A toy model to understand the dynamics of the vortical motions in turbulent boundary layers

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Recent studies indicate that the structure of the turbulent boundary layer at high Reynolds number (Re) is composed of large uniform momentum zones segregated by fissures of concentrated vorticity. Experiments reveal that the dimensionless fissures thickness (scaled by boundary layer thickness) is $\mathcal{O}(1/\sqrt{Re})$ and the dimensionless streamwise velocity jump across a fissure scales with the friction velocity $\mathcal{O}(u_\tau)$. A toy model that captures the essential elements of the turbulent boundary layer structure at high Re is constructed to evaluate the long-time averaged flow statistics of the boundary layer. First, a "master" instantaneous streamwise velocity profile in the wall-normal direction is constructed by placing a discrete number of fissuress across the boundary layer thickness. The number of fissures and their wall-normal locations follow scalings informed by the Mean Momentum Balance (MMB) theory. Next, the wall-normal positions of the fissures are allowed to randomly move in the wall-normal direction creating a statistically independent second instantaneous velocity profile. This process is then repeated to create an ensemble of instantaneous velocity profiles from which average statistics of the turbulent boundary layer can be computed and assessed. The statistics of the toy model are compared to statistics acquired in turbulent boundary layers at high Re.

NUMERICAL METHODS

A stream-wise master velocity profile is represented by a set of discrete steps uniformly spaced according with Eqs. 1 and 2,

$$U^+_{i+1} = U^+_i + \phi_c^2 ln(\phi_c), \tag{1}$$
$$y^+_{i+1} = \phi_c y^+_i. \tag{2}$$

These relationships are derived from the MMB theory [1], where Eq. 1 determine the increments in the stream-wise normalized velocity U^+, the width of the steps in the x coordinate and Eq. 2 determine the increments in the normalized wall normal position y^+, the height of the steps in the y coordinate (See Fig. 1). The initial wall normal position was set to $y_0^+ = 100$ in order to coincide with the boundary edge of the log-layer in the traditional law-wall theory and $U_0^+ = 0.5U_\infty^+$ to be the half of the free-stream velocity. The constant factor ϕ_c is given by $\phi_c = (1+\sqrt{5})/2$ and since the thickness of the vortical fissures scales like $\mathcal{O}(1/\sqrt{Re})$, it is considered negligible for this model. The last position y^+_{i+1} of the vortical fissure and its associated velocity U^+_{i+1} are bounded by the turbulent boundary layer thickness δ or its respective Reynolds number. It is defined like

$$\delta^+ = \frac{\delta}{\nu/u_\tau}, \tag{3}$$

where $u_\tau = \sqrt{\tau_\omega/\rho}$ is the friction velocity (τ_ω is the mean wall shear stress and ρ is the mass density respectively) and ν is the kinematic viscosity. Fig. 1 depicts the step turbulent master profile with a grid of 5481 linearly spaced points in the wall normal direction each one associated to a velocity. The dot circles are the positions and velocities of the vortical fissures computed using Eqs. 2 and 1 respectively. The zones of uniform momentum are created assigning the same velocity of the vortical fissure to the grid points between the previous vortical fissure and the actual vortical fissure. This velocity remains characteristic for each vortical fissure, thus the number of vortical fissures dictates the number of uniform momentum zones.

The instantaneous multiple velocity profiles are created by simulate a random motion in the wall normal direction of the vortical structures. This is accomplished by add a Gaussian perturbation of the actual height of the uniform momentum zone to the current position of the vortical fissure (Black dots in Fig. 1) in the step turbulent master profile. Once the new wall normal positions have

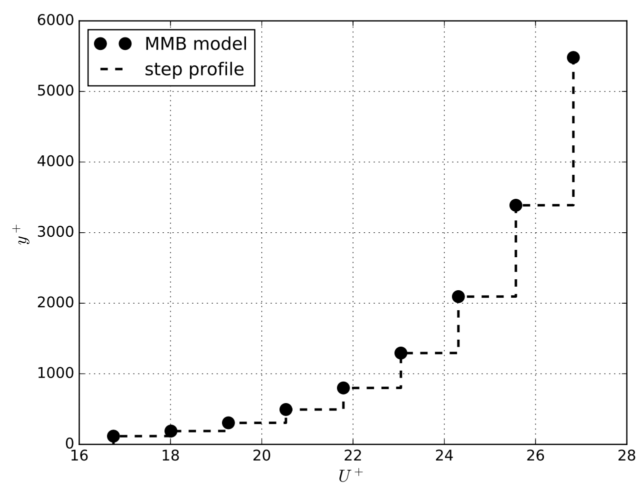

FIG. 1. Step turbulence master profile for $\delta^+ = 5200$ and $U^+_\infty = 26.5$.

been computed, the grid velocity is filled with the attached velocity corresponding to the new positions of the vortical fissures. This allows create velocity profiles with different uniform momentum zones distributions trough the boundary layer. Fig. 2 shows five instantaneous turbulent velocity profiles, it can be seen how the vortical fissures changes their position trough the boundary layer for the different profiles, the time units are arbitrary, i.e. they just illustrate different instants of time. These vortical fissure are allowed to change their positions with their respective associated velocity, this physically means that the vortical fissures can cross each other in the turbulent boundary layer profile generating zones of negative vorticity. The multiple instantaneous velocity profiles are independent each other, thus to create statistically consistency 5000 independent realizations were created and averaged. As a consequence a mean turbulent profile is created (See Fig. 3).

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[1] J. Klewicki, P. Fife, T. Wei, and P. McMurtry, Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences **365**, 823 (2007).

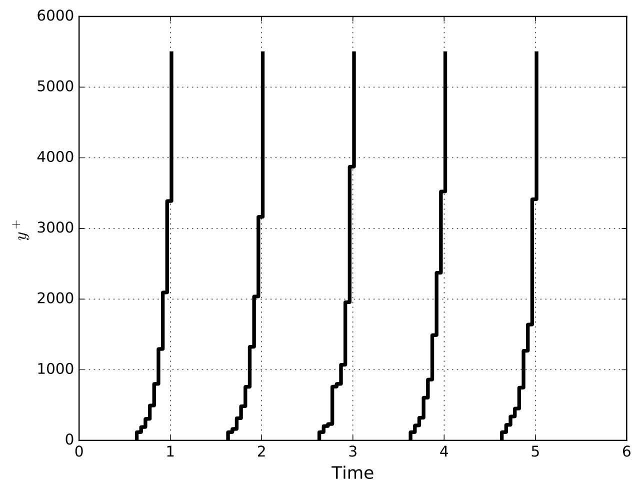

FIG. 2. Multiple instantaneous velocity profiles with a gaussian perturbation of mean $\mu = 0$ and standard deviation $\sigma = 0.4$.

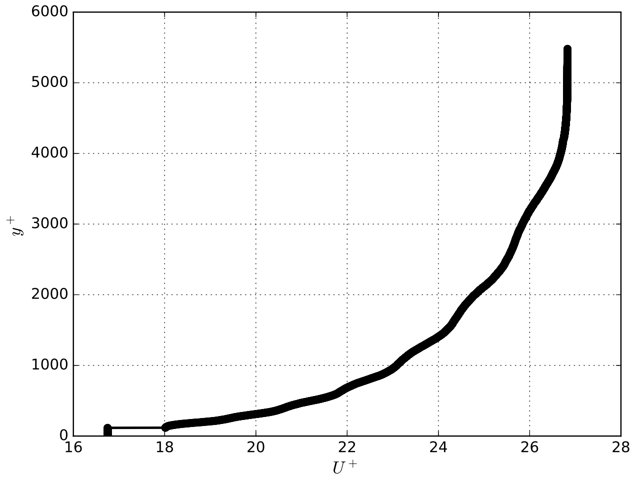

FIG. 3. Mean turbulent stream-wise velocity profile for 5000 independent realizations with a gaussian perturbation of $\mu = 0$ and $\sigma = 0.4$.