MATH 131 Homework 4
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1. **Prove** $\exists N : n > N \implies s_n > a$

   Let $\lim s_n = L$. Then $\forall k \in \mathbb{N}\, \exists N \in \mathbb{N}\, \forall n \in \mathbb{N} : n > N \implies |s_n - L| < \frac{1}{k+1}$.

   Take $k = \lceil a \rceil$. Then $\exists N \in \mathbb{N}\, \forall n \in \mathbb{N} : n > N \implies |s_n - L| < \frac{1}{a+1} \implies -\frac{1}{a+1} < s_n - L < \frac{1}{a+1}$.

   Since $L > a \implies -\frac{1}{a+1} < s_n - a < \frac{1}{a+1}$.

   Taking only the LHS and adding a, we get $a - \frac{1}{a+1} < s_n$. But $a - \frac{1}{a+1} > a \implies a < s_n \implies \exists N \in \mathbb{N}\, \forall n \in \mathbb{N} : n > N \implies s_n > a$.

2. **TODO**

3. **Show** $\lim \frac{a^n}{n!} = 0 \forall a \in \mathbb{R}$

   Fix $a \in \mathbb{R}$. Consider $\lim |\frac{s_{n+1}}{s_n}|$.

$$\frac{s_{n+1}}{s_n} = \frac{a^{n+1}}{(n+1)!}\left(\frac{n!}{a^n}\right) = \frac{a}{n}$$

   Then $\lim |\frac{s_{n+1}}{s_n}| = \lim |\frac{a}{n}| = |a| \lim \frac{1}{n} = |a|0 = 0 < 1$.

   Then by 9.12a we get $\lim \frac{a^n}{n!} = 0$. Since we did not specify a particular a, this holds $\forall a \in \mathbb{R}$.

4.

5.