STAT 631 Homework 7

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1)

```r
load("HW07.RData")
n = dim(Rt)[1];N = dim(Rt)[2]
```

The first thing we must do is create our vectors for $Y_{jt} = R_{jt} - \mu_{ft}$, the excess returns on the jth security. And $Y_{Mt} = R_{Mt} - \mu_{ft}$ the excess returns on the market portfolio.

```r
Yt = apply(Rt, 2, function(u) u - Rf)
YM = RM - Rf
```

Next we will compute the linear model:

$$Y_t = \alpha + \beta(Y_{Mt}) + e_t$$

```r
fit = lm(Yt ~ YM)
sfit = summary(fit)
```

a)

The estimates of beta for each asset is below:

```r
beta_est <- fit$coefficients[2,]
beta_est
```

```
     ABEV      AVGO       IBM       JNJ       MCD        MU       PHG       PII
0.8518551 1.2833012 0.8497390 0.5601800 0.6919863 1.4923087 0.9836818 1.3299907
```

The assets with the highest betas are Micron Technology, Inc. and Polaris Industries Inc. with $\hat{\beta} = 1.4923087, 1.3299907$ respectively.

b)

The square risk is computed through the below formula:

$$\sigma_j^2 = \beta_j^2 \sigma_M^2 + \sigma_{e_j}^2$$

Which has two components, market of systematic component of risk $\beta_j^2 \sigma_M^2$ and unsystematic component of risk $\sigma_{e_j}^2$. In the regression context the proportion of square risk that is due to the systematic risk is equivalent to R^2 for each regression.

```
R.Sq = c()
for(i in 1:N){
  R.Sq = c(R.Sq, sfit[[i]]$r.sq)
}
names(R.Sq) = syb; R.Sq
```

```
     ABEV      AVGO       IBM       JNJ       MCD        MU       PHG       PII
0.1872660 0.3831049 0.3654584 0.3067195 0.3750489 0.3166774 0.2952846 0.3340980
```

c)

If we assume that each Y_{it} follows CAPM that means we are assuming that $\alpha_i = 0$ for each. Then the estimate of excess return is simply $\hat{Y_{it}} = \hat{\beta}_i \bar{Y}_{Mt}$.

```
average_market_return <- mean(YM)
excess_return <- beta_est*average_market_return
excess_return
```

```
      ABEV       AVGO        IBM        JNJ        MCD         MU        PHG
0.14716762 0.22170483 0.14680204 0.09677744 0.11954848 0.25781324 0.16994218
       PII
0.22977097
```

d)

To test if the CAPM holds for each individual asset we test:

$$H_0 : \alpha_i = 0, H_1 : \alpha_i \neq 0$$

This is automatically computed through the lm function.

```
Alpha = c()
for(i in 1:N){
  Alpha = rbind(Alpha, sfit[[i]]$coef[1,])
}
rownames(Alpha) = syb; Alpha
```

```
         Estimate Std. Error     t value    Pr(>|t|)
ABEV -0.308534833  0.1727557 -1.78595994 0.074647681
AVGO  0.420904913  0.1585249  2.65513498 0.008153696
IBM  -0.057389884  0.1089980 -0.52652232 0.598734198
JNJ   0.025545568  0.0819848  0.31158906 0.755469023
MCD   0.104518882  0.0869558  1.20197719 0.229881306
MU    0.002434994  0.2133954  0.01141071 0.990899836
PHG  -0.180169557  0.1479323 -1.21791885 0.223768633
PII  -0.308860114  0.1827843 -1.68975228 0.091632919
```

At a significance level of $\alpha = 0.05$ we can say that every asset except Broadcom Inc. cannot reject the null hypothesis. For Broadcom Inc the estimate of α is 0.4209049 . Since this is greater than zero we can conclude that this security was under-priced in the past.

2)

a)

If we assume $y_1, ..., y_m$ are i.i.d k x 1 random vectors and $y_i \sim N_k(0, \Omega)$ Then $S = \sum_{i=1}^{m} y_i y_i^T \sim W_k(m, \Omega)$, the Wishart distribution with N degrees of freedom. Then:

$$\frac{m-k+1}{k} y^T S^{-1} y \sim F_{k,m-k+1}$$

We are testing:

$$H_0 : \alpha = 0, H_1 : \text{Any } \alpha_i \neq 0$$

For the Wald test we have the below:

$$F_W = \frac{n-N-1}{nN} \frac{1}{m_{11}} \hat{\alpha}^T \hat{\Sigma}_e^{-1} \hat{\alpha} \sim F_{N,n-N-1}$$

```r
alpha = Alpha[, "Estimate"]
et = resid(fit)
Sig = 1/n*t(et)%*%et
m11 = sfit[[1]]$cov.unscaled[1,1]
wald = (n - N - 1)/(n*N)*1/m11*t(alpha)%*%solve(Sig)%*%alpha
cat("Wald test:");c(statistic = wald, p.value = 1 - pf(wald, N, n - N - 1))
```

```
Wald test:

 statistic    p.value
1.90298192 0.05724853
```

The likelihood ratio test statistic is the discrepancy between the full and reduced model under H_0:

$$\text{LRT} = 2\{\ell(\hat{\alpha}, \hat{\beta}, \hat{\Sigma}_e) - \ell(0, \hat{\beta}_0, \hat{\Sigma}_{0e})\}$$

Through simplification and a non-normality adjustment we have:

$$\mathfrak{I}_{LR} = (n - N/2 - 2)\{\log|\hat{\Sigma}_{0e}| - \log|\hat{\Sigma}_e|\} \sim X^2_N$$

```r
fit0 <- lm(Yt ~ YM - 1)
et0 <- resid(fit0)
Sig0 = 1/n*t(et0)%*%et0
lr = (n-N/2 - 2)*(log(det(Sig0))-log(det(Sig)))
cat("Likelihood ratio test:"); c(statistic = lr, p.value = 1 - pchisq(lr,N))
```

```
Likelihood ratio test:

  statistic     p.value
15.09932233  0.05724273
```

Both tests fail the reject the null hypothesis that the CAPM model holds at significance level $\alpha = 0.05$. This is different than what we would expect as we previously found that individually Broadcom Inc. did not follow CAPM. However these test the overall p-value as opposed to the individual p-values from earlier.

b)

We have been unable to reject the null hypothesis that the CAPM model holds. In this case the total risk of Y_t is:

$$\sigma_j = \sqrt{\beta_j^2 \sigma_M^2 + \sigma_{e_j}^2}$$

We can find the systematic component of risk through $\sqrt{\beta_j^2 \sigma_M^2}$, along with the covariance: $\sigma_{jj'} = \beta_j \beta_{j'} \sigma_M^2$:

```
systematic <- beta_est%*%t(beta_est)*as.numeric(var(YM))
systematic
```

```
         ABEV      AVGO      IBM      JNJ      MCD        MU      PHG       PII
[1,] 3.829201  5.768608 3.819689 2.518083 3.110570  6.708124 4.421779  5.978483
[2,] 5.768608  8.690283 5.754278 3.793437 4.686006 10.105644 6.661315  9.006456
[3,] 3.819689  5.754278 3.810200 2.511828 3.102843  6.691461 4.410795  5.963632
[4,] 2.518083  3.793437 2.511828 1.655891 2.045511  4.411263 2.907762  3.931451
[5,] 3.110570  4.686006 3.102843 2.045511 2.526805  5.449202 3.591938  4.856494
[6,] 6.708124 10.105644 6.691461 4.411263 5.449202 11.751520 7.746223 10.473311
[7,] 4.421779  6.661315 4.410795 2.907762 3.591938  7.746223 5.106061  6.903669
[8,] 5.978483  9.006456 5.963632 3.931451 4.856494 10.473311 6.903669  9.334131
```

The non-systematic component of risk is $\sigma_{e_j}^2$, this is the variance of errors $\hat{\Sigma}$:

```
Sig
```

```
           ABEV       AVGO          IBM        JNJ        MCD           MU
ABEV 16.5891042 -0.8464808  0.448048482  0.1538744  0.3952642  1.550929832
AVGO -0.8464808 13.9685955 -0.215315411 -1.0944358 -0.3346543  5.530083537
IBM   0.4480485 -0.2153154  6.603817732  0.6600797  0.5642478 -0.004424773
JNJ   0.1538744 -1.0944358  0.660079685  3.7361533  0.6589814 -2.256064697
MCD   0.3952642 -0.3346543  0.564247818  0.6589814  4.2029577 -1.861156341
MU    1.5509298  5.5300835 -0.004424773 -2.2560647 -1.8611563 25.312090499
PHG   1.8873867  0.1067565  0.222725300  0.2758891  0.1590913  0.700376200
PII  -0.1029834 -1.3589304  0.721575079 -0.7126409 -0.1908340  0.827640894
            PHG        PII
ABEV  1.8873867 -0.1029834
AVGO  0.1067565 -1.3589304
IBM   0.2227253  0.7215751
JNJ   0.2758891 -0.7126409
```

```
MCD   0.1590913 -0.1908340
MU    0.7003762  0.8276409
PHG  12.1642147 -0.2917898
PII  -0.2917898 18.5710101
```

c)

We can find the minimum non-systematic variance portfolio by using Σ from our regression in the minimum variance portfolio formula.

$$w_{min.v} = \frac{\Sigma^{-1}1}{1^T\Sigma^{-1}1}$$

```
ones = rep(1, N)
IS = solve(Sig)
a = as.numeric((t(ones)%*%IS%*%ones))
w.min = 1/a*(IS%*%ones)
w.min
```

```
           [,1]
ABEV 0.04766569
AVGO 0.09709975
IBM  0.10700354
JNJ  0.31011904
MCD  0.23252018
MU   0.06018579
PHG  0.06956478
PII  0.07584122
```

d)

```
w_equal <- rep(1/8,N)
```

To test both portfolios we create 8 x 2 matrix W:

```
W = cbind(w.min,w_equal)
```

Then the hypothesis to test if these two portfolios follow the CAPM model is:

$$H_0 : W^T\alpha = 0, H_A : W^T\alpha \neq 0$$

Then we have the test statistic:

$$\frac{m-k+1}{k} y^T S^{-1} y \sim F_{k,m-k+1}$$

Where $y = W^T\hat{\alpha}, S = n\widehat{var}(W^T\hat{\alpha}), m = n-2$. With $var(\hat{\alpha}) = m_{11}\Sigma_e$. And m_{11} is $(X^TX)^{-1}_{11}$.

```
y = t(W)%*%alpha
S = n*m11*t(W)%*%Sig%*%W
m = n - 2
k = 2

test_stat <- (m-k+1)/k*t(y)%*%solve(S)%*%y
cat("Wald test statistic: "); c(statistic = test_stat,
                               p.value = 1 - pf(test_stat, k, n - k - 1))
```

```
Wald test statistic:

statistic   p.value
1.3329267 0.2645427
```

We cannot reject the null hypothesis that both portfolios follow CAPM.