Assignment 1

(Due Friday, **August 30th** 11:59 pm)

Question 1 [2+2+2+2=8 points]: Solve exercise 1 on page 38 in our textbook. Notice that to answer these questions correctly, you should be thinking like a statistician and talking about population parameters, not only sample statistics. That is, do some inference in every part, using as much everyday layperson's terminology as possible. For example, part b should not just say "The intercept is (or is not) $10,000$." What does an intercept mean in context to someone who is selling movie tickets?

Question 2 [4 points]: Show that $Var(Y_i|X_i = x_i) = Var(\epsilon_i)$ in the simple linear regression model. Don't overthink this; the answer is simple. What did you assume in answering this?

Question 3 [2 points]: Define using only words what the least squares criterion is.

Question 4 [1+2+2+5+2=12 points]: Solve exercise 4 on page 40 in our textbook, except do:

(a) Setup:

   i. Write down your design matrix $\mathbf{X}$.

   ii. Show, using matrix notation and starting with the principle of least squares, that the least squares estimate of β is
$$\hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}$$

(b) As in text.

Question 5 [2+4=6 points]: Show that the least-squares criterion applied to the "intercept-only" model, i.e.,
$$y_i = \beta_0 + \epsilon_i, i = 1, 2, ..., n$$
results in the least squares estimator of $\beta_0 : \hat{\beta}_0 = \overline{y}$ by following these steps:

(a) Write down your design matrix $\mathbf{X}$. (It won't be the same as any we've used in class.) Double check: does $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ give the set of equations listed above? Notice this model has no predictor variable.

(b) Use the previously derived formula $\hat{\boldsymbol{\beta}} = (\mathbf{X}^{'}\mathbf{X})^{-1}\mathbf{X}^{'}\mathbf{y}$ to get the least squares estimator.

Question 6 [5 points]: Prove that $Cov(aX, bY) = abCov(X, Y)$.

Question 7 [3 points]: Solve exercise 7 on page 42 in our textbook.

Question 8 [5 points]: Using $\hat{\boldsymbol{\beta}} = (\mathbf{X}^{'}\mathbf{X})^{-1}\mathbf{X}^{'}\mathbf{y}$, finish our algebra from class and show that $\hat{\beta}_0 = \overline{y} - \frac{SXY}{SXX}\overline{x}$ for the simple linear regression case.

Question 9 [5 points]: Show that for the usual regression model $\mathbf{y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon}$, where the usual regression assumptions from question 4 apply, $Var(\ \boldsymbol{a'\hat{\beta}}|\boldsymbol{X}) = \sigma^2\ \boldsymbol{a'(X'X)^{-1}}\ \boldsymbol{a}$, where $\boldsymbol{a}$ is a constant vector.