STAT 631 Exam 2

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```
load("Mid2_2024.Rdata")
```

1)

We begin by using the Fama-French 3 factor model for R_t.

First we find $Y_t = R_t - \mu_f 1_n$ which is the excess return. We have the risk free rate from the FF5 data set.

```
Yt= apply(Rt,2, function(x) x - FF5[,6]);dimnames(Yt)[[2]] =syb;
n= dim(Yt)[1]; N= dim(Yt)[2]; p=3
```

Next we fit the FF3-factor model through the following regression:

$$Y_t = \alpha + B^T F_t + e_t$$

Which has the usual assumptions (expected value of residual = 0 , diagonal residual matrix).

This uses three factors, excess returns on market portfolio, Small minus Big, High minus low. Each is available in the FF5 dataset.

```
Mkt.RF = FF5[,1]
SMB = FF5[,2]
HML = FF5[,3]
fit= lm(Yt~Mkt.RF+SMB+HML);sfit = summary(fit)
```

Now with our fitted model we can find R^2 for all 32 asset returns, grabbing it from the summary of each fit:

```r
R.Squared = c(); for(i in 1:N) R.Squared[i] = sfit[[i]]$r.squared
```

b)

To test if the FF3-factor model holds for all 32 assets as a whole we use the Wald test. Our hypothesis is that:

$$H_0 : \alpha = 0, H_a : \alpha \neq 0$$

Where α is the length N vector, the intercept of our model.

To test this we employ the helpful wald.fun function from Factor_Tests from Homework 8:

```r
source("FM_Functions.R")
source("Factor_Tests.R")
```

```r
Alpha = c()
for(i in 1:N){
  Alpha = rbind(Alpha, sfit[[i]]$coef[1,])
}
alpha = Alpha[,"Estimate"]
m11 = sfit[[1]]$cov.unscaled[1,1]
Sig = 1/n*t(resid(fit))%*%resid(fit)
var.alpha = m11*Sig
wald.fun(est = alpha, est.var = var.alpha, n =  n, p = p)
```

```
         Wald       p.value          df1          df2
    0.7580343     0.8338924   32.0000000 2417.0000000
```

We cannot reject the null hypothesis that the α vector is equal to 0.

c)

To use the variance-covariance estimate of R_t we use the below:

That:

$$\hat{\Sigma_R} = \hat{B}^T \hat{\Sigma_F} \hat{B} + diag(\hat{\Sigma}_e)$$

Then:

```r
Bhat = fit$coefficients[-1,]
Sigma.F = var(FF5[,c(-4,-5,-6)])
Sig.R = t(Bhat)%*%Sigma.F%*%Bhat + diag(diag(Sig))
Sig.R[1:5,1:5]
```

```
          AIR       AMD      AMGN       AMT      AOSL
AIR  7.926135  2.407331 1.1662137 1.1265777  3.321113
AMD  2.407331 13.374723 1.6321114 1.6530776  3.553892
AMGN 1.166214  1.632111 2.4275812 0.7418677  1.387595
AMT  1.126578  1.653078 0.7418677 2.5735015  1.346481
AOSL 3.321113  3.553892 1.3875947 1.3464807 11.829295
```

d)

The explicit solution for the minimum variance portfolio, allowing short selling, is below:

$$w_{min.v} = \frac{\Sigma^{-1}1}{1^T\Sigma^{-1}1}$$

For this problem we use Σ from part c, the weights are as follows:

```r
ones = rep(1,N)
IS = solve(Sig.R)
a = as.numeric((t(ones)%*%IS%*%ones))
w.min = 1/a*(IS%*%ones)
w.min
```

```
              [,1]
AIR  -0.0104841931
AMD  -0.0285132886
AMGN  0.0595912261
AMT   0.0411559247
AOSL -0.0007512568
APPS -0.0056102630
BCE   0.1905558969
CAE  -0.0231004879
CINF -0.0451460030
CLS  -0.0092191570
DCO   0.0112159317
DXC  -0.0411870885
EQIX  0.0375655854
```

```
GD     0.0489679348
HII    0.0425148199
HSBC   0.0476333793
HXL   -0.0227415218
JNJ    0.1860571486
KO     0.1725510223
LMT    0.0834251305
MCD    0.0959874033
MTSI  -0.0169268808
MXL   -0.0173561680
PEP    0.1259630983
RTX   -0.0070645310
TDG   -0.0254072893
TDY    0.0098128173
TRI    0.1292946107
TRV    0.0179924113
UCTT  -0.0182096929
WCC   -0.0391724101
WRB    0.0106058908
```

e)

Now for each step we are taking a model based on the return of each portfolio combined and testing if (now a vector with length three) α is equal to 0. In other words:

$$H_0 : \alpha = 0, H_a : \alpha \neq 0$$

To get the returns of the portfolio i. we simply use:

$$Y_t w_a$$

So:

```
Y.a = Yt%*%w.min
```

To get the portfolio for ii. we simply take weights equal to 1/32, and perform the same computation:

```
w.b = rep(1/32, 32)
Y.b = Yt %*% w.b
```

To get the portfolio for iii. we take weights equal to R.sq/sum(R.sq) and perform the same calculation:

```
w.c = R.Squared/sum(R.Squared)
Y.c = Yt %*% w.c
```

Now that we have our three returns we bind them together and similarly to part b we start with fitting the FF3 model:

```
Yt.e = cbind(Y.a, Y.b, Y.c)
```

```
fit.e = lm(Yt.e~Mkt.RF+SMB+HML);sfit.e = summary(fit.e)
```

Now that we have fit this model we continue what we did with the wald test, grab our appropriate arguments for the wald function:

```
Alpha.e = c()
for(i in 1:3){
  Alpha.e = rbind(Alpha.e, sfit.e[[i]]$coef[1,])
}
alpha.e = Alpha.e[,"Estimate"]
m11 = sfit.e[[1]]$cov.unscaled[1,1]
Sig.part.e = 1/n*t(resid(fit.e))%*%resid(fit.e)
var.alpha.e = m11*Sig.part.e
wald.fun(est = alpha.e, est.var = var.alpha.e, n =  n, p = p)
```

```
        Wald      p.value          df1          df2
   0.7891953    0.4998213    3.0000000 2446.0000000
```

For these three portfolios we cannot reject the null hypothesis that all three conform to the FF3 factor model.

2)

a)

Given the multivariate T distribution which fit is provided by data mt we can find the minimum variance portfolio through a similar formula from before. In fact we have a better estimate of the Covariance matrix from our fit of FF3 model. We extract that better estimate below:

```r
Sig.a = Sig.R[syb6,syb6]
Sig.a
```

```
           TRV       MXL       DCO       BCE        KO      HSBC
TRV  2.3403112  1.525634 1.4770971 0.7310328 0.7048881 1.0639061
MXL  1.5256339 12.859049 2.9518047 1.1836239 1.0213772 1.5164120
DCO  1.4770971  2.951805 8.5909783 0.9847274 0.8083684 1.4796522
BCE  0.7310328  1.183624 0.9847274 1.3865728 0.4855591 0.6912802
KO   0.7048881  1.021377 0.8083684 0.4855591 1.2865052 0.6513248
HSBC 1.0639061  1.516412 1.4796522 0.6912802 0.6513248 2.5621780
```

Now we use the same explicit formula from earlier:

$$w_{min.v} = \frac{\Sigma^{-1}1}{1^T\Sigma^{-1}1}$$

```r
ones = rep(1,6)
IS = solve(Sig.a)
a = as.numeric((t(ones)%*%IS%*%ones))
w.min.a = 1/a*(IS%*%ones)
w.min.a
```

```
            [,1]
TRV   0.08678846
MXL  -0.02481772
DCO  -0.01114030
BCE   0.39923117
KO    0.44947422
HSBC  0.10046417
```

b)

First we create the excess return vector, utilizing the center estimate from the multivariate t distribution:

```r
mu.f = 4.72/260
m.ex = mt$center - mu.f
m.ex
```

```
       TRV        MXL        DCO        BCE         KO       HSBC
0.06722061 0.10973920 0.02421123 0.02725641 0.05587342 0.02763245
```

Now we use the explicit formula to solve for this portfolios weights:

```
aT = as.numeric((t(ones)%*%IS%*%m.ex))
w.T = 1/aT*(IS%*%m.ex)
w.T
```

```
            [,1]
TRV   0.44472408
MXL   0.10918909
DCO  -0.09437795
BCE  -0.03104572
KO    0.71588351
HSBC -0.14437302
```

c)

Now we have Jennifers portfolio. Let's first create the mean, scale vector reflecting this through Yw:

```
mean <- .2*mu.f + .3*mt$center%*%w.min.a + .5*mt$center%*% w.T
mean
```

```
           [,1]
[1,] 0.06801509
```

And its scale is:

```
scale_port <- .2*0 + .3* t(w.min.a)%*%mt$scale%*%w.min.a + .5*t(w.T)%*%mt$scale%*%w.T
scale_port
```

```
          [,1]
[1,] 0.4510626
```

We have a portfolio that has a t distribution with the mean, scale values above and nu = 3.48 from the multivariate fit.

To find VAR, ES parametrically we use:

$$VaR(\alpha) = -S(\hat{\mu} + \lambda F_{\nu}^{-1}(\alpha))$$

and:

The longer formula for ES which is:

$$ES(\alpha) = S(-\mu + \lambda \frac{f_v(F_\nu^{-1}(\alpha))}{\alpha} * \frac{\nu + (F_\nu^{-1}((\alpha))^2}{\nu - 1})$$

which I could compute, but sadly I have run out of time.