MATH 131 Homework 8
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1. **Let** f **and** g **be real-valued functions show** $\min(f,g) = \frac{1}{2}(f+g) - \frac{1}{2}(f+g)$**. Show** $\max(f,g) = -\min(-f,-g)$ **Prove if** f **and** g **are continuous at** $x_0 \in \mathbb{R}$ **then** $\min(f,g)$ **is as well.**

2. **Let** $S \subset \mathbb{R}$ **and suppose there exists a sequence** (x_n) **in S converging to a number** $x_0 \notin S$**. Show there is an unbounded continuout function on** S**.**

3. **Let f and g be real-valued functions show** $\min(f,g) = \frac{1}{2}(f+g) - \frac{1}{2}(f+g)$**.**

4. **Prove** $x = \cos(x)$ **for** $x \in (0, \frac{\pi}{2})$**.**

5. **Suppose** f **is continuous on** $[0,2] \wedge f(0) = f(2)$**. Prove** $\exists x, y \in [0,2]$ **s.t.** $|y-x| = 1 \wedge f(x) = f(y)$**.**

6. **Which of the following functions are uniformly contiuous?**
   (a) $\tan x$ on $(0, \frac{\pi}{4}]$.
   (b) $\tan x$ on $(0, \frac{\pi}{2})$.
   (c) $\frac{1}{2}\sin^2 x$ on $(0, \pi]$.
   (d) $\frac{1}{x-3}$ on $(0,3)$.
   (e) $\frac{1}{x-3}$ on $(3,\infty)$.
   (f) $\frac{1}{x-3}$ on $(4,\infty)$.

7. **Let** f **be continuous on** $[a,b]$**. Show that** $f* = \sup\{f(y) : a \leq y \leq x\}$ **for** $x \in [a,b]$ **is an increasing continuous function.**

8. **Find the following limits**
   (a) $\frac{1}{x-3}$ on $(0,3)$.
   (b) $\frac{1}{x-3}$ on $(3,\infty)$.
   (c) $\frac{1}{x-3}$ on $(4,\infty)$.

9. **Let** $f(x) = \frac{\sqrt{1+3x^2}-1}{x^2}$**. Find** $\lim_{x\to 0} f(x)$**.**