STAT 631 Risk

Jack Cunningham (jgavc@tamu.edu)

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```
load("Risk.RData")
```

1)

We use the profile likelihood method to fit the multivariate t distribution.

```
library(MASS)
library(mnormt)

df = seq(1,8, 0.01)
loglik_p = c()
for(i in 1:length(df)){
  fit = cov.trob(y4 , nu = df[i])
  loglik_p[i] = sum(dmt(y4, mean = fit$center, S = fit$cov, df = df[i], log = T))
}

nu = df[which.max(loglik_p)]
cat("The MLE of degrees of freedom:", paste(nu))
```

```
The MLE of degrees of freedom: 4.45
```

Our estimates are:

```
est = cov.trob(y4, nu = nu, cor = T)
names(est)
```

```
[1] "cov"    "center" "n.obs"  "cor"    "call"   "iter"
```

The MLE for the mean vector is:

```r
est$center
```

```
      CPB       CVS         K        PG 
0.1905486 0.3139376 0.2375338 0.2751555 
```

The MLE of the scale matrix Lambda is:

```r
est$cov
```

```
         CPB      CVS        K       PG
CPB 5.378719 1.750477 2.680701 1.633501
CVS 1.750477 6.628764 1.721214 1.710757
K   2.680701 1.721214 3.677831 1.633538
PG  1.633501 1.710757 1.633538 3.232732
```

The MLE of Covariance is:

```r
est$cov*nu/(nu - 2)
```

```
         CPB       CVS        K       PG
CPB 9.769510  3.179437 4.869028 2.966971
CVS 3.179437 12.040000 3.126288 3.107294
K   4.869028  3.126288 6.680142 2.967039
PG  2.966971  3.107294 2.967039 5.871696
```

b)

The tangency portfolio allowing short selling has the following explicit solution:

$$w_t = \frac{\Sigma^{-1}\mu_{ex}}{1^T\mu_{ex}}$$

Where Σ is the covariance matrix:

```r
y.4.S = est$cov*nu/(nu - 2)
y.4.S
```

```
          CPB       CVS        K       PG
CPB 9.769510  3.179437 4.869028 2.966971
CVS 3.179437 12.040000 3.126288 3.107294
K   4.869028  3.126288 6.680142 2.967039
PG  2.966971  3.107294 2.967039 5.871696
```

And mu.ex is the excess return, taking the MLE of the mean vector and subtracting by the risk free rate:

```
mu.f = 3.5/52
m.ex = est$center - mu.f
ones = rep(1,4)
```

We can now calculate w_t:

```
IS = solve(y.4.S)
aT = as.numeric((t(ones)%*%IS%*%m.ex))
w4.T = 1/aT*(IS%*%m.ex)
mu4.T = as.numeric(t(w4.T)%*%est$center)
s4.T = sqrt(as.numeric(t(w4.T)%*%y.4.S%*%w4.T))
cat("Tangency Portfolio for y4:"); w4.T[,1];
```

```
Tangency Portfolio for y4:
```

```
       CPB        CVS          K         PG
-0.1158143  0.2711458  0.2758663  0.5688021
```

```
cat("Portfolio return is:", mu4.T, "\t with risk", s4.T)
```

```
Portfolio return is: 0.2850912   with risk 2.209045
```

c)

If we choose to have 20% of assets on the risk free asset and 80% on risky assets we choose a point on the tangency line.

```
w4.c = c(w4.T*.8,.2)
names(w4.c) = c(syb4, "RF")
w4.c
```

```
       CPB        CVS          K         PG         RF
-0.0926514  0.2169166  0.2206931  0.4550417  0.2000000
```

Then we can compute the expected return and risk easily, through:

$$\mu_p = \mu_f + w(\hat{\mu_T} - \mu_f), \hat{\sigma}_p = w\hat{\sigma}_T$$

```
mu.4.c = .8*mu4.T + .2*mu.f
S.4.c = .8*s4.T
cat("Portfolio for y4, 20% in RF:"); w4.c;
```

```
Portfolio for y4, 20% in RF:

       CPB        CVS          K         PG         RF
-0.0926514  0.2169166  0.2206931  0.4550417  0.2000000
```

```
cat("Portfolio return is:", mu.4.c, "\t with risk", S.4.c)
```

```
Portfolio return is: 0.2415345   with risk 1.767236
```

We use the following to find the distribution of the portfolio:

$$w^T Y \sim t_\nu(w^T \mu, w^T \Lambda w)$$

We need to take the risk from our last step, square it and transition it to the scale parameter:

```
scale.c = sqrt(S.4.c^2 * (nu - 2)/nu)
```

So the distribution of the portfolio is: $t_{\hat{\nu}}(.2415, 1.311287)$ where $\hat{\nu} = 4.45$.

d)

Using this distribution we can calculate the one-week VaR and ES with S = 50000.

```
S = 50000
alpha = c(.05,0.01)
q.t = qt(alpha, df = nu); VaR.t = -S*(mu.4.c + scale.c*q.t);
ES.t = S*(-mu.4.c + scale.c*dt(q.t, nu)/alpha*(nu+q.t^2)/(nu-1))
VaR.t = VaR.t/100; ES.t = ES.t/100
output <- rbind(alpha, VaR.t, ES.t)
output
```

```
         [,1]     [,2]
alpha    0.050    0.010
VaR.t 1237.325 2204.151
ES.t  1870.834 3032.390
```