STAT 608 HW 4

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1)

The linear model is:

$$Y = X\beta + e$$

With design matrix X:

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

And response vector Y:

$$Y = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 \end{bmatrix}^T$$

And coefficient vector β:

$$\beta = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \end{bmatrix}^T$$

b)

$$X^T X = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

$$X^T y = \begin{bmatrix} y_1 + y_4 + y_5 + y_7 \\ y_2 + y_4 + y_6 + y_7 \\ y_3 + y_5 + y_6 + y_7 \end{bmatrix}$$

c)

We know that $(X^T X)(X^T X)^{-1} = I_3$ so $A(X^T X)(X^T X)^{-1} = I_3$:

$$A(X^T X)(X^T X)^{-1} = c \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = I_3$$

So $c = \frac{1}{8}$.

d)

$$\hat{\beta} = (X^T X)^{-1} X^T y = \frac{1}{8} \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} y_1 + y_4 + y_5 + y_7 \\ y_2 + y_4 + y_6 + y_7 \\ y_3 + y_5 + y_6 + y_7 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 3y_1 - y_2 - y_3 + 2y_4 + 2y_5 - 2y_6 + y_7 \\ -y_1 + 3y_2 - y_3 + 2y_4 - 2y_5 + 2y_6 + y_7 \\ -y_1 - y_2 + 3y_3 - 2y_4 + 2y_5 + 2y_6 + y_7 \end{bmatrix}$$