Using spin glass models to simulate linguistic change in speech communities

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Abstract

In the 1700s Grimms law suggested that a primary vehicle for language change was sound change, a process which occurs across a linguistic community with great regularity. When a sound changes in a given phonological triggering environment, all occurrences of that sound in that environment change across the language. For centuries it was speculated that sound changes such as these occur randomly and take hold across a linguistic community via unknown and perhaps unobservable mechanisms.

It was shown in the 1960s by William Labov that sound change proliferates in the context of social pressures and can occur over the time-period of only a couple generations. In his 1963 work "On the Mechanism of Linguistic Change" he introduces two landmark datasets showing the development of two sound changes, one on Martha's Vineyard and one in New York City between the 1930s and 1960s.

Labov's work showed that, while linguistic change is primarily motivated by arbitrary acts of phonological change, community-level adoption of novel variants only occurs within the context of some social pressure to either assimilate or dissimilate to status quo pronunciations.

While Labov's works studied linguistic change from the top-down, cataloguing the changes as they occurred, aggregating results according to demography and results of sociological surveys, we attempt to model linguistic change as the product of individual, randomized interactions, with the hope that bulk properties identical to those observed by Labov will arise out of sufficiently robust models.

We take spin glasses as the basis of our model. A spin glass is a representation of an N particle system in which every particle takes on a binary spin value -1 or +1. The particles interact with one another to influence one another's spin. If two particles interact ferromagnetically then it will decrease their energy for them to align spins with one another, otherwise they are antiferromagnetic and they will disalign spins to minimize energy. Particles in a spin glass are also often subject to an external field, where the external field biases particles to take on either spin +1 or -1 according to the direction of the field. A spin glass is typefied by interactions between particles being nonuniform across the system. A spin glass is specified by its Hamiltonian H. Where H is a function that assigns energy values to particular configurations of the N particles according to the energies introduced via the particle-by-particle interactions and the energy introduced by each particle's interaction with the external field. In general, spin glass models will have Hamiltonian which involves sums of interactions of individual spins (either ferromagnetic or antiferromagnetic according to each unique pair i,j) and interactions of individual spins with the field. The system can then be studied by studying both the time dynamics of the

system as the system attempts to find lower energy configurations from some initial conditions, and the ground state (i.e. lowest energy state) of the system. The beauty of the mathematics of spin glass modeling is that bulk properties, such as the expected spin of individual particles or the sum of all spins arise solely out of a simple description out of how pairs of individual particles interact.

Spin glasses are ideal models for linguistic change in this sense, because we can accomplish Labov's task of predicting bulk properties but do so by modeling only individual members of a speech community and their attitudes towards one another and towards the status quo. We will develop a number of Hamiltonians in this paper all of which, in different capacities, encode the following 5 assumptions about sound change

1. Sound change, on an *individual level*, occurs via either accommodation or via dissimilation due to *interactions* between individuals in the community as a whole
2. Individuals have attitudes towards members of their in-group and members of out-groups which help decide the extent to which they either accommodate or dissimilate upon contact with out-groups
3. New group membership can also emerge spontaneously as a result of linguistic change, leading the influence of group identity to be fluid and influenced by sound change itself
4. There is some general linguistic environment which establishes what is considered "prestige", "orthodox", or "correct' pronunciation and individuals have some pressure to conform with that environment
5. The "orthodox" pronunciation can change when a high prestige group undergoes sound change

and we will show that from simply modeling the language community as an ensemble of individuals each of which interact with one another according to the assumptions above, we will observe several bulk phenomena which confirm some of Labov's key results but also challenge his assumptions regarding the key motivating social factors underlying sound change.

1 Linguistic mechanisms of sound change

In the 1700s Grimms law suggested that a primary vehicle for language change was sound change, a process which occurs across a linguistic community with great regularity. When a sound changes in a given triggering environment, all occurrences of that sound in that environment change across the language. For centuries it was speculated that sound changes such as these occur randomly and take hold across a linguistic community via unknown and perhaps unobservable mechanisms.

It was shown in the 1960s by William Labov that sound change proliferates in the context of social pressures and can occur over the time-period of only a couple generations. In his 1963 work "On the Mechanism of Linguistic Change" he introduces two landmark datasets showing the development of two sound changes, one on Martha's Vineyard and one in New York City between the 1930s and 1960s.

They stand together as fascinating complementary studies because they showcase the evolution of sound change within ethnic and socioeconomic subgroups at distinct phases of their evolution and which both show the influence of in-group/out-group processing but in different ways.

The Martha's Vineyard study showcased in depth study of a small linguistic community of native Vineyarders. A sound change arose between 1930 and 1960 which caused centralization of certain vowels. It was found that the sound change arose amongst white Yankee fishermen on the upland part of Martha's Vineyard and became a salient and well-regarded marker of in-group identitiy as their maritime lifestyle became threatened by tourism and declining fishing stocks. It was most vigorously adopted by those amongst the young community who saw themselves as wanting to return to Martha's Vineyard to raise their families. Furthermore, though it initially began amongst multi-generational white fisherman, it broke through ethnic barriers to third and fourth generation Portuguese immigrants as well as native Wampanoag all of whom considered themselves alike the Yankees in their solidarity against the threats to their economic well-being posed by influence from the mainland. Adoption of the sound change was more highly correlated with attitudes towards the island than it was with any ethnic distinction. Even more importantly, it arose as a part of a community of relative prestige on the island but within the context of out-group dissimilation with the contemporary linguistic trends on the mainland. In fact, those who had worked or received an education on the mainland previously and planned to return to the mainland had nearly no adoption of the sound change whatsoever, presumable because they had no negative attitudes towards Boston English and therefore very little reason to reactively dissimilate in order to mark their identity as a Vineyarder.

The New York City case tracked the development of the /*aw*/ vowel from the lower vowel /*a*/ in words like "law" and "caught". It became most notable adopted by lower middle class Jews and Italians. What was incredibly notable about this sound change is that it likely began as a form of hypercorrection by second-generation Jewish and Italian immigrants to adjust for atypical pronunciations their parents inherited from Yiddish and Italian respectively. Because it was adopted by a lower prestige group it was became stigmatized and developed registerial variation. In particular, those who used it frequently in casual speech used it in less in formal speech than any other group. Furthermore, those who used it in casual speech stigmatized it more than any other group. The reason Labov found for this was that the lower middle class Italians and Jews who participated in the sound change were actually completely unaware that they used it in casual speech and actually heard themselves as using the prestige variant. Thus, in this study we see a contrast in which a minority group developed a non-prestige linguistic innovation that became widespread below the threshold of consciousness for most casual speakers. It is unclear, based on Labov's data alone, why it became increasingly common despite it being stigmatized by those who used it, though Labov believes there is evidence that a clustering effect regarding group membership, friendship, kinship, etc. caused it to proliferate amongst the low prestige community even though they recognize it as being low prestige they didn't believe they personally participated in. It is important to note that in this study the pronunciation didn't cross racial lines to non-whites and that it didn't penetrate the ranks of either the very poor or the upper middle class.

An important conclusion here is that linguistic innovation can either be subconscious or conscious and can occur in the context of social pressure by prestige groups stigmatizing the change or can occur within minority groups that explicitly want to identify themselves as an in-group to some majority's out-group.

In his conclusion to the work described above, Labov outlines a 13 step mechanism for sound change viewed through the lens of the social dynamics. In particular, he studies the emergence of innovative sound changes which is traditionally viewed as a contrasting and inverse process as that by which dialects become more similar to one another (e.g. when a group migrates to a different region and loses characteristics of their prior vernacular).

Lightly paraphrased and shortened for brevity are the core steps of Labov's sound change.

1. **Sound change originates with a restricted sub-group of the community, at a time when the separate identity is weakened by social pressures. Typically the linguistic form which is the source of the wide-scale sound change is an irregularly distributed form which is stereotypically a marker of in-group status**

2. **Succeeding generations of the linguistic group responding to the same social pressure carry the linguistic variable further along the process of change, beyond the model set by their parents. The variable is now defined as a marker both of group membership and generation**

3. The sound change spreads from one sub-group to all other members who identify with the same value group (this could be based on economic or social class, occupation, geographical region, race, religion, or some other basis of shared identity) and the linguistic variable becomes a normative marker of the group

4. Other subgroups which enter into the speech community in the interim adopt the older sound change and treat the newer sound change as a catalyst for stage 1, this creates a process of recycling within the community.

5. Eventually, if the change began with a low status group, the change is recognized as being stereotypical and is stigmatized through the prestige group's control of institutions.

6. Speakers who casually use the changed value begin another phase of hypercorrection in which performative and careful speech prompts them to use the *prestige* form and they begin to believe that they actually use the *prestige* form in all speaking environments

7. Under extreme stigmatization the form may become to overt topic of social comment and disappear entirely.

8. **If the change originated with a prestige group it may become a model for all members of the speech community and may become adopted by all other groups in proportion to their contact with users of the prestige model**

There is an important note I'd like to make about Labov's axioms. Primarily, Labov's model suggests that in-group *affinity* is the primary driver of linguistic change. Secondly, Labov's model doesn't really give an account of pure linguistic innovation. In his first axiom he implicitly assumes that the emergence of a novel form uses an already irregularly distributed linguistic variable as its nucleus. In the case of the New York sound change it arose out of novel pronunciations introduced by immigration from Italian and Yiddish native speakers. Finally he leaves the circumstances under which language spread from the minority prestige group to the highly unspecified.

The results in this paper will challenge the assumption that in-group affinity is a primary catalyst for change. It will further suggest that linguistic innovation *can* happen in a vacuum but will highlight that Labov's observed dynamics/predictions heavily rely on his assumption that language change is initiated by contact with new groups and doesn't give a true account of spontaneous innovation. Finally, my results will show that spread from one group to another is difficult to simulate *in vitro*. It is very likely that such spread only occurs under complex population dynamics where social groups and their relations change over time or under circumstances in which confounding linguistic variables and contact with new groups triggers diffusion of sound change from minority groups to an entire community.

Our modeling will focus primarily on the bolded axioms above. It will do so in a way that focuses on what we know about the relationship between sound change and *individual* interactions and points of view and will explore mechanisms by which those individual preferences result in bulk phenomena characteristic of language change. The complex inter-generational, time-based population dynamics interactions described in step 4 are beyond the scope of this paper. Furthermore, complex interaction which may arise due to social mobility or geographic migrations will also be ignored in this paper. We will also be ignoring the phonological reality that sound change along one variable may be influenced by hypercorrection of another variable. We will consider our variables to be binary and not admit the possibility that there could be multiple phonological variables which confound one another. Though as noted above, our results show that Labov's axioms are likely underdescriptive. In order to elucidate the true mechanisms of linguistic change we would need a much more robust understanding of how populations change, social strata change, and languages make contact with one another.

For examples of how phonological variables can confound one another see chain shifts or more specifically the Great Vowel Shift.

Later researchers (Auer and Kerswili, 2005) have elaborated on the role individual attitudes impact sound change by showing that personal changes over the course of individual's lifetime occur due to either accommodation to stereotyped idealizations of the speech patterns of those they interact with or dissimilation from the stereotyped idealizations of those they interact with according to speakers' attitudes towards members of their immediate community. Thus individuals will lose their dialect if they move into a new community towards which they have a positive attitude but they may differentiate further if they have a negative view of their new environment. This vehicle for language change on the timescale of an individual's life is known as *change-by-accommodation* and acts as a function of speaker attitudes towards linguistic communities they interact with (whether they be in-groups or out-groups).

Furthermore, people may even change their idea of what group they belong to on the basis of some newly discovered shared value or on their belief that they are part of a common linguistic community, this is certainly the case among transplants who begin adopting new regional dialects and then eventually consider themselves members of the linguistic in-group defined by the dialect of their new home. Finally, much more recent work has found that during daily-interactions individuals also show differing levels of plasticity in their pronunciations which toggles their receptiveness to medium-term and long-term acquisition of sound changes (Sonderegger et al 2017).

1.1 Scope of this work

We would like to design computational models that achieve the same population dynamics as those defined by Labov's mechanisms for sound change, taking into account some of the newer developments mentioned above.

We will focus our work specifically on linguistic *innovation* as opposed to dialect leveling (i.e. the loss of a dialect in the presence of a high prestige majority group, which Auer and Kerswili focus on). That is, we will study the kind of sound change detailed by Labov, in which a community, over the course of several generations, develops a new sound change contrary to the speech patterns of some other linguistic group. We will distill the above discussion into several key axioms:

1. Sound change, on an *individual level*, occurs via either accommodation or via dissimilation due to *interactions* between individuals in the community as a whole

2. Individuals have attitudes towards members of their in-group and members of out-groups which help decide the extent to which they either accommodate or dissimilate upon contact with out-groups

3. New group membership can also emerge spontaneously as a result of

linguistic change, leading the influence of group identity to be fluid and influenced by sound change itself

4. There is some general linguistic environment which establishes what is considered "prestige", "orthodox", or "correct' pronunciation and individuals have some pressure to conform with that environment

5. The "orthodox" pronunciation can change when a high prestige group undergoes sound change

It is important to note that this represents an incredibly simple model of language dynamics in vitro. It focuses its expression solely on the attitudes of individuals and considers the social dynamics of society to be more or less fixed.

2 Spin glasses as models of many-body interactions

Spin systems were developed as a way of modeling energy interactions between magnetic particles in a physical systems. In a spin system, N particles are represented as having either spin $\{+1, -1\}$. In physical systems, spin can represent magnetic orientation of an individual particle. Thus it is interesting to study the relationship between typical values of bulk magnetization (sum of magnetization over all particles) and the ways in which individual particles interact.

Each assignment of individual spins is known as a configuration. In an N-particle system there are 2^N possible configurations. We can generate interesting models of interaction by defining an energy function associated with each configuration, known as a Hamiltonian, written $H(\sigma)$.

In physical systems, the probability of a given configuration is equal to

$P(\sigma) = \frac{e^{-\beta H(\sigma)}}{Z}$

where β is $1/T$ and T is the temperature of the system. Z is a normalization constant required for the probabilities to make up a valid probability distribution, it is known as the partition function and is studied in its own right due to the many interesting properties derivable from it.

A standard form for $H(\sigma)$ is

$H(\sigma) = -\sum_{(i,j)\in E} J_{i,j}\sigma_i\sigma_j - h_i \sum_i \sigma_i$

Where E defines the set of which particles interact with which. The simplest spin model is the 1D Ising model, in which the only edges in E are

$(i,\ i+1)$. This model corresponds to a line of particles which interact only with their immediate neighbors.

In the above Hamiltonian the parameter $J_{i,j}$ is called the "interaction" parameter. If all $J_{i,j} = J > 0$ then interactions are considered ferromagnetic and spins tend to align with one another. If all $J_{i,j} = J < 0$ interactions are antiferromagnetic and spins desire to disalign with their neighbors. In general $J_{i,j}$ can be all different values (taking on either ferromagnetic or antiferromagnetic values on a by-particle basis). If $J_{i,j}$ are all possibly distinct values then we call our model a spin glass.

The term h_i is called the "external field" parameter and represents the influence of an external magnetic field. Typically h_i are all equal to h, though in general one could imagine very complicated external fields which vary on by-particle basis or perhaps even are a function of other attributes of the system. If h is positive then particles will be biased towards spin 1, otherwise they will be biased towards spin -1.

For any given system, defined by its Hamiltonian, there are a number of ground-states and expected values of each particle (also known as the magnetization of the particle) which can potentially be random variables if the $J_{i,j}$ or h_i are random variables.

In typical studies on spin-glasses, the Hamiltonian itself is the only thing parameterizing the numerical model under investigation and one proceeds directly to deriving resultant properties from the Hamiltonian alone (e.g. magnetization of each particle, bulk magnetization, magnetization of different clusters, free energy density, phase transitions, etc.).

We will be studying our systems via time-series processes called Markov-chain Monte Carlos and will therefore devote some interest to the effects that different spin-update policies have on the time-evolution of our system.

We will thus introduce both a number of different Hamiltonians and two different algorithms for finding minimal energy states. We will observe the time evolution of each model under numerical simulation and see how well it achieves the predictions linguistic theory makes about sound change in different communities.

We will describe our method of representation in the next section and describe how it may be used to accomplish the predictions of sound change dynamics laid out above.

3 Linguistic communities as spin glasses

We will consider group dynamics only within populations with a single minority community A and a majority community B. Consider a population of size N with minority community fraction $\alpha \in [0, .5)$. We will refer to this community as community A and represent it as the first αN bits in a configuration vector $\sigma \in \{-1, 1\}^N$. The remaining $N - \alpha N$ members of the majority community, community B, will be represented by the $N - \alpha N$ bits of the configuration σ. In general, we are agnostic as to whether spins represent an individual for a single generation or perhaps just an abstract individual who is perhaps not unique over generations. Furthermore, the relationship between time steps in our algorithm and time in the real world is not intended to have any deterministic correspondence.

In this model, we will represent some abstract sound change that takes on binary values. People either say 1 or they say the alternative expression, -1. A concrete example would be the vowel shift in NY dialects of English where the /o/ in dog is pronounced /aw/ as opposed to the more common /a/. We would represent individuals who take the unmarked pronunciation with value (also known as spin) +1 and individuals who take the NY variant with spin -1.

Each model we will consider will have interaction parameters $J_{A,B}$, $J_{A,A}$, $J_{B,B}$ and $J_{B,A}$ where each $J_{X,Y}$ is a distribution from which interaction parameters between individuals of groups X and Y will be drawn. We will assign each individual i a plasticity p_i which determines their susceptibility to sound change. Finally, all our models will have an external field h_i which indicates each individual's influence to conform to some external "prestige" or "orthodox" pronunciation. For each of our basic models of group dynamics we will also have two alternatives, one in which the fixed groups are the only interaction parameters, and another in which there are interaction parameters between individuals who have the same alignment. We will also introduce two variants of prestige influence, one in which the external field is a constant positive constant h and another in which h is equal to the average magnetization of the "prestige" group (either group A or B). Thus for each of the two basic models of group dynamics there will be a variant with only the fixed groups and a variant both with fixed group membership and with linguistic group membership and for each of those two variants a version with fixed external field and one with dynamic field.

We will perform our time dynamics simulation via the Markov chain Monte carlo method known as simulated annealing.

Simulated annealing
Carry out the following simulated annealing algorithm where $\beta(t)$ is updated such that the initial temperature is decreased continuously to zero at the end of each epoch.
Choose τ to be some constant $O(1)$

1. first initialize the population to be configuration $\sigma_0 = \{1\}^N$.
2. for epoch $t \in [0, t_{max}]$ do
   (a) Perform $\tau \cdot N$ successive Glauber updates with transition probability
   $P(\sigma_{t+1}) = p_i \cdot min(exp\{-\beta(t)[H(\sigma_{t'}) - H(\sigma_t)]\}, 1)$
   where t' is got by randomly choosing and index i and flipping its value.

We will chart the relation between epochs **T** ad the fraction of each group A and group B taking on value -1. We will call these fractions X_A and X_B - they are equivalent to the average magnetization of each group.

We will call the event that over half of a group takes on a spin -1 for epochs $>$ **T**, for some **T** a "sound change", as it represents an enduring change in the polarity of that group with respect to the abstract language feature whose distribution is being modeled here.

For various values of α and for each of the 3 Hamiltonians described below, I will run the two MCMC algorithms above and measure three quantities per epoch: the fraction of community A with spin -1, the fraction of community B with spin -1, and the overall fraction with spin -1, resp $\chi_A, \chi_B, \chi_{pop}$ (χ_* as shorthand for all of them).

I will now introduce our basic Hamiltonian, the variant we will use to model sound change originating in a prestige group, and the variant we will use to model the way social dynamics may themselves be a reflex of sound change.

Our experiments will include those with just the simple antagonistic model, those with the antagonistic model + fluctuating institutional pressure, and those with both fluctuating institutional pressure and a term to model social dynamics evolving under as a reflex of sound change. Thus we will have three primary models of increasing complexity.

3.1 Simple antagonistic model with constant institutional pressure

The Hamiltonian of a configuration is

$$\begin{aligned} H(\sigma) = \frac{1}{N} \cdot [&-(J_{A,A} \cdot \sum_{i,j \in A} \sigma_i \cdot \sigma_j + J_{B,B} \cdot \sum_{i,j \in B} \sigma_i \cdot \sigma_j) \\ &+ (J_{A,B} \cdot \sum_{i \in A \ j \in B} \sigma_i \cdot \sigma_j + J_{B,A} \cdot \sum_{i \in B \ j \in A} \sigma_i \cdot \sigma_j) \\ &- h \cdot \sum_i \sigma_i] \end{aligned}$$

Where each (i, j) interaction is drawn from one of the following distributions as appropriate:

$J_{A,A} \sim J_{B,B} \sim \mathcal{N}(\frac{c_1}{N}, \frac{1}{N})$

for $c_1 > 0$ indicating on average, ferromagnetic intra-community interactions.

$J_{A,B} \sim J_{B,A} \sim \mathcal{N}(\frac{c_2}{N}, \frac{1}{N})$

for $c_2 > 0$ indicating on average, anti-ferromagnetic inter-community interactions.

Note that this model features a constant external field term $h > 0$ representing institutional pressure to tend towards the more “orthodox” or “prestige” spin of 1.

3.1.1 With dynamic external field

We will also attempt to model the effects of sound change in a prestige minority group on the linguistic community as a whole by permitting h to vary according to X_A by setting $h = -1 \cdot \chi_{prestige} + 1 \cdot (1 - \chi_{prestige}) = 1 - 2 \cdot \chi_{prestige}$ where the prestige population is set to minority group A.

3.1.2 With fluid linguistic in-groups

We will also attempt a simple model of social dynamics by having an additional interaction term that biases individuals to ferromagnetically interact with individuals who had the same spin as they did last epoch and to interact antiferromagnetically with those who had a different spin (this is analogous to us having social-group-based attitudes with others in addition to attitudes based on perceived linguistic similarity).

The Hamiltonian is identical to the one above except that it has an interaction parameter J_{in} for particles with like-spin and J_{out} for particles with unlike-spin.

This gives the additional term:

$$\sum_{i,j} \mathbb{1}_{\sigma_i \neq \sigma_j}(J_{out}\sigma_i\sigma_j) - \mathbb{1}_{\sigma_i = \sigma_j}(J_{in}\sigma_i\sigma_j)$$

where $\mathbb{1}_{\sigma_i = \sigma_j}$ is the indicator function that $\sigma_i = \sigma_j$ and $\mathbb{1}_{\sigma_i = \sigma_j}$ is the indicator for $\sigma_i \neq \sigma_j$

and where each in-group and out-group interaction parameter for i, j is drawn from $J_{in} \sim J_{out} \sim \mathcal{N}(\frac{c_3}{N}, \frac{1}{N})$ for positive c_3, indicating ferromagnetic behavior for spins which were equal in the previous epoch and antiferromagnetic between pairs which were not.

3.2 Correspondence to linguistic model

The first, simplest model, appeals to the traditional belief that our speech is derived from accommodation to those we have positive attitudes towards and dissimilation from those we have negative attitudes towards. This model will show that that assumption alone is sufficient for language change to occur spontaneously sweep across a minority group in a community with two antagonistic and unequally sized subgroups. Our second model models Labov's prediction that if the minority group is a prestige group then language change may occur across the whole population, a prediction which we show is incorrect absent some more substantive corollary explanations. Our third model attempts to correct the shortcomings of the second by positing that language change can spread across the whole population if language change itself, and the chauvinism surrounding it, reshapes individual-to-individual attitudes. Our results show that this holds true only under a very narrow range of parameters and that it is likely we'd need wholesale restructuring of the social hierarchy for population-wide language change to be observable.

4 Results

We report three substantive results from our numerical simulation. In each simulation we used value $\tau = .3$ and $N = 300$. Labov's axiom that sound change occurs in the minority group was confirmed in every simulation run during this experiment. All line graphs in this section display epochs, or iterations of the outer loop of the MCMC algorithm, as their x-axis.

4.1 The myth of ingroup affinity

The graphs in figure 1 show the amount of time required for more than half of the minority group to take on spin -1 and for that fraction to never again dip below half. This is what we refer to as substantive sound change within a group. The axes demonstrate the relationship between the time for such sound change to occur and the values of ingroup affinity and outgroup antipathy ($J_{X,X}$ and $J_{X,Y}$ resp.). Simulations occurred over 200 time epochs. In each epoch$\tau \cdot N$ updates occurred. Values of 200 in the above graph indicate that sound change never occurred.

One of Labov's key predictions was that ingroup affinity (represented in our model as $J_{A,A}$ and $J_{B,B}$) is the primary catalyst of linguistic change. As you can see, the level of ingroup affinity actually has no bearing on the level of antipathy required for sound change to occur. There is a clear phase transition line at around $J_{A,B} = 1.6$. For levels of antipathy below this, no amount of ingroup affinity will cause spontaneous sound change to occur. This is because very high levels of ingroup affinity reinforce the initial conditions and make change in the population unlikely. This point highlights the important role interactions with new populations plays in real world sound change scenarios.

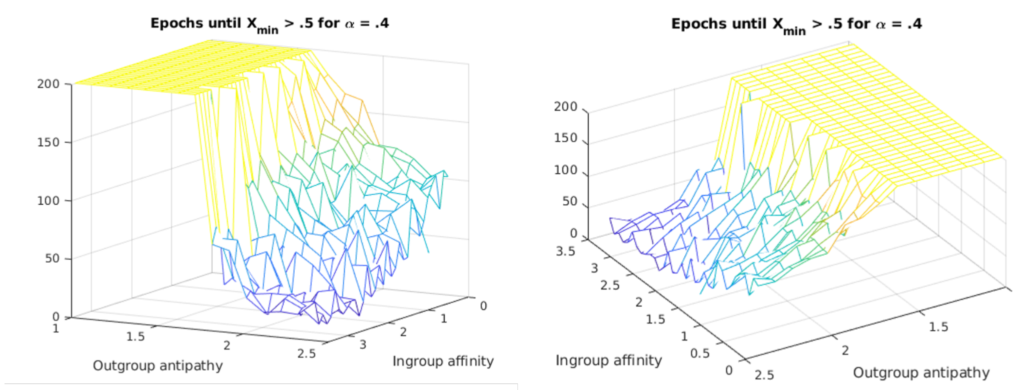

Figure 1: Phase transition lines appear at values of around 1.6 for outgroup antipathy

This of course begs the question: does ingroup affinity play any role, and if so, when?

As Labov noted, sound change began in his case studies due to the irregular distribution of some linguistic variable. This would correspond to the initial conditions vector having some fraction of -1s at the outset.

The graphs in figure 2 tell us that increasing affinity dampens sound change, as the phase transition line increases lightly to 1.7 by the end of the range. However, the curve greatly increases in steepness as affinity increases. This indicates that affinity becomes the driving force for sound change once $X_A \geq .5$ but below .5 it has a dampening effect.

This indicates that even in Labov's theory where there is a complex set of initial conditions, antipathy, or the desire to dissimilate from out groups, is the driving factor of sound change, and only after over half the population has adopted the irregular variable does affinity substantially aid the spread of the sound change.

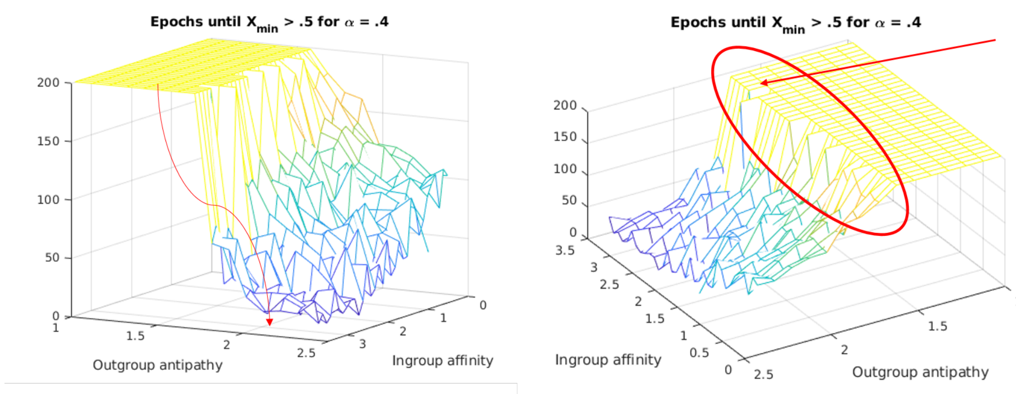

Figure 2: The shape of the graph indicates that below $X_A = .5$ increased affinity dampens sound change (see the phase transition being pushed upwards to $J_{A,B}$ levels of around 1.7) but once over half the population takes on spin -1, sound change becomes driven primarily by the affinity value.

Figure 3 further demonstrates the new understanding of the role affinity plays in the social model of linguistic change exemplified here. For sufficiently high antipathy, increased affinity quickens the rate of sound change, but it is neither necessary nor sufficient for such sound change to occur.

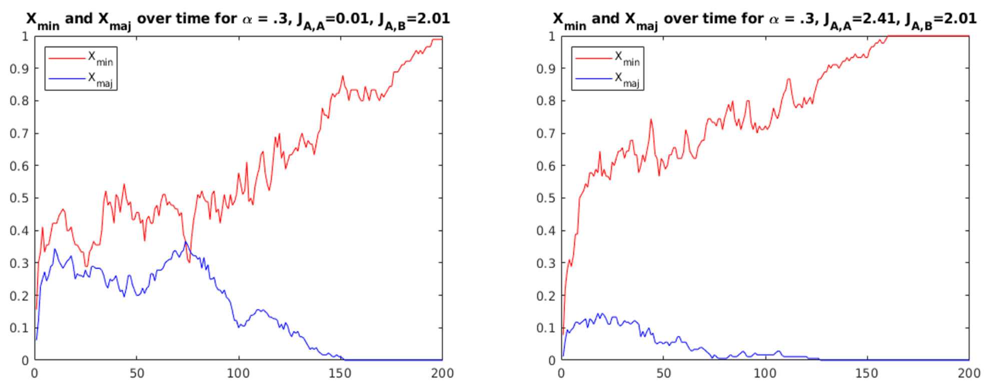

Figure 3: For fixed levels of antipathy above the phase transition increased affinity causes overall sound change to proceed much more rapidly

4.1.1 The unmentioned role of minority size

Though it hasn't been noted in the literature, the size of the minority group should be expected to play a role in the rate at which sound change occurs. In particular, larger minority groups should be relatively insulated from the antiferromagnetic effects of the majority group and should show longer time dynamics and greater resistance to change overall. This can be seen in figure 4 in which the effects of interclass antagonism are severely blunted for large values of α.

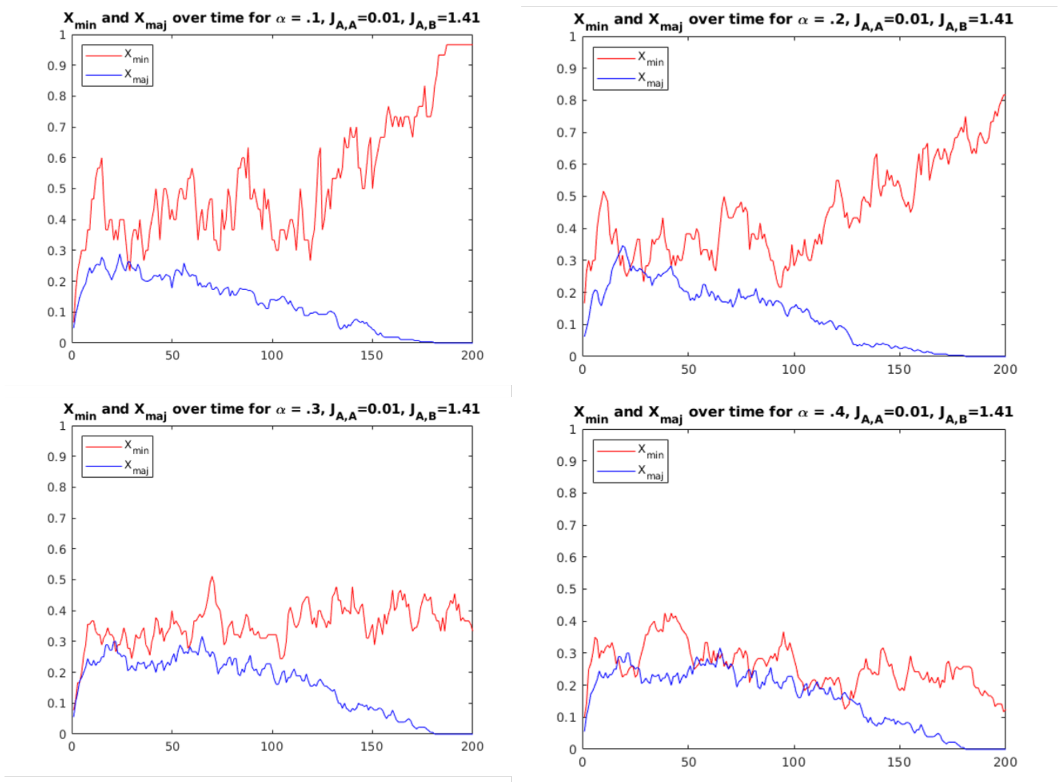

Figure 4: Smaller minority groups are more adversely influenced by antiferromagnetic interactions with the majority group. Larger groups are more stable and the effects of class antagonism are blunted.

4.2 The insufficiency of the "prestige" argument

Our model elucidated another key deficiency in Labov's theory. Labov hypothesized that if the minority group is the prestige group then sound change can spread to the population at large. Our data shows that this condition is necessary but not sufficient for population-wide sound change and that stable sound change across populations requires more conditions

Our data generated by the simple Hamiltonian model with dynamic external field, introduced above, demonstrate that absent some further conditions, sound change originating in a minority population is inherently unstable. The figures below shows that a predator-prey style dynamics arises in which the minority group initially diverges as a dissimilatory reaction to the majority. The majority then conforms to the minority due to prestige effects. But then, due to the large size of the majority which has conformed to the minority, the conditions for spontaneous sound change once again exist resulting in an unstable cycle. The chaotic and unpredictable end distributions can be seen in figure 5

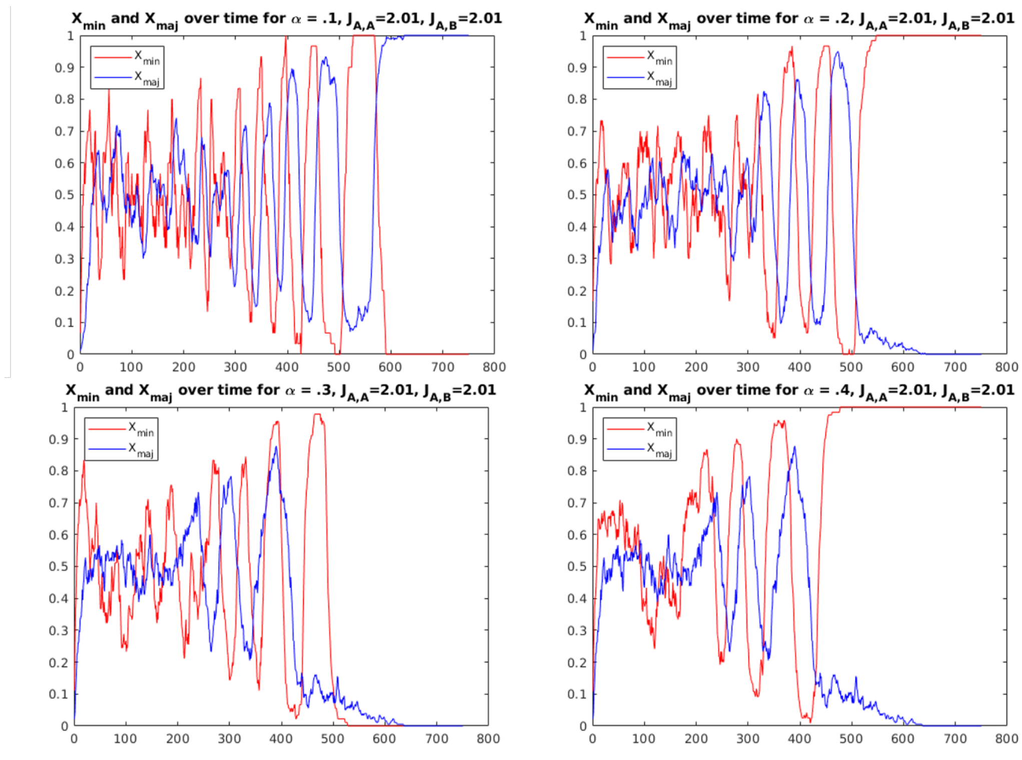

Figure 5: The dynamic external field creates a chaotic system resulting in convergence to either A, B, or neither having complete sound change by the end. Note the oscillations have greater period when the minority group is larger. This is due to the general stabilizing impact of the minority group being larger that was noted at the end of the last section.

The end distributions get locked, somewhat randomly, in metastable states due to the temperature decrease in the annealing schedule of the MCMC algorithm used. With no annealing, a true predator-prey model develops, as in figure 6 which never exhausts itself.

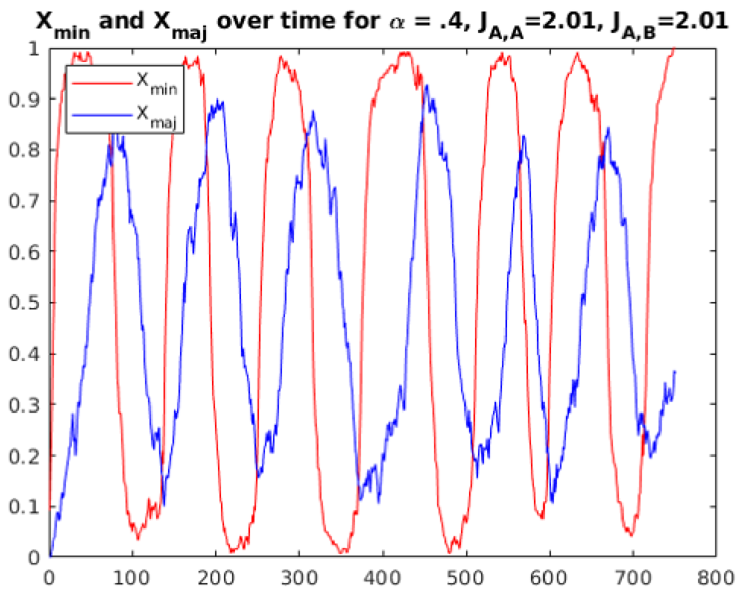

Figure 6: Removing the annealing schedule shows that our simple prestige model truly shows no signs of convergence, indicating some revisions are necessary to account for language change from the prestige minority group to the majority.

4.3 Correcting the "prestige" model with population dynamics

The simple prestige model given in the previous section, inspired by the underspecified role of prestige in Labov's theory never seems to tire itself out. It perpetually provokes widescale change then settles into functionally circumstances as the initial conditions which provoked change in the first place. This is due to antipathy's chief role in sound change for our models. As discussed in section 4.1, it is unlikely that affinity alone could account for sound change so we are unlikely to get out this modeling issue by simply discarding our notion of interclass antagonism.

A more likely suggestion is that populations evolve over time. Though an exhaustive sociological account of how class dynamics may evolve over time is outside the scope of this paper, I offer a couple of different scenarios which could provide necessary and sufficient conditions for sound change to spread from the minority to the group as a whole.

As noted, somehow, once the linguistic change has touched all groups, the social landscape must have evolved enough for the initial conditions to not once again trigger another round of sound change. I posit that large scale social change may occur for a number of reasons:

a) new groups may enter the community creating solidarity among previous groups

b) the groups may be sufficiently antagonistic that they separate from one another entirely when it comes to individual-to-individual contact, and further contact is only established via shared participation in institutions controlled by the prestige group

c) an existential threat or major upheaval may cause the groups to recalibrate their relations with one another

d) the sound change itself becomes a marker of in-group status which can have the effect of blunting the antagonistic interactions which may exist on the underlying social stratum.

This paper models only the last possibility, and it does so via introduction of the term discussed in section 3.1.2, reproduced below:

$$\sum_{i,j} \mathbb{1}_{\sigma_i \neq \sigma_j}(J_{out}\sigma_i\sigma_j) - \mathbb{1}_{\sigma_i = \sigma_j}(J_{in}\sigma_i\sigma_j)$$

Under this model, individuals feel a kinship with people who spoke the same as they did in the previous epoch, a kinship which exists as another layer on top of the social stratum underlying the language change.

This model only achieves success within a very narrow range of parameters. The reason for this is that if linguistic affinity is to powerful from the onset, no change occurs in the first place, or if it does it doesn't spread to the majority group. For a narrow range of parameters around those in figure 7 we see sound change spread to close to 97% of the population in the end. This is only reproducible only for values of $\alpha \leq .2$, and as you can see in figure 8 is not reproducible for different values of the parameters.

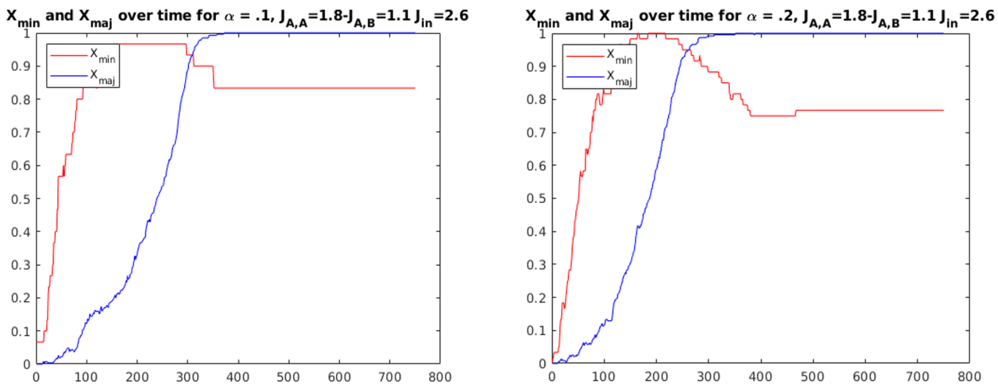

Figure 7: Sound change first spreads to the minority group, the majority group then adopts it, and then the new affinity caused by linguistic ingroup creates a slightly stable equilibrium in which the minority group dissimilates slightly in response to the new behavior of the majority group

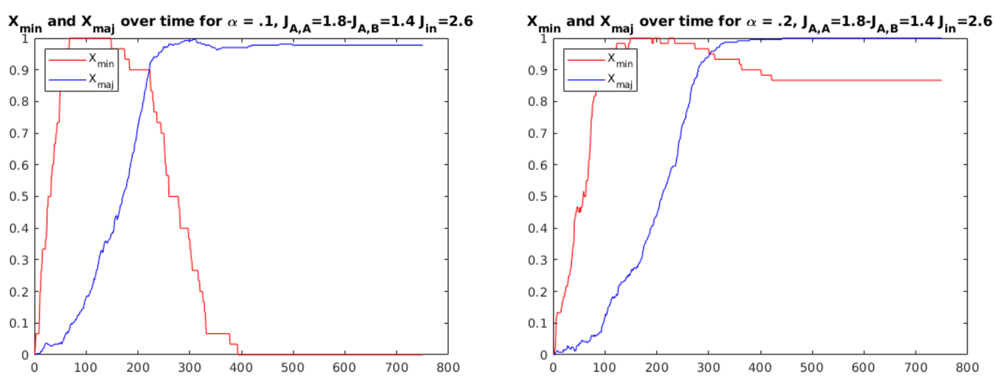

Figure 8: Even for nearly identical values of the parameters the results in figure 7 are not reproducible

As shown in the figures, the instability of this wide-scale sound change and the narrowness of the valid parameters under which this result is reproducible suggest that further refinements to linguistic theory are likely needed to explain Labov's conditions under which sound change spreads from a minority prestige group to the community as a whole.

In the narrow range of parameters where our model would predict spontaneous sound change is capable of starting in a minority group and then spreading to the majority group, we should also expect that this phenomena happens primarily only for small minority groups ($\alpha < .2$) and that once the sound change has become adopted by the majority there is some reactionary backlash causing X_A decline somewhat as it has in the figures above.

5 Conclusion

This article has shown that modeling sound change based solely on the interactions of individuals in a population can be a fruitful exercise largely confirming the expectations set by the fieldwork literature in sociolinguistics. We have noted results which suggest revisions to Labov's core theory of sound change. In particular

a) we highlight that his model does not give an account for circumstances under which true, spontaneous sound change would occur

b) ingroup affinity makes change more rapid once widespread ($X_A > .5$) change has already been achieved

c) outgroup antipathy, and not ingroup affinity, is the leading catalyst for spontaneous sound change

d) populations with larger minority groups are less susceptible to sound change and generally show greater stability

e) the social landscape must evolve substantially over time in order for sound change originating in the minority prestige group to cause population-wide change. This is because linguistic convergence of both groups recreates the initial conditions of sound change unless the social landscape has evolved

f) the emergence of new linguistic ingroups may be a valid model for sound change which occurs across the whole population, but only for small minority fractions

All of these results are made with the major caveats that they do not model confounding linguistic variables, they do not model complex social dynamics of the type tentatively mentioned in section 4.3. Finally, and perhaps most damningly, this study makes no assumptions about the underlying social network graph besides, the implausible one, that it is a complete graph, in which all individuals weigh some influence on all others.

This study, however, has presented ecologically plausible results which in part confirm current orthodoxy on the subject and contradict or revise some commonly accepted principles. This has resulted both in new insights for the relationship between individual attitudes and emergent properties in linguistic communities, as well as insights into faithful modeling of toy linguistic environments one may be interested in studying.

This study has shown that spin glass modeling is a computational environment with which we may experiment with the consequences of different sociological assumptions. With precise direction from field data describing the social network, the ways populations evolve over time, and the relationships between confounding linguistic variables one could easily construct more complex Hamiltonians which capture any and all of the possible extensions to this study suggested here.

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