Preliminary Examination

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prelim exam

Dirac equation and charge conjugation

$$\mathcal{L} = \frac{i}{2}\left[\,\overline{\psi}\gamma^{\mu}\left(\partial_{\mu}\psi\right) - \left(\partial_{\mu}\overline{\psi}\right)\gamma^{\mu}\psi\right] - m\overline{\psi}\psi$$

$$(i\partial\!\!\!/ - m)\,\psi = 0$$

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$$

$$e^{-} \iff e^{+}$$

$$\hat{\psi} \to \hat{\psi}^{C} = C\gamma^{0}\hat{\psi}^{*}$$

$$C = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

The new quantum mechanics, when applied to the problem of the structure of the atom with point-charge electrons, does not give results in agreement with experiment. The discrepancies consist of " duplexity " phenomena, the observed number of stationary states for an electron in an atom being twice the number given by the theory. To meet the difficulty, Goudsmit and Uhlenbeck have introduced the idea of an electron with a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton. This model for the electron has been fitted into the new mechanics by Pauli,* and Darwin,† working with an equivalent theory, has shown that it gives results in agreement

Majorana fermions and neutrinos

- Majorana fermions can be seen as solutions to the Dirac equation with an extra constraint
- $\psi = \psi^C$
- implies these fermions are **chargeless**, and their **own anti-particles**

The left and right handed components of the spinor are no longer independent

$$\Psi_{majorana} = \psi_L + \psi_R$$

mass terms:

$$\left(\overline{\hat{\psi}^C}_L \hat{\psi}_L + \overline{\hat{\psi}}_L \hat{\psi}_L^C\right)_{Majorana}$$

$$\left(\overline{\hat{\psi}}_L \hat{\psi}_R + \overline{\hat{\psi}}_R \hat{\psi}_L\right)_{Dirac}$$

why is it still unknown if neutrinos are Dirac or Majorana fermions?

- Both types of fermions are compatible with observed features of neutrinos, *including* neutrino oscillations. majorana neutrinos and their anti-matter partners are only distinguishable by their chirality.
- In the ultra-relativistic limit, helicity and chirality (lepton number). we can never boost into a frame where the neutrino has right handed helicity. (does the goldhaber experiment measure helicity or chirality? chirality is the sign of the phase of the wavefunction for the dirac fermion. it describes which path it takes in the complex plane.
- expand the fermionic propagator in the limit p¡¡m, and the

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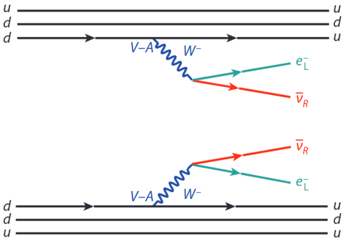
$$\left(\overline{\hat{\psi}}_L \hat{\psi}_R + \overline{\hat{\psi}}_R \hat{\psi}_L\right)_{Dirac}$$

SM and BSM double-beta decay

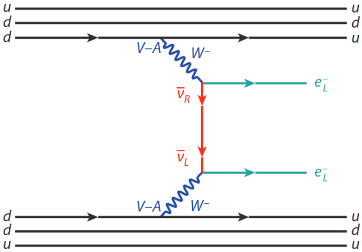
$2\nu\beta\beta$: the neutrinos are Dirac fermions, and have distinct anti-particles ν, $\overline{\nu}$



extremely rare process, observed in direct detection experiments (CUORE-0, GERDA, XENON, EXO, NEMO)

$T_{1/2}^{2\nu\beta\beta} \sim 10^{19} - 10^{24} yr$

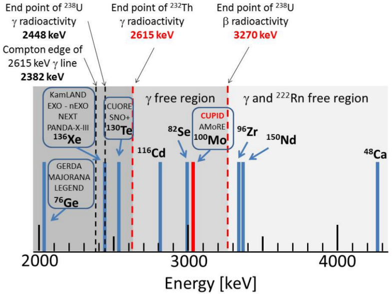
$0\nu\beta\beta$: the neutrinos are Majorana fermions and are absorbed as virtual particles

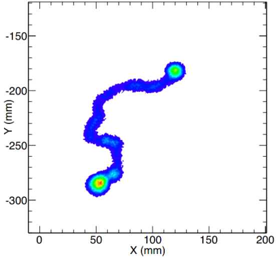


Never before observed in any experiment: $T_{1/2}^{0\nu\beta\beta} > 10^{26} yr$ (Lepton number violation)

experimental signature and detection

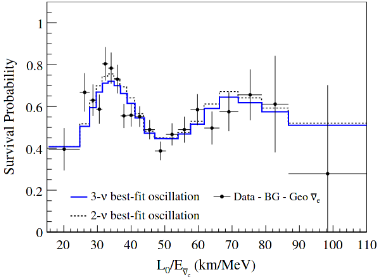
- mono-energetic peak with 2 electrons leaving the reaction.
- tomography of electron tracks can help reduced backgrounds, except for the $2\nu\beta\beta$ irreducible background
- experimental goal is to measure $T_{1/2}^{0\nu\beta\beta}$ to constrain the effective neutrino mass $m_{\beta\beta}$.
- this measurement complements measurements of Σ and m_β from cosmology and kinematic experiments, respectively.





neutrino mass and oscillations

- Homestake Mine and the solar neutrino anomaly
  $^{37}Cl + \nu_e \rightarrow \; ^{37}Ar + e^-$
  observes factor of 3 discrepancy in ν_e flux.
- Sudbury Neutrino Observatory (SNO) measures total neutrino flux: ν_μ, ν_τ account for ν_e deficit.
- KAMLAND, a reactor neutrino experiment shows survival probability of electron neutrinos
  $P = 1 - sin^2(2\theta)sin^2(\Delta m^2 L/4E)$

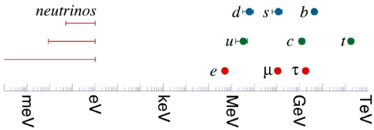


mass scale

- Physicists believed neutrinos to be massless for decades, due to Standard Model predictions (Higgs mechanism for leptons) and experimental data

- Neutrinos are orders of magnitude lighter than any other fundamental particle. This in addition to them being chargeless, makes it hard to measure their presence in experiments.

- The common model for describing BSM neutrino mass states is the light-neutrino mixing model (3 mass eigenstates)



neutrino mixing

A given flavor eigenstate is a linear combination of mass eigenstates (PMNS-matrix)

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$$\times \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

parameters: angles $\theta_{12}, \theta_{13}, \theta_{23}$ CP-violating phases $\delta_{CP}, \alpha_1, \alpha_2$, and neutrino masses m_1, m_2, m_3

mass hierarchy

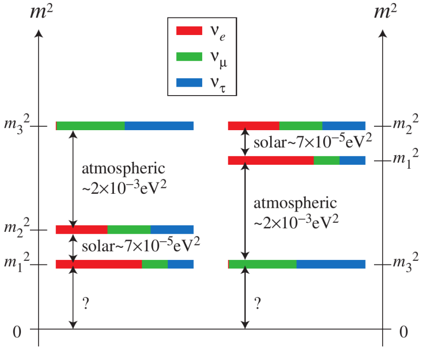
Neutrino oscillation experiments are sensitive to $\Delta m_{ij}^2 = m_i^2 - m_j^2$ This, coupled with the neutrinos' very small mass scale, leads to ambiguity in their ordering.

normal ordering

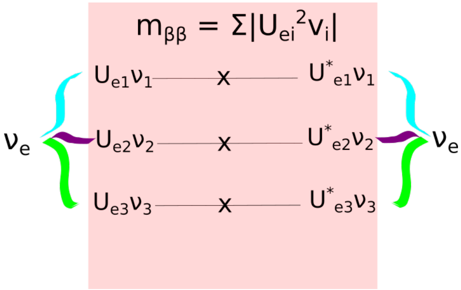
$\implies m_3^2 > m_2^2 > m_1^2$

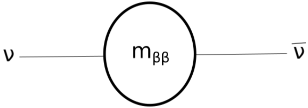
inverted ordering

$\implies m_2^2 > m_1^2 > m_3^2$

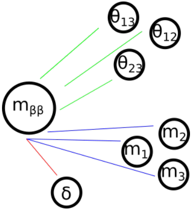


effective Majorana neutrino mass





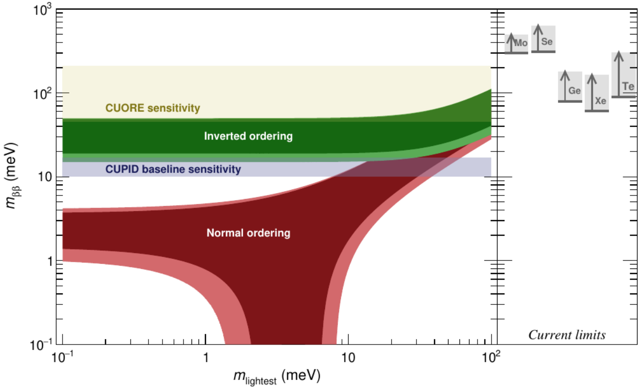
$m_{\beta\beta}\bar{\psi}\psi$



zero neutrino half-life

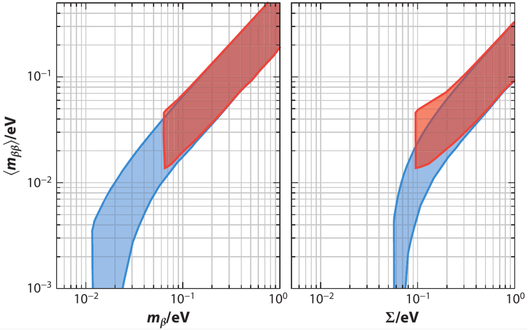
$$T_{1/2}^{0\nu\beta\beta} = \left(G|\mathcal{M}|^2\langle m_{\beta\beta}\rangle\right)^{-1}$$

$$\langle m_{\beta\beta}\rangle = \Sigma|U_{ej}^2 m_j|$$



complementary limits

- Limits on the effective $0\nu\beta\beta$ halflife are also constrained by kinematic and cosmological searches
- $m_\beta = \sqrt{U_{ei}^2 m_i^2}$ and $\Sigma = m_1 + m_2 + m_3$
- $m_\beta < 1.1\text{eV}$ $\Sigma < 0.3\text{eV}$



inverted normal

designing a direct-detection experiment

otope	$Q_{\beta\beta}$ (keV)	I.A. (%)	$G^{0\nu}$	$H^{0\nu}$
Ca	4272	0.187	24.81	826.2
Ge	2039	7.8	2.36	49.6
Se	2995	8.73	10.16	198.1
Zr	3350	2.8	20.58	342.7
Mo	3034	9.63	15.92	254.5
Pd	2018	11.72	4.82	70.0
Cd	2814	7.49	16.70	230.1
Sn	2287	5.79	9.04	116.5
Te	866	31.69	0.59	7.4
Te	2527	33.8	14.22	174.8
Xe	2458	8.9	14.58	171.4
Nd	1929	5.76	10.10	109.1
Nd	3371	5.64	63.03	671.7
Sm	1215	22.7	3.02	31.3
Gd	1730	21.86	9.56	95.5
Pt	1047	7.2	7.56	61.0

$$Rate\left(\mathcal{O}_{majorana}\right) \propto \frac{m_\nu^2}{Q_{\beta\beta}}$$

$$T_{1/2}^{0\nu\beta\beta} = \left(G|\mathcal{M}|^2\langle m_{\beta\beta}\rangle\right)^{-1}$$

$$\simeq 10^{27-28}\left(\frac{0.01\ eV}{\langle m_{\beta\beta}\rangle}\right)^2 \text{years}$$

$$F_{0\nu} = \tau_{1/2}^{bkg.fluct.} = \ln(2) N_{\beta\beta}\epsilon\frac{t}{n_B}$$

far reaching consequences of observing $0\nu\beta\beta$

- baryogenesis through leptogenesis
- charge parity violating process in the standard model
- lepton number conservation (accidental symmetry of the standard model)

$$m_\nu = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T$$

$$\langle m_{\beta\beta} \rangle = |U_{ij}^2 m_j|$$

$0\nu\beta\beta$ half-life and sensitivity

- direct detection experiments typically characterize the half-life in terms of the effective neutrino mass $m_{\beta\beta}$

$$T_{1/2}^{0\nu\beta\beta} = \left(G|\mathcal{M}|^2\langle m_{\beta\beta}\rangle\right)^{-1} \simeq 10^{27-28}\left(\frac{0.01\ eV}{\langle m_{\beta\beta}\rangle}\right)^2 \text{years}$$

$$F_{0\nu} = \tau_{1/2}^{bkg.fluct.} = \ln(2) N_{\beta\beta}\epsilon\frac{t}{n_B}$$

$0\nu\beta\beta$ half-life and sensitivity

- to probe

$$T_{1/2}^{0\nu\beta\beta} = \left(G|\mathcal{M}|^2\langle m_{\beta\beta}\rangle\right)^{-1}$$
$$\simeq 10^{27-28}\left(\frac{0.01\ eV}{\langle m_{\beta\beta}\rangle}\right)^2 \text{years}$$

$$T_{1/2}^{0\nu\beta\beta} \sim \ \epsilon\sqrt{\frac{M_{iso}t}{b\Delta E}}$$