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In [1]: import numpy as np
import scipy.linalg
from sklearn.linear_model import LinearRegression
import matplotlib.pyplot as plt
import time
```

```
In [2]: # Best practice source:
# https://numpy.org/doc/stable/reference/random/generated/numpy.random.seed.h
from numpy.random import MT19937
from numpy.random import RandomState, SeedSequence
rs = RandomState(MT19937(SeedSequence(72730)))
```

```
In [3]: # Generate simulated data
# Must have rows > cols
rows=100
cols = 10
sigma_true = .5
x = np.random.normal(size=(rows, cols))
beta_true = np.multiply(np.random.normal(size=cols), np.random.uniform(-6,6,c
y = np.matmul(x, beta_true) + sigma_true*np.random.normal(size=rows)
```

```
In [4]: # Use scipy qr decomposition to solve for betas
decomp = scipy.linalg.qr(x)
print("Dimensions of Q: ", decomp[0].shape)
print("Dimensions of R: ", decomp[1].shape)
```

```
Dimensions of Q: (100, 100)
Dimensions of R: (100, 10)
```

```
In [5]: # Verify that Q is orthogonal
print("Q'Q: ", np.matmul(decomp[0], decomp[0].T).round(10))
print("Det of Q: ", scipy.linalg.det(decomp[0]))
```

```
Q'Q: [[ 1. -0.  0. ... -0.  0. -0.]
 [-0.  1. -0. ... -0.  0. -0.]
 [ 0. -0.  1. ... -0.  0. -0.]
 ...
 [-0. -0. -0. ...  1. -0.  0.]
 [ 0.  0.  0. ... -0.  1.  0.]
 [-0. -0. -0. ...  0.  0.  1.]]
Det of Q: 1.00000000000000029
```

```
In [6]: # Verify that we can recover X
np.allclose(np.dot(decomp[0], decomp[1]), x)
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Out[6]: True
```

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In [7]: # Solve for beta using  $R\beta = Q^t y$ 
qty = np.matmul(decomp[0].T, y.reshape(-1,1))
# Need to use the linearly independent part of R, corresponding  $Q^t Y$ 
beta_hat = scipy.linalg.solve_triangular(decomp[1][:cols, :cols], qty[:cols])
```

```
In [8]: # Verify that we're close to the true beta
print("Difference in Beta Hat vs Beta True:")
np.subtract(beta_hat, np.reshape(beta_true, (-1,1)))
```

Difference in Beta Hat vs Beta True:

```
Out[8]: array([[ 0.03062098],
               [-0.01686938],
               [-0.00668422],
               [ 0.10214225],
               [-0.02472997],
               [-0.03930336],
               [ 0.06000058],
               [-0.0130657 ],
               [-0.00876292],
               [-0.01727208]])
```

```
In [9]: # Verify that we're close to the fitted beta from sklearn
lfit = LinearRegression() #L2 reg is 1/C
# need numpy 2dim array for X
# X = x.reshape((n,1)).copy()
lfit.fit(x,y)
print("Differences in sklearn vs backsolve: ")
np.subtract(beta_hat, np.reshape(lfit.coef_, (-1,1)))
# Looking good!
```

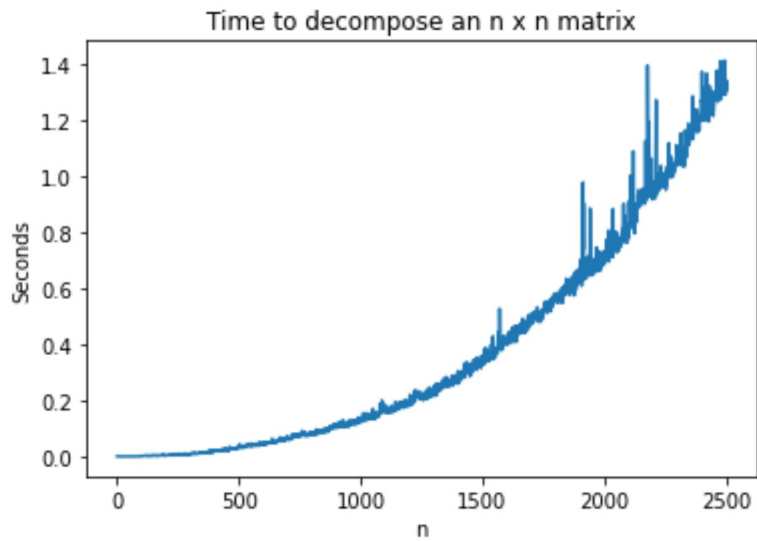
Differences in sklearn vs backsolve:

```
Out[9]: array([[ 2.50484113e-03],
               [-4.05947403e-03],
               [ 1.86747191e-03],
               [-3.68102959e-04],
               [-1.77282626e-04],
               [ 6.26297593e-04],
               [ 2.80990643e-03],
               [-1.11652546e-03],
               [ 4.49906040e-05],
               [ 8.06336569e-04]])
```

```
In [10]: # Time the qr decomp as the size of a square matrix increases
row_space = np.linspace(1, 2500, 2500, dtype=int)
outputs = np.zeros(row_space.shape[0])
for i in range(row_space.shape[0]):
    n = row_space[i]
    data = np.random.normal(size=(n,n))
    start = time.time()
    tmp = scipy.linalg.qr(data)
    outputs[i] = time.time() - start
```

```
In [11]: plt.plot(row_space, outputs)
plt.xlabel("n")
plt.ylabel("Seconds")
plt.title("Time to decompose an n x n matrix")
```

Out[11]: Text(0.5, 1.0, 'Time to decompose an n x n matrix')



In []: