MATH 131 Homework 3
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1. **Let $a, b \in R$. Show if $a \leq b_1$ for every $b_1 > b$, then $a \leq b$.**

   Suppose $a > b$. Then by the denseness of $\mathbb{Q}$ (Thm 4.7) $\exists b_1 : a > b_1 > b$ but this is a contradiction, as we said $a \leq b_1$ for every $b_1 > b$. Therefore $a \leq b$ if $a \leq b_1$ for every $b_1 > b$.

2. **Prove that for any $A, B \subset \mathbb{R} : \sup(A \cup B) = \max\{\sup(A), \sup(B)\}$.**

   WLOG Suppose $\sup(A) > \sup(B)$ then $\forall b \in B : \sup(B) > b \implies \forall b \in B : \sup(A) > \sup(B) > b$.

   So $\forall x \in A \cup B : \sup(A) > x \implies \sup(A)$ is an upper bound on $A \cup B$. Now we will prove that $\sup(A)$ is the least upper bound.

   Suppose $\exists x : x$ is a upper bound $\wedge\, x < \sup A$. But then $x < \sup(A) \wedge \forall a \in A : x > a$. But this is a contradiction, as by definition $\sup(A)$ is the least upper bound. So therefore $\sup(A) = \sup(A \cup B)$.

   Note when $\sup(A) = \sup(B)$ either choice satisfies max.

3. **Prove every nonempty set $F \subset P(E)$ admits $\sup F$ and $\inf F$ and that $\sup(F) = \cap F$** Define $A \geq B$ if $A \subset B$ and $A \leq B$ if $B \subset A$.

   Let $F \subset P(E)$. Then take $\forall f \in F : \cap F \subset f \implies \cap F > f$ so $\cap F$ is an uppper bound. Suppose $\exists x \in P(E) : (\forall f \in F : x > f) x < \cap F$. But then $\forall f \in F$

4. **Determine $\lim s_n$ where $s_n = \sqrt{n^2+1} - n$**

   Claim: $\lim s_n = 0$

   Proof: Fix $k \in \mathbb{N}$. Then take $n_0 = \sqrt{k^2 + 2k}$. Then $|\sqrt{n^2+1} - n| < |\sqrt{n^2+1} - n|$

5. **Find $\lim \frac{4n+3}{7n-5}$**

   Claim: $\lim \frac{4n+3}{7n-5} = \frac{4}{7}$

   Proof: Fix $k \in \mathbb{N}$. Then take $n_0 = \max(5, 1+k)$

   If $n \geq n_0$ then $|\frac{4n+3}{7n-5} - \frac{4}{7}| = \frac{7(4n+3)-4(7n-5)}{7(7n-5)} = \frac{41}{7(7n-5)}$ but since $n \geq 5 \implies \frac{41}{7(7n-5)} \leq \frac{1}{1+k}$. So this is indeed a limit.

6. **Determine if $\lim_{n\to\inf}$**

7. **Determine if $\lim_{n\to\inf}$**

8. **Find $\lim \sqrt[3]{n^3+n^2+1} - \sqrt[3]{n^3+1}$**

   Claim: This limit is 0

   Proof: Fix k. Then take $n_0 =$

9. **Determine if $\lim_{n\to\inf}$**