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Measurement of the Earth's rotational speed via Doppler shift of solar absorption lines

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This paper describes an experiment regularly presented to advanced undergraduate Physics students at the Universidad de los Andes in Bogotá, Colombia. The purpose of the experiment is to use high-resolution solar spectra to measure the horizontal speed of the laboratory caused by terrestrial rotation. Using this result, the radius of the Earth can be deduced. It is also possible to observe the Earth's motion towards or away from the Sun, and hence compute our planet's orbital eccentricity.
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I. INTRODUCTION

The ability to take high-resolution spectra opens up a wide range of topics for student experiments. One of these topics is the Doppler shift, available in a variety of astronomical objects including the Sun. With the aid of a telescope or long-focus converging lens, it is possible to use the Doppler shift to measure solar rotation. But even without such additional equipment, one can use the Doppler shift to measure the rotation of the Earth. Because the Earth's rotation results in an Eastward motion, we move toward the sun in the morning and away from the sun in the afternoon. Thus, morning spectra are blueshifted, while afternoon spectra are redshifted.

In order to collect enough data for a reasonable result, roughly one spectrum should be taken every hour for an entire day. Each spectrum must then be carefully analyzed to obtain the radial velocity (V_R) at the time of exposure. This radial velocity is the component of the velocity of the Earth's surface (V) along the Earth-Sun line (Fig. 1).

From the geometry in Fig. 1, we see that

$$V_R = V\sin(h), \tag{1}$$

where h is the solar hour angle. Thus, plotting the measured values of V_R versus $sin(h)$ should give a linear graph with slope V, the speed of the laboratory. Theoretically, the laboratory speed is

$$V = \frac{2\pi R\cos(\Lambda)}{T}, \tag{2}$$

where R is the radius of the Earth, Λ is the latitude, and T is one solar day. For our latitude of 5° we find $V = 462$ m/s. For a reasonable student experiment, we would like to be able to measure to within one-tenth of this value.

As $V \ll c$, the corresponding Doppler shift is quite small. The relation between speed and wavelength shift is[1]

$$\Delta\lambda = \lambda - \lambda_0 = (V/c)\lambda_0, \tag{3}$$

where λ is the observed wavelength and λ_0 is the natural, or rest-frame, wavelength. Equation (3) is the low-speed limit to the relativistic Doppler formula. In our case $V/c \sim 10^{-6}$, and if the wavelength is, say, 6000 Å, the Doppler shifts will be smaller than 0.01 Å.

II. DATA ACQUISITION

In order to measure such small Doppler shifts, the first requirement is a good spectrograph with a linewidth of about a tenth of an Angstrom. A spectrograph is simply a device that records the spectrum using a photographic camera, while the term linewidth refers to the width of a recorded spectral line from a monochromatic light source. We use the locally built ESPARTACO spectrograph available at our campus observatory. This instrument features a 127-cm length collimator, a reflective diffraction grating of 2400 lines/mm, and a camera focal length of 91 cm. Light is introduced by a 50-μm optical fiber with no slit, and the image is recorded using a CCD camera. The resulting dispersion is about 33 pixels per Angstrom. Additional details of this low cost, high-resolution spectrograph can be found in Ref. 2. For this experiment, it is not necessary to build or acquire a similar instrument; a temporary setup on a good optical bench should perform satisfactorily and may itself provide a nice student project in optics.

In order to determine the Doppler shifts, an absolute wavelength standard is required. This is normally accomplished by taking a reference spectrum from a calibration lamp before and/or after each solar spectrum. Unfortunately, this method is not reliable enough for our purpose because the spectrograph parameters, such as camera position and grating angle, have been observed to vary slightly between one exposure and the next. Therefore, we make use of the atmospheric oxygen absorption lines, which are included in the solar spectrum itself; they are abundant in a narrow band near 6300 Å.

We collect sunlight directly into the optical fiber without using a telescope. In this way, incoming light is integrated over the entire solar disk, conveniently canceling the effect of solar rotation. Nevertheless, the fiber should not be pointed directly at the sun because such a highly collimated input produces intermodal interference effects within the fiber.[3] Instead, the fiber may be pointed at a bright cloud, a white wall, or even a cloudy sky—sunny weather is not required!

III. SPECTRAL ANALYSIS

After obtaining the spectra, the first step is to reduce each CCD image (Fig. 2) to an intensity-position profile (Fig. 3). To accomplish this reduction, we use a freely available astronomical image-processing program called IRIS.[4]

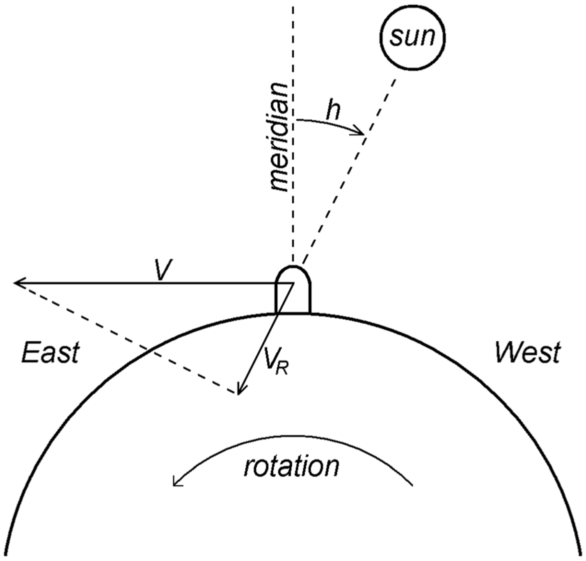


Fig. 1. Schematic view of the Earth-Sun geometry looking South. V is the observer's speed due to terrestrial rotation and h is the solar hour angle.

After reducing the images, we need to identify all possible lines in each spectrum. We use Charlotte Moore's classic solar catalog,[5] but alternatives are available, such as the Spectroweb facility.[6] A visual comparison of the experimental spectrum (Fig. 3) with the literature spectrum allows one to identify each recorded absorption line, thus giving its natural wavelength and the chemical element that produces it. Any misidentification will show up as an outlier as described below.

Next, the position of each line on the spectrum must be measured. A visual way of doing this is to zoom into a spectral line and draw a straight vertical line that bisects its area. This procedure gives positions accurate to around a tenth of a pixel, which corresponds to a wavelength uncertainty of a few mÅ. Though not necessary, more precise readings can be obtained by fitting each spectral line to a Gaussian profile. When the rest-frame wavelengths are plotted against these positions, a very linear graph is obtained (Fig. 4); any outlying points represent misidentified spectral lines.

Although most students are readily satisfied with a linear fit to these data, there is actually a small curvature due to the non-linear dispersion of the spectrograph, and this must be accounted for in order to measure the small Doppler shifts. Thus, a second-order polynomial is fitted to the atmospheric lines and represents the observed wavelengths. The fitting polynomial may be the same for all spectra or it may be optimized for each individual spectrum; the latter gives slightly smaller error bars but essentially the same results. In either case, the oxygen lines must be measured every time in order to correct for possible instrumental shifts as mentioned above.

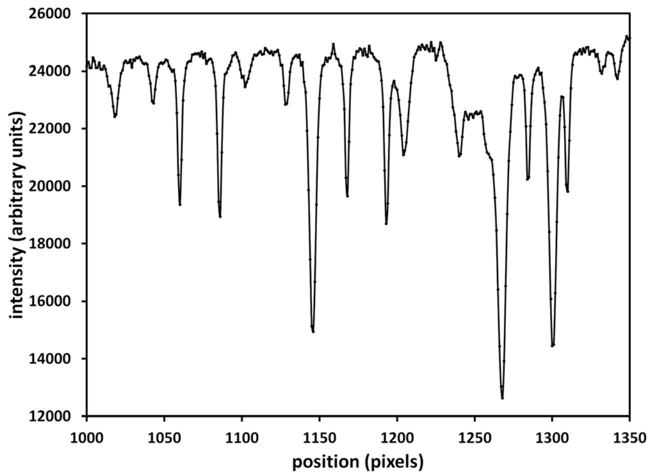


Fig. 3. Intensity profile of the spectral portion shown in Fig. 2.

After subtraction of the optimal parabolic fit, the offset between solar and telluric lines becomes evident and can be measured as shown in Fig. 5. The solar spectral lines show a difference between their observed and natural wavelengths. This vertical displacement between the two resulting groups of spectral lines is the desired spectral shift $\Delta\lambda$.

The spectral shift can now be converted to a radial velocity using Eq. (3). Because λ varies only slightly within the studied portion of the spectrum (note that the spectral range is only about 50 Å), a single mean value for λ may be used. The radial velocity is taken to be positive if the spectrum is redshifted, i.e., if observed wavelengths are greater than the natural wavelengths.

An uncertainty may be estimated for each radial velocity from the dispersions of the two groups of residuals. Using standard error-propagation techniques, we obtain a typical error of around 40 m/s.

IV. RESULTS

A plot of V_R versus $\sin(h)$ is shown in Fig. 6; the linear relationship is evident. The observer's eastward speed, as read from the slope of a linear fit, is $V = 472 \pm 27$ m/s. Note that the data in Fig. 6 have a significant vertical offset that surprises many students. This offset implies motion towards or away from the Sun and hence an elliptical orbit. Our planet reaches perihelion (nearest to the sun) on January 3 each year, and aphelion (farthest from the sun) on July 5. The data presented here were taken between September 7 and 10, 2009 and appropriately exhibit a movement toward the Sun with a radial velocity of -455 ± 17m/s. For comparison, a group of students who took their data on April 20, 2010 obtained a radial velocity of $+408$ m/s, indicating motion away from the sun.

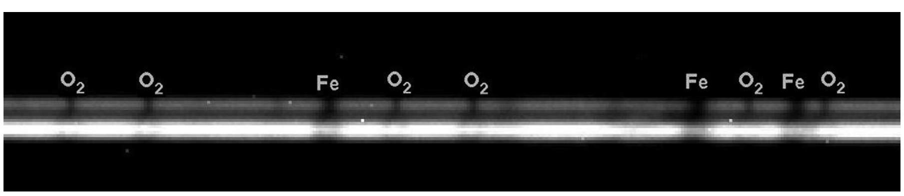


Fig. 2. Portion of the solar spectrum as imaged by the spectrograph. The upper band is properly focused while the lower, brighter band is out of focus; this is caused by a flaw in our self-built instrument.

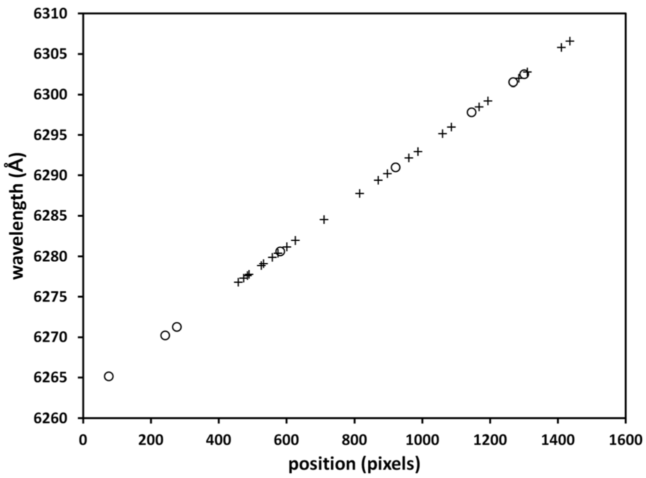


Fig. 4. Correspondence between wavelength and position for 24 atmospheric oxygen lines (crosses) and 8 solar iron lines (circles). This graph is important to verify the correct identifications of all spectral lines.

Knowing the speed of the Earth's surface, we can immediately determine the Earth's radius using Eq. (2). Using our laboratory speed measurement, we find $R = 6,500 \pm 370$ km.

In addition to the radius of the Earth and the speed of its surface, we can also determine the eccentricity of the Earth's orbit. We begin with the equation for an ellipse in polar coordinates (r, ϕ), given by

$$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos(\phi)}, \tag{4}$$

where a is the semi-major axis and ε is the eccentricity of the orbit. Differentiating with respect to time gives

$$\frac{dr}{dt} = \frac{\varepsilon\sin(\phi)}{a(1-\varepsilon^2)}\left(r^2\frac{d\phi}{dt}\right). \tag{5}$$

Now, $(1/2)r^2\, d\phi/dt$ is the time derivative of the area swept out by the Sun-Earth line, and from Kepler's second law we know this is constant and therefore equal to the total area of the ellipse $A = \pi a^2\sqrt{1-\varepsilon^2}$ divided by the total orbital period T_o (1 year). Equation (5) then becomes

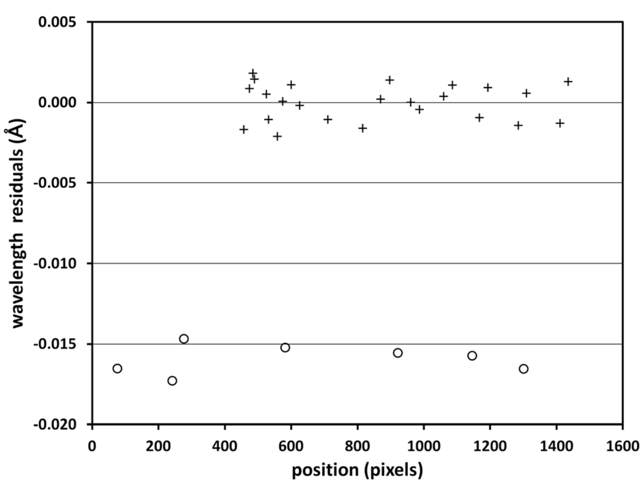


Fig. 5. Wavelength residuals after subtracting the optimal quadratic fit to the atmospheric lines (crosses). The solar lines (circles) show a blueshift of 0.016 Å.

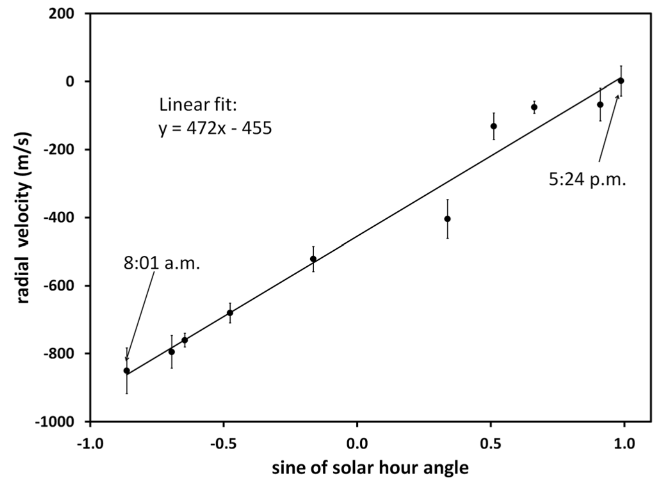


Fig. 6. Plot of the measured radial velocities. The slope gives the Earth's rotational speed while the vertical offset indicates an additional constant velocity, in this case directed towards the sun, due to the Earth's orbital eccentricity.

$$\frac{dr}{dt} = \frac{\varepsilon\sin(\phi)}{\sqrt{1-\varepsilon^2}}\frac{2\pi a}{T_o}, \tag{6}$$

which, for small values of ε, gives

$$\frac{dr}{dt} \approx V_o\varepsilon\sin(\phi), \tag{7}$$

where $V_o = 2\pi a/T_o$ is the (approximate) mean orbital speed and ϕ is our orbital position measured from perihelion. Using $a = 1$ AU, the above data give $\varepsilon = 0.017$ for the September measurement and $\varepsilon = 0.014$ for the April result. The accepted value is $\varepsilon = 0.0167$. For these calculations, we have approximated ϕ as being proportional to time. While not strictly true, this approximation is accurate enough for our purposes.

V. FINAL COMMENTS

It is interesting to compare a spectral profile (Fig. 3) taken at noon with another one taken near sunset. While the solar absorption lines have similar strengths (or depths) in both spectra, the oxygen lines vary significantly. Late in the afternoon they become much stronger because the sunlight has to travel through much more of the atmosphere. This effect helps to address a frequent question from students: How can you tell which spectral lines come from the Sun and which are produced in the Earth's atmosphere?

As a demonstration of the optical Doppler effect, this experiment is conceptually simpler than some previously reported procedures[7,8] that involve the interference, or "beat," phenomenon to measure the frequency shift. Here, the Doppler shift is measured directly without invoking any other effect. One drawback, however, is that the speed we are measuring cannot be independently controlled.

Viewed from a different perspective, this experiment provides pedagogical "proof" of the Earth's rotation as well as its non-circular orbital motion. Although a Foucault pendulum generally provides a simpler and more elegant example of the Earth's rotation, it is not very successful at our nearly equatorial location. Thus, it seems fitting that this is precisely the location where the speed of the Earth's surface has its greatest value.

ACKNOWLEDGMENTS

The author thanks the referees for their careful proofreading and their numerous important comments, which helped to significantly improve the manuscript.

[1]See, for example, HyperPhysics on Relativistic Doppler Effect at http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/reldop2.html.

[2]B. Oostra and D. Ramírez, "ESPARTACO, a high-resolution, low-cost spectrograph for students," Rev. Colomb. Física **43**(2), 312–317 (2011), available online at http://revcolfis.org/ojs/index.php/rcf/article/view/430224/pdf.

[3]P. Hlubina, "Measuring dispersion between two modes of an optical fibre using low-coherence spectral interferometry with a Michelson interferometer," J. Mod. Optic **46**(11), 1595–1604 (1999).

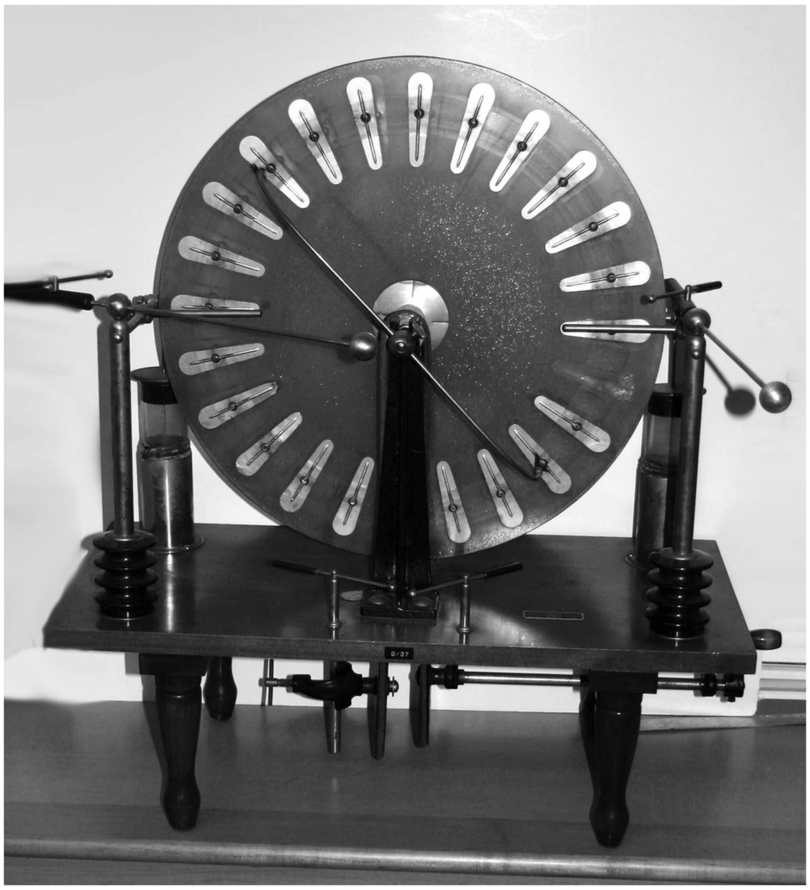
[4]IRIS software is superbly documented and can be downloaded from http://astrosurf.com/buil/us/iris/iris.htm.

[5]C. E. Moore, M. G. J. Minnaert, and J. Houtgast, "The solar spectrum 2935 to 8770 Angstrom," National Bureau of Standards Monograph (1966).

[6]Spectroweb: The Interactive Database of Spectral Standard Star Atlases is available online at http://spectra.freeshell.org/spectroweb.html.

[7]T. D. Nichols, D. C. Harrison, and S. S. Alpert, "Simple laboratory demonstration of the Doppler shift of laser light," Am. J. Phys. **53**(7), 657–660 (1985).

[8]R. H. Belansky and K. H. Wanser, "Laser Doppler velocimetry using a bulk optic Michelson interferometer: A student laboratory experiment," Am. J. Phys. **61**(11), 1014–1019 (1993).



Wimshurst Machine. While American college physics departments may have newer Wimshurst machines, this big Cenco machine with twenty-inch glass plates is a sturdy survivor and I have found examples in many departments. A slightly smaller ancestor with vulcanite plates is in the 1909 catalogue at $48; by 1937 the machine had grown to full size and cost $80. Replacement glass plates with all of the sectors attached cost $7.50 each. The machine generated a spark 14 cm long and could be used for x ray demonstrations (!) This well-preserved example is in the physics department at Washington & Lee University in Lexington, Virginia. (Notes and photograph by Thomas B. Greenslade, Jr., Kenyon College)