1. R-Lab§7.13.2, Problems 3-6. For Problem 6, part (b), set random seed, `set.seed(200128)` and use `rt()` to generate. **Note:** This is not a suggestion, you are required to use R's basic function `rt()` to simulate.

2. Problems 1&2 of R-Lab§7.13.1 with a different set of data that you will create.

   The data set is weekly returns of 4 companies, General Mills, Keurig Dr Pepper, Coca-Cola, Procter & Gamble, from Jan 1, 2011 to Aug 31, 2024. Use the following R code to create the data.

```
> library(quantmod)
> syb = c("GIS", "KDP","KO", "PG")
> d = length(syb)
> rt = c()
> for(i in 1:d) {
  +     getSymbols(syb[i], from = "2011-01-01", to = "2024-08-31")
  +     rt = cbind(rt, weeklyReturn(Ad(get(syb[i])), type = "log")[-1])
>  }
>  colnames(rt) = syb
>  rt = rt*100  ## covertto
```

   (a) Problem 1 of R-Lab§7.13.1 using `rt`. Include a scatterplot matrix.

   (b) Before proceeding Problem 2 of R-Lab§7.13.1, give a justification that a multivariate t is a suitable candidate model for rt.

   (c) Problem 2 of R-Lab§7.13.1 using `rt`.

   (d) Compute the AIC and BIC of the multivariate t fit.

   (e) Fit the skewed multivariate t distribution of Azzalini & Capitanio to the data `rt`. The fit requires `sn` package. If `nu` estimate is greater than 4, transform the direct parameter estimates to the central parameter estimates with the following command,

   ```
   > dp2cp(fit_st$dp, "st")
   ```

   where `fit_st` is the returned value from the fit. The output `$beta` is the estimate of the means, `$var.cov` is the estimate of covariance matrix, `$gamma1` is the estimate of skewnesses, and `$gamma2M` estimates the common kurtosis of the four marginal distributions.

   (f) Compute the AIC and BIC of the skewed multivariate-t fit in part(d). Which model, multivariate-t or skewed multivariate-t, is selected by the AIC? And by the BIC?

3. This question was one of the midterm questions in 2019. Please load the data file `HW03.RData`:

   > load("/path_you_saved/HW03.RData")

   The file contains 2 `R` objects, `xt` and `univ.t`:

   - `xt` – daily log returns of 4 companies in percentage(%), the ticker symbols are `ACM`, `AXP`, `CVX` and `JNJ` from April 2, 2009 to September 30, 2019.

```
head(xt,1)

##                ACM    AXP     CVX      JNJ
## 2009-04-02 4.2858 3.6714 2.90041 -0.13208

tail(xt,1)

##                  ACM      AXP     CVX      JNJ
## 2019-09-30  0.72145 -0.26174  0.0000  0.60470
```

- `univ.t` – the MLE obtained by fitting each return series with a univariate t, `univ.t$mle` contains estimates and `univ.t$se` contains corresponding standard error of each estimate. Both are 4×3 matrices.

```
univ.t$mle

##         mean   scale     df
## ACM 0.047592 1.37137 4.0215
## AXP 0.102297 0.96991 2.6400
## CVX 0.056037 0.98899 4.0902
## JNJ 0.065948 0.66911 4.0123

univ.t$se

##         mean    scale      df
## ACM 0.031602 0.032859 0.31996
## AXP 0.023429 0.025792 0.16124
## CVX 0.022674 0.025331 0.35859
## JNJ 0.015373 0.016845 0.33831
```

(a) Is a multivariate-t model appropriate for `xt`? Why? Give an informal statistical justification.

(b) Create a data set, say `x` which has either 2, 3, or 4 series of `xt` that can be modeled with a multivariate-t distribution. For example, if only `ACM` and `JNJ` are to be modeled by a multivariate-t, then `x = xt[, c(1,4)]`. Your `x` should include all series that are suitable and be consistent with your answer in part (a). Fit a multivariate t model to `x` with the MLE.

(c) State the estimated marginal distributions of each series in `x` from the model in part (b).

(d) Give a likelihood ratio test for $H_0 : \nu = 4$, where ν is the degrees of freedom (or shape parameter) of the multivariate-t model of `x`. Please include the test statistic, p-value and conclusion.