Preliminary Examination

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prelim exam

Dirac equation and charge conjugation

$$\mathcal{L} = \frac{i}{2} \left[\, \overline{\psi} \gamma^\mu \left(\partial_\mu \psi \right) \right.$$

$$\left. - \left(\partial_\mu \overline{\psi} \right) \gamma^\mu \psi \right] - \boxed{m \overline{\psi} \psi}$$

$$\left(i \slashed{\partial} - m \right) \psi = 0$$

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$$

The Quantum Theory of the Electron.

By P. A. M. DIRAC, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

The new quantum mechanics, when applied to the problem of the structure of the atom with point-charge electrons, does not give results in agreement with experiment. The discrepancies consist of "duplexity" phenomena, the observed number of stationary states for an electron in an atom being twice the number given by the theory. To meet the difficulty, Goudsmit and Uhlenbeck have introduced the idea of an electron with a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton. This model for the electron has been fitted into the new mechanics by Pauli,* and Darwin,† working with an equivalent theory, has shown that it gives results in agreement

$$e^- \Longleftrightarrow e^+$$

$$\hat{\psi} \to \hat{\psi}^C = C \gamma^0 \hat{\psi}^*$$

$$C = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

Majorana fermions and neutrinos

The left and right handed components
of the spinor are no longer independent

- Majorana fermions can be seen
  as solutions to the Dirac
  equation with an extra
  constraint

$$\Psi_{majorana} = \psi_L + \psi_R$$

- $\psi = \psi^C$
- implies these fermions are
  chargeless, and their **own
  anti-particles**

mass terms:

$$\left(\overline{\hat{\psi^C}}_L \hat{\psi}_L + \overline{\hat{\psi}}_L \hat{\psi}_L^C\right)_{Majorana}$$

$$\left(\overline{\hat{\psi}}_L \hat{\psi}_R + \overline{\hat{\psi}}_R \hat{\psi}_L\right)_{Dirac}$$

why is it still unknown if neutrinos are Dirac or Majorana fermions?

- Both types of fermions are compatible with observed features of neutrinos, *including* neutrino oscillations. majorana neutrinos and their anti-matter partners are only distinguishable by their chirality.

- In the ultra-relativistic limit, helicity and chirality (lepton number). we can never boost into a frame where the neutrino has right handed helicity. (does the goldhaber experiment measure helicity or chirality? chirality is the sign of the phase of the wavefunction for the dirac fermion. it describes which path it takes in the complex plane.

- expand the fermionic propagator in the limit p ≫ m, and the

The left and right handed components of the spinor are no longer independent
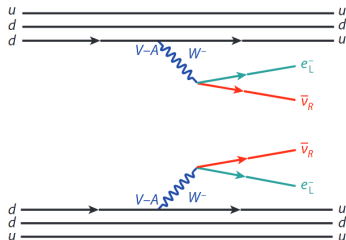
$$\Psi_{majorana} = \psi_L + \psi_R$$

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$$\left(\overline{\hat{\psi}}_L \hat{\psi}_R + \overline{\hat{\psi}}_R \hat{\psi}_L \right)_{Dirac}$$

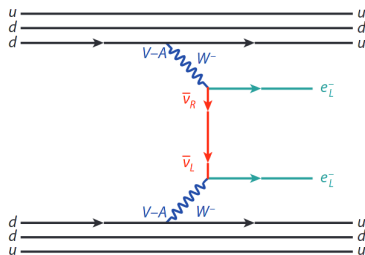4 / 24

SM and BSM double-beta decay

$2\nu\beta\beta$: the neutrinos are Dirac fermions, and have distinct anti-particles ν, $\overline{\nu}$

extremely rare process, observed in direct detection experiments (CUORE-0, GERDA, XENON, EXO, NEMO)
$T_{1/2}^{2\nu\beta\beta} \sim 10^{19} - 10^{24} yr$

$0\nu\beta\beta$: the neutrinos are Majorana fermions and are absorbed as virtual particles

Never before observed in any experiment: $T_{1/2}^{0\nu\beta\beta} > 10^{26} yr$ (Lepton number violation)

experimental signature and detection

- mono-energetic peak with 2 electrons leaving the reaction.

- tomography of electron tracks can help reduced backgrounds, except for the $2\nu\beta\beta$ irreducible background

- experimental goal is to measure $T_{1/2}^{0\nu\beta\beta}$ to constrain the effective neutrino mass $m_{\beta\beta}$.

- this measurement complements measurements of Σ and m_β from cosmology and kinematic experiments, respectively.
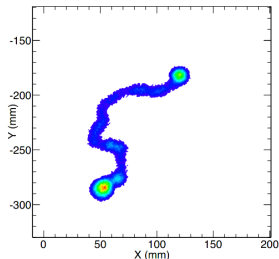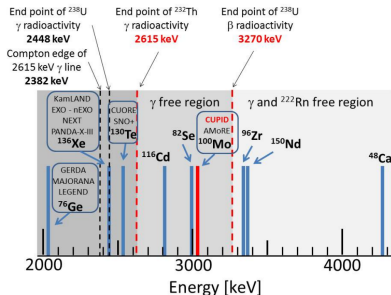
neutrino mass and oscillations

- Homestake Mine and the solar neutrino anomaly
  $^{37}Cl + \nu_e \rightarrow \, ^{37}Ar + e^-$
  observes factor of 3 discrepancy in ν_e flux.

- Sudbury Neutrino Observatory (SNO) measures total neutrino flux: ν_μ, ν_τ account for ν_e deficit.

- KAMLAND, a reactor neutrino experiment shows survival probability of electron neutrinos
  $P = 1 - sin^2(2\theta)sin^2(\Delta m^2 L/4E)$
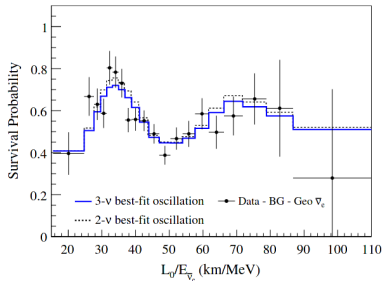
neutrino mass and the β-spectrum

- Tritium (^3H) is a viable isotope for directly probing the neutrino mass - low Q value (decays in region $E_0 - E$ scale with $(E_0 - E)^3/Q^3$ and short-life to reduce line broadening......?

- Measure deviation from the mass-less spectrum's linearity at large energies.

- This method is most-direct and model independent probe of the neutrino's mass. Constructing a sensitive experiment is very challenging (KATRIN may be last of its kind)
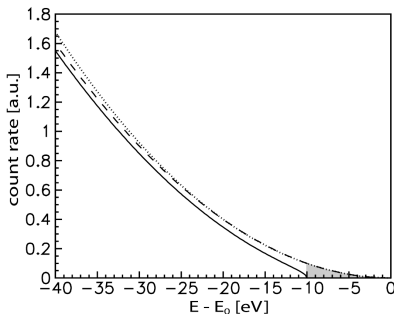
mass scale

- Physicists believed neutrinos to be massless for decades, due to Standard Model predictions (Higgs mechanism for leptons) and experimental data

- Neutrinos are orders of magnitude lighter than any other fundamental particle. This in addition to them being chargeless, makes it hard to measure their presence in experiments.

- The common model for describing BSM neutrino mass states is the light-neutrino mixing model (3 mass eigenstates)
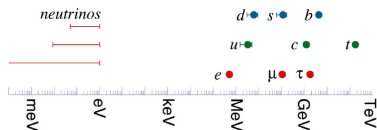
neutrino mixing

A given flavor eigenstate is a
linear combination of mass
eigenstates (PMNS-matrix)

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

parameters: angles
$\theta_{12}, \theta_{13}, \theta_{23}$ CP-violating
phases $\delta_{CP}, \alpha_1, \alpha_2$, and
neutrino masses m_1, m_2, m_3

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$$\times \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
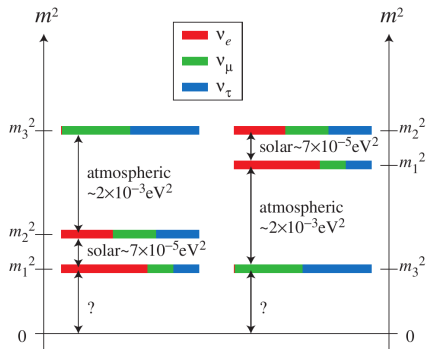
mass hierarchy

Neutrino oscillation experiments are sensitive to $\Delta m_{ij}^2 = m_i^2 - m_j^2$ This, coupled with the neutrinos' very small mass scale, leads to ambiguity in their ordering.
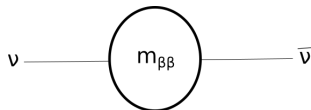**normal ordering**
$\implies m_3^2 > m_2^2 > m_1^2$
inverted ordering
$\implies m_2^2 > m_1^2 > m_3^2$

effective Majorana neutrino mass
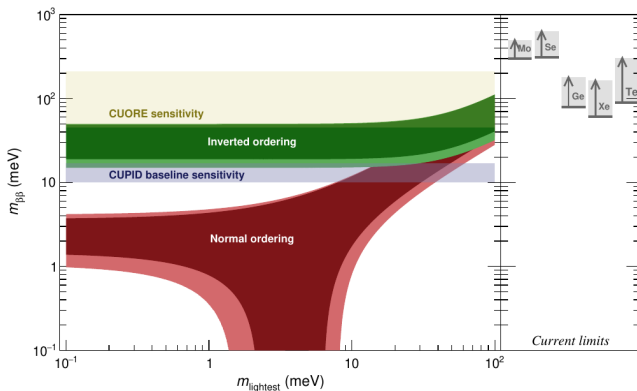

$$m_{\beta\beta} = \Sigma |U_{ei}^2 v_i|$$
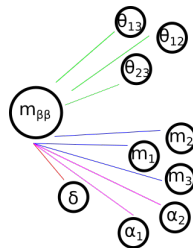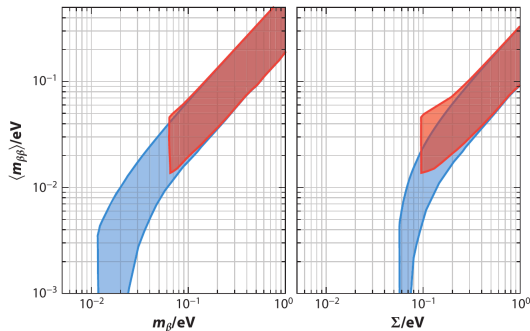
zero neutrino half-life

$$T_{1/2}^{0\nu\beta\beta} = \left(G|\mathcal{M}|^2\langle m_{\beta\beta}\rangle\right)^{-1}$$

$$\langle m_{\beta\beta}\rangle = \Sigma|U_{ej}^2 m_j|$$

complementary limits

- Limits on the effective $0\nu\beta\beta$ halflife are also constrained by kinematic and cosmological searches

- $m_\beta = \sqrt{U_{ei}^2 m_i^2}$ and $\Sigma = m_1 + m_2 + m_3$

- $m_\beta < 1.1 \text{eV}$
  $\Sigma < 0.3 \text{eV}$

inverted normal

designing a direct-detection experiment

| Isotope	$Q_{\beta\beta}$ (keV)	I.A. (%)	$G^{0\nu}$	$H^{0\nu}$
^{48}Ca	4272	0.187	24.81	826.2
^{76}Ge	2039	7.8	2.36	49.6
^{82}Se	2995	8.73	10.16	198.1
^{96}Zr	3350	2.8	20.58	342.7
^{100}Mo	3034	9.63	15.92	254.5
^{110}Pd	2018	11.72	4.82	70.0
^{116}Cd	2814	7.49	16.70	230.1
^{124}Sn	2287	5.79	9.04	116.5
^{128}Te	866	31.69	0.59	7.4
^{130}Te	2527	33.8	14.22	174.8
^{136}Xe	2458	8.9	14.58	171.4
^{148}Nd	1929	5.76	10.10	109.1
^{150}Nd	3371	5.64	63.03	671.7
^{154}Sm	1215	22.7	3.02	31.3
^{160}Gd	1730	21.86	9.56	95.5
^{198}Pt	1047	7.2	7.56	61.0

$$T_{1/2}^{0\nu\beta\beta} = \left(G|\mathcal{M}|^2\langle m_{\beta\beta}\rangle\right)^{-1}$$

$$F_{0\nu} = \tau_{1/2}^{bkg.fluct.} = \ln(2)N_{\beta\beta}\epsilon\frac{t}{n_{bkg}}$$

$$F_{0\nu} = \ln(2) \times \frac{N_A}{W}\left[a\epsilon\sqrt{\frac{Mt}{b\Delta}}\right]$$

$$Rate\,(\mathcal{O}_{majorana}) \propto \frac{m_\nu^2}{Q_{\beta\beta}}$$

backgrounds

- Primordial radioisotopes: ^{238}U and ^{232}Th generate chain of β, γ near $Q_{\beta\beta}$.
- ^{222}Rn can exist in gaseous state produces $\beta + \gamma$ impinging on $Q_{\beta\beta}$
- ^{208}Tl emits γ at 2615 keV, affecting a number of candidate source isotopes.
- the intrinsically irreducible background $2\nu\beta\beta$: $\Gamma_{2\nu\beta\beta}$ is negligible in some isotopes (^{130}Te), but not others (^{100}Mo)
- muons enter the backgrounds in a number of ways: prompt signal + long-lived isotope activation + neutron production

Next Generation 0νββ experiments
goal: probe 0νββ to a half life of 10^{28} *years*
requirement: $B.I. =$ 0.1 *counts* / (*FWHM · tonne · year*)
CUPID conservative goal: $B.I. =$ 0.5 *counts* / (*FWHM · tonne · year*)

CUORE experiment

- Array of 988 TeO_2 bolometers instrumented with NTD sensors.
- CUORE cryostat maintains about 200kg of ^{130}Te around 10mK.
- ^{208}Tl emits γ at 2615 keV, affecting a number of candidate source isotopes.
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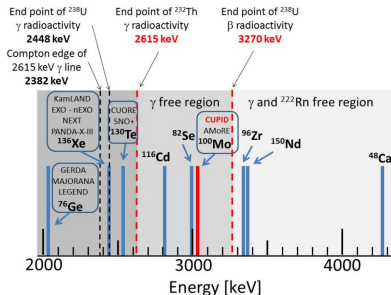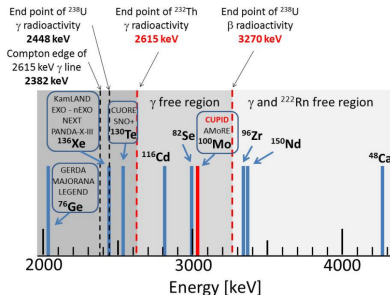
Next Generation $0\nu\beta\beta$ experiments
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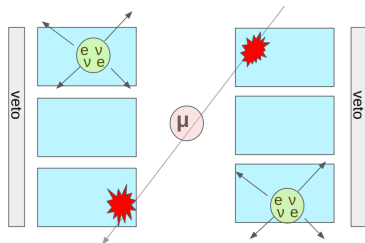**CUPID conservative goal**: $B.I. =$ 0.5 $counts / (FWHM \cdot tonne \cdot year)$

$2\nu\beta\beta$ events and muons

- The rate of $2\nu\beta\beta$ events is not negligible[1] in the CUPID array

- Minimizing the distance cut helps avoid mis-labelling random $2\nu\beta\beta$ coincidences as multiplicity 2.

- Assuming a simple muon veto geometry, increasing the distance cut rejects more muon events.
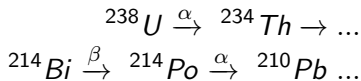
$$T_{1/2}^{2\nu\beta\beta} = 7.1 * 10^{18} yr$$
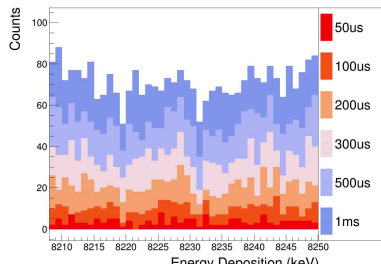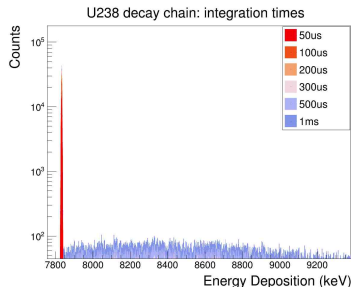
$\rightarrow$ single crystal rate $\sim 3mHz$

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[1]**arxiv.org/abs/1912.07272**.

Integration time in the CUPID array

$$^{238}U \xrightarrow{\alpha} {}^{234}Th \rightarrow ...$$
$$^{214}Bi \xrightarrow{\beta} {}^{214}Po \xrightarrow{\alpha} {}^{210}Pb ...$$

- 100,000 Uranium-238 events (full chain)

- $T_{1/2}$ ^{214}Po $\sim 160\mu$s

- $T_{1/2}$ ^{214}Bi ~ 20 minutes

$2\nu\beta\beta$ tagging simulation

- 40 million events generated in crystal volume.

- expect $\sim 90\%$ efficiency for $2\nu\beta\beta$ tagging, when using approximate CUORE parameters

- larger distance cut causes more random coincidences and more variation with integration time

- bremsstrahlung, escape, random coincidences are primary contributors
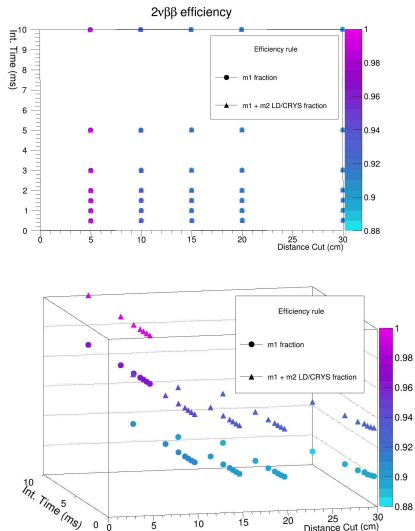
implementation of muon background

- incorporating scintillating muon veto geometry into monte-carlo simulations

- muon flux at LNGS is $3 \cdot 10^{-8}$ muons $/ (s \cdot cm^2)$

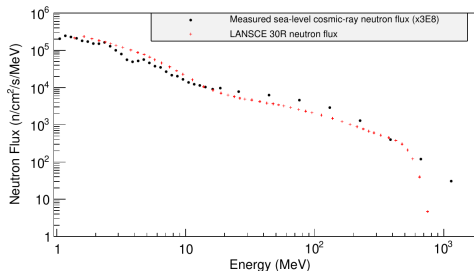- muon suppression versus distance cut optimization.
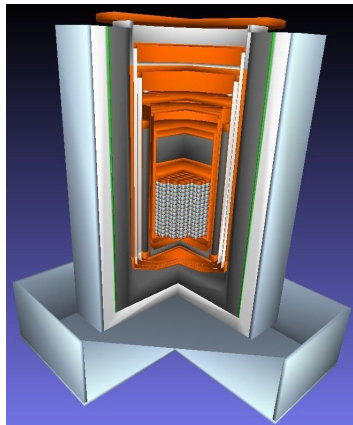
preliminary muon veto geometry

spallation in ^{100}Mo

$\sigma = \sigma_0 f(A)f(E)e^{-P\Delta A}$

$\times exp\left(-R|Z| - SA + TA^2 + UA^3\right|^\nu \Omega\eta\xi)$

- semi-empirical cross section formula

- at LANL, a beam bombarding protons on a tungsten target mimics the cosmic ray spectrum.

- $3\cdot 10^8$ scaling in flux $\implies$ 2 days of beam time equates to 1 million years of sea-level exposure.

far reaching consequences of observing $0\nu\beta\beta$

- baryogenesis through leptogenesis

- charge parity violating process in the standard model

- lepton number conservation (accidental symmetry of the standard model)

$$m_\nu = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T$$

$$\langle m_{\beta\beta} \rangle = |U_{ij}^2 m_j|$$