Optimization with Process Limits and Application Requirements for Force Sensors

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***Abstract*—Piezoresistive cantilevers are commonly used for force sensing. However, the design of piezoresistive cantilevers is complicated by coupling between many design parameters as well as design and operation constraints. Here, we discuss analytical models and optimization techniques for piezoresistive cantilevers.**

I. Introduction

Piezoresistivity is a common transduction mechanism for force sensors [1]-[7]. Piezoresistive force sensors have advantages such as high dynamic range, relatively small size, simple fabrication, and straightforward signal conditioning circuitry. However, piezoresistive sensor design is challenging due to many coupled parameters such as cantilever and piezoresistor dimensions and fabrication process parameters. These parameters must be set together to optimize force resolution.

Many researchers have focused on improving the resolution of piezoresistive sensors [3]-[5]. However, previous optimization models have employed approximations to simplify the analysis, such as ignoring variations in dopant concentration and nonlinear phenomena such as fluid damping. Thus, existing analytical models and optimization techniques breakdown for cases such as thin, ion implanted devices and high bandwidth devices operating in liquid.

In this paper, we review analytical models and optimization techniques for piezoresistive cantilevers that we have recently reported [8]-[10]. We demonstrate improved analytical models and introduce an efficiency factor, β^*, which captures the reduction in sensitivity due to the dopant atoms being spread across the thickness. We also implement both analytical and numerical optimization techniques to demonstrate the choice of optimal design parameters that satisfy many commonly encountered design constraints. The design approach we describe is general and can easily be applied to other piezoresistive sensors.

II. Analytical Model

A. Force Sensitivity

Piezoresistivity (π) describes the change in the electrical resistivity (ρ) of a material due to applied mechanical stress (σ) [11][12],

$$\frac{\Delta\rho}{\rho} = \pi\sigma. \tag{1}$$

To calculate the change in resistance due to mechanical stress, we consider arbitrary profiles of electrical resistivity and mechanical stress. Because resistivity, carrier mobility, and piezoresistivity are functions of dopant concentration, these variables are defined locally. Therefore, it is necessary to average the change in resistance by integrating all local variables over the three dimensions.

We model the piezoresistor as a composite of many thin slices connected in parallel (Fig. 1), where the resistivity and piezoresistance coefficient of each thin slice are constant. We calculate the change in resistance of each slice, then integrate the conductance to compute the overall resistance change due to the applied force (see [8] for detailed calculations). With signal conditioning circuitry using a balanced 1/4-active

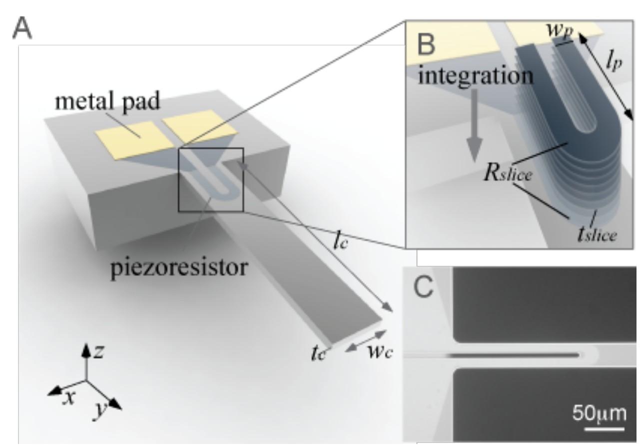

Fig. 1. Piezoresistive cantilever. (A) Geometry of cantilever. (B) Geometry of piezoresistor. We model the piezoresistor as a composite of many thin slices connected in parallel. We calculate change in resistance (R_{slice}) of each slice and compute the overall resistance change by integrating the conductance. (C) Typical piezoresistive cantilever with U-shpaed piezoresistor. Reprinted from Park et al. [9] © 2010 IEEE

Wheatstone bridge configuration, the force sensitivity, S, of a piezoresistive cantilever (length l_c, width w_c, and thickness t_c) with a U-shaped piezoresistor (length l_p and width w_p) is

$$S = \frac{3(l_c - 0.5 l_p)\pi_{l.\max}}{2 w_c t_c^2} \gamma V_{bridge} \beta^* . \tag{2}$$

$\pi_{l,\max}$ is the maximum longitudinal piezoresistivity at 300 K, V_{bridge} is the Wheatstone bridge bias voltage, and γ is a geometric factor defined as the ratio of the resistance of the strained region in the piezoresistor to the total resistance including unstrained regions, interconnectors, and contact pads. β^* is an efficiency factor defined by

$$\beta^* = \frac{2}{t_c} \frac{\int_{-t_c/2}^{t_c/2} q\mu p P z dz}{\int_{-t_c/2}^{t_c/2} q\mu p dz},. \tag{3}$$

where q and P are the elementary charge, and longitudinal piezoresistance factor, respectively. β^* describes how the dopant profile affects the sensitivity of the device. If dopants are close to the surface and the dopant concentration is low enough to maintain a high piezoresistance factor across the resistor, then β^* is close to one. If the dopants are uniformly distributed through the thickness of the cantilever or the dopant concentration is very high, then β^* approaches zero. β^* is a function of only cantilever thickness and process parameters.

B. Noise

Piezoresistive cantilever performance is limited by four primary sources of noise: Johnson (V_J), 1/f (V_H), amplifier (V_A), and thermomechanical noise (V_M) [9][10]. These noise sources are uncorrelated, so the overall root mean square voltage noise can be computed from

$$\overline{V_{total}} = \sqrt{\overline{V_J^2} + \overline{V_H^2} + \overline{V_A^2} + \overline{V_M^2}} . \tag{4}$$

Johnson noise (V_J) is the result of the thermal motion of carriers within a resistive element and is independent of frequency. 1/f noise (V_H) is a fluctuation in resistor conductance that varies inversely with frequency. There are many possible 1/f noise mechanisms, but piezoresistor 1/f noise is generally believed to be due to defects in the bulk of the material. The noise inherent to signal conditioning circuitry (V_A) and intrinsic to the cantilever due to the thermal energy of the silicon atoms (V_M) must considered as well.

C. Force Resolution

The force sensitivity and noise of the piezoresistive cantilever determine the minimum detectable force or force resolution,

$$F_{\min} = \frac{\overline{V_{total}}}{S} . \tag{5}$$

Experimental results agree well with the analytical model. For example, the analytical prediction of ten different cantilevers fabricated with various process conditions is comparable to the experimental results in Fig. 2.

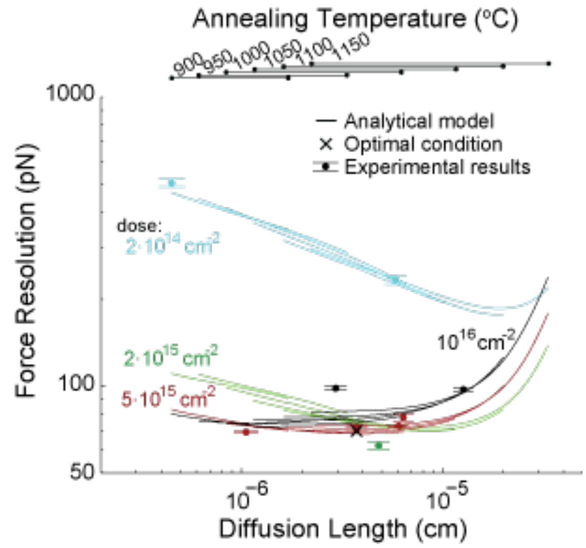

Fig. 2. Force resolution of sample piezoresistive cantilever with 2-mW maximum power dissipation and 2-V maximum bias voltage. We calculated the force resolution for a variety of process parameters. The experimental results (three cantilevers each wafer) agree well with the analytical model. The experimental cantilevers achieve minimum force resolutions of 69.8 pN, which are comparable to the optimal force resolution of 68.1 pN, which is predicted by analytical optimization technique. Reprinted from Park et al. [9] © 2010 IEEE

D. Design Constraints

The performance of the piezoresistive cantilever is constrained by several factors such as power dissipation, bandwidth, spring constant.

1) Power dissipation (W): Force resolution improves with power dissipation. However, there is a limit to the maximum power dissipation sustained by the cantilever due to Joule heating. Thus, we achieve the optimal force resolution by increasing the power dissipation to the maximum which the cantilever can sustain.

2) Bandwidth (f_{bw}): The resonant frequency (f_0) determines the upper limit of the measurement bandwidth. The resonant frequency depends on cantilever dimensions and the fluid environment [13].

3) Spring constant (k_c): The spring constant is determined by the dimensions of the cantilever and elastic modulus of the cantilever. When measuring material properties with a cantilever, cantilever stiffness should typically be comparable to that of the sample in order to sufficiently deform it without damaging it.

III. Optimization

To optimize the performance of piezoresistive cantilevers, we should choose design and process parameters to achieve the best resolution within constrains discussed in the previous section. There are four parameter types: cantilever dimensions, piezoresistor dimensions, fabrication process and operating parameters. Here, we summarize analytical and numerical techniques for choosing optimal cantilever design parameters.

A. Analytical Technique

The optimization technique uses an analytical model of force resolution (5) with constraints. We summarize the analytical technique in Fig. 3 and more information is available in [9]. The optimal parameters are usually determined from design and operating constraints. For example, the cantilever dimensions are chosen by the measurement bandwidth and desired stiffness of cantilever. The optimal piezoresistor dimensions can be determined based on efficiency factor, sheet resistance, and number of carrier for a variety of process conditions. Fabrication process and operating parameters can be chosen by considering maximum power dissipation and maximum bias voltage.

The advantage of this technique is that we can easily see how parameters affect cantilever performance. In optimization of sample piezoresistive cantilever with 2-mW maximum power dissipation and 2-V maximum bias voltage (Fig. 2), we find that the optimal force resolution is obtained for an intermediate ion implantation dose and diffusion length due to interplay between the efficiency factor, the maximum bias voltage, and the maximum power dissipation. Although an analytical approach is useful for most piezoresistive cantilever designs, it is difficult to apply to nonlinear phenomena, which are not modeled analytically, such as fluid damping, which are more readily modeled numerically.

B. Numerical Technique

To overcome the problem of analytical optimization technique, we combine a primarily analytical model for

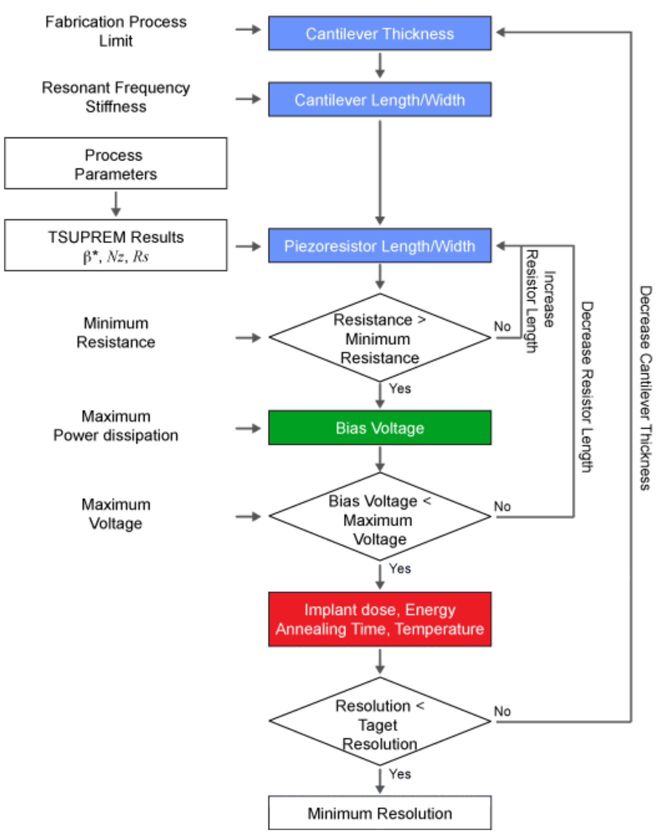

Fig. 3. Analytical optimization flowchart. The optimal parameters are chosen by design constraints. Reprinted from Park et al. [9] © 2010 IEEE

cantilever performance with an iterative optimizer [10]. The optimal parameters to achieve the best force resolution are found iteratively using a quasi-Newtonian optimization method. Because the piezoresistive cantilever design problem is not convex, the local optimum found by the optimizer is not guaranteed to be the global optimum. A large number ($>10^5$) of convergence tests of the optimizer with randomly generated starting points found that the global optimum is found more than 99% of the time. In practice, the global optimum is readily found by seeding the optimizer with several random starting conditions, as shown in Fig. 4.

The benefit of numerical optimization is that constraints are applied as bounds rather than fixed parameter values, leading to find the global optimal conditions while satisfying many complex nonlinear constraints such as the frequency response of the cantilever in liquid. This numerical optimization is easily extensible to other loading conditions, sensor geometries, transduction methods, and optimization goals [14].

We implemented numerical optimization method and nonlinearity analysis [9], to calculate dynamic range of typical n-type (phosphorus) diffused cantilever with 1-1000 Hz bandwidth in Fig. 5. We also applied both design constraints ($w_c > 3t_c$, and $l_c > 10w_c$) and operation constraints ($V_{bridge} < 10$ V, $f_0 > 5f_{bw}$, and change in temperature of piezoresistor should be smaller than 25 degree C.) We calculated minimum and maximum detectable forces of the diffused cantilever with various cantilever thickness (0.5 to 25 μm) and stiffness. We found that the diffused cantilever can detect sub-pN to mN force and tens of pm to tens of mm displacement. The upper and lower bounds of cantilever stiffness with given thickness are determined by constraints ($w_c > 3t_c$, $l_c > 10w_c$, and $f_0 > 5f_{bw}$).

IV. CONCLUSION

We have summarized an improved analytical model as well as analytical and numerical optimization techniques for

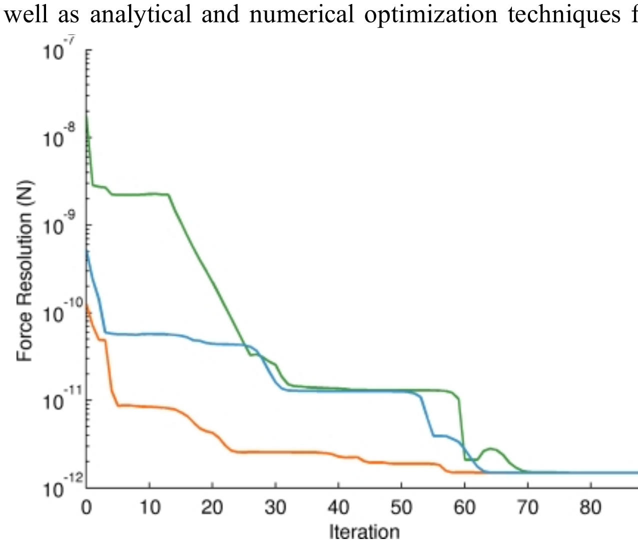

Fig. 4. Numerical optimization with three randomly generated starting points. Force resolution improves as the optimizer iterates. Despite the non-convex nature of the problem, the global optimum is readily found from repeated calculations. Reprinted from Doll et al. [10] © 2009 American Institute of Physics.

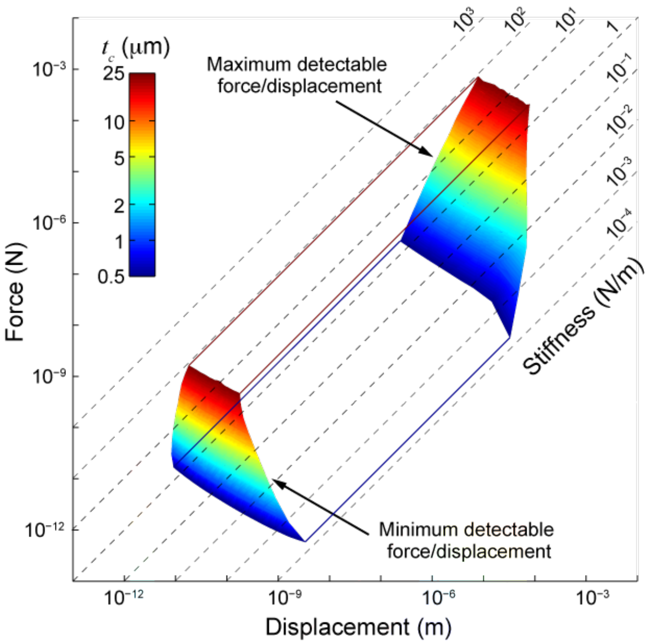

Fig. 5. Dynamic range of typical n-type (phosphorus) diffused cantilever with 1-1000 Hz bandwidth for a range of device thickness (t_c) and stiffness (k_c) values. Piezoresistive cantilevers can detect forces and displacements from the atomic scale (sub-nm and pN) to the macroscale (mN and mm). Thinner cantilever can have a broader range of stiffness values while satisfying the resonant frequency requirement. For example, 0.5-μm thick cantilever has stiffness from 0.17 mN/m to 1.8 N/m, while a 25-μm thick cantilever ranges in stiffness from 2.6 to 89 N/m. As the device thickness becomes thinner, the minimum detectable force and displacement improve. Additionally, for a fixed thickness, soft cantilevers have better force resolution while stiff cantilevers have better displacement resolution.

piezoresistive cantilevers. The results have been experimentally verified for a range of process conditions. We

have also demonstrated that analytical and numerical optimization techniques can achieve optimal cantilever design by balancing the competing requirements of low noise and high sensitivity. We have also implemented optimization technique to calculate dynamic range of diffused cantilever. The results indicate that piezoresistive cantilever design can be tuned to detect force and displacement signals at the atomic scale (sub-nm and pN) to the macroscale (mN and mm) with a single cantilever.

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