

The `Midterm22.RData` contains 4 objects, `rt`, `Rf`, `Mkt` and `syb`.

`rt`: a  $608 \times 18$  numerical matrix. It contains weekly stock returns (in %) of 18 stocks, from 1/1/2011 to 8/31/2022. Their ticker symbols can be found in the object `syb`.

`Mkt`: a  $608 \times 1$  numerical matrix. It is weekly return (in %) of S&P 500 index of the same period as `rt`

`Rf`: a  $608 \times 1$  numerical matrix. It is weekly data of 3-month US T-Bill.

Your solutions must be in a pdf file which includes written text, code and results from knitting the RMD file. Please write down your answers clearly with proper labelling. I will not search your R code or output for your answers. Please kindly remove any numerical errors of zeros.

1. One way to diversify a stock portfolio is through grouping. In this question, you will create two portfolios, aggressive and nonaggressive portfolios. All portfolio weights should have their corresponding companies clearly labelled. **The current risk free rate is 3.31% ( weekly rate 3.31/52).**

- (a) Use the data provided to estimate stock betas for all 18 companies. To continue the rest of questions, it is convenient to order the betas and our data accordingly. Suppose `betas` is the vector of all betas.

```
> ind = order(betas, decreasing = T)
> betas = betas[ind]
> rt = rt[,ind]
> syb = syb[ind]
```

Show the beta estimates. You should be able to make two sets of weekly returns, `rt.A` for aggressive stocks and `rt.B` for nonaggressive stocks based on the beta estimates.

- (b) The aggressive portfolio is the minimum variance portfolio without short sales of the aggressive stocks. Give the estimates of the portfolio weights, return and risk.
- (c) The non-aggressive portfolio is the tangency portfolio allowing short sales of the non-aggressive stocks. Give the estimates of the portfolio weights, return and risk.
- (d) Let  $A$  be the aggressive portfolio and  $B$  be the non-aggressive portfolio. Define the bivariate random variable  $P = (A, B)^T$ , give the estimates of the mean and variance-covariance matrix of  $P$ .
- (e) The final risky asset portfolio consists of  $w_p A + (1 - w_p) B$  such that the portfolio yields the highest Sharpe ratio among such combinations. Find the estimates of  $(w_p, 1 - w_p)^T$ , portfolio return and portfolio risk.
- (f) Show the portfolio in Part (e) in terms of individual stocks and test (with the Wald test) if the portfolio follow the CAPM model. (Reminder: you have reordered the data).
- (g) If I am going to invest 15% on risk-free asset and 85% on risky asset. Give the estimates of return and risk of my investment.