

Summary of Logical Equivalences

Theorem 2.1.1 Logical Equivalences

Given any statement variables p , q , and r , a tautology \mathbf{t} and a contradiction \mathbf{c} , the following logical equivalences hold.

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| 1. <i>Commutative laws:</i> | $p \wedge q \equiv q \wedge p$ | $p \vee q \equiv q \vee p$ |
| 2. <i>Associative laws:</i> | $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | $(p \vee q) \vee r \equiv p \vee (q \vee r)$ |
| 3. <i>Distributive laws:</i> | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ |
| 4. <i>Identity laws:</i> | $p \wedge \mathbf{t} \equiv p$ | $p \vee \mathbf{c} \equiv p$ |
| 5. <i>Negation laws:</i> | $p \vee \sim p \equiv \mathbf{t}$ | $p \wedge \sim p \equiv \mathbf{c}$ |
| 6. <i>Double negative law:</i> | $\sim(\sim p) \equiv p$ | |
| 7. <i>Idempotent laws:</i> | $p \wedge p \equiv p$ | $p \vee p \equiv p$ |
| 8. <i>Universal bound laws:</i> | $p \vee \mathbf{t} \equiv \mathbf{t}$ | $p \wedge \mathbf{c} \equiv \mathbf{c}$ |
| 9. <i>De Morgan's laws:</i> | $\sim(p \wedge q) \equiv \sim p \vee \sim q$ | $\sim(p \vee q) \equiv \sim p \wedge \sim q$ |
| 10. <i>Absorption laws:</i> | $p \vee (p \wedge q) \equiv p$ | $p \wedge (p \vee q) \equiv p$ |
| 11. <i>Negations of \mathbf{t} and \mathbf{c}:</i> | $\sim \mathbf{t} \equiv \mathbf{c}$ | $\sim \mathbf{c} \equiv \mathbf{t}$ |