Week 7: Linear Regression Pt. 2

03/03/2020

Jake Campbell

Expanding on Simple Linear Regression

- A lot of the time you'll have access to several variables you can use to develop a model
- Adding additional variables can have a positive impact on our regression model
- Adding too many predictors can cause our model to overfit
- There are additional assumptions and things to take into account when dealing with multiple independent variables

Developing a Linear Model with Multiple Variables

- Like in simple linear regression, we use `lm()` and `R`'s formula interface to develop a linear model
- To add additional independent variables to the formula interface, we use a `+`
- Think about variables that you are putting in the model; does their relationship with the response make sense?
  - Adding too many independent variables to our model can cause it to overfit to our data, creating issues in predicting future values

What Does our Output Mean?

- Our summary output is very similar to the output in simple linear regression
- Coefficient estimates are the change in y with each additional unit of x WITH all other independent predictors held constant
- R squared never decreases with each additional variable
  - Adding variables of little to no importance can inflate our R squared value
  - Adjusted R squared takes into account the number of variables in the model and penalizes additional variables that don't improve model performance

Model Assumptions

- The assumptions from simple linear regression apply to multiple regression
- In addition, we have to think about multi-collinearity
  - This is when our independent variables are highly correlated amongst each other
  - Doesn't necessarily cause issues with predictions or adjusted R squared
  - Does mess with our coefficients, making relationships difficult to interpret

Detecting and Dealing with Multicollinearity

$$VIF_k = \frac{1}{1 - R_k^2}$$

- The variance inflation factor can detect high collinearity
  - Use the `vif()` function from the `car` library
- VIF values above `10` indicate a variable with strong collinearity; above `5` could also signal problems
- Essentially the VIF is looking at how well the independent variables can be used to predict each other
- Adjust for multi-collinearity by combining correlated variables or removing them from the model altogether

Comparing Nested Models

- We can use a partial F-test to see if a complex model is significantly better than a simple model that uses a subset of the variables in the complex model
- Is the improvement created by developing a more complex model actually substantial enough to warrant making the model more complex?
- Run `anova()` using both models as your arguments
- A p-value less than alpha `.05` indicates that our full, complex model significantly improves on the reduced model

Okay, So Now What?

- Linear regression is a good introduction to the modeling process, but it has issues
  - There are a lot of things that we want to model that aren't linear
  - Being a parametric model, linear regression has a lot of assumptions
- What can we do to make linear models more flexible and address assumption issues?

Words of Caution

- When relationships between variables start becoming much more advanced than a linear model can determine, might be easier to go non-parametric route
  - Ensemble tree models: boosting, random forest
- These non-parametric approaches don't need to meet a laundry list of assumptions
- They can better identify non-linear patterns and complex interactions
- Their main con: difficult to explain results to others

9/16

Transforming the Response

- Sometimes the response isn't normally distributed
  - This can lead to issues in residual normality
  - Can lead to non-constant variance
  - Common issue when predicting monetary values

Box-Cox Transformation

- Searches all possible values of lambda for the one that maximizes the likelihood that y comes from a normal distribution
- Instead of looking at the lambda that produces the most normal y, look at the range to see if it includes a more common value
  - Using a specific lambda can make our results more confusing

11/16

Common Box-Cox Transformations

LAMBDA	Y_TRANSFORMATION
-2.0	1/(Ysquared)
-1.0	1/Y
-0.5	1/sqrt(Y)
0.0	log(Y)
0.5	sqrt(Y)
1.0	Y
2.0	Ysquared

12/16

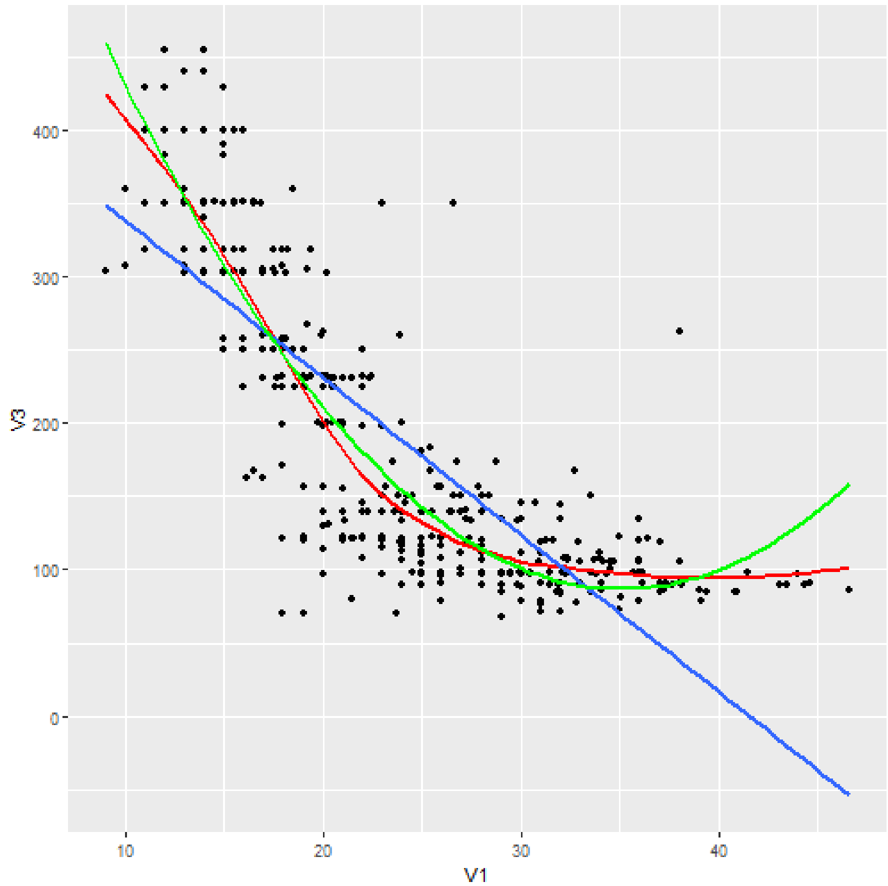
Maybe Your Data Isn't Linear

- Linear regression fits a straight line to your data
- Maybe your data has a curved relationship
  - Polynomial Regression
- Maybe your data has a complicated, hard to determine relationship (maybe a sin wave)
  - Splines
- Maybe your data is categorical
  - Logistic Regression

Polynomial Regression

- If we see a curved relationship between our predictor and response we can use polynomial regression to get a better fit
- If we miss the curved relationship in the initial graphs, we can also identify it in the residuals vs. fitted plot

Polynomial Regression



Polynomial Regression

- Polynomial regression can take on different degrees
  - 2 is quadratic, 3 is cubic; not suggested going beyond cubic to avoid overfit
- Each coefficient is an additional degree; for example, consider a model predicting salary by age to the second degree.
  - $PredictedSalary = B_0 + B_1(age) + B_2(age^2)$
- `poly()` can be used to take the polynomial in R
  - Specify `degree = x` to set degree
  - Orthogonal or raw return: What's the difference?