STAT 631 Homework 9

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```r
load("HW09.RData")
```

```r
alpha <- seq(0.05, 0.01, -0.01)
S  <- 50000
```

1)

a)

The non-parametric estimates of VaR and estimated shortfall are as follows:

$$\hat{\text{VaR}}^{\text{np}}(\alpha) = -\text{S} \times \hat{q}(\alpha)$$

$$\hat{\text{ES}}^{\text{np}}(\alpha) = -\text{S} \times \frac{\sum_{i=1}^{n} R_i 1\{R_i < \hat{q}(\alpha)\}}{\sum_{i=1}^{n}\{R_i < \hat{q}(\alpha)\}}$$

```r
q = quantile(rt, alpha)
VaR.np = -S*q; ES.np = rep(0,5); for(i in 1:5){ES.np[i] <-  -S*mean(rt[rt < q[i]])}
VaR.np = VaR.np/100; ES.np = ES.np/100

stats_np <- rbind(q,VaR.np, ES.np)
rownames(stats_np) <- c("Sample Quantile", "VaR.np","ES.np")
stats_np
```

```
                        5%          4%          3%          2%         1%
Sample Quantile   -4.24646   -4.632041   -5.243954   -6.234252   -7.30261
VaR.np          2123.23013 2316.020396 2621.977064 3117.125865 3651.30513
ES.np           3170.55689 3398.170028 3716.953655 4146.982608 4913.70225
```

b)

Using the Peaks over Thresholds approach we have the following conditional distribution of $y = x_t - \eta$ when $x_t > \eta$:

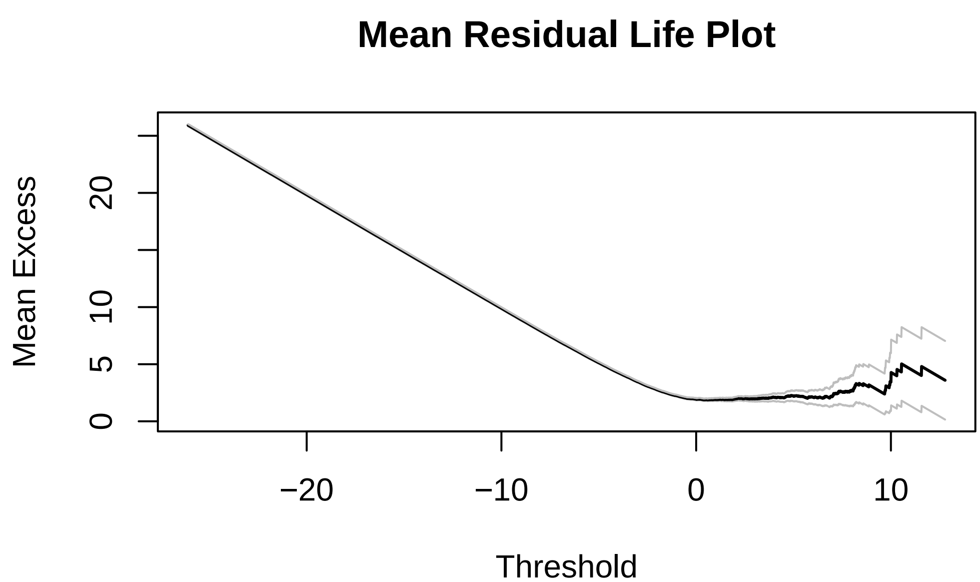
$$G_{\xi,\psi(\eta)}(y) = \begin{cases} 1 - (1 + \frac{\xi y}{\psi(\eta)})^{-1/\xi}, & \xi \neq 0 \\ 1 - \exp(-y/\psi(\eta)), & \xi = 0 \end{cases}$$

We need to choose an appropriate threshold η, we can do this through mean excess plots. Since for a fixed ξ the mean excess function is the linear function of $y = \eta - \eta_0$. In these plots we are looking for horizontal line and erring on the side of a lower threshold so we have a larger sample to estimate the parameters of G.

```
library(POT)
```

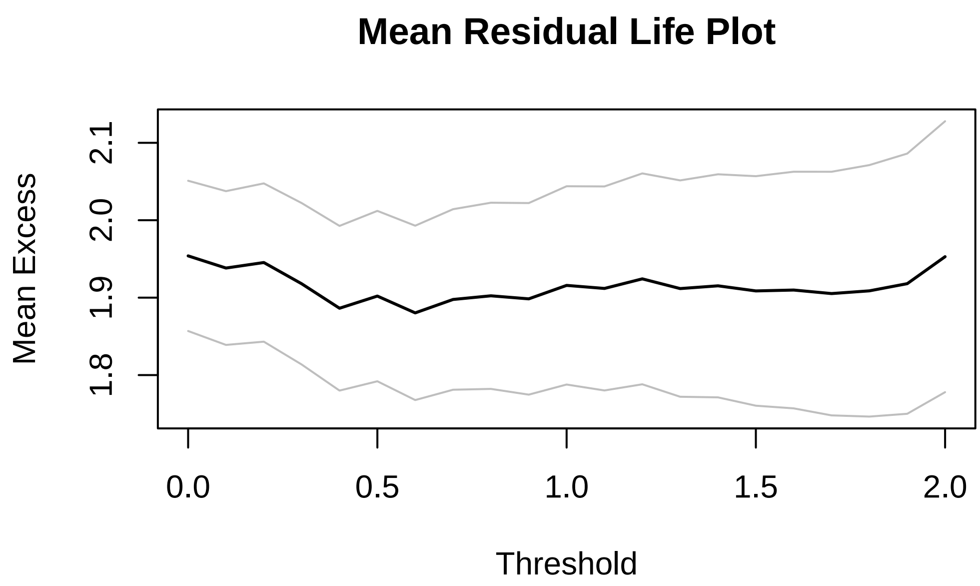
```
Warning: package 'POT' was built under R version 4.3.3
```

```
xt = - rt
mrlplot(xt)
```



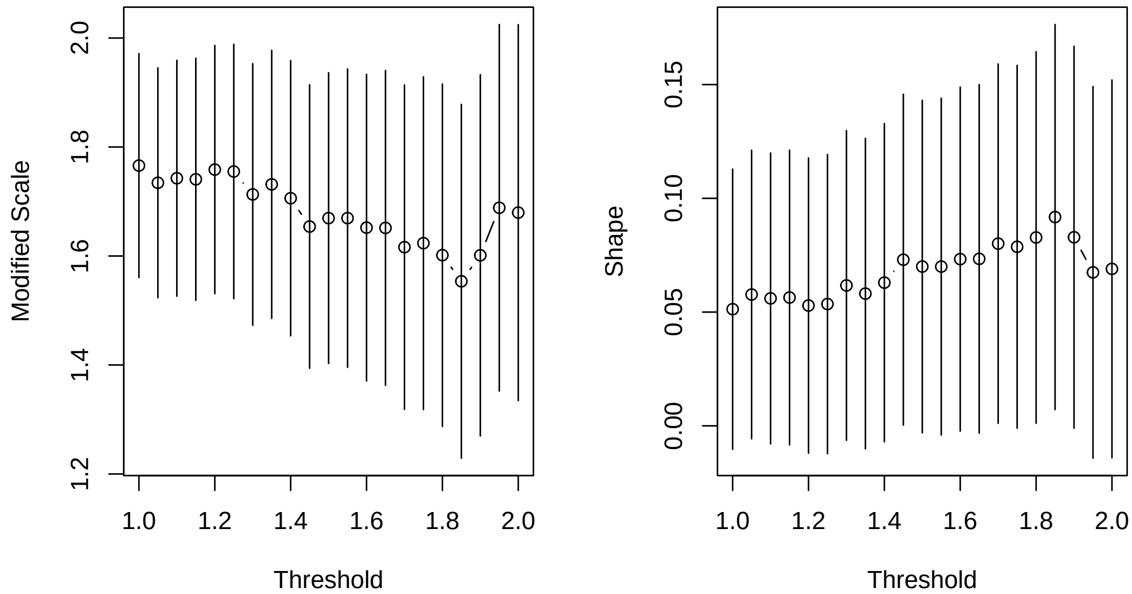
We limit the interval to examine:

```r
us = seq(0,2,.1)
mrlplot(xt, range(us), nt = length(us))
```



We can see stability in the range of 1.3 to 1.8 especially. Before we decide we will take a look at the parameter stability plots:

```r
par(mfrow = c(1,2));us = seq(1,2,.05)
tcplot(xt, range(us), cmax = T, nt = length(us))
```



The scale and shape parameters are stable in the interval of (1.5,1.7) range especially. So we would select $\eta_0 = 1.5$. We now calculate the MLEs of $G_{\eta,\psi(\eta)(\cdot)}$:

```r
library(evir)
```

```
Attaching package: 'evir'

The following objects are masked from 'package:POT':

    dgpd, pgpd, qgpd, rgpd
```

```r
mle = gpd(xt, threshold = 1.5)
cat("Number of Exceedances:", mle$n.exceed);
```

```
Number of Exceedances: 757
```

```
cat("estimates of scale and shape:");mle$par.ests
```

```
estimates of scale and shape:

        xi       beta 
0.06998916 1.77471856 
```

After reviewing the residual plots (which are not shared here due to it being an interactive menu) we can say this distribution fits pretty well despite struggling to maintain a straight line in the middle portion of the qqplot.

With this distribution fit we can now compute value at risk and expected shortfall using the below functions:

$$\mathrm{VaR}_p = \eta - \frac{\psi(\eta)}{\xi}\{1 - [\frac{n}{n_\eta}(1-p)]^{-\xi}\}$$

$$\mathrm{ES}_q = \frac{\mathrm{VaR}_q}{1-\xi} + \frac{\psi(\eta) - \xi\eta}{1-\xi}$$

Since our shape parameter ξ is between 0 and 1.

We use the riskmeasures function from library evir for these calculations:

```
risk = S/100*riskmeasures(mle, p = 1 - alpha)[,2:3];
colnames(risk) = c("VaR","ES"); rownames(risk) = c("5%","4%","3%","2%","1%");risk
```

```
        VaR       ES
5% 2124.185 3181.739
4% 2345.377 3419.578
3% 2635.690 3731.739
2% 3054.916 4182.514
1% 3799.725 4983.374
```