Fitting spatial random field models using `Stan` and the SPDE approach: implementation via `TMB` and a comparative study of two different parametrizations.

Joaquin Cavieres[1], Paula Moraga[2]

[1] Instituto de Estadística, Universidad de Valparaíso
joaquin.cavieres@uv.cl

[2]Computer, Electrical, and Mathematical Sciences and Engineering Division, King Abdullah University of Science and Technology (KAUST)
paula.moraga@kaust.edu.sa

Abstract

`Stan` is a statistical software that uses Hamiltonian Monte Carlo (HMC) for Bayesian inference and is widely used to solve a range of problems that arise in scientific disciplines, including astronomy, ecology, biology and engineering. `Stan` has been used to analyze problems of moderate size in a particular spatial domain, based on observed data in a regular/irregular grid (lattice data) or point referenced data (geostatistical). However, the use of `Stan` for the analysis of big spatial and spatio-temporal datasets is still limited due to its inability to deal with sparse matrices and the high computational costs when solving problems in these settings.

An alternative to fit spatial models in a Bayesian framework is to use the integrated nested Laplace approximation (INLA) and the stochastic partial differential equation (SPDE) approaches which are implemented in the R-INLA package for approximated Bayesian inference in latent Gaussian models. The computational efficiency is achieved when the SPDE method approximates a Gaussian random field by a Gaussian Markov random field using a discretization of the region of the study. A version of the SPDE/GMRF parametrization allow us to obtain approximations of a continuous Gaussian random field using a parameter that controls the variance (τ) and a spatial parameter of scale (κ). Another version uses the parameters σ (marginal standard deviation) and ρ (range) which is the distance at which observations are no longer spatially correlated.

In this work, we implement a C++ routine that enables to fit spatial and spatio-temporal models via `Stan` using the SPDE approach. Specifically, we use Template Model Builder (`TMB`) and perform inference using the R package `tmbstan` which facilitates the linkage between `TMB` and `Stan`. The use of `tmbstan` allows us perform inference in a Bayesian framework through No-U-Turn Sampler (NUTS), sampling the posterior distribution of the parameters of the Gaussian random field. We also compare the newly implemented parametrization of the SPDE method (σ and ρ) with the existing parametrization in TMB (τ and κ), and show the performance of both parametrizations using several goodness of fit diagnostics obtained by means of a simulation study in different settings.

This work allows us to fit spatial and spatio-temporal models using Bayesian inference and MCMC through the R package `tmbstan` in a friendly way and in a reasonable computational time.

Keywords: Spatial modeling; Gaussian random field; `Stan`; Template Model Builder (`TMB`); Stochastic partial differential equation (SPDE)