Department of Mathematics and Statistics, California State University, Long Beach

STAT 572 : Computational Statistics, Fall 2014
Midterm exam : Oct. 30

Note: You have to provide problem statement, algorithm, result, and discussion. Insert a page break between problems. Hand-written report will not be accepted.

1. Cosider the following probability distribution function

$$f(x) = \begin{cases} \frac{1-\exp\{-y^2/2\}}{y^2\sqrt{2\pi}}, y \neq 0 \\ \frac{1}{2\sqrt{2\pi}}, \quad\quad y = 0. \end{cases}$$

This distribution is called Slash distribution which has been used widely in simulation study. The shape of the distribution is similar to the standard normal density but havier tails. We are interested in generating random sample from the density.

   (a) Use the alternative CDF method to generate a random sample of size 200 from the density. Plot the density histogram of the random sample. Also provide the emperical and the theoretical CDF and compare.

   (b) Using the random sample in (a), we are interested in a Monte Carlo test for

$$H_0 : m \leq 1 \quad vs. \quad H_1 : m > 1.$$

   Here m is the population median. Carefully describe how you would perform the Monte Carlo test. Calculate the Monte Carlo *p-value* and draw a conclusion.

2. Cosider the following probability distribution function

$$f(x) = \begin{cases} \frac{x-2}{2}, & 2 \leq x < 3 \\ \\ \frac{2-\frac{x}{3}}{2}, & 3 \leq x \leq 6. \end{cases}$$

Use an acception-rejection method to generate a random sample of size 300 from the density. For candidate distributions use Unif(2,6). Provide the density histogram of the random samples with the true density superimposed. Give the figure showing the accepted and rejected variates as in Figure 4.3 and calculate the probability of rejection.

3. Suppose we want to estimate θ, where

$$\theta = \int_0^1 e^{x^2} dx$$

We want to argue that generating a random variable U from $Uniform(0, 1)$ and then using the estimate $e^{U^2}(1 + e^{1-2U})/2$ is better than generating two uniform random numbers and using $(e^{U_1^2} + e^{U_2^2})/2$. Demonstrate this by a simulation.

4. Let $X_1, X_2, ..., X_n$ be i.i.d. random variables having unknown mean μ. For given constant $a < b$, we are interested in estimating $p = Prob[a < \frac{\bar{X}-\mu}{SE(\bar{x})} < b]$. Explain how we can use the bootstrap approach to estimate p. Use the forearm data to estimate p with $a = -2, b = 2$. Provide the 95% bootstrap percentile CI for p. Discuss your findings.