Jesse Cai
MATH 164 Homework 3
304634445

15.1 Minimize $f(x_1, x_2) = -2x_1 - x_2$ such that

$x_1 - s_1 = 2$

$x_1 + x_2 + s_2 = 3$

$x_1 + 2x_2 + s_3 = 5$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

15.3 We can rewrite the original problem as follows

Minimize $\sum_{i=1}^{n} c_i(x_i^+ - x_i^-)$ such that

$$A(x^+ - x^-) = b, x^+, x^- \geq 0$$

However for it to be true standard form we'll need to rewrite A and c as $A' = [A, -A], c' = [c, c]$.

15.5 Let number of units shipped be x_1, x_2, x_3, x_4 respectively for AC, AD. BC, BD.

Minimize $f(x) = x_1 + 2x_2 + 3x_3 + 4x_4$ s.t.

$x_1 + x_2 + s_1 = 70$ $x_3 + x_4 + s_2 = 80$ $x_1 + x_3 = 50$ $x_2 + x_4 = 60$

15.7 Let x_i be the weight of item i used.

Then our goal is to minimize the total cost $f(x) = 2x_1 + 4x_2 + x_3 + 2x_4$ such that.

$x_1 + x_2 + x_3 + x_4 = 1000$

$3x_1 + 8x_2 + 16x_3 + 4x_4 = 10000$

$6x_1 + 46x_2 + 9x_3 + 9x_4 = 2000$

$20x_1 + 5x_2 + 4x_3 = 5000$

However the only solution is $\begin{bmatrix} 179 \\ -175 \\ 573 \\ 422 \end{bmatrix}$ which is infeasible, so there is no solution.

15.9 The matrix has full rank 3, so there are $\binom{5}{3} = 10$ basic solutions.

They are as follows:

$x^*_{1,2,3} = \frac{1}{17}\begin{bmatrix} -4 & -80 & 83 & 0 & 0 \end{bmatrix}$

$x^*_{1,2,4} = \begin{bmatrix} -10 & 49 & 0 & -83 & 0 \end{bmatrix}$

$x^*_{1,2,5} = \frac{1}{31}\begin{bmatrix} 105 & 25 & 0 & 0 & 83 \end{bmatrix}$

$x^*_{1,3,4} = \frac{1}{11}\begin{bmatrix} -12 & 0 & 49 & -80 & 0 \end{bmatrix}$

$x^*_{1,3,5} = \frac{1}{35}\begin{bmatrix} 100 & 0 & 25 & 0 & 80 \end{bmatrix}$

$x^*_{1,4,5} = \frac{1}{18}\begin{bmatrix} 65 & 0 & 0 & 25 & 49 \end{bmatrix}$

$x^*_{2,3,4} = \begin{bmatrix} 0 & -6 & 5 & 2 & 0 \end{bmatrix}$

$x^*_{2,3,5} = \frac{1}{23}\begin{bmatrix} 0 & -100 & 105 & 0 & 4 \end{bmatrix}$

$x^*_{2,4,5} = \begin{bmatrix} 0 & 13 & 0 & -21 & 2 \end{bmatrix}$

$x^*_{3,4,5} = \frac{1}{19}\begin{bmatrix} 0 & 0 & 65 & -100 & 12 \end{bmatrix}$

15.10 TODO

16.2 (a)

$$A = \begin{bmatrix} 3 & 1 & 0 & 1 \\ 6 & 2 & 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 5 \end{bmatrix}, c = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

(b) The tableau is given by

$$\begin{bmatrix} 3 & 1 & 0 & 1 & 4 \\ 6 & 2 & 1 & 1 & 5 \\ 2 & -1 & -1 & 0 & 0 \end{bmatrix}$$

we need to pivot about $(2,3)$ and $(1,4)$ and to get

$$\begin{bmatrix} 3 & 1 & 0 & 1 & 4 \\ 3 & 1 & 1 & 0 & 1 \\ 5 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) The basic feasible solution corresponding to this is $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}$ with cost -1.

(d) The coefficients are $\begin{bmatrix} 5 & 0 & 0 & 0 \end{bmatrix}$.

(e) Yes it is optimal, as all the reduced row coefficients are positive.

(f) Yes, there is a basic feasible solution.

(g) TODO

16.3 The tableau for this problem is given by

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ -1 & -1 & -3 & 0 \end{bmatrix}$$

By $R_3 + R_2 + R_1 \to R_3$ we can get the cannonical form.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -1 & 3 \end{bmatrix}$$

And then we pivot about -1 by $R_1 + R_3 \to R_3; R_2 - R_1 \to R_1$.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 5 \end{bmatrix}$$

Then the solution given is $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ for a total cost of 5.

16.4 We know there will be 3 slack variables because we have three inequalities. The tableau for this problem is given by

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 & 0 & 7 \\ 1 & 1 & 0 & 0 & 1 & 9 \\ -2 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

There's a negative reduced cost coefficient, so we pivot along $(1,1)$ via $R_3 - R_1 \to R_3$ and $R_4 + 2R_1 \to R_4$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 & 0 & 7 \\ 0 & 1 & -1 & 0 & 1 & 4 \\ 0 & -1 & 2 & 0 & 0 & 10 \end{bmatrix}$$

Again we pivot along $(3,2)$ via $R_2 + R_4 \to R_2$ and $R_4 + R_3 \to R_4$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 2 & 1 & 0 & 17 \\ 0 & 1 & -1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & 1 & 14 \end{bmatrix}$$

Now our reduced row coefficients are all positive so our solution is $x = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ for a total cost of 14.

16.5 (a) TODO

16.6 TODO

16.8 (a) $\begin{bmatrix} 6 \\ 0 \\ 7 \\ 5 \\ 0 \end{bmatrix}$ is the basic feasible solution for this tableau, with $f(x) = 8$

(b) $\begin{bmatrix} 0 & 4 & 0 & 0 & -4 \end{bmatrix}$

(c) Yes, as we can get any negative value since the last column is all negative.

(d) We would neeed to pivot about $(3, 2)$ to get

$$\begin{bmatrix} 0 & 0 & -\frac{1}{3} & 1 & 0 & \frac{8}{3} \\ 1 & 0 & -\frac{2}{3} & 0 & 0 & \frac{4}{3} \\ 0 & 1 & \frac{1}{3} & 0 & -1 & \frac{7}{3} \\ 0 & 0 & -\frac{4}{3} & 0 & 0 & -\frac{4}{3} \end{bmatrix}$$

(e) TODO

(f) Note from the tableau $a_2 = a_4 + 2a_1 + 3a_3$ and $a_5 = -a_4 - 2a_1 - 3a_3$ so therefore $\begin{bmatrix} 2 \\ -1 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -3 \\ -1 \\ -1 \end{bmatrix} \in$ $kernel(A)$. By RN theorem we know that $nullity(A) = 2$ Since these two vectors are linearly independent, they forma basis for the kernel of A.

16.9

16.10

16.11

16.12

16.14