STAT 639 HW 4

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8/20/24

$$\begin{bmatrix} 1 & 2 & 3 \\ a & b & c \end{bmatrix}$$

Matrix Algebra Review

$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 2 & 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 3 & 0 \\ 2 & 1 \\ 0 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

1.

$A' = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 0 \\ 3 & -2 \end{bmatrix}$

2.

$A' + B = \begin{bmatrix} 1 & -2 \\ 3 & 2 \\ 4 & 1 \\ 3 & -4 \end{bmatrix}$

3.

$AB = \begin{bmatrix} 4 & -5 \\ 6 & 5 \end{bmatrix}$

4.

$BA = \begin{bmatrix} 1 & -2 & 0 & 2 \\ 3 & 0 & 6 & 9 \\ 1 & 2 & 4 & 4 \\ 2 & -4 & 0 & 4 \end{bmatrix}$

$AB \neq BA$

5.

AB is not singular. The matrix is invertible due to the fact that the determinant is not zero.

$\det(AB) = |AB| = (4)(5) - (-5)(6) = 50$

6.

The trace is the sum of diagonal elements. $\text{Tr}(AB) = 4 + 5 = 9$.

7.

$(AB)' = B'A'$

8.

$(AB)^{-1} = \frac{1}{50} \begin{bmatrix} 5 & 5 \\ -6 & 4 \end{bmatrix}$

9.

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

10.

$I_2 A = A$. The identity matrix multiplied by a matrix does not change the matrix.

11.

The column space of C is a plane on XY.

12.

The projection matrix for C is:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

13.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

14.

The vector d pointing at $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ has been projected down to the XY plane losing its Z axis direction. It now is a line with components $\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$.

15.

No vector d and f are not orthogonal. The dot product $d'f = 2$. In order for two vectors to be orthogonal the dot product must equal zero.

16.

The dot product of $1 \cdot 1$ where $1 = \begin{bmatrix} 1 & 1 & \cdot & \cdot & 1 \end{bmatrix}$ with length n is:

$1 \cdot 1 = 1_1 1_1 + 1_2 1_2 + ... 1_n 1_n = \sum_{i=1}^n 1_i 1_i = \sum_{i=1}^n 1_i = n$

17.

The dot product of $1 \cdot X = 1_1 X_1 + 1_2 X_2 + ... + 1_n X_n = \sum_{i=1}^n X_i$.

18.

The dot product of $X \cdot X = X_1 X_1 + X_2 + ... + X_n X_n = \sum_{i=1}^n X_i^2$.

19.

The first eigenvector is multiplied by λ when multiplied by A. We stay on the same eigenvector space changing direction/magnitude each time it is transformed by A.

II. Calculus Review

1.

$6x + 2y^2$

2.

$4xy - 1$

III. Log Review

1.

$\log(e) = 1$

2.

$\log(\frac{x}{y}) = \log(x) - \log(y)$

3.

$\log(x^n) = n\log(x)$

4.

$\log(x) = y, x = e^y$

IV. Statistics and Linear Regression Review

1.

$\hat{y} = 1.1667x - 129.1667$

2.

```
1.1667*160 - 129.1667
```

```
[1] 57.5053
```

3.

As height increases by 1 cm the prediction of weight increases by 1.1667 kg.

4.

The standard error of the slope is the measure of spread in the distribution of the height parameter estimate.

5.

$t_{n-2} = \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta})}$

```
t_statistic = (1.1667 - 0)/0.1521
t_statistic
```

```
[1] 7.670611
```

6.

Yes, height and weight are linearly associated. The height parameter estimate is statically significantly different than zero.

7.

There is evidence as height increases weight tends to increase.

8.

```
t_alpha <- qt(1 - 0.05, 8 - 2)
lower <- 1.1667 - (0.1521/sqrt(8))*t_alpha
```

```
upper <- 1.1667 + (0.1521/sqrt(8))*t_alpha
c(lower, upper)
```

```
[1] 1.062205 1.271195
```

9.

If we were to repeat this experiment again we are 95% confident that the estimate of the height parameter would rest between 0.86878 and 1.46462.