Parcial N° 1

Ejercicio N° 1:

Punto $(-2, 3, 2)$

$$L_1 \begin{cases} x + 3y = -7 \\ 2y + z = 7 \end{cases}$$

Para L_1:

$y = t$

$x + 3t = -7$

$x = -7 - 3t$

$2t + z = 7$

$z = 7 - 2t$

$$\boxed{L_1 \begin{cases} x = -7 - 3t \\ y = t \\ z = 7 - 2t \end{cases}}$$ → Paramétrica L_1

$$L_2 \begin{cases} -x + 2y + 3z = 5 \\ x - y - 3z = 4 \end{cases}$$

$y = 9$

↳ No se puede pasar a paramétrica

Ejercicio N° 2:

$$L1 \begin{cases} x = 3 + 10t \\ y = 2 - 2t \\ z = -4 + 6t \end{cases} \qquad L2 \begin{cases} x = -2 - 20r \\ y = 5 + 4r \\ z = 45 - 12r \end{cases}$$

$$\vec{v1} = \langle 10, -2, 6 \rangle \qquad \vec{v2} = \langle -20, 4, -12 \rangle$$

Sacando el vector del plano.

$$\vec{n} \cdot \vec{v1} = 0 \qquad \langle a, b, c \rangle \cdot \langle 10, -2, 6 \rangle = 0$$

$$10a - 2b + 6c = 0$$

$$\vec{n} \cdot \vec{v2} = 0 \qquad \langle a, b, c \rangle \cdot \langle -20, 4, -12 \rangle = 0$$

$$-20a + 4b - 12 = 0$$

Despejando en terminos de c.

$$-20a + 4b = 12c$$
$$10a - 2b = -6c$$

$$10a = -6c + 2b$$

$$a = \frac{-6c + 2b}{10}$$

Sacando b:

$$a - 20\left(\frac{-6c+2b}{10}\right)$$

$$12 - 4b - 2b = -6$$

$$-6b = -12c - 6c$$

$$b = \frac{-18}{6}c$$

$$b = 3c$$

Sustituyendo $b = 3c$ en a

$$a = \frac{-6c + 2(3)}{10}$$

$$a = \frac{6c+6}{10}$$

$$\vec{n}\left(\frac{-6c+6}{10}, 3, c\right) \rightarrow \quad c = 3$$

$$\vec{n}\left(\frac{-18+6}{10}, 3, 3\right)$$

Sustituyendo

$$-\frac{6}{5}(x-3) + 3(y-2) + 3(z+4) = 0$$

$$-\frac{6}{5}x + \frac{18}{5} + 3y - 6 + 3z + 12 = 0$$

$$-\frac{6}{5}x + 3y + 3z = -\frac{48}{5}$$

Ejercicio N°3:

Buscando centro y radio de las esferas.

Ecuación 1: $x^2+y^2+z^2-28x-8y-6z+196=0$

$$x^2-28x+196+y^2-8y+16+z^2-6z+9 = -196+196+16+9$$

$$(x-14)^2+(y-4)^2+(z-3)^2 = 25$$

Centro: $(14, 4, 3)$ radio $= 5$

Ecuación 2: $\frac{3x^2}{3}+\frac{3y^2}{3}+\frac{3z^2}{3}-\frac{44x}{3}-\frac{4y}{3}+\frac{22z}{3}+\frac{128}{3}=0$

$$x^2+y^2+z^2-\frac{44}{3}x-\frac{4}{3}y+\frac{22}{3}z+\frac{128}{3}=0$$

$$x^2-\frac{44}{3}x+\frac{484}{9}+y^2-\frac{4}{3}y+\frac{4}{9}+z^2+\frac{22}{3}z+\frac{121}{9} = -\frac{128}{3}+\frac{484}{9}+\frac{4}{9}+\frac{121}{9}$$

$$\left(x-\frac{22}{3}\right)^2+\left(y-\frac{2}{3}\right)^2+\left(z+\frac{11}{3}\right)^2 = 25$$

Centro: $\left(\frac{22}{3}, \frac{2}{3}, -\frac{11}{3}\right)$ radio $= 5$

Ecuación 3: $\frac{3x^2}{3}+\frac{3y^2}{3}+\frac{3z^2}{3}-\frac{20x}{3}-\frac{22y}{3}-\frac{38z}{3}+\frac{119}{3}=0$

$$x^2+y^2+z^2-\frac{20}{3}x-\frac{22}{3}y-\frac{38}{3}z+\frac{119}{3}=0$$

$$x^2-\frac{20}{3}x+\frac{100}{9}+y^2-\frac{22}{3}y+\frac{121}{9}+z^2-\frac{38}{3}z+\frac{361}{9} = -\frac{119}{3}+\frac{100}{9}+\frac{121}{9}+\frac{361}{9}$$

$$\left(x-\frac{10}{3}\right)^2+\left(y-\frac{11}{3}\right)^2+\left(z^2-\frac{19}{3}\right)^2 = 25$$

Centro: $\left(\frac{10}{3}, \frac{11}{3}, \frac{19}{3}\right)$ radio $= 5$

Encontrando las ecuaciones de las esferas desconocidas:

$$PM_{c1\,c4}\left(\frac{x+14}{2}, \frac{y+4}{2}, \frac{z+3}{2}\right)=\left(\frac{22}{3}, \frac{2}{3}, -\frac{11}{3}\right)$$

$$\frac{x+14}{2}=\frac{22}{3} \qquad \frac{y+4}{2}=\frac{2}{3} \qquad \frac{z+3}{2}=-\frac{11}{3}$$

$$x=\frac{2}{3} \qquad y=-\frac{8}{3} \qquad z=\frac{-31}{3}$$

$$C_4\left(\frac{2}{3}, -\frac{8}{3}, -\frac{31}{3}\right)$$

Ecuacion canonica de la esfera 4:

$$\left(x-\frac{2}{3}\right)^2+\left(y+\frac{8}{3}\right)^2+\left(z+\frac{31}{3}\right)^2=25$$

Para buscar el centro 5: Primero hay que buscar la recta que esta entre las esferas que estan arriba.

$$\vec{V_{c1\,c2}}=\left\langle \frac{22}{3}-14, \frac{2}{3}-4, \frac{-11}{3}-3\right\rangle$$

$$\vec{V_{c1\,c2}}=\left\langle -\frac{20}{3}, \frac{-10}{3}, \frac{-20}{3}\right\rangle \left(-\frac{3}{10}\right)$$

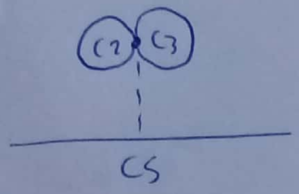
$$\vec{V_{c1\,c2}}=\langle 2,1,2\rangle$$

$$L1\begin{cases} x=\frac{10}{3}+2t \\ y=\frac{11}{3}+t \\ z=\frac{19}{3}+2t \end{cases}$$

PM ente el C2 y el C4:

$$\left(\frac{\frac{22}{3} + \frac{2}{3}}{2}, \frac{\frac{2}{3} + \frac{8}{3}}{2}, \frac{-\frac{11}{3} - \frac{31}{3}}{2} \right)$$

$$= (4, -1, -7)$$



$$\left.\begin{cases} x = \frac{10}{3} + 2t & = \frac{10}{3} + 2\left(-\frac{10}{3}\right) = -\frac{10}{3} \\ y = \frac{11}{3} + t & = \frac{11}{3} - \frac{10}{3} = \frac{1}{3} \\ z = \frac{14}{3} + 2t & = \frac{19}{3} + 2\left(-\frac{10}{3}\right) = \frac{1}{3} \end{cases}\right\} \left(-\frac{10}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$\langle x-4, y+1, z+7 \rangle \langle 2,1,2 \rangle$$

$$2x - 8 + y + 1 + 2z + 14 = 0$$

$$2x + y + 2z + 7 = 0$$

$$2\left(\frac{10}{3} + 2t\right) + \left(\frac{11}{3} + t\right) + 2\left(\frac{19}{3} + 2t\right) + 7 = 0$$

$$\frac{20}{3} + 4t + \frac{11}{3} + t + \frac{38}{3} + 4t + 7 = 0$$

$$9t + 30 = 0$$

$$t = -\frac{30}{9}$$

$$\boxed{t = -\frac{10}{3}}$$

Ecuacion de la esfera S_1

$$(x+\tfrac{10}{3})^2+(y-\tfrac{1}{3})^2+(z-\tfrac{1}{3})^2=25$$

Para L2 y L3:

Para L2:

C1 y C3 $(14,4,3)$ y $\left(\frac{10}{3},\frac{11}{3},\frac{19}{3}\right)$

$$\overrightarrow{V_{C1C3}} = \left\langle \frac{10}{3}-14,\ \frac{11}{3}-4,\ \frac{19}{3}-3 \right\rangle$$

$$\overrightarrow{V_{C1C3}} = \left\langle -\frac{32}{3},\ -\frac{1}{3},\ \frac{10}{3} \right\rangle$$

$$L2\begin{cases} x = 14-\frac{32}{3}t \\ y = 4-\frac{1}{3}t \\ z = 3-\frac{10}{3}t \end{cases}$$

Para L3:

C4 y C5 $\left(\frac{2}{3},-\frac{8}{3},\frac{31}{3}\right)$ $\left(-\frac{10}{3},-\frac{1}{3},-\frac{1}{3}\right)$

$$\overrightarrow{V_{C4C5}} = \left\langle -\frac{10}{3}-\frac{2}{3},\ -\frac{1}{3}-\left(-\frac{8}{3}\right),\ -\frac{1}{3}-\frac{31}{3} \right\rangle$$

$$\overrightarrow{V_{C4C5}} = \left\langle -4,\ \frac{7}{3},\ -\frac{32}{3} \right\rangle$$

$$L3\begin{cases} x = \frac{2}{3}-4t \\ y = -\frac{8}{3}+\frac{7}{3}t \\ z = \frac{31}{3}-\frac{32}{3}t \end{cases}$$