A toy model to understand the dynamics of the vortical motions in turbulent boundary layers

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Recent studies indicate that the structure of the turbulent boundary layer at high Reynolds number (Re) is composed of large uniform momentum zones segregated by fissures of concentrated vorticity. Experiments reveal that the dimensionless fissures thickness (scaled by boundary layer thickness) is $\mathcal{O}(1/\sqrt{Re})$ and the dimensionless streamwise velocity jump across a fissure scales with the friction velocity $\mathcal{O}(u_\tau)$. A toy model that captures the essential elements of the turbulent boundary layer structure at high Re is constructed to evaluate the long-time averaged flow statistics of the boundary layer. First, a "master" instantaneous streamwise velocity profile in the wall-normal direction is constructed by placing a discrete number of fissuress across the boundary layer thickness. The number of fissures and their wall-normal locations follow scalings informed by the Mean Momentum Balance (MMB) theory. Next, the wall-normal positions of the fissures are allowed to randomly move in the wall-normal direction creating a statistically independent second instantaneous velocity profile. This process is then repeated to create an ensemble of instantaneous velocity profiles from which average statistics of the turbulent boundary layer can be computed and assessed. The statistics of the toy model are compared to statistics acquired in turbulent boundary layers at high Re.

NUMERICAL METHODS

A stream-wise master velocity profile is represented by a set of discrete steps, each one with an evenly distributed width of U_i^+ units according with Eq. 1 and a height of y_i^+ units according with Eq. 2. Since the thickness of the vortical fissures is $\mathcal{O}(1/\sqrt{Re})$, it is considered negligible in this model. Eqs. 1 and 2 are derived from the MMB theory [1]. Table I, summarize the wall normal positions and the speed increments in the four-layer structure given by[1]. These increments can be also obtained by the recurrence formulas

$$U_{i+1}^+ = U_i^+ + \phi_c^2 ln(\phi_c), \tag{1}$$
$$y_{i+1}^+ = \phi_c y_i^+, \tag{2}$$

where $\phi_c = (1+\sqrt{5})/2$ is a constant factor while the initial wall normal position and stream-wise normalized velocity have been set to $y_0^+ = 3.5$ and $U_0^+ = 0.5U_\infty^+$ respectively. In this paper different free-stream normalized velocities U_∞^+ are used. Using the information described above a master profile is constructed, the input parameters are the Reynolds number associated with the boundary layer thickness

$$\delta^+ = \frac{\delta}{\nu/u_\tau}, \tag{3}$$

where $u_\tau = \sqrt{\tau_\omega/\rho}$ is the friction velocity (τ_ω is the mean wall shear stress and ρ is the mass density respectively) and U_∞^+. Fig. 1 depicts the step turbulent master profile with a grid of 5481 linearly spaced points in the wall normal direction each one associated to a velocity. The dot circles are the positions of the vortical fissures computed using Eqs. 2 and 1 respectively. The zones of uniform momentum are created assigning the same velocity of the vortical fissure to the grid points spanned between previous vortical fissure and the actual vortical fissure. Thus

TABLE I. Scaling associated to four layer structure from MMB theory. Δy represents the wall normal position of the vortical fissure in the turbulent boundary layer while Δu represents the velocity associated to each layer.

Layer	Δy	Δu
I	$\mathcal{O}(\nu/u_\tau)(\lesssim 3)$	$\mathcal{O}(u_\tau)(\lesssim 3)$
II	$\mathcal{O}(\sqrt{\nu\delta/u_\tau})(\sim 1.6)$	$\mathcal{O}(U_\infty)(\sim 0.5)$
III	$\mathcal{O}(\sqrt{\nu\delta/u_\tau})(\sim 1.0)$	$\mathcal{O}(u_\tau)(\sim 1)$
IV	$\mathcal{O}(\delta)(\to 3)$	$\mathcal{O}(U_\infty)(\to 0.5)$

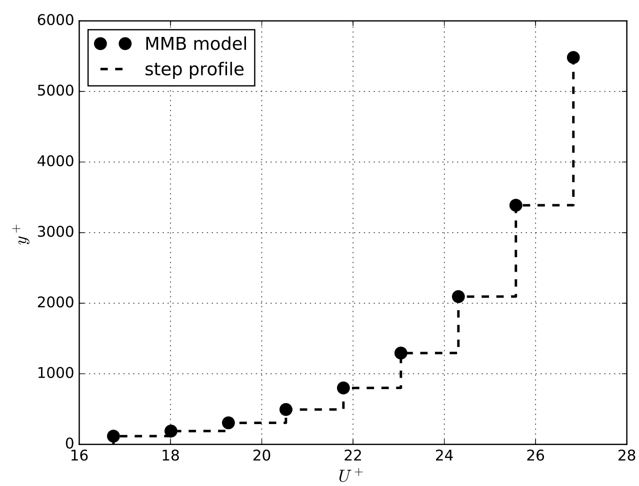

FIG. 1. Step turbulence master profile for $\delta^+ = 5200$ and $U_\infty^+ = 26.5$.

the number of vortical fissures dictates the number of uniform momentum zones. The instantaneous multiple velocity profiles are created by add a gaussian perturbation of the actual height to the positions of the vortical fissures (Black dots in Fig. 1) in the step turbulent master profile. Once the new wall normal positions for the vortical fissures are obtained the grid is filled with uniform momentum zones similarly to the process described above. Fig. 2 shows five instantaneous turbulent velocity profiles, it can be seen how the vortical fissures changes their position trough the boundary layer for the different profiles, the time units are arbitrary, i.e. they just illustrate different instants of time. These vortical fissure are allowed to change their positions with their respective associated velocity, this physically means that the vortical fissures can cross each other in the turbulent boundary layer profile generating zones of negative vorticity. The multiple instantaneous velocity profiles are independent each other, thus to create statistically consistency 5000 independent realizations were created and averaged. As a consequence a mean turbulent profile is created (See Fig. 3).

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[1] J. Klewicki, P. Fife, T. Wei, and P. McMurtry, Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences **365**, 823 (2007).

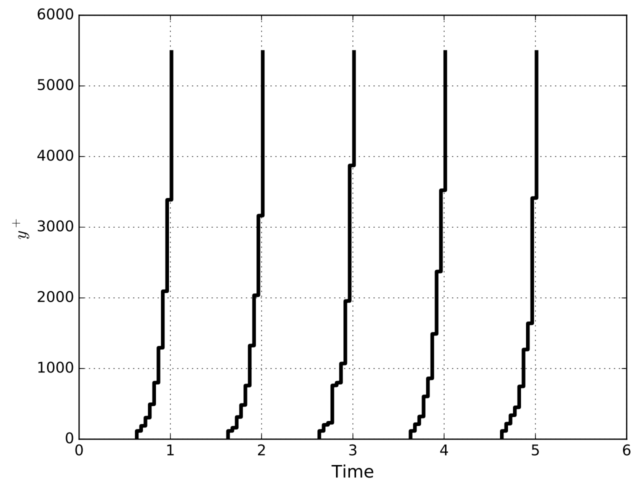

FIG. 2. Multiple instantaneous velocity profiles with a gaussian perturbation of mean $\mu = 0$ and standard deviation $\sigma = 0.4$.

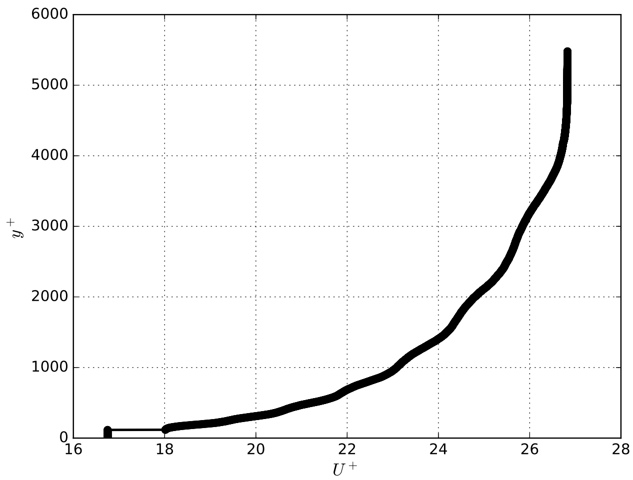

FIG. 3. Mean turbulent stream-wise velocity profile for 5000 independent realizations with a gaussian perturbation of $\mu = 0$ and $\sigma = 0.4$.