NOTATING TEMPERAMENTS

A STUDY OF NOTATION SYSTEMS FOR TEMPERAMENTS FROM THE RENAISSANCE TO THE PRESENT

Submitted: September 13, 2001

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NOTATING TEMPERAMENTS

Musical theorists through the ages have explored the issues surrounding temperament, Pythagoras and Aristoxenus in Ancient Greece, Boethius in medieval Europe. The resurgence of Greek philosophy in the Renaissance led to a renewed interest in temperament. Both Zarlino and Vicentino produced major theoretical works surrounding the issues of just and mean-tone temperaments while a number of composers, such as Gesualdo, explored the realm of extreme chromaticism. The rise of the keyboard in the Baroque period prompted further development of mean-tone by such theorists as Werkmeister, Kirnberger, Vallotti and Young.

The exploration of temperament continued through the eighteenth and nineteenth century but it wasn't until the twentieth century that the music world saw another proliferation of tuning theory. Inspired by the breakdown of the common practice tradition composers started to build their own instruments to explore new tuning systems. The advent of the electronic medium allowed even greater freedom that was hitherto impossible.

One characteristic of twentieth century tuning theory was the extensive use of microtonality, use of tones smaller than a half step. This had of course been explored in the past, the most comprehensive work being Nicola Vicentino's *Antica musica ridotta alla moderna prattica* of 1576, but the twentieth century took divisions of octave much further.

It stands to reason that in order to properly represent non-twelve tone temperaments new notation systems need to be developed. In truth conventional notation can easily and logically represent for both twelve (Figures 1.1 and 1.2) or seventeen tone systems (Figure 1.2).

Figure 1.1 Twelve-tone System Notated with Sharps

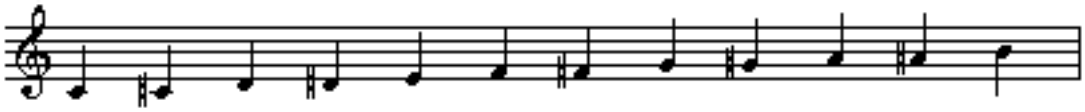


Figure 1.2 Twelve-tone System Notated with Flats

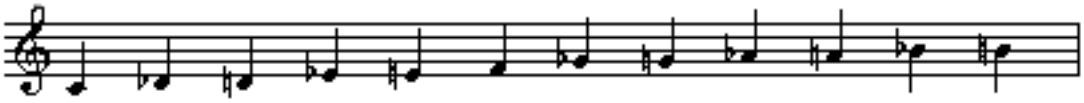
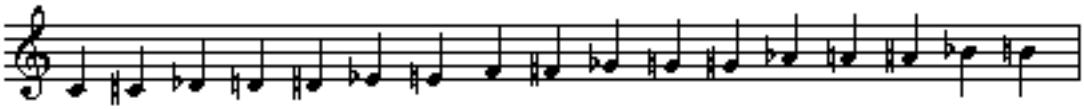


Figure 1.3 Seventeen-tone System



And yet with advent if dodecaphonic music in the early twentieth century even conventional notation for twelve-tone temperament came under scrutiny. Once one goes beyond twelve or seventeen tones the temperament can still expressed with traditional symbols but the resulting notation system is not a comprehensive or intuitive use of available symbols.

This paper examines six types of notation: (I) Systems which use only the five traditional accidentals; (II) Systems which use additional accidentals; (III) Systems using altered noteheads (IV) Systems which use numbers; (V) Systems which use an altered staff; and (VI) Systems which use color.[1]

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[1] In 1990 Gardner Read published a book entitled *20th-Century Microtnal Notation.* In it he identifies the following options as the best ways of dealing with microtonal notation

1. "modify the existing accidentals, or invent new symbols to take their place"
2. "altering notehead shapes to distinguish one microtone from another"
3. " enlarge the normal staff five-line staff so as to accommodate more than the usual twelve pitches to the octave" (Read p 10)

In his discussion of quarter-tones Read raises the additional possibilities of "auxiliary numerals specifying the microtone from another" and "abstract signs related to standard noteheads or to accompanying accidentals"[1]. Read regards modified accidentals as the best solution.

I have used *20th-Century Microtnal Notation* as the springboard for this paper and as such the categorization of notation systems is quite close to Read's.

(I) Systems which use only the five traditional accidentals

This first category is certainly not the most populated (that honor belongs to the "additional accidentals" group) but it is the most logical starting point as it works with what is already available. As mentioned above the use of tradition accidentals is problematic because of their long standing association with the equal tempered twelve-tone scale. It does however provide a way of presenting information without needing to introduce a new system and as such as been favored in some circumstances by theorists. Both Easley Blackwood[2] and Rudolph Rasch[3] have devised a number of systems exclusively using standard accidentals.

Figure 2.1 Easley Blackwood "Modes and Chord Progressions in Equal Tunings" - 19 Note EQ Notation

Positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Notes	C	C♯	D♭	D	D♯	E♭	E	E♯	F	F♯	G♭	G	G♯	A♭	A	A♯	B♭	B	B♯	C
		D𝄫	C𝄪		E𝄫	D𝄪		F♭		G𝄫	F𝄪		A𝄫	G𝄪		B𝄫	A𝄪		C♭	

Figure 2.2 Easley Blackwood "Modes and Chord Progressions in Equal Tunings" - 17 Note EQ Notation

Positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Notes	C	D♭	C♯	D	E♭	D♯	E	F	G♭	F♯	G	A♭	G♯	A	B♭	A♯	B	C
		B♯	E𝄫		C𝄪	F♭			E♯	A𝄫		F𝄪	B𝄫		G𝄪	C♭		

[2] Blackwood is a contemporary theorist/composer who has focused on equal tempered divisions of the octave. Read cites him as the "most conspicuous proponent" of the belief "that expanded microtonal systems based on equal temperament tunings are as valid acoustically and musically, and technically challenging as functional division of the octave adhering to just or to mean-tone intonation principles" (Read p 5). Blackwood's major works include the book *The Structure of Recognizable Diatonic Tunings* and the set of compositions entitled *Twelve Microtonal Etudes*.

[3] A theorist, composer, music editor and musicologist Rasch's approach to notation systems of equal divisions of the octaves is based on the works of Wyschengradsky and Haba.

Figure 2.3 Easley Blackwood "Modes and Chord Progressions in Equal Tunings"
- 16 Note EQ Notation

Positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Notes	C	C♯	D♭	D	D♯	D𝄪	E	F	F♯	F𝄪	G	G♯	A	A♯	B♭	B	C
		D𝄫			E♭					G♭				B𝄫			

The logic for using standard accidentals to express a nineteen-tone system has already been addressed above. Blackwood also favored standard accidentals for sixteen and seventeen tone systems because it allowed the diatonic structures available in these systems to be easily recognizable.[4] In terms of enharmonic equivalents Blackwood used double sharps and flats quite extensively in the nineteen tone scale, the number decreasing to a certain degree with the seventeen tone system and even more so with the sixteen tone system.

Figure 2.4 Rudolf Rasch *Relations between Multiple Divisions of the Octave*
- 19-tone System

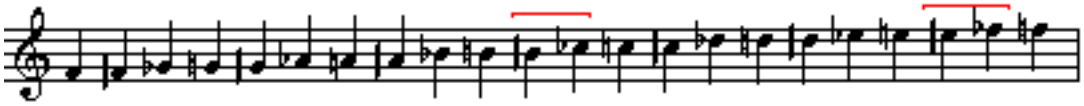
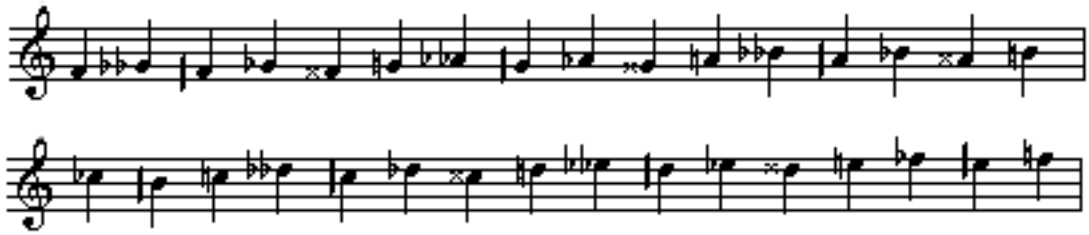


Figure 2.5 Rudolf Rasch *Relations between Multiple Divisions of the Octave*
- 31 tone System



[4] Blackwood outlined his view on notating non-twelve-tone equal temperaments in his article "Modes and Chord Progressions" (*Perspectives of New Music* 29:2 (1991)):

"What is wanted is a notation that is compatible with the existing notation, and with musical habits. To meet these requirements, I think it essential that the five-line staff should be preserved, and that octaves should have the same appearance in all the equal tunings. In addition, the usual direction and approximate distance associated with sharps and flats should be retained." (p181)

Rasch also uses traditional accidentals exclusively for the nineteen-tone system. In terms of enharmonic equivalents he only recognizes B sharp/C flat and E sharp/F flat eschewing double sharps and flats. Rasch expands his repertoire to include the double sharps and flats with his notation of the thirty-one-tone system in which he hasn't allowed any enharmonic equivalents.

(II) Systems which use additional accidentals

This is the most popular form of modified notation. The use of additional accidently allows the noteheads and staff itself to remain the same and as such provides the least amount of disruption to standard notation practices. Logically the first and easiest step for an expanded repertoire of accidentals is to address the twenty-four tones per octave, quarter-tone, temperament. This can be achieved simply and elegantly with the addition of a couple of new accidentals to represent the quarter-step up or down. The issue with quarter-tone notation is the lack of consensus regarding which accidents should be used to represent quarter-steps and how they should be used.

(i) Quarter-Tone Notation

One could argue that the popularity of a notation system can be gauged by its use by third parties and one way of measuring this is to examine the most common examples of microtonal notation found in notation manuals. (There is of course an obvious flaw to this argument given the possibility that the author of the manual or the notation may be acting with some personal agenda). Kurt Stone's *Manual of Twentieth Century Notation*[5] provides a brief, four page, discussion of microtonal notation, focusing on quarter tones. Stone

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[5] Stone, Kurt *Music Notation in the Twentieth Century* New York: W.W. Norton & Company, 1980 p67-70

demonstrates three methods of quarter-tone notation: arrowed accidentals, altered sharps, and altered flats (see Figure 3.1).[6]

Figure 3.1 Quarter-tone notation symbols described in Kurt Stone's *Manual of Twentieth Century Notation*

System	Symbols	Notes
Arrowed Accidentals	𝄬 𝄮 𝄯 𝄭 𝄮 𝄰	- "C sharp-arrow down, C natural-arrow up, D flat-arrow down same (*choice of enharmonic equivalent used should be based on tonal context*)"
Altered Sharps	𝄲 𝄪	- 1 line = ¼, 2 lines = reg, 3 lines = ¾ - this method first recorded by Tartini in 1756 lacks flats
Altered Flats	𝄳	- ¼ flat to be combined with Tartini's ¼ sharp

In 1967 the International Musicological Society propsed the following symbols become standard[7]

Figure 3.2 International Musicological Quarter-tone proposals 1967

Semi-sharp	
Sesqui-sharp	
Semi-flat	
Sesqui-flat	

While in 1974 the International Conference on New Music Notation endorsed the use of arrows. Other popular quarter-flat notation include quarter sharps drawn with one vertical and one horizontal line ( ), two vertical and one horizontal line ( ), and quarter flats represented by the slashed flat (𝄳) and filled in flat (𝄳).

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[6] All of the symbols used above are available for score creating in *Finale* through the font *Tamburo*. *Finale* has an interesting and powerful approach to the issue of microtonal notation as it allows the entire system to be specified in terms of the number of steps per tone and the symbol used to represent each one.

[7] Interesting enough none of these symbols other than the sesqui-sharp are available in Finale

Figure 3.3 Julián Carrillo – Quarter-tone Notation

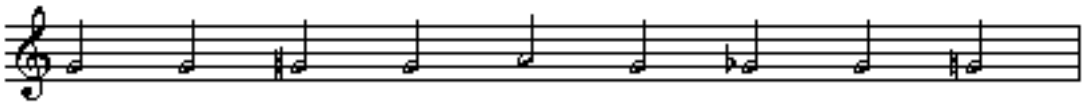


Figure 3.4 Ivan Wyschengradsky *Manual d'harmonie à quarts de ton*

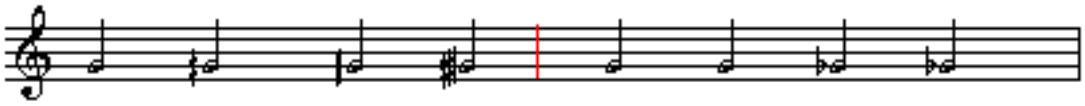


Figure 3.5 Easley Blackwood *Twelve Microtonal Etudes* – Quarter-tone Notation

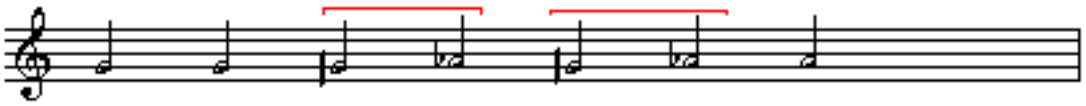
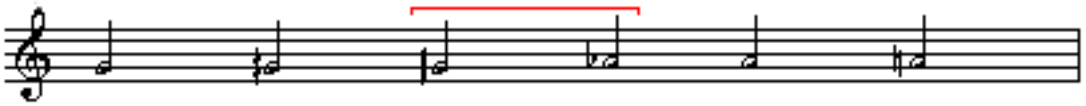


Figure 3.6 Rudolf Rasch *Relations between Multiple Divisions of the Octave*



Carrillo,[8] Wyschengradsky,[9] Blackwood, and Rasch adopted similar approaches to the quarter-tone notation. All systems implement different notation for quarter-tone and three-quarter tones sharp. The main differences are in the type of symbols chosen to present the quarter and three-quarter steps and whether or not there is quarter and three-quarter flats (as there is in the Carillo and Wyschengradsky but not the Blackwood or Rasch). Carrillo's system differs also from the other in that the solid obliques indicating a quarter-

[8] Julián Carrillo was a Mexican theorist and composer at the turn of the century. His journal *El Sonido 13* (*The Thirteenth Sound*, referring to sounds smaller then a half-step) provided a forum for him to discuss his tuning experimentation. Starting from the quarter-tone he worked on further equal divisions of twelve-tone equal temperament. His work culminated in 1924 with sixty-fourth tones (384 divisions of the octave).

[9] Wyschengradsky was a tuning theorist in the early twentieth century. According to Read he "believed in the strictly determinate or systematic building of ultrachromatic sacles. His notation for third-, sixth-, and twelfth-tones illustrates his methodical approach to microtonal scale ///, at once both logical and practical." (Read p 101)

step appear after the notehead of the altered pitch (which could prove to be problematic in a performance context).

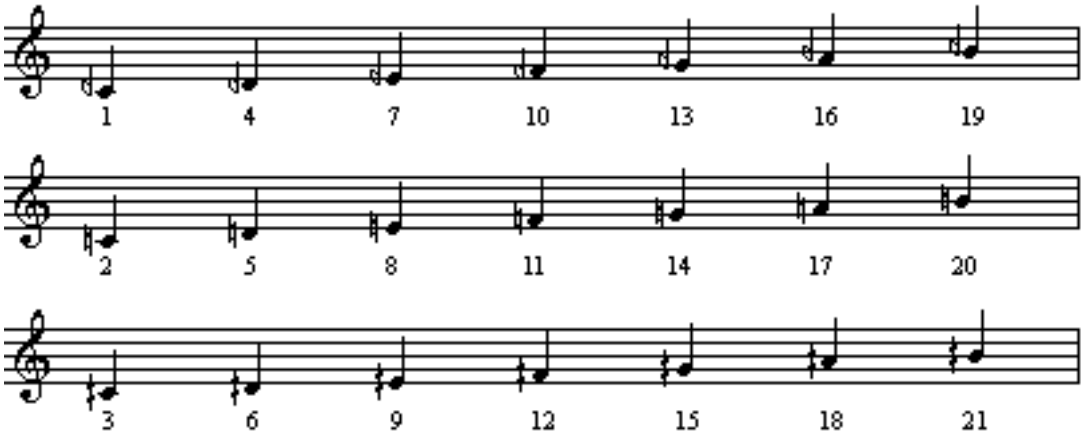
(ii) 15, 21, 31 and 24 Notes per Octave

Figure 3.7 Easley Blackwood "Modes and Chord Progressions in Equal Tunings"
- 15 Note EQ Notation

Positions	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Notes	C	C	D	D	D	E	E	F	F♯	G	G	A	A	A	B	C
	B		C♯		E♭						A♭		B♭	B♭		B

Unlike his sixteen and seventeen-tone notations Blackwood opted to use his quarter-tone symbol in his fifteen-tone system notation. He used conventional notation for the five pitches which divided the octave into five equal parts and then two new accidentals, representing up and representing down, to notate the subdivision each of these divisions into three equal parts.

Figure 3.8 Ivor Darreg 21 Tone Scale from *XF1, Volume 1, Number 12*[10]

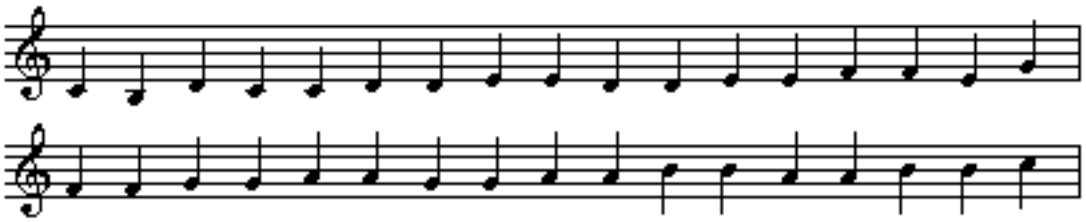


Used in the twelfth piece of the first volume of his collection of forty-four pieces entitled *Beyond the Xenharmonic Frontier* Ivor Darreg's 21 scale is most easily represented by the three seven tone scales above (the numbers represent the scale-steps) – not only does it

[10] McLaren, B "An Analysis of Ivor Darreg's XF1, Volum3 1, Number 12" *Xenharmonikon* 14 (1993): 40.

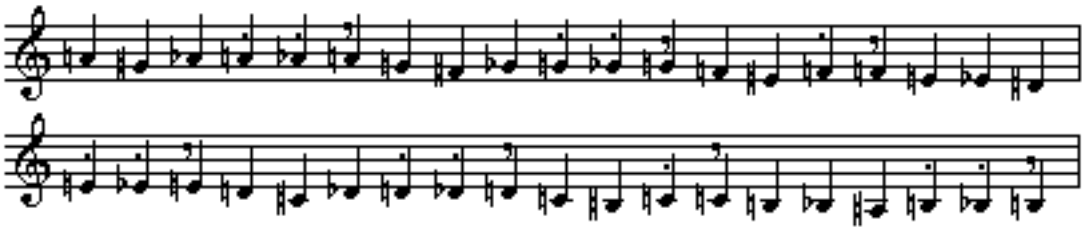
make it easier to read but it also illustrates the choices made in terms of which accidentals to use and when.

Figure 3.9 Gary Morrison "34-Tone Notation System Based on Adjusted 17-Tone"



Morrison's equal tempered thirty-four-tone system is based on the notation for a seventeen-tone equal tempered scale with additional accidentals created for dividing the tones found in the seventeen-tone scale in half.

Figure 3.10 Nicola Vicentino's *Antica musica ridotta alla moderna prattica* of 1576
Names of the Keys on the Archicembalo in Descending Order



In terms of accidentals placed before the notes Vicentino[11] didn't deviate from traditional notation or employ double sharps or flats, rather he introduced the use of periods and commas above the notes to indicate which version of the note should be played. This system would not work well in a contemporary context as the period (though placed stem-

11 Nicola Vicentino was a sixteenth century Italian theorist, keyboardist, and composer. His *Antica musica ridotta alla moderna prattica* of 1576 was an expansive treaty explaining his tuning system which consisted of six levels for each scale degree (36 pitches in total). In order to train singers in his tempoerament Vicentino build an archicembalo. His notation system for his temperament was based on standard accidentals and was supplemented by a period and a comma over noteheads to help distinguish five steps per whole-tone.

side) could potentially be confused with a staccato and (like the problem with Carillo's obliques) most performers are used to reading pitch information before the note.

(iii) Twelfth-tone Notation

Figure 3.11 Alois Haba – Twelve tone notation

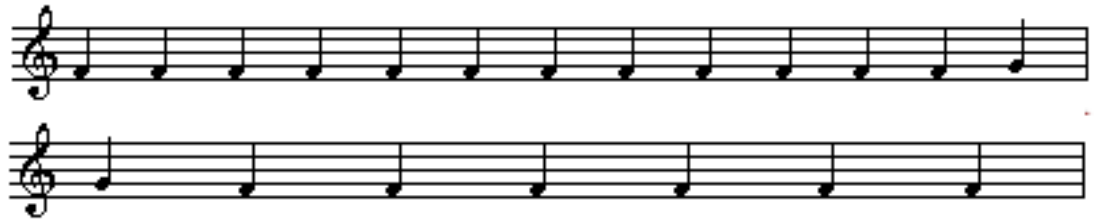


Figure 3.12 Rudolf Rasch *Relations between Multiple Divisions of the Octave* 12th-tone

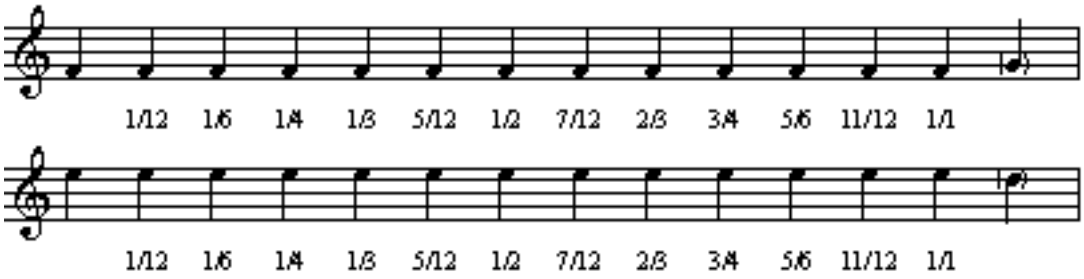


Figure 3.13 Ivan Wyschengradsky *Manual d'harmonie à quarts de ton* – 12th tones

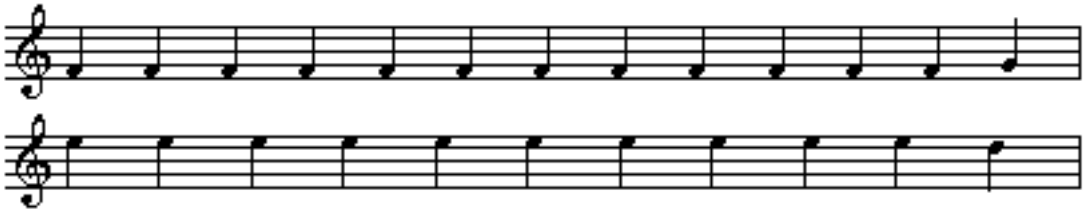
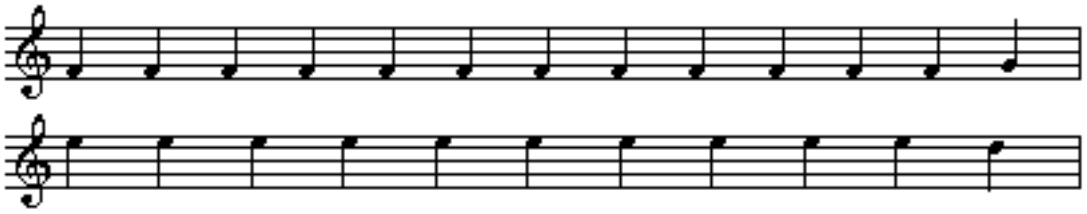


Figure 3.14 Erza Sims "A Question of Microtonal Notation" – Twelve tone notation



Haba,[12] Rasch, Wyschengradsky, and Sims[13] all developed notations for ascending and descending twelfth, sixth, quarter, and third-tone steps. All of these systems are fundamentally the same theoretically and differ only in the accidentals created by the theorist/composer to denote the various micro-steps.

Ben Johnston[14] developed a complex set of symbols which indicate variations in cents from traditional pitches.

Figure 3.15 Ben Johnston's Notation System

	+70c less 53c = +17C		-70c less 49c less 22c = -141c
	+70c less 53c plus 22c = +39c		+140c less 53c = +87c
	+70c plus 49c = +119c		+140c less 53c less 22c = +65c
	+70c less 53c less 22c = -05c		-49c less 53c = 102c
	-70c plus 53c =-17c		-49 c less 53c plus 22c = -80c
	-70c plus 53c plus 22c = +05c		+49c plus 22c = +71c
	-70c less 49c = -119c		-140c less 22c = -162

(III) Systems using altered noteheads

Alternated noteheads were present in the nineteenth century with some whole-tone notations which shifted the implications of filled or hollow notes from the domain of rhythm to that of pitch. In the twentieth century Harry Partch[15] has created a system based

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12 Haba was an early twentieth century tuning theorist. He saw quarter-tones as the natural expansion of Schoenberg's twelve-tone serial technique. Haba viewed quarter-tones as "plastic and independent intervals, not shadings of the old intervals" (Read p5).

13 Sims is a contemporary theorist/composer. In his article "A Question of Microtonal Notation" *Xenharmonikon* XI he rejected Haba's twelfth-tone notation as being "too fussy and his graphical distinctiosn to minute for assimilation, hence impractical." (Read p 102)

14 Ben Johnston was a student of Harry Partch only briefly during the fifties and has been one of the key proponents of his theories and of just-intonation in the twentieth century. His approach to notation is highly personalized using plus and minus signs, arrows, other accidentals as well as vibration ratios and cents to indicate the sizes of intervals.

15 Harry Partch is credited with the resurrection of Just Intonation theory in the twentieth century. Using ratios limited by prime-11 he developed a forty-three tone scale for which he not only developed his own notation systems but built his own instruments to realize. Partch's notations can be divided into sixth main categories: systems which only use ratios; systems which use ratios and conventional noteheads; systems which use different shaped noteheads; systems which use color bands; systems which circle noteheads in color; system which use numbers (not ratios). Partch's *Genesis of Music* stands not only as an explanation for his own system but as one of the key works in twentieth century tuning theory.

on altered noteheads and ratios (Partch's notations were often piece specific and the example below is actually a propsal of notation by Rodney Will based on Partch's 43-tone scale and the composer's theoretical concepts surrounding it)[16] while Erhard Karkoschka has used altered noteheads on a modified staff and Alain Danielion has have explored altered noteheads in combination with accidentals and ratios.

Figure 4.1 Wholetone Notation

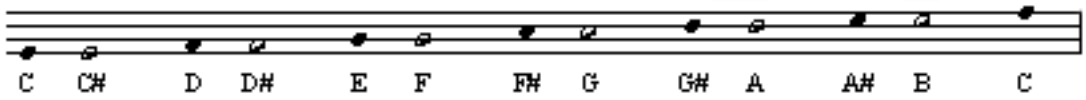


Figure 4.2 Partch/Will: *A Proposed Graphic Notation*

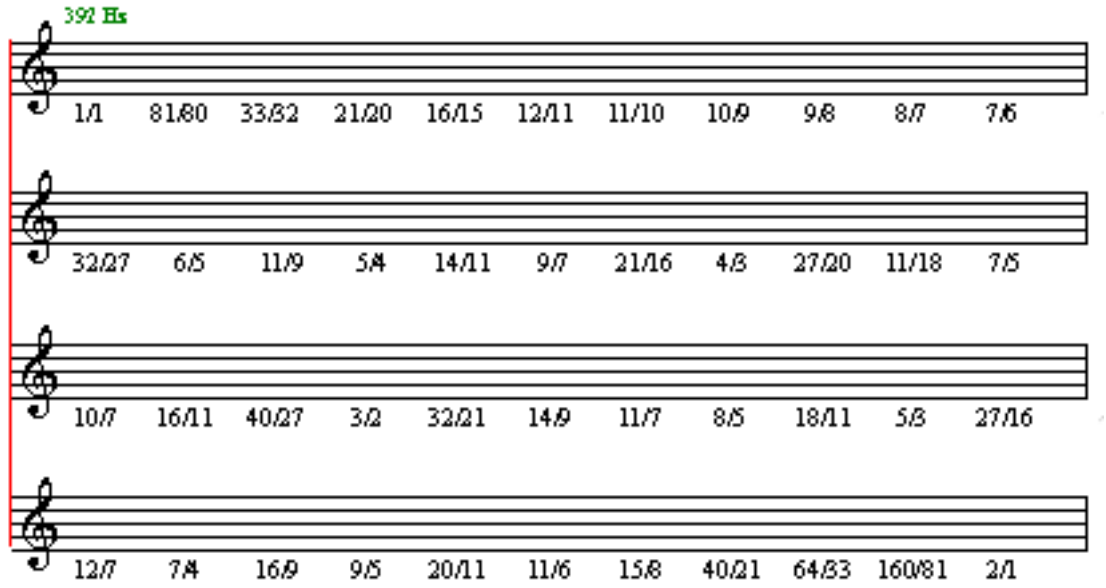
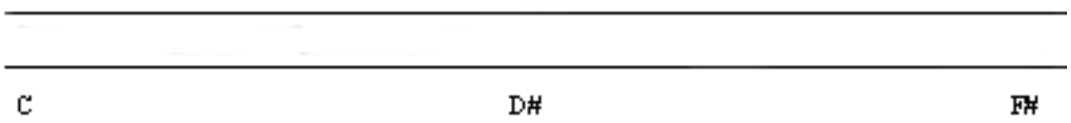
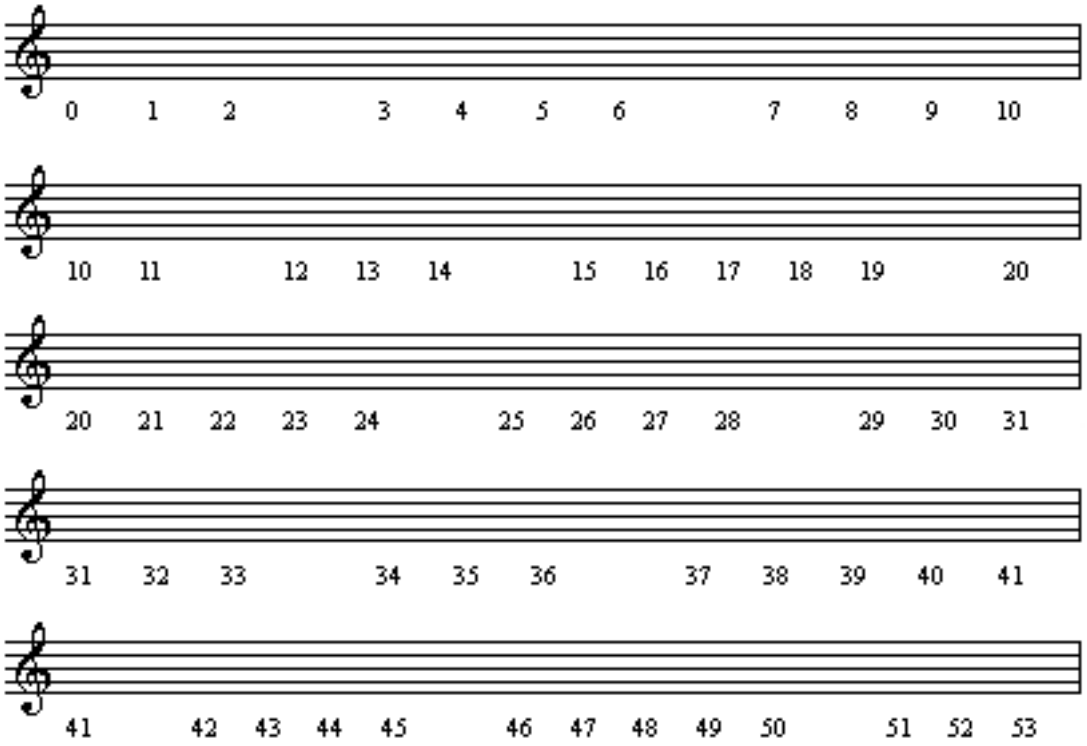


Figure 4.3 Erhard Karkoschka: *Notation in New Music*



[16] Read p132

Figure 4.4 Alain Daniélou - Notaiton for 53 tone system[17]



(IV) Systems which use numbers

Another system for microtonal notation is the use of numbers. The use of numbers in series instead of note names was first proposed by Rousseau but it's microtonal application was first explored by Carrillo who proposed the following system which would represent pitches a sixteenth of a tone or larger. His method relied on numbers to represent pitch rather than position on the staff. C was always represented a 0 and proceeding

17

Frequencies Corresponding to the numbers in the Daniélou example													
0	1/1	8	10/9	16	100/81		(62)/(45)	33	192/125	41	128/75	49	243/128
1	81/80	9	9/8	17	5/4	25	25/18		(48)/(31)		(1024)/(595)	50	48/25
2	128/125	10	256/225	18	81/64	26	45/32	34	25/16	42	125/72		(29)/(15)
	(30)/(29)	11	144/125	19	32/25	27	64/45	35	128/81	43	255/128	51	125/64
3	25/24		(595)/(512		(31)/(24)	28	36/25	36	8/5	44	16/9	52	160/81
4	256/243	12	) 75/64	20	125/96		(45)/(31)		(50)/(31)	45	9/5	53	2/1
5	16/15	13	32/27	21	320/243	29	375/256	37	81/50		(248)/(135)		
6	27/25	14	6/5	22	4/3	30	40/27	38	400/243	46	729/400		
	(135)/(124)		(98)/(81)	23	27/20	31	3/2	39	5/3	47	50/27		
7	800/729	15	243/200	24	512/375	32	243/160	40	27/16	48	15/8		

sixteenth tone was assigned the next consecutive number to 95. The following chart (Figure 4.1) demonstrates how the system worked for sixteenth, eighth and quarter-tones.

Figure 5.1 Julián Carrillo's Microtonal Notation System

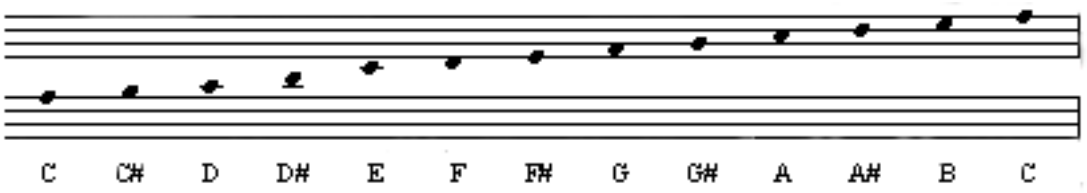
Divisions of Octave	Numbering Protocol
Sixteenth-tone	Every number
Eighth-tone	Every other number
Quarter-tone	Every fourth number * if the scale only consists of QT then the numbers 0-23 could be used

A second, and more popular, application of numbers has been in the specification of ratios. The pitch of the note is represented traditionally and either the note or the relationship between two notes is written as a ratio and its cents equivalent. This method has been used by Partch, Jim Tenney and Lou Harrison. Karheinz Stockhausen used fractional intervals in the context of a graph representing frequency in some of his electronic compositions.

(V) Systems which use an altered staff

The whole tone notation system reduced the number of staff lines to four (see Figure 4.1). Likewise the Ailler-Brennink Chromatic Notation system proposed by Albert Brennink, and was partially based on the work of Johann Ailler in the early twentieth century. It is an attempt at a truly pictorial notation which represents twelve-tone equal temperament without any enharmonic equivalents and where are intervals are represented proportionally. As mentioned above the proposed system has only four lines per staff and each line or space is assigned to a particular step of the twelve-tone scale, thus eliminating the need for accidentals.

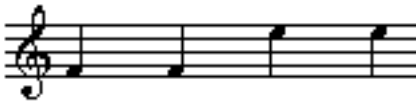
Figure 6.1 Ailler-Brennink Chomromatic Noatation



(VI) Systems which use color

Partch used color in some of his scores to indicate relative detunings of pitches. In the following example orange meant slight sharp, red meant slightly flat, blue meant more sharp, and violet meant more flat.

Figure 7.1 Partch – Colored Notation



Partch's system is interesting because it ties into the common musical vocabulary of colouring, in this context in terms of exact pitch. Because the pitches are exact colour is a good way to imply the mood of the pitch, the main issue that needs to be investigated is if the majority of people perceive the colouring of pitch in the same way – if that is true then certain colours related to certain deviations from the notated pitch can be seen as intuitive, if not it becomes yet another system of notation which must be learned.

Enrique Moreno proposed a theory to the application his book *Expanded Tunings*.

Figure 7.2 Moreno – Proposed Rules for Color Notation

Rules for the use of color in music notation:
1. A tuning has as many pitch classes as it has steps per cycle 2. To every pitch class, a different color, and only one different color shall be assigned 3. Color representing pitch. No enharmonic spelling shall alter the one-to-one absolute correspondence between color and pitch. 4. Only note heads are to be colored. **Exception:** Any tuning containing twelve steps per cycle shall be notated in black and white.

His rules seem quite logical and implemental, however without providing an adequate working example they can only be judged in terms of their potential and not in terms of their practicality.

Conclusion

This paper has briefly surveyed of some of the main methods of notating alternate tuning systems. The main conclusion that can be drawn from a critique of the various systems of microtonal notation presented is that the simpler the better. This is of course not a new idea and Read among others has argued that the most effective method of notation microtonal music is to complicate the matter as little as possible, usually by deviating as little as possible from standard notation. Of course there are a number of ways this deviation can take place but modified accidentals are the most popular approach and the most practical. The issue then is what the new accidentals should look like. Should they be completely new symbols, or should they be modified versions of existing symbols. Personally I lean towards new symbols because modified symbols can confuse the issue even more as they can be mistaken for the existing accidental symbols. The issue of which symbols is ultimately left to the choice of the composer as it appears unlikely that there will be any consensus made within the music community in general. This is not however a major issue as long as the accidental symbols chosen are easy to recognize and used consistently throughout the piece. This is quite a small deviation from conventional notation is would not be hard for the performer to adapt.

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