Assignment 0

(Review - Not for a grade)

I. Matrix Algebra Review

Define matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ as follows:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 2 & 0 & -2 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 0 & -1 \\ 3 & 0 \\ 2 & 1 \\ 0 & -2 \end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

1. Calculate $\boldsymbol{A}'$, the transpose of matrix $\boldsymbol{A}$.
2. Calculate $\boldsymbol{A}' + \boldsymbol{B}$.
3. Calculate $\boldsymbol{AB}$, where $\boldsymbol{AB}$ is the matrix product, or matrix multiplication.
4. Calculate $\boldsymbol{BA}$. Is $\boldsymbol{AB} = \boldsymbol{BA}$?
5. Is the matrix $\boldsymbol{AB}$ singular? Why or why not? (Invertible means nonsingular; check out the Wikipedia page for invertible matrices for a review.)
6. Calculate the trace of $\boldsymbol{AB}$.
7. Write $(\boldsymbol{AB})'$ in another form algebraically: remove the parentheses.
8. Calculate $(\boldsymbol{AB})^{-1}$.
9. Write $\boldsymbol{I}_2$, the 2×2 identity matrix.
10. What is $\boldsymbol{I}_2\boldsymbol{A}$? Why?
11. Describe geometrically the space spanned by $\boldsymbol{C}$. That is, the space spanned by the two vertical vectors in the matrix $\boldsymbol{C}$. Assume we're working in three dimensional space defined by axes xyz.
12. Calculate the projection matrix for $\boldsymbol{C}$. That is, what is the matrix that projects a vector in x, y, z space onto the x, y plane?
13. Project the vector $\boldsymbol{d} = [2\ 2\ 2]'$ onto the space spanned by $\boldsymbol{C}$.
14. Describe geometrically what you did in the previous step.
15. Are the vectors $\boldsymbol{d}$ defined above and $\boldsymbol{f} = [1\ 0\ 0]'$ orthogonal? Why or why not? (Talk about a dot product in your answer.)
16. Calculate the dot product $\mathbf{1} \cdot \mathbf{1}$, where the vector $\mathbf{1} = [1\ 1 \cdots 1]'$ is of length n.
17. Calculate the dot product $\mathbf{1} \cdot \boldsymbol{X}$, where $\mathbf{1}$ is defined as above and the vector $\boldsymbol{X} = [x_1\ x_2 \cdots x_n]'$.
18. Calculate the dot product $\boldsymbol{X} \cdot \boldsymbol{X}$, where $\boldsymbol{X}$ is defined as above.
19. Describe geometrically what the first eigenvector (sorted in order from highest eigenvalue to lowest) would tell you about a vector space. Feel free to assume real positive eigenvalues.

II. Calculus Review

Define

$$f(x, y) = 3x^2 + 2xy^2 - y$$

1. Calculate $\frac{\partial}{\partial x} f(x, y)$.
2. Calculate $\frac{\partial}{\partial y} f(x, y)$.

III. Log Review

1. Calculate $\log(e)$. (Note that statisticians usually write "log" instead of "ln" when they really mean log base e)
2. Rewrite $\log\left(\frac{x}{y}\right)$ in terms of a difference.
3. Rewrite $\log(x^n)$ in terms of a product.
4. Solve $\log(x) = y$ for x.

IV. Statistics and Linear Regression Review

After regressing eight patients' weights (in kg) on their height (in cm), a doctor found the following output:

Coefficient	Estimate	Std. Error	t-value	$Pr(> \|t\|)$
(intercept)	-129.1667	24.3610	-5.302	0.001826
Height	1.1667	0.1521	?????	0.000257

1. Write down the least squares regression line using $\hat{y} = \textit{predicted weight}$ and $x = \textit{height}$.
2. What weight does the model predict for someone who is 160 cm tall.
3. Interpret the slope of the line in the context of this model.
4. Interpret the standard error of the slope in the context of this model.
5. Calculate the t-statistic for testing whether the slope is significant.
6. Are height and weight linearly associated? Explain. (Assume assumptions are met.)
7. A journal article might report that height is a significant predicator of weight. Explain what this means in context, as if to someone with no statistical background.
8. Calculate a 95% confidence interval for the slope.
9. Interpret your interval above in context.