

3.5. Exercises

```
1 def example1(S):
2     """Return the sum of the elements in sequence S."""
3     n = len(S)
4     total = 0
5     for j in range(n):
6         total += S[j]
7     return total
```

$\sim n$ operations

$O(n)$

```
9 def example2(S):
10     """Return the sum of the elements with even index in sequence S."""
11     n = len(S)
12     total = 0
13     for j in range(0, n, 2):
14         total += S[j]
15     return total
```

$\frac{n}{2}$ operations

$O(n)$
 ~~$O(\frac{n}{2})$~~

```
17 def example3(S):
18     """Return the sum of the prefix sums of sequence S."""
19     n = len(S)
20     total = 0
21     for j in range(n):
22         for k in range(1+j):
23             total += S[k]
24     return total
```

$\sim n$ operations
 $\sim n$ operations

~~$O(n^2)$~~
 $O(n^2)$

```
26 def example4(S):
27     """Return the sum of the prefix sums of sequence S."""
28     n = len(S)
29     prefix = 0
30     total = 0
31     for j in range(n):
32         prefix += S[j]
33         total += prefix
34     return total
```

$2n$ operations

$O(n)$

```
36 def example5(A, B):
37     """Return the number of elements in B equal to the sum of prefix sums in A."""
38     n = len(A)
39     count = 0
40     for i in range(n):
41         total = 0
42         for j in range(n):
43             for k in range(1+j):
44                 total += A[k]
```

assume that A and B have equal length

for each n -loop, n^2 lines

$(n)(n^2)$

~~$O(n^2)$~~

$O(n^3)$

Exercises

Tests are in excel

R-3.2

$$A \\ O(8n \log n)$$

$$B \\ O(2n^2)$$

$$n_0 = ?$$

$$8n \log n = 2n^2$$

$$\log_2 n = \frac{n}{4}$$

$$2^{\frac{n}{4}} = n$$

$$n_0 \approx 1.24 \quad n_0 = 2 \quad \boxed{n_0 = 2}$$

$$f_{3.3.4} \quad A \\ 40n^2 = 2n^3$$

$$\frac{1}{20} n$$

$$\boxed{n_0 = 20}$$

$$\text{test: } A \\ n = 20 \\ n = 19 \\ n = 21 \\ 40(20^2)$$

tests
in excel

R-3.6

$$\sum_{i=0}^n 2i = 0 + 2 + 4 + \dots + 2n = 2(0 + 1 + 2 + \dots + n)$$

$$= 2 \left(\frac{n(n+1)}{2} \right)$$

$$= n(n+1)$$

$$\sum_{i=0}^n (a_i + a_n) = \frac{n}{2} (a_1 + a_n) = \frac{n}{2} (0 + 2n) = n^2$$

test:

R-3.8

$$4n \log n + 2n \quad O(n \log n) \times$$

$$2^{10} \quad O(1) \times$$

$$3n + 100 \log n \quad O(\log n) \checkmark$$

$$n^2 + 10n \quad O(n^2)$$

$$4n \quad O(n) \checkmark$$

$$n^3 \quad O(n^3)$$

$$2^{\log n} = n \quad O(n) \times$$

$$n \log n \quad O(n \log n) \times$$

ordered (fastest to slowest)

~~1)~~ 1) $f(n) = 2^{10}$

2) $f(n) = 3n + 100 \log n$

3) $f(n) = 2^{\log n}$

4) $f(n) = 4n$

5) $f(n) = n \log n$

6) $f(n) = 3n + 100 \log n$

7) $f(n) = n^2 + 10n$

8) $f(n) = n^3$