**Project Rules:**

1. This project is to be done individually.
2. Each student must develop his/her own code and submission material
3. The code needs to be written in either Matlab, or Python.
4. The student must submit a written report with plots and explaining what each plot means and the assumptions employed
5. The report/code will not be graded on efficiency, style, or writing quality, but both should be clear enough to be reasonably understandable.

**Objective:** To explore, and understand the matched filter, as well as creating a receiver operating characteristics (ROC) curve.

# Problem 1

Consider a RADAR system with a pulse width of 1 $\mu$s, and a center frequency of 1 GHz. The RADAR can transmit an unmodulated pulse, as well as employ pulse compression. The time bandwidth product of the pulse compression waveform is $TBP = \beta\tau = 13$. The pulse compression waveform used is a Barker-13 biphase code. The receiver noise figure is 3 dB. Assume the following:

1. RADAR Peak transmit power of 10 dBW
2. Target RCS of 30 dBsm
3. Transmit and Receive antenna gain of 20 dB each
4. Assume no system, or atmospheric losses

Plot the following:

- The matched filter output for the unmodulated pulse, and the Barker-13 code (note you have to figure out the appropriate sampling frequency).Assume the same sampling frequency is used for both scenarios. Plot the matched filter output without noise, and with noise, assuming the target is at a range of 10 Km. (The range is only there to enable calculating the received power in order to have the appropriate amplitude).
- Hint: A received data sample can be modeled in the following manner $y(n) = \sqrt{P_r} x_{signal}(n) + x_{noise}(n)$, where $x_{noise} = \sqrt{\frac{\sigma^2}{2}} [x_{noise,real} + j x_{noise,imag}]$. Recall $x_{noise}$ is distributed $CN(0, \sigma^2)$

# Problem 2

Consider the system in Problem 1. The goal of this problem is to produce ROC curves with the matched filter outputs you created. The ROC curves will be created empirically. This means, you will have to create multiple samples of the received data in order to build the probability density function (PDF) of the signal plus noise data, and the noise only data. You will sweep the detection threshold and determine if a detection occurred, or a false alarm occurred. The detection threshold will be a multiple of the matched filter output noise power.

We have talked about the matched filter dealing with continuous signals. In this project (as well as in digital systems) you will be dealing with discrete signals. The cross correlation (matched filtering operation) of a discrete signal of length N is given by

$$XC[k] = \sum_{n=0}^{2N-1} \alpha x[n]x^*[n+k] \tag{1}$$

The matched filter is defined as $h[n] = \alpha x^*[n]$, $\forall\ n \in [0, N-1]$ and zero otherwise. This means that the matched filter peak (waveform energy) for discrete matched filtering is given by $XC[0] = \sum_{n=0}^{2N-1} \alpha x[n]x^*[n+0] = \alpha(N)$. When matched filtering noise only, the output power will be $P_{noise} = \sigma^2|\alpha|^2 N$

Choose one of the two waveforms mentioned above and follow these steps:

1. Produce the matched filter output when there is signal and noise present (already did this in problem 1)
2. Produce the matched filter output when there is noise only (send the same noise used to degrade the signal)
3. Collect the matched filter output at the expected delay ($zero^{th}$), for both the signal + noise, and noise only cases.
4. Repeat this 100000 times (you would be running this 100000 trials of this experiment). It is important that each trial produces a new noise signal of the same power.
5. After collecting all 100000 samples of matched filter peaks for noise, and signal plus noise calculate the probability of detection and the probability of false alarm for the following threshold multipliers: $THR = [1 : 5 : 100]$ (from 1 to 100 in steps of 5). These thresholds will be applied as follows, 1) you will calculate the noise power after matched filtering analytically, 2) You will then use a detection threshold that is the THR multiplier times the analytical noise power after matched filtering
6. Plot $P_d$ vs $P_{fa}$

In addition to the plot above, make sure the matched filter output peak for the case when there is signal + noise yields the expected result. Furthermore, calculate the theoretical single sample SNR prior to matched filtering, and the SNR after matched filtering. Compare those results with the average SNR after matched filtering across the 100000 trials. It should be very close.