STAT 510 Quiz 3

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In [1]:
```
# import libraries
import pandas as pd
from pandas import Series, DataFrame
import numpy as np

import statsmodels.api as sm
from statsmodels.formula.api import ols

import seaborn as sns
import matplotlib.pyplot as plt
%matplotlib inline
```

Question 1

Part a.

Hald Data

For this data we begin by conducting a model selection analysis based on the R^2, adjusted R^2, the AIC, and Mallow's C_p.

To construct all possible regression models, I wrote a script in python that produces the necesary results:

In [2]:
```
run construct_models
```

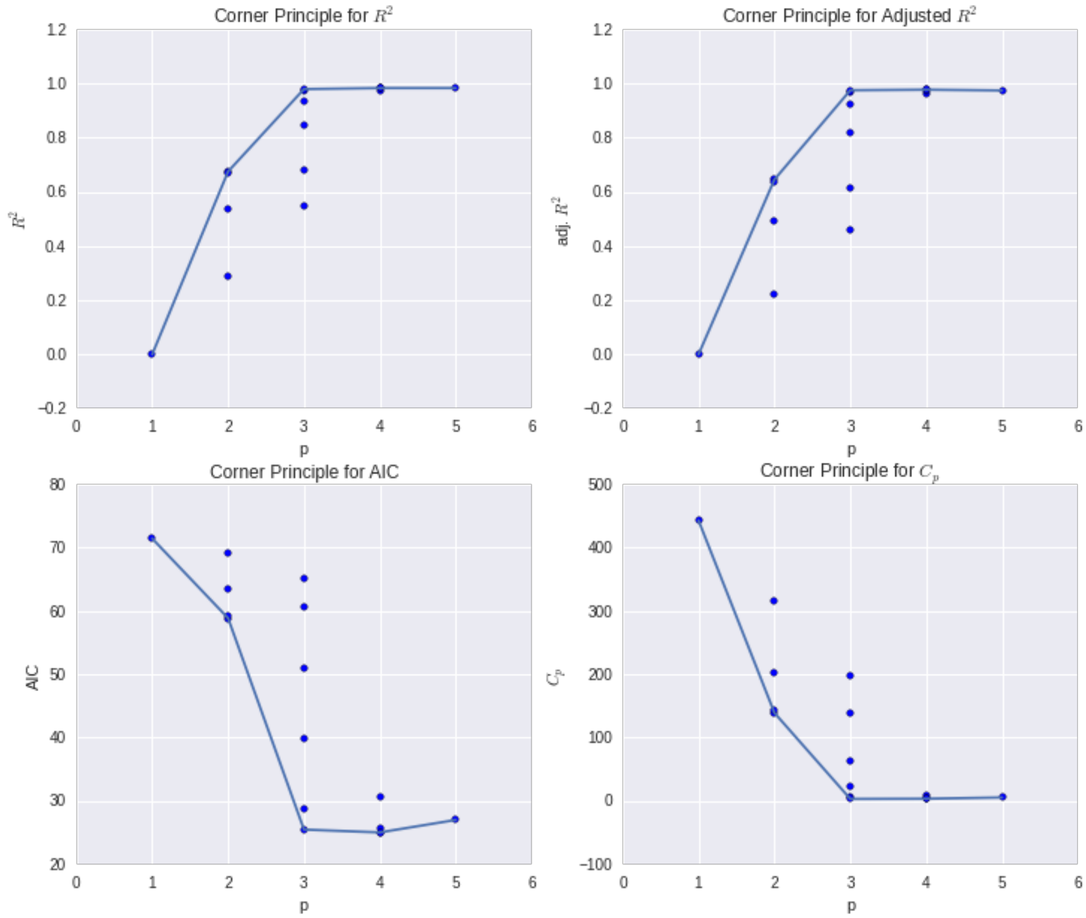
```
These are the first few rows of the Hald dataset:

       Y  X1  X2  X3  X4       mean
0   78.5   7  26   6  60  95.423077
1   74.3   1  29  15  52  95.423077
2  104.3  11  56   8  20  95.423077
3   87.6  11  31   8  47  95.423077
4   95.9   7  52   6  33  95.423077

The possible models are:

Y ~ X1 + X2 + X3 + X4
Y ~ X2 + X3 + X4
Y ~ X1 + X3 + X4
Y ~ X3 + X4
Y ~ X1 + X2 + X4
Y ~ X2 + X4
Y ~ X1 + X4
Y ~ X4
Y ~ X1 + X2 + X3
Y ~ X2 + X3
Y ~ X1 + X3
Y ~ X3
Y ~ X1 + X2
Y ~ X2
Y ~ X1
Y ~ mean

The following plots summarize our analysis:
```


Corner Principle for R^2
R^2
p
Corner Principle for Adjusted R^2
adj. R^2
p
Corner Principle for AIC
AIC
p
Corner Principle for C_p
C_p
p

In [3]:
```
results_tbl
```

Out[3]:

	model	P	R-sq	Adj. R-sq	AIC	Cp
0	model00	5	0.9824	0.9736	24.9443	3.0000
1	model01	4	0.9728	0.9638	28.5759	5.3375
2	model02	4	0.9813	0.9750	23.7276	1.4968
3	model03	3	0.9353	0.9223	37.8526	20.3731
4	model04	4	0.9823	0.9764	22.9739	1.0182
5	model05	3	0.6801	0.6161	58.6293	136.2259
6	model06	3	0.9725	0.9670	26.7417	3.4959
7	model07	2	0.6745	0.6450	56.8516	136.7308
8	model08	4	0.9823	0.9764	23.0112	1.0413
9	model09	3	0.8470	0.8164	49.0371	60.4377
10	model10	3	0.5482	0.4578	63.1167	196.0947
11	model11	2	0.2859	0.2210	67.0674	313.1543
12	model12	3	0.9787	0.9744	23.4200	0.6782
13	model13	2	0.6663	0.6359	57.1780	140.4864
14	model14	2	0.5339	0.4916	61.5195	200.5488
15	model15	1	0.0000	0.0000	69.4444	440.9167

In [4]:
```
print 'The models to be considered are model 06: {} and model 12: {}.'.format(
models[6], models[12])
```

```
The models to be considered are model 06: Y ~ X1 + X4 and model 12: Y ~ X1 + X2
.
```

The above analysis indicates that a model with p = 3 might be adequate. The corner principle plots indicate that , models with P > 3, do not produce improvements in R^2, adjusted R^2, the AIC, or Mallow's C_p. In particular, model 06: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_4 + \epsilon$ and model 12: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$, shows high R^2, low AIC and C_p that is small and relatively close to p=3. The other models with p=3, show large values of C_p which is an indication of bias, thus they will not be considered.

Part b.

Now we implement Forward, Backward, and Stepwise selection and compare with the results in a. above. For this I wrote the module mselector. The python code is included as an appendix.

Import mselector and create new dataset.

In [5]:
```
import mselector as s

dfnew = df.iloc[:,:5]
```

Forward Selection

Run the froward selection algorithm:

In [6]:
```
s.foresel(dfnew,'Y')
```

```
The optimal model is Y ~ X4 + X1

/home/jcapitz/anaconda/lib/python2.7/site-packages/scipy/stats/stats.py:1277: U
serWarning: kurtosistest only valid for n>=20 ... continuing anyway, n=13
  "anyway, n=%i" % int(n))
```

Out[6]:

OLS Regression Results

Dep. Variable:	Y	**R-squared:**	0.972
Model:	OLS	**Adj. R-squared:**	0.967
Method:	Least Squares	**F-statistic:**	176.6
Date:	Tue, 01 Dec 2015	**Prob (F-statistic):**	1.58e-08
Time:	13:30:42	**Log-Likelihood:**	-29.817
No. Observations:	13	**AIC:**	65.63
Df Residuals:	10	**BIC:**	67.33
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[95.0% Conf. Int.]**
Intercept	103.0974	2.124	48.540	0.000	98.365 107.830
X4	-0.6140	0.049	-12.621	0.000	-0.722 -0.506
X1	1.4400	0.138	10.403	0.000	1.132 1.748

Omnibus:	0.408	**Durbin-Watson:**	1.788
Prob(Omnibus):	0.816	**Jarque-Bera (JB):**	0.432
Skew:	-0.331	**Prob(JB):**	0.806
Kurtosis:	2.400	**Cond. No.**	97.0

Backward Selection

Run the backward selection algorithm:

In [7]:
```
s.backsel(dfnew, 'Y')
```

```
The optimal model is Y ~ X2 + X1
```

Out[7]:

OLS Regression Results

Dep. Variable:	Y	**R-squared:**	0.979
Model:	OLS	**Adj. R-squared:**	0.974
Method:	Least Squares	**F-statistic:**	229.5
Date:	Tue, 01 Dec 2015	**Prob (F-statistic):**	4.41e-09
Time:	13:30:42	**Log-Likelihood:**	-28.156
No. Observations:	13	**AIC:**	62.31
Df Residuals:	10	**BIC:**	64.01
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[95.0% Conf. Int.]**
Intercept	52.5773	2.286	22.998	0.000	47.483 57.671
X2	0.6623	0.046	14.442	0.000	0.560 0.764
X1	1.4683	0.121	12.105	0.000	1.198 1.739

Omnibus:	1.509	**Durbin-Watson:**	1.922
Prob(Omnibus):	0.470	**Jarque-Bera (JB):**	1.104
Skew:	0.503	**Prob(JB):**	0.576
Kurtosis:	1.987	**Cond. No.**	175.

Stepwise Selection

Run the stepwise selection algorithm:

In [8]:
```
s.stepwsel(dfnew, 'Y')
```

```
The optimal model is Y ~ X1 + X2
```

Out[8]:

OLS Regression Results

Dep. Variable:	Y	**R-squared:**	0.979
Model:	OLS	**Adj. R-squared:**	0.974
Method:	Least Squares	**F-statistic:**	229.5
Date:	Tue, 01 Dec 2015	**Prob (F-statistic):**	4.41e-09
Time:	13:30:42	**Log-Likelihood:**	-28.156
No. Observations:	13	**AIC:**	62.31
Df Residuals:	10	**BIC:**	64.01
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[95.0% Conf. Int.]
Intercept	52.5773	2.286	22.998	0.000	47.483 57.671
X1	1.4683	0.121	12.105	0.000	1.198 1.739
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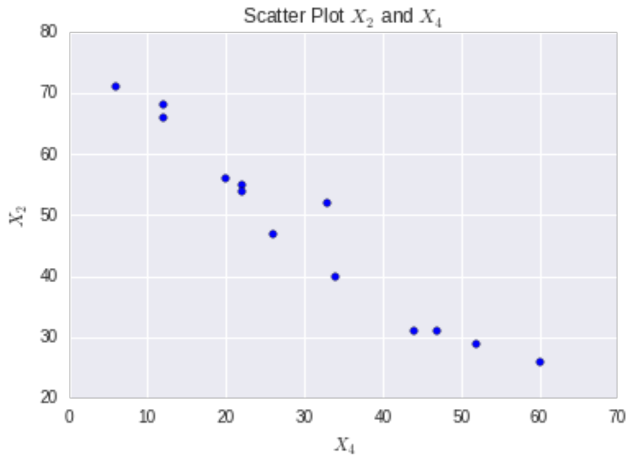
Omnibus:	1.509	**Durbin-Watson:**	1.922
Prob(Omnibus):	0.470	**Jarque-Bera (JB):**	1.104
Skew:	0.503	**Prob(JB):**	0.576
Kurtosis:	1.987	**Cond. No.**	175.

Conclusion

Based on the results of the analysis in part a. and b. above the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$ seems most appropriate. In part a. this particular model shows high values of R^2 and adjusted-R^2 as well as low values of AIC and C_p statistics. In addition, both the backward selection and stepwise selection algorithms selected this particular model. In a sense, this result is satisfactory because the backward selection and the stepwise selection algorithm take into account interactions between the explanatory variables, which is the reason the stepwise algorithm "dropped" X_4 in favor of X_2 in the selection process. The reason this variable gets dropped is because of the correlation between X_2 and X_4, which we can see in the graph below:

In [9]:
```
plt.scatter(x=df['X4'], y=df['X2'])
plt.title('Scatter Plot $X_2$ and $X_4$')
plt.ylabel('$X_2$')
plt.xlabel('$X_4$')
plt.show()
```



Question 2

The DFFITS statistic is composed of two factors:

$$e_i \left[\frac{n-p-1}{SSE\,(1-h_{ii}) - e_i^2} \right]^{1/2}$$

and

$$\left(\frac{h_{ii}}{1-h_{ii}} \right)^{1/2}$$

The first factor, known as the Studentized Deleted Residual, is effective in capturing extreme Y observations. If and observation y_i is an outlier, this factor will be large in absolute value. Notice that the residual e_i, will also tend to be large.

The second factor is a function of the leverage h_{ii}, with the property that $0

APPENDIX: Code

construct_models.py Script

In [ ]:
```
#!/usr/bin/python

# import libraries
import pandas as pd
from pandas import Series, DataFrame
import numpy as np

import statsmodels.api as sm
from statsmodels.formula.api import ols

import seaborn as sns
import matplotlib.pyplot as plt

# hald data
filename = '~/Documents/LinearRegression/STAT510/quiz3/hald.txt'
df = pd.read_table(filename, delim_whitespace=True)
df['mean'] = np.mean(df['Y'])
print 'These are the first few rows of the Hald dataset:'
print ''
print df.head()

# powerset
def powerset(seq):
    """
    Returns all the subsets of this set. This is a generator.
    """
    if len(seq) <= 1:
        yield seq
        yield []
    else:
        for item in powerset(seq[1:]):
            yield [seq[0]]+item
            yield item

v = [df.columns[i] for i in range(1,5)]
r = [x for x in powerset(v)]

# initialization
n = len(df)
modeldict = {'model15': ols(formula = 'Y ~ mean', data = df)}
modelp = {'model15' : 1}
paxis = [1]
models = []
rsqvals = [ols(formula = 'Y ~ mean', data = df).fit().rsquared]
adj_rsqvals = [ols(formula = 'Y ~ mean', data = df).fit().rsquared_adj]
aic = [n * np.log(ols(formula = 'Y ~ mean', data = df).fit().ssr) - n * np.log
(n) + 2 * 1]
full_mse = ols(formula = 'Y ~ X1 + X2 + X3 + X4', data = df).fit().mse_resid
Cp = [(ols(formula = 'Y ~ mean', data = df).fit().ssr/full_mse) - n + 2 * 1]
counter = 0

for l in r[:-1]: # lists in r
    reg = 'Y ~ ' + l[0]
    pminus = len(l)
    p = pminus + 1

    for m in range(1,len(l)): # build each model l to input in ols
        reg += ' + ' + l[m]

    if counter < 10:
        modeldict['model' + '0' + str(counter)] = ols(formula = reg, data = df
) # create dict with model objects
        modelp['model' + '0' + str(counter)] = p
```

mselector.py Module

In [ ]:
```
#!/usr/bin/python

from statsmodels.formula.api import ols

def foresel(df, response, alpha = 0.1):
    ''' Performs forward selection for regression.
    args:
        df = data frame with response and covariates
        alpha = a float indicating confidence level
        response = string that represents the response variable
            e.g. 'Y'
    attributes:
        summary = ols(formula,data).fit().summary()
    '''
    # initial assignments
    covariates = set(df.columns)
    covariates.remove(response)
    candidates = []

    while True:

        oldpval = alpha
        rejects = set()

        for variable in covariates:
            candidatesubset = candidates + [variable]
            formula = '{} ~ {}'.format(response, ' + '.join(candidatesubset))
            pval = ols(formula,df).fit().pvalues[-1]
            if pval < oldpval:
                var2add = variable
                oldpval = pval
                optmodel = formula
            else:
                rejects.add(variable)

        candidates.append(var2add)
        covariates.remove(var2add)

        if covariates == rejects:

            print 'The optimal model is {}'.format(optmodel)

            break

    return ols(optmodel,df).fit().summary()


def backsel(df, response, alpha = 0.1):
    '''
    Performs backward selection for regression.
    args:
        df = data frame with response and covariates
        alpha = a float indicating confidence level
        response = string that represents the response variable
            e.g. 'Y'
    attributes:
        summary = ols(formula,data).fit().summary()
    '''
    # initial assignments
    covariates = set(df.columns)
    covariates.remove(response)
    formula = '{} ~ {}'.format(response,' + '.join(list(covariates)))
```