1. §5.20 Exercise 3. Equation (5.15) is on page 41 of Handout 3.
2. §5.20 Exercise 5:(a) and (b)
3. In this problem, we will fit distributions to the weekly returns of S&P 500 in the same time frame as the last homework,

```
> getSymbols("^GSPC", from = "2005-01-01", to = "2024-08-01")
> x = weeklyReturn(Ad(GSPC), type = "log")*100  ## convert to %
> n = dim(x)[1]  ## sample size
```

(a) Fit standardized t, skewed t, GED, skewed GED, Normal Inverse Gaussian and Johnson S_U distributions to the weekly returns of S&P 500 `rt` using `fitdist()` of R's package `rugarch`.

Tips: The `fitdist()` of `rugarch` is convenient for fitting several distributions.

```
> dists = c("std", "sstd", "ged", "sged", "nig", "jsu")
> fits = vector("list", 6)
> for(i in 1:6) fits[[i]] = fitdist(dists[i], rt)
```

The `std` fit is stored in `fits[[1]]`, etc. You can compute/plot parts (b) and (c) easily with a loop. The object `dists` can be used for labelling plots.

(b) For each fitted distribution, plot the density curve of fitted distribution with estimated parameter values from part (a) overlaying the kernel density estimate curve (`density()`). Please refer to Problem 6 of §R-Lab 4.10 in the last homework. Use different colors for the two curves. Make the 6 plots on a 2×3 layout. Clearly label them. Examine the plots, which distribution is the best fit?

If necessary, zoom in the plot by setting `xlim` to be shorter. Also mind the limits of y-axis, your plots should not be cut off.

Tips: Store the kernel estimates, `den = density(x)`. The return values are `den$x` and `den$y`, both are of the default length 512. The vector `den$y` consists of kernel estimates evaluated at `den$x`. You can plot the kernel density curve with them. For the parametric density curve, use `ddist()` evaluate density at `den$x` with the MLE estimates for the arguments. For example, the `std` density sequence can be obtained by

```
> est = fits[[1]]$pars
> ys = ddist(dists[1], den$x, mu = est["mu"], sigma = est["sigma"],
             skew = est["skew"], shape = est["shape"])
```

The six plots can be done with a loop in R code.

(c) For each fitted distribution, plot the QQ plot of sample quantiles versus quantiles of the fitted parametric distribution. Please also include a reference line. Make the 6 plots on a 2×3 layout. Clearly label them. Examine the plots, which distribution is the best fit?

Tips: you should be able to plot all 6 plots in a loop like part (b), use `qdist()` instead of `ddist`. You n sample quantiles against n parametric quantiles as you did in HW 1. You can use the quantiles, say `qs = (1:n)/(n + 1)` as in HW 1 or `qs = ((1:n) - 1/2)/n` to avoid 0 and 1.

(d) Compute the AIC and BIC criteria for all 6 models. Which model is selected by the AIC? The BIC? Which model would you choose? Why? Your answer should based on parts (b)-(d).

(e) For the two skewed distributions, skewed-t and skewed-GED, construct the 95% confidence intervals for the skewness parameter.

(f) Depending on the model you select in part (d), do one of the following tests.

- If the model you select is t or skewed-t, please do this part.
  Consider the skew-t distribution for the S&P 500 daily returns. We would like to test for the distribution symmetry based on the MLE of skew-t distribution. Compute the likelihood ratio statistic and give the test. Please include the null, alternative hypotheses, test statistic, p-value and conclusion.
- If the model you select is GED or skewed-GED, please do this part.
  Consider the skew-GED distribution for the S&P 500 daily returns. We would like to test for the distribution symmetry based on the MLE of skew-GED distribution. Compute the likelihood ratio statistic and give the test. Please include the null, alternative hypotheses, test statistic, p-value and conclusion.
- If your model is normal inverse Gaussian or Johnson S_U distribution, skip this question.