

## Juan Carlos Apitz

### STAT510 Homework 4

#### Imports

```
In [1]: #PANDAS
import pandas as pd
from pandas import DataFrame, Series

#NUMPY
import numpy as np

#SCIPY t and F distributions
from scipy.stats import t
from scipy.stats import f
from scipy.stats import norm
from scipy.stats import chi2

#STATMODELS
import statsmodels.api as sm
from statsmodels.formula.api import ols

#SEABORN plotting
import seaborn as sns

#MATPLOTLIB plotting
import matplotlib.pyplot as plt
%matplotlib inline
```

#### Problem 6.16

```
In [2]: filename = '~/Documents/LinearRegression/STAT510/Kutner/CH6DS/CH06PR15.txt'

patient = pd.read_table(filename, delim_whitespace=True, names=['satisfact
ion','age','severity', 'anxiety'])
patient.head()
```

```
Out[2]:
```

|   | satisfaction | age | severity | anxiety |
|---|--------------|-----|----------|---------|
| 0 | 48           | 50  | 51       | 2.3     |
| 1 | 57           | 36  | 46       | 2.3     |
| 2 | 66           | 40  | 48       | 2.2     |
| 3 | 70           | 41  | 44       | 1.8     |
| 4 | 89           | 28  | 43       | 1.8     |

**a.**

Test the following hypothesis:

$$H_o : \beta_1 = \beta_2 = \beta_3 = 0$$

$$H_a : \text{at least one } \beta_i = 0$$

To test the above hypothesis we fit the model and conduct an F test.

```
In [3]: model = ols(formula="satisfaction ~ age + severity + anxiety", data=patient).fit()
```

```
In [4]: dfn = model.df_model
        dfd = model.df_resid
        fcrit = f.ppf(0.9, dfn, dfd)
        print 'The 10 percent critical F-value with 3 and 42 degrees of freedom is
        %.2f and the model F-statistic is %.2f' %(fcrit,model.fvalue)
        print 'Therefore since the F-statistic is greater than the critical F-value
        we reject the null hypothesis at the 10 percent level. We can conclude that
        there exist a regression relation.'
        print 'The p-value of the test is %.11f.' %model.f_pvalue
```

The 10 percent critical F-value with 3 and 42 degrees of freedom is 2.22 and the model F-statistic is 30.05

Therefore since the F-statistic is greater than the critical F-value we reject the null hypothesis at the 10 percent level. We can conclude that there exist a regression relation.

The p-value of the test is 0.00000000015.

In [5]: `model.summary()`

Out[5]: OLS Regression Results

|                          |                  |                            |          |
|--------------------------|------------------|----------------------------|----------|
| <b>Dep. Variable:</b>    | satisfaction     | <b>R-squared:</b>          | 0.682    |
| <b>Model:</b>            | OLS              | <b>Adj. R-squared:</b>     | 0.659    |
| <b>Method:</b>           | Least Squares    | <b>F-statistic:</b>        | 30.05    |
| <b>Date:</b>             | Thu, 29 Oct 2015 | <b>Prob (F-statistic):</b> | 1.54e-10 |
| <b>Time:</b>             | 09:26:42         | <b>Log-Likelihood:</b>     | -169.36  |
| <b>No. Observations:</b> | 46               | <b>AIC:</b>                | 346.7    |
| <b>Df Residuals:</b>     | 42               | <b>BIC:</b>                | 354.0    |
| <b>Df Model:</b>         | 3                |                            |          |
| <b>Covariance Type:</b>  | nonrobust        |                            |          |

|                  | <b>coef</b> | <b>std err</b> | <b>t</b> | <b>P&gt; t </b> | <b>[95.0% Conf. Int.]</b> |
|------------------|-------------|----------------|----------|-----------------|---------------------------|
| <b>Intercept</b> | 158.4913    | 18.126         | 8.744    | 0.000           | 121.912 195.071           |
| <b>age</b>       | -1.1416     | 0.215          | -5.315   | 0.000           | -1.575 -0.708             |
| <b>severity</b>  | -0.4420     | 0.492          | -0.898   | 0.374           | -1.435 0.551              |
| <b>anxiety</b>   | -13.4702    | 7.100          | -1.897   | 0.065           | -27.798 0.858             |

|                       |        |                          |       |
|-----------------------|--------|--------------------------|-------|
| <b>Omnibus:</b>       | 5.219  | <b>Durbin-Watson:</b>    | 2.183 |
| <b>Prob(Omnibus):</b> | 0.074  | <b>Jarque-Bera (JB):</b> | 2.074 |
| <b>Skew:</b>          | -0.098 | <b>Prob(JB):</b>         | 0.354 |
| <b>Kurtosis:</b>      | 1.978  | <b>Cond. No.</b>         | 782.  |

**b.**

```
In [6]: g = 3; p = len(model.params); n = model.nobs; adj_alpha = 1-0.1/(2*g)
        tcrit = t.ppf(adj_alpha, n-p)

        b1 = model.params[1]; b1low = b1 - tcrit * model.bse[1]; b1high = b1 + tcrit * model.bse[1]
        b2 = model.params[2]; b2low = b2 - tcrit * model.bse[2]; b2high = b2 + tcrit * model.bse[2]
        b3 = model.params[3]; b3low = b3 - tcrit * model.bse[3]; b3high = b3 + tcrit * model.bse[3]

        print 'The 90 percent joint CI for beta 1 is: (%.2f, %.2f)' %(b1low, b1high)
        print 'The 90 percent joint CI for beta 2 is: (%.2f, %.2f)' %(b2low, b2high)
        print 'The 90 percent joint CI for beta 3 is: (%.2f, %.2f)' %(b3low, b3high)

The 90 percent joint CI for beta 1 is: (-1.61, -0.67)
The 90 percent joint CI for beta 2 is: (-1.52, 0.64)
The 90 percent joint CI for beta 3 is: (-29.09, 2.15)
```

### C.

```
In [7]: print 'The coefficient of multiple determination is: %.2f' %model.rsquared
        print 'This coefficient measures the proportionate reduction of total variation associated with the model.'

The coefficient of multiple determination is: 0.68
This coefficient measures the proportionate reduction of total variation associated with the model.
```

## Problem 6.19

```
In [8]: filename = '~/Documents/LinearRegression/STAT510/Kutner/CH6DS/CH06PR18.txt'

        rental = pd.read_table(filename, delim_whitespace=True, names=['rate', 'age', 'opex', 'vacancy', 'sqf'])
        rental.head()
```

```
Out[8]:
```

|   | rate | age | opex  | vacancy | sqf    |
|---|------|-----|-------|---------|--------|
| 0 | 13.5 | 1   | 5.02  | 0.14    | 123000 |
| 1 | 12.0 | 14  | 8.19  | 0.27    | 104079 |
| 2 | 10.5 | 16  | 3.00  | 0.00    | 39998  |
| 3 | 15.0 | 4   | 10.70 | 0.05    | 57112  |
| 4 | 14.0 | 11  | 8.97  | 0.07    | 60000  |

**a.**

Test the following hypothesis:

$$H_o : \beta_1 = \beta_2 = \beta_3 = 0$$

$$H_a : \text{at least one } \beta_i = 0$$

To test the above hypothesis we fit the model and conduct an F test.

```
In [9]: model = ols(formula="rate ~ age + opex + vacancy + sqf", data=rental).fit()
```

```
In [10]: dfn = model.df_model
         dfd = model.df_resid
         fcrit = f.ppf(0.95, dfn, dfd)
         print 'The 5 percent critical F-value with %.0f and %.0f degrees of freedom is %.2f and the model F-statistic is %.2f' %(dfn,dfd,fcrit,model.fvalue)
         print 'Therefore since the F-statistic is greater than the critical F-value we reject the null hypothesis at the 5 percent level. We can conclude that there exist a regression relation.'
         print 'The p-value of the test is %.15f.' %model.f_pvalue
         print ''
         print 'The results of the test imply that at least one of the beta parameters is non-zero, which is an indication that at least one of the explanatory variables influences the rental rate.'
```

The 5 percent critical F-value with 4 and 76 degrees of freedom is 2.49 and the model F-statistic is 26.76

Therefore since the F-statistic is greater than the critical F-value we reject the null hypothesis at the 5 percent level. We can conclude that there exist a regression relation.

The p-value of the test is 0.0000000000000073.

The results of the test imply that at least one of the beta parameters is non-zero, which is an indication that at least one of the explanatory variables influences the rental rate.

In [11]: `model.summary()`

Out[11]: OLS Regression Results

|                          |                  |                            |          |
|--------------------------|------------------|----------------------------|----------|
| <b>Dep. Variable:</b>    | rate             | <b>R-squared:</b>          | 0.585    |
| <b>Model:</b>            | OLS              | <b>Adj. R-squared:</b>     | 0.563    |
| <b>Method:</b>           | Least Squares    | <b>F-statistic:</b>        | 26.76    |
| <b>Date:</b>             | Thu, 29 Oct 2015 | <b>Prob (F-statistic):</b> | 7.27e-14 |
| <b>Time:</b>             | 09:26:43         | <b>Log-Likelihood:</b>     | -122.75  |
| <b>No. Observations:</b> | 81               | <b>AIC:</b>                | 255.5    |
| <b>Df Residuals:</b>     | 76               | <b>BIC:</b>                | 267.5    |
| <b>Df Model:</b>         | 4                |                            |          |
| <b>Covariance Type:</b>  | nonrobust        |                            |          |

|                  | coef      | std err  | t      | P> t  | [95.0% Conf. Int.] |
|------------------|-----------|----------|--------|-------|--------------------|
| <b>Intercept</b> | 12.2006   | 0.578    | 21.110 | 0.000 | 11.049 13.352      |
| <b>age</b>       | -0.1420   | 0.021    | -6.655 | 0.000 | -0.185 -0.100      |
| <b>opex</b>      | 0.2820    | 0.063    | 4.464  | 0.000 | 0.156 0.408        |
| <b>vacancy</b>   | 0.6193    | 1.087    | 0.570  | 0.570 | -1.545 2.784       |
| <b>sqf</b>       | 7.924e-06 | 1.38e-06 | 5.722  | 0.000 | 5.17e-06 1.07e-05  |

|                       |       |                          |          |
|-----------------------|-------|--------------------------|----------|
| <b>Omnibus:</b>       | 1.922 | <b>Durbin-Watson:</b>    | 1.580    |
| <b>Prob(Omnibus):</b> | 0.383 | <b>Jarque-Bera (JB):</b> | 1.301    |
| <b>Skew:</b>          | 0.148 | <b>Prob(JB):</b>         | 0.522    |
| <b>Kurtosis:</b>      | 3.545 | <b>Cond. No.</b>         | 1.74e+06 |

**b.**

```
In [12]: g = 4; p = len(model.params); n = model.nobs; adj_alpha = 1-0.05/(2*g)
tcrit = t.ppf(adj_alpha, n-p)

b1 = model.params[1]; b1low = b1 - tcrit * model.bse[1]; b1high = b1 + tcrit * model.bse[1]
b2 = model.params[2]; b2low = b2 - tcrit * model.bse[2]; b2high = b2 + tcrit * model.bse[2]
b3 = model.params[3]; b3low = b3 - tcrit * model.bse[3]; b3high = b3 + tcrit * model.bse[3]
b4 = model.params[4]; b4low = b4 - tcrit * model.bse[4]; b4high = b4 + tcrit * model.bse[4]

print 'The 90 percent joint CI for beta 1 is: (%.2f, %.2f)' %(b1low, b1high)
print 'The 90 percent joint CI for beta 2 is: (%.2f, %.2f)' %(b2low, b2high)
print 'The 90 percent joint CI for beta 3 is: (%.2f, %.2f)' %(b3low, b3high)
print 'The 90 percent joint CI for beta 4 is: (%.7f, %.7f)' %(b4low, b4high)
print ''
print 'The above results indicate that with the exception of beta 3 (since its confidence interval contains 0), all the coefficients are significant. This means that there exist a linear relation between rental rates and the variables age, operating expense, and square footage.'
```

```
The 90 percent joint CI for beta 1 is: (-0.20, -0.09)
The 90 percent joint CI for beta 2 is: (0.12, 0.44)
The 90 percent joint CI for beta 3 is: (-2.16, 3.40)
The 90 percent joint CI for beta 4 is: (0.0000044, 0.0000115)
```

The above results indicate that with the exception of beta 3 (since its confidence interval contains 0), all the coefficients are significant. This means that there exist a linear relation between rental rates and the variables age, operating expense, and square footage.

### C.

```
In [13]: print 'The R-squared of the model is: %.2f' %model.rsquared
print 'This coefficient measures the proportionate reduction of total variation associated with the model. Therefore, our model "explains" approximately 58 percent of the variation in the data.'
```

```
The R-squared of the model is: 0.58
This coefficient measures the proportionate reduction of total variation associated with the model. Therefore, our model "explains" approximately 58 percent of the variation in the data.
```

## Problem 7.5

### a.

**Fit the full and partial models:**

```
In [14]: model1 = ols(formula="satisfaction ~ age ", data=patient).fit()
model2 = ols(formula="satisfaction ~ age + severity", data=patient).fit()
modelf = ols(formula="satisfaction ~ age + severity + anxiety", data=patient).fit()
```

### Calculate the appropriate sum of squares:

```
In [15]: SSRF = modelf.ess
SSRX1 = model1.ess
SSRX2GX1 = model2.ess - model1.ess
SSRX3GX1X2 = modelf.ess - model2.ess
SSE = modelf.ssr
SST = modelf.centered_tss
```

### Create the ANOVA table dataframe:

```
In [16]: SS = Series([SSRF, SSRX1, SSRX2GX1, SSRX3GX1X2, SSE, SST])
DF = Series([modelf.df_model, 1, 1, 1, modelf.df_resid, modelf.df_model +
modelf.df_resid ])

aovtbl = DataFrame()
aovtbl['SS'] = SS
aovtbl['df'] = DF
aovtbl['MS'] = SS/DF

aovtbl.index = ['Full Model', 'X1', 'X2 given X1', 'X3 given X2, X1', 'Error', 'Total']

aovtbl['F'] = aovtbl['MS']/aovtbl.loc['Error', 'MS']

aovtbl.loc['Total', ['MS', 'F']] = 'NA'
aovtbl.loc['Error', 'F'] = 'NA'
#aovtbl['MS'] = aovtbl
#/#aovtbl.loc['Error', 'MS']
```

### The ANOVA table that decomposes the regression sum of squares:

```
In [17]: aovtbl
```

```
Out[17]:
```

|                        | SS           | df | MS       | F        |
|------------------------|--------------|----|----------|----------|
| <b>Full Model</b>      | 9120.463666  | 3  | 3040.155 | 30.05208 |
| <b>X1</b>              | 8275.388851  | 1  | 8275.389 | 81.80263 |
| <b>X2 given X1</b>     | 480.915294   | 1  | 480.9153 | 4.753871 |
| <b>X3 given X2, X1</b> | 364.159521   | 1  | 364.1595 | 3.599735 |
| <b>Error</b>           | 4248.840682  | 42 | 101.1629 | NA       |
| <b>Total</b>           | 13369.304348 | 45 | NA       | NA       |



**b.**

Test the following hypothesis:

$$H_o : \beta_3 = 0$$

$$H_a : \beta_3 \neq 0$$

To test the above hypothesis we calculate the F-value and compare it with  $F_{0.975,1,42}$ 

```
In [18]: fcrit = f.ppf(0.975,aovtbl.loc['X3 given X2, X1','df'],aovtbl.loc['Error',
'df'])
print 'The F-critical value is: %.2f' %fcrit
```

The F-critical value is: 5.40

```
In [19]: fvalue = aovtbl.loc['X3 given X2, X1','MS']/aovtbl.loc['Error','MS']
pvalue = 1 - f.cdf(fvalue,aovtbl.loc['X3 given X2, X1','df'],aovtbl.loc['E
rror','df'])

print 'The calculated F-value for the test is: %.2f.' %fvalue
print ''
print 'Since the F-value is smaller than the F-critical value we fail to r
eject the null hypothesis. We conclude that the variable anxiety can be
dropped from the model'
print ''
print 'The pvalue of the test is: %.4f' %pvalue
```

The calculated F-value for the test is: 3.60.

Since the F-value is smaller than the F-critical value we fail to reject the null hypothesis. We conclude that the variable anxiety can be dropped from the model

The pvalue of the test is: 0.0647

## Problem 7.6

Test the following hypothesis:

$$H_o : \beta_2 = \beta_3 = 0$$

$$H_a : \beta_2 \neq 0 \text{ or } \beta_3 \neq 0 \text{ or both.}$$

To test the above hypothesis we calculate the F-value and compare it with  $F_{0.975,2,42}$ 

```
In [20]: fcrit = f.ppf(0.975,aovtbl.loc['X3 given X2, X1','df'] + aovtbl.loc['X2 gi
ven X1','df'], aovtbl.loc['Error','df'])
print 'The F-critical value is: %.2f' %fcrit
```

The F-critical value is: 4.03

```
In [21]: fvalue = ((aovtbl.loc['X3 given X2, X1', 'SS'] + aovtbl.loc['X2 given X1', 'SS'])/2)/aovtbl.loc['Error', 'MS']
pvalue = 1 - f.cdf(fvalue, aovtbl.loc['X3 given X2, X1', 'df'] + aovtbl.loc['X2 given X1', 'df'], aovtbl.loc['Error', 'df'])

print 'The calculated F-value for the test is: %.2f.' %fvalue
print ''
print 'Since the F-value is greater than the F-critical value we reject the null hypothesis. We conclude that one or both of the variables anxiety or severity cannot be dropped from the model'
print ''
print 'The pvalue of the test is: %.4f' %pvalue
```

The calculated F-value for the test is: 4.18.

Since the F-value is greater than the F-critical value we reject the null hypothesis. We conclude that one or both of the variables anxiety or severity cannot be dropped from the model

The pvalue of the test is: 0.0222

## Problem 7.10

Test the following hypothesis:

$$H_o : \beta_1 = -0.1, \beta_2 = 0.4$$

$$H_a : \text{not both equalities in } H_o \text{ hold}$$

To test the above hypothesis we calculate the F-value and compare it with  $F_{0.99,2,76}$

```
In [22]: rental['mod_dep'] = rental['rate'] + 0.1*rental['age'] - 0.4*rental['opex']
rental.head()
```

```
Out[22]:
```

|   | rate | age | opex  | vacancy | sqf    | mod_dep |
|---|------|-----|-------|---------|--------|---------|
| 0 | 13.5 | 1   | 5.02  | 0.14    | 123000 | 11.592  |
| 1 | 12.0 | 14  | 8.19  | 0.27    | 104079 | 10.124  |
| 2 | 10.5 | 16  | 3.00  | 0.00    | 39998  | 10.900  |
| 3 | 15.0 | 4   | 10.70 | 0.05    | 57112  | 11.120  |
| 4 | 14.0 | 11  | 8.97  | 0.07    | 60000  | 11.512  |

## Fit the full and reduced model

```
In [23]: modelf = ols(formula="rate ~ age + opex + vacancy + sqf", data=rental).fit()
modelr = ols(formula="mod_dep ~ vacancy + sqf ", data=rental).fit()
```

```
In [24]: fcrit = f.ppf(0.99, modelr.df_resid - modelf.df_resid, modelf.df_resid)
print 'The F-critical value is: %.2f' %fcrit
```

The F-critical value is: 4.90

```
In [25]: fvalue = ((modelr.ssr - modelf.ssr)/(modelr.df_resid - modelf.df_resid))/modelf.mse_resid
pvalue = 1 - f.cdf(fvalue, modelr.df_resid - modelf.df_resid, modelf.df_resid)

print 'The calculated F-value for the test is: %.2f.' %fvalue
print ''
print 'Since the F-value is smaller than the F-critical value we fail to reject the null hypothesis. We conclude that one or both of the equalities in the null hypothesis do not hold.'
print ''
print 'The pvalue of the test is: %.4f' %pvalue
```

The calculated F-value for the test is: 4.61.

Since the F-value is smaller than the F-critical value we fail to reject the null hypothesis. We conclude that one or both of the equalities in the null hypothesis do not hold.

The pvalue of the test is: 0.0129

## Problem 14.4

**a.**

**Create the mean response function:**

```
In [26]: def mrf(beta, X):
        prob = np.exp(np.dot(X,beta))/(1 + np.exp(np.dot(X,beta)))
        return prob
```

**Set up the design matrix and the coefficients vector:**

```
In [27]: X = np.hstack((np.ones(1000).reshape(1000,1),np.linspace(90,160,1000).reshape(1000,1)))
        beta = np.array([[-25],[0.2]])
```

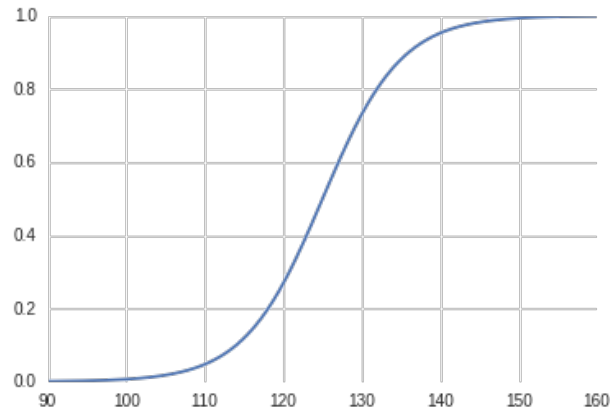
**Calculate the values for the mean response function:**

```
In [28]: Y = mrf(beta,X)
```

**Plot the function:**

```
In [29]: plt.plot(X[:,1],Y)
```

```
Out[29]: [<matplotlib.lines.Line2D at 0x7eff1dd52bd0>]
```



**b.**

Solving the equation below for  $X_i$ :

$$\frac{e^{-25+0.2X_i}}{1 + e^{-25+0.2X_i}} = 0.5$$

We obtain the solution  $X_i = 125$

**c.**

**Odds when X = 150:**

```
In [30]: odds1 = mrf(beta,np.array([[1,150]]))[0][0]/(1-mrf(beta,np.array([[1,150]]
))[0][0])
```

```
print 'The odds when X = 150 are: %.2f' %odds1
```

The odds when X = 150 are: 148.41

**Odds when X = 151**

```
In [31]: odds2 = mrf(beta,np.array([[1,151]]))[0][0]/(1-mrf(beta,np.array([[1,151]]
))[0][0])
```

```
print 'The odds when X = 151 are: %.2f' %odds2
```

The odds when X = 151 are: 181.27

**Ratio of the odds:**

```
In [32]: oddsratio = odds2/odds1

print 'The ratio of the odds is: %.3f' %oddsratio

The ratio of the odds is: 1.221
```

**Calculate:**

$$e^{0.2}$$

```
In [33]: print 'The result is %.3f, which means the two quantities match as they should.' %np.exp(beta[1])[0]

The result is 1.221, which means the two quantities match as they should.
```

## Problem 14.9

```
In [34]: filename = '~/Documents/LinearRegression/STAT510/Kutner/CH14DS/CH14PR09.txt'

performance = pd.read_table(filename, delim_whitespace=True, names=['perform', 'stability',])
performance.insert(1, 'intercept', 1)
performance.head()
```

Out[34]:

|   | perform | intercept | stability |
|---|---------|-----------|-----------|
| 0 | 0       | 1         | 474       |
| 1 | 0       | 1         | 432       |
| 2 | 0       | 1         | 453       |
| 3 | 1       | 1         | 481       |
| 4 | 1       | 1         | 619       |

**a.**

**Fit the model to obtain MLEs for  $\beta_0$  and  $\beta_1$ .**

```
In [35]: model = sm.Logit(performance['perform'], performance[['intercept', 'stability']]).fit()

Optimization terminated successfully.
Current function value: 0.541514
Iterations 6
```

**The MLEs are given below:**

```
In [36]: DataFrame(model.params, columns = ['MLEs'])
```

```
Out[36]:
```

|           | MLEs       |
|-----------|------------|
| intercept | -10.308925 |
| stability | 0.018920   |

The response function is given by:

$$\pi_i = \frac{e^{-10.309+0.019X_i}}{1 + e^{-10.309+0.019X_i}}$$

**c.**

```
In [37]: ans1 = np.exp(model.params[1])

print 'The value of exp(b1) is %.3f.' %ans1
print ''
print 'This value means that the chance of performance increases by about
1.9 percent given a unit change in the stability score.'
```

The value of exp(b1) is 1.019.

This value means that the chance of performance increases by about 1.9 per cent given a unit change in the stability score.

**d.**

```
In [38]: ans2 = model.predict(exog=np.array([1,550]))[0]

print 'The estimated probability of performance given a stability score of
550 is %.3f.' %ans2
```

The estimated probability of performance given a stability score of 550 is 0.524.

**e.**

Solving the equation below for  $X_i$ :

$$\frac{e^{-10.309+0.019X_i}}{1 + e^{-10.309+0.019X_i}} = 0.7$$

We obtain the solution  $X_i = \frac{\ln(\frac{7}{3}) - b_0}{b_1} \Big|_{b_0=-10.309, b_1=0.019} = 582$

```
In [39]: ans3 = (np.log(7/3)-model.params[0])/model.params[1]

print 'We verify this numerically and obtain %.2f.' %ans3

We verify this numerically and obtain 581.51.
```

## Problem 14.20

```
In [40]: filename = '~/Documents/LinearRegression/STAT510/Kutner/CH14DS/CH14PR14.txt'

flu = pd.read_table(filename, delim_whitespace=True, names=['received', 'age', 'awareness', 'gender'])
flu.insert(1, 'intercept', 1)

flu.head()
```

```
Out[40]:
```

|   | received | intercept | age | awareness | gender |
|---|----------|-----------|-----|-----------|--------|
| 0 | 0        | 1         | 59  | 52        | 0      |
| 1 | 0        | 1         | 61  | 55        | 1      |
| 2 | 1        | 1         | 82  | 51        | 0      |
| 3 | 0        | 1         | 51  | 70        | 0      |
| 4 | 0        | 1         | 53  | 70        | 0      |

**b.**

Test the following hypothesis:

$$H_o : \beta_3 = 0$$

$$H_a : \beta_3 \neq 0$$

To test the above hypothesis we fit the model and conduct an F test.

**Fit the model to obtain MLEs for  $\beta_3$ .**

```
In [41]: modelf = sm.Logit(flu['received'], flu[['intercept', 'age', 'awareness', 'gender']]).fit()

Optimization terminated successfully.
Current function value: 0.330482
Iterations 7
```

In [42]: `model.summary()`

Out[42]: Logit Regression Results

|                       |                  |                          |           |
|-----------------------|------------------|--------------------------|-----------|
| <b>Dep. Variable:</b> | received         | <b>No. Observations:</b> | 159       |
| <b>Model:</b>         | Logit            | <b>Df Residuals:</b>     | 155       |
| <b>Method:</b>        | MLE              | <b>Df Model:</b>         | 3         |
| <b>Date:</b>          | Thu, 29 Oct 2015 | <b>Pseudo R-squ.:</b>    | 0.2212    |
| <b>Time:</b>          | 09:26:46         | <b>Log-Likelihood:</b>   | -52.547   |
| <b>converged:</b>     | True             | <b>LL-Null:</b>          | -67.470   |
|                       |                  | <b>LLR p-value:</b>      | 1.486e-06 |

|                  | coef    | std err | z      | P> z  | [95.0% Conf. Int.] |
|------------------|---------|---------|--------|-------|--------------------|
| <b>intercept</b> | -1.1772 | 2.982   | -0.395 | 0.693 | -7.023 4.668       |
| <b>age</b>       | 0.0728  | 0.030   | 2.396  | 0.017 | 0.013 0.132        |
| <b>awareness</b> | -0.0990 | 0.033   | -2.957 | 0.003 | -0.165 -0.033      |
| <b>gender</b>    | 0.4340  | 0.522   | 0.832  | 0.406 | -0.589 1.457       |

```
In [43]: b3 = model.params[3]
se3 = model.bse[3]

zstat = b3/se3

print 'The calculated Z-statistic is: %.3f.' %zstat

The calculated Z-statistic is: 0.832.
```

```
In [44]: zcrit = norm.ppf(0.975)

print 'The Z-critical value is: %.3f.' %zcrit
print ''
print 'Since the calculated Z-statistic is less than the Z-critical, we fail to reject the null hypothesis and conclude that we can drop gender from the model.'
```

The Z-critical value is: 1.960.

Since the calculated Z-statistic is less than the Z-critical, we fail to reject the null hypothesis and conclude that we can drop gender from the model.

**C.**

**Now we calculate the reduced model**



```
In [45]: modelr = sm.Logit(flu['received'], flu[['intercept','age','awareness']]).fit()
```

Optimization terminated successfully.  
Current function value: 0.332690  
Iterations 7

```
In [46]: modelr.summary()
```

Out[46]: Logit Regression Results

|                       |                  |                          |           |
|-----------------------|------------------|--------------------------|-----------|
| <b>Dep. Variable:</b> | received         | <b>No. Observations:</b> | 159       |
| <b>Model:</b>         | Logit            | <b>Df Residuals:</b>     | 156       |
| <b>Method:</b>        | MLE              | <b>Df Model:</b>         | 2         |
| <b>Date:</b>          | Thu, 29 Oct 2015 | <b>Pseudo R-squ.:</b>    | 0.2160    |
| <b>Time:</b>          | 09:26:47         | <b>Log-Likelihood:</b>   | -52.898   |
| <b>converged:</b>     | True             | <b>LL-Null:</b>          | -67.470   |
|                       |                  | <b>LLR p-value:</b>      | 4.690e-07 |

|                  | coef    | std err | z      | P> z  | [95.0% Conf. Int.] |
|------------------|---------|---------|--------|-------|--------------------|
| <b>intercept</b> | -1.4578 | 2.915   | -0.500 | 0.617 | -7.172 4.256       |
| <b>age</b>       | 0.0779  | 0.030   | 2.622  | 0.009 | 0.020 0.136        |
| <b>awareness</b> | -0.0955 | 0.032   | -2.946 | 0.003 | -0.159 -0.032      |

Calculate:

$$G^2 = -2 [\log L(R) - \log L(F)]$$

```
In [47]: Gsq = -2*(modelr.llf - modelf.llf)
G_pval = 1 - chi2.cdf(Gsq,1)

print 'The G squared statistic is: %.3f' %Gsq
print ''
print 'The approximate p-value is: %.3f' %G_pval
```

The G squared statistic is: 0.702

The approximate p-value is: 0.402

```
In [48]: Chicrit = chi2.ppf(0.95,1)

print 'The chi squared critical value with 1 degree of freedom is: %.3f' % Chicrit
print ''
print 'Since the G squared statistic is less than the Chi Squared critical value, we fail to reject the null hypothesis and conclude that gender can be dropped from the model.'
print ''
print 'This indicates that Wald test conducted in part b. agrees with the likelihood ratio test conducted in c.'
```

The chi squared critical value with 1 degree of freedom is: 3.841

Since the G squared statistic is less than the Chi Squared critical value, we fail to reject the null hypothesis and conclude that gender can be dropped from the model.

This indicates that Wald test conducted in part b. agrees with the likelihood ratio test conducted in c.