Exercise 5: WAIC

Jason Doll

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Objective

This exercise will demonstrate how to do Bayesian model comparison using Watanabe's Information Criterion (WAIC)

Background

In many applications, it is neccessary to compare two or more models to determine which model "best" describes the data. Although there are many methods, this exercise will demonstrate how to modify a Stan model to calculate WAIC. You are given the R code to generate simulated data and two basic Stan programs. The first file, "EX5_WAIC_LR.stan", estimates paramters of a linear regression model and the second file, "Ex5_WAIC_ME.stan", esimtates parameters of a mixed effect model. The "EX5_WAIC_LR.stan" model you will fit to observations i is:

$$\begin{aligned}
\text{Model:} \qquad & \mathrm{y}_i = \alpha + \beta \mathrm{X}_i + \epsilon_i \\
& \epsilon_i \sim \text{Normal}(0, \sigma) \\
\text{Priors:} \qquad & \alpha \sim \text{Normal}(0, 100) \\
& \beta \sim \text{Normal}(0, 100) \\
& \sigma \sim \text{half-cauchy}(0, 5)
\end{aligned}$$

The "EX5_WAIC_ME.stan" model you will fit to observations i and group j is:

$$\begin{aligned}
\text{Model:} \qquad & \mathrm{y}_i = \alpha_{j[i]} + \beta \mathrm{X}_i + \epsilon_i \\
& \epsilon_i \sim \text{Normal}(0, \sigma) \\
\text{Priors:} \qquad & \alpha_j \sim \text{Normal}(\mu, \tau) \\
& \mu \sim \text{Normal}(0, 100) \\
& \tau \sim \text{half-cauchy}(0, 5) \\
& \beta \sim \text{Normal}(0, 100) \\
& \sigma \sim \text{half-cauchy}(0, 5)
\end{aligned}$$

You will modify the two Stan programs to generate a new quantity to hold the log-likelihood. This parameter will be extracted and WAIC calculate using the loo package.

R packages required for this exercise

1. rstan
2. shinystan
3. loo

Directions

Add a "generated quantities block" with the following code to the two existing files "Ex5_WAIC_LR.stan" and "Ex6_WAIC_ME.stan". Note, you MUST use the quantity "log_lik".

Stan code for centered parameterization

```
generated quantities{
    vector[n] log_lik;

    for ( i in 1:n ) {
        log_lik[i] = normal_lpdf( y[i] | alpha + beta * x[i], sigma_y);
    }
}
```

Stan code for non-centered parameterization

```
generated quantities{
    vector[n] log_lik;

    for ( i in 1:n ) {
        log_lik[i] = normal_lpdf( y[i] | gpmu[gp[i]] + beta * x[i], sigma_y);
    }
}
```

R Code

After modifying both .stan files to include the "generated quantities" block above, open "Ex5_WAIC.R" and run lines 1 through the lines below. This should be around line 100-107.

```
#Noncentered Example without Divergent transitions
non_Center_WAIC_ME = stan(file = 'Ex5_WAIC_ME.stan',
                  data = dataList ,
                  init = initslst,
                  chains = nchains,
                  iter = 1000 ,
                  warmup = 500 ,
                  thin = 2,
                  control = list(adapt_delta=0.99, max_treedepth=15))
```

After executing both models the first step to calculate WAIC is to extract the log_lik parameter from both stan objects.

```
log_likFE = extract_log_lik(WAIC_FE)
log_likME = extract_log_lik(non_Center_WAIC_ME)
```

Finally, calculate the WAIC value

```
waic(log_likFE)
```

```
##
## Computed from 750 by 600 log-likelihood matrix
##
##           Estimate   SE
```

```
## elpd_waic  -2937.2 18.4
## p_waic         3.0  0.3
## waic        5874.5 36.8
```

```r
waic(log_likME)
```

```
##
## Computed from 750 by 600 log-likelihood matrix
##
##           Estimate   SE
## elpd_waic  -2880.3 18.5
## p_waic         8.0  0.6
## waic        5760.6 36.9
```

Your results should show that the mixed effects model (one that produced the log_likME object above) will have the lowest WAIC value.