STAT 2201 HW1 Changcheng Ji

Q1. It's generally a circle, while more condensed next to the origin and getting sparse when moving far from origin.

Q2: Yes. consider point $P(x,y,z)$, ~~the~~ s.t. $\|P-O\|=r$, r is given. The points P with the same r (i.e. $x^2+y^2+z^2$) will have same prob. to have the random point. Thus everything would look like a sphere.

HS - Ex 2.2. $|A|=0$, it's impossible that all λ_i of A are positive.
$\det(A)=\prod_{i=1}^{n}\lambda_i=0$, which is contradict to $\lambda_i>0\ \forall i$. $\square$

HS - Ex 2.3 $\lambda_i\neq 0\ \forall i$. $\det(A)=\prod_{i=1}^{n}\lambda_i\neq 0$. $\Rightarrow$ A is nonsingular matrix.
$\Rightarrow$ Thus A is invertible, A^{-1} exists. $\square$

Iz - Ex. 3 20. Calculate $\det(\cdot)$ $(\ )^{-1}$ of $R=\begin{pmatrix}1 & & \rho\\ & \ddots & \\ \rho & & 1\end{pmatrix}$

$$\det(R)=\begin{vmatrix}1 & & \rho\\ & \ddots & \\ \rho & & 1\end{vmatrix}=\begin{vmatrix}1+(n-1)\rho & \cdots & 1+(n-1)\rho\\ \rho & 1 & \vdots\\ \vdots & \ddots & 1\\ \rho & \cdots\ 1 & 1\end{vmatrix}=(1+(n-1)\rho)\begin{vmatrix}1 & 1 & \cdots & 1\\ 0 & 1-\rho & & 0\\ \vdots & & \ddots & \\ 0 & \cdots & & 1-\rho\end{vmatrix}=(1-\rho)^{n-1}(1+(n-1)\rho)$$

$$R^{-1}=\left(\mathrm{diag}(1-\rho,\cdots,1-\rho)+\begin{pmatrix}\rho\\ \vdots\\ \rho\end{pmatrix}\mathbb{1}'_{1\times n}\right)^{-1}=D^{-1}-\frac{1}{1+\mathbb{1}'D^{-1}\cdot\rho\cdot\mathbb{1}}\cdot D^{-1}\cdot\rho\cdot\mathbb{1}\cdot\mathbb{1}'\cdot D^{-1}$$
$$=\mathrm{diag}(1-\rho,\cdots,1-\rho)-\frac{1-\rho}{n\rho}\cdot\frac{\rho}{(1-\rho)^2}\cdot\mathbb{1}_{n\times n}=(1-\rho)I-\frac{1}{n(1-\rho)}J.$$

Q6 Set new matrix(vec) $M=\begin{pmatrix}Y\\ Z\end{pmatrix}=\begin{pmatrix}AX\\ BX\end{pmatrix}$ $\quad Cov(M)=\begin{pmatrix}A\Sigma A' & A\Sigma B'\\ B\Sigma A' & B\Sigma B'\end{pmatrix}$

as. $Y \perp\!\!\!\perp Z$, $A\Sigma B'=0$.

Q7. Set $Y=\begin{pmatrix}X_1\\ X_2\end{pmatrix}$ $Z=(X_3)$. $X=\begin{pmatrix}Y\\ Z\end{pmatrix}\sim N_p\left(\begin{pmatrix}\mu_Y\\ \mu_Z\end{pmatrix},\begin{pmatrix}\Sigma_{YY} & \Sigma_{YZ}\\ \Sigma_{ZY} & \Sigma_{ZZ}\end{pmatrix}\right)$ $\quad \Sigma_{YY}=\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}$ $\quad \Sigma_{YZ}=\cancel{(\rho\ \rho^2)}\ \begin{pmatrix}\rho^2\\ 0\end{pmatrix}$

~~$X_3 = Z \sim$~~ $\begin{pmatrix}X_1\\ X_2\end{pmatrix}\sim\left(\begin{pmatrix}\mu_1+\rho^2(X_3-\mu_3)\\ \mu_2\end{pmatrix},\begin{pmatrix}1-\rho^4 & \rho\\ \rho & 1\end{pmatrix}\right)$

$$=\left(\mu_Y+\Sigma_{YZ}\cdot\Sigma_{ZZ}^{-1}(X_3-\mu_Z),\ \Sigma_{YY}-\Sigma_{YZ}\Sigma_{ZZ}^{-1}\Sigma_{ZY}\right)$$
$$=\left(\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}+\begin{pmatrix}\rho^2\\ 0\end{pmatrix}\cdot 1\cdot(X_3-\mu_3),\ \begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}-\begin{pmatrix}\rho^2\\ 0\end{pmatrix}\cdot 1\cdot(\rho^2\ 0)\right)$$
$$=\left(\begin{pmatrix}\mu_1+\rho^2(X_3-\mu_3)\\ \mu_2\end{pmatrix},\begin{pmatrix}1-\rho^4 & \rho\\ \rho & 1\end{pmatrix}\right)=RHS\quad\square.$$

Stat 2221
Advanced Multivariate Analysis
Spring 2024
Jason Fine

Homework #1
Assigned January 25, 2024.
Due in Canvas be end of day Friday, February 2, 2024.

1. Härdle and Simar Exercise 1.19.
2. Härdle and Simar Exercise 1.20. (You may *imagine* the 3D point cloud, without actually rotating using a computer software.)
3. Härdle and Simar Exercise 2.2.
4. Härdle and Simar Exercise 2.3.
5. Izenman Exercise 3.20. Inverse and det of (A+uv')
6. Show that for $X \sim N_p(\mu, \Sigma)$, non-random matrices $A_{(q\times p)}$ and $B_{(r\times p)}$, $Y = AX$ and $Z = BX$ are independent if and only if $A\Sigma B = 0$.
7. Suppose $X = (X_1, X_2, X_3)'$ has mean $(\mu_1, \mu_2, \mu_3)'$ and covariance matrix

$$\Sigma = \begin{matrix} 1 & \rho & \rho^2 \\ \rho & 1 & 0 \\ \rho^2 & 0 & 1 \end{matrix}.$$

Show that the conditional distribution of $(X_1, X_2)'$ given X_3 has mean $(\mu_1 + \rho^2(X_3 - \mu_3), \mu_2)'$ and covariance matrix

$$\begin{matrix} 1-\rho^4 & \rho \\ \rho & 1 \end{matrix}.$$