Department of Mathematics and Statistics, California State University, Long Beach

STAT 572 : Computational Statistics, Fall 14
Final : Due by midnight Th, Dec. 11

Note: You have to provide both algorithm and MATLAB code for each problem followed by discussions. Insert a page break for each problem. Hand-written report will not be accepted. Electronic copy (in pdf format) is due by midnight on Dec. 11 (Thursday night).

1. Consider the attached finalpr1 data. To load the data use Import Data under File. The sample of size 200 has been taken iid from the estimated mixture normal distribution

$$\delta N\left(\mu_1, \sigma_1^2\right) + (1-\delta) N\left(\mu_2, \sigma_2^2\right).$$

   We are interested in Monte Carlo inference on δ.

   (a) Assuming a Unif(0,1) prior distribution for δ use MCMC techniques to construct a Markov chain of size 1000 whose stationary distribution equals the posterior density of δ. For μ's and δ's use the estimates from the finite mixture. After 10% burn-in provide the histogram of the chain, Monte Carlo estimate of the mean $E(\delta)$ and the variance $Var(\delta)$. Construct an approximate 90% CI for $E(\delta)$. Also give the mix rate.

   (b) Repeat (a) using a Beta density as the prior. Choose parameters of your proposal density carefully so that the chain moves quickly away from the starting value and converges to it stationary distribution. Carefully compare the results with (a)

2. It is known that the distribution of waiting times between events in a Poisson process with intensity λ are $Exp(\lambda)$. We would like to use this fact to generate random numbers from $Poisson(\lambda)$. Generate events $X_i = \sum_{j=1}^{i} Y_j,\ Y_j \sim Exp(\lambda)$ and then take $Z = \#\{X_i : X_i \in [l-1, l)\}, l \geq 1$, as $Poisson(\lambda)$ pseudo-rv's. Write a function to generate N such random numbers. Using your function generate N=1000 such random numbers with $\lambda = 1$. Count the outcomes in the calagories 0, 1, 2, ..., 10, 10+. Provide the density histogram and compare with the theoretical density. Discuss your findings.

   (a) Describe how you might do this using a resampling method.

   (b) Describe how you might do this using a kernel density estimation method.

   For each method you used, describe your procedure in detail, write a MATLAB function, and provide results. Compare the results both graphically and numerically.

3. Consider random variables X and Y having the following conditional distributions.

$$f(x|y) = ye^{-xy}, \quad 0 < x < 10$$

$$f(y|x) = xe^{-xy}, \quad 0 < y < 10$$

   Use Gibbs sampler to generate MC of size 1000 from the marginal distribution$f(x)$. Use your own starting values and burin-in period. Estimate mean, variance, skewness, and kurtosis of the marginal distribution. Afer burn-in plot the MC and give the histogram of the chain. Estimate the marginal pdf $\hat{f(x)}$ at $x = 0.1,\ 1.8,\ 3.5,\ 9.2$.
   Repeat with different starting values and compare the results.