The `Mid2_2024.RData` contains 5 objects – 2 time series data `FF5` and `Rt`, 2 vectors of ticker symbols `syb` and `syb.6`, and the MLE of a multivariate t, `mt`.

Both `FF5` and `Rt` are daily data from Jan 1, 2015 to September 30, 2024, sample size $n = 2452$. They are numerical matrices without any "class".

`FF5` – an $n \times 6$ matrix of the Fama-French 5 factors and the last column is the daily risk-free rate in %.

```
head(FF5,3);tail(FF5,1)

##          Mkt.RF   SMB   HML   RMW   CMA RF
## 20150102  -0.12 -0.57  0.08 -0.25  0.12  0
## 20150105  -1.84  0.25 -0.68  0.16 -0.08  0
## 20150106  -1.04 -0.78 -0.31  0.53  0.02  0
##          Mkt.RF   SMB  HML  RMW   CMA   RF
## 20240927  -0.08  0.71 0.55 0.09 -0.07 0.02
## 20240930   0.36 -0.11 0.15 0.43  0.00 0.02
```

`Rt` – an $n \times 32$ matrix of log daily return of 32 assets in %. Show only the first 5 below,

```
head(Rt,3)[,1:5]; tail(Rt,2)[,1:5]

##                AIR      AMD     AMGN      AMT     AOSL
## 2015-01-02 -2.0732  0.00000  0.37597  0.82612 -0.11306
## 2015-01-05 -1.7424 -0.37523 -1.19543 -1.45530 -0.90910
## 2015-01-06 -2.4992 -1.13423 -3.27477 -0.26504  0.11409
##               AIR      AMD     AMGN      AMT     AOSL
## 2024-09-27 1.3856 -1.89253  1.11256 -0.62758 -1.27797
## 2024-09-30 1.0458 -0.16442 -0.14267  0.96786 -0.53735
```

`syb` – a 32×1 vector of ticker symbols of `Rt`.

`syb6` – a 6×1 vector of ticker symbols of subset of `Rt` that have similar degrees of freedom that can be model with a multivariate t distribution. Its MLE is given in `mt`.

```
syb6

## [1] "TRV"  "MXL"  "DCO"  "BCE"  "KO"   "HSBC"
```

`mt` – The MLE of dimension 6 multivariate t distribution, including the estimates of degrees of freedom `mt$nu`, mean vector `mt$center` and scale matrix Λ, `mt$scale`.

```
names(mt)

## [1] "nu"     "center"   "scale"
```

You should retrieve any number to be used, not copy and paste from an output.

All portfolios calculated in this exam are without constraints other than the total weights is 1.

1. Let $\boldsymbol{R}_t$ be the daily return of the 32 assets at time t with the sample `Rt`. Consider the Fama-French 3 factor model for $\boldsymbol{R}_t$. Fit the regression model using the given dat to get the estimates of the model.

   (a) Find the R-Squared for all 32 asset returns. Please store them in a vector, say `R.Sq`.

   (b) Test if the FF3-factor model holds for the 32 assets as a whole with the Wald test.

   (c) Instead of the sample variance (high parameter count), estimate the variance-covariance estimate of $\boldsymbol{R}_t$ with the FF3-factor model based estimator. Please show only the first 5×5, i.e., `[1:5,1:5]`.

   (d) Find the minimum variance portfolio of $\boldsymbol{R}_t$ using the variance estimate from part (c).

   (e) Consider three portfolios for R_t,

      i. the minimum portfolio in part (d);

      ii. the naive equal weight portfolio, $1/32, \ldots, 1/32$

      iii. the R-Squared portfolio, that is, using the R code: `R.Sq/sum(R.Sq)`

   Give a Wald test to check if the FF3-factor models for all three portfolios.

2. The subset of $\boldsymbol{R}_t$ of 6 asset returns are a multivariate t distritution and the MLE are given in `mt`. We will use full parametric method to compute the VaR and ES of portfolios of these 6 assets.

```
mt$nu
##   nu
## 3.48

mt$center
##      TRV      MXL      DCO      BCE       KO     HSBC
## 0.085374 0.127893 0.042365 0.045410 0.074027 0.045786

mt$scale
##          TRV     MXL     DCO     BCE      KO    HSBC
## TRV  0.96560 0.45974 0.57710 0.25462 0.34850 0.43696
## MXL  0.45974 5.52459 1.24697 0.37215 0.23961 0.75310
## DCO  0.57710 1.24697 3.57023 0.27554 0.27056 0.61603
## BCE  0.25462 0.37215 0.27554 0.58663 0.23631 0.27667
## KO   0.34850 0.23961 0.27056 0.23631 0.55816 0.21365
## HSBC 0.43696 0.75310 0.61603 0.27667 0.21365 1.27604
```

   (a) Find the minimum variance portfolio of these 6 assets.

   (b) The current annul risk free rate is 4.72%, convert it to daily by 4.72/260. Find the tangency portfolio of these 6 assets.

   (c) Jennifer is going to invest $S = \$50,000$ to these 6 assets by distributing 20% to risk free asset, 30% to minimum variance portfolio and 50% to tangency portfolio. Find the distribution of her portfolio and compute her one -day VaR and ES at $\alpha = 5\%$.