Department of Mathematics and Statistics, California State University, Long Beach

STAT 572 : Computational Statistics, Spring 20
Final : Due by midnight Th, May 14 in the Dropbox

Note: You have to provide both procedure and MATLAB code for each problem followed by discussions (note the instruction that I sent earlier). Insert a page break for each problem. Hand-written report will not be accepted. Electronic copy (in pdf format) is due by midnight on May 14 (Thursday night). Put your file in the Dropbox on the class BeachBoard. Late submission will not be accepted.

1. Consider the following probability distribution function

$$f(x) = \frac{1}{b}\exp[-(x-a)/b]\exp\{-\exp[-(x-a)/b]\}, b > 0.$$

This distribution is called Extreme Value distribution (EV(a,b)) with the location parameter a and the scale parameter b. This distribution has been used widely to model extreme events, for example, annual maximum rainfall. We are interested in generating random sample from the density with $a = 2$ and $b = 1$.

   (a) Use any method of your choice to generate a random sample of size 100 from the density. Plot the density histogram of the random sample with the true density superimposed. Also provide the empirical and the theoretical CDF and compare.

   (b) Using the random sample in (a), construct a MC for the location parameter a assuming the scale parameter b is known as 1. Plot the MC and the density histogram after burn-in. Give the mean, standard deviation, and 95% percentile interval for a. Discuss your findings.

2. It is known that the distribution of waiting times between events in a Poisson process with intensity λ are $Exp(\lambda)$. We would like to use this fact to generate random numbers from $Poisson(\lambda)$. Generate events $X_i = \sum_{j=1}^{i} Y_j,\ Y_j \sim Exp(\lambda)$ and then take $Z = \#\{X_i : X_i \in [l-1, l)\}, l \geq 1$, as $Poisson(\lambda)$ pseudo-rv's. Write a function to generate N such random numbers. Using your function generate N=1000 such random numbers with $\lambda = 1$. Count the outcomes in the calagories 0, 1, 2, ..., 10, 10+. Provide the density histogram and compare with the theoretical density. Discuss your findings.

   (a) Describe how you might do this using a resampling method.

   (b) Describe how you might do this using a kernel density estimation method.

   For each method you used, describe your procedure in detail, write a MATLAB function, and provide results. Compare the results both graphically and numerically.

3. Use Gibbs sampler to generate bivariate MC, $\begin{bmatrix} X \\ Y \end{bmatrix}$ of size 5000 from a bivariate mixture normal with

$\mu = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$, $v = \begin{bmatrix} 15 \\ 10 \end{bmatrix}$, $\Sigma_1 = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, and $p = 0.3$.

After burn-in 10% plot each MC and histogram. Give the scatterplot of X vs Y.