

The data `HW07.RData` contains 4 objects, `Rt`, `RM` and `syb`:

`RM`: Weekly returns of S&P 500, Jan 1, 2014 to Sep 31, 2024.

`Rf`: Weekly data of US 3 month Treasury Bill for the same period of `RM`.

`Rt`: Weekly returns of 8 stocks, Ambev S.A. (ABEV), Broadcom Inc. (AVGO), IBM Co. (IBM), Johnson & Johnson (JNJ), McDonald's Co. (MCD), Micron Technology, Inc. (MU), Koninklijke Philips N.V. (PHG) and Polaris Industries Inc. (PII).

`syb`: the ticker symbols of `Rt`.

Important: Both `RM` and `Rf` are matrices of dimension $n \times 1$. When apply R's `var()`, the return values is a 1×1 matrix. You can remove the matrix with `as.vector()` or `as.numeric()`. Eg. `YM = RM-Rf; as.vector(var(YM))`.

1. Let Y_{jt} be the excess return of stock j and $\mathbf{Y}_t = (Y_{1t} \dots, Y_{Nt})^T$, where $N = 8$. Fit the excess return model of (7.9) on page 165 of Handout 7.
 - (a) Give the estimates of betas for all 8 assets. Which two assets have the highest betas?
 - (b) Give the proportion of the square risk that is due to the systematic risk for each asset.
 - (c) Suppose each Y_{it} , $i = 1, \dots, 8$ follows the CAPM, give the estimates for the excess return based on the model.
 - (d) Test if the CAPM holds for each individual asset. What are the hypotheses of the tests? State your conclusion.
2. In Question 1, we consider the CAPM for each individual asset. In this question, we will consider $\mathbf{Y}_t = (Y_{1t}, \dots, Y_{Nt})^T$ as a whole.
 - (a) Test if the CAPM holds for the 8 assets as a whole using both the Wald and likelihood ratio tests. Are the test results the same as what you expected?
 - (b) What are the estimated systematic component and unique component of the risk of \mathbf{Y}_t ? (Please note: both are matrices).
 - (c) For a CAPM, the systematic component is unavoidable risk and the unique component can be diversified out. Find the portfolio that minimizes the unique risk of \mathbf{Y}_t allowing short selling. (this was a midterm question in Spring 2020).
 - (d) Consider 2 portfolios, one is from part(c) and the other is an equal weight portfolio (i.e., $w_i = 1/8, i = 1, \dots, 8$, in R: `rep(1/8, 8)`). Give a Wald test to check if the CAPM holds for the both portfolios.
3. The `Midterm22.RData` is the data set used in the Midterm Fall, 2022. The question file is attached.