Particle Swarm Optimization: Exploring the Necessity of Velocity Clamping in Lower Dimensional Optimization Problems

*An evaluation on the merits of velocity clamping when convergence conforming control parameter configurations are employed.

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***Abstract*—Velocity clamping is widely cited as a useful technique to combat undesirable particle roaming behaviour, by restricting extreme velocity acceleration. It has however been shown that if certain convergence conditions are satisfied, particle velocities will reach equilibrium. This paper examines the performance of particle swarm optimization when these conditions are adhered to. A comparison is made between the success of traditional velocity clamping under these conditions, and the lack there of. The study concluded that when convergence conditions are obeyed, competitive solutions can be obtained without the need to restrict velocity growth in any capacity. It is also shown that under these convergence conditions, some desirable performance characteristics are exhibited. Most notably, particles display advantageous trade-off between exploration and exploitation.**

I. Introduction

Particle Swarm Optimization (PSO), as established by Kennedy and Eberhart in 1995 [4], is a nature-inspired stochastic optimization algorithm. In PSO a population of particles traverse a search space guided by both global and local topology. The position of a particle during this procedure represents a candidate solution to a predetermined optimization problem. The quality of a particular solution is assessed by means of an objective function evaluation. The movement characteristic of particles greatly depend on the values of key model parameters. Poor choices for these parameters may cause the particles to exhibit roaming behaviour [5]. *Roaming behaviour* is the propensity of a particle to explore beyond the search domain. An initial explosion of particle velocity is often the cause of particle divergence. Velocity clamping is credibly cited to curtail particle roaming. *Velocity clamping* refers to the restriction of particle velocities to not exceed a predetermined threshold. It has been shown both empirically and theoretically that if model parameters are chosen to satisfy certain conditions, the particles will eventually converge innately [2] [11] [13] [8]. In light of this finding, the relevance of velocity clamping is immediately challenged. This paper empirically address the validity of this conclusion.

The remainder of this paper is organized as follows: Section II supplies an overview of PSO and the influence each control parameters imposes on the swarms behaviour. Section III outlines the PSO evaluation metrics used to analyze the quality of PSO performance. Section IV discloses the benchmark functions and experimental procedure followed to obtain results. Subsequently, the results obtained are presented in Section V. Lastly, section VI concludes the paper.

II. Background

Various PSO techniques have been developed since the initial contributions of Kennedy and Eberhart. This paper uses the inertia weight model of PSO, developed by Shi and Eberhart [12]. Intertia weight PSO establishes a proportional relationship between a particle's prior velocity and its successive velocity. According to this model, the equations to update the velocity and position of a particle are given by equations (1) and (2) below.

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{v}_i^{t+1} \tag{1}$$
$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + c_1\mathbf{r}_1(\mathbf{y}_i^t - \mathbf{x}_i^t) + c_2\mathbf{r}_2(\hat{\mathbf{y}}_i^t - \mathbf{x}_i^t) \tag{2}$$

It is common practice to denote vectors using a bold-face font. Bold symbols in the above equations represent n dimensional vectors, where n corresponds to the dimensionality of the search domain. The syllabary of equations (1) and (2) are clarified below:

- $\mathbf{x}_i^t$ and $\mathbf{x}_i^{t+1}$ specify the position of the i^{th} particle during iteration t and $t+1$ respectively. The position of a particle represents a potential solution to the optimization problem.
- $\mathbf{v}_i^t$ and $\mathbf{v}_i^{t+1}$ denote the velocity of the i^{th} particle during iteration t and $t+1$ respectively. It is important to note that the velocity vector of a particle specifies both the direction this particle is moving, as well as the magnitude of the movement. The velocity of all particles are initialized to the zero vector at iteration one.
- $\mathbf{y}_i^t$ represents the position which yields the best objective function evaluation, that has been visited by particle i.

This point is commonly referred to as a particles personal best vector. The quality of a particles current position is evaluated upon every iteration of the simulation. The evaluation is compared to that of the personal best vector, if the current position is found to be of higher quality, the personal best vector of this particle is updated to reflect the current position.

- $\hat{\mathbf{y}}_i^t$ denotes the position which yields the best objective function evaluation that any particle in the neighborhood has visited. In PSO, the *neighborhood* of a particle specifies a cluster of entities within the swarm that share locally observed topological information. This point can be interpreted as the "best personal best" of any particle in this neighborhood.
- $\mathbf{r_1}$ and $\mathbf{r_2}$ are n dimensional random vectors, where each element in the vector is chosen from a *uniform(0,1)* distribution. The non-deterministic characteristics of these vectors are critical to facilitating exploration of search space. If $\mathbf{r_1}$ and $\mathbf{r_2}$ were not randomly initialized, it is not possible for particles to explore any location in the search domain that is not in the span of the particles randomly initialized locations. In short- the randomness ensures proper exploration capability. The values for these vectors are re-computed for every particle on each iteration.
- c_1 is a control coefficient for the term $\mathbf{r}_1(\mathbf{y}_i^t - \mathbf{x}_i^t)$. Collectively this term is referred to as the "cognitive component", which quantifies performance of the particles current position relative to that of the particles personal best. This interaction has aptly been coined as "nostalgia" by Kennedy and Eberhart [4].
- c_2 is the control coefficient for the term $\mathbf{r}_2(\hat{\mathbf{y}}_i^t - \mathbf{x}_i^t)$. Collectively this term is referred to as the "social component", which quantifies performance of the particles current position relative to that of the particles neighborhood best. This interaction has similarly been referred to as "envy" by Kennedy and Eberhart [4].
- w regulates the affect that previous velocity imposes on the calculation of the velocity in the subsequent iteration. The velocity of the previous iteration is often regarded as the "inertia" or "momentum" of that particle, thus the fitting name for this PSO model - Interia Weight PSO.

A. Communication Topology techniques

The information sharing methodology implemented has great effect on the performance characteristics of the swarm. Each particle imposes a social influence unto other particles within it's neighborhood. The way in which said neighborhood is globally defined is regarded as the "communication topology" of the swarm. Communication topology determines how fast (or slow) good solutions disseminate to other neighborhoods [6].

The two most common communication approaches are *ring* topology, and *star* topology [9]. This paper will employ the *star* topology technique- in which search information is shared globally throughout the swarm. If *star* topology is used, $\hat{\mathbf{y}}_i^t$ is often regarded as the "global best" position. This paper will refer to $\hat{\mathbf{y}}_i^t$ as such.

B. Velocity Clamping

Velocity clamping was introduced to regulate surges in particle accelerations, which may result in particles roaming beyond the confines of the search domain [3]. Multiple methods of velocity limitation have been proposed, this discussion will focus on the restriction of velocity magnitude. The maximum velocity magnitude is often chosen as a fraction of the search domain's width. This fraction is referred to as k. Small values of k ensure that multiple "steps" are needed in order for a particle to roam beyond the restrictions of the search domain. This result demonstrates that the granularity of the traversal procedure is highly depended on chosen clamping coefficient k. Equations (3) defines the calculation of the maximum velocity threshold. Equation (4) exemplifies the clamping procedure.

$$\mathbf{V}_{max} = k(\mathbf{x}_{max} \text{ - } \mathbf{x}_{min}) \tag{3}$$

$$\mathbf{v_i(t+1)} = \begin{cases} \mathbf{v_i'(t+1)} & if \;\; \mathbf{v_i'(t+1)} < \mathbf{V_{max}} \\ \mathbf{V_{max}} & if \;\; \mathbf{v_i(t+1)} \geq \mathbf{V_{max}} \end{cases} \tag{4}$$

In this equation, $\mathbf{v_i'(t+1)}$ is calculated using equation (2).

It is important to note that velocity clamping does not affect the direction of particle movement, but only the magnitude of said movement. As a result, the inertial, social, and cognitive components of (1) are conserved.

C. Particle Convergence Condition

If the control parameters w, c_1, c_2 are chosen to satisfy the following equation, it has been both theoretically and empirically demonstrated that particles will eventually reach an equilibrium state. That is- the particles will stop moving.

$$c_1 + c_2 < \frac{24(1-w^2)}{7-5w} \quad \text{for} \;\; w \in [-1, 1] \tag{5}$$

The region defined by Equation (5) was derived by Poli and Broomhead, and Jiang et al. independently [11] [13]. In light of this equations derivation, the relevance of velocity clamping is uncertain. If control parameter values are selected such that this condition is satisfied, particle velocities are guaranteed to converge to zero. With this property in mind, there is no need to assert the granularity of the search process.

D. Exploration vs Exploitation

An important factor contributing to the success of PSO is the balance between exploration and exploitation of the swarm [10]. A successful PSO implementation should explore the search space early on in the simulation, and eventually exploit promising solutions. A useful metric to quantify the exploratory tendencies of a swarm is *swarm diversity*. Swarm diversity refers to the average euclidean distance between a particle, and the position of the average particle. High swarm diversity implies the particles are far from one another, while low diversity implies particles are converging to a singularity.

E. The PSO Algorithm Used

This section will highlight the PSO algorithm followed throughout the remainder of this paper. The iterative-improvement nature of PSO clearly illustrated in the figure below.

Algorithm 1 Standard PSO Algorithm

Create and initialize an n_x-dimensional swarm, S;
repeat
    for each particle i=1,...,$S.n_s$ **do**
        if $f(S.x_i) < f(S.y_i)$ **then**
            $S.y_i = S.x_i$;
        end if
        if $f(S.y_i) < f(S.\hat{y}_i)$ **then**
            $S.\hat{y}_i = S.y_i$;
        end if
    end for
    for each particle i=1,...,$S.n_s$ **do**
        update the velocity;
        update the position;
    end for
until stopping condition is true

III. Evaluation Metrics

PSO is a non-deterministic process, which implies that multiple independent trials under the same control configurations may yield different results. This non-determinism stems from the random initialization positions of particles in the swarm, as well as $\mathbf{r_1}$ and $\mathbf{r_2}$ being iteratively sampled from an n-dimensional $Uniform(0, 1)$ distribution. In effort to minimize performance variance between simulations, each simulation will be ran 25 times. The results of each simulation are iteratively averaged, as the performance over time is of interest to this investigation.

Multiple performance metrics will be tracked during each iteration of the PSO simulation. The metrics of importance are noted below:

1) Average velocity magnitude of the swarm
2) Percentage of particles outside the search domain
3) Global best position
4) Particle Diversity

The merit to each of the above metrics are discussed below:

- *Average velocity magnitude* refers to the intensity of acceleration that particles within the swarm are exposed to on average. This is a useful metric as it indicates the breadth of step-sizes that particles are taking with each subsequent iteration. This metric gives strong insight into the granularity of the search process.
- The *percentage of particles outside the search domain* gives an indication into the stability and search success of the simulation. With poor control parameter choices, nearly all the particles leave the search domain in the first few iterations.
- The *Global best position* represents the quality of the best solution obtained by the simulation. The merit of this as a performance metric is trivial.
- *Particle diversity* quantifies the explorative and exploitative tendencies of the swarm. When diversity is high, particles are exploring the search space, when diversity is low, the particles are converging to a point of interest. It is critical that particle diversity expand in the early stages of the simulation, but eventually contract as the iterations go on.

The above metrics are evaluated at each iteration of the simulation, and averaged across all 25 simulations. The median and variance of each metric at each iteration, across the 25 simulations is also noted.

This evaluation procedure is applied to various control parameter configurations. Two sets of control parameter configurations will violate the Poli-Stability condition of equation (5), whilst two satisfy the condition. Performance for all sets of control parameter configurations will be measured using various velocity clamping intensities (k), as well as without velocity clamping.

For all simulations, a swarm of 30 particles will traverse a 30-Dimensional search space for 5000 iterations. The intention is to gather data on the performance characteristics of PSO when various intensities of velocity clamping are implemented. If there is no significant improvement between simulations where velocity clamping is employed, it can be argued that velocity clamping is unnecessary, provided that the control parameter configurations satisfy the Poli-Stability conditions of equation (5).

IV. Simulation Procedure

This section highlights the empirical process used to compare the performance of velocity clamping against that of its absence. To diversify the testing procedure, a variety of benchmark functions from R. Lang and A. Engelbrecht [7] have been selected to gauge PSO performance.

A. Selected Benchmark Functions

Five benchmark functions will be used to evaluate the performance of the PSO implementation. In the following functions, D denotes the dimension of the search space. The benchmark functions selected have a global minimum at the point $\mathbf{x}* = (0, 0, ..., 0)$.

1) Exponential Function:

$$f(\mathbf{x}) = -exp(-0.5 \sum_{n=1}^{D} x_i^2) \;\; where \;\; -1 \leq x_i \leq 1 \tag{6}$$

2) Elliptic Function:

$$f(\mathbf{x}) = \sum_{n=1}^{D} (10^6)^{\frac{i-1}{d-1}} (\mathbf{x_i}^2) \;\; where \;\; -10 \leq x_i \leq 10 \tag{7}$$

3) Step Function:

$$f(\mathbf{x}) = \sum_{n=1}^{D} (\lfloor x_i^2 \rfloor) \;\; where \;\; -100 \leq x_i \leq 100 \tag{8}$$

4) Brown Function:

$$f(\mathbf{x}) = \sum_{n=1}^{D-1} (x_i^2)^{(x_{i+1}^2+1)} + (x_{i+1}^2)^{(x_i^2+1)} \quad where \ -1 \leq x_i \leq 4 \tag{9}$$

5) Cosine-Mixture Function:

$$f(\mathbf{x}) = 0.1 \sum_{n=1}^{D} \cos(5\pi x_i) - \sum_{n=1}^{D} x_i^2 \ \ where \ \ -1 \leq x_i \leq 1 \tag{10}$$

The global best particle was updated synchronously as suggested in the original literature [3]. A Synchronous global update strategy performs neighborhood best updates separately from particle position updates. This implies changes in the global best position are only absorbed into the model on the following iteration [9]. To ensure that infeasible solutions don't entice the swarm out of the search domain, personal best updates require the candidate solution to satisfy the feasibility constraints imposed by the optimization function.

B. Simulation parameters

Multiple control parameter configurations will be used to emulate various PSO behaviour characteristics. The control parameter assignments used in this study are tabulated below.

w	c_1	c_2
1.0	2.0	2.0
0.7	1.4	1.4
0.9	2.0	2.0
0.9	0.7	0.7

Notice that two sets of configurations do not satisfy the stability conditions of equation (5), whilst two sets do. This is critical as performance should be evaluated both under favorable conditions, as well as when divergent behaviour is exhibited.

To gauge the effects of of velocity clamping on PSO performance, three different intensities of velocity clamping are examined. The most intense form of velocity clamping used in this paper is $k = 0.1$, followed by $k = 0.3$ and $k = 0.5$ in decreasing order of intensity.

The performance of PSO on the benchmark functions given by equations (6) (7) (8) (9) (10) will be evaluated using the metrics discussed in the prior section. The results of each independent trial are aggregated into one representative sample.

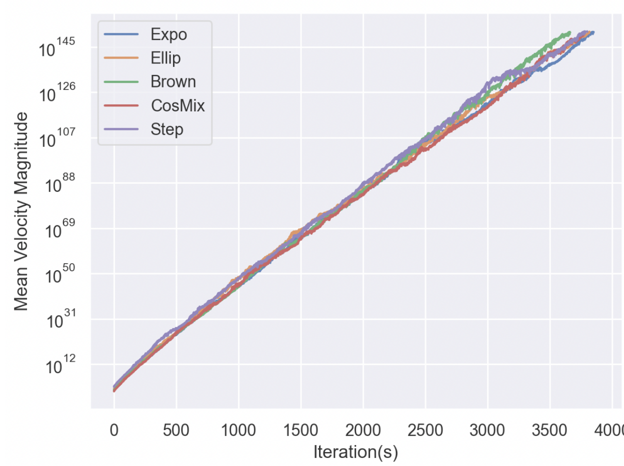
V. Simulation Results

This section presents empirical results and observations regarding the comparisons between various degrees of velocity clamping, and that of no velocity limitation. Subsection (A) exhibits the adverse behavioural characteristics of PSO when poor control parameter selections are made. Subsection (B) highlights performance when stability-adhering control parameter configurations are chosen.

A. Performance of PSO with non-conforming control parameter configurations

As discussed, the performance of PSO is highly dependent on the control parameter values chosen. When poor parameter selections are made, PSO exhibits extremely undesirable behaviour. An explosion of particle velocity is illustrated in figure 1. Note that this behaviour is highly consistent throughout the various benchmark functions selected. In such a scenario, particles are so far out of the feasible solution space that candidate solutions represented by the particles positions are valueless. Very similar results are obtained using control configuration ($w = 0.9, c_1 = 2, c_2 = 2$). The percentage of particles beyond the confines of feasible space is shown in figure 2. The speed at which particles leave the search space, and remain outside the search space, is unfavorable. Figure 2 suggests that after only a few iterations nearly all the particles are exhibiting roaming behaviour.

Fig. 1. Explosion in mean velocity magnitude ($w = 1, c_1 = 2, c_2 = 2$)



Velocity clamping is effective in mitigating particle roaming when the control parameters do not adhere to the stability conditions of equation (5). A plot of velocity magnitude for various velocity clamping intensities is given by figure 3. Velocity clamping intensity is defined for various values of k, as defined in equation (3). This figure demonstrates the degree to which velocity clamping has restricted velocity explosion in the Brown benchmark function (9). The results were similar across all benchmark functions.

Figure 3 highlights why velocity clamping has longly been regarded as a successful tool to suppress particle velocity. Although figure 3 suggests that employing velocity clamping when poor control parameters are selected is successful, Figure 3 does not provide much detail as to how the various velocity clamping intensities perform against one another. A more detailed visualization on the performance of various velocity clamping intensities is given in figure 4.

Figure 4 demonstrates that the performance of a velocity-restricted PSO algorithm is highly dependent on the degree to

Fig. 2. Particles exhibiting roaming behaviour (%) ($w = 1, c_1 = 2, c_2 = 2$)

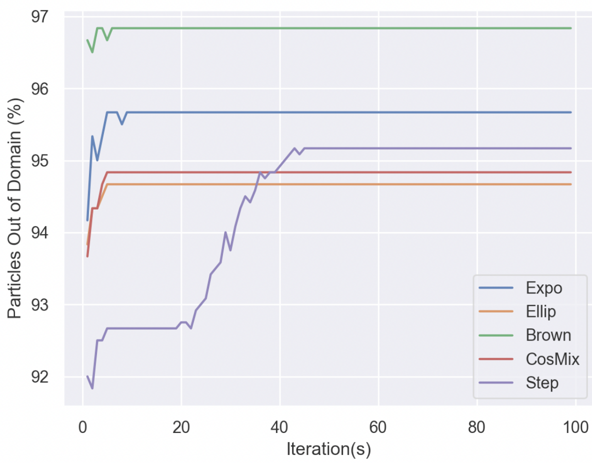
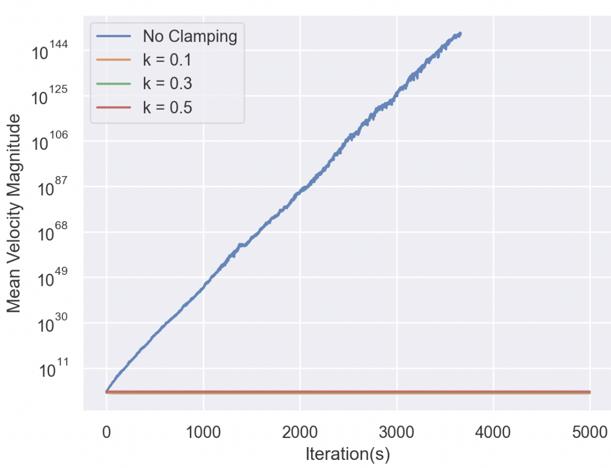


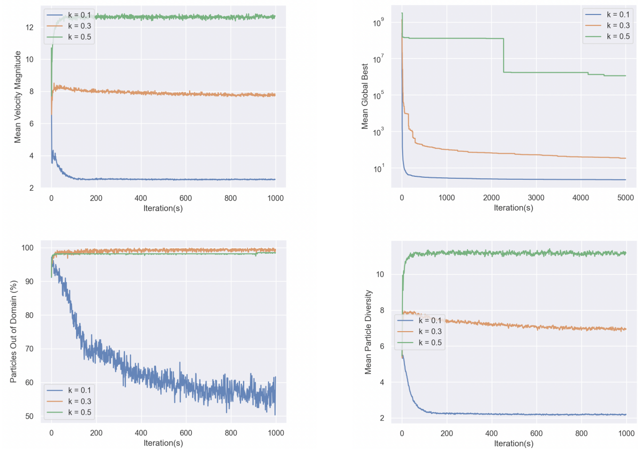
Fig. 3. Velocity clamping mitigating velocity explosion in Brown benchmark function ($w = .9, c_1 = 2, c_2 = 2$)



which velocity is restricted. This result is consistent regardless of the benchmark function used, though some benchmark functions are effected more highly. The consequence of this finding is that PSO algorithms with poor control parameter configurations need to be tuned. That is- a problem-specific optimal value of k should be found. This is a computationally demanding process.

In the case of the Brown benchmark function, only severe velocity clamping ($k = 0.1$) kept particles from leaving the search domain in the long term. Consequential to this finding, the global best position improved consistently between iterations, until a near-optimal value was found. Unfortunately, a clamping coefficient of ($k = 0.3$ and $k = 0.5$) was not restrictive enough to keep the particles from leaving the search domain, resulting in global best stagnancy. In the case of

Fig. 4. Performance of velocity clamping using the Brown benchmark function ($w = .9, c_1 = 2, c_2 = 2$)



the 30-dimensional brown benchmark function, a clamping coefficient of $k = 0.1$ will sufficiently suppress velocity explosion such that favourable search behaviour is exhibited. The "sufficient" value for k will vary between problems however. For the Exponential and Cosine-Mixture benchmark function given by equations (6) and (10) respectively, all clamping intensities ($k = 0.1$, $k = 0.3$, $k = 0.5$) successfully induced good behavioural characteristics.

B. Performance of PSO with stability-adhering control parameter configurations, compared to velocity clamping

The behaviour of PSO with control parameters conforming to equation (5) demonstrate behaviour very different to that of the previous section. Figure 5 is the analogue to figure 1, using control parameters that conform to equation (5). Notice the convergence in velocity magnitude. If particles automatically avoid roaming behaviour, there is no need to restrict particle velocities. From an optimization perspective this is useful as it cuts back on the computational expensive task of tuning the model.

The quality of the solutions obtained when using control parameters that conform to equation (5) is often competitive between good clamping coefficients, and no velocity clamping. It is important to note that some velocity clamping coefficients often yield poor results. Figure 6 demonstrates this finding. This suggests that in order for velocity clamping to yield competitive results, model tuning is required to ensure that a "good" value for k is employed.

Stability in performance is also a desirable PSO characteristic. It is preferable that PSO find good solutions reliably. In some cases, variance in global best fitness between independent trials is minimized if the selected control parameter configurations satisfy the stability condition of equation (5). Figure 7 illustrates non-restrictive methods finding desirable points in the search space with increased reliability, compared to the various velocity restricting techniques. Similar results

Fig. 5. Magnitude of particle velocity using control parameters that satisfy equation (5) ($w = 0.7, c_1 = 1.4, c_2 = 1.4$)

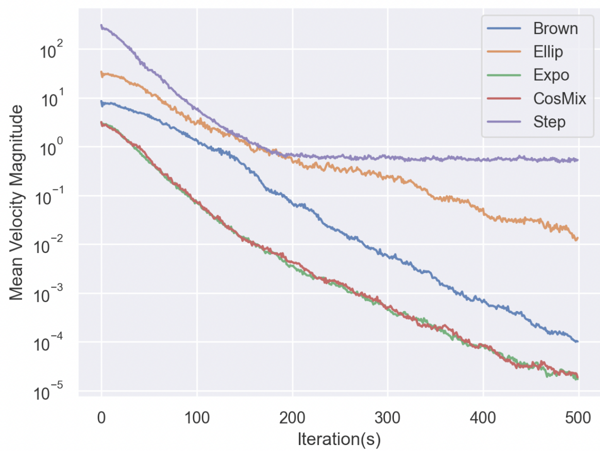
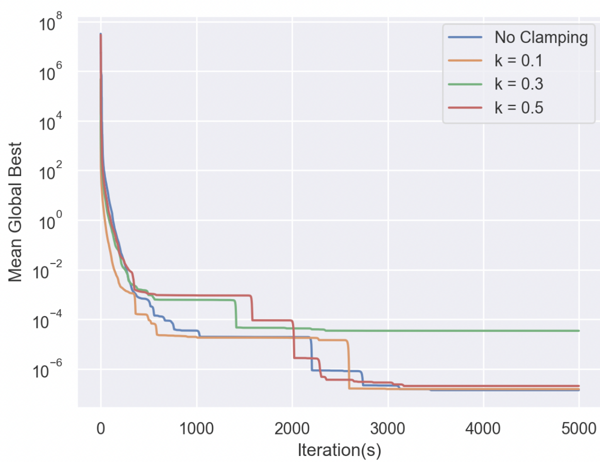


Fig. 6. Mean global best fitness between independent simulations of the Brown benchmark function ($w = 0.7, c_1 = 1.4, c_2 = 1.4$)



are obtained in the Cosine-Mixture, Elliptic, and Exponential benchmark functions.

In the case of the Step benchmark function, mild-moderate ($k = 0.5$, $k = 0.3$) velocity clamping lead to lower variance in global best fitness. This is shown in figure 8. Although velocity clamping may yield better PSO performance in some benchmark functions, the majority of simulations demonstrated increased consistency when no clamping was employed.

All clamping strategies (as well as no velocity clamping) were successful in limiting roaming behaviour. By the final iterations all particles converged to a point within the bounds of the search space. It is noticeable however, that the absence of velocity clamping results in mean particle diversity being higher for longer, compared to when velocity restrictive measures were used. Figure 9 demonstrates this

Fig. 7. Variance of global best fitness between independent simulations using the Brown benchmark function and control parameters conforming to equation (5) ($w = 0.9, c_1 = 0.7, c_2 = 0.7$)

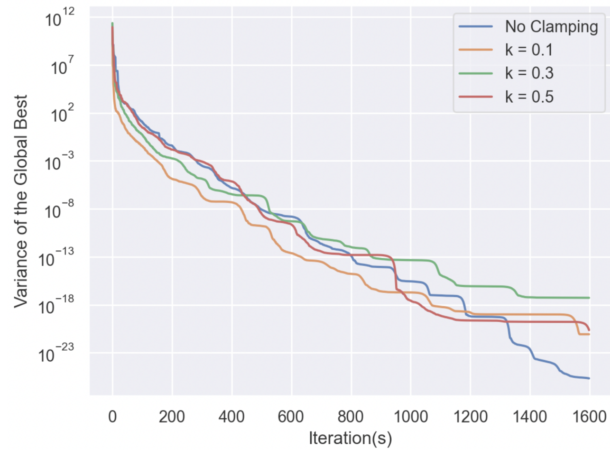
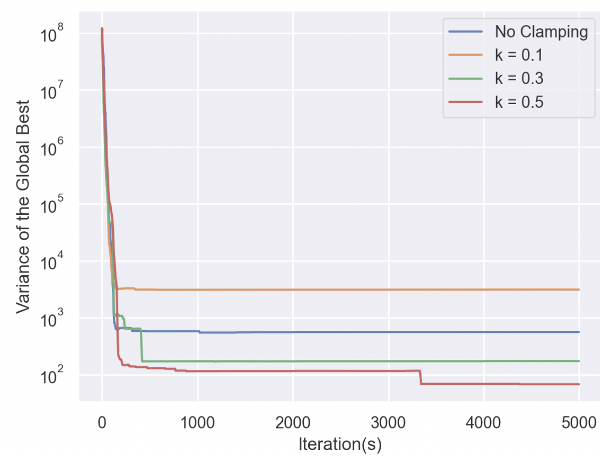


Fig. 8. Variance of global best fitness between independent simulations using the Step benchmark function and control parameters conforming to equation (5) ($w = 0.7, c_1 = 1.4, c_2 = 1.4$)

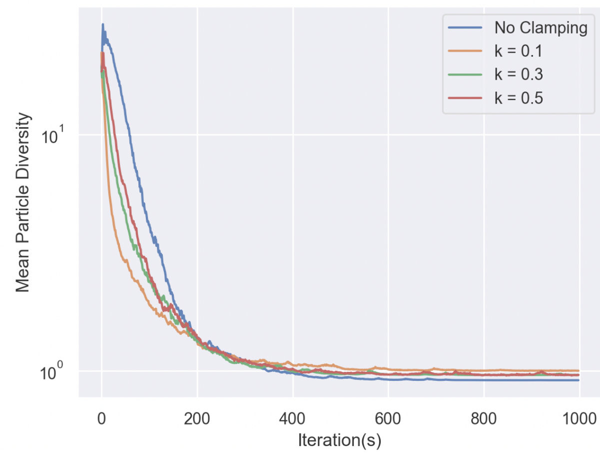


finding. This result suggests that explorative behaviour is exhibited for a longer duration, before exploitative behaviour is exhibited. This outcome was common when the control parameter configurations conformed to equation (5).

The switch from explorative to exploitative behaviour is highly desirable in PSO. It was found that the degree to which velocity was clamped, had a direct relationship on the intensity of exploitation in the swarm. When no form of velocity restriction was imposed on the swarm, particles explore more in the early iterations, and exploit more in the later iterations. Figure 10 illustrates the transition from explorative to exploitative behaviour.

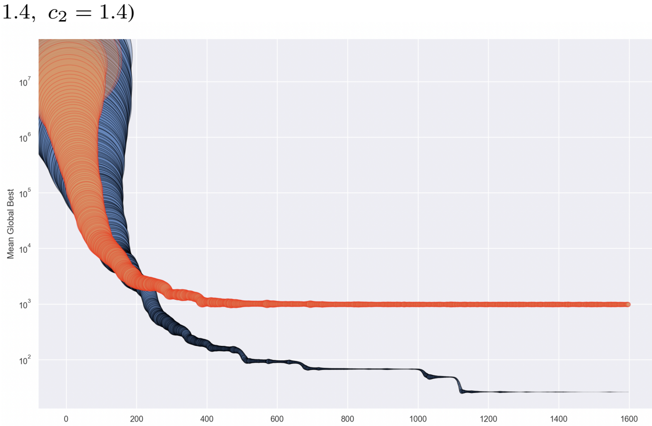
In figure 10 intense velocity clamping is directly compared to the absence of velocity clamping. The radius of a point is

Fig. 9. Mean particle diversity in Elliptic benchmark function using control parameters conforming to equation (5) ($w = 0.7, c_1 = 1.4, c_2 = 1.4$)



directly related to the mean velocity of particles during that iteration. This convention gives good insight to the granularity of the search process. Early on in the search procedure, particles should exhibit explorative behaviour (low granularity), transitioning to exploitative tendency (high granularity) during later iterations. As evident in the figure, velocity magnitude in early iterations is much larger when there is no velocity restriction. The explorative behaviour of the swarm in the early iterations allows particles to locate promising points in the search domain that are not yet explored by clamped particles. This empowers unrestricted particles to exploit better solutions when velocities are not confined, in the case of the Elliptic function.

Fig. 10. Mean global best performance on Elliptic benchmark function where point radius is directly related to velocity magnitude. ($w = 0.7$, $c_1 = 1.4$, $c_2 = 1.4$)



The question regarding the necessity of velocity clamping has yet to be directly addressed. It is shown in figure 4 that velocity clamping does not unconditionally mitigate particle roaming behaviour. Although effective- the clamping coefficient k acts as yet another model-specific control parameter that must be tuned, in order to perform optimally. It has also been shown in figure 7 that when good control parameter values are selected, desirable global best fitness evaluations are obtained with minimal variance. That being said, simply using control parameter configurations that represents solutions to equation (5) is not enough to ensure optimal PSO performance. It has however been demonstrated that if control parameters are carefully selected, particle velocities do not need to be restricted in order for PSO to yield solutions that are competitive with those obtained using velocity clamping. Furthermore, figure 6 has shown that if a poor value is chosen for the clamping coefficient, the model may yield worse solutions. With these findings in consideration, this study suggests that velocity clamping is not required for PSO to demonstrate favorable behavioural characteristics. The degree of velocity clamping is highly model specific, and if chosen incorrectly, the solutions may not be satisfactory. The findings of this study suggest that for 30-dimensional optimization problems, carefully selected control parameters cause the PSO model to perform competitively.

VI. Conclusion

The intention of this paper was to investigate the necessity of velocity clamping as a tool to mitigate the adverse effects of particle roaming in particle swarm optimisation (PSO), provided that control parameters satisfy a stability condition. To investigate this question, evaluation metrics were established as a means to evaluate the quality of PSO performance. With these evaluation techniques in place, simulations were conducted using various control parameter configurations and various velocity clamping techniques. The simulation results suggest that PSO performance quality depends highly on the degree to which velocity was restricted, and the control parameter configurations used. For configurations satisfying the stability condition, competitive solutions can be obtained. These solutions were often better than those obtained by employing velocity restrictive measures. This is likely as a result of the premature exploitative tendency that velocity restriction foists upon the model. For 30 dimensional search spaces and convergence satisfying control parameters, inaccurate velocity clamping intensity catalyzes highly granular searching behaviour in the early iterations of the simulation, hindering proper particle exploration. As a result it is concluded that velocity clamping is a non-essential technique, provided that the control configurations of the PSO model adhere to the noted convergence conditions.

Future work includes a study regarding a performance comparison between dynamic velocity clamping techniques, and control parameter tuning, for high-dimensional PSO problems. It was demonstrated in this paper that PSO performance is highly dependent to both these fields, it will be interesting to study which optimization technique yields superior solutions in higher dimensional problems.

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