GammaPBHPlotter: A public code for calculating the complete Hawking evaporation gamma-ray spectra from primordial black holes

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Abstract

We present `GammaPBHPlotter`, a public Python package for calculating and plotting the Hawking-radiation gamma-ray spectra of primordial black holes (PBHs) in the mass range of 10^{14}–10^{19} g. The code computes instantaneous and time-averaged spectra and includes all four photon components: direct Hawking emission, secondary emission from hadronization/decays (via `BlackHawk`+`PYTHIA`), final-state radiation, and in-flight annihilation. For speed, spectra at 56 logarithmically spaced PBH masses are precomputed and interpolated in log space. Users can generate spectra for monochromatic, Gaussian, non-Gaussian-collapse, and log-normal PBH mass distributions, visualize dN_γ/dE_γ and $E_\gamma^2 dN_\gamma/dE_\gamma$, and save results for comparison and downstream analysis.

CONTENTS

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I. SUMMARY

Hawking radiation [1] remains an unobserved property of black holes. As the temperature of black holes is inversely proportional to the square of their mass, conventional stellar mass black holes are expected to emit too little radiation to ever be detected. However, primordial black holes (PBHs) that could have formed from the collapse of primordial perturbations in the early universe, can provide detectable signals [2]. PBHs with mass less than 10^{14} grams would have evaporated via Hawking radiation long before the present age of the universe. Upcoming gamma-ray telescopes such as e-ASTROGAM [3] and AMEGO-X [4] will be sensitive enough in the MeV range to detect the Hawking spectra of PBHs lying between this lower bound and 10^{19} grams. We have developed `GammaPBHPlotter`, an open-source software to simulate the exact gamma-ray spectra produced from different PBH mass-distributions. This document includes a complete overview of the underlying physics and operations of this software, as well as user instructions on how to best utilize it.

II. HAWKING SPECTRA

A. Modeling the emission components

The gamma-ray spectrum of a PBH within the relevant mass range consists of four primary components; direct/primary Hawking radiation, secondary radiation, final-state radiation, and in-flight annihilation.

Direct Hawking radiation accounts for all kinematically allowed elementary particles formed at the event Horizon [1], including gamma-ray photons. Secondary radiation originates from the decay of unstable particles and contributes significantly at lower energies. We rely on `BlackHawk` [5, 6] to evaluate the gamma-ray primary and secondary spectral components. `BlackHawk` uses `PYTHIA` [7] for the modeling of the hadronization and decay processes leading to the secondary spectra. Final-state radiation originates from relativistic electrons and positrons (e^+) and has a differential spectrum given by [8],

$$\frac{dN_\gamma^{\rm FSR}}{dE_\gamma} = \frac{\alpha}{2\pi}\int dE_{e^+}\frac{dN_{e^+}}{dE_{e^+}}\left(\frac{2}{E_\gamma}+\frac{E_\gamma}{E_{e^+}^2}-\frac{2}{E_{e^+}}\right)\left[\ln\left(\frac{2E_{e^+}+(E_{e^+}-E_\gamma)}{m_{e^+}^2}\right)-1\right], \tag{1}$$

where $\alpha = 137.037$ is the fine structure constant, E_{e+} is the kinetic energy of a given positron, E_γ is the energy of the emitted photon, $m_{e^+} = 0.511$ MeV is the rest mass of the electron, and $\frac{dN_{e+}}{dE_{e+}}$ the differential spectrum of emitted electrons/positrons. In addition to the previously mentioned components, gamma-rays can be produced through pair-annihilation of positrons with interstellar medium electrons. This is known as in-flight annihilation and its differential spectrum is [8],

$$\begin{aligned}\frac{dN_\gamma^{\rm IA}}{dE_\gamma} &= \frac{\pi\alpha^2 n_H}{m_e}\int_{m_e}^{\infty} dE_{e^+}\frac{dN_{e^+}}{dE_{e^+}}\int_{E_{\rm min}}^{E_{e^+}}\frac{dE}{dE/dx}\frac{P_{E_{e^+}\to E}}{(E^2-m_e^2)} \\ &\times\left(-2-\frac{(E+m_e)(m_e^2(E+m_e)+E_\gamma^2(E+3m_e)-E_\gamma(E+m_e)(E+3m_e))}{E_\gamma^2(E-E_\gamma+m_e)^2}\right).\end{aligned} \tag{2}$$

We take $n_H = 1\,{\rm cm}^{-3}$ as the density of interstellar medium hydrogen (and by extension electrons). E_{e^+} is again the initial positron total energy, E is the final positron total energy, dE/dx is the rate of positron energy lost over distance via the Bethe-Bloch formula [9], E_γ is the resulting photon energy from annihilation, and $P_{E_{e^+}\to E}$ is the probability of a particular positron of a given initial and final energy to decay. This probability matrix can be calculated as [8],

$$P_{E_{e^+}\to E} = \exp\left(-n_H\int_E^{E_{e^+}}\sigma_{\rm ann}(E')\,\frac{dE'}{dx}\,dE'\right), \tag{3}$$

where σ_{ann} is the cross section of annihilation for positrons of a given energy.

In Fig. 1, we give the individual gamma-ray spectral components as well as their sum for a PBH of mass 3×10^{15} grams.

B. Interpolation

The process of performing these simulations can be quite cumbersome in terms of time and power constraints. In order to produce a more convenient experience for users, we opted to interpolate our data points rather than

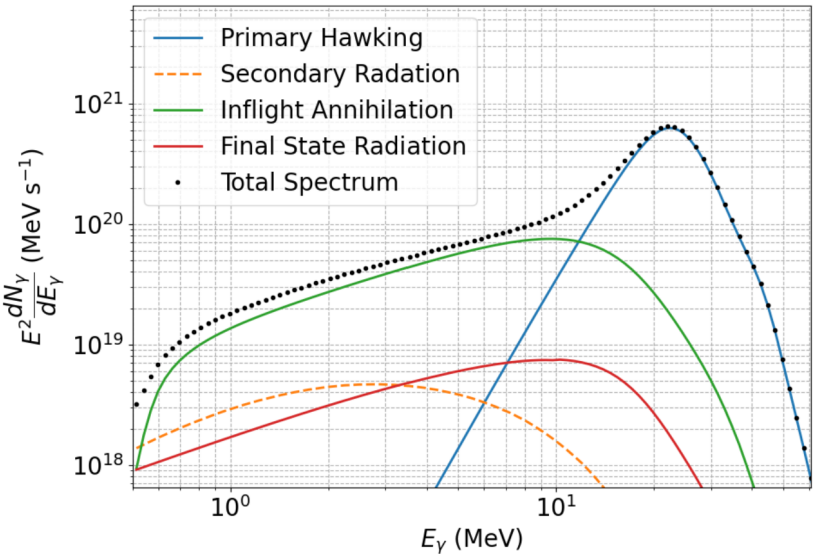

FIG. 1. The total gamma-ray spectrum of a 3×10^{15} grams PBH as well as its components.

render them in full for every PBH simulated. We rendered the gamma-ray hawking spectra of 56 different masses logarithmically spaced throughout our range. Those masses, in units of grams, are as follows.

$5 \times 10^{13},\ 6 \times 10^{13},\ 7 \times 10^{13},\ 8 \times 10^{13},\ 9 \times 10^{13},$
$1 \times 10^{14},\ 1.5 \times 10^{14},\ 2 \times 10^{14},\ 3 \times 10^{14},\ 4 \times 10^{14},\ 5 \times 10^{14},\ 6 \times 10^{14},\ 7 \times 10^{14},\ 8 \times 10^{14},\ 9 \times 10^{14},$
$1 \times 10^{15},\ 1.5 \times 10^{15},\ 2 \times 10^{15},\ 3 \times 10^{15},\ 4 \times 10^{15},\ 5 \times 10^{15},\ 6 \times 10^{15},\ 7 \times 10^{15},\ 8 \times 10^{15},\ 9 \times 10^{15},$
$1 \times 10^{16},\ 1.5 \times 10^{16},\ 2 \times 10^{16},\ 3 \times 10^{16},\ 4 \times 10^{16},\ 5 \times 10^{16},\ 6 \times 10^{16},\ 7 \times 10^{16},\ 8 \times 10^{16},\ 9 \times 10^{16},$
$1 \times 10^{17},\ 1.5 \times 10^{17},\ 2 \times 10^{17},\ 3 \times 10^{17},\ 4 \times 10^{17},\ 5 \times 10^{17},\ 6 \times 10^{17},\ 7 \times 10^{17},\ 8 \times 10^{17},\ 9 \times 10^{17},$
$1 \times 10^{18},\ 1.5 \times 10^{18},\ 2 \times 10^{18},\ 3 \times 10^{18},\ 4 \times 10^{18},\ 5 \times 10^{18},\ 6 \times 10^{18},\ 7 \times 10^{18},\ 8 \times 10^{18},\ 9 \times 10^{18},\ 1 \times 10^{19}$

For each of those masses, we calculated the gamma-ray spectra of each of the four components (direct Hawking, secondary, inflight annihilation, final-state radiation) mentioned previously. With those data points, a linear spline in log space is used to interpolate the individual components and the total gamma ray spectrum for any mass of PBH which lies within our range.

III. PBH DISTRIBUTION

Users can calculate the gamma-ray spectra from four types of PBH mass distributions. Those are, i) a monochromatic distribution with a mass to be set in the range of 5×10^{13} to 1×10^{19} grams, ii) a Gaussian distribution of PBH masses originating from a Gaussian distribution of density perturbations [10], iii) a more realistic non-Gaussian PBH mass distribution from [10] and iv) a log-normal distribution of PBH masses. In Fig. 2, we give the gamma-ray spectra from monochromatic and Gaussian PBH mass-distributions.

IV. SOFTWARE CONTENT

`GammaPBHPlotter` was written in Python version 3.9 and is capable of running on Windows, Linux, and Mac. The main code uses five modules in its routine (`colorama` [11], `NumPy` [12], `Matplotlib` [13], `tqdm` [14], and `SciPy` [15]). Since the software automatically checks and downloads all missing modules, this requirement should not be a concern for the user.

Inside the folder labeled the software's current name and edition are 5 items. Listed below for each item are brief descriptions, as well as an explanation of where they can read more about each of those items)

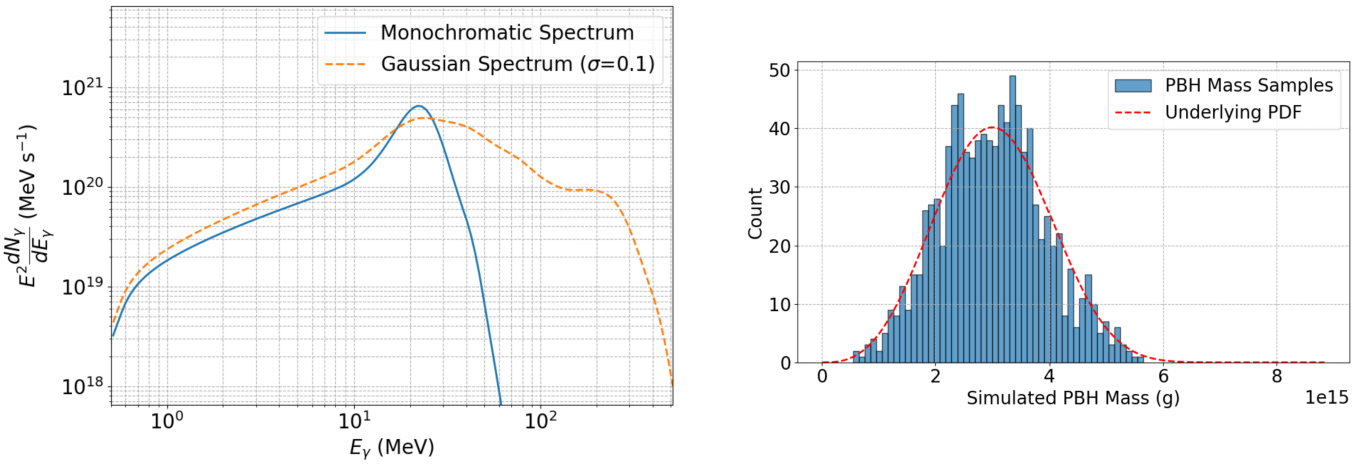

FIG. 2. Left: The total gamma-ray spectrum per PBH for a monochromatic PBH with mode mass 3×10^{15} g (blue) and for a Gaussian distribution with the same mode (orange). Right: Histogram of 1000 sampled PBH masses used to generate the figure on the left.

The file `"BlackHawk_data"` contains 56 sets of pre-rendered black hole spectral data by which this software uses for interpolation (See Section V on page 8).

The file `"results"` contains the data for simulations generated and saved using this software. (See Section VII on page 9).

The executable "Main.py" is the source code of this software. Run it in order to perform its routines. (See section VIII on page 9)

"Readme.txt" is a brief text file that provides general instructions of how to properly use this software.

"Manual.pdf", i.e. this document.

V. INPUTS

Upon activating the main python executable (main.py), the user is presented with a start screen. This includes the software's name, version, purposes, and other acknowledgments. Underneath the start screen is a main menu.

A. Menu Navigation

Whenever a user starts the program or completes a simulation, the user is presented with the following text. If the user enters 0, they will instantly close the program. If at any point, they wish to go back to a previous screen, they can enter `"b"` or "back" to do so.

$$\textit{Select}: 1 : \textit{Monochromatic spectra} \tag{4}$$
$$2 : \textit{Distributed spectra (Gaussian Distribution)} \tag{5}$$
$$3 : \textit{Distributed spectra (Non} - \textit{Gaussian Collapse)} \tag{6}$$
$$4 : \textit{Distributed spectra (Log} - \textit{Normal Distribution)} \tag{7}$$
$$5 : \textit{View previous spectra} \tag{8}$$
$$0 : \textit{Exit} \tag{9}$$

Entering the numbers 1, 2, 3, or 4 will direct the user to simulate the spectrum of PBHs of the listed distribution method.

B. User entered data

For monochromatic distribution, the user only needs to enter the peak masses to generate the desired spectra. One mass for a single simulation may be entered, but the user is also able to run multiple simulations at different peak

masses if they are entered with comma separation. For example, if the entered text reads `"3.5e15, 4.2e14, 3e17"`, then the program would generate all 3 of those spectra in that order. For monochromatic spectra, this is the only value that the user needs to enter.

For any continuous distribution, the sample size (or N) also needs to be entered after the peak masses. N determines how many individual masses will be generated by the desired probability distribution function. A good minimum value to ensure precision is N $\geq$ 1000.

Finally, the software will ask for a particular value of the σ which denotes a standard-deviation parameter of the underlying variable. This variable is the unitless parameter of density perturbations (δ_l) for the Gaussian/non-Gaussian cases, and the natural log of PBH mass ($\ln M_{PBH}$) for the log-normal case). Due to specific limitations within the models developed by Biagetti et al.[10], the σ values used within the Gaussian and non-Gaussian collapse methods must be within a limited range.

With the Gaussian collapse method, σ_G must fall within this range,

$$0.03 < \sigma_G < 0.255. \tag{10}$$

With the non-Gaussian collapse method σ_G must fall within this range,

$$0.04 < \sigma_{NG} < 0.16. \tag{11}$$

For Log-normal distribution σ may be any value greater than 0. However, the binning structure for histograms begins to break down past usability at values greater than 2. Multiple different values of sigma may be entered with commas separating them similarly to mass.

Once N PBH spectra are generated, these spectra are summed together before being divided by N to produce a mean average spectrum. This provides the user with the ability to observe the mean contribution of individual PBHs and normalize it to the amplitude of spectra generated by a monochromatic distribution.

C. BlackHawk Data

Located within the folder labeled `blackhawk_data` there are 56 sub folders, each dedicated to one of the 56 reference masses and the pre-rendered spectral data associated with them. Each folder is named with the scheme (`"1.0e+14"`, `"1.5e+14"`, `"2.0e+14"`, etc.). Within each subfolder there are 9 .txt files which are listed and described in descending alphanumerical order.

`{mass}`: It lists the parameters used to input the data into BlackHawk and generate the direct and secondary radiation spectra.

`final_State_radiation_{prim/sec}`: Two files display the photon energy `E_gamma (MeV)` and the $\frac{dN_\gamma}{dE_\gamma}$ differential flux of the final state radiation `Final State Radiation in MeV`$^{-1}$ `s`$^{-1}$. The `final_State_radiation_prim` gives the spectrum calculated from the primary positrons, while the `final_State_radiation_sec` from the secondary positrons.

`inflight_annihilation_{prim/sec}`: Two files respectively give the inflight annihilation spectra calculated by primary and secondary positrons (`IFA_{primary/secondary}_masked) in MeV`$^{-1}$ `s`$^{-1}$.

`Instantaneous_primary_spectra`: This file is generated by `BlackHawk`. It lists in the header the first column as `energy/particle` which serves a similar, but more generalized, parameter to the `E_gamma` variable listed in previous files in units of GeV. From there, each subsequent column is labeled by the associated fundamental particle of which it represents the direct Hawking flux (photons, gluons, higgs, etc.) in units of GeV^{-1}s^{-1}. Note that as this file was generated by BlackHawk, it uses a different set of units from this software. The particle energy is multiplied by 10^3 to convert from GeV to MeV and particle flux is multiplied by 10^{-3} to convert GeV^{-1}s^{-1} to MeV^{-1}s^{-1}.

`Instantaneous_secondary_spectra`: This is a file generated to isolate the secondary flux of positrons and gamma rays in units of MeV^{-1}s^{-1} compared to the particle's energy in units of MeV.

`Instantaneous_total_spectra`: This is the second file generated by `BlackHawk`. The only way it differs from the aforementioned `instantaneous_primary_spectra` file is that is that the spectra displayed here are the sum of the direct and secondary radiation. Note that as this file was generated by BlackHawk, it uses a different set of units from this software. The particle energy must be multiplied by 10^3 to convert from GeV to MeV and particle flux must be multiplied by 10^{-3} to convert GeV^{-1}s^{-1} to MeV^{-1}s^{-1} before either can be compared to any results from this software.

VI. GRAPHS

After all inputs have been entered, the user will see a second window open that displays their spectra in the form of a graph. The x-axis displays the energy of the emitted photons E_γ while the y-axis displays the Hawking radiation

spectra $\frac{dN_\gamma}{dE_\gamma}$. The graphs will have limits of 500 keV to 5 GeV and the highest three orders of magnitude of the spectrum's peak along the y-axis.

Whenever a non-monochromatic spectrum is generated, a histogram will be displayed afterward. This shows the distribution of masses in 50 bins and the PDF that was used to generate them.

VII. SAVED FILES

Once all graphs have been displayed in a given simulation, the software will prompt the users asking if they wish to save any of the spectra. If they select `"y"` or `"yes"` in response, they will be presented with an indexed list of all spectra generated. The user needs only to enter the corresponding number or numbers (separated by commas) to decide which ones they desire to save. If `n` or `no` are entered, the user will return to the main menu.

Once saved, the user will read a confirmation message reading `"Saved → path"` with `"path"` referring to the file path to the destination folder. This folder will depend on which distribution method was used to generate the spectrum.

Monochromatic distribution goes to `.../results/monochromatic`
Gaussian distribution goes to `.../results/gaussian`
Log-normal distribution goes to `.../results/lognormal`
Non-Gaussian collapse goes to `.../results/non_gaussian`

Monochromatic spectra are each saved in one file under the name of `mass_spectrum.txt`. `"mass"` is a placeholder for the monochromatic mass value entered for that particular simulation. This file contains 7 columns. One to represent the photon energy in MeV, and 6 to represent the different components of the gamma-ray, Hawking radiation (Direct, secondary, inflight annihilation, final state radiation, and the total sum) in $\mathrm{MeV}^{-1}\ \mathrm{s}^{-1}$.

Saving non-monochromatic spectra instead produces two files inside a sub folder named `mass_sigma.txt`. `"sigma"` is another placeholder referring to the σ value entered for this particular simulation. The file named `"distributed_spectrum.txt"` has two columns comparing photon energy and the total gamma-ray spectrum. Meanwhile `mass_distribution.txt"` has one column which lists every single mass generated in the simulation in units of grams. This second file also lists the distribution's mean mass in the header.

VIII. ROUTINES

In this section, there is an overview of the source code and its routines. Each routine encapsulates a clear responsibility—dependency management, user I/O, data loading, numerical calculation, visualization, or file I/O—so the codebase remains maintainable and extensible.

`_ensure_packages(modules)`: Ensures that required Python packages are present at startup. It attempts to import each requested module and, on failure, installs the corresponding PyPI package via `pip`. This prevents runtime errors from missing dependencies.

`discover_mass_folders(data_dir)`: Scans `blackhawk_data` for subfolders that parse as floating-point masses and contain all required data files. Returns sorted lists of masses and folder names used elsewhere for preloading and interpolation.

`info(msg)`, `warn(msg)`, `err(msg)`: Convenience printers using `colorama` for consistent CLI status messages (information, warnings, and errors).

`user_input(prompt)`: Wraps `input()` for consistent interaction. Trims whitespace and cleanly exits if the user types `q` or `exit`; otherwise returns the entered string.

`list_saved_runs(base_dir)`: Returns a sorted list of subdirectories (each representing a saved run) if present; otherwise an empty list. Used by the "view previous" flow.

`snap_to_available(mval, available, tol=1e-12)`: Matches a requested mass to the nearest precomputed mass in logarithmic space if it is within a small relative tolerance; otherwise returns `None`. This aids reuse of exact grids when possible.

`parse_float_list_verbose(s, ...)`: Parses a comma-separated list of numbers with optional bounds and positivity constraints. Invalid tokens are skipped with human-readable warnings; duplicates are ignored. Used to robustly read peak masses and σ lists.

`delta_l(mass_ratio, kappa, delta_c, gamma)`: Maps a dimensionless mass ratio to the primordial density perturbation variable δ_l via the model's analytic transformation, clipping internal arguments to remain real.

`mass_function(,`δ_l`, `σ_x`, `δ_c`, `γ`)`: Implements a Gaussian-collapse PDF over δ_l with the appropriate Jacobian factor that converts from collapse variable to mass probability up to a scale factor.

`mass_function_exact(,`δ_l`, `σ_X`, `σ_Y`, `δ_c`, `γ`)`: Implements the full non-Gaussian PDF (including error-function and square-root terms) with an explicit Jacobian. This captures the shape required for non-Gaussian collapse modeling.

`mass_function_lognormal(x, `μ`, `σ`)`: Standard log-normal PDF in mass space; extremely small inputs are clipped to avoid underflow in `log`.

`load_data(filepath, skip_header=0)`: Thin wrapper over NumPy's `genfromtxt`. Verifies existence and loads numeric arrays while respecting header lines.

`load_spectra_components(directory)`: Loads the six component files for a given mass directory, converts energy units where needed (GeV $\to$ MeV), interpolates component grids to common E_γ arrays, and returns a dictionary with aligned arrays for primary/secondary direct photons, in-flight annihilation (primary+secondary), and final-state radiation (primary+secondary).

`monochromatic_spectra()`:

- Discovers available precomputed masses and prompts for one or more masses within that range.
- Pre-loads component matrices across all reference masses and builds log–log interpolators via `RectBivariateSpline` (linear in $\log M$, cubic in $\log E$).
- For an exact mass folder, uses the pre-rendered components; otherwise interpolates in $(\log M, \log E)$.
- Derives a conservative high-energy cutoff for in-flight annihilation by comparing available component maxima and zeroing the tail. Also guards rare terminal anomalies on the right edge before summation.
- Plots per-mass component curves and their total dN_γ/dE_γ; then overlays $E_\gamma^2 dN_\gamma/dE_\gamma$ curves for a comparison panel, including a dotted "Summed" line across all selections.
- Optionally saves each selected spectrum to `results/monochromatic/{mass_label}_spectrum.txt` with columns for E_γ and all components plus the total.

`_trim_right_spike(x_line, y_line, up_thresh, down_thresh, max_trim_frac)`: Utility to trim only a terminal up-spike or down-cliff at the far right of a line, walking left until ratios normalize or a small fraction of points has been removed. Used when overlaying analytic PDFs against histograms to avoid spurious final bins.

`distributed_spectrum(distribution_method)`:

1. Prompts for peak masses and a target sample size N. For Gaussian collapse, reads $\sigma_x \in [0.03, 0.255]$; for non-Gaussian, reads $\sigma_x \in [0.04, 0.16]$ with a fixed ratio $\sigma_y/\sigma_x = 0.75$; for log-normal, reads log-space $\sigma > 0$.
2. Builds the underlying PDF on a fixed collapse grid for Gaussian/non-Gaussian cases (normalizes to a PMF on x), or samples directly from a log-normal for log-normal runs.
3. Converts collapse samples to masses by scaling the grid so that its mode matches the requested peak, then, for each draw, either uses a pre-rendered mass bin or interpolates components in $(\log M, \log E)$.
4. Zero-floors tiny values, applies a conservative in-flight cutoff using the nearest available reference bins, and aggregates running sums for direct, secondary, in-flight, and final-state components.
5. Averages over N to obtain $\langle dN/dE \rangle$, then plots overlay panels for dN/dE and $E^2 dN/dE$ across all requested parameter sets.
6. For each run, also shows a 50-bin mass histogram with an analytic PDF overlay converted to expected bin counts; the overlay's terminal artifact, if present, is trimmed using `_trim_right_spike`.
7. Optionally saves spectra and sampled masses to `results/{gaussian|non_gaussian|lognormal}/{peak__sigma}/`, writing `distributed_spectrum.txt` and `mass_distribution.txt` (with the mean mass noted in the header).

`generate_monochromatic_for_mass(mval, DATA_DIR, MONO_RESULTS_DIR, tol)`: Given a requested mass, chooses an exact folder if available (within tolerance) or interpolates from the preloaded grid. Writes a single file `Spectrum_components_and_total.txt` containing E_γ, each component, and the total into a labeled subdirectory under `results/monochromatic/`.

`view_previous_spectra()`: Interactive browser for saved outputs. Users can queue:

- Monochromatic: generates (or reuses) a components file via `generate_monochromatic_for_mass` and queues the total.

- Distributed: lists saved Gaussian, non-Gaussian, or log-normal runs and queues their `distributed_spectrum.txt`.

Once queued, the routine plots comparison panels for dN/dE and E^2dN/dE with shared axis limits.

`show_start_screen()`: Prints a brief, styled banner describing the tool, authorship line, and simple navigation instructions.

`main()`: Application entry point. Displays the top-level menu, dispatches to monochromatic/distributed flows, opens the "view previous" browser, or exits. All exceptions are caught at the top level to print a traceback and pause for user acknowledgment.

IX. CONCLUSION

`GammaPBHPlotter` is an open source program designed first and foremost to allow for an intuitive, comprehensive, and efficient means of calculating the gamma-ray Hawking spectra of low-mass primordial black holes in the range of 5×10^{13} to 5×10^{19} grams. It does this by interpolating pre-rendered spectral data of PBHs as a product of their mass. Then, via a number of distribution methods, users can calculate the expected gamma ray spectrum PBHs would emit under different circumstances. With these pieces of data at hand, researchers can now more easily compare observed excess gamma radiation to probe for potential PBH signals.

X. ACKNOWLEDGMENTS

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