Math 104A Homework #2 *

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General Instructions: Please write your homework papers neatly. You need to turn in both your code and descriptions on Canvas with the appropriate form that the TA requires. Write your own code, individually. Do not copy codes!

1. (a) Write the Lagrangian form of the interpolating polynomial $P_2(x)$ corresponding to the data in the table below:

x_j	$f(x_j)$
0	1
1	1
3	-5

   (b) Use $P_2(x)$ you obtained in (a) to approximate $f(2)$.

2. (a) Implement the Barycentric Formula for evaluating the interpolating polynomial for arbitrarily distributed nodes $x_0, ..., x_n$; you need to write a function or script that computes the barycentric weights $\lambda_j^{(n)} = 1/\Pi_{k\neq j}(x_j - x_k)$ first and another code to use these values in the Barycentric Formula. Make sure to test your implementation.

   (b) Consider the following table of data

x_j	$f(x_j)$
0.00	0.0000
0.25	0.7071
0.50	1.0000
0.75	0.7071
1.25	-0.7071
1.50	-1.0000

   Use your code in (a) to find $P_5(2)$ as an approximation of $f(2)$.

3. *The Runge Example.* Let

$$f(x) = \frac{1}{1+x^2}, \quad x \in [-5, 5]. \tag{1}$$

Using your Barycentric Formula code (Prob. 3) and (2) and (3) below, evaluate and plot the interpolating polynomial of $f(x)$ corresponding to

(a) the equidistributed nodes $x_j = -5 + j(10/n)$, $j = 0, ..., n$ for $n = 4$, 8, and 12.

(b) the nodes $x_j = 5\cos(\frac{j\pi}{n})$, $j = 0, ..., n$ for $n = 4$, 8, 12, and 100.

(c) Repeat (a) for $f(x) = e^{-x^2/5}$ for $x \in [-5, 5]$ and comment on the result.

Remark 1. *It can be shown that for equidistributed nodes one can use the barycentric weights*

$$\lambda_j^{(n)} = (-1)^j \binom{n}{j}, \quad j = 0, ..., n, \tag{2}$$

where $\binom{n}{j}$ *is the binomial coefficient (nchoosek(n,j) in Matlab). It can be shown that for the nodes* $x_j = \frac{a+b}{2} + \frac{b-a}{2}\cos(\frac{j\pi}{n})$, $j = 0, ..., n$, *in* $[a, b]$, *one can use*

$$\lambda_j^{(n)} = \begin{cases} \frac{1}{2}(-1)^j & \textit{for } j = 0 \textit{ or } j = n \\ (-1)^j & \textit{for } j = 1, ..., n-1. \end{cases} \tag{3}$$

Make sure to employ (2) and (3) in your Barycentric Formula code for this problem. To plot the corresponding $P_n(x)$ *evaluate* $P_n(x)$ *at a large number of points* $\bar{x}$ *to have a good plotting resolution, e.g.* $\bar{x}_k = -5 + k(10/n_e)$, $k = 0, ..., n_e$ *with* $n_e = 5000$. *Note that your Barycentric Formula cannot be used to evaluate* $P_n(x)$ *when* x *coincides with an interpolating node! Plot also* f *for comparison. Compare (a) and (b) and comment on the result in view of what you observed in Prob. 2.*