

# Status Update on High-Frequency Cantilever Design for Fluid Operation

Joey Doll, June 2008

# Overview

- **Cantilevers in Fluid**
  - Analytical modeling
  - Implications for design
- Numerical optimization of epitaxial cantilevers
  - Comparison with Previous Approaches
  - Numerical model
  - Comparison with experimental data (Harley)
  - Implementation
  - Results
- Piezoelectric Actuation Model
- Summer and Future Work

# Fluid Modeling

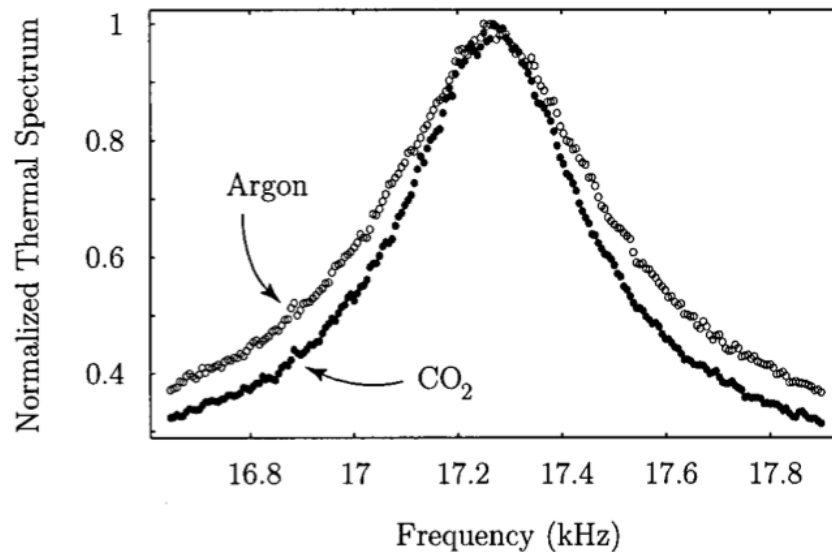
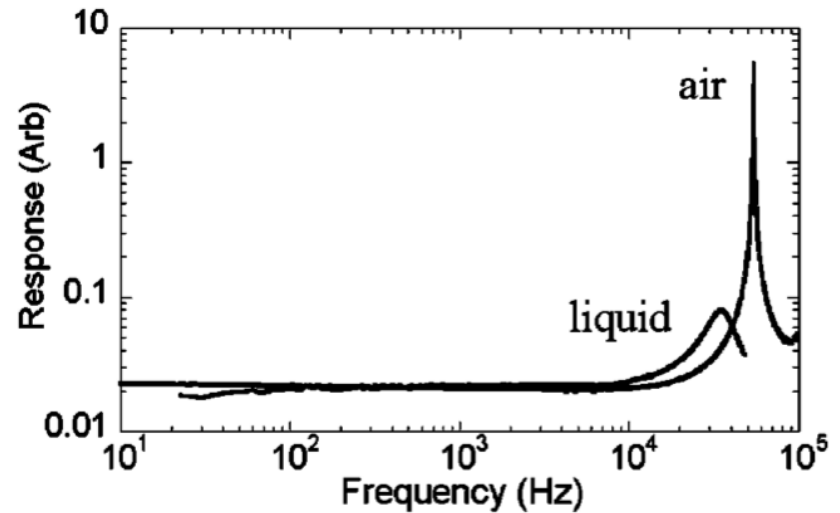
- Oscillations of a thin blade on fluid
  - W. Chu, 1963
- Extended to cantilevers for AFM
  - J. Sader, 1993 - Ongoing
- Analytical approximations for frequency spectra of cantilevers in fluid

# Fluid Modeling

- Assumptions
  - $t \ll w \ll L$
  - Rectangular, uniform cross-section
  - Vibrations smaller than any dimensions
  - Fluid damping dominates internal dissipation
  - Incompressible, infinite viscous fluid
- Variations on this analysis exist (e.g. inviscid)

# Fluid Modeling

**High-speed atomic force microscopy in liquid,**  
Sulcheck, Hsieh, Adams, Minne, Quate,  
Adderton, Rev. Sci. Instr. (2000)



**Rheological measurements using  
microcantilevers,** Boskovic, Chon, Mulvaney,  
Sader, Soc. Of Rheology (2002)

# Fluid Modeling

$$\text{Re} = \frac{\rho \omega w^2}{\eta} \quad \kappa = C_n \frac{w}{L}$$

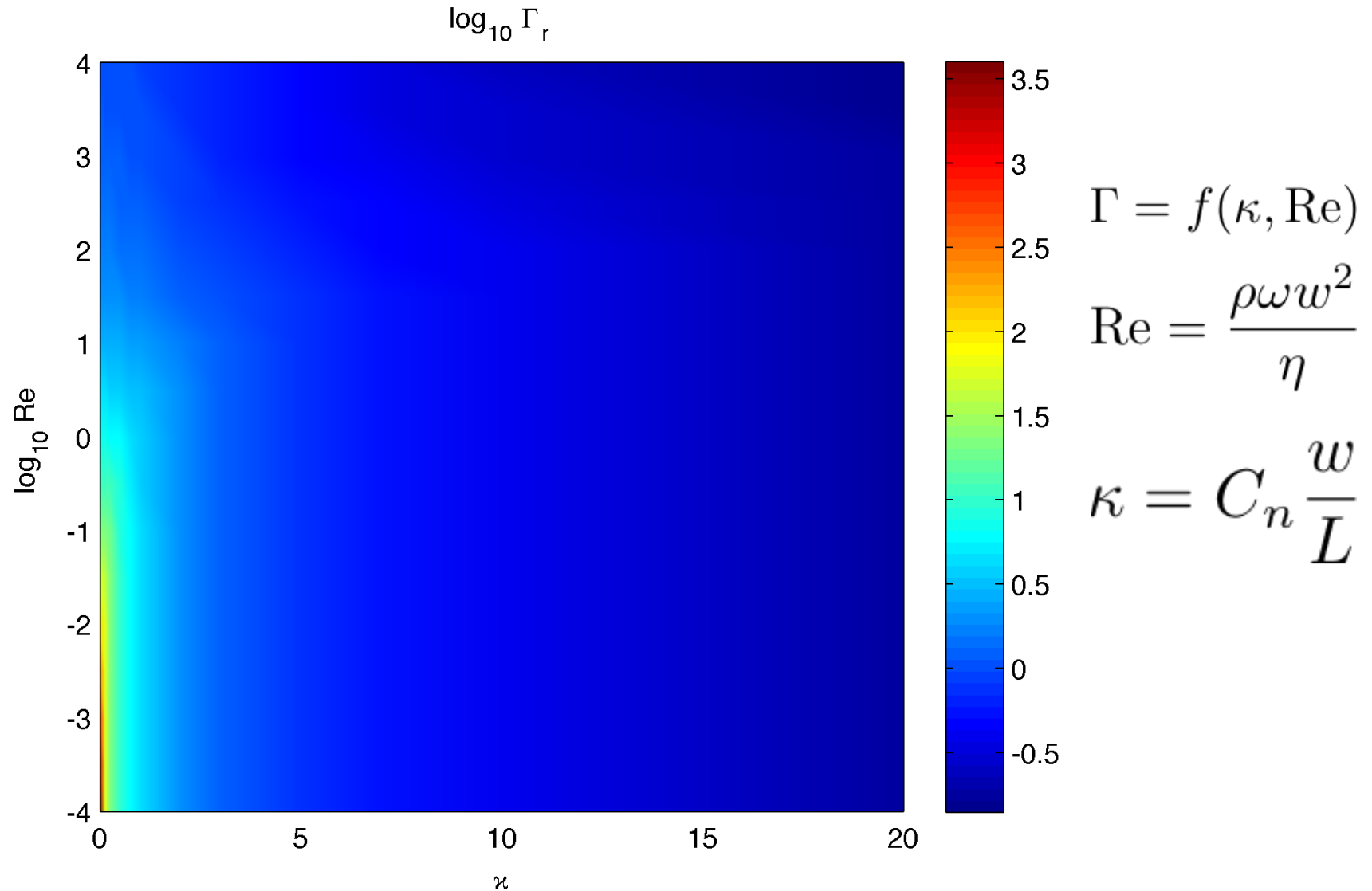
$$\omega_{damped} = \omega_{vac} \left[ 1 + \frac{\pi \rho_f w}{4 \rho_c h} \Gamma_r(\omega_{damped}, n) \right]^{-1/2}$$

$$Q_n = \frac{\frac{4 \rho_c h}{\pi \rho_f w} + \Gamma_r(\omega_{damped}, n)}{\Gamma_i(\omega_{damped}, n)}$$

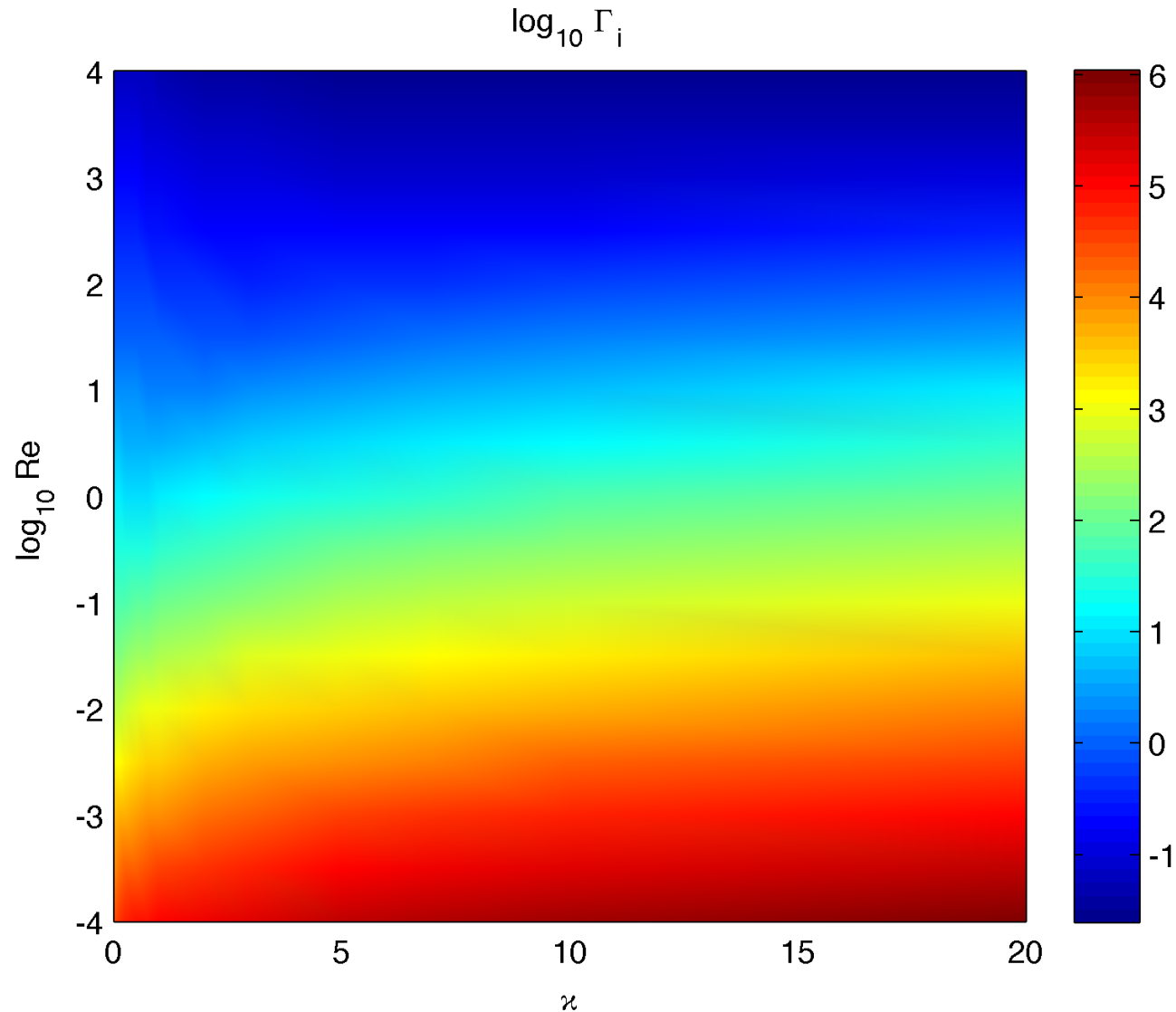
Frequency response of cantilever beams immersed in  
viscous fluids with applications to the atomic force  
microscope: Arbitrary mode order. Eysden and Sader (2007)

Valid for  $Q \gg 1$  or  $\text{Re} \ll 1$

# Hydrodynamic Function



# Hydrodynamic Function



$$\Gamma = f(\kappa, \text{Re})$$

$$\text{Re} = \frac{\rho \omega w^2}{\eta}$$

$$\kappa = C_n \frac{w}{L}$$



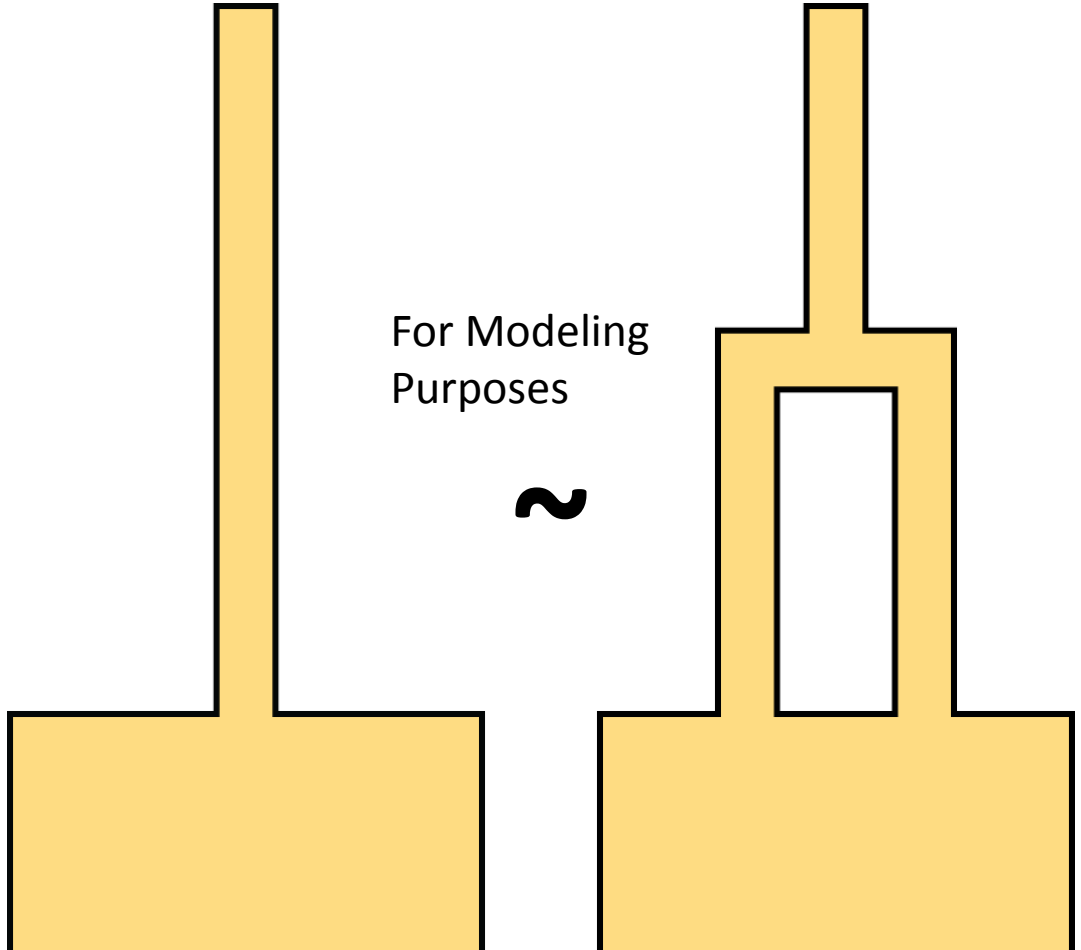
# Fluid Modeling

$L, w, t$

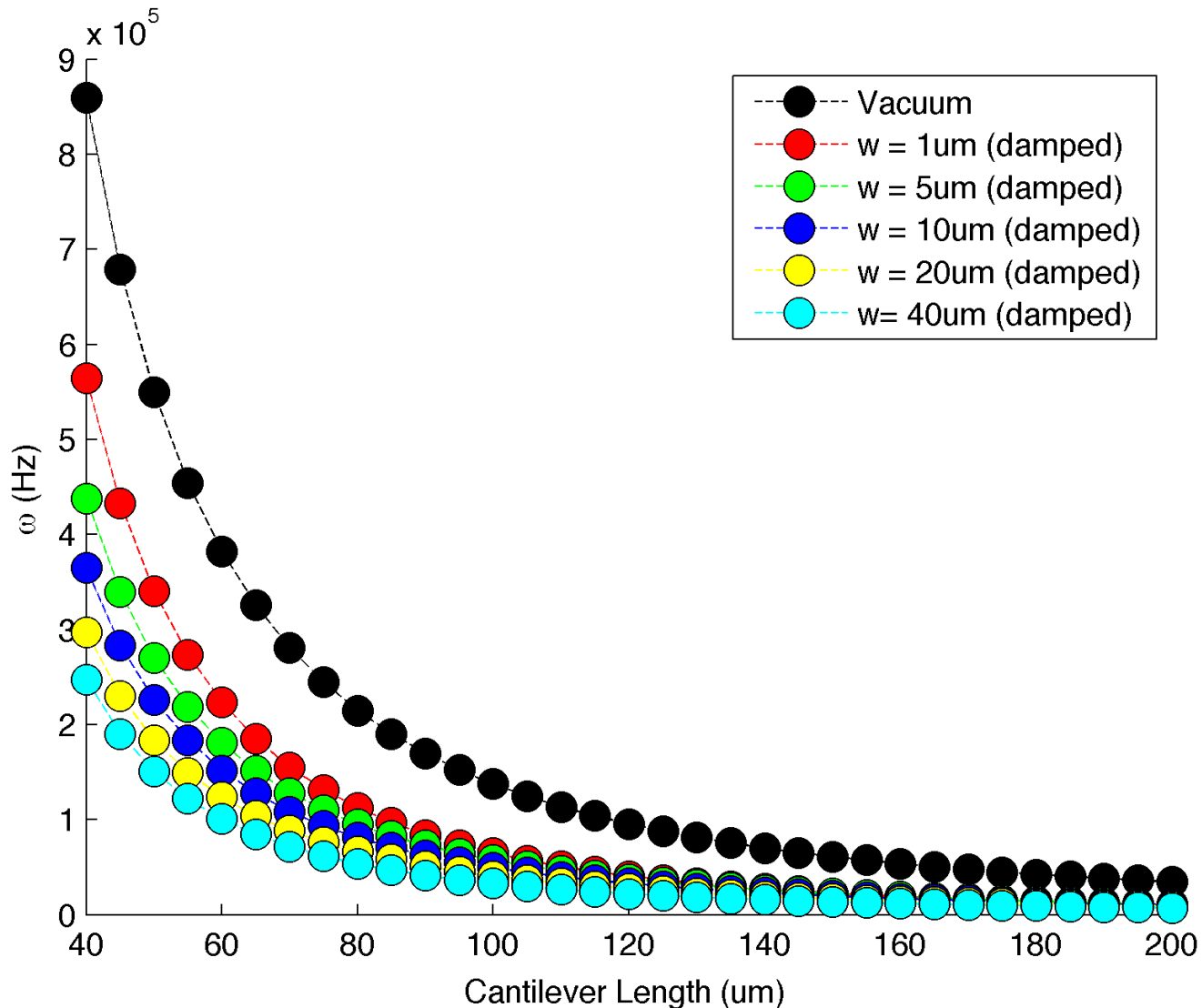
Damping  $\sim w^2$

For Modeling  
Purposes

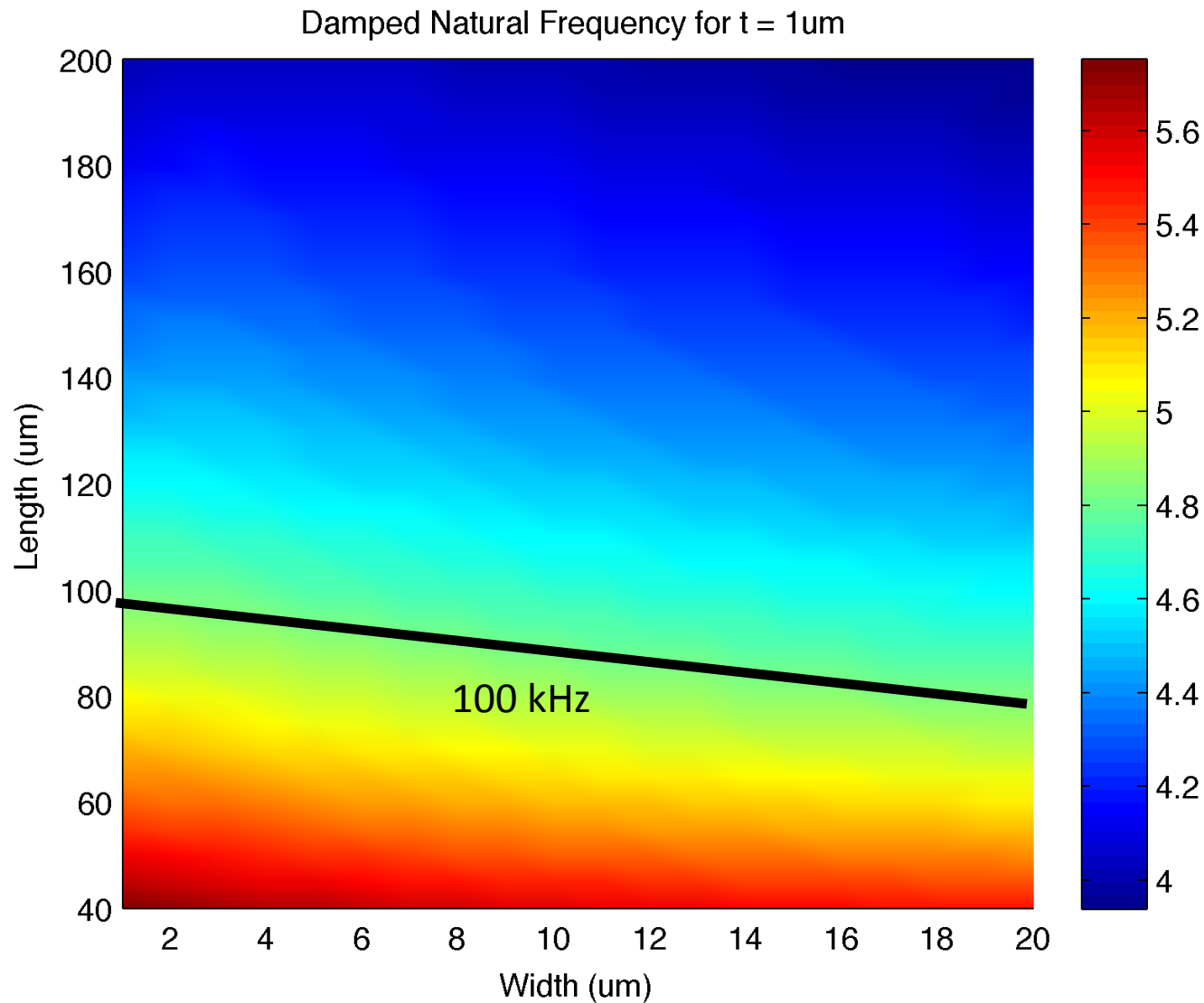
$\sim$



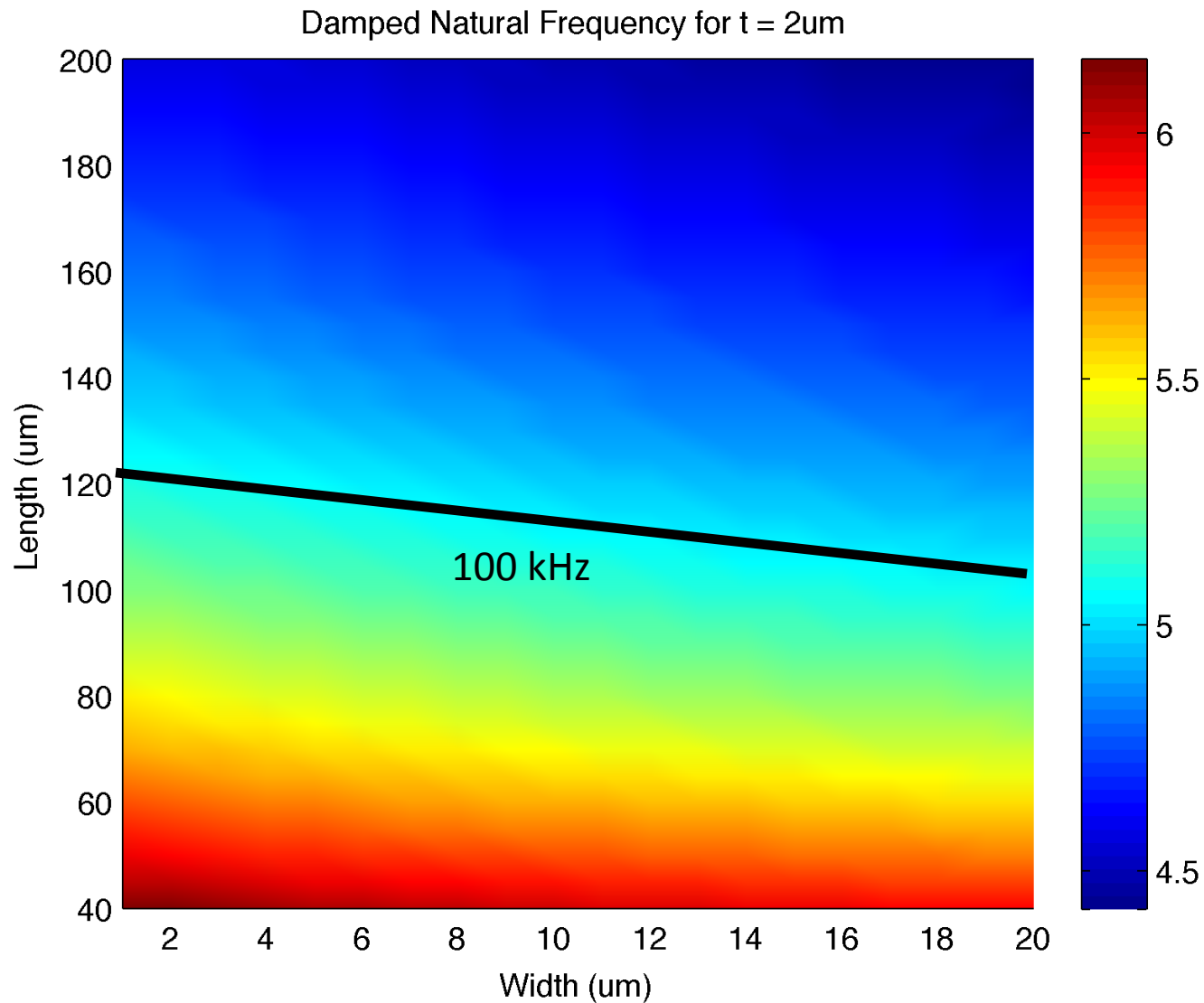
# Vacuum vs. Liquid Frequencies



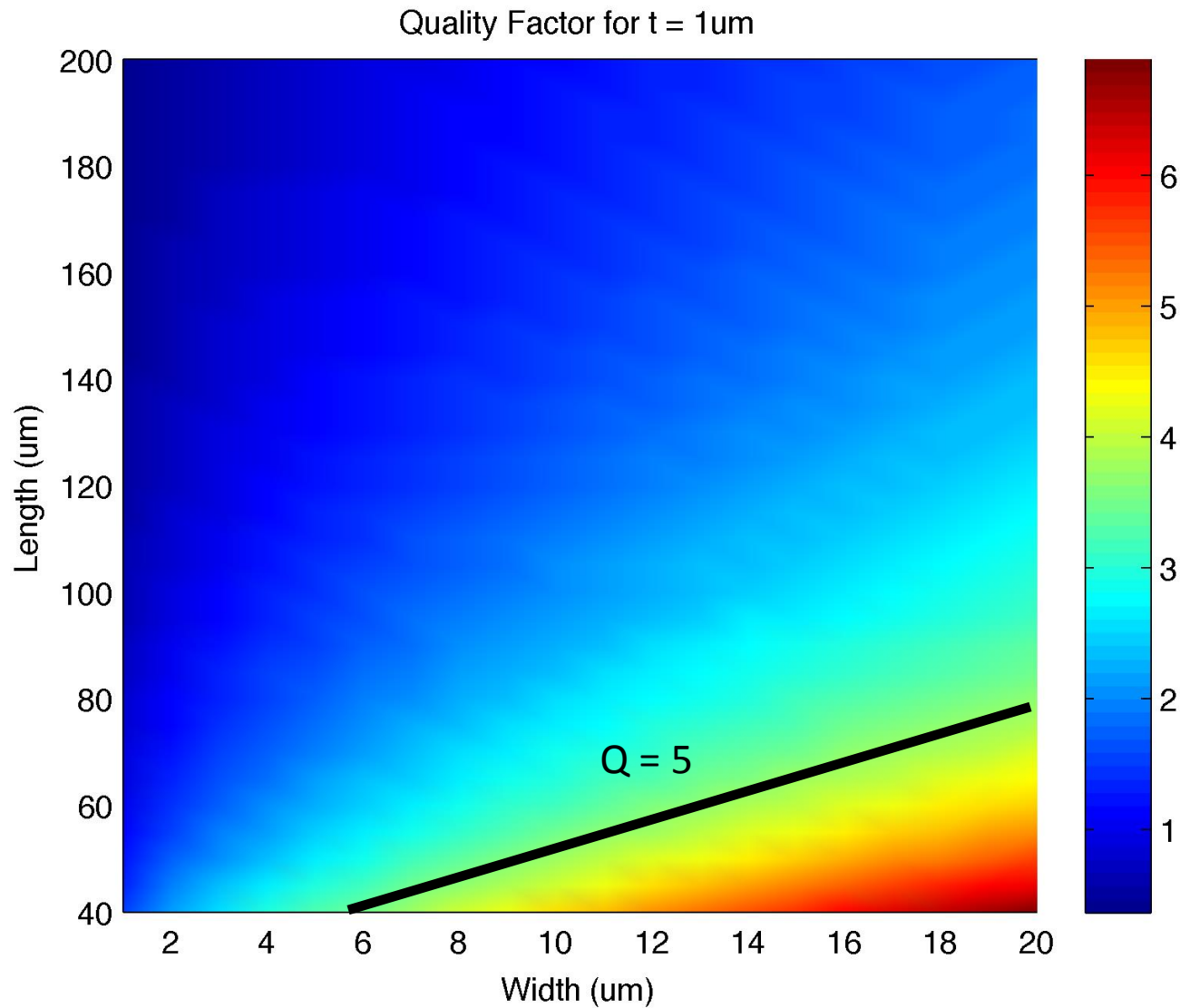
# Damped Natural Frequency



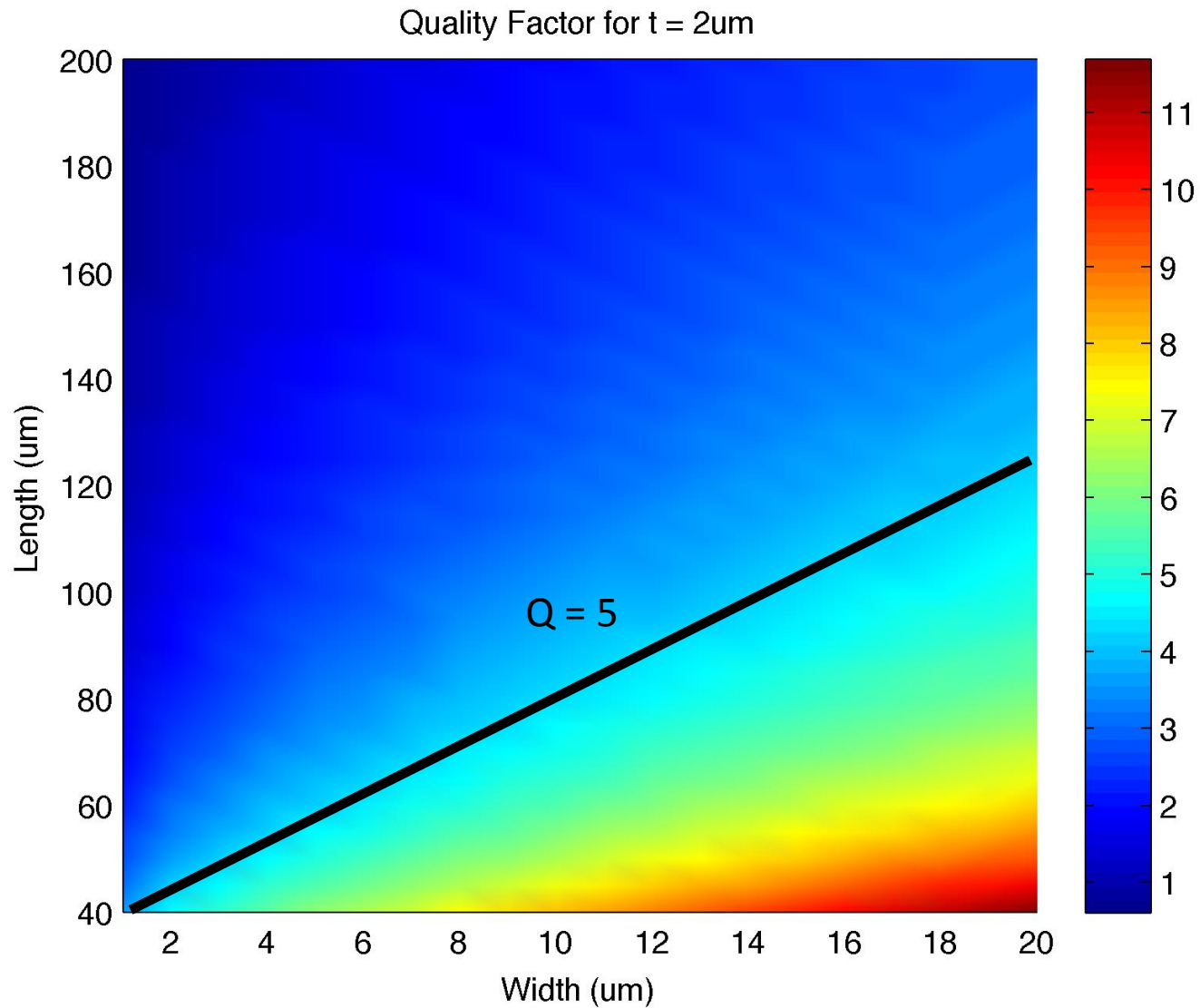
# Damped Natural Frequency



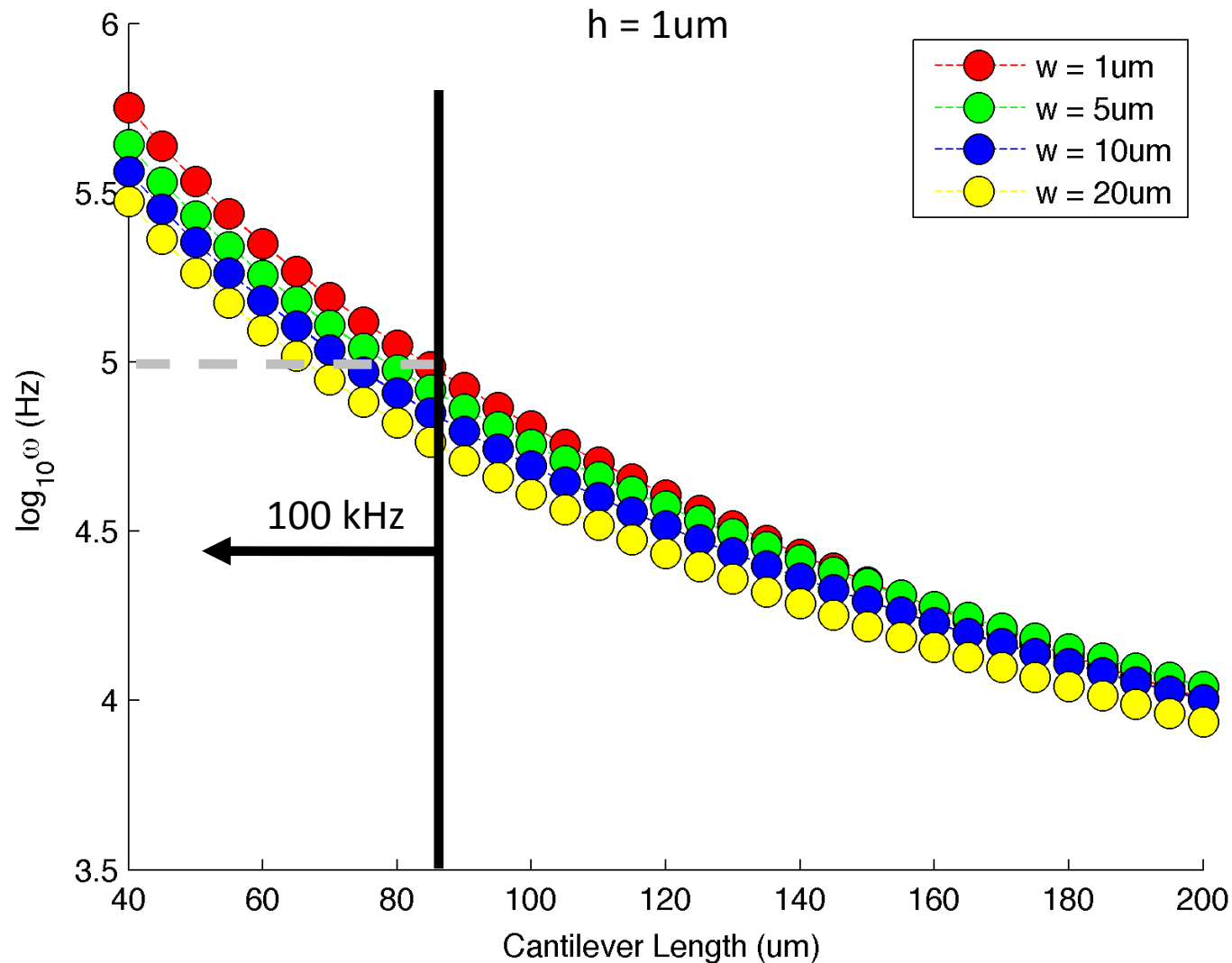
# Quality Factor



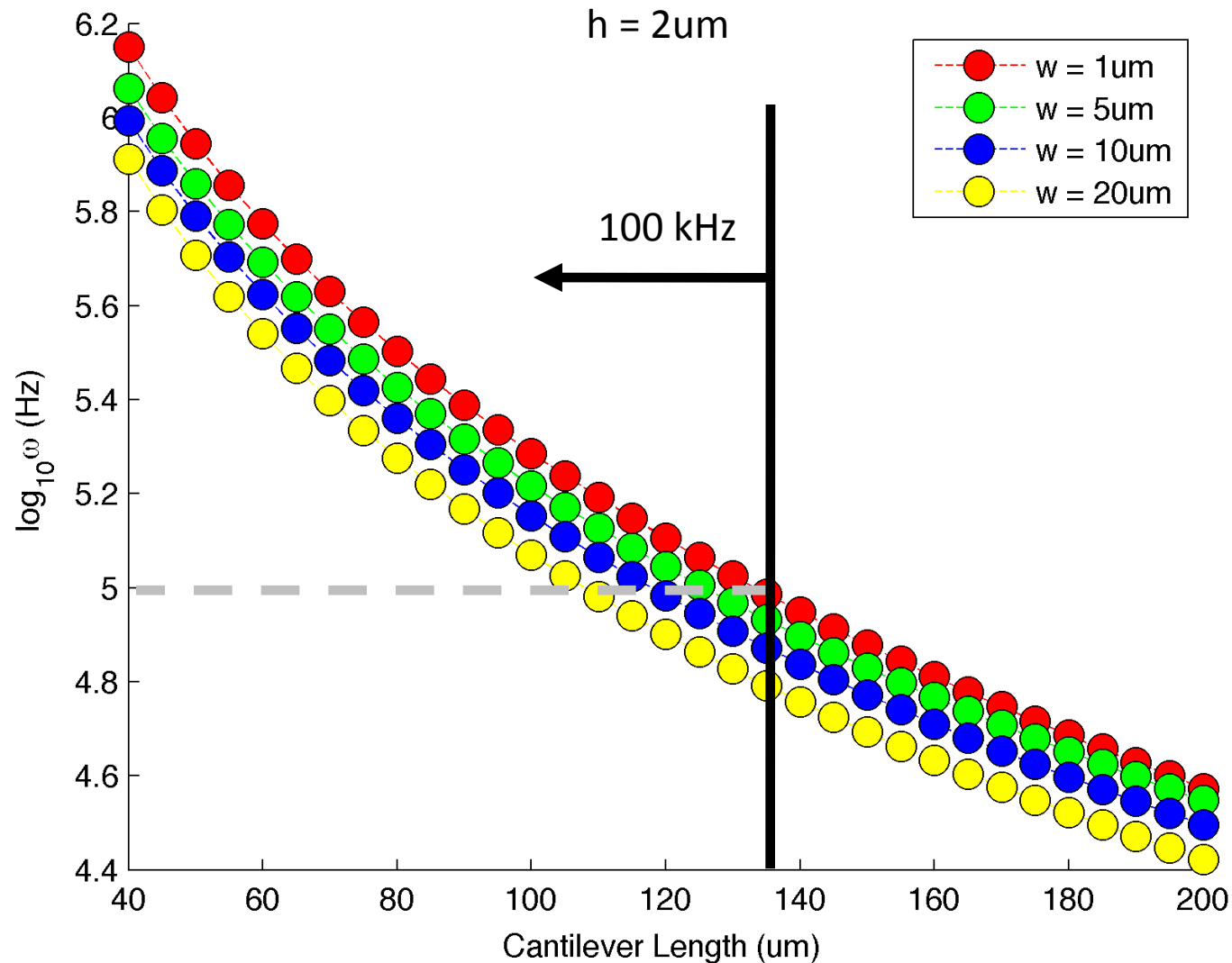
# Quality Factor



# Damped Natural Frequency

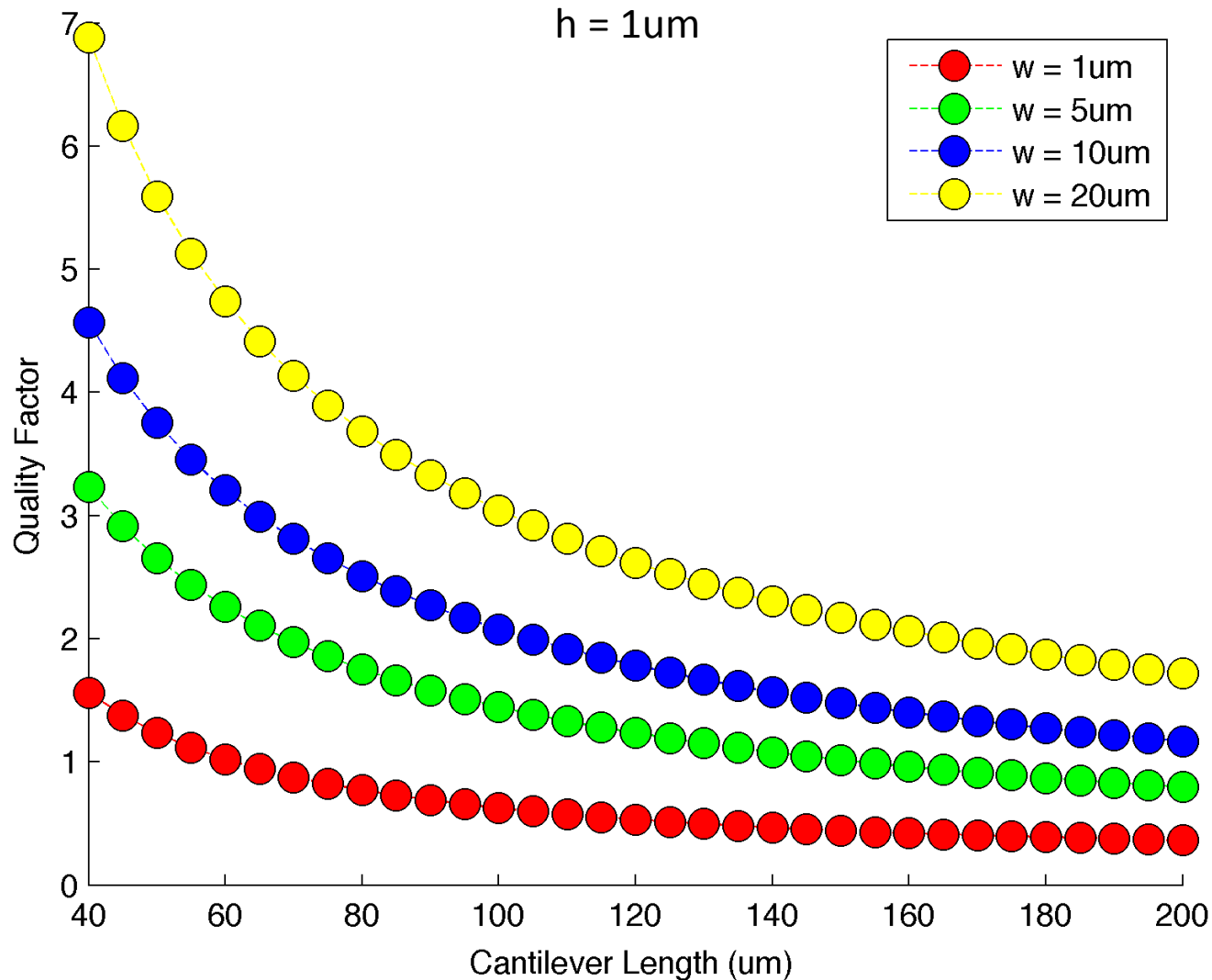


# Damped Natural Frequency

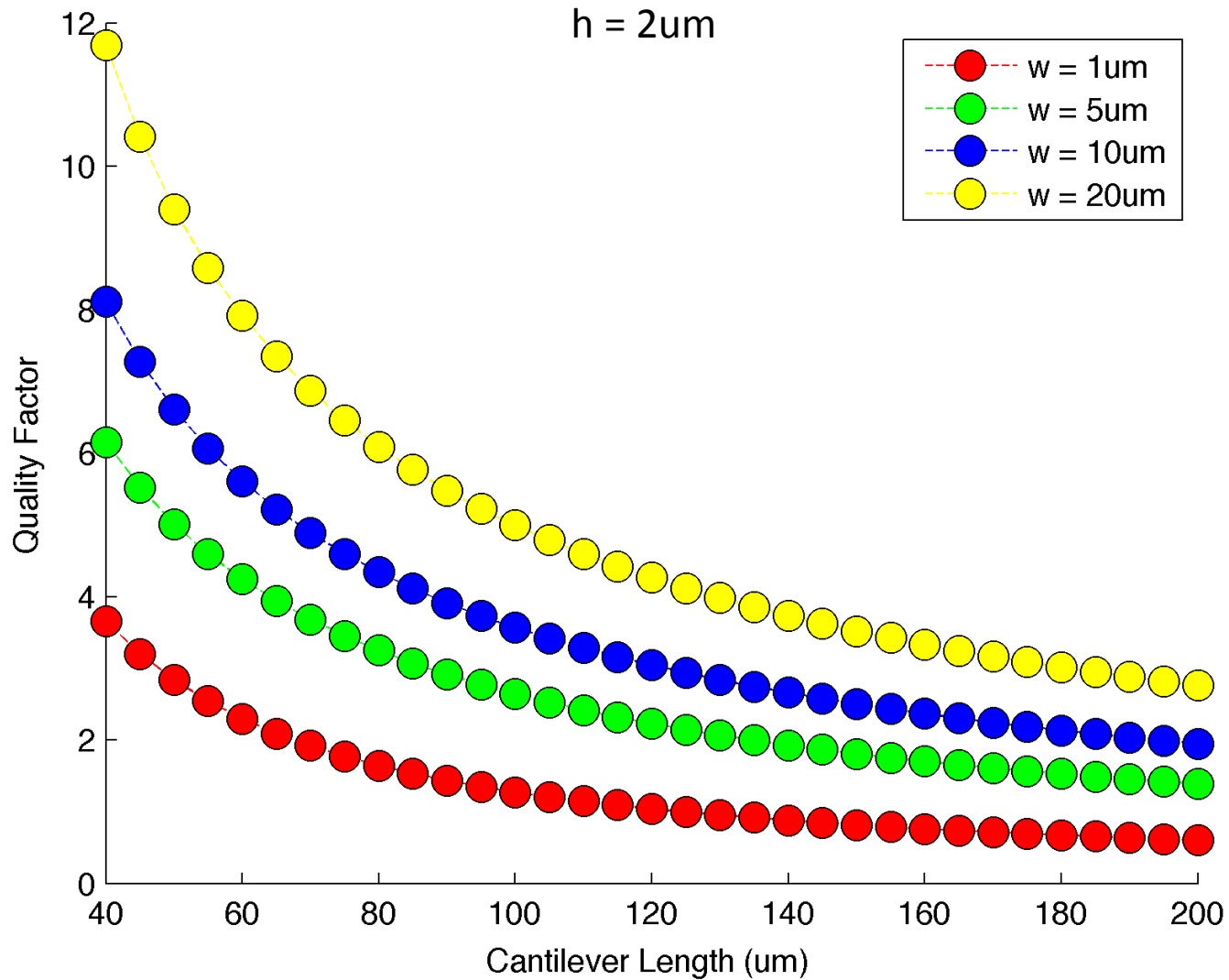




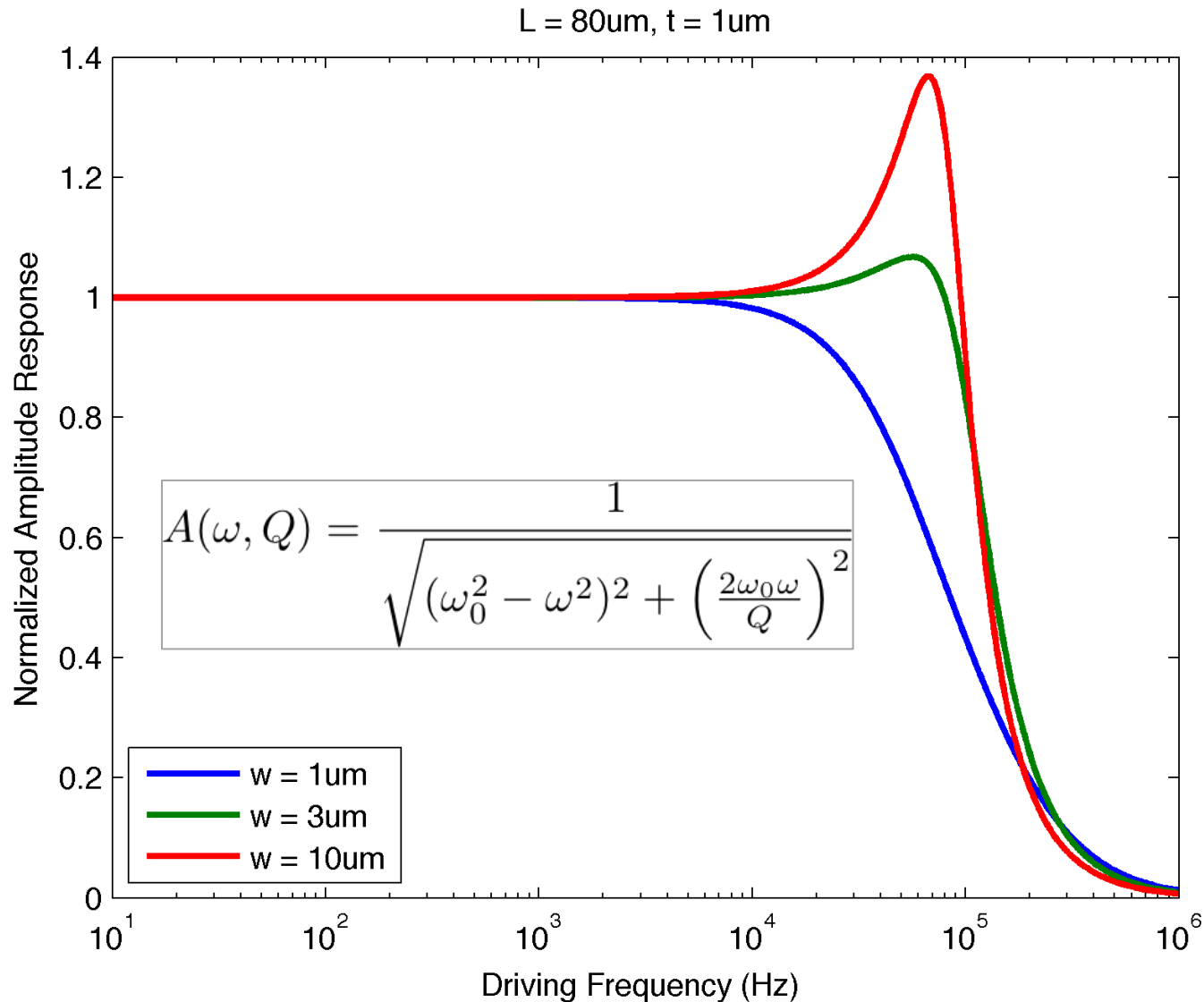
# Quality Factor



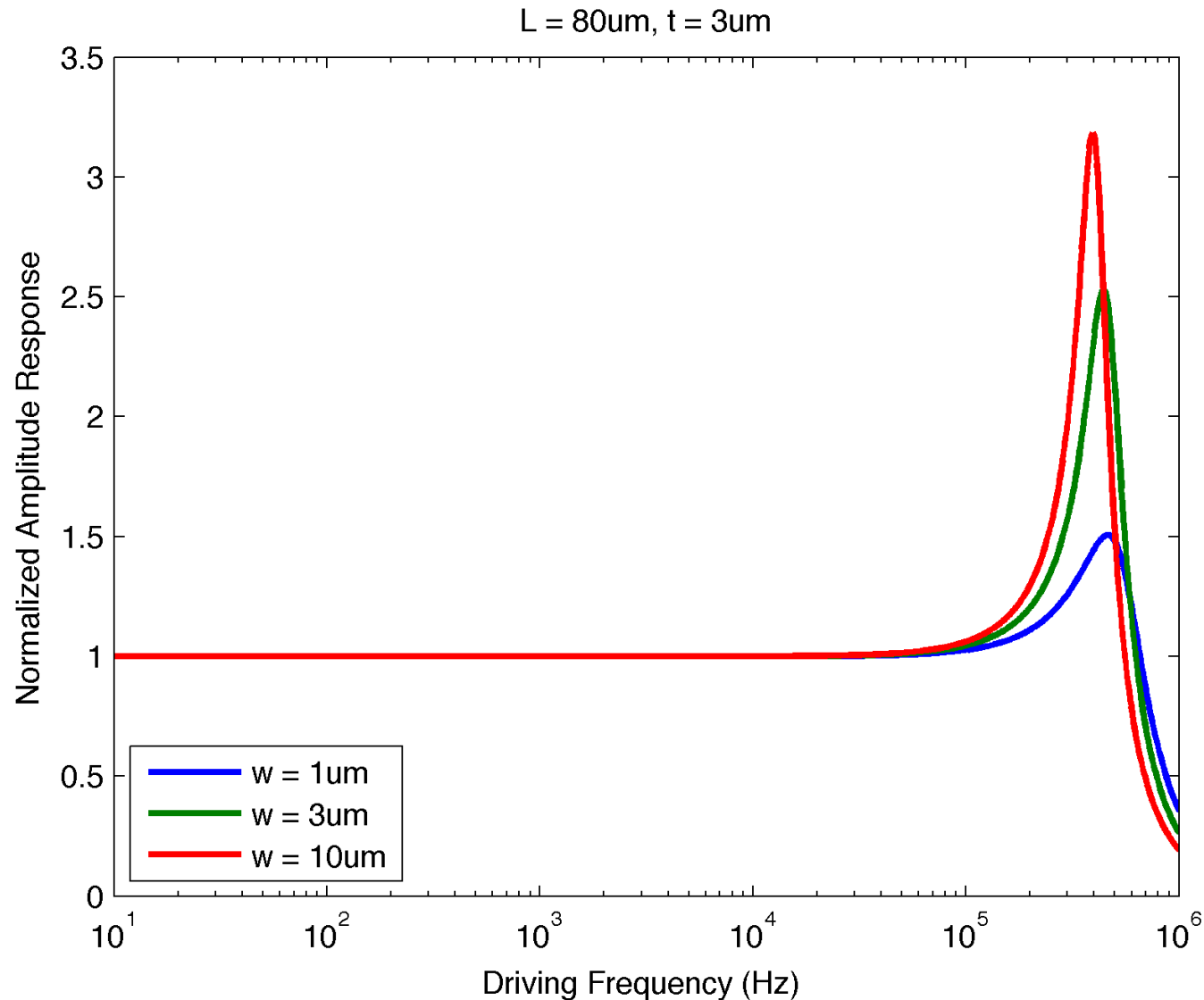
# Quality Factor



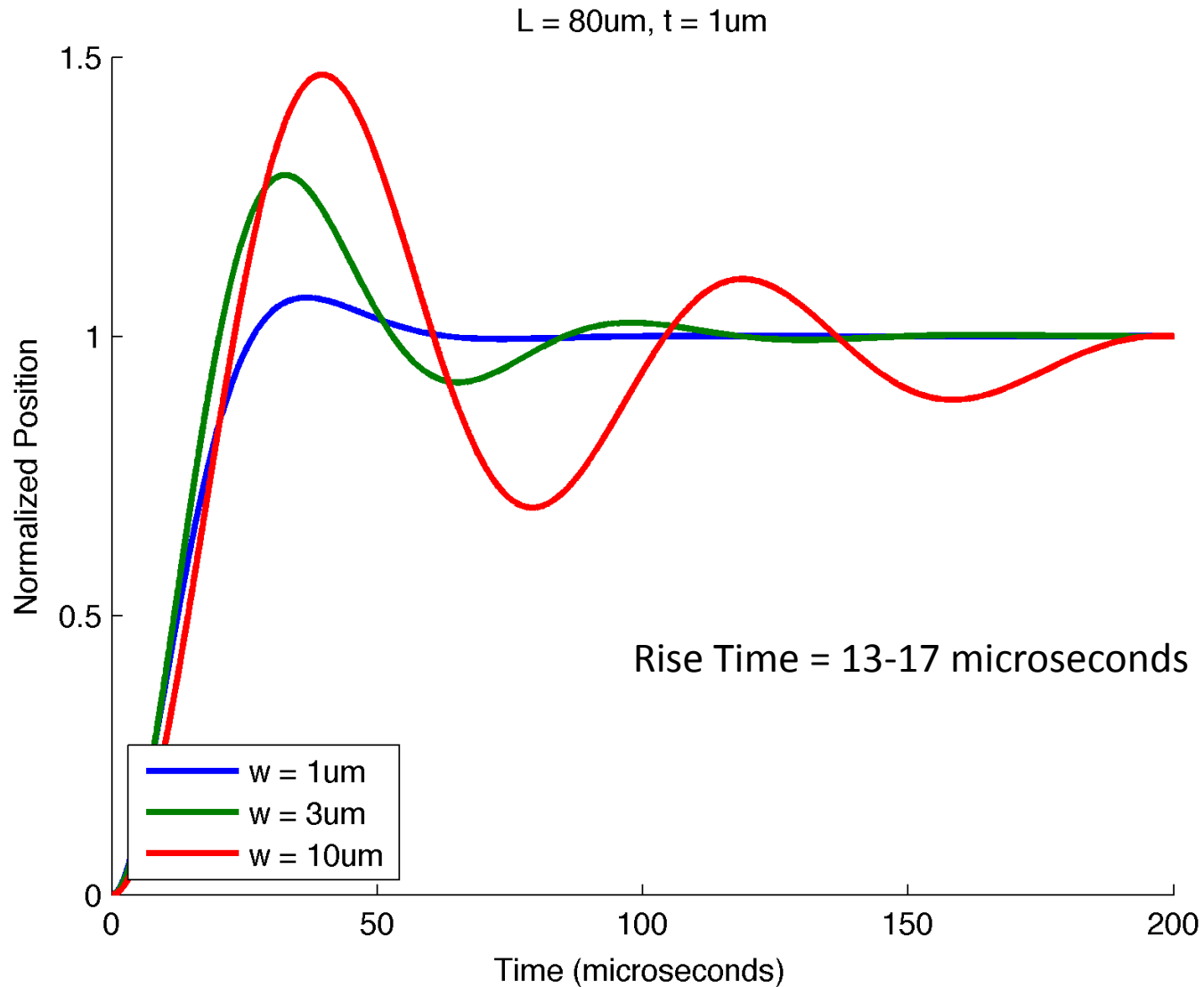
# Frequency Response (SHO)



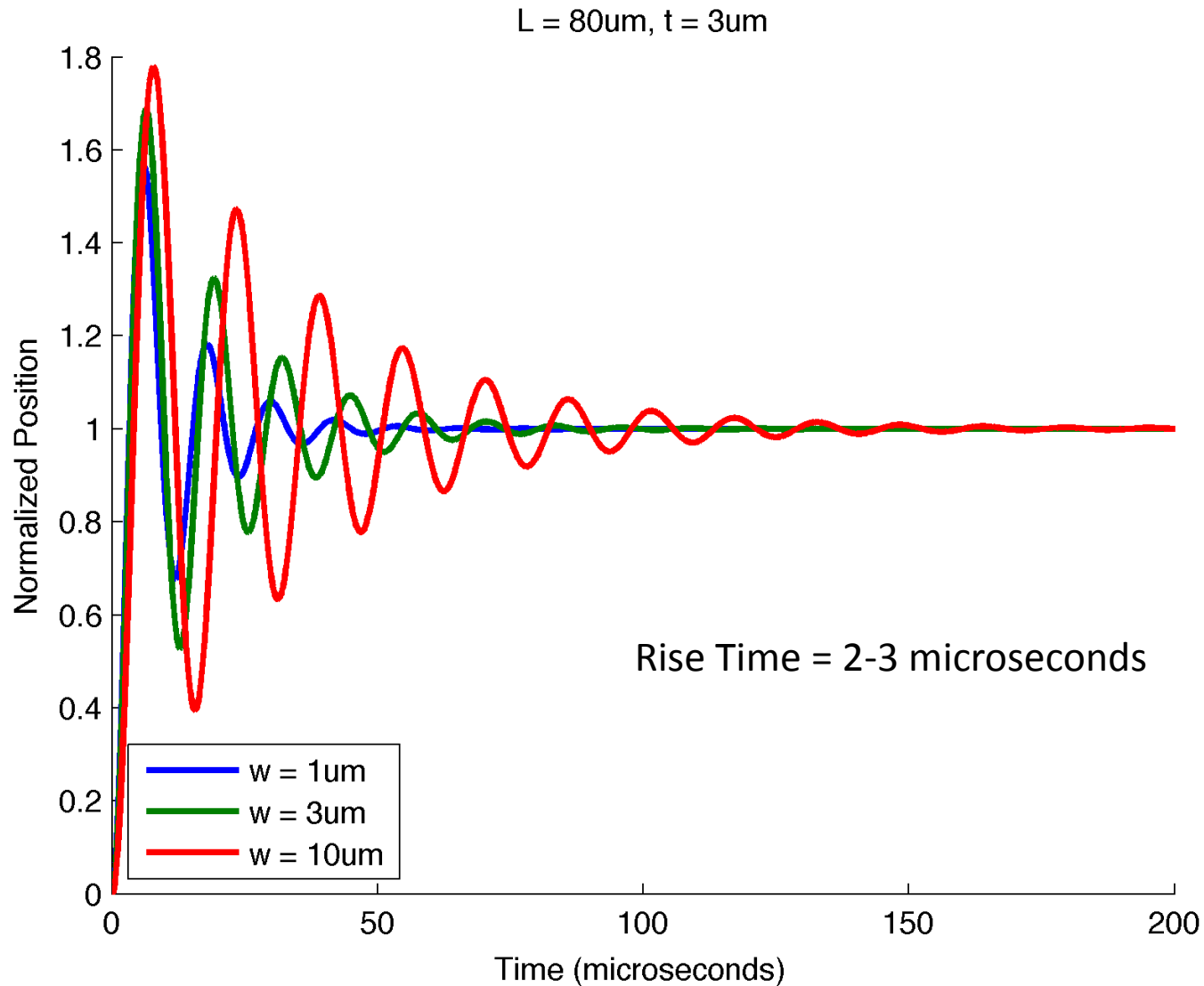
# Frequency Response (SHO)



# Open-Loop Step Response



# Open-Loop Step Response



# Fluid Modeling Conclusions

- Flat frequency response to 100 kHz requires damped natural frequency  $\sim 500$  kHz
- Width matters in fluid. Reducing width:
  - Increases first resonant mode frequency
  - Reduces quality factor
- But won't want to reduce w/t too much
- Microsecond rise time is feasible, independent of Q
- Cantilever near a surface
  - Surface doesn't matter if gap  $>$  cantilever width
  - If gap  $<$  cantilever width, dissipation increases dramatically

# Overview

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# Comparison with Other Approaches

- Normal process
  - Pick a thickness
  - Choose length and width based upon natural frequency constraints
  - Choose piezoresistor dimensions from Harley analysis of  $1/f$  noise considerations or Sung-Jin's work for ion implantation
  - Choose doping & maximize bias voltage

# Comparison with Other Approaches

- Can we write the problem more generally?
- Problem statement: Minimize force resolution given
  - Frequency span of signal to be measured
  - Fabrication constraints
  - Power dissipation (quasi-constraint)
- Other optimizations
  - Minimize rise time
  - Maximize effective bandwidth (e.g.  $< 5\%$  frequency response deviation)
  - Minimize noise at a particular frequency

# Model and Goal

- Optimize cantilever inputs to maximize performance
  - Cantilever geometry
  - Piezoresistor Geometry
  - Piezoresistor doping
  - Bias voltage
- Constants
  - Fluid media
  - Cantilever mechanical properties
- Constraints
  - Resonant frequency
  - Power dissipation
- Goals
  - Force resolution

# Electrical Modelling

- Piezoresistor
  - Epitaxial only (for now, see conclusions)
  - Concentration dependent mobility
  - Piezoresistive coefficient from Harley data
  - Voltage drop completely across the piezoresistor (reasonable for metal lines)
- Noise
  - 1/f, Johnson, amplifier (no thermomechanical, fluid yet)
  - $\alpha = 10^{-5}$  (approximated from Harley data)

# How Well does the Model Fit?

- Compare model to epi cantilever experimental data
  - Harley's 89nm thick cantilever

Cantilever

<b>Thickness</b>	89 nm
<b>Width</b>	44 $\mu\text{m}$
<b>Length</b>	300 $\mu\text{m}$

Piezoresistor

<b>Thickness</b>	30 nm
<b>Width</b>	44 $\mu\text{m}$
<b>Length</b>	45 $\mu\text{m}$
<b>Gap</b>	3 $\mu\text{m}$

Other

<b>Freq Min</b>	10 Hz
<b>Freq Max</b>	1 kHz
<b>Bias</b>	5 V
<b>Doping</b>	$4\text{e}19/\text{cm}^3$
<b>Alpha</b>	$1\text{e}-5$

# How Well does the Model Fit?

	Calculated	Measured
Force Resolution (pN)	0.35	0.5
Resistance (Ohms)	2003	-
Integrated Noise (V)	5.2e-7	1.14e-6
Stiffness (N/m)	4.8e-5	3e-5
Knee Frequency (Hz)	1534	1000
Piezo Factor (m <sup>2</sup> /N)	3.7e-10	4e-10
Johnson Noise Density (V)	5.7e-9	1.5e-8
Power Dissipation (mW)	12.5	-
Natural Freq, Vacuum (Hz)	1359	-
<b>Natural Freq, Water (Hz)</b>	<b>34</b>	-
Quality Factor	0.5	

Pretty good

# The Optimization Problem

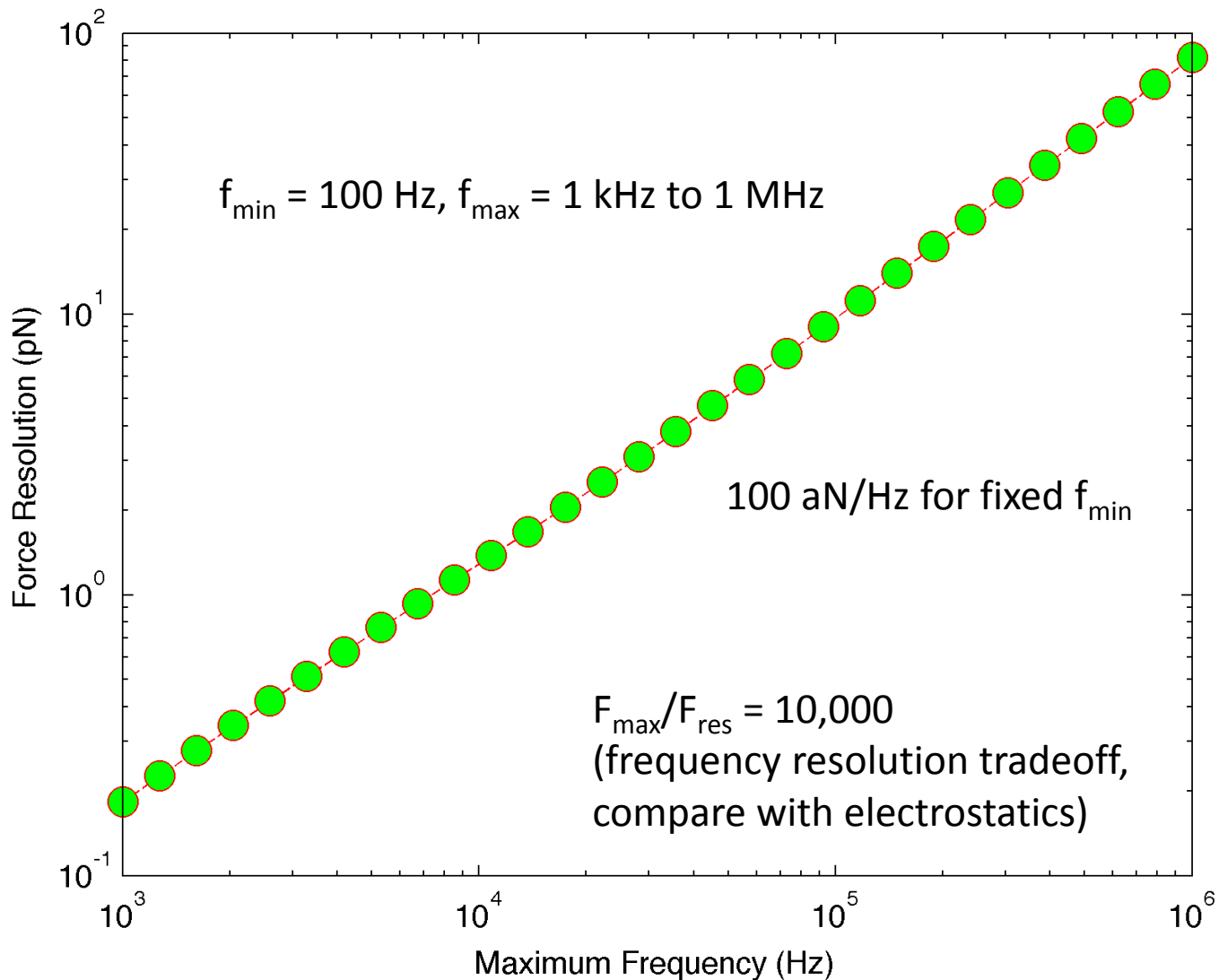
- Optimization characteristics
  - Smooth functions
  - Suggest single global optimum (proof?)
- Constraints
  - Simple boundaries on geometry, doping dictated by fabrication capabilities
  - Nonlinear constraints (e.g. frequency, power, other)
- Options
  - Brute force global ( $10^n$  = infeasible)
  - Guess-and-check (heuristics)
  - Constrained, nonlinear optimization

# Optimization Problems

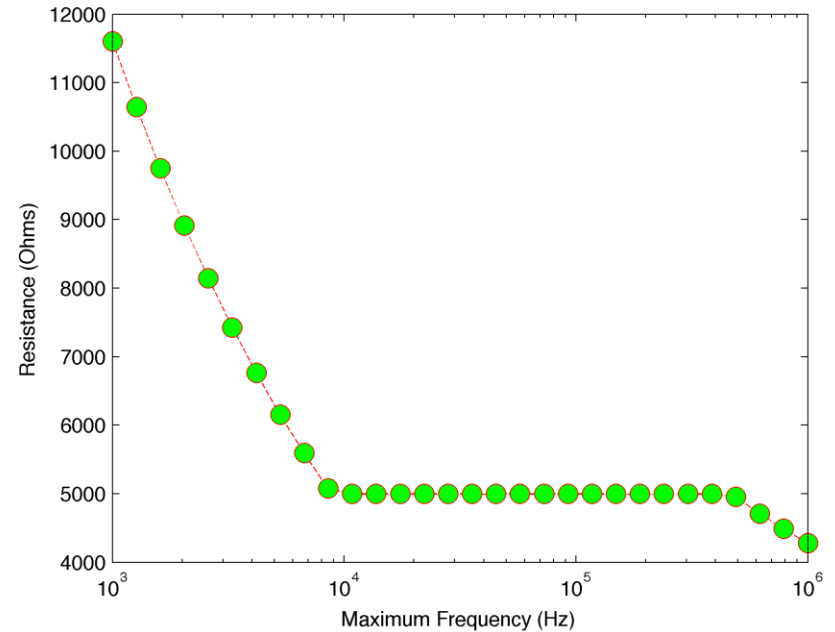
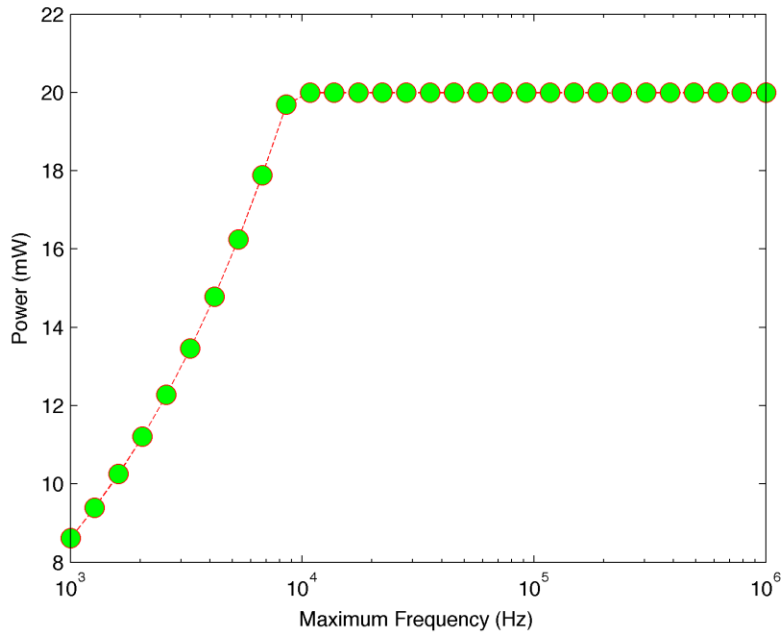
- Fixed  $f_{\min}$ , varying  $f_{\max}$  (resonance and noise integration)
  - $f_{\min} = 100 \text{ Hz}$
  - $t_{\min} = 1 \mu\text{m}$
  - $w_{\min} = 1 \mu\text{m}$
  - $N_{\max} = 1e20/\text{cc}$
  - $V_{\max} = 5\text{V}$
- Fixed  $f_{\max}$ , varying  $f_{\min}$ 
  - $f_{\max} = 100 \text{ kHz}$
  - Rest same as above
- Both optimizations performed for vacuum
- ~100 iterations per data point => 30 minutes
- Very early results, from the last few days



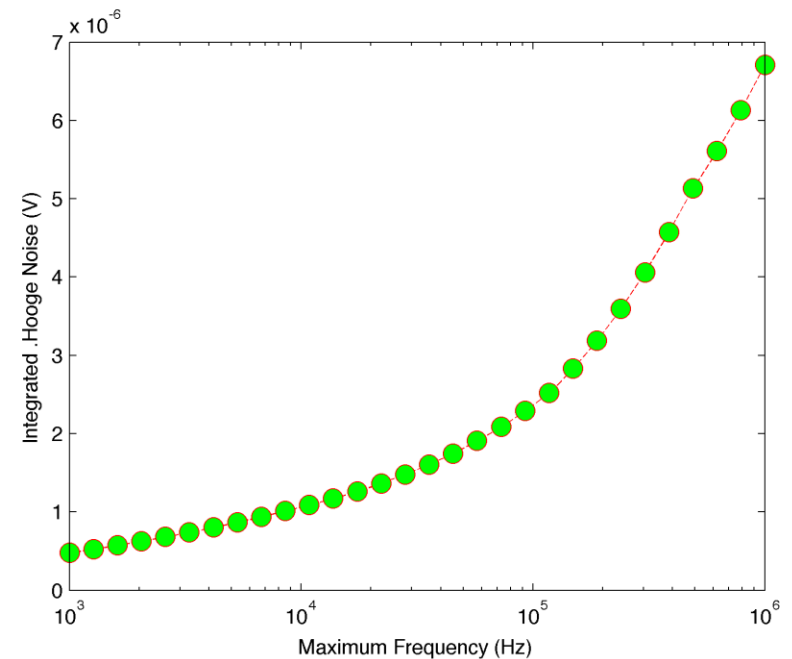
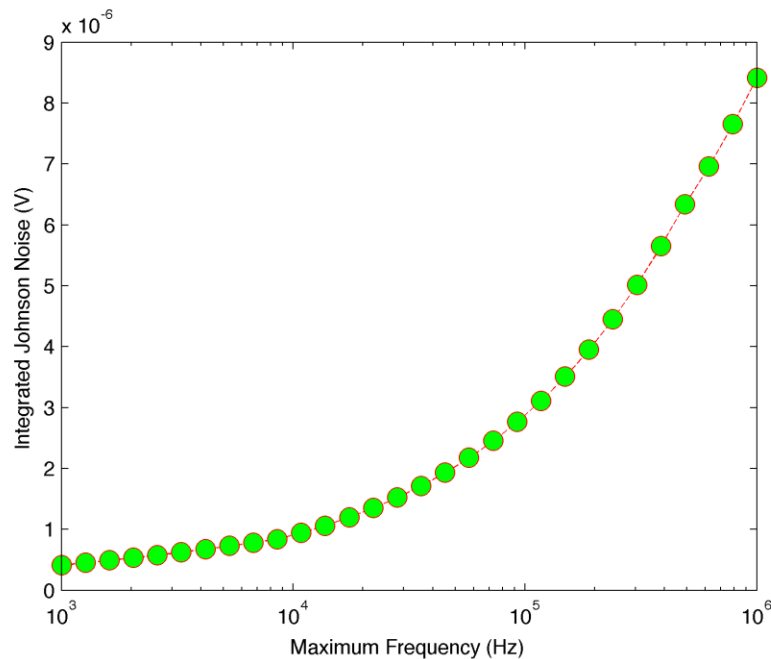
# Force Resolution



# Power and Resistance

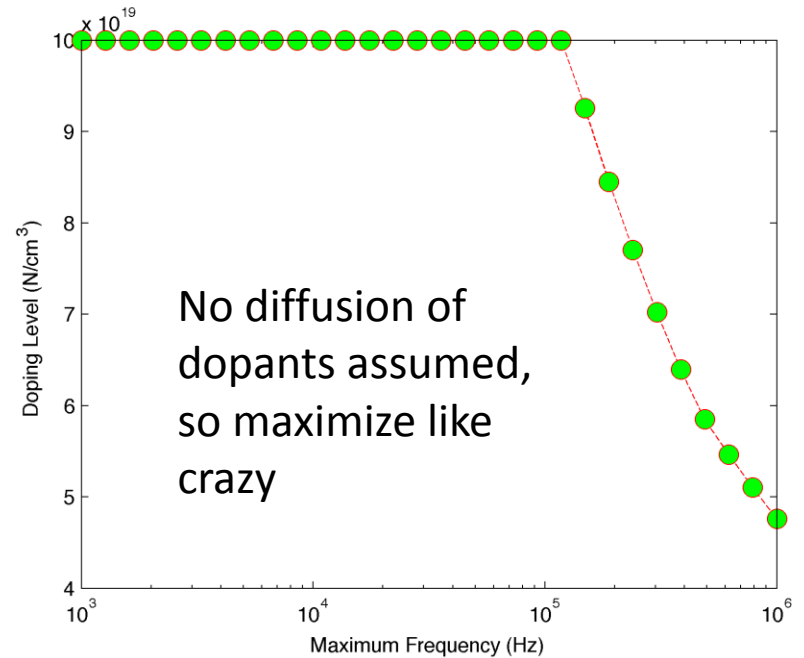
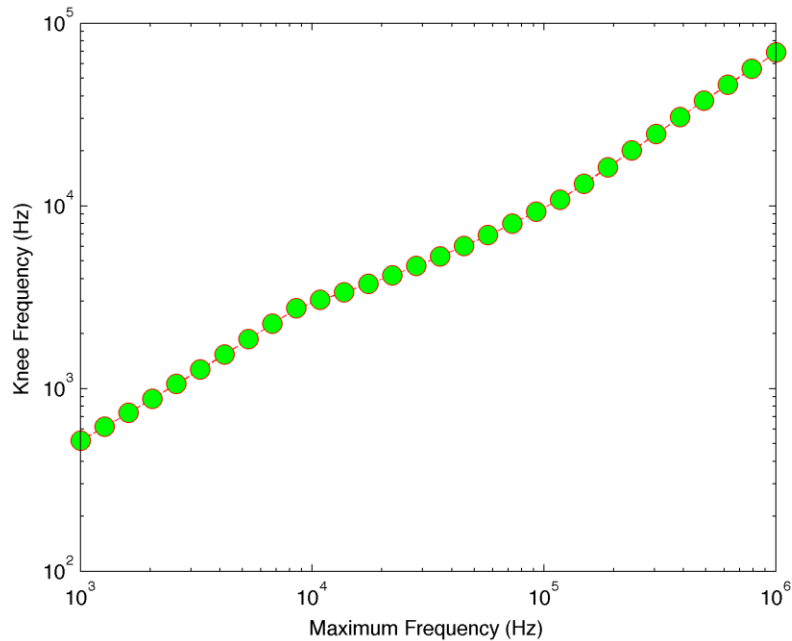


# Johnson and Hooge Noise



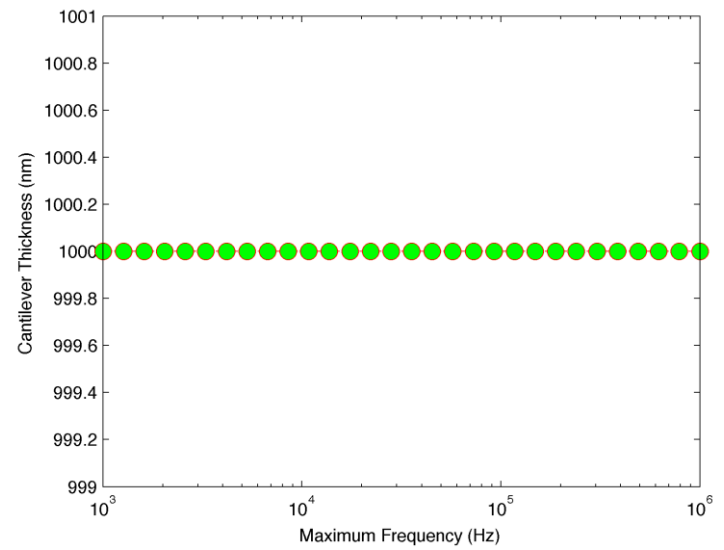
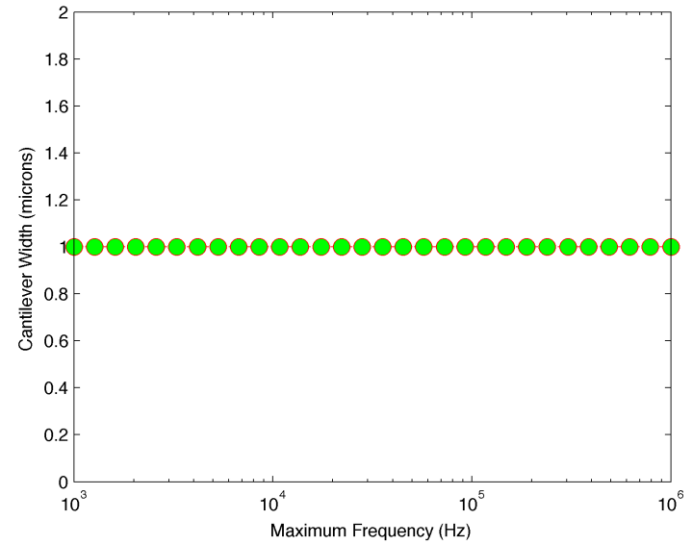
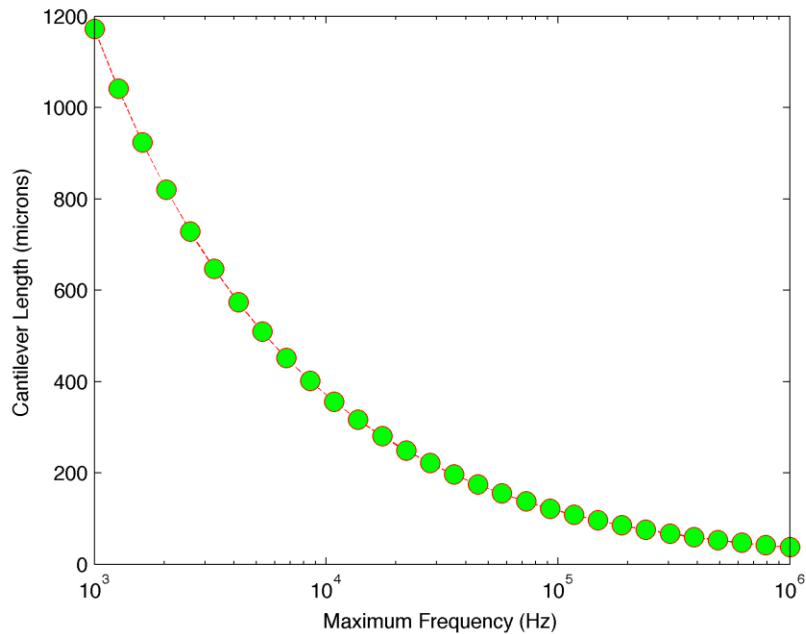
Roughly comparable Johnson and  $1/f$  noise (need to prove that it's the global minimum and not just the local minimum)

# Knee Frequency and Doping

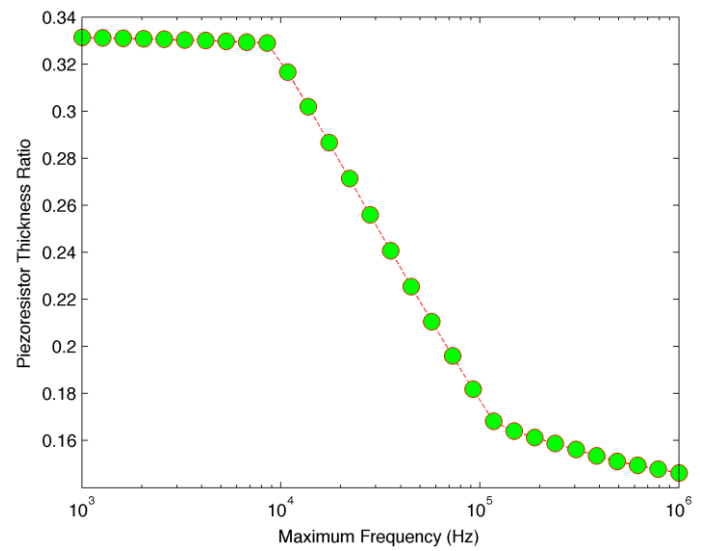
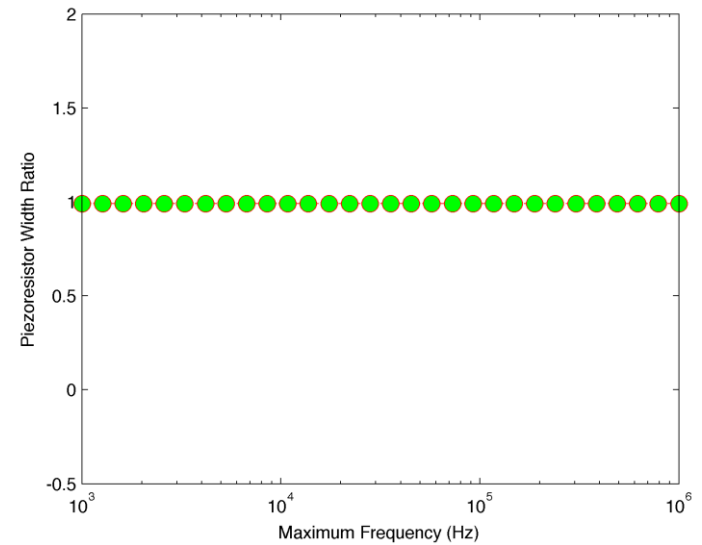
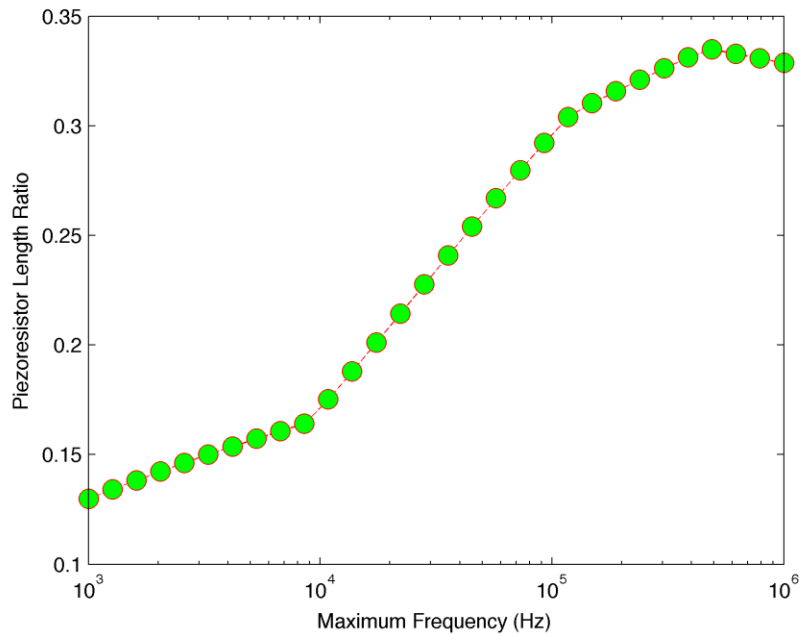


Compare knee frequency and  $f_{\text{max}}$ . High doping levels.

# Cantilever Geometry

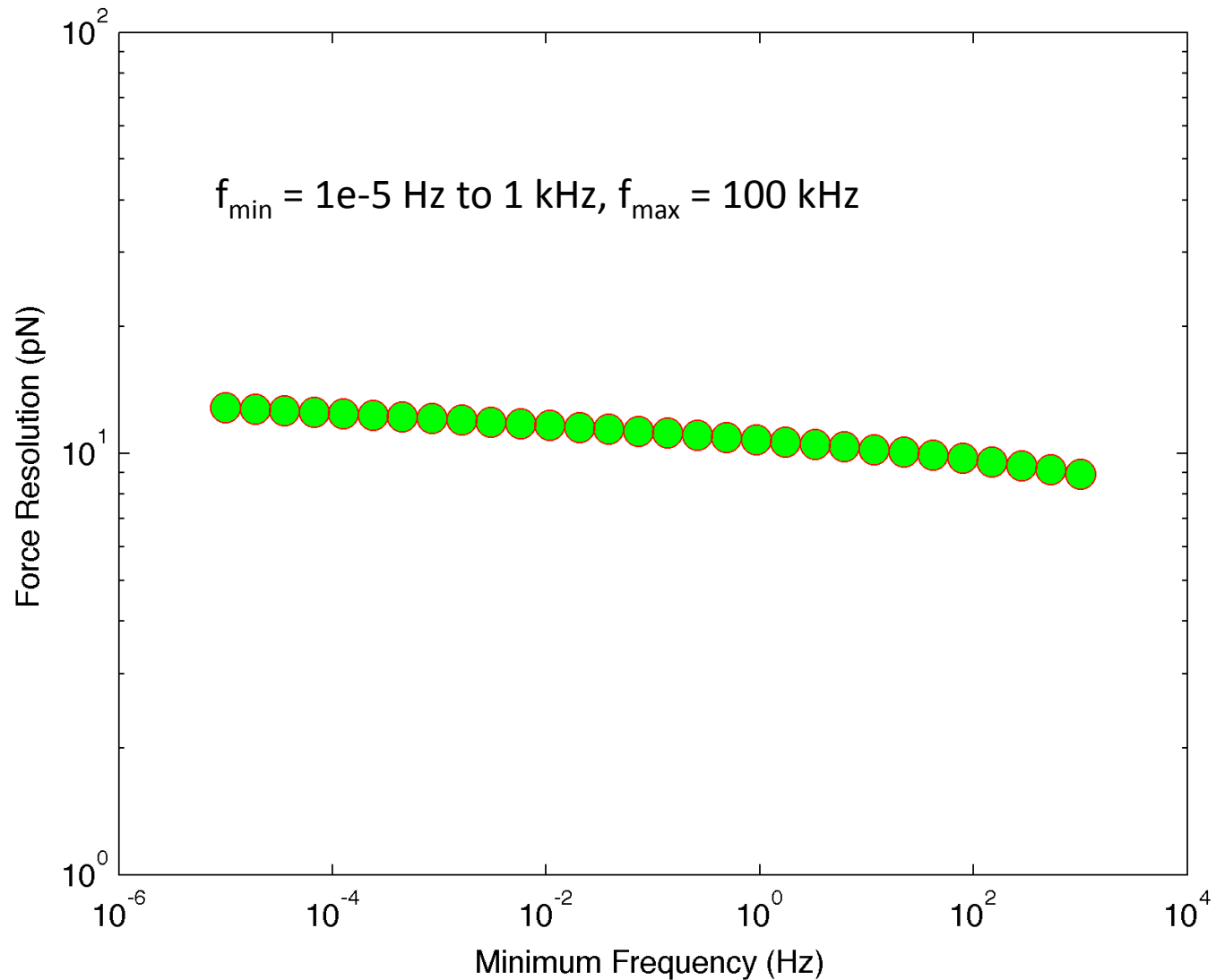


# Piezoresistor Geometry

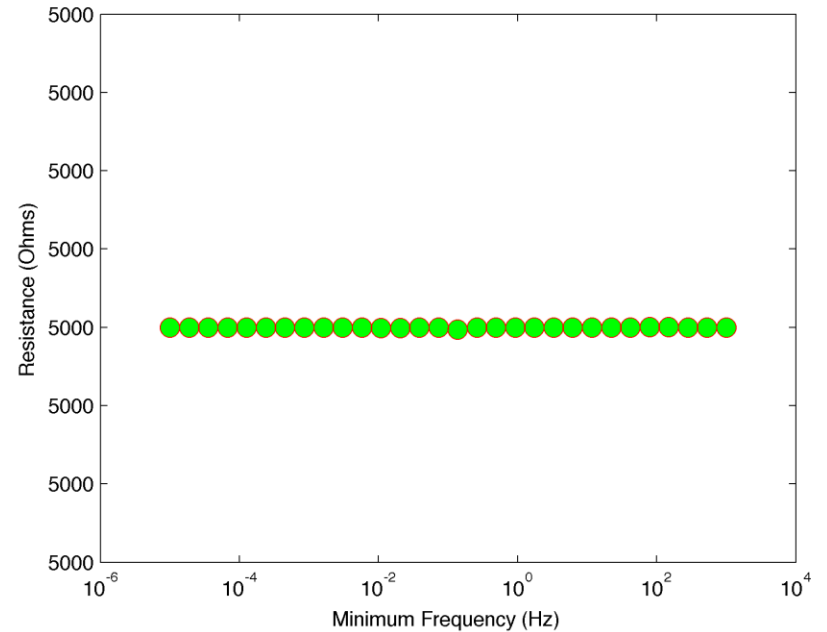
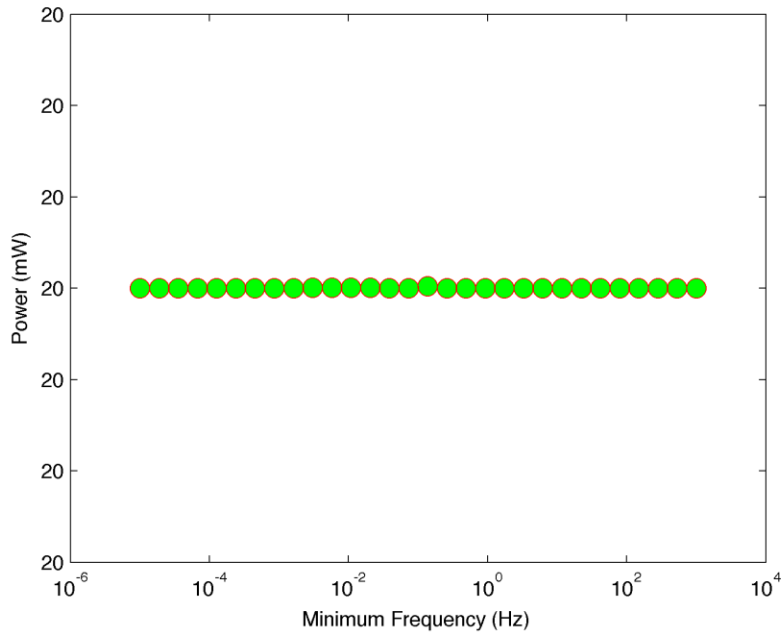


Not the design parameters described by Harley

# Force Resolution

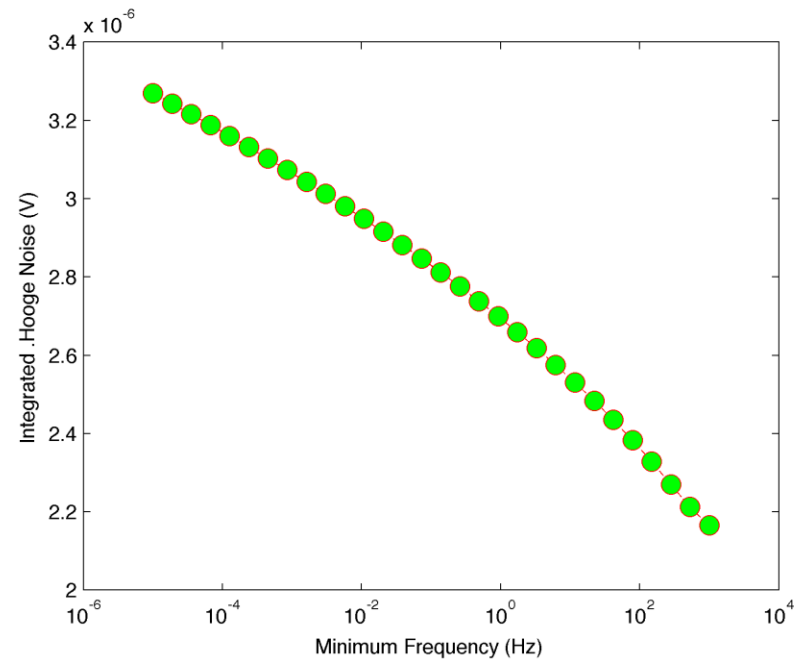
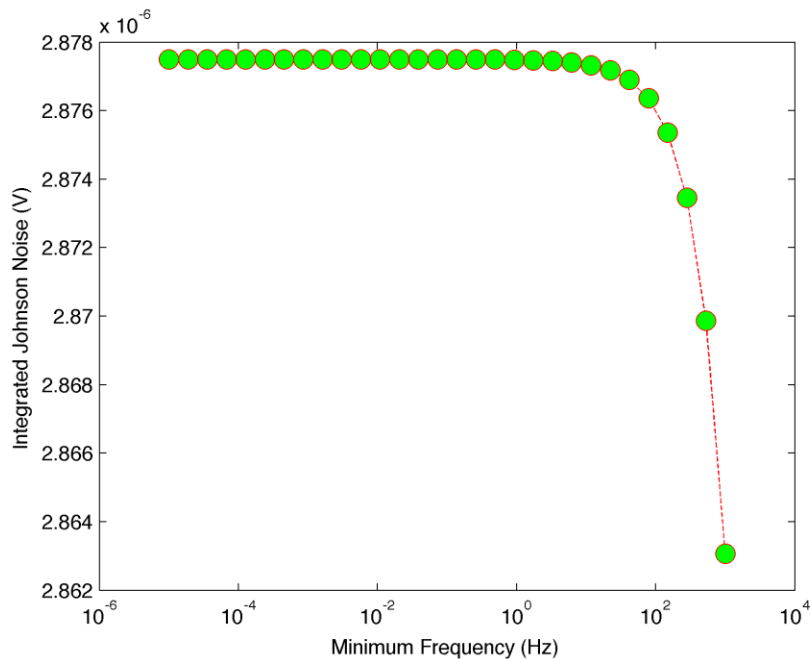


# Power and Resistance



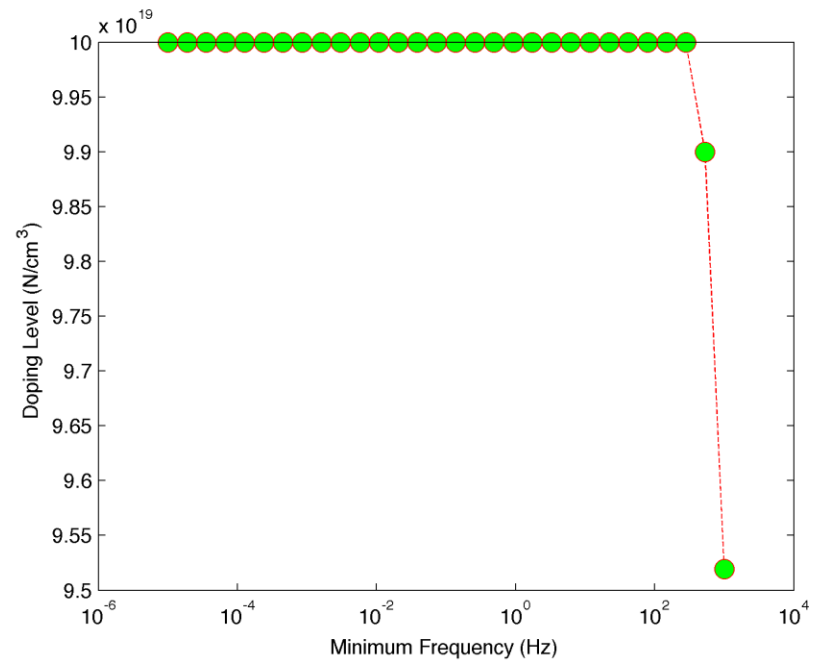
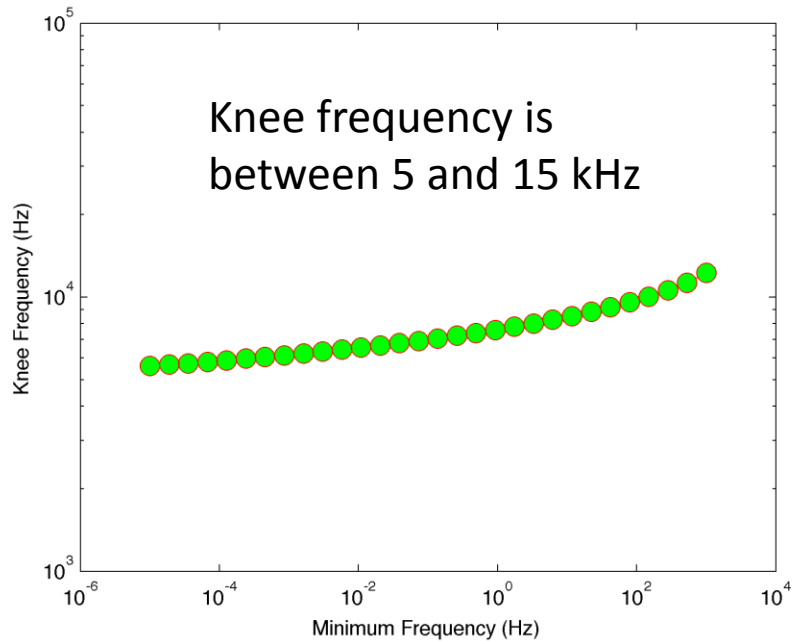


# Johnson and Hooge Noise

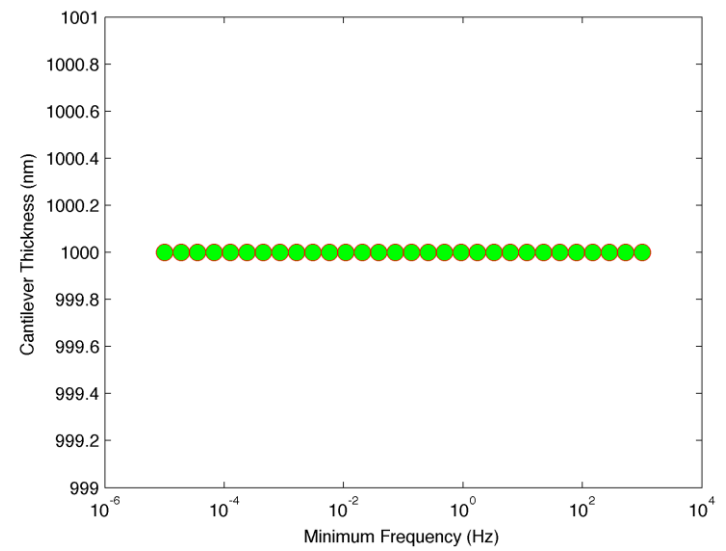
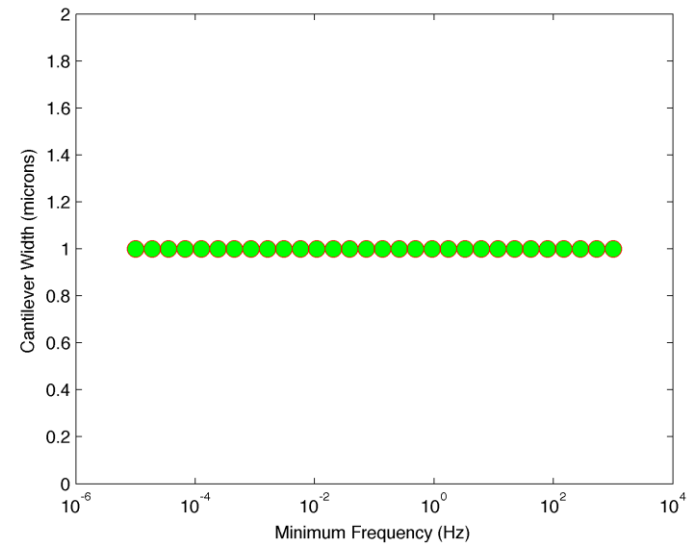
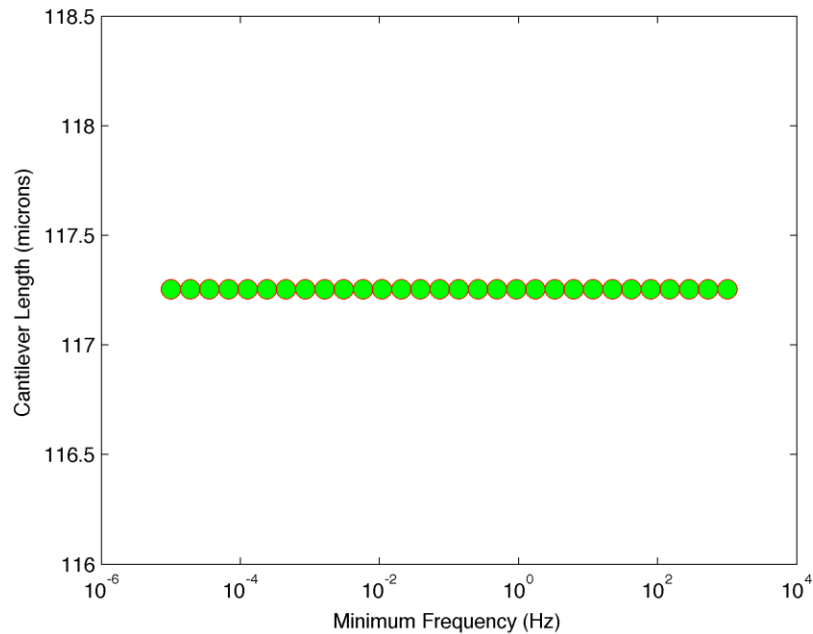


Little change in integrated 1/f noise by changing piezoresistor dimensions.

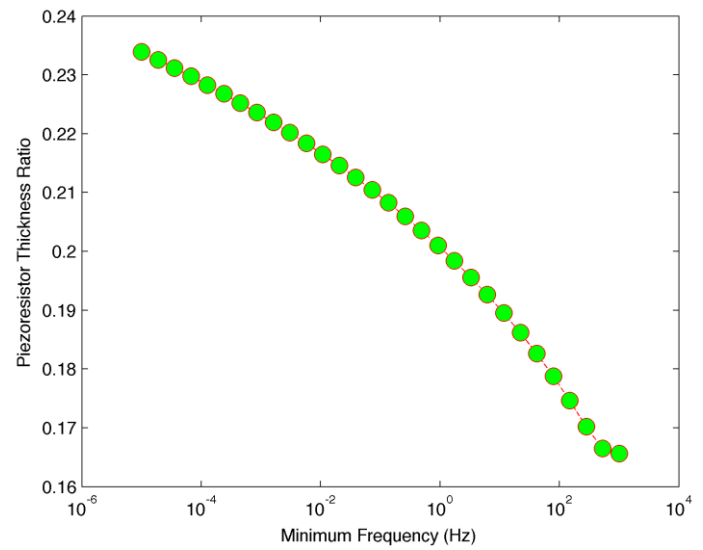
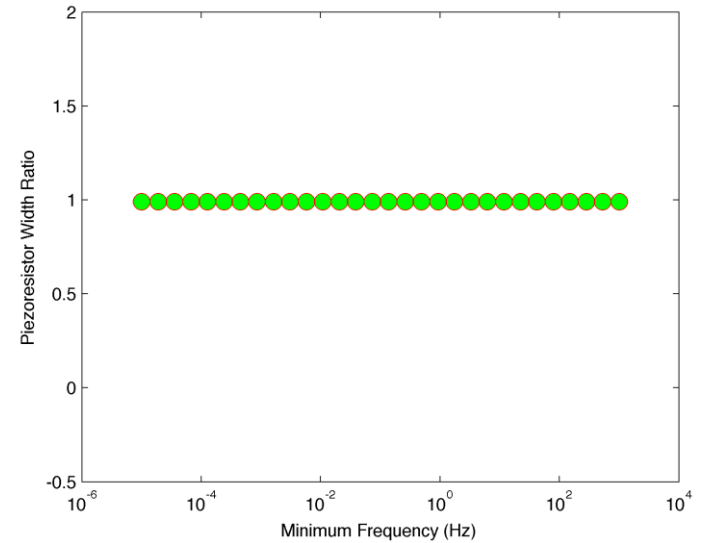
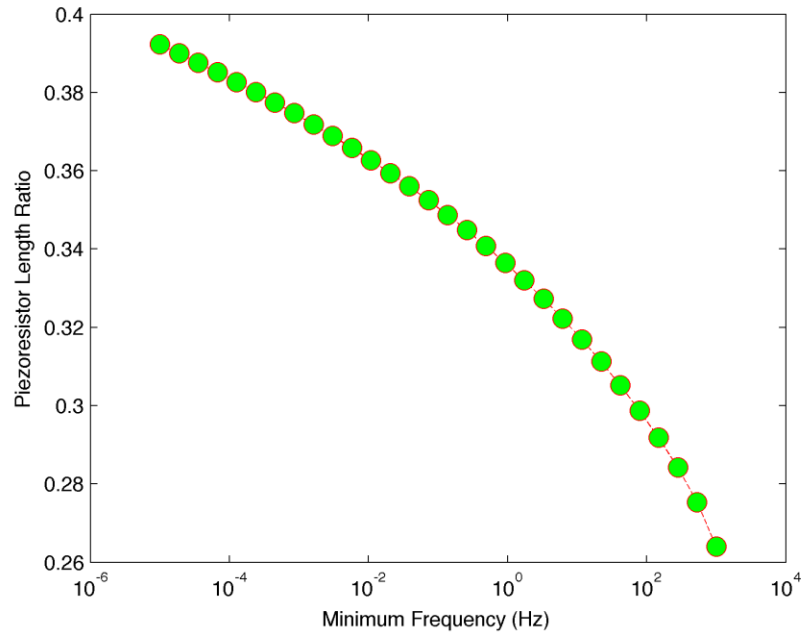
# Knee Frequency and Doping



# Cantilever Geometry



# Piezoresistor Geometry



Again, not the design parameters described by Harley

# Optimization Conclusions

- High doping is preferred
  - Even without taking TCF into account
  - Based upon Harley fit to experimental data
- Maximize piezoresistor width
- Piezoresistor thickness and length ratios don't agree with prior optimizations
  - Still need to definitively prove that these global optima (in progress)
- Model agrees with ultra thin epitaxial cantilevers
- Numerical optimization is more flexible and simple than heuristic design
- More on extending the optimization at the end

# Future Design Work

- Optimization
  - Thermal model rather than power dissipation
  - Integration with TSUPREM for ion implantation, won't need lookup tables
  - Integration with Comsol for complete model
  - Optimize rise time, useful frequency range
- Modeling
  - Complete electromechanical model (e.g. piezoelectric charging, power electronics)

# Optimization Code and Model

- Originally written in Python using SciPy (March)
  - Optimizers did not reliably converge, lots of headaches
- Switched to Matlab using object oriented features introduced in R2008a (May)
- Uses local, nonlinear constrained optimization
  - L-BFGS-B (fmincon)
- Code is online
  - [http://microsystems.stanford.edu/git/cantilever\\_optimization.git](http://microsystems.stanford.edu/git/cantilever_optimization.git)
    - [http://microsystems.stanford.edu/wiki/Version\\_Control\\_with\\_Git](http://microsystems.stanford.edu/wiki/Version_Control_with_Git)
  - Reasonably clean code, will have README soon

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# Piezoelectric Actuation Model

	Stiffness ( $10^{10}$ N / m <sup>2</sup> )	Strain Coefficient ( $10^{-12}$ C / N)	Relative Permittivity	Coupling Coefficient K <sup>2</sup> (%)	Velocity (m / s)	Density (kg / m <sub>3</sub> )
Aluminum Nitride (AlN)	33.0	5.6 (d <sub>33</sub> )	8.6	6.0	11,300	3.26
Barium Titanate (BaTiO <sub>3</sub> ) *	11.0 - 27.5	82-145 (d <sub>33</sub> )	625-1350	39 – 46	4460	5.85
Lithium Niobate (LiNbO <sub>3</sub> )	24.5	19.2 (d <sub>33</sub> )	44	17.2 †	4379 †	4.64
Lithium Tantalate (LiTaO <sub>3</sub> )	23.3	8.0 (d <sub>33</sub> )	41	4.7 †	4112 †	7.64
P(VDF–TrFE)	0.3	-12.0 (d <sub>31</sub> )	13	0.18	2400	1.88
Quartz (SiO <sub>2</sub> )	10.7	2.3 (d <sub>11</sub> )	4.5	0.11 †	3948 †	2.65
PZT (PbZrTiO <sub>3</sub> )*	4.8 – 13.5	240-550 (d <sub>33</sub> )	1100-3200	66 - 73	4600	7.55
Zinc Oxide (ZnO)	21.0	10-12 (d <sub>33</sub> )	8.5	7.5	6,080	5.60



SAW Substrates



Thickness mode, thin film



Ferroelectric ceramic,  
bulk material



Ferroelectric polymer

$$k^2 = \sqrt{\frac{\text{mechanical energy stored}}{\text{electrical energy applied}}}$$

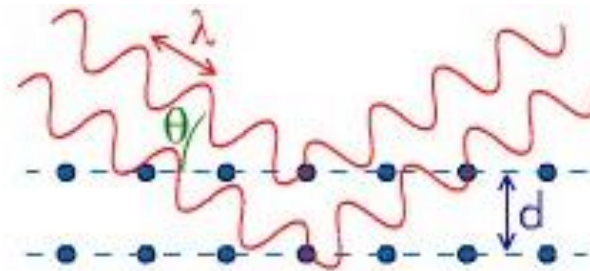
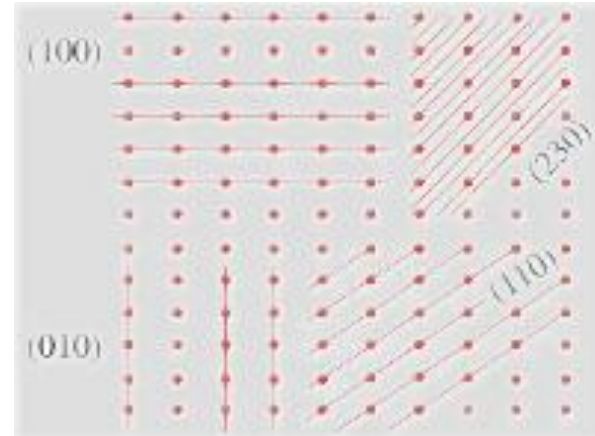
Borrowed from Justin Black, UC Berkeley

# X-Ray Diffraction

- ◆ Diffraction occurs only when the distance travelled by the rays reflected from successive planes differs by a complete number  $n$  of wavelengths:

$$n\lambda = 2d \sin\theta$$

- ◆ By varying the angle  $\theta$ , the Bragg's Law conditions are satisfied by different  $d$ -spacings in polycrystalline materials.
- ◆ A perfect crystal would give peaks that were delta functions.



$\lambda$  is the x-ray wavelength

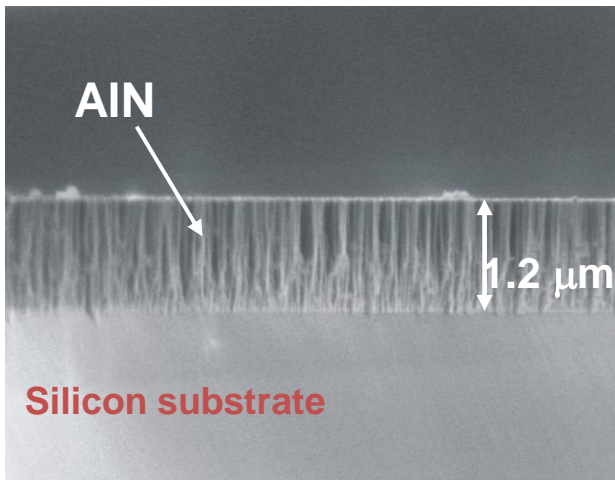
$\theta$  is the angle between the incident ray and crystal surface

$d$  is the spacing between the layers of atoms

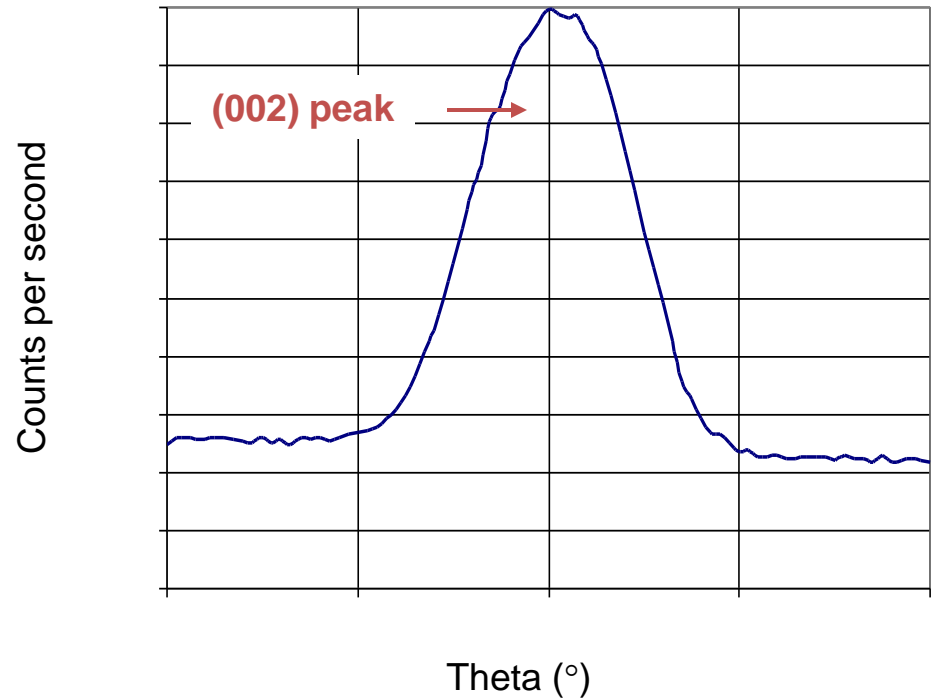
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# Aluminum Nitride

- ◆ A normal coupled scan gives the stoichiometry (relative composition) of different crystal orientations (e.g. (002), (110))
- ◆ For AlN, the piezoelectric (002) crystal peak orientation occurs at  $36.1^\circ$
- ◆ A rocking curve measures the alignments of the (002) crystallites

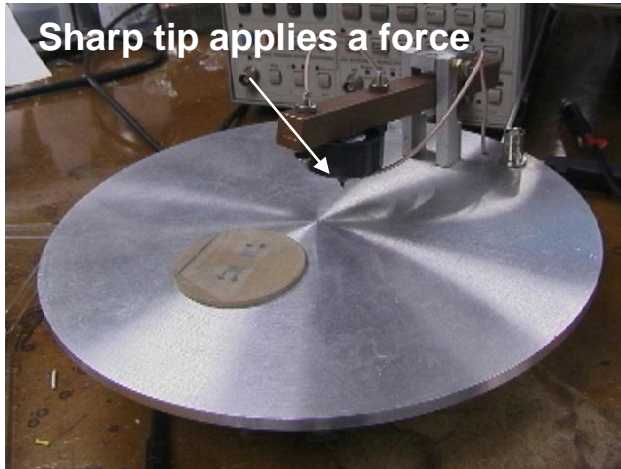


$\theta$ -2 $\theta$  Scan (or normal coupled)



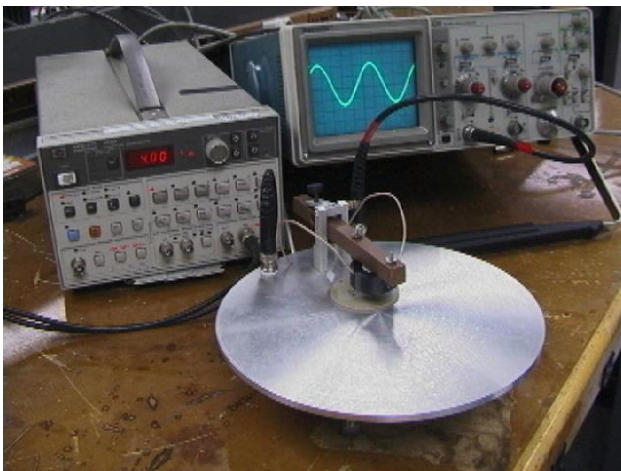
# $d_{31}$ and $d_{33}$ Measurement

Commercial versions cost \$3500 +



Stress induced charge collected from the sample surface

This charge is capacitively divided between piezoelectric sample and  $C_{in}$  of the oscilloscope



Current plan is to actuate cantilevers and measure tip displacement with LDV (indirect), but other methods include applying a force with a conducting AFM tip and collecting the charge

Borrowed from Justin Black, UC Berkeley

# Piezoelectric Actuation Model

117 nm (want 500 nm) static tip deflection for:

- $d_{31} = 3 \text{ pC/N}$
- $w_{\text{cantilever}} = w_{\text{piezoelectric}} = 10\mu\text{m}$
- $t_{\text{cantilever}} = 1\mu\text{m}, t_{\text{piezoelectric}} = 500\text{nm}$
- $l_{\text{cantilever}} = 150\mu\text{m}, l_{\text{piezoelectric}} = 30\mu\text{m}$
- $V_{\text{bias}} = 5\text{V}$

$$\delta(x) = \frac{x^2 d_{31} \mathbf{E}_p (t_e + t_p) A_e E_e A_p E_p}{(t_e + t_p)^2 A_e E_e A_p E_p + 4(A_e E_e + A_p E_p)(E_e I_e + E_p I_p)}$$

More complicated for multimorphs, electrode and insulating layers can drastically change things (also in their paper)

# Overview

- Cantilevers in Fluid
  - Analytical modeling
  - Implications for design
- Numerical optimization of epitaxial cantilevers
  - Comparison with Previous Approaches
  - Numerical model
  - Comparison with experimental data (Harley)
  - Implementation
  - Results
- Piezoelectric Actuation Model
- **Summer and Future Work**

# Plan for the Summer

- XRD on aluminum nitride for process optimization
- ASML mask design
- Fabrication of actuating cantilevers to characterize mechanical resonance,  $d_{31}$  (July/August)
- Mask design again
- Fabrication of actuating, sensing cantilevers (August/September)
- Arsenic piezoresistors (July?)

# Arsenic Piezoresistors

- Arsenic Piezoresistors
  - Lower modulus in  $\langle 100 \rangle$  versus  $\langle 110 \rangle$
  - Higher mobility for N-type dopants (lower R for fixed N)
  - Higher piezoresistive coefficient (from Kanda, limited experimental data)
  - Lower diffusivity in silicon (300nm junctions achievable via ion implantation)
  - Would like to test with the class cantilever process to get  $P(N)$  and  $\alpha(\sqrt{Dt})$ , expected results...