HW 4. Shixiang Zhu GTID # 903280826.

Problem 5.1.

$$p_i(x) = f(x_i) + (x - x_i)\frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} + O(x_{i+1} - x_i)^3$$

$$= f(x_i) + (x - x_i)\frac{f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2}(x_{i+1} - x_i)^2 - f(x_i)}{x_{i+1} - x_i}$$

$$= f(x_i) + f'(x_i)(x - x_i) + \frac{f''(x_i)}{2}(x_{i+1} - x_i)(x - x_i) + O(x_{i+1} - x_i)$$

$$= f(x_i) + f'(x_i)(x - x_i) + \frac{f''(x_i)}{2}(x_{i+1} - x_i)(x - x_i) + O(n^3)$$

Problem 5.2. let $t = \frac{x_i + x_{i+1}}{2}$

then $p_{i0}(x) = \frac{(x - t)}{x_i - t}\frac{(x - x_{i+1})}{x_i - x_{i+1}}$

Therefore $A_{i0} = \frac{1}{(x_i - t)(x_i - x_{i+1})}\int_{x_i}^{x_{i+1}} x^2 - (t + x_{i+1})x + t x_{i+1}\, dx$

$$= \frac{(x_{i+1} - x_i)(2x_{i+1}^2 + 2x_{i+1}x_i + 2x_i^2 - \frac{3}{2}(3x_{i+1}^2 + 4x_i x_{i+1} + x_i^2) + 3(x_{i+1}^2 + x_i x_{i+1})}{6(x_i - t)(x_i - x_{i+1})}$$

$$= \frac{x_{i+1} - x_i}{6}.$$

similarly $p_{i2}(x) = \frac{x - t}{x_{i+1} - t}\cdot\frac{x - x_i}{x_{i+1} - x_i}$

and p_{i2} and p_{i0} are symmetrily identical.

then $A_{i2} = \int_{x_{i+1}}^{x_i} p_{i2}(x)\,dx = -\frac{x_i - x_{i+1}}{6} = \frac{x_{i+1} - x_i}{6}.$

Finally $p_{i1}(x) = \frac{x - x_i}{t - x_i}\cdot\frac{x - x_{i+1}}{t - x_{i+1}}.$

$$A_{i1} = \int_{x_i}^{x_{i+1}} p_{i1}(x)\,dx$$

$$= \frac{(x_{i+1} - x_i)(2x_{i+1}^2 + 2x_i x_{i+1} + 2x_i^2 - 3x_i^2 - 3x_{i+1}^2 - 6x_{i+1}x_i + 6x_{i+1}x_i}{-6\cdot\frac{1}{4}(x_{i+1} - x_i)^2}$$

$$= \frac{2}{3}(x_{i+1} - x_i).$$