Invariant property of of copulas One of the most important properties of the copula is that a copula is invariant under increasing and continuous transformations of the variates. Suppose g_j, $j = 1, \ldots, d$ are strictly increasing functions and $X_j = g_j(Y_j)$. Then $\boldsymbol{X} = (X_1, \ldots, X_d)$ and $\boldsymbol{Y}$ have the same copulas. That is,

$$C_X(u_1, \ldots, u_d) = C_Y\{u_1, \ldots, u_d\}\ .$$

$$\begin{aligned} F_X(x_1, \ldots, x_d) &= P\left\{g_1(Y_1) \le x_1, \ldots, g_d(Y_d) \le x_d\right\} \\ &= P\left\{Y_1 \le g_1^{-1}(x_1), \ldots, Y_d \le g_d^{-1}(x_d)\right\} \\ &= F_Y\left\{g_1^{-1}(x_1), \ldots, g_d^{-1}(x_d)\right\} \end{aligned}$$

The marginal CDF of X_j is $F_{X_j}(x_j) = F_{Y_j}\left\{g_j^{-1}(x_j)\right\} \Longrightarrow F_{X_j}^{-1}(u) = g_j\left\{F_{Y_j}^{-1}(u)\right\}$. By (5.1) and the above two equations, the copula of $\boldsymbol{X}$ is

$$\begin{aligned} C_X(u_1, \ldots, u_d) &= F_X\left\{F_{X_1}^{-1}(u_1), \ldots, F_{X_d}^{-1}(u_d)\right\} \\ &= F_Y\left[g_1^{-1}\left\{F_{X_1}^{-1}(u_1)\right\}, \ldots, g_d^{-1}\left\{F_{X_d}^{-1}(u_d)\right\}\right] \\ &= F_Y\left\{F_{Y_1}^{-1}(u_1), \ldots, F_{Y_d}^{-1}(u_d)\right\} \\ &= C_Y\{u_1, \ldots, u_d\}\ . \end{aligned}$$