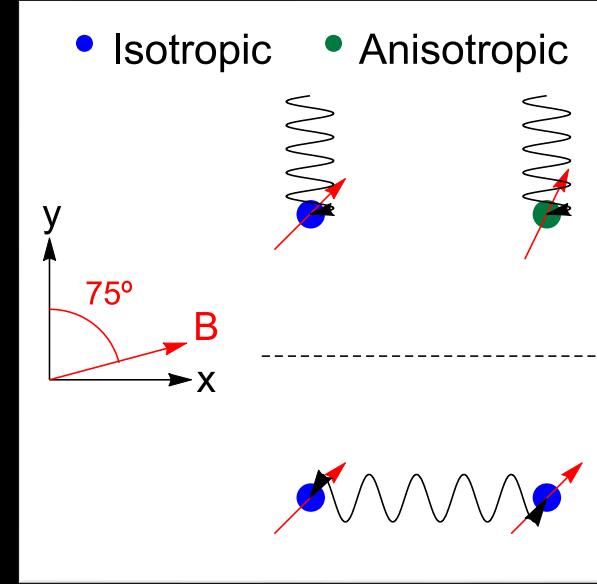
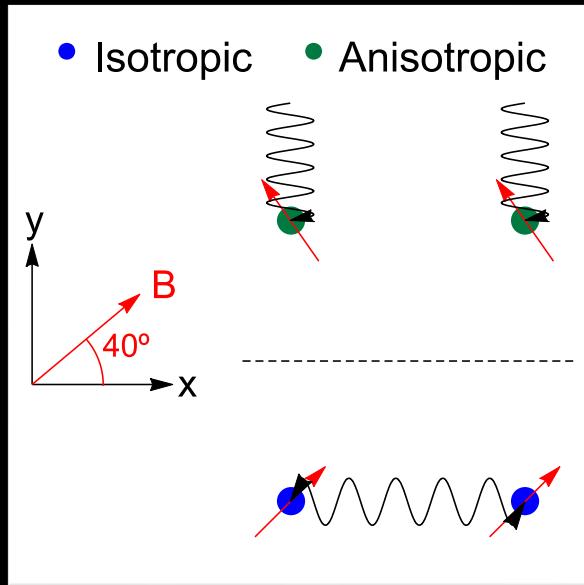
ELECTRICAL CONTROL OF TWO-QUBIT OPERATIONS

- Two-qubit operations are tunable in one direction
- In the other direction cQED can be used
- We devise two different protocols

- Isotropic
- Anisotropic



ELECTRICAL CONTROL OF TWO-QUBIT OPERATIONS



Protocol 1:

$$-\phi = 40^\circ \ (50^\circ)$$

- Single qubit operations in ASS-ASS
- Two qubit operations in ISS-ISS

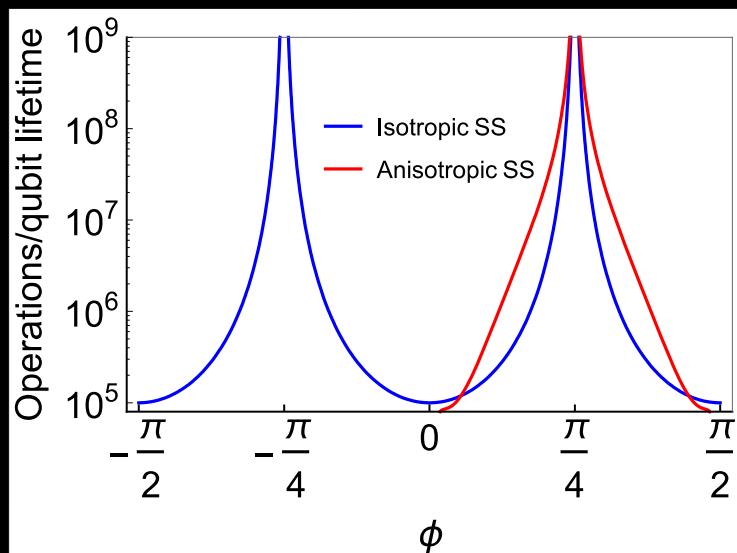
Protocol 2:

$$-\phi = 15^\circ \ (75^\circ)$$

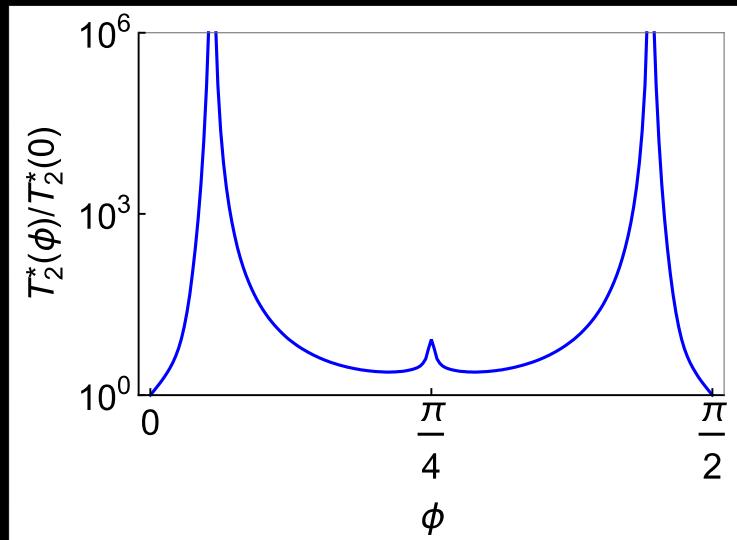
- Single qubit operations in ISS-ASS
- Two qubit operations in ISS-ISS

PROTOCOL 1 VS PROTOCOL 2

- The angles in P1 is near DFS:
Enhanced single-qubit operations



- The angles in P2 implies ISS very close to ASS:
Reduced charge noise exposure during the adiabatic sweep



CONCLUSIONS

Despite the strong SOC acceptor qubits allow fast operations and yet have desirable coherence properties: Holes are coherent!

The lower local symmetry of the acceptor + spin 3/2 physics of the GS give rise to interesting magnetic phenomena:

- Dramatic in-plane g-factor anisotropy
- Decoherence Free Subspace
- Two qubit coupling anisotropy

This makes the in-plane magnetic field orientation an unexpected knob that can be used after to:

- Extend the qubit lifetime
- Modulate the two-qubit couplings by changing gate voltages