A toy model to understand the dynamics of the vortical motions in turbulent boundary layers

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Recent studies indicate that the structure of the turbulent boundary layer at high Reynolds number (Re) is composed of large uniform momentum zones segregated by fissures of concentrated vorticity. Experiments reveal that the dimensionless fissures thickness (scaled by boundary layer thickness) is $\mathcal{O}(1/\sqrt{Re})$ and the dimensionless streamwise velocity jump across a fissure scales with the friction velocity $\mathcal{O}(u_\tau)$. A toy model that captures the essential elements of the turbulent boundary layer structure at high Re is constructed to evaluate the long-time averaged flow statistics of the boundary layer. First, a "master" instantaneous streamwise velocity profile in the wall-normal direction is constructed by placing a discrete number of fissuress across the boundary layer thickness. The number of fissures and their wall-normal locations follow scalings informed by the Mean Momentum Balance (MMB) theory. Next, the wall-normal positions of the fissures are allowed to randomly move in the wall-normal direction creating a statistically independent second instantaneous velocity profile. This process is then repeated to create an ensemble of instantaneous velocity profiles from which average statistics of the turbulent boundary layer can be computed and assessed. The statistics of the toy model are compared to statistics acquired in turbulent boundary layers at high Re.

I. NUMERICAL METHODS

A step master stream-wise velocity profile is represented by a set of discrete steps uniformly spaced according with Eqs. 1 and 2,

$$U_{i+1}^+ = U_i^+ + \phi_c^2 ln(\phi_c), \tag{1}$$

$$y_{i+1}^+ = \phi_c y_i^+. \tag{2}$$

These relationships are derived from the MMB theory [1], where Eq. 1 determine the increments in the stream-wise normalized velocity U^+, the width of the steps in the x coordinate and Eq. 2 determine the increments in the normalized wall normal position y^+, the height of the steps in the y coordinate (See Fig. 1). The constant factor ϕ_c is given by $\phi_c = (1+\sqrt{5})/2$ and since the thickness of the vortical fissures scales like $\mathcal{O}(1/\sqrt{Re})$, it is considered negligible at high Re. The initial wall normal position was set to $y_0^+ = \phi_c\sqrt{\delta+}$ in order to match with the onset of the logarithmic region according with the MMB theory and $U_0^+ = 0.5U_\infty^+$ to be the half of the normalized free-stream velocity U_∞^+.

The last position y_N^+ of the vortical fissure and its associated velocity U_N^+ is bounded by the turbulent boundary layer thickness δ or its respective Reynolds number $\delta^+ = \frac{\delta}{\nu/u_\tau}$, where $u_\tau = \sqrt{\tau_\omega/\rho}$ is the friction velocity (τ_ω is the mean wall shear stress and ρ is the mass density respectively) and ν is the kinematic viscosity. Fig. 1 depicts the step master turbulent velocity profile with a grid of 5481 linearly spaced positions in the wall normal direction each one associated to a streamwise velocity. The dot circles are the positions and velocities of the vortical fissures computed using Eqs. 2 and 1 respectively. The zones of uniform momentum are created allocating the same velocity of the vortical fissure to the grid points between the previous vortical fissure and the current vortical fissure. This velocity remains characteristic for each vortical fissure, thus the number of vortical fissures establishes the number of uniform momentum zones.

Then instantaneous multiple velocity profiles are created by simulate a random motion in the wall normal direction of the vortical structures. This is accomplished by add a Gaussian perturbation of the actual height of the uniform momentum zone to the current position of the vortical fissure (black dots in Fig. 1) in the step turbulent master profile. Once the new wall normal positions

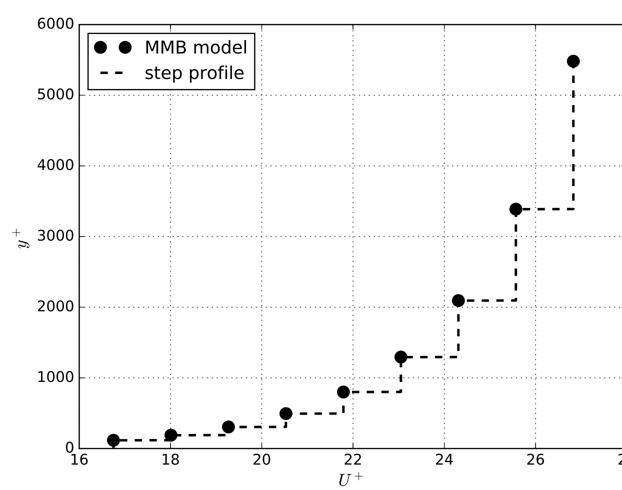

FIG. 1. Step turbulence master profile for $\delta^+ = 5200$ and $U_\infty^+ = 26.5$.

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have been computed, the grid velocity is filled with the attached velocity of each vortical fissure corresponding to their new wall normal positions. This physically means that the vortical fissures can cross each other through the turbulent boundary layer step profile generating zones of negative vorticity. Fig. 2 shows five instantaneous turbulent velocity step profiles, it can be seen how the vortical fissures changes their position trough the turbulent boundary layer for the different profiles. The time units in Fig. 2 are arbitrary, i.e. they just illustrate different instants of time. The multiple instantaneous step velocity profiles are considered independent events since they are the result of the Gaussian perturbation in the wall normal positions of the master profile. Thus to achieve statistical convergence the mean turbulent stream-wise velocity profile is constructed by average over 5000 independent realizations.

II. MEAN TURBULENT STATISTICS ANALYSIS

To prove the accurateness of our model, the high order moments such as the mean, variance, skewness and kurtosis of the mean turbulent streamwise velocity profile were computed. Like it was described in I, not only a Gaussian perturbation was used to randomize the positions of the vortical fissure but also an uniform distribution. The main purpose was explore the relationship between the perturbation distribution and the statistics of our model. This could give us some insight about how this process occurs in a real turbulent flow.

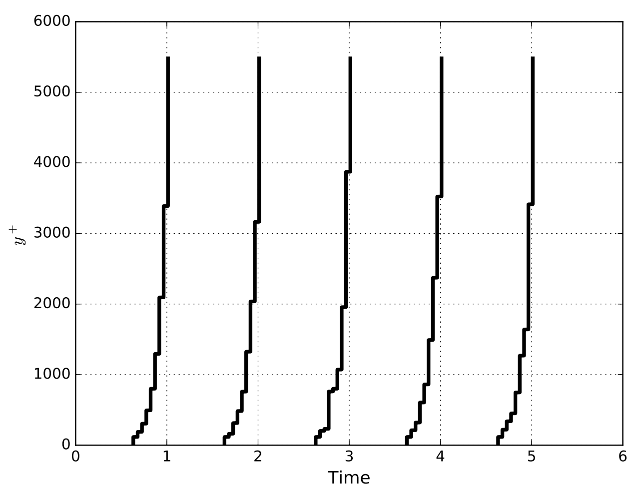

FIG. 2. Multiple instantaneous velocity profiles with a gaussian perturbation of mean $\mu = 0$ and standard deviation $\sigma = 0.4$.

A. Gaussian perturbation

Two scenarios for the Gaussian perturbation were considered, for $\sigma = 0.4$ and $\sigma = 1$ with $\mu = 0$ respectively. First scenario considers that the random variation in the position of the vortical fissures ranges between -120% and 120% of their current positions, being 3σ events less likely. Fig 3).

B. Uniform distribution

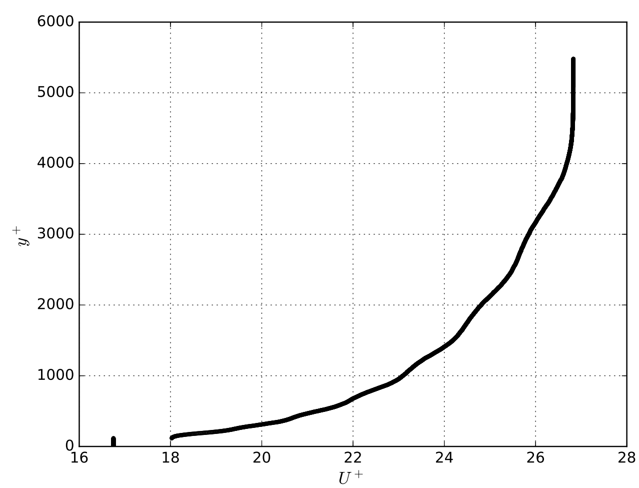

FIG. 3. Mean turbulent stream-wise velocity profile for 5000 independent realizations with a Gaussian perturbation of $\mu = 0$ and $\sigma = 0.4$.

[1] J. Klewicki, P. Fife, T. Wei, and P. McMurtry, Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences **365**,

823 (2007).

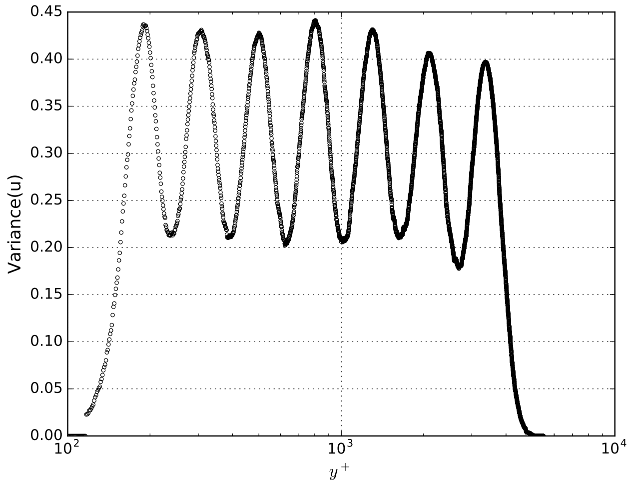

FIG. 4. Mean turbulent stream-wise velocity profile for 5000 independent realizations with a Gaussian perturbation of $\mu = 0$ and $\sigma = 0.4$.

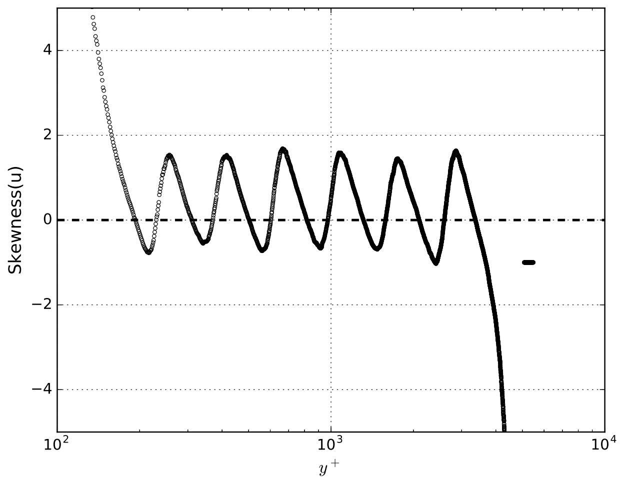

FIG. 5. Mean turbulent stream-wise velocity profile for 5000 independent realizations with a Gaussian perturbation of $\mu = 0$ and $\sigma = 0.4$.

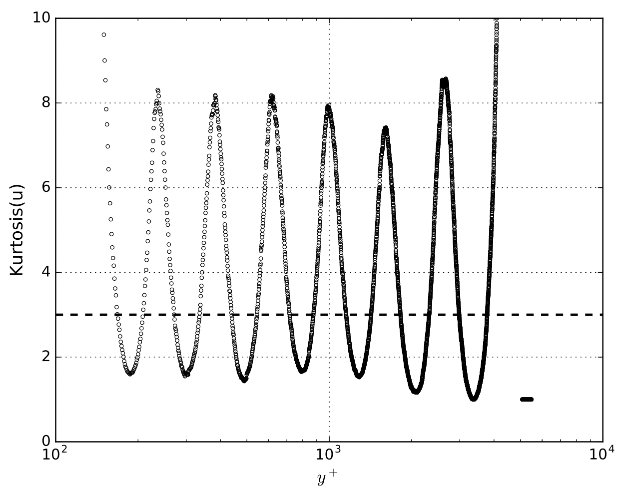

FIG. 6. Mean turbulent stream-wise velocity profile for 5000 independent realizations with a Gaussian perturbation of $\mu = 0$ and $\sigma = 0.4$.