MATH 131 Homework 3
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1. **Let** $a, b \in R$. **Show if** $a \leq b_1$ **for every** $b_1 > b$**, then** $a \leq b$**.**

   Suppose $a > b$. Then by the denseness of $\mathbb{Q}$ (Thm 4.7) $\exists b_1 : a > b_1 > b$ but this is a contradiction, as we said $a \leq b_1$ for every $b_1 > b$. Therefore $a \leq b$ if $a \leq b_1$ for every $b_1 > b$.

2. **Prove that for any** $A, B \subset \mathbb{R} : \sup(A \cup B) = \max\{\sup(A), \sup(B)\}$**.**

   WLOG Suppose $\sup(A) > \sup(B)$ then $\forall b \in B : \sup(B) > b \implies \forall b \in B : \sup(A) > \sup(B) > b$.

   So $\forall x \in A \cup B : \sup(A) > x \implies \sup(A)$ is an upper bound on $A \cup B$. Now we will prove that $\sup(A)$ is the least upper bound.

   Suppose $\exists x : x$ is a upper bound $\wedge\ x < \sup A$. But then $x < \sup(A) \wedge \forall a \in A : x > a$. But this is a contradiction, as by definition $\sup(A)$ is the least upper bound. So therefore $\sup(A) = \sup(A \cup B)$.

   Note when $\sup(A) = \sup(B)$ either choice satisfies max.

3. **Determine** $\lim s_n$ **where** $s_n = \sqrt{n^2 + 1} - n$

4. **Find** $\lim \frac{4n+3}{7n-5}$

   Claim: $\lim \frac{4n+3}{7n-5} = \frac{4}{7}$

   Fix $k \in \mathbb{N}$. Then take $n_0 = \frac{4k}{7}$

   If $n \geq n_0$ then

   $|\frac{4n+3}{7n-5} - \frac{4}{7}| = \frac{1}{k+1}$

5. **Determine if** $\lim_{n \to \inf}$

6. **Determine if** $\lim_{n \to \inf}$

7. **Determine if** $\lim_{n \to \inf}$

8. **Determine if** $\lim_{n \to \inf}$