Extended Comment on 'Statistical Modelling of Citation Exchange between Statistics Journals'

1) Introduction:

In the Discussion on 'Statistical Modelling of Citation Exchange Between Statistics Journals' by Cristiano Varin, Manuela Cattelan and David Firth, we briefly consider a networks analysis of the statististics journal citation data discussed in the article.

The following is an extended discussion, expanding on the network models and adding visualizations and details of implementation in R.

2) Visualization:

Julyan Arbel writes in a blog post that he is surprised to see no "networks representation" of the citation data presented in the article by Varin, Cattelan, and Firth (2016; Arbel 2015). Plotting the network of journals offers a succinct summary of relationships, clustering and centrality. Of course, our interpretation of such a plot is sensitive to the plotting algorithm, parameters, and any underlying model.

The first figure in Arbel's post is a network plot prepared using the Gephi software (http://gephi.github.io/). It shows the traffic between journals via the edge weights, although for visual clarity only the top decile of weightiest edges is included. However, the distances between journals are not meaningful and are instead (I believe) optimized for appearance.

Similarly, Figure 1 below presents a thinned down version of the citation network. An edge $(i \to j)$ is only visible if j receives at least seven percent of the citations given by i. It shows that several journals are "parents" of some beneath them, but many journals are on the same level. It is evident why *Journal of the Royal Statistical Society, Series B* (JRSS-B), *Annals of Statistics* (AoS), *Biometrika* (Bka) and *Journal of the American Statistical Association* (JASA) are consistently the top four ranked journals, affirming "diffuse opinion within the statistical community" (Varin, Cattelan, and Firth 2016).

Figures? and? below provide examples of citation network plots in which distances between journals are model-based. The discussion describes the underlying models and corresponding code.

3) Network Modelling:

The Stigler model estimates "export scores", u_i , such that c_{ij} is assumed binomially distributed with $E(c_{ij}) = t_{ij} \exp(\alpha_i + \beta_j)$ and $u_i = \alpha_i - \beta_i$, as in 'quasi-symmetry' formulation (4). Note that in Varin et al. the data is stored in an 47 x 47 cross-citation matrix C, in which c_{ij} is the number of times journal i is cited by journal j excluding self-citations. T is the symmetric total citation matrix, i.e. $t_{ij} = c_{ij} + c_{ji}$ for $i \neq j$ and 0 otherwise. To faciliate modeling in this paper I have transposed the data to fit a standard network framework such that c_{ij} is the number of times journal i cites journal j. This change is reflected in the notation below. In this notation, the 'quasi-symmetry' formulation (4) is expressed $E(c_{ij}) = t_{ij} \exp(\alpha_j + \beta_i)$ and $u_i = \alpha_i - \beta_i$.

We can place these assumptions in the context of an exponential random graph model (ERGM) on a valued network, where edge weights are directed citation counts. A direct extension of the Stigler model would retain the assumption of binomially distributed citations. However, for ease of modelling we assume Poisson-distributed citatons with mean c_{ij} , thereby modelling a count instead of a proportion. We include sender (β_i) and receiver (α_i) effects, so that our assumption is $c_{ij} \sim pois(\lambda_{ij} = exp(\beta_i + \alpha_j))$. The model formula is below.

Computational note: We can easily implement this (Poisson) network model using the latentnet package in R (Krivitsky and Handcock 2015). The Poisson model is used because in latentnet I was unable to set

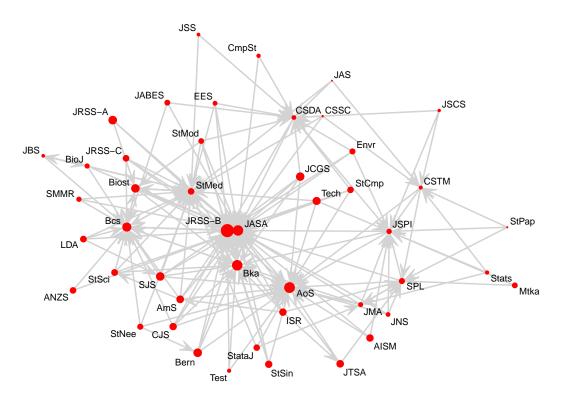


Figure 1: Statistics journal citation network. Edges only pictured if receiver accounts for at least seven percent of sender's total citations sent. Vertex sizes are scaled to export scores under the Stigler model.

unique binomial trials for each ij, although the ergmm helper functions seems to indicate that it is possible. (See details in the binomial_error code chunk in the markdown version of this document.) It may seem natural to use the ergm.count (Krivitsky 2015) package for weighted networks instead, but that package seems not to have sender and receiver terms implemented. In addition, using the latentnet package sets up later analysis.

```
latent.srp1 = ergmm(Cnet ~ sender(base=0) + receiver(base=0) - 1, response = "citations",
family="Poisson.log", control = control.ergmm(pilot.runs=1), seed = 123)
```

The Stigler export scores reported in Table 5 of Varin et al. are highly correlated (.95) with the corresponding estimates from this model $(\alpha_i - \beta_i)$. The plot below compares rankings by the two methods, showing their similarity. Differences may be due in most part to the switch from binomial to Poisson expectations. (Code for a plot comparing scores instead of ranks is in the latent_sr_analysis code chunk in the markdown version of this document.)

4) Equivalence to GLM:

It is interesting to note that the estimates of export scores in Table 5 of Varin et al. are, up to scaling, identical to estimates from a binomial glm with logit link on proportional citation counts. The response is a vector of $\{\frac{c_{ij}}{t_{ij}}\}$ with zeros for diagonal elements. The zero-sum constraint on the export scores described in Section 5.1 of Varin et al. is not implemented so one score is NA. (For details, see the glm code chunk in the markdown version of this document.)

5) Network Model Extensions:

A benefit of the network model described above is extensibility, both theoretically and computationally.

Rank Comparison

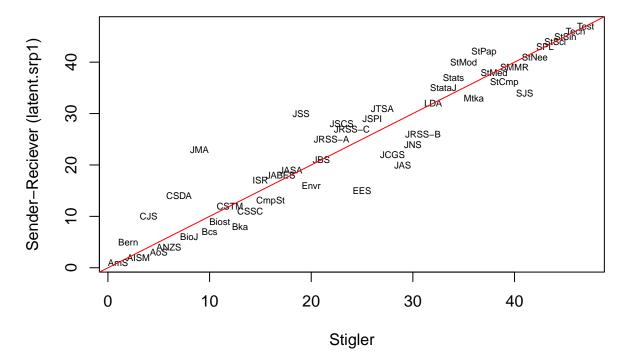


Figure 2: Comparison of rankings by the Stigler and sender-receiver (latent.srp1) model. There is more differentiation towards the middle due to scores being more tightly clustered.

We can make minor adjustments to the latent.srp1 model above, such as including an intercept term for dyad-wise distributions, i.e., $\lambda_{ij} = \exp(\theta + \beta_i + \alpha_j)$. We can also make more significant changes, such as locating journals in latent space and adding cluster labels. These models and corresponding visualizations provide a deeper understanding of the landscape of journals.

5.1) Latent Space Model:

As a preliminary extension, consider the example of a two-dimensional latent space model using the latentnet package (Krivitsky and Handcock 2015). This model posits distances between journals as latent variables that affect edge weights (i.e., citation counts). Below, latent.srp2 adds to latent.srp1 that journals reside in two-dimensional euclidean space, i.e., $\lambda_{i,j} = \exp(\|Z_i - Z_j\| + \beta_i + \alpha_j)$. (See formulation in Equation 4 of Krivitsky et al. (2009). For background see Hoff, Raftery, and Handcock (2002); Krivitsky and Handcock (2015).)

```
latent.srp2 = ergmm(Cnet~euclidean(d=2) + sender(base=0) + receiver(base=0) - 1,
    response = "citations", family="Poisson.log", seed=123,
    control=ergmm.control(interval=200, sample.size=10000, burnin=100000))
```

Estimates of export scores from the Stigler model are very highly correlated (.99) with the corresponding estimates from this model (receiver minus sender coefficient). The rankings differ only slightly, as shown in Figure?. (Code for a three-dimensional latent space model is in the latent_sr3 code chunk and plotted against the Stigler model in the latent_sr23_analysis chunk. The correlation is greater than .99)

Below (Figure?, left) is a plot of estimated journal positions from latent.srp2. Node sizes are scaled to receiver minus sender coefficient. The coloring corresponds to the clustering model of Varin et al. (see their

Rank Comparison

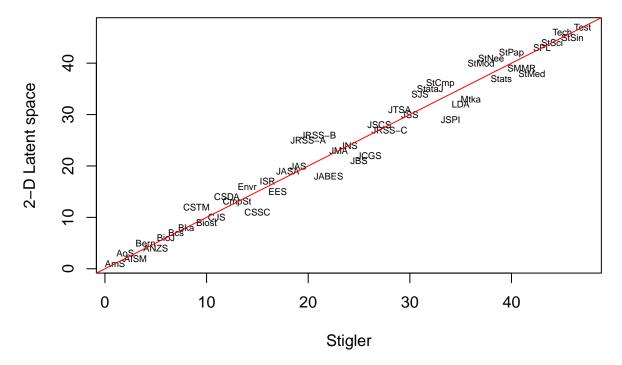


Figure 3: Comparison of rankings by the Stigler and two-dimensional latent space (latent.srp2) model.

Section 3), as do the cluster labels on the right. Although there is no clustering term in our latent space model, the clustering of the authors is fairly well captured. The plot shows how the clusters fit together, and which journals are neighbors. However, we should be careful not to put too much stock in the exact positions. The right-hand plot displays the uncertainty in the positions using a sample of draws from the model.

Figure? gives a visual aid to the observation (see Section 7.2 of Varin et al.) that many journals are not significantly different in rank, and therefore 'grouped' rankings are more appropriate than traditional ordering. We see a periphery of mostly low-ranked journals on the right and a small cluster of leading journals around JRSS-B. Roughly, journals decrease in rank from left to right, but middle-ranked journals are dispersed widely from top to bottom. Centrality does not equate to rank or prestige, as we can see with *Biometrics* (BSC) and Bernoulli (Bern) on the edge of the plot.

Further, the two-dimensional latent space model (latent.srp2) accounts better for topical connections between journals than the sender-receiver only model (latent.srp1). The residuals of latent.srp2 have range (-26,27) with standard deviation 3.7, while those of latent.srp1 have range (-128,79) with standard deviation 8.7. The largest residual of latent.srp1 corresponds to citations from Biometrics (Bcs) to Statistics in Medicine (StMed). There is a clear topical connection between those journals that is not captured by their general tendency to import/export citations. In the latent.srp2 model they are positioned near eachother and that residual drops to 3.

5.2) Latent Space Cluster Model:

We can further extend the model to include a clustering term for a fixed number of clusters. In addition to estimating cluster assignments this provides probabilities of cluster membership for each journal. (See Krivitsky and Handcock (2015), Krivitsky and Handcock (2008)). The resulting model reveals that divisions between clusters are soft and many journals should be thought to straddle two or more clusters.

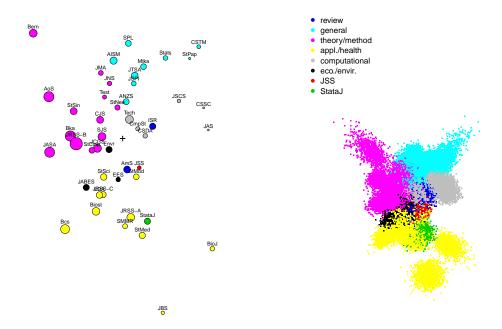


Figure 4: Estimated journal positions from the two-dimensional latent space model. Left: Point estimates with node size scaled to receiver minus sender coefficient. Right: Sample of positions from the model. Colouring is due to the hierarchical clustering of Varin et al.

Figure ? (left) shows the output of a three-cluster latent space model in three-dimensional space (latent.srp3.3). For visual clarity, an edge $i \to j$ is only shown if it accounts for at least seven percent of j's total citations received. In comparison, the three-cluster model in two-dimensional space (latent.srp2.3, Figure ? top right) does not separate clusters well, as shown by the overlap in the variance circles (radii equals the square root of intracluster variance (Krivitsky and Handcock 2015)). The middle-right plot of Figure ? shows the probabilistic group membership underlying the left-hand plot. Finally, the bottom-right plot uses the latent.srp3.3 estimated positions, but colors nodes by the hiearchical clustering of Varin et al. The latent.srp3.3 model agglomerates the hierarchical clusters into two major groups, one that is mostly applications and one that is theoretical, general and computational. The hierarchical 'review' cluster straddles the latent space clusters, reflecting its broad subject matter. (Code for a few additional models and plots is in the latent_sr?? code chunks in the the markdown version of this document. I was unable to reliably fit a latent space model with six or more clusters for direct comparison to the hierarchical clustering.)

The scores and ranks calculated from the cluster models are very similar to earlier models. (Code to compare models is in the compare_clustered code chunk in the markdown version of this document.) Only a few journal ranks change by more than a few places. We reiterate that ranking differences may not be significant, but reference them for ease of comparison.

6) Network Structure:

The Stigler model offers a method to test for significant difference in the export scores of two journals, but does not give an overall picture of network hierarchy or structure. Along with visualization, descriptive network statistics help to analyze network structure.

The dispersed hierarchy of statistics journals is reflected in the triad census. In the thinned network of Figure 1, 021U triads make up the majority of triads other than the single-edge type, 012, which suggests hierarchy and overall sparsity. However, there are also significant counts of 102 and 111D type triads, indicating more lateral connections. (See Figure? for triad labels.) There are no cyclic triads of type O30C and 120C, which would indicate a lack of hierarchy, and very few of the full (300) and near-full (210) types which also contain cycles.

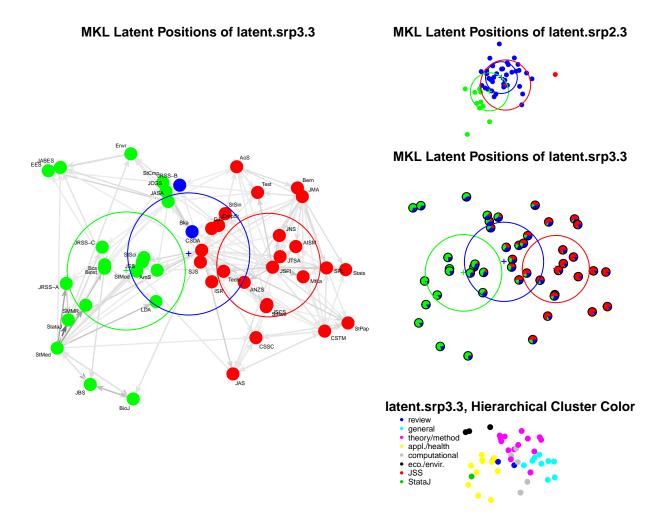


Figure 5: Comparison of two- and three-dimensional latent space cluster models. (Left) Edges only pictured if sender accounts for at least seven percent of receiver's total citations received.

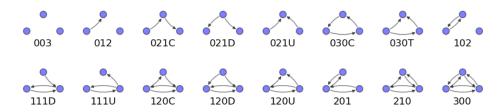


Figure 6: Triad types

In contrast, in the full network the most popular triads are by far the full (300) or almost-full (210) types, indicative of the network density. The R output below shows all network triad statistics for the thinned down (Cnet.strong) and full network (Cnet).

summary.statistics(Cnet.strong~triadcensus())

```
triadcensus.012 triadcensus.021D triadcensus.021U
##
##
              3626
## triadcensus.021C triadcensus.111D triadcensus.111U triadcensus.030T
                                189
##
                92
                                                  5
  triadcensus.030C
                    triadcensus.201 triadcensus.120D triadcensus.120U
##
##
                 0
                                  3
                                                 88
                                                                   7
## triadcensus.120C
                    triadcensus.210
                                    triadcensus.300
```

summary.statistics(Cnet~triadcensus())

```
triadcensus.012 triadcensus.102 triadcensus.021D triadcensus.021U
##
##
                998
                                 794
                                                   301
  triadcensus.021C triadcensus.111D triadcensus.111U triadcensus.030T
##
                376
                                1365
                                                   788
##
                     triadcensus.201 triadcensus.120D triadcensus.120U
## triadcensus.030C
##
                 47
                                1557
                                                   860
                                                                     491
## triadcensus.120C
                     triadcensus.210 triadcensus.300
##
                663
                                3596
                                                  3131
```

7) Conclusions:

References

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Hoff, Peter D., Adriean E. Raftery, and Mark S. Handcock. 2002. "Latent Space Approaches to Social Network Analysis." *Journal of the American Statistical Association* 97 (460): 1090–1098.

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