Suaray **FALL 2013**

STAT 410/510: REGRESSION ANALYSIS-MIDTERM 1

You are allowed 1.5 hours to complete this exam. There are a total of 110 possible points, including 10 extra credit points. You must show all work to get credit. Please do not hesitate to raise your hand if you have a question.

1. What is the simple linear regression model? Identify all variables. What are the assumptions made for this model? What additional assumption needs to be made in order to find the maximum likelihood estimators? Use random vector and matrix notation. **(10 Points)**

2. Consider the simple linear regression model. Suppose that the value of the predictor variable X is replaced by cX, where c is some nonzero constant. State how the following are affected by this change:

- **a.** b_0 and b_1
- **b.** MSE
- **c.** The result of the test H_0:$\beta_0=0$.

3. Detailed interviews were conducted with 15 street vendors to study the factors influencing their incomes. The data appears in the following table:

Vendor Number	Monthly Earnings	Age	Hours Worked Per Day
1	2841	29	12
2	1876	21	8
3	2934	62	10
4	1552	18	10
5	3065	40	11
6	3670	50	11
7	2005	65	5
8	3215	44	8
9	1930	17	8
10	2010	70	6
11	3111	20	9
12	2882	29	9
13	1683	15	5
14	1817	14	7
15	4066	33	12

a. Provide a function for estimating mean monthly earnings based on age alone. Produce a 95% confidence interval for the slope coefficient here.
b. Provide a function for estimating mean monthly earnings based on hours worked per day alone. Produce a 95% confidence interval for the slope coefficient here.
c. Continue using the model from b. Write the general formula for a 95% prediction interval (use variables). Now find each piece of this formula for this problem if $x_h=10$ hours per day.
d. If you had to choose only one predictor, which would it be? Now use **both** age and hours per day to predict earnings. Would you come to the same conclusion?
e. Again, using both predictors, explain the meaning of the R^2 and F statistics in the ANOVA table.

4. Let X be a nxp matrix. Assume that the inverse $(X'X)^{-1}$ exists, and define $A=(X'X)^{-1}X'$ and $H=XA$.

a. Show that $(I-H)(I-H)=I-H$ and $HX=X$

b. Find $A(I-H)$ and $(I-H)A'$