ECE501b – Course Project

WEIGHTING THE COLLEY COLLEGE RANKING METHOD USING CONFERENCE HISTORY

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I. Introduction

The purpose of this project was to build upon the foundations of the Colley matrix method (1) of calculating NCAA Division 1 FBS football rankings. The Colley matrix method of calculating the rankings is an iterative method of calculating ranking by solely using the number of wins and losses each team has and their relative performance against one another with these wins and losses factored in. The author makes the assumptions that:

(i) The margin of victory or defeat should not factor into the ranking.
(ii) All non-Division 1 team games played are inconsequential and therefore should not be included in the ranking calculations.
(iii) There is no conference weighting for strength of schedule calculations. Strength of schedule is determined strictly by the opponents wins and losses.

This project explored the effect of factoring assumptions (ii) and (iii) into the ranking algorithm. Additionally, rather than compare the results with the current AP rankings, validation of the results was done by verifying whether a higher ranked team defeated a lower ranked team. If they did, the model was successful, and the results validated for that comparison. If not, the ranking was a false prediction. This project was chosen due to my personal interest in sports and in particular, football.

II. Background

As mentioned earlier the Colley matrix method utilizes only the wins and losses for every team in Division 1 FBS football rather than any other variable. The author explains that this simplifies the process of ranking and allows for fairness in ranking for schools not in well known and high caliber conferences. Additionally, the author claims that runaway scores and ad hoc adjustments for home and away are not necessary for using this method. The author uses a Laplacian statistical method in order to keep all of the rating calculations non-zero and interdependent with one another. Simply put, by dividing 1 plus the number of wins by the 2 plus the total number games, the ranking always is non-zero. (i.e. 0.5 with no games played) (Equation 2.1)

$$r = {}^{1 + Wins}/_{2 + TotalGames} \qquad \mathbf{2.1}$$

As, each team plays in more games, their ratings either increase or decrease, but never equal zero or is greater than or equal to 1. Additionally, with some algebra, the author shows that the number of effective wins can be weighted by the rankings of the other teams played.

$$n_{w,i}^{eff} = {}^{(n_{w,i} - n_{l,i})}/_{2} + \sum_{j=1}^{n_{tot,i}} r_j^i \qquad \mathbf{2.2}$$

The author further extrapolates this to a representative linear matrix equation. The columns and rows are symmetric representations of all 130 Division 1 teams. The intersection of two teams in the Colley matrix being the number games played against one another multiplied by negative one and where the diagonal of the Colley matrix is defined as the number of games played plus 2. This matrix is then multiplied by a 1x130 column vector of the ranking for each team equaling another 1x130 column vector, b, that is defined equation 2.6.

$$\boldsymbol{C}\vec{r} = \vec{b} \qquad \mathbf{2.3}$$

$$c_{i,j} = -n_{j,i} \qquad \mathbf{2.4}$$

$$c_{i,i} = 2 + n_{tot,i} \qquad \mathbf{2.5}$$

$$b_i = 1 + {}^{(n_{w,i} - n_{l,i})}/_{2} \qquad \mathbf{2.6}$$

The r vector was then solved iteratively by the author until a steady state value is found for the rankings. The first component of this project was determining the AP ranking by using this recursive method. The respective schedules and their outcomes for the 2019 season (2) were entered into an excel document. (including non-Division 1 games) Using MATLAB, this data was then uploaded into a matrix, parsed into the Colley matrix C and vector b using the above equations (non-Division 1 FBS games were ignored), and a final ranking was calculated for each team by using the linsolve() function. This function was used rather than iteratively solving manually.

The second portion of this project involved creating a weighting function that took into account the effects of inter conference team records on the rankings of each individual team. I decided it was best to use the same Laplacian method as with the Colley matrix to determine this weighting vector. However, this output vector was determined not by a team's wins and losses but rather the inter-conference

records for all of history (3). Rather than parse the data through MATLAB, a matrix was created manually with the same structure as the Colley matrix.

$$\boldsymbol{C_{conf}}\vec{r}_{conf} = \vec{b}_{conf} \qquad \mathbf{2.3}$$

$$c_{i,j} = -n_{j,i} \qquad \mathbf{2.4}$$

$$c_{i,i} = 2 + n_{tot,i} \qquad \mathbf{2.5}$$

$$b_i = 1 + {}^{(n_{w,i} - n_{l,i})}/_{2} \qquad \mathbf{2.6}$$

The final ranking was then determined by multiplying the conference weighting vector with the Colley ranking vector.

$$\vec{r}_{final} = \vec{r}_{conf}{}^{T}\vec{r} \qquad \mathbf{2.7}$$

Additionally, the non-Division 1 teams were accounted for by using this weighting vector. A catch all team was first added to the team matrix so that the weighting vector of it could influence the ranking vector and an OTH conference was integrated into the Conference weighting vector by subtracting 0.005 from each conference's weight and adding it to the OTH conference. This resulted in the conference weight of the non-Division 1 teams being significantly less than that of the other conferences. This inherently make it very costly to a team that lost to a non-Division 1 team and granted points marginally to a victory over such.

Finally, these ranking values were then compared with one another not by their similarity to the AP poll, but by their effectiveness in predicting the winner in the matchups between two different teams. If a team is ranked higher than another team and then logically that team should defeat the other team.

This project is representative of the eigenvalue / eigenvector portion of the course. Where we are solving for the eigenvectors of the system given a certain set of eigenvalues.

III. Results

Initially the ranking list was not as expected. The author mentioned that there would be conservation of the average rating equaling 0.5 and that the matrix C should be positive definite. The initial results of running just the Colley model did not have those properties. It was assumed due to the large amount of data manually entered (5hrs) for the entirety of the 2019 season for all 130 teams that

human error was to blame. Therefore, I included error checking code to find which potions of the matrix were incorrect. According to paper the matrix should be symmetric, therefore a check was done to ensure that cells $c_{i,j} = c_{j,i}$. I found approximately 30 mismatches here where I had correctly entered a team name in one instance but incorrectly in another. All of error checking code I wrote for the diagonal yielded no inconsistencies. After having done this, most of the discrepancies in the ranking vectors were resolved. There still existed some errors, but these were limited to the few last ranked teams being negatively weighted. Although this output was incorrect, I compensated by ordering the list of vectors in the end by value. Similar issues were encountered with the Conference team matrix. The Inter-Conference matrix had both a correct average value and was positive definite.

```
%Error check the Colley matrix
%due to large data entry (human error)
%all cells should equal 0 if correct
tot_val = zeros(length(ALL),1);
for i=1:ALL_length
    for j=1:ALL_length
        tot_val(i) = tot_val(i) + C(i,j);
    end
    tot_val(i) = tot_val(i) - 2;
end
error_mat = zeros(length(ALL),2);
index = 1;

%Error checking for matrix structure
%indexes are correclated to team names
for i=1:ALL_length
    for j=1:ALL_length
        if (C(i,j) ~= C(j,i)) && (i < j)
            error_mat(index,1) = i;
            error_mat(index,2) = j;
            index = index + 1;
        end
    end
end
for i=1:ALL_length
   if (error_mat(i,1) > 0)
       error_teams(i,1) = ALL(error_mat(i,1));
       error_teams(i,2) = ALL(error_mat(i,2));
   end
end
```

An initial evaluation of the rankings difference between the two methods shows a distinct difference even in the top 10 rankings. Additionally, the correlating AP

ranking was close to both the Colley and Conference rankings for some teams. Overall however, the Colley method better matched the AP rankings than did the Conference method. A full comparison between the Colley and Conference methods can be found in Appendix A.

Rank	Colley Method	Colley with Conference	AP
1	Ohio State	Ohio State	LSU
2	LSU	LSU	OSU
3	Clemson	Wisconsin	Clemson
4	Georgia	Penn State	Georgia
5	Oklahoma	Michigan	Utah
6	Memphis	Oklahoma	Oklahoma
7	Penn State	Auburn	Florida
8	Wisconsin	App State	Baylor
9	Utah	Baylor	Alabama
10	Notre Dame	Minnesota	Wisconsin
11	Florida	Oregon	Auburn
12	Baylor	Clemson	Penn State
13	Boise State	LA Lafayette	Oregon
14	Cincinnati	USC	Notre Dame
15	Auburn	Notre Dame	Minnesota
16	Michigan	Cincinnati	Memphis
17	Oregon	Memphis	Michigan
18	Navy	Indiana	Iowa
19	Minnesota	Oklahoma State	Boise State
20	App State	Texas AM	App State
21	Alabama	MI State	Cincinnati
22	SMU	Tennessee	Virginia
23	Iowa	Kansas State	Navy
24	Air Force	SMU	USC
25	USC	Navy	Air Force

Table 3.1 – Top 25 Ranking Comparison

Many top rank teams both in the AP and the Colley method are ranked much lower in the Conference method. This was likely due to the negative effect of playing non-Divisional 1 teams. For example, Utah was ranked 9th and 5th respectively in the Colley and AP rankings. However, it was ranked 62nd in the Conference method due to a much weaker strength of schedule. Conversely, Oregon was ranked 11th by the Conference method and 17th and 13th respectively in the Colley and AP rankings. According to the Conference ranking, Oregon should have handedly beaten Utah whereas the other rankings predicted a closer game with Utah winning. Here the Conference method successfully predicted the Oregon upset.

Team A	Team B	Colley Prediction	Conference Prediction	AP Prediction	Winning Team
Baylor	Oklahoma	**Oklahoma**	**Oklahoma**	**Oklahoma**	**Oklahoma**
LA-Lafayette	App State	**App State**	**App State**	**App State**	**App State**
Cincinnati	Memphis	**Memphis**	Cincinnati	**Memphis**	**Memphis**
Georgia	LSU	**LSU**	**LSU**	**LSU**	**LSU**
Hawaii	Boise State	**Boise State**	**Boise State**	**Boise State**	**Boise State**
Virginia	Clemson	**Clemson**	**Clemson**	**Clemson**	**Clemson**
Ohio State	Wisconsin	**Ohio State**	**Ohio State**	**Ohio State**	**Ohio State**
Utah	Oregon	Utah	**Oregon**	Utah	**Oregon**

Unfortunately, for the purposes of performance analysis, the three models predicted identically for the majority of the week 15 games. There were only 2 games that the Conference method differed from the other two rankings, and the results of these games were split. The aforementioned Utah/Oregon game was correctly predicted by the Conference method where the Cincinnati / Memphis game was correctly predicted by the Colley method. I had planned on using the last 2 games of the season for every team to be used as a test set where the first 10 games were to be used as training data for the rankings. Unfortunately, I encountered the issue where some teams' week 10 games were other teams' week 11 game. This caused issues with the development of the Colley matrix and maintaining its symmetry to function properly. A better test of these two methods would involve an additional analysis of the upcoming bowl games as well.

IV. Conclusions

Although the Conference method outperformed the other two methods, this result may not necessarily be the trend as more bowl games are played at the end of the season. The limited number of games and the similar prediction between the two methods may be why the author had originally used the AP rankings themselves as a baseline of measure for the accuracy of his prediction. I think that a less weighted version of the Conference method would likely still be able to predict upsets as it did in week 15, but still maintain some continuity of the predictions found both in the AP and Colley ranking methods.

V. References

(1) Colley, W. M. (date). Colley's Bias Free College Football Ranking Method: The Colley Matrix Explained. Princeton University, Princeton, NJ

(2) (2019, December 1). Retrieved from https://www.espn.com/college-football/teams

(3) (2019, December 1). Retrieved from http://mcubed.net/ncaaf/tvc/index.shtml

APPENDIX A

Rank	Colley Method	Colley with Conference	Rank	Colley Method	Colley with Conference	Rank	Colley Method	Colley with Conference
1	Ohio State	Ohio State	51	Cal	Hawaii	101	Maryland	San Jose State
2	LSU	LSU	52	MS State	FSU	102	Northwestern	Eastern MI
3	Clemson	Wisconsin	53	Kentucky	UCLA	103	Texas Tech	NC State
4	Georgia	Penn State	54	FSU	TCU	104	Troy	Ball State
5	Oklahoma	Michigan	55	GA Southern	Purdue	105	Coastal	Georgia Tech
6	Memphis	Oklahoma	56	LA Tech	South Carolina	106	NC State	FIU
7	Penn State	Auburn	57	North Carolina	Florida	107	Northern IL	Fresno State
8	Wisconsin	App State	58	Tulane	Oregon St	108	Fresno State	Texas St
9	Utah	Baylor	59	Miami OH	Utah State	109	Army	Army
10	Notre Dame	Minnesota	60	Arkansas State	LA Monroe	110	Rutgers	Northern IL
11	Florida	Oregon	61	Wyoming	FL Atlantic	111	Kansas	CO State
12	Baylor	Clemson	62	Miami FL	Utah	112	Middle Tenn	UNLV
13	Boise State	LA Lafayette	63	UAB	North Carolina	113	Vanderbilt	Ohio
14	Cincinnati	USC	64	Boston College	Stanford	114	CO State	East Carolina
15	Auburn	Notre Dame	65	Central MI	Boston College	115	UNLV	Middle Tenn
16	Michigan	Cincinnati	66	Illinois	West Virginia	116	East Carolina	Miss
17	Oregon	Memphis	67	Colorado	Georgia	117	Texas St	Texas
18	Navy	Indiana	68	Washington State	Miami FL	118	Rice	South Alabama
19	Minnesota	Oklahoma State	69	Georgia State	Tulane	119	North Texas	Rice
20	App State	Texas AM	70	Nebraska	Virginia	120	UTSA	North Texas
21	Alabama	MI State	71	Southern Miss	Wyoming	121	Arkansas	UTSA
22	SMU	Tennessee	72	Nevada	Marshall	122	Bowling Green	Arkansas
23	Iowa	Kansas State	73	Buffalo	Northwestern	123	South Alabama	Bowling Green
24	Air Force	SMU	74	Missouri	Nevada	124	New Mexico	UCONN
25	USC	Navy	75	Charlotte	Tulsa	125	UCONN	New Mexico
26	LA Lafayette	Boise State	76	TCU	Miami OH	126	NM State	Old Dominion
27	Virginia	Alabama	77	Western MI	Duke	127	Old Dominion	NM State
28	UCF	Iowa State	78	Oregon St	Arizona	128	UMass	UTEP
29	FL Atlantic	Washington	79	South Carolina	LA Tech	129	Akron	Akron
30	Oklahoma State	GA Southern	80	UCLA	Western Kentucky	130	UTEP	UMass
31	Kansas State	MS State	81	Duke	Central MI			
32	Texas	Arizona State	82	Tulsa	Maryland			
33	Wake Forest	Air Force	83	West Virginia	UAB			
34	Temple	Cal	84	Kent State	Troy			
35	Indiana	Iowa	85	Stanford	Coastal			
36	Western Kentucky	Arkansas State	86	Purdue	Buffalo			
37	Louisville	Illinois	87	South Florida	Western MI			
38	San Diego St	UCF	88	LA Monroe	South Florida			
39	Texas AM	Wake Forest	89	Syracuse	Texas Tech			
40	Tennessee	Nebraska	90	Houston	Houston			
41	Hawaii	Louisville	91	Ohio	Rutgers			
42	Washington	Georgia State	92	Georgia Tech	Southern Miss			
43	Iowa State	Colorado	93	Toledo	Syracuse			
44	Virginia Tech	Temple	94	Liberty	Charlotte			
45	Marshall	Pitt	95	Arizona	Liberty			
46	Arizona State	Missouri	96	FIU	Kent State			
47	Utah State	Virginia Tech	97	Miss	Toledo			
48	MI State	San Diego St	98	Eastern MI	Kansas			
49	Pitt	Washington State	99	San Jose State	Vanderbilt			
50	BYU	BYU	100	Ball State	Kentucky			