

## Min-Max Heap

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## 1 Namespace Documentation

### 1.1 \_\_mmheap Namespace Reference

#### Functions

- `size_t parent (size_t i)`
- `size_t has_parent (size_t i)`
- `size_t left (size_t i)`
- `size_t right (size_t i)`
- `size_t gparent (size_t i)`
- `bool has_gparent (size_t i)`
- `bool child (size_t i, size_t c)`
- `uint64_t log_2 (uint64_t i)`
- `bool min_level (size_t i)`
- `std::pair< bool, size_t > min_child (int *heap_array, size_t i, size_t right_index)`
- `std::pair< bool, size_t > min_gchild (int *heap_array, size_t i, size_t right_index)`
- `std::pair< bool, size_t > min_child_or_gchild (int *heap_array, size_t i, size_t right_index)`
- `std::pair< bool, size_t > max_child (int *heap_array, size_t i, size_t right_index)`
- `std::pair< bool, size_t > max_gchild (int *heap_array, size_t i, size_t right_index)`
- `std::pair< bool, size_t > max_child_or_gchild (int *heap_array, size_t i, size_t right_index)`
- `void sift_down_min (int *heap_array, size_t sift_index, size_t right_index)`
- `void sift_down_max (int *heap_array, size_t sift_index, size_t right_index)`
- `void sift_down (int *heap_array, size_t sift_index, size_t right_index)`
- `void bubble_up_min (int *heap_array, size_t bubble_index)`
- `void bubble_up_max (int *heap_array, int bubble_index)`
- `void bubble_up (int *heap_array, int bubble_index)`

### 1.1.1 Detailed Description

The `_mmheap` namespace contains functions that are only intended for internal use by the "public-facing" functions in the `mmheap` namespace. None of the functions in `_mmheap` : : should be necessary externally.

### 1.1.2 Function Documentation

#### 1.1.2.1 `void _mmheap::bubble_up ( int * heap_array, int bubble_index )`

perform min-max heap bubble-up on an element (at `bubble_index`)

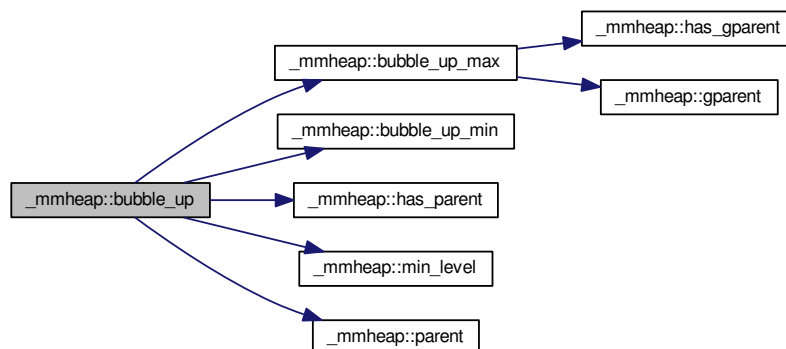
##### Parameters

<i>heap_array</i>	the heap
<i>bubble_index</i>	the index of the element that should be bubbled up

Definition at line 372 of file `mmheap.h`.

References `bubble_up_max()`, `bubble_up_min()`, `has_parent()`, `min_level()`, and `parent()`.

Here is the call graph for this function:



#### 1.1.2.2 `void _mmheap::bubble_up_max ( int * heap_array, int bubble_index )`

perform min-max heap bubble-up on an element (at `bubble_index`) that is on a max-level

##### Parameters

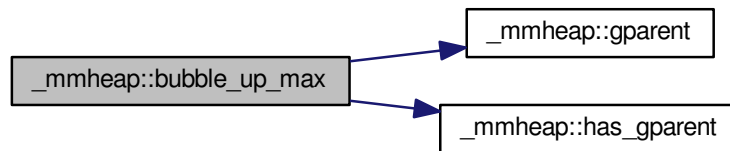
<i>heap_array</i>	the heap
<i>bubble_index</i>	the index of the element that should be bubbled up

Definition at line 354 of file `mmheap.h`.

References `gparent()`, and `has_gparent()`.

Referenced by `bubble_up()`.

Here is the call graph for this function:



Here is the caller graph for this function:



#### 1.1.2.3 `void _mmheap::bubble_up_min ( int * heap_array, size_t bubble_index )`

perform min-max heap bubble-up on an element (at `bubble_index`) that is on a min-level

Parameters

<i>heap_array</i>	the heap
<i>bubble_index</i>	the index of the element that should be bubbled up

Definition at line 336 of file [mmheap.h](#).

Referenced by [bubble\\_up\(\)](#).

Here is the caller graph for this function:



#### 1.1.2.4 `bool _mmheap::child ( size_t i, size_t c ) [inline]`

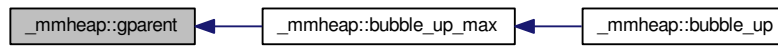
Definition at line 63 of file [mmheap.h](#).

#### 1.1.2.5 `size_t_mmheap::gparent ( size_t i )` [inline]

Definition at line 61 of file [mmheap.h](#).

Referenced by [bubble\\_up\\_max\(\)](#).

Here is the caller graph for this function:

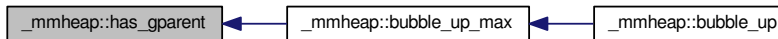


#### 1.1.2.6 `bool_mmheap::has_gparent ( size_t i )` [inline]

Definition at line 62 of file [mmheap.h](#).

Referenced by [bubble\\_up\\_max\(\)](#).

Here is the caller graph for this function:



#### 1.1.2.7 `size_t_mmheap::has_parent ( size_t i )` [inline]

Definition at line 58 of file [mmheap.h](#).

Referenced by [bubble\\_up\(\)](#).

Here is the caller graph for this function:



#### 1.1.2.8 `size_t_mmheap::left ( size_t i )` [inline]

Definition at line 59 of file [mmheap.h](#).

1.1.2.9 `uint64_t _mmheap::log_2 ( uint64_t i )`

Definition at line 71 of file [mmheap.h](#).

1.1.2.10 `std::pair<bool, size_t> _mmheap::max_child ( int * heap_array, size_t i, size_t right_index )`

get a pair consisting of an indication of whether `i` has any children, and if so, the index of the child containing the maximum value.

Parameters

<i>heap_array</i>	the heap
<i>i</i>	the index (parent) for which to find the max-child
<i>right-index</i>	the index of the right-most element that is part of the heap

Returns

a pair where the first element is `true` if `i` has children (`false` otherwise), and the second element is the index of the child whose value is largest (only if the first element is `true`)

Definition at line 187 of file [mmheap.h](#).

1.1.2.11 `std::pair<bool, size_t> _mmheap::max_child_or_gchild ( int * heap_array, size_t i, size_t right_index )`

get a pair consisting of an indication of whether `i` has any children, and if so, the index of the child or grandchild containing the maximum value.

Parameters

<i>heap_array</i>	the heap
<i>i</i>	the index (parent) for which to find the max-(grand)child
<i>right-index</i>	the index of the right-most element that is part of the heap

Returns

a pair where the first element is `true` if `i` has children (`false` otherwise), and the second element is the index of the child or grandchild whose value is largest (only if the first element is `true`)

Definition at line 243 of file [mmheap.h](#).

1.1.2.12 `std::pair<bool, size_t> _mmheap::max_gchild ( int * heap_array, size_t i, size_t right_index )`

get a pair consisting of an indication of whether `i` has any grandchildren, and if so, the index of the grandchild containing the maximum value.

Parameters

<i>heap_array</i>	the heap
<i>i</i>	the index (parent) for which to find the max-grandchild
<i>right-index</i>	the index of the right-most element that is part of the heap

Returns

a pair where the first element is `true` if `i` has grandchildren (`false` otherwise), and the second element is the index of the grandchild whose value is largest (only if the first element is `true`)

Definition at line 211 of file [mmheap.h](#).

1.1.2.13 `std::pair<bool, size_t> _mmheap::min_child ( int * heap_array, size_t i, size_t right_index )`

get a pair consisting of an indication of whether `i` has any children, and if so, the index of the child containing the minimum value.

## Parameters

<i>heap_array</i>	the heap
<i>i</i>	the index (parent) for which to find the min-child
<i>right-index</i>	the index of the right-most element that is part of the heap

## Returns

a pair where the first element is `true` if `i` has children (`false` otherwise), and the second element is the index of the child whose value is smallest (only if the first element is `true`)

Definition at line 112 of file [mmheap.h](#).

1.1.2.14 `std::pair<bool, size_t> _mmheap::min_child_or_gchild ( int * heap_array, size_t i, size_t right_index )`

get a pair considering of an indication of whether `i` has any children, and if so, the index of the child or grandchild containing the minimum value.

## Parameters

<i>heap_array</i>	the heap
<i>i</i>	the index (parent) for which to find the min-(grand)child
<i>right-index</i>	the index of the right-most element that is part of the heap

## Returns

a pair where the first element is `true` if `i` has children (`false` otherwise), and the second element is the index of the child or grandchild whose value is smallest (only if the first element is `true`)

Definition at line 167 of file [mmheap.h](#).

1.1.2.15 `std::pair<bool, size_t> _mmheap::min_gchild ( int * heap_array, size_t i, size_t right_index )`

get a pair considering of an indication of whether `i` has any grandchildren, and if so, the index of the grandchild containing the minimum value.

## Parameters

<i>heap_array</i>	the heap
<i>i</i>	the index (parent) for which to find the min-grandchild
<i>right-index</i>	the index of the right-most element that is part of the heap

## Returns

a pair where the first element is `true` if `i` has grandchildren (`false` otherwise), and the second element is the index of the grandchild whose value is smallest (only if the first element is `true`)

Definition at line 135 of file [mmheap.h](#).

1.1.2.16 `bool _mmheap::min_level ( size_t i ) [inline]`

returns `true` if `i` is on a Min-Level



**Parameters**

<i>i</i>	index into the heap
----------	---------------------

**Returns**

`true` if `i` is on a min-level

Definition at line 97 of file [mmheap.h](#).

Referenced by [bubble\\_up\(\)](#).

Here is the caller graph for this function:



#### 1.1.2.17 `size_t _mmheap::parent ( size_t i )` [inline]

Definition at line 57 of file [mmheap.h](#).

Referenced by [bubble\\_up\(\)](#).

Here is the caller graph for this function:



#### 1.1.2.18 `size_t _mmheap::right ( size_t i )` [inline]

Definition at line 60 of file [mmheap.h](#).

#### 1.1.2.19 `void _mmheap::sift_down ( int * heap_array, size_t sift_index, size_t right_index )`

perform min-max heap sift-down on an element (at `sift_index`)

**Parameters**


---

<i>heap_array</i>	the heap
<i>sift_index</i>	the index of the element that should be sifted down
<i>right_index</i>	the index of the right-most element that is part of the heap

Definition at line 321 of file [mmheap.h](#).

1.1.2.20 void mmheap::sift\_down\_max ( int \* heap\_array, size\_t sift\_index, size\_t right\_index )

perform min-max heap sift-down on an element (at `sift_index`) that is on a max-level

Parameters

<i>heap_array</i>	the heap
<i>sift_index</i>	the index of the element that should be sifted down
<i>right_index</i>	the index of the right-most element that is part of the heap

Definition at line 290 of file [mmheap.h](#).

1.1.2.21 void mmheap::sift\_down\_min ( int \* heap\_array, size\_t sift\_index, size\_t right\_index )

perform min-max heap sift-down on an element (at `sift_index`) that is on a min-level

Parameters

<i>heap_array</i>	the heap
<i>sift_index</i>	the index of the element that should be sifted down
<i>right_index</i>	the index of the right-most element that is part of the heap

Definition at line 259 of file [mmheap.h](#).

## 1.2 mmheap Namespace Reference

### Functions

- void [make\\_heap](#) (int \*heap\_array, size\_t size)  
*make an arbitrary array into a heap (in-place)*
- void [heap\\_insert](#) (int value, int \*heap\_array, size\_t &count, size\_t max\_size)
- int [heap\\_max](#) (int \*heap\_array, size\_t count)
- int [heap\\_min](#) (int \*heap\_array, size\_t count)
- std::pair< bool, int > [heap\\_insert\\_circular](#) (int value, int \*heap\_array, size\_t &count, size\_t max\_size)  
*add to heap, rotating the maximum value out if the heap is full*
- int [heap\\_replace\\_at\\_index](#) (int new\_value, size\_t index, int \*heap\_array, size\_t count)
- int [heap\\_remove\\_at\\_index](#) (size\_t index, int \*heap\_array, size\_t &count)
- int [heap\\_remove\\_min](#) (int \*heap\_array, size\_t &count)
- int [heap\\_remove\\_max](#) (int \*heap\_array, size\_t &count)

### 1.2.1 Detailed Description

The `mmheap` namespace defines functions that are useful for building and maintaining a Min-Max heap. All necessary ("public-facing") functionality is in this namespace.

### 1.2.2 Function Documentation

1.2.2.1 void mmheap::heap\_insert ( int *value*, int \* *heap\_array*, size\_t & *count*, size\_t *max\_size* )

insert a new value to the heap (and update the `count`)

## Parameters

	<i>value</i>	the new value to insert
	<i>heap_array</i>	the heap
<i>in, out</i>	<i>count</i>	the current number of items in the heap (will update)
	<i>max_size</i>	the physical storage allocation size of the heap

## Exceptions

<i>std::runtime_error</i>	if the heap is full prior to the insert operation
---------------------------	---

Definition at line 425 of file [mmheap.h](#).

1.2.2.2 `std::pair<bool, int> mmheap::heap_insert_circular ( int value, int * heap_array, size_t & count, size_t max_size )`

add to heap, rotating the maximum value out if the heap is full

Add to the min-max heap in such a way that the maximum value is removed at the same time if the heap has reached its storage capacity.

## Parameters

	<i>value</i>	new value to add
	<i>heap_array</i>	the heap
<i>in, out</i>	<i>count</i>	number of values currently in the heap (will update)
	<i>max_size</i>	maximum physical size allocated for the heap

## Returns

a pair consisting of a flag and a value; the first element is a flag indicating that overflow occurred, and the second element is the value that rotated out of the heap (formerly the maximum) when the new value was added (set only if an overflow occurred)

Definition at line 483 of file [mmheap.h](#).

1.2.2.3 `int mmheap::heap_max ( int * heap_array, size_t count )`

get the maximum value in the heap

## Parameters

<i>heap_array</i>	the heap
<i>count</i>	the current number of values contained in the heap

## Returns

the maximum value in the heap

## Exceptions

<i>std::runtime_error</i>	if the heap is empty
---------------------------	----------------------

Definition at line 444 of file [mmheap.h](#).

1.2.2.4 `int mmheap::heap_min ( int * heap_array, size_t count )`

get the minimum value in the heap

**Parameters**

<i>heap_array</i>	the heap
<i>count</i>	the current number of values contained in the heap

**Returns**

the minimum value in the heap

**Exceptions**

<i>std::runtime_error</i>	if the heap is empty
---------------------------	----------------------

Definition at line 461 of file [mmheap.h](#).

1.2.2.5 `int mmheap::heap_remove_at_index ( size_t index, int * heap_array, size_t & count )`

remove and return value at a given index

**Parameters**

	<i>index</i>	index to remove
	<i>heap_array</i>	the heap
<i>in, out</i>	<i>count</i>	current number of values in the heap (will update)

**Returns**

the value being removed

**Exceptions**

<i>std::runtime_error</i>	if the heap is empty
<i>std::range_error</i>	if the index is out of range

Definition at line 566 of file [mmheap.h](#).

1.2.2.6 `int mmheap::heap_remove_max ( int * heap_array, size_t & count )`

remove and return the maximum value in the heap

**Parameters**

<i>heap_array</i>	the array
<i>count</i>	the current number of values in the heap (will update)

**Returns**

the maximum value in the heap

**Exceptions**

<i>std::runtime_error</i>	if the heap is empty
---------------------------	----------------------

Definition at line 609 of file [mmheap.h](#).

1.2.2.7 `int mmheap::heap_remove_min ( int * heap_array, size_t & count )`

remove and return the minimum value in the heap

## Parameters

<i>heap_array</i>	the array
<i>count</i>	the current number of values in the heap (will update)

## Returns

the minimum value in the heap

## Exceptions

<i>std::runtime_error</i>	if the heap is empty
---------------------------	----------------------

Definition at line 587 of file [mmheap.h](#).

1.2.2.8 `int mmheap::heap_replace_at_index ( int new_value, size_t index, int * heap_array, size_t count )`

replace and return the value at a given index with a new value

## Parameters

<i>new_value</i>	new value to insert
<i>index</i>	index of the value to replace
<i>heap_array</i>	the heap
<i>count</i>	number of values currently stored in the heap

## Returns

the old value being replaced

## Exceptions

<i>std::runtime_error</i>	if the heap is empty
<i>std::range_error</i>	if the index is out of range

Definition at line 521 of file [mmheap.h](#).

1.2.2.9 `void mmheap::make_heap ( int * heap_array, size_t size )`

make an arbitrary array into a heap (in-place)

Applies Floyd's algorithm (adapted to a min-max heap) to produce a heap from an arbitrary array in linear time.

## Parameters

<i>heap_array</i>	the array that will become a heap
<i>size</i>	the number of elements in the array

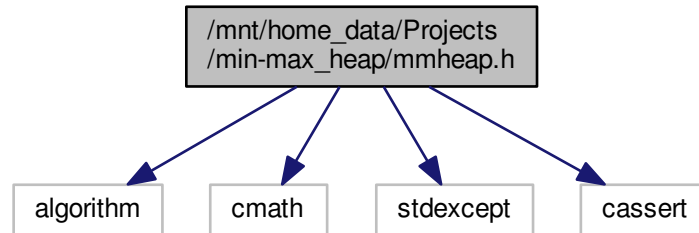
Definition at line 408 of file [mmheap.h](#).

## 2 File Documentation

### 2.1 /mnt/home\_data/Projects/min-max\_heap/mmheap.h File Reference

```
#include <algorithm>
#include <cmath>
#include <stdexcept>
#include <cassert>
```

Include dependency graph for mmheap.h:



### Namespaces

- [\\_mmheap](#)
- [mmheap](#)

### Functions

- [size\\_t \\_mmheap::parent](#) (size\_t i)
- [size\\_t \\_mmheap::has\\_parent](#) (size\_t i)
- [size\\_t \\_mmheap::left](#) (size\_t i)
- [size\\_t \\_mmheap::right](#) (size\_t i)
- [size\\_t \\_mmheap::gparent](#) (size\_t i)
- [bool \\_mmheap::has\\_gparent](#) (size\_t i)
- [bool \\_mmheap::child](#) (size\_t i, size\_t c)
- [uint64\\_t \\_mmheap::log\\_2](#) (uint64\_t i)
- [bool \\_mmheap::min\\_level](#) (size\_t i)
- [std::pair< bool, size\\_t > \\_mmheap::min\\_child](#) (int \*heap\_array, size\_t i, size\_t right\_index)
- [std::pair< bool, size\\_t > \\_mmheap::min\\_gchild](#) (int \*heap\_array, size\_t i, size\_t right\_index)
- [std::pair< bool, size\\_t > \\_mmheap::min\\_child\\_or\\_gchild](#) (int \*heap\_array, size\_t i, size\_t right\_index)
- [std::pair< bool, size\\_t > \\_mmheap::max\\_child](#) (int \*heap\_array, size\_t i, size\_t right\_index)
- [std::pair< bool, size\\_t > \\_mmheap::max\\_gchild](#) (int \*heap\_array, size\_t i, size\_t right\_index)
- [std::pair< bool, size\\_t > \\_mmheap::max\\_child\\_or\\_gchild](#) (int \*heap\_array, size\_t i, size\_t right\_index)
- [void \\_mmheap::sift\\_down\\_min](#) (int \*heap\_array, size\_t sift\_index, size\_t right\_index)
- [void \\_mmheap::sift\\_down\\_max](#) (int \*heap\_array, size\_t sift\_index, size\_t right\_index)
- [void \\_mmheap::sift\\_down](#) (int \*heap\_array, size\_t sift\_index, size\_t right\_index)
- [void \\_mmheap::bubble\\_up\\_min](#) (int \*heap\_array, size\_t bubble\_index)
- [void \\_mmheap::bubble\\_up\\_max](#) (int \*heap\_array, int bubble\_index)
- [void \\_mmheap::bubble\\_up](#) (int \*heap\_array, int bubble\_index)
- [void mmheap::make\\_heap](#) (int \*heap\_array, size\_t size)  
*make an arbitrary array into a heap (in-place)*
- [void mmheap::heap\\_insert](#) (int value, int \*heap\_array, size\_t &count, size\_t max\_size)
- [int mmheap::heap\\_max](#) (int \*heap\_array, size\_t count)
- [int mmheap::heap\\_min](#) (int \*heap\_array, size\_t count)

- `std::pair< bool, int > mmheap::heap_insert_circular` (int value, int \*heap\_array, size\_t &count, size\_t max\_size)  
*add to heap, rotating the maximum value out if the heap is full*
- `int mmheap::heap_replace_at_index` (int new\_value, size\_t index, int \*heap\_array, size\_t count)
- `int mmheap::heap_remove_at_index` (size\_t index, int \*heap\_array, size\_t &count)
- `int mmheap::heap_remove_min` (int \*heap\_array, size\_t &count)
- `int mmheap::heap_remove_max` (int \*heap\_array, size\_t &count)

### 2.1.1 Detailed Description

Defines functions for maintaining a Min-Max Heap, as described by Adkinson: M. D. Atkinson, J.-R. Sack, N. Santoro, and T. Strothotte. 1986. Min-max heaps and generalized priority queues. Commun. ACM 29, 10 (October 1986), 996-1000. DOI=<http://dx.doi.org/10.1145/6617.6621>

This file defines two namespaces:

- The `mmheap` namespace defines functions that are useful for building and maintaining a Min-Max heap. All necessary ("public-facing") functionality is in this namespace.
- The `_mmheap` namespace contains functions that are only intended for internal use by the "public-facing" functions in the `mmheap` namespace. None of the functions in `_mmheap` : : should be necessary externally.

### Author

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Definition in file [mmheap.h](#).

## 2.2 /mnt/home\_data/Projects/min-max\_heap/mmheap.h

```
00001 #ifndef MMHEAP_H
00002 #define MMHEAP_H
00003 /**
00004  * @file mmheap.h
00005  *
00006  * Defines functions for maintaining a Min-Max Heap,
00007  * as described by Adkinson:
00008  *      M. D. Atkinson, J.-R. Sack, N. Santoro, and T. Strothotte. 1986.
```



```

00009 *      Min-max heaps and generalized priority queues.
00010 *      Commun. ACM 29, 10 (October 1986), 996-1000.
00011 *      DOI=http://dx.doi.org/10.1145/6617.6621
00012 *
00013 * @details
00014 *      This file defines two namespaces:
00015 *      * The 'mmheap' namespace defines functions that are useful for building and
00016 *      maintaining a Min-Max heap. All necessary ("public-facing") functionality
00017 *      is in this namespace.
00018 *      * The '_mmheap' namespace contains functions that are only intended for
00019 *      internal use by the "public-facing" functions in the 'mmheap' namespace.
00020 *      None of the functions in '_mmheap::' should be necessary externally.
00021 *
00022 * @author Jason L Causey
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00041 *      OUT OF OR IN CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN
00042 *      THE SOFTWARE.
00043 */
00044
00045 #include <algorithm>
00046 #include <cmath>
00047 #include <stdexcept>
00048 #include <cassert>
00049
00050 /**
00051 * The '_mmheap' namespace contains functions that are only intended for internal
00052 * use by the "public-facing" functions in the 'mmheap' namespace. None of the
00053 * functions in '_mmheap::' should be necessary externally.
00054 */
00055 namespace _mmheap{
00056
00057     inline size_t parent(size_t i)          { return (i - 1) / 2; }
00058     inline size_t has_parent(size_t i)      { return i > 0; }
00059     inline size_t left (size_t i)          { return 2*i + 1; }
00060     inline size_t right (size_t i)         { return 2*i + 2; }
00061     inline size_t gparent(size_t i)         { return parent(parent(i)); }
00062     inline bool has_gparent(size_t i)       { return i > 2; }
00063     inline bool child(size_t i, size_t c) { return c == left(i) || c == right(i); }
00064
00065     /*
00066     * fast log-base-2 based on code from:
00067     * http://stackoverflow.com/a/11398748
00068     * @param i value to compute the log_2 for (must be > 0)
00069     * @return log-base-2 of 'i'
00070     */
00071     uint64_t log_2(uint64_t i) {
00072         static const uint64_t tab64[64] = {
00073             63, 0, 58, 1, 59, 47, 53, 2,
00074             60, 39, 48, 27, 54, 33, 42, 3,
00075             61, 51, 37, 40, 49, 18, 28, 20,
00076             55, 30, 34, 11, 43, 14, 22, 4,
00077             62, 57, 46, 52, 38, 26, 32, 41,
00078             50, 36, 17, 19, 29, 10, 13, 21,
00079             56, 45, 25, 31, 35, 16, 9, 12,
00080             44, 24, 15, 8, 23, 7, 6, 5
00081         };
00082         i |= i >> 1;
00083         i |= i >> 2;

```

```

00084         i |= i >> 4;
00085         i |= i >> 8;
00086         i |= i >> 16;
00087         i |= i >> 32;
00088         return tab64[(uint64_t)((i - (i >> 1))*0x07EDD5E59A4E28C2)) >> 58];
00089     }
00090
00091     /**
00092     * returns 'true' if 'i' is on a Min-Level
00093     *
00094     * @param i index into the heap
00095     * @return 'true' if 'i' is on a min-level
00096     */
00097     inline bool min_level(size_t i) {
00098         return i > 0 ? log_2(++i) % 2 == 0 : true;
00099     }
00100
00101     /**
00102     * get a pair consisting of an indication of whether 'i' has any children, and
00103     * if so, the index of the child containing the minimum value.
00104     *
00105     * @param heap_array the heap
00106     * @param i the index (parent) for which to find the min-child
00107     * @param right_index the index of the right-most element that is part of the heap
00108     * @return a pair where the first element is 'true' if 'i' has children ('false'
00109     * otherwise), and the second element is the index of the child whose value
00110     * is smallest (only if the first element is 'true')
00111     */
00112     std::pair<bool, size_t> min_child(int*
heap_array, size_t i, size_t right_index){
00113         std::pair<bool, size_t> result{false, 0};
00114         if(left(i) <= right_index){
00115             auto m = left(i);
00116             if(right(i) <= right_index && heap_array[right(i)] < heap_array[m]){
00117                 m = right(i);
00118             }
00119             result = {true, m};
00120         }
00121         return result;
00122     }
00123
00124     /**
00125     * get a pair consisting of an indication of whether 'i' has any grandchildren, and
00126     * if so, the index of the grandchild containing the minimum value.
00127     *
00128     * @param heap_array the heap
00129     * @param i the index (parent) for which to find the min-grandchild
00130     * @param right_index the index of the right-most element that is part of the heap
00131     * @return a pair where the first element is 'true' if 'i' has grandchildren
00132     * ('false' otherwise), and the second element is the index of the
00133     * grandchild whose value is smallest (only if the first element is 'true')
00134     */
00135     std::pair<bool, size_t> min_gchild(int*
heap_array, size_t i, size_t right_index){
00136         std::pair<bool, size_t> result{false, 0};
00137         auto l = left(i);
00138         auto r = right(i);
00139         if(left(l) <= right_index){
00140             auto m = left(l);
00141             if(right(l) <= right_index && heap_array[right(l)] < heap_array[m]){
00142                 m = right(l);
00143             }
00144             if(left(r) <= right_index && heap_array[left(r)] < heap_array[m]){
00145                 m = left(r);
00146             }
00147             if(right(r) <= right_index && heap_array[right(r)] < heap_array[m]){
00148                 m = right(r);
00149             }
00150             result = {true, m};
00151         }
00152         return result;
00153     }
00154
00155     /**
00156     * get a pair consisting of an indication of whether 'i' has any children, and
00157     * if so, the index of the child or grandchild containing the minimum value.
00158     *
00159     * @param heap_array the heap
00160     * @param i the index (parent) for which to find the min-(grand)child
00161     * @param right_index the index of the right-most element that is part of the heap
00162     * @return a pair where the first element is 'true' if 'i' has children

```

```

00163      *          ('false' otherwise), and the second element is the index of the
00164      *          child or grandchild whose value is smallest (only if the first
00165      *          element is 'true')
00166      */
00167      std::pair<bool, size_t> min_child_or_gchild(int*
heap_array, size_t i, size_t right_index){
00168          auto m = min_child(heap_array, i, right_index);
00169          if(m.first){
00170              auto gm = min_gchild(heap_array, i, right_index);
00171              m.second = gm.first && heap_array[gm.second] < heap_array[m.second] ? gm.second : m.second
;
00172          }
00173          return m;
00174      }
00175
00176      /**
00177      * get a pair consisting of an indication of whether 'i' has any children, and
00178      * if so, the index of the child containing the maximum value.
00179      *
00180      * @param heap_array the heap
00181      * @param i           the index (parent) for which to find the max-child
00182      * @param right-index the index of the right-most element that is part of the heap
00183      * @return a pair where the first element is 'true' if 'i' has children ('false'
00184      *         otherwise), and the second element is the index of the child whose value
00185      *         is largest (only if the first element is 'true')
00186      */
00187      std::pair<bool, size_t> max_child(int*
heap_array, size_t i, size_t right_index){
00188          std::pair<bool, size_t> result {false, 0};
00189          if(left(i) <= right_index){
00190              auto m = left(i);
00191              if(right(i) <= right_index && heap_array[right(i)] > heap_array[m]){
00192                  m = right(i);
00193              }
00194              result = {true, m};
00195          }
00196          return result;
00197      }
00198
00199      /**
00200      * get a pair consisting of an indication of whether 'i' has any grandchildren, and
00201      * if so, the index of the grandchild containing the maximum value.
00202      *
00203      * @param heap_array the heap
00204      * @param i           the index (parent) for which to find the max-grandchild
00205      * @param right-index the index of the right-most element that is part of the heap
00206      * @return a pair where the first element is 'true' if 'i' has grandchildren
00207      *         ('false' otherwise), and the second element is the index of the
00208      *         grandchild whose value is largest (only if the first element is 'true')
00209      */
00210
00211      std::pair<bool, size_t> max_gchild(int*
heap_array, size_t i, size_t right_index){
00212          std::pair<bool, size_t> result{false, 0};
00213          auto l = left(i);
00214          auto r = right(i);
00215          if(left(l) <= right_index){
00216              auto m = left(l);
00217              if(right(l) <= right_index && heap_array[right(l)] > heap_array[m]){
00218                  m = right(l);
00219              }
00220              if(left(r) <= right_index && heap_array[left(r)] > heap_array[m]){
00221                  m = left(r);
00222              }
00223              if(right(r) <= right_index && heap_array[right(r)] > heap_array[m]){
00224                  m = right(r);
00225              }
00226              result = {true, m};
00227          }
00228          return result;
00229      }
00230
00231      /**
00232      * get a pair consisting of an indication of whether 'i' has any children, and
00233      * if so, the index of the child or grandchild containing the maximum value.
00234      *
00235      * @param heap_array the heap
00236      * @param i           the index (parent) for which to find the max-(grand)child
00237      * @param right-index the index of the right-most element that is part of the heap
00238      * @return a pair where the first element is 'true' if 'i' has children
00239      *         ('false' otherwise), and the second element is the index of the

```

```

00240     *           child or grandchild whose value is largest (only if the first
00241     *           element is 'true')
00242     */
00243     std::pair<bool, size_t> max_child_or_gchild(int*
heap_array, size_t i, size_t right_index){
00244         auto m = max_child(heap_array, i, right_index);
00245         if(m.first){
00246             auto gm = max_gchild(heap_array, i, right_index);
00247             m.second = gm.first && heap_array[gm.second] > heap_array[m.second] ? gm.second : m.second
;
00248         }
00249         return m;
00250     }
00251
00252     /**
00253     * perform min-max heap sift-down on an element (at 'sift_index') that is on a min-level
00254     *
00255     * @param heap_array the heap
00256     * @param sift_index the index of the element that should be sifted down
00257     * @param right_index the index of the right-most element that is part of the heap
00258     */
00259     void sift_down_min(int* heap_array, size_t sift_index, size_t right_index){
00260         bool sift_more = true;
00261         while(sift_more && left(sift_index) <= right_index){           // if a[i] has
children
00262             sift_more = false;
00263             auto mp = min_child_or_gchild(heap_array, sift_index, right_index);           // get min
child or grandchild
00264             auto m = mp.second;
00265             if(child(sift_index, m)){           // if the min
was a child
00266                 if(heap_array[m] < heap_array[sift_index]){
00267                     std::swap(heap_array[m], heap_array[sift_index]);
00268                 }
00269             }
00270             else{           // min was a
grandchild
00271                 if(heap_array[m] < heap_array[sift_index]){
00272                     std::swap(heap_array[m], heap_array[sift_index]);
00273                     if(heap_array[m] > heap_array[parent(m)]){
00274                         std::swap(heap_array[m], heap_array[parent(m)]);
00275                     }
00276                     sift_index = m;
00277                     sift_more = true;
00278                 }
00279             }
00280         }
00281     }
00282
00283     /**
00284     * perform min-max heap sift-down on an element (at 'sift_index') that is on a max-level
00285     *
00286     * @param heap_array the heap
00287     * @param sift_index the index of the element that should be sifted down
00288     * @param right_index the index of the right-most element that is part of the heap
00289     */
00290     void sift_down_max(int* heap_array, size_t sift_index, size_t right_index){
00291         bool sift_more = true;
00292         while(sift_more && left(sift_index) <= right_index){           // if a[i] has
children
00293             sift_more = false;
00294             auto mp = max_child_or_gchild(heap_array, sift_index, right_index);           // get max
child or grandchild
00295             auto m = mp.second;
00296             if(child(sift_index, m)){           // if the max
was a child
00297                 if(heap_array[m] > heap_array[sift_index]){
00298                     std::swap(heap_array[m], heap_array[sift_index]);
00299                 }
00300             }
00301             else{           // max was a
grandchild
00302                 if(heap_array[m] > heap_array[sift_index]){
00303                     std::swap(heap_array[m], heap_array[sift_index]);
00304                     if(heap_array[m] < heap_array[parent(m)]){
00305                         std::swap(heap_array[m], heap_array[parent(m)]);
00306                     }
00307                     sift_index = m;
00308                     sift_more = true;
00309                 }
00310             }
00311         }

```

```

00311     }
00312 }
00313
00314 /**
00315  * perform min-max heap sift-down on an element (at 'sift_index')
00316  *
00317  * @param heap_array the heap
00318  * @param sift_index the index of the element that should be sifted down
00319  * @param right_index the index of the right-most element that is part of the heap
00320  */
00321 void sift_down(int* heap_array, size_t sift_index, size_t right_index){
00322     if(min_level(sift_index)){
00323         sift_down_min(heap_array, sift_index, right_index);
00324     }
00325     else{
00326         sift_down_max(heap_array, sift_index, right_index);
00327     }
00328 }
00329
00330 /**
00331  * perform min-max heap bubble-up on an element (at 'bubble_index') that is on a min-level
00332  *
00333  * @param heap_array the heap
00334  * @param bubble_index the index of the element that should be bubbled up
00335  */
00336 void bubble_up_min(int* heap_array, size_t bubble_index){
00337     bool finished = false;
00338     while(!finished && has_gparent(bubble_index)){
00339         finished = true;
00340         if(heap_array[bubble_index] < heap_array[gparent(bubble_index)]){
00341             std::swap(heap_array[bubble_index], heap_array[gparent(bubble_index)]);
00342             bubble_index = gparent(bubble_index);
00343             finished = false;
00344         }
00345     }
00346 }
00347
00348 /**
00349  * perform min-max heap bubble-up on an element (at 'bubble_index') that is on a max-level
00350  *
00351  * @param heap_array the heap
00352  * @param bubble_index the index of the element that should be bubbled up
00353  */
00354 void bubble_up_max(int* heap_array, int bubble_index){
00355     bool finished = false;
00356     while(!finished && has_gparent(bubble_index)){
00357         finished = true;
00358         if(heap_array[bubble_index] > heap_array[gparent(bubble_index)
00359     ]){
00359             std::swap(heap_array[bubble_index], heap_array[gparent
00360 (bubble_index)]);
00361             bubble_index = gparent(bubble_index);
00362             finished = false;
00363         }
00364     }
00365 }
00366 /**
00367  * perform min-max heap bubble-up on an element (at 'bubble_index')
00368  *
00369  * @param heap_array the heap
00370  * @param bubble_index the index of the element that should be bubbled up
00371  */
00372 void bubble_up(int* heap_array, int bubble_index){
00373     if(min_level(bubble_index)){
00374         if(has_parent(bubble_index) && heap_array[bubble_index] > heap_array[
00375 parent(bubble_index)]){
00375             std::swap(heap_array[bubble_index], heap_array[parent
00376 (bubble_index)]);
00376             bubble_up_max(heap_array, parent(bubble_index)
00377 );
00377         }
00378         else{
00379             bubble_up_min(heap_array, bubble_index);
00380         }
00381     }
00382     else{
00383         if(has_parent(bubble_index) && heap_array[bubble_index] < heap_array[
00384 parent(bubble_index)]){
00384             std::swap(heap_array[bubble_index], heap_array[parent
00385 (bubble_index)]);

```

```

00385         bubble_up_min(heap_array, parent(bubble_index
00386     ));
00387     }
00388     else{
00389         bubble_up_max(heap_array, bubble_index);
00390     }
00391 }
00392 }
00393
00394 /**
00395  * The 'mmheap' namespace defines functions that are useful for building and
00396  * maintaining a Min-Max heap. All necessary ("public-facing") functionality
00397  * is in this namespace.
00398  */
00399 namespace mmheap{
00400     /**
00401      * @brief make an arbitrary array into a heap (in-place)
00402      * @details Applies Floyd's algorithm (adapted to a min-max heap) to produce
00403      * a heap from an arbitrary array in linear time.
00404      *
00405      * @param heap_array the array that will become a heap
00406      * @param size the number of elements in the array
00407      */
00408 void make_heap(int* heap_array, size_t size){
00409     if(size > 1){
00410         for(int current = _mmheap::parent(size-1); current >= 0; --current){
00411             _mmheap::sift_down(heap_array, current, size-1);
00412         }
00413     }
00414 }
00415
00416 /**
00417  * insert a new value to the heap (and update the 'count')
00418  *
00419  * @param value the new value to insert
00420  * @param heap_array the heap
00421  * @param[in,out] count the current number of items in the heap (will update)
00422  * @param max_size the physical storage allocation size of the heap
00423  * @throws std::runtime_error if the heap is full prior to the insert operation
00424  */
00425 void heap_insert(int value, int* heap_array, size_t& count, size_t max_size){
00426     if(count < max_size){
00427         heap_array[count++] = value;
00428         _mmheap::bubble_up(heap_array, count-1);
00429     }
00430     else{
00431         throw std::runtime_error("Cannot insert into heap - allocated size is full.");
00432     }
00433 }
00434
00435 /**
00436  * get the maximum value in the heap
00437  *
00438  * @param heap_array the heap
00439  * @param count the current number of values contained in the heap
00440  *
00441  * @return the maximum value in the heap
00442  * @throws std::runtime_error if the heap is empty
00443  */
00444 int heap_max(int* heap_array, size_t count){
00445     if(count < 1){
00446         throw std::runtime_error("Cannot get max value in empty heap.");
00447     }
00448     auto m = _mmheap::max_child(heap_array, 0, count-1);
00449     return m.first ? heap_array[m.second] : heap_array[0];
00450 }
00451
00452 /**
00453  * get the minimum value in the heap
00454  *
00455  * @param heap_array the heap
00456  * @param count the current number of values contained in the heap
00457  *
00458  * @return the minimum value in the heap
00459  * @throws std::runtime_error if the heap is empty
00460  */
00461 int heap_min(int* heap_array, size_t count){
00462     if(count < 1){
00463         throw std::runtime_error("Cannot get min value in empty heap.");
00464     }

```

```

00465         return heap_array[0];
00466     }
00467
00468     /**
00469     * @brief add to heap, rotating the maximum value out if the heap is full
00470     * @details Add to the min-max heap in such a way that the maximum value is removed
00471     *          at the same time if the heap has reached its storage capacity.
00472     *
00473     * @param value new value to add
00474     * @param heap_array the heap
00475     * @param[in,out] count number of values currently in the heap (will update)
00476     * @param max_size maximum physical size allocated for the heap
00477     *
00478     * @return a pair consisting of a flag and a value; the first element is a flag
00479     *          indicating that overflow occurred, and the second element is the value
00480     *          that rotated out of the heap (formerly the maximum) when the new value
00481     *          was added (set only if an overflow occurred)
00482     */
00483     std::pair<bool, int> heap_insert_circular(int value, int* heap_array,
size_t& count, size_t max_size){
00484         int max_value = 0;
00485         bool overflowed = count == max_size ? true : false;
00486         if(!overflowed){
00487             heap_insert(value, heap_array, count, max_size);
00488         }
00489         else{
00490             // if the heap is full, replace the
max value with the new add...
00491             auto m = max_size > 1 ? _mmheap::max_child(heap_array, 0, max_size-1).second : 0;
00492             max_value = heap_array[m];
00493             if(value < max_value){
00494                 // if the new value is larger than
the one rotating out, just rotate the new value
00495                 heap_array[m] = value;
00496                 if(max_size > 1){
00497                     // if this is non-trivial
// check that the new value isn't
the new min
00498                     if(value < heap_array[0]){
00499                         std::swap(heap_array[0], heap_array[m]); // (if it is, make it so)
00500                     }
00501                     _mmheap::sift_down(heap_array, m, max_size-1); // sift the new item down
00502                 }
00503             }
00504             else{
00505                 max_value = value;
00506             }
00507             return std::pair<bool, int>{overflowed, max_value};
00508         }
00509     }
00510
00511     /**
00512     * replace and return the value at a given index with a new value
00513     *
00514     * @param new_value new value to insert
00515     * @param index index of the value to replace
00516     * @param heap_array the heap
00517     * @param count number of values currently stored in the heap
00518     *
00519     * @return the old value being replaced
00520     * @throws std::runtime_error if the heap is empty
00521     * @throws std::range_error if the index is out of range
00522     */
00523     int heap_replace_at_index(int new_value, size_t index, int* heap_array,
size_t count){
00524         if(count == 0){
00525             throw std::runtime_error("Cannot replace value in empty heap.");
00526         }
00527         if(index > count){
00528             throw std::range_error("Index beyond end of heap.");
00529         }
00530         int old_value = heap_array[index];
00531         heap_array[index] = new_value;
00532         if(_mmheap::min_level(index)){
00533             if(new_value < old_value){
00534                 _mmheap::bubble_up_min(heap_array, index);
00535             }
00536             else{
00537                 if(_mmheap::has_parent(index) && heap_array[_mmheap::parent(index)] < new_value){
00538                     _mmheap::bubble_up(heap_array, index);
00539                 }
00540                 _mmheap::sift_down(heap_array, index, count-1);
00541             }
00542         }
00543     }

```

```

00541         else{
00542             if(new_value > old_value){
00543                 _mmheap::bubble_up_max(heap_array, index);
00544             }
00545             else{
00546                 if(_mmheap::has_parent(index) && new_value < heap_array[_mmheap::parent(index)]){
00547                     _mmheap::bubble_up(heap_array, index);
00548                 }
00549                 _mmheap::sift_down(heap_array, index, count-1);
00550             }
00551         }
00552         return old_value;
00553     }
00554
00555     /**
00556     * remove and return value at a given index
00557     *
00558     * @param      index      index to remove
00559     * @param      heap_array the heap
00560     * @param[in,out] count    current number of values in the heap (will update)
00561     *
00562     * @return     the value being removed
00563     * @throws     std::runtime_error if the heap is empty
00564     * @throws     std::range_error  if the index is out of range
00565     */
00566     int heap_remove_at_index(size_t index, int* heap_array, size_t& count){
00567         if(count == 0){
00568             throw std::runtime_error("Cannot remove value in empty heap.");
00569         }
00570         if(index > count){
00571             throw std::range_error("Index beyond end of heap.");
00572         }
00573         int old_value = heap_replace_at_index(heap_array[count-1], index, heap_array, count);
00574         --count;
00575         return old_value;
00576     }
00577
00578     /**
00579     * remove and return the minimum value in the heap
00580     *
00581     * @param heap_array the array
00582     * @param count      the current number of values in the heap (will update)
00583     *
00584     * @return the minimum value in the heap
00585     * @throws std::runtime_error if the heap is empty
00586     */
00587     int heap_remove_min(int* heap_array, size_t& count){
00588         if(count == 0){
00589             throw std::runtime_error("Cannot remove from empty heap.");
00590         }
00591         int value = heap_array[0];
00592         std::swap(heap_array[0], heap_array[count-1]);
00593         --count;
00594         if(count > 0){
00595             _mmheap::sift_down(heap_array, 0, count-1);
00596         }
00597         return value;
00598     }
00599
00600     /**
00601     * remove and return the maximum value in the heap
00602     *
00603     * @param heap_array the array
00604     * @param count      the current number of values in the heap (will update)
00605     *
00606     * @return the maximum value in the heap
00607     * @throws std::runtime_error if the heap is empty
00608     */
00609     int heap_remove_max(int* heap_array, size_t& count){
00610         if(count == 0){
00611             throw std::runtime_error("Cannot remove from empty heap.");
00612         }
00613         auto value = heap_array[0];
00614         auto m = _mmheap::max_child(heap_array, 0, count-1);
00615         if(m.first){
00616             value = heap_array[m.second];
00617         }
00618         else{
00619             m.second = 0;
00620         }
00621         heap_remove_at_index(m.second, heap_array, count);

```



```
00622         return value;
00623     }
00624 }
00625
00626 #endif
```

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