

Week 5: Linear Regression Pt. 1

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What is Linear Regression?

- Statistical model used to identify a relationship between a predictor, x , and a response, y .
- Linear regression identifies a line of best fit between the predictor and the response.
- Not only can we identify relationships between x and y , we can also predict future relationships using the regression line.

The Linear Regression Equation

$$\hat{Y} = b_0 + b_1x + i$$

- \hat{Y} is the predicted value of the dependent variable
- b_0 is the y-intercept term
 - This is what \hat{Y} equals when x is 0
- x is the value of the independent variable
- b_1 is the slope (referred to as the coefficient) of x
 - For every 1 unit increase in x , \hat{Y} increases by b_1
- There is also random error, i , that encapsulates the randomness that the model can't catch

Ordinary Least Squares

- Goal is to identify coefficients that minimize the sum of squared differences between the actual and predicted y values
 - Also known as residuals
- A perfect model would have no difference between actual values and predictions
- Influential outliers can have a large impact on the line of best fit

Developing a Linear Model in R

- In R, we can use `lm()` to develop a linear regression model
- We use the `y ~ x` formula interface while specifying the data we are using
- We need to call `summary()` to see model output

```
pres.lm1 <- lm(prestige ~ education, data = prestige)
```

Model Output in R

- First we get the spread of the residuals
- The residual is the difference between actual and predicted y-value
- Next we get coefficient info
 - Slope estimates, standard error, and p-values
- The final block of text includes several additional pieces of model information, mainly used to validate our model

Coefficient Estimates

```
coefficients(pres.lml)
```

```
## (Intercept)    education  
##   -10.731982     5.360878
```

- The coefficient estimates are the constants of the linear regression formula
- In this case, our model would look as follows:
 - $\hat{Y} = -10.732 + 5.361(x)$
- The intercept value suggests that when education is 0, our predicted value for prestige is **-10.732**
- The education estimate suggests that for each additional point of education, prestige increases by **5.361**

How Good is Our Model???

- The R-squared value is a measure between 0 and 1 showing how much variance the model explains
 - The closer the value is to 1, the more the model explains
- Mathematically, it's 1 minus the ratio of the sum of squared errors and the total sum of squares
 - SST is the total error between the mean of y and its specific observations
 - SSE is the unexplained error; the difference between the prediction and the observations
- The F-value explains whether the model fits the data better than random guessing
 - Random guessing would just be predicting the mean value of y for all observations

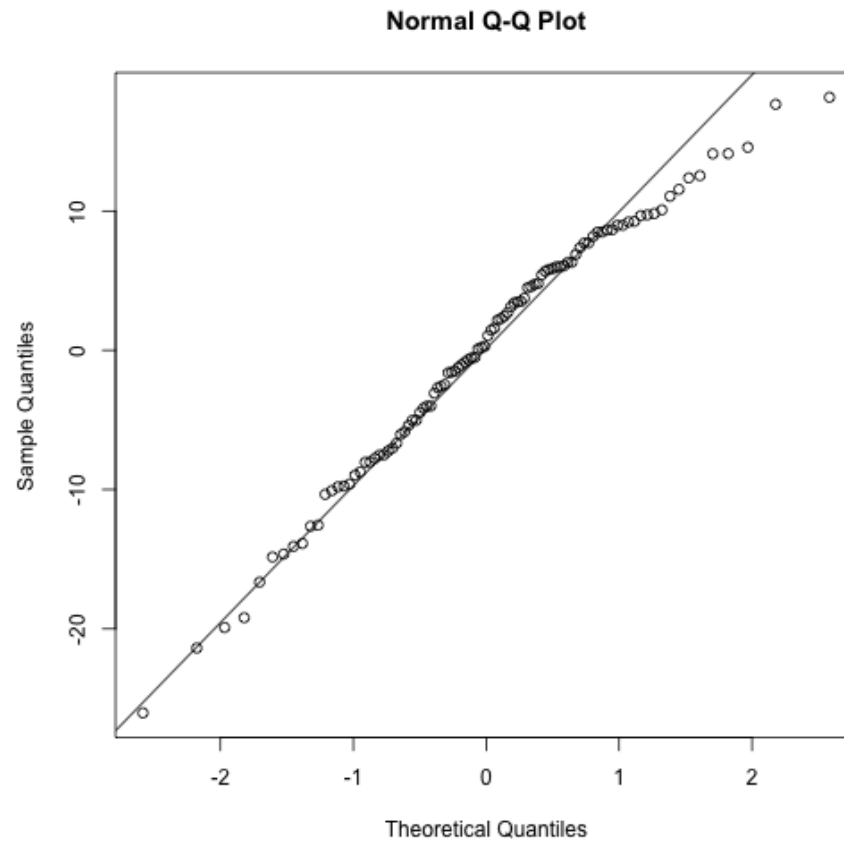
Linear Regression Assumptions

- Normality of Residuals: use a QQ plot to determine normality of model residuals
- Constant Variance: variance of the residuals are the same for different values of x
- Linearity: Relationship between x and y is linear
- Independent observations: observations don't influence each other

Normality

- We can use a QQ plot to determine if our residuals follow a normal distribution
- The points should follow along the straight QQ line
 - This line represents perfectly normal data; don't expect all of your residuals to follow it exactly

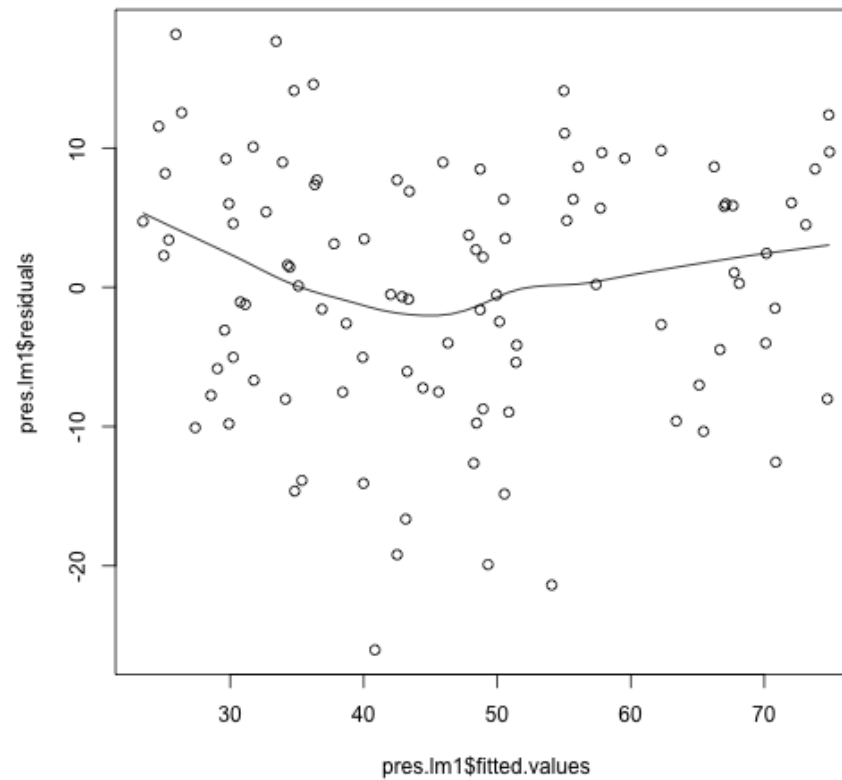
Normality



Constant Variance and Linearity

- We can check out constant variance and linearity by plotting residuals vs. fitted values
- If the points are spread out evenly around 0, we can assume constant variance
- If the points show no real pattern or trend, sticking close to 0, we can assume linearity

Constant Variance and Linearity



What if Our Model Doesn't Meet Assumptions?

- It doesn't necessarily mean we have a bad model, it just means that we could improve upon it
- Can improve through transforming variables, getting additional data, or even going with a non-linear model!

Making Predictions

- One of the benefits of linear regression is that we can predict new data
- Use the `predict()` function to make these predictions
 - The first argument should be the name of the model
 - Without additional arguments, this will just predict on the data used to train the model
 - Specify the argument `newdata = x` where `x` is the new data to make additional predictions

Linear Regression with Categorical Data

- Categorical data is treated a little differently than numeric data
- Categorical data are treated as dummy variables
 - Acting as a numeric flag for the different categories

Coefficient Output of a Categorical Predictor

```
pres.lm2 <- lm(prestige ~ type, data = prestige)
```

```
coefficients(pres.lm2)
```

```
## (Intercept)      typeprof      typewc  
##    35.527273    32.321114     6.716206
```

- The first factor level is represented by the intercept
 - Baseline or reference level
- The coefficients of the other two factor levels would be added to the intercept if the observation is one of those levels; if it is the baseline level, nothing is added