Week 6: Linear Regression Pt. 1

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What is Linear Regression?

- Statistical model used to identify a relationship between a predictor, x, and a response, y.
- Linear regression identifies a line of best fit between the predictor and the response.
- Not only can we identify relationships between x and y, we can also predict future relationships using the regression line.

The Linear Regression Equation

$\hat{y} = b_0 + b_1 x + i$

- $\hat{y}$ is the predicted value of the dependent variable
- b_0 is the y-intercept term
    - This is what $\hat{y}$ equals when x is 0
- x is the value of the independent variable
- b_1 is the slope (referred to as the coefficient) of x
    - For every 1 unit increase in x, $\hat{y}$ increases by b_1
- There is also random error, i, that encapsulates the randomness that the model can't catch

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Ordinary Least Squares

- Goal is to identify coefficients that minimize the sum of squared differences between the actual and predicted y values
    - Also known as residuals
- A perfect model would have no difference between actual values and predictions
- Influential outliers can have a large impact on the line of best fit

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Developing a Linear Model in R

- In R, we can use `lm()` to develop a linear regression model
- We use the `y ~ x` formula interface while specifying the data we are using
- We need to call `summary()` to see model output

```
pres.lm1 <- lm(prestige ~ education, data = prestige)
```

Model Output in R

- First we get the spread of the residuals
- The residual is the difference between actual and predicted y-value
- Next we get coefficient info
  - Slope estimates, standard error, and p-values
- The final block of text includes several additional pieces of model information, mainly used to validate our model

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Coefficient Estimates

```
coefficients(pres.lm1)
```

```
## (Intercept)   education
##  -10.731982    5.360878
```

- The coefficient estimates are the constants of the linear regression formula
- In this case, our model would look as follows:
    - $\hat{y} = -10.732 + 5.361(x)$
- The intercept value suggests that when education is `0`, our predicted value for prestige is `-10.732`
- The education estimate suggests that for each additional point of education, prestige increases by `5.361`

How Good is Our Model???

- The R-squared value is a measure between `0` and `1` showing how much variance the model explains
  - The closer the value is to `1`, the more the model explains
- Mathematically, it's `1` minus the ratio of the sum of squared errors and the total sum of squares
  - SST is the total error between the mean of y and its specific observations
  - SSE is the unexplained error; the difference between the prediction and the observations
- The F-value explains whether the model fits the data better than random guessing
  - Random guessing would just be predicting the mean value of y for all observations

Linear Regression Assumptions

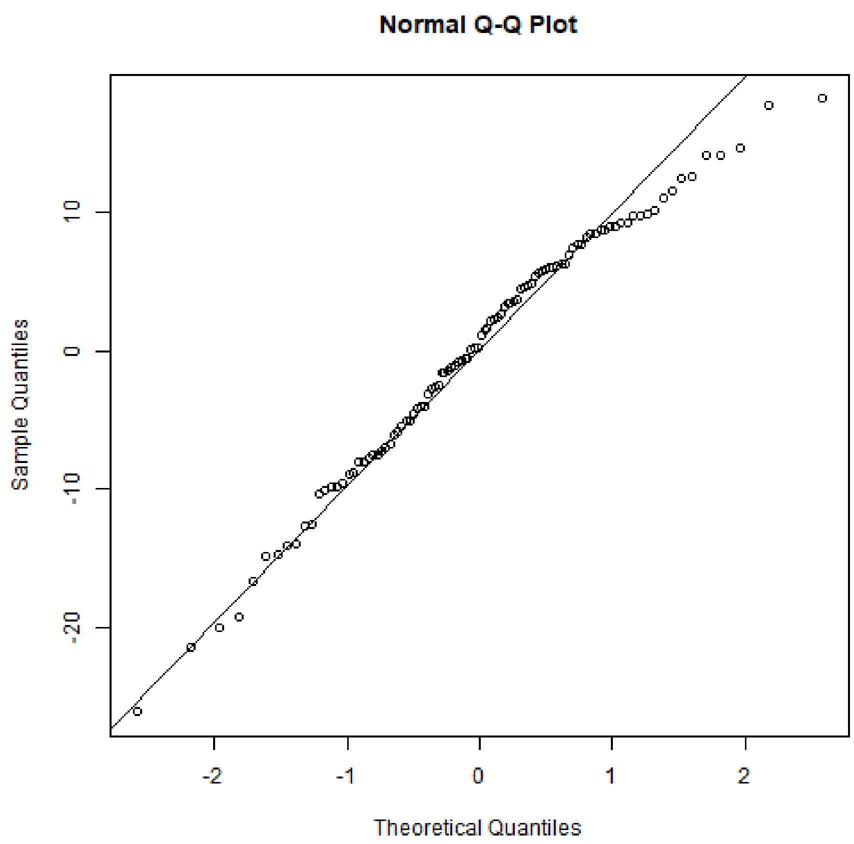
- Normality of Residuals: use a QQ plot to determine normality of model residuals
- Constant Variance: variance of the residuals are the same for different values of x
- Linearity: Relationship between x and y is linear
- Independent observations: observations don't influence each other

Normality

- We can use a QQ plot to determine if our residuals follow a normal distribution
- The points should follow along the straight QQ line
  - This line represents perfectly normal data; don't expect all of your residuals to follow it exactly

Normality

```
qqnorm(pres.lm1$residuals)
qqline(pres.lm1$residuals)
```

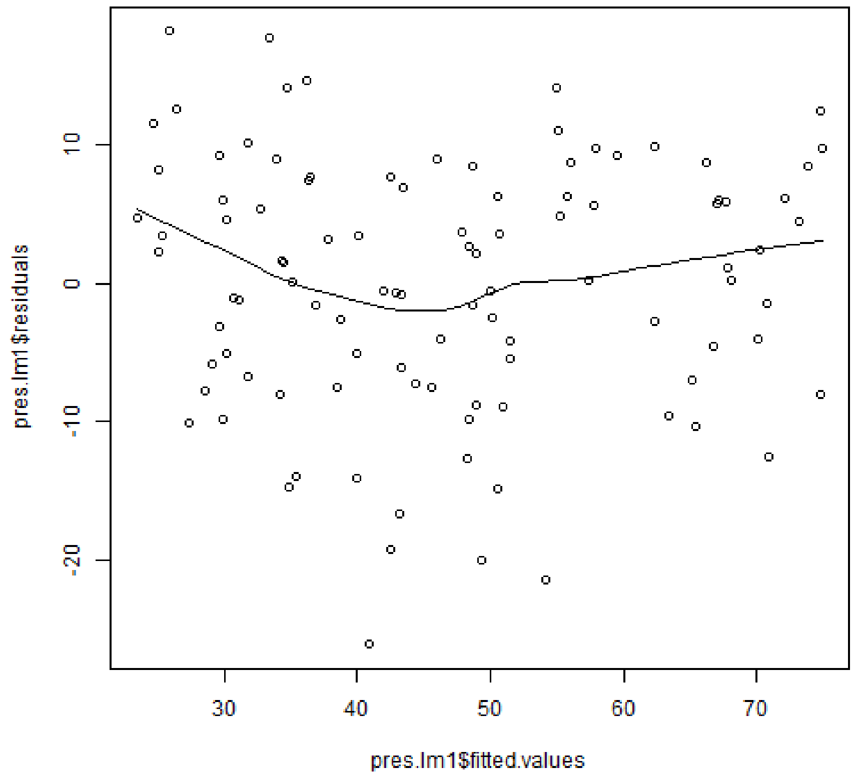


Constant Variance and Linearity

- We can check out constant variance and linearity by plotting residuals vs. fitted values
- If the points are spread out evenly around `0`, we can assume constant variance
- If the points show no real pattern or trend, sticking close to `0`, we can assume linearity

Constant Variance and Linearity

```
scatter.smooth(pres.lm1$fitted.values, pres.lm1$residuals)
```



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What if Our Model Doesn't Meet Assumptions?

- It doesn't necessarily mean we have a bad model, it just means that we could improve upon it
- Can improve through transforming variables, getting additional data, or even going with a non-linear model!

Making Predictions

- One of the benefits of linear regression is that we can predict new data
- Use the `predict()` function to make these predictions
  - The first argument should be the name of the model
  - Without additional arguments, this will just predict on the data used to train the model
  - Specify the argument `newdata = x` where x is the new data to make additional predictions

Linear Regression with Categorical Data

- Categorical data is treated a little differently than numeric data
- Categorical data are treated as dummy variables
    - Acting as a numeric flag for the different categories

Coefficient Output of a Categorical Predictor

```
pres.lm2 <- lm(prestige ~ type, data = prestige)

coefficients(pres.lm2)
```

```
## (Intercept)    typeprof      typewc
##   35.527273   32.321114    6.716206
```

- The first factor level is represented by the intercept
  - Baseline or reference level
- The coefficients of the other two factor levels would be added to the intercept if the observation is one of those levels; if it is the baseline level, nothing is added