Please load the data "Risk.RData" for this assignment. This practice is from part of old HW questions including old exam question, some of the data you may not use.

There are 4 sets of stock returns in percentage (%), from Jan 1, 2011 to Sep 31, 2022.

`y1` : Daily returns of Oracle (ORCL).

`y2` : Weekly returns of American Express (AXP) and Intel (INTC). We have worked on them in Homework 4 with earlier starting time.

`y4` : Weekly returns of Campbell Soup (CPB), CVS, Kellogg (K) and Procter & Gamble (PG). We have worked on them in Homework 3 with slightly shorter time frame.

`y8` : Weekly returns of 8 stocks, Amazon (AMZN), Coke (KO), Nike (NKE), Pfizer (PFE), Tesla (TSLA), United Health (UNH), United Rentals, VISA (V). We have worked on them in Homework 4 with an ending date a week earlier.

There are also `syb1`, `syb2`, `syb4` and `syb8` which contain ticker symbols of the 4 sets of returns.

Note: The annual risk free rate is 3.5%, convert to weekly by 3.5/52. The amount of investment $S = 50000$, through out this homework.

1. We will use `y4` in this question. In our previous study (pretending we did), `y4` is a multivariate t model.
   (a) Refit the multivariate-t to `y4`. Show your estimates.
   (b) Use the MLE of `y4` to calculate the tangency portfolio of `y4` allowing short selling. Named the tangency portfolio w4.T.
   (c) Consider a portfolio which consists of 20% of risk free asset and 80% of risky assets, CPB, CVS, K and PG, with the tangency portfolio weights. Risk-free asset return is 3.5/52. Find the distribution of this portfolio.
   (d) Compute the one-week VaR and ES with $S = 50000$ investment on Part (c) at $\alpha = 0.05, 0.01$.

2. This is an old exam question. The data set will be used in this question is `y8`. Often we have a portfolio of $N = 8$ stocks that do not have a simple multivariate model. A formula of approximation for a portfolio $\boldsymbol{w} = (w_1, \ldots, w_N)^T$ is as follows
$$\text{VaR} = \sqrt{\sum_{i=1}^{N} w_i^2 \text{VaR}_i^2 + 2 \sum_{i<j} w_i w_j \rho_{ij} \text{VaR}_i \text{VaR}_j} \ , \tag{1}$$
where VaR_i is the VaR of asset i and ρ_{ij} is the Spearman's correlation between the returns of asset i and asset j. Similar formula for expected shortfall.
   (a) Compute one-week VaR and ES for each return series with t distribution at $\alpha = 0.05$ and investment $S = 50000$. You will need to fit a t distribution to each series first. You will find `MASS`'s `fitdistr()` convenient for this question.
   (b) Use sample mean and sample variance-covariance to calculate the tangency portfolio of `y8` allowing short selling. Named the tangency portfolio `w8.T`.
   (c) Compute the one-week VaR and ES of the tangency portfolio at $\alpha = 0.05$ with investment $S = 50000$ using the formula (1).

3. The object `y1` is the daily returns of Oracle. For an investment of $S = 50000$, compute the estimated one-day VaR and ES at $\alpha = 0.05$ and 0.01 using the following procedures. Justify your choice of bandwidth and threshold.

(a) Nonparametric estimation.

(b) Parametric estimation with a t distribution, which has been selected from a list of candidate models. Nevertheless, plot the Q-Q (sample quantiles against model quantiles) to see the fit.

(c) Semiparametric regression estimation. Use the following candidate bandwidths $m = n^s, s = 0.5, 0.55, \ldots, 0.75, 0.8$.

`R` commands for superimposing a fitted line on a scatter plot of `x` and `y`

```
> plot(x,y)
> abline(lsfit(x,y)$coef)
```

Remind again: Our data are in %. Remember to convert the investment to $S/100$.