HW 3 - Kevin de Szendeffy

February 23, 2021

[1]:
```
##################################################
### imports
import numpy as np
import pandas as pd

import matplotlib.pyplot as plt
import seaborn; seaborn.set()

import scipy as sp
import scipy.stats as sps

import math
from time import time

from sklearn.linear_model import LinearRegression
import statsmodels.api as sm

import itertools
```

1 Problem 1

[2]:
```
p = 5
rho = 0.8
Sigma = np.full((p,p), rho)+(-rho+1)*np.eye(p)
```

[3]:
```
Sigma
```

[3]:
```
array([[1. , 0.8, 0.8, 0.8, 0.8],
       [0.8, 1. , 0.8, 0.8, 0.8],
       [0.8, 0.8, 1. , 0.8, 0.8],
       [0.8, 0.8, 0.8, 1. , 0.8],
       [0.8, 0.8, 0.8, 0.8, 1. ]])
```

[4]:
```
L = np.linalg.cholesky(Sigma)
```

[5]:
```
L
```

[5]:
```
array([[1.        , 0.        , 0.        , 0.        , 0.        ],
       [0.8       , 0.6       , 0.        , 0.        , 0.        ],
       [0.8       , 0.26666667, 0.53748385, 0.        , 0.        ],
       [0.8       , 0.26666667, 0.16537965, 0.51140831, 0.        ],
       [0.8       , 0.26666667, 0.16537965, 0.12033137, 0.49705012]])
```

[6]:
```
L1 = L[0:2, 0:2]
```

1.0.1 a) $(Y_1, Y_2) \sim N((0,0), \Sigma_{12})$ where Σ_{12} is

[7]:
```
L1@np.transpose(L1)
```

[7]:
```
array([[1. , 0.8],
       [0.8, 1. ]])
```

1.0.2 b)

[8]:
```
x = np.transpose([0.23, -0.65, -0.3])
```

First, we note it is equivalent to find the conditional distribution of (Y_4, Y_5) given $Y_1 = 0.23$, $Y_2 = -0.65$, and $Y_3 = -0.3$. Then we define L_1, A, and L_2 as follows.

[9]:
```
L1 = L[0:3, 0:3]
L2 = L[3:5, 3:5]
A = L[3:5, 0:3]
```

Thus the conditional distribution of (Y_4, Y_5) given $Y_1 = 0.23$, $Y_2 = -0.65$, and $Y_3 = -0.3$ is $N(\mu_{45|123}, \Sigma_{45|123})$ where $\mu_{45|123}$ is

[10]:
```
A@np.linalg.inv(L1)@x
```

[10]:
```
array([-0.22153846, -0.22153846])
```

and $\Sigma_{45|123}$) is

[11]:
```
L2@np.transpose(L2)
```

[11]:
```
array([[0.26153846, 0.06153846],
       [0.06153846, 0.26153846]])
```

1.0.3 c) L in $\Sigma = LL'$ is

[12]:
```
L
```

[12]:
```
array([[1.        , 0.        , 0.        , 0.        , 0.        ],
       [0.8       , 0.6       , 0.        , 0.        , 0.        ],
       [0.8       , 0.26666667, 0.53748385, 0.        , 0.        ],
       [0.8       , 0.26666667, 0.16537965, 0.51140831, 0.        ],
       [0.8       , 0.26666667, 0.16537965, 0.12033137, 0.49705012]])
```

1.0.4 d) L^{-1} is

[13]:
```
np.linalg.inv(L)
```

[13]:
```
array([[ 1.        ,  0.        ,  0.        ,  0.        ,  0.        ],
       [-1.33333333,  1.66666667,  0.        ,  0.        ,  0.        ],
       [-0.82689823, -0.82689823,  1.86052102,  0.        ,  0.        ],
       [-0.60165684, -0.60165684, -0.60165684,  1.95538472,  0.        ],
       [-0.47338107, -0.47338107, -0.47338107, -0.47338107,  2.01186954]])
```

1.0.5 e) $A = PD^{\frac{1}{2}}$

[14]:
```
ev, P = np.linalg.eigh(Sigma)
```

[15]:
```
D = np.diag(ev)
```

[16]:
```
P@np.sqrt(D)
```

[16]:
```
array([[ 3.20020123e-01,  0.00000000e+00,  0.00000000e+00,
         2.39973167e-01,  9.16515139e-01],
       [-3.12358051e-01,  0.00000000e+00, -5.58570184e-17,
         2.49864860e-01,  9.16515139e-01],
       [-2.55402400e-03,  0.00000000e+00,  3.65148372e-01,
        -1.63279342e-01,  9.16515139e-01],
       [-2.55402400e-03, -3.16227766e-01, -1.82574186e-01,
        -1.63279342e-01,  9.16515139e-01],
       [-2.55402400e-03,  3.16227766e-01, -1.82574186e-01,
        -1.63279342e-01,  9.16515139e-01]])
```

1.0.6 f)

[17]:
```
ysim = L @ np.random.normal(size=(5, 10000))
df = pd.DataFrame(ysim).T
ybar = df.apply(np.mean, axis=0).values
```

[18]:
```
sigmahat = np.zeros(Sigma.shape)
for i, r in df.iterrows():
    hold = (r.values-ybar)[:, np.newaxis]
    sigmahat += hold @ hold.T
sigmahat/= df.shape[0]
```

[19]:
```
print(ybar)
print(sigmahat)
```

```
[0.0096619  0.00767152 0.00338953 0.00290112 0.00266758]
[[0.99358867 0.80595063 0.80931672 0.79806052 0.80461073]
 [0.80595063 1.01499931 0.8164054  0.80298267 0.81187738]
 [0.80931672 0.8164054  1.01646978 0.81051097 0.81420219]
```

```
 [0.79806052 0.80298267 0.81051097 1.00016302 0.80852963]
 [0.80461073 0.81187738 0.81420219 0.80852963 1.01087406]]
```

[20]:
```
mu_diff = np.linalg.norm(ybar-np.zeros(p))
sigma_diff = np.linalg.norm(sigmahat - Sigma)
print("n = ", 10000)
print("Mu diff: ", mu_diff)
print("Sigma diff: ", sigma_diff)
```

```
n =  10000
Mu diff:  0.013387528088009627
Sigma diff:  0.05057825761213993
```

1.0.7 g)

[21]:
```
ysim = L @ np.random.normal(size=(5, 50))
df = pd.DataFrame(ysim).T
ybar = df.apply(np.mean, axis=0).values
sigmahat = np.zeros(Sigma.shape)
for i, r in df.iterrows():
    hold = (r.values-ybar)[:, np.newaxis]
    sigmahat += hold @ hold.T
sigmahat/= df.shape[0]
mu_diff = np.linalg.norm(ybar-np.zeros(p))
sigma_diff = np.linalg.norm(sigmahat - Sigma)
print("n = ", 50)
print("Mu diff: ", mu_diff)
print("Sigma diff: ", sigma_diff)
```

```
n =  50
Mu diff:  0.20541184830511416
Sigma diff:  1.0208336116865688
```

2 Problem 3

[22]:
```
x = np.linspace(-2, 2, 100)
fx = x**3
Io = list(range(75))
Io.extend(list(range(86, 100)))
Iu = list(range(76, 86))
X = x[Io]
```

[23]:
```
sigf = 0.1
l = 0.1
```

[24]:
```
Sigma = np.zeros([100, 100])
```

[25]:
```
for i in range(100):
    for j in range(100):
        Sigma[i, j] = (sigf**2)*np.exp(-1/(2*l**2)*(x[i]-x[j])**2)
```

[26]:
```
Sigma
```

[26]:
```
array([[1.00000000e-002, 9.21618123e-003, 7.21446326e-003, …,
        0.00000000e+000, 0.00000000e+000, 0.00000000e+000],
       [9.21618123e-003, 1.00000000e-002, 9.21618123e-003, …,
        0.00000000e+000, 0.00000000e+000, 0.00000000e+000],
       [7.21446326e-003, 9.21618123e-003, 1.00000000e-002, …,
        1.18575755e-322, 0.00000000e+000, 0.00000000e+000],
       …,
       [0.00000000e+000, 0.00000000e+000, 1.18575755e-322, …,
        1.00000000e-002, 9.21618123e-003, 7.21446326e-003],
       [0.00000000e+000, 0.00000000e+000, 0.00000000e+000, …,
        9.21618123e-003, 1.00000000e-002, 9.21618123e-003],
       [0.00000000e+000, 0.00000000e+000, 0.00000000e+000, …,
        7.21446326e-003, 9.21618123e-003, 1.00000000e-002]])
```

[27]:
```
sigxx = Sigma[Io, :][:, Io]
sigyy = Sigma[Iu, :][:, Iu]
sigxy = Sigma[Io, :][:, Iu]
sigyx = np.transpose(sigxy)
```

[28]:
```
mu_y = sigyx@np.linalg.inv(sigxx)@fx[Io]
sig_y = sigyy - np.linalg.multi_dot([sigyx, np.linalg.inv(sigxx), sigxy])
```

[29]:
```
sample = np.random.multivariate_normal(mu_y, sig_y, 1000)
y = np.mean(sample, axis = 0)
```

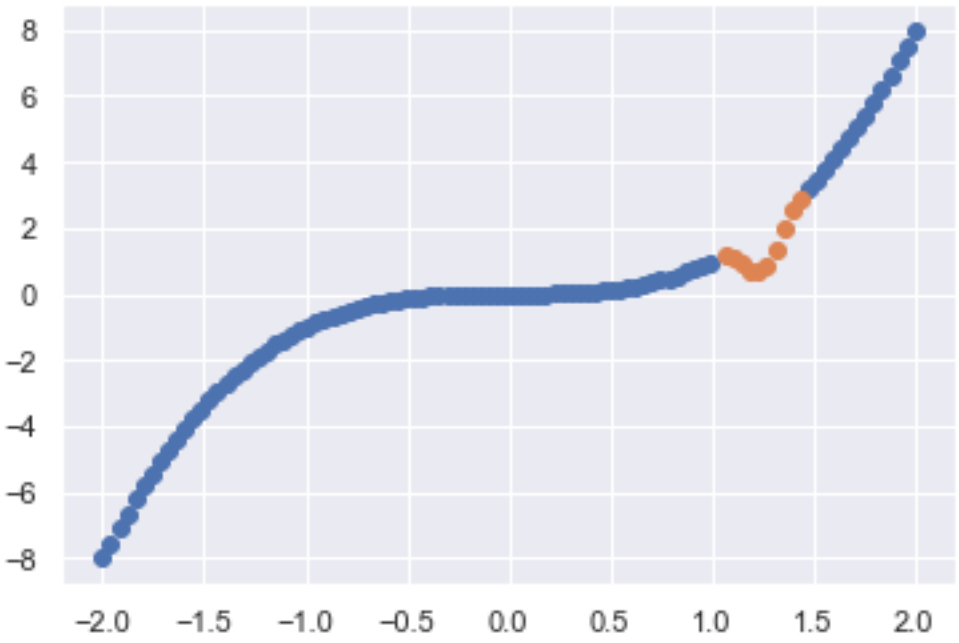
```
<ipython-input-29-e7bda8616836>:1: RuntimeWarning: covariance is not positive-
semidefinite.
  sample = np.random.multivariate_normal(mu_y, sig_y, 1000)
```

[30]:
```
y
```

[30]:
```
array([1.14823311, 1.09979373, 0.92636519, 0.72902555, 0.67512681,
       0.89091209, 1.37456763, 1.98331579, 2.52786137, 2.91890331])
```

[31]:
```
plt.scatter(x[Io], fx[Io])
plt.scatter(x[Iu], y)
```

[31]:
```
<matplotlib.collections.PathCollection at 0x243a2a075b0>
```



[32]:
```
np.linalg.norm(y-fx[Iu])
```

[32]: 2.2899929886872075