STAT 631 Homework 8

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11/1/24

```
source("FM_Functions.R")
source("Factor_Tests.R")
load("HW08.RData")
attach(FF5)
```

1)

The Fama-French 3 factor model is the below:

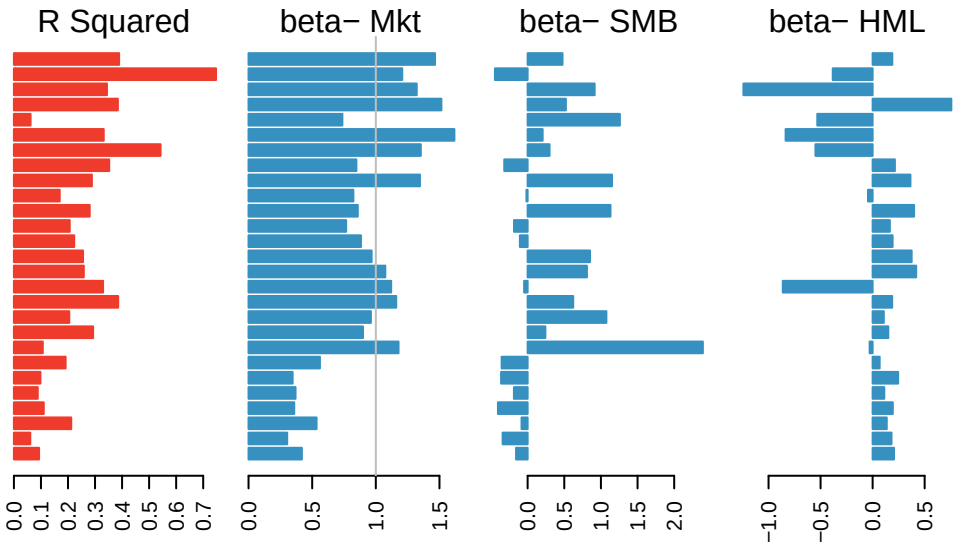
$$Y_t = \alpha + B^T F_t + \epsilon_t, \quad E[\epsilon_t | F_t] = 0, \quad E[\epsilon_t \epsilon_t^T | F_t] = \Sigma_\epsilon$$

Where $F = \big[\text{Excess Return Market Portfolio} \quad \text{Small Minus Big} \quad \text{High Minus Low}\big]$, these are each economic vectors with length n.

```
Yt = apply(Rt,2, function(x) x-RF); dimnames(Yt)[[2]] = syb;
n = dim(Yt)[1]; N = dim(Yt)[2]; p = 3
fit = lm(Yt ~ Mkt.RF + SMB + HML); sfit = summary(fit)
```

a)

```
betas = coef(fit)[-1,]
R.Squared = c(); for(i in 1:N) R.Squared[i] = sfit[[i]]$r.squared
coef.plot(R.Squared, coef(fit)[-1,],labs = syb)
```



From the R squared plot we see that the three factor Fama French model performance varies greatly. Let's take a look at the breakdown by industry:

```
table(Hi_R.Sq = R.Squared > 0.5, by_industry)
```

```
       by_industry
Hi_R.Sq Ent Food HCare Tech
  FALSE   7    7     6    5
  TRUE    0    0     0    2
```

Generally R-Squared isn't very high for these assets. There are only two that exceed 0.5, both are in the technology industry Microsoft and Autodesk.

```
table(Hi_R.Sq = R.Squared < 0.2, by_industry)
```

```
       by_industry
Hi_R.Sq Ent Food HCare Tech
  FALSE   6    1     5    6
  TRUE    1    6     1    1
```

R-Squared is particularly low for the Food industry, six of the seven stocks have an R-Squared beneath 0.2. This indicates that the three factor Fama French model does not perform well for this industry.

```
table(Aggressive = coef(fit)[2,] > 1, by_industry)
```

```
          by_industry
Aggressive Ent Food HCare Tech
     FALSE   3    7     5    1
     TRUE    4    0     1    6
```

On an industry level we see that that Food and Heath Care are not aggressive compared to market returns while Technology generally is. Entertainment is more of a mixed bag.

```
library(tidyverse)
```

```
-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.2     v readr     2.1.4
v forcats   1.0.0     v stringr   1.5.0
v ggplot2   3.4.2     v tibble    3.2.1
v lubridate 1.9.2     v tidyr     1.3.0
v purrr     1.0.2
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to beco
```

```
compare <- data.frame(
  Stock = syb,
  Beta = coef(fit)[2,],
  Industry = by_industry
)
compare |>
  group_by(Industry) |>
  summarise(Average_Beta = mean(Beta))
```

```
# A tibble: 4 x 2
  Industry Average_Beta
  <chr>           <dbl>
```

```
1 Ent             1.05
2 Food            0.413
3 HCare           0.921
4 Tech            1.32
```

By taking a look at the average Beta we can see that the Food Industry has a Beta of 0.4132 on average. Healthcare, despite being not aggressive compared to the market, is far closer to 1 in comparison.

b)

To identify the individual assets that don't follow the FF-3-factor model we use the t-test for $H_0 : \alpha_i = 0$ that is automatically computed from the lm function.

```
Alpha = c()
for(i in 1:N){
  Alpha = rbind(Alpha, sfit[[i]]$coef[1, ])
}
dimnames(Alpha)[[1]] = syb
Alpha_df <- data.frame(Alpha, Industry = by_industry)
Alpha_df |>
  filter(Pr...t.. < .05)
```

```
         Estimate Std..Error   t.value   Pr...t.. Industry
LBTYA -0.08814986 0.03719910 -2.369677 0.01789040      Ent
PARA  -0.11743008 0.05692884 -2.062752 0.03925461      Ent
WBD   -0.10721466 0.05359548 -2.000442 0.04557659      Ent
MD    -0.12999721 0.05171332 -2.513805 0.01201531    HCare
```

There are four individual assets that do not follow the FF-3 factor model, Live Nation Entertainment, Paramount, Warner Brothers Discovery and Pediatric Medical Group. The first three are in the entertainment industry and the last is in healthcare.

c)

We are testing the hypothesis that $H_0 : \alpha = 0$. If we reject this hypothesis this indicates that the FF-3 factor does not hold for all 27 assets. We perform the Wald and Likelihood Ratio Tests.

```
alpha <- coef(fit)[1, ]
res = resid(fit); Sig.e = 1/n*t(res)%*%res
m11 = sfit[[1]]$cov.unscaled[1,1]
var.alpha = m11*Sig.e
```

```r
p = 3

wald.fun(est = alpha, est.var = var.alpha, n =  n, p = p)
```

```
      Wald      p.value          df1          df2
 1.1490913    0.2718611   27.0000000 2150.0000000
```

```r
res.0 = resid(lm(Yt~Mkt.RF + SMB + HML - 1))
Sig.e0 = 1/n*t(res.0)%*%res.0
lrt.fun(sig = Sig.e, sig0 = Sig.e0,n = n)
```

```
       LRT    p.value         df
31.0114868  0.2706672 27.0000000
```

Both the Wald and Likelihood test ratios have a similar result with p value $\approx .271$. We cannot reject the null hypothesis that the FF-3 factor does not hold for all 27 assets.

```r
wald = c(); lrt = c()
for(i in industry){
  ind = which(by_industry == i)
  wald = rbind(wald, wald.fun(alpha[ind], m11*Sig.e[ind,ind],n = n, p = p))
  lrt = rbind(lrt, lrt.fun(Sig.e[ind,ind], Sig.e0[ind,ind], n = n))
}

rownames(wald) = rownames(lrt) = industry
cat("Wald test by industry:"); wald
```

```
Wald test by industry:
```

```
           Wald    p.value df1  df2
Food  0.2752167 0.96372392   7 2170
Ent   1.7837085 0.08628134   7 2170
HCare 1.8053977 0.09425210   6 2171
Tech  1.3190840 0.23693005   7 2170
```

```r
cat("LRT by industry:"); lrt
```

```
LRT by industry:
```

```
            LRT    p.value df
Food   1.929655 0.96363090  7
Ent   12.475994 0.08595263  7
HCare 10.825360 0.09392620  6
Tech   9.233106 0.23635053  7
```