Authors' Response to Reviews of

An Improved Minimum-Distance Texture Estimator for Speckled Data under the $\mathcal{G}^0$ Model

Julia Cassetti, Alejandro C. Frery
Journal of Mathematical Imaging and Vision,

RC: ***Reviewers' Comment***, AR: Authors' Response, ☐ Manuscript Text

1. Reviewer #1

1.1. Summary

> This paper addresses the proposal of new estimators based on the minimum distance for the roughness parameter of the $\mathcal{G}^0$ distribution, as an extension of [1]. Particularly, authors combine the triangular distance with the gamma and log-normal asymmetric kernels to develop their estimators. Synthetic and real experiments indicate the proposals are more robust than the estimators based on maximum likelihood and log cumulant.
>
> In my opinion, the problem to be tackled is very interesting and the paper is well written and presents convincent practical results. However, some questions in the presentation of the novel method need to be answered. My recommendation is "accepted before major revisions".

AR: We would like to thank the reviewer for the careful analysis of our work. We agree with the suggestions, and we have proceeded accordingly.

1.2. Critical comments

> - In works of statistical inference, the proposal of estimators need to be accompanied of their respective standard error. I think that author should explore meaningfully this aspect in your proposal. I know that authors presented the Figure 13 on which it is made a study on asymptotic confidence interval via bootstrap for the proposed estimators, but several issues need to be answered.
>   - It is known that the maximum likelihood estimators has minimum asymptotic variance. What about $\widehat{\alpha}_{LC}$, $\widehat{\alpha}_{\Gamma}$, and $\widehat{\alpha}_{LN}$?
>   - What about the asymptotic distributions on the considered estimators? Please furnish a discussion about this point.
>   - Why were the intervals due to $\widehat{\alpha}_{ML}$ and $\widehat{\alpha}_{LC}$ not approached?
>   - When increasing n of 9 to 25, should the length of intervals not diminish?
>
>   For instance, Tables 4–7 must present their respective standard errors.

- We added Table 5 with the sample variance of the estimators for several cases of L and of α, and its corresponding analysis.
- We included Figs. 5 and 6. The former displays the estimators' sample density for $\alpha = -3$, $L = 3$ and for several sample sizes including $n = 500$. The latter exibits the sample density function for different values of the texture parameter, and for samples of size 500. We also present the values of skewness, kurtosis in Tables 4 and 5, respectively. We analyzed these figures and tables, and draw conclusions about the finite-sample and asymptotic distributions of the estimators.

- Thank you very much for this careful observation. There was an error in Fig. 15. We fixed this error and also added Table 9, which shows the length of the bootstrap confidence intervals.

- It can be seen in Table 9 that the length of the confidence intervals decrease as the sample size increases. It is important to note that for $n = 9$ ML and LC methods did not converge. For this reason the confidence intervals for these cases are not shown. Tables $4 - 7$ were transformed in Tables $6 - 10$. All tables showing estimates are accompanied by their respective confidence intervals as a measure of estimation error.

1.3. Detailed comments

- (page, column, line)=(p,c,l)=(1,2,33): Present citation for the next sentence:

  "...which appears from the use of coherent illumination [?,?]. "
- (page, column, line)=(p,c,l)=(2,1,16): I think the next phrase is not always true. Please, rewrite.

  "Tison et al. [50] showed that estimators based on logcu- mulants outperform the ML estimator for the parameters of the amplitude G 0 distribution. "
- (page, column, line)=(p,c,l)=(2,1,16): Replace "...proposed an MDE estimator ..." by "...proposed a MDE estimator ...".
- Define Ω in Equation (1).
- (page, column, line)=(p,c,l)=(2,2,18): What does the term "distribution functions" mean?
- (page, column, line)=(p,c,l)=(3,1,55): $\alpha \in (1, -3)$? Is it correct?
- (page, column, line)=(p,c,l)=(3,2,17): It is known that the variance of $X \sim \mathcal{G}^0$ is infinity for $\alpha \geq -2$. If authors go to consider the interval $\alpha \in (-2, -1)$, it is important to mention this problem.
- Equation (6): Is it correct?
- (page, column, line)=(p,c,l)=(3,2,24-35): It is not enough to mention only the used interactive method. How was it made the initial input?
- Equation (8): This expression should be rewritten, the integration operation is pretty informal.
- (page, column, line)=(p,c,l)=(4,1,45-52): This text was mentioned in the introduction section. Please remove it.

2

- (page, column, line)=(p,c,l)=(4,2,35-39): With respect "It can be seen that these kernels fit well the theoretical density function even for a small sample size.", it is not clear why the IG kernel is a good proposal.
- (page, column, line)=(p,c,l)=(4,2,58): Please, rewrite "It is essential to have measures that quality the quality of the estimation."
- (page, column, line)=(p,c,l)=(9,1,43): Replace "...techniques methods..." by "...methods..."

AR: We have made the following changes:

- We have added the reference by Yue et al. (2021), which is a modern and comprehensive survey about speckle.

- Than you very much for this deep observation. We have rephrased our statement, and now we say that

Tison et al. [50] showed that estimators based on logcumulants are well-suited to model amplitude single-look data from urban areas, where very large values are frequently observed. Nevertheless, that work overlooks the fact that estimation under the $\mathcal{G}^0$ distribution is more sensitive to very small values rather than to extremely large ones. In this work, we explore this fact and propose suitable solutions using kernels.

- We struggled with this point, as English is not our native language, and we found the following answer in `https://forum.wordreference.com/threads/an-or-a-before-an-acronym-starting-with-m-h.2554395/`

  > There is more than one type of acronym. See the Wikipedia article here. Assuming you have in mind only the type of acronym called an "initialism," the choice of *a* or *an* depends upon the first sound pronounced when saying the initialism. If it is a consonant sound, use *a*. If it is a vowel sound, use *an*. Thus, it's "an MTR station" (*m* is pronounced "em") and "an HSBC branch" (*h* is pronounced "aitch").

  According this recommendation, we write "an MDE".

- We added "and Ω is the parameter space." to the sentence after (1).

- We changed the text as follows:

  In particular, several measures have been proposed to reflect the closeness ~~between two distribution functions.~~ among the models that describe samples.

- It was wrong; we corrected it and now it reads "$\alpha \in (-3, -1)$".

- Falta, creo que me dijiste que te la deje

- The equation 6 is correct. We only consider the terms that depend on α, which are those involved in the minimization.

- We added the following text to explain which was the initial point that was considered in the algorithm.

  taking as a starting point α_0 the alpha moment estimator when it exists. Otherwise we consider $\alpha_0 = -1.5$.

- We rewrite equation 8 as

$$d_{\mathrm{T}}(f_{\mathrm{V}}, f_{\mathrm{W}}) = \int_S \frac{(f_{\mathrm{V}}(x) - f_{\mathrm{W}}(x))^2}{f_{\mathrm{V}}(x) + f_{\mathrm{W}}(x)} dx, \qquad (1)$$

- Following your request this paragraph was deleted.

- We explain this in the text as follows:

> It can be seen that these kernels fit reasonably well the theoretical density function even for a small sample size. The IG kernel presents a good approximation to the true density function at the center of the function although heavier tails are observed. The Γ and LN kernels show a better fit, with the latter kernel having a higher kurtosis than the other kernels.

- We changed the text as follows:

> It is essential to have measures that ~~quality~~ quantify the quality of the estimation.

- We deleted the word “techniques”.

References

Yue, D.-X., Xu, F., Frery, A. C. & Jin, Y.-Q. (2021), ‘SAR image statistical modeling Part I: Single-pixel statistical models’, *IEEE Geoscience and Remote Sensing Magazine* **9**(1), 82–114.