

The `Midterm21.RData` contains 3 objects, `yt`, `uni.t.est`, and `syb`.

`yt`: a 730×2 numerical matrix. It contains weekly stock returns (in %) of Bristol-Myers Squibb (BMS) and Merck (MRK), from the week of 1/12/2007 to the week of 12/31/2020. Their ticker symbols can be found in the object `syb`.

`uni.t.est`: The MLEs and standard errors of location, scale and DF of fitting univariate t model to each series of `yt` using the R function `fitdistr()`.

Use the following commands to check the list of data.

```
> head(yt,2); tail(yt,2)
> syb
> uni.t.est
```

Your solutions must be in a pdf or doc file which includes written text, code and results from knitting the RMD file. Please write down your answers clearly with proper labelling. I will not search your R code or output for your answers.

1. Denote the weekly returns of BMY and MRK by the random vector $\mathbf{Y}_t = (Y_{1t}, Y_{2t})^T$. Then `yt` is a sample of \mathbf{Y}_t . Consider a bivariate- t distribution for modelling `yt`.
 - (a) Based on the information in the object `uni.t.est`, is the bivariate- t a suitable candidate model for fitting `yt`? Please justify briefly.
 - (b) Fit a bivariate- t to `yt`. Report the MLEs of all the model parameters. Please also include $\hat{\rho}$, the MLE of correlation between the two component series (set `cor = T` in `cov.trob()`). Please store these estimates.
 - (c) State the marginal distributions of the bivariate- t in part(b) including the MLE estimates of the parameters. Please store the scale estimates of the 2 marginal distributions.
2. If \mathbf{Y}_t is a bivariate- t , then by definition, its copula \mathbf{U}_t is a t -copula with parameters ρ (correlation) and ν (degrees of freedom) that have been estimated in part (b) of Question 1.
 - (a) Give an estimate for the coefficient of tail dependence of the t -copula \mathbf{U}_t .
 - (b) Since `yt` is a sample of \mathbf{Y}_t , find the corresponding sample copulas (name the object `ut`). Plot the histogram of each series of `ut` and the scatter plot of the 2 series.
 - (c) Use `ut` to generate a set of bivariate meta- t data `xt` having marginal distributions $t_2(0.25, 2.45^2)$ and $t_5(0.25, 3^2)$, which are the t -distributions with degrees of freedom 2 and 5 respectively. Their means are both 0.25 and scales are 2.45 and 3. Plot the histogram of each series of `xt` and the scatter plot of the 2 series.