Homework 1

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2.1)

a)

$$p_{Y_1}(y_1) = \sum_{y_2 \in Y_2} p_{Y_1,Y_2}(y_1, y_2)$$

Which leaves us with the marginal probability distribution of the father being:

```
father_marginal_distribution <- rowSums(occupation_data)
names(father_marginal_distribution) <- occupation_labels
father_marginal_distribution
```

```
     farm operatives craftsmen sales professional
    0.110      0.279     0.277 0.099        0.235
```

b)

Similarly we have:

$$p_{Y_2}(y_2) = \sum_{y_1 \in Y_1} p_{Y_1,Y_2}(y_1, y_2)$$

Which leaves us with the marginal probability distribution of the son being:

```
son_marginal_distribution <- colSums(occupation_data)
names(son_marginal_distribution) <- occupation_labels
son_marginal_distribution
```

```
     farm operatives craftsmen sales professional
    0.023      0.260     0.240 0.125        0.352
```

c)

The conditional distribution of a son's occupation given that the father is a farmer is:

$$p_{Y_2|Y_1}(y_2|y_1 = \text{farmer}) = \frac{Pr(\{Y_1 = \text{farmer}\} \cap \{Y_2 = y_2\})}{Pr(Y_1 = \text{farmer})}$$

```
occupation_data["farm",]/father_marginal_distribution["farm"]
```

```
      farm   operatives    craftsmen        sales professional
0.16363636   0.31818182   0.28181818   0.07272727   0.16363636
```

d)

The conditional distribution of a father's occupation given that the son is a farmer is:

$$p_{Y_1|Y_2}(y_1|y_2 = \text{farmer}) = \frac{Pr(\{Y_1 = y_1\} \cap \{Y_2 = \text{farmer}\})}{Pr(Y_2 = \text{farmer})}$$

```
occupation_data[,"farm"]/son_marginal_distribution["farm"]
```

```
      farm   operatives    craftsmen        sales professional
0.78260870   0.08695652   0.04347826   0.04347826   0.04347826
```

2.2)

Since the two random variables are independent we have:

a)

$$E[a_1Y_1 + a_2Y_2] = a_1\mu_1 + a_2\mu_2 \quad Var(a_1Y_1 + a_2Y_2) = a_1^2\sigma_1^2 + a_2^2 + \sigma_2^2$$

b)

$$E[a_1Y_1 - a_2Y_2] = a_1\mu_1 - a_2\mu_2 \quad Var(a_1Y_1 - a_2Y_2) = a_1^2\sigma_1^2 + a_2^2 + \sigma_2^2$$