Campaign Resource Allocations Under the Electoral College*

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In a recent essay Steven Brams and Morton Davis offer a game-theoretic analysis of campaign resource allocations under the Electoral College that yields the nonobvious prediction that: "The Electoral College . . . induces candidates to allocate campaign resources roughly in proportion to the 3/2's power of the electoral votes of each state."[1] They argue, moreover, that the 3/2's rule predicts the candidates' allocations (measured by campaign appearance data) as well as if not better than a simple proportional rule. Hence, they offer both a theoretical and an empirical justification for believing that the Electoral College establishes a bias that favors larger states. We argue, however, that the apparent empirical support for the 3/2's rule is an artifact of: (1) the empirical relationship between size and the competitiveness of states, (2) the existence of corner solutions to the candidate's maximization problem, and (3) the campaign's sequential nature. Looking at the frequency of campaign appearances as well as Stanley Kelley, Jr.'s estimates of the time candidates spend in states, we conclude that a modified proportional rule predicts allocations better generally than the 3/2's rule. We do not dispute the fact that resource allocations under the Electoral College typically favor larger states, but our theoretical rationale for this bias differs markedly from Brams and Davis's argument.

Section One reviews briefly the theoretical problem Brams and Davis address as well as their solution to it. Section Two presents an analysis of the data so as to make the strongest case for the validity of the 3/2's hypothesis. Section Three, however, examines both theoretically and empirically the strategic effects of competitiveness. Section Four considers the effects of "corner solutions" and sequential campaign planning. Briefly, we conclude that while no simple allocation rule is likely to account adequately for observed allocations, the validity of the 3/2's hypothesis is at best highly suspect.

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[1] Steven J. Brams and Morton D. Davis, "The 3/2's Rule in Presidential Campaigning," *American Political Science Review*, 68 (March 1974), 113.

Resource Allocations and the 3/2's Rule

Turning first to a theoretical definition of election competition, if candidates seek to maximize their respective probabilities of winning, then two-candidate elections can be modeled as two-person zero-sum games. To construct such a model, we introduce some essential notation:

- E_j : the electoral vote of state j, $j=1, \cdots, 51$;
- θ_i : candidate i's strategy, $i=1, 2$;
- $\phi_{ij}(\theta_1, \theta_2)$: candidate i's popular vote plurality in state j;
- $P_{ij}(\theta_1, \theta_2)$: the probability that candidate i receives all of state j's electoral vote, which, under the unit rule system equals $\Pr[\phi_{ij}>0]$;
- $\bar{E}_i(\theta_1, \theta_2)$: the total electoral vote realized by candidate i.

Candidate 1's decision task, then, is to find an equilibrium solution to the following problem:

$$(1) \quad \max_{\theta_1} \min_{\theta_2} \Pr\left[\bar{E}_1(\theta_1, \theta_2) - \bar{E}_2(\theta_1, \theta_2) > 0\right],$$

subject to appropriate constraints on θ_1 and θ_2. If θ_1 and θ_2 are alternative allocations of some scarce resource (e.g., time) across states, and if we denote the resources candidate i allocates to state j by θ_{ij}, the logical constraint is,

$$(2) \qquad \sum_{j=1}^{51} \theta_{ij} \leq B_i, \qquad i = 1, 2 \quad \text{and}$$
$$\theta_{ij} \geq 0 \qquad \text{for all } i, j$$

where B_i is candidate i's budget.[2] It is straightforward to show, moreover, that if the variance of the random variable $\bar{E}_1 - \bar{E}_2$ is constant for all θ_1 and θ_2, maximizing probability of winning is equivalent to maximizing expected electoral vote.[3] That is, expression (1) is equivalent to

[2] In spatial models of party competition, however, strategies are not resource allocations but positions on "issues," and the constraint should be $\theta_{i1} = \theta_{i2} = \cdots = \theta_{i51}$. For a discussion of this distinction and its implications see Melvin J. Hinich and Peter C. Ordeshook, "The Electoral College: A Spatial Analysis," *Political Methodology* (forthcoming).

[3] See Peter H. Aranson, Melvin J. Hinich, and Peter C. Ordeshook, "Election Goals and Strategies. Equivalent and Nonequivalent Candidate Objectives," *American Political Science Review*, 68 (March, 1974), 135–152.

(3) $$\max_{\theta_1} \min_{\theta_2} \sum_{j=1}^{51} E_j P_{1j}(\theta_1, \theta_2)$$

For most general assumptions about P_{1j}, however, an equilibrium solution in pure strategies does not exist for the zero-sum game described by expressions (2) and (3). If, for example,

(4) $$P_{1j} = \begin{Bmatrix} +1 \\ \frac{1}{2} \\ 0 \end{Bmatrix} \text{ if and only if } \theta_{1j} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \theta_{2j},$$

the election model is a Colonel Blotto game. The unfortunate feature of such games is that they possess only complicated mixed strategy solutions that are not subject to any substantively meaningful interpretation.[4]

Brams and Davis's resolution of this problem is this: First, they assume that the candidates match strategies in each state, i.e., $\theta_{1j} = \theta_{2j}$ for all j. Second, they derive the probability that candidate i carries state j, $P_{ij}(\theta_1, \theta_2)$, from the assumption that citizens do not abstain and that

(5) $$p_{1j} = \frac{\theta_{1j}}{\theta_{1j} + \theta_{2j}}$$

where p_{ij} is the probability that a citizen in state j votes for candidate i (hence, $p_{2j} = \theta_{2j}/[\theta_{1j} + \theta_{2j}]$). The probability $P_{ij}(\theta_1, \theta_2)$, then, is (for an even number, n_j, of citizens in state j),

(6) $$P_{1j}(\theta_1, \theta_2) = \sum_{k=n_j/2+1}^{n_j} \binom{n_j}{k} p_{1j}^{k} (1 - p_{1^j})^{n_j - k}$$

Owing to the law of large numbers (and certainly even Alaska's population is a large number statistically), P_{1j} is essentially described by expression (4). However, P_{1j} remains a differentiable function of θ_{ij} since only in the limit, $n_j \rightarrow \infty$, is P_{1j} restricted to the values 1, 1/2, 0. Hence, the calculus can still be applied to solve for minimax strategies.

With these assumptions Brams and Davis show that

(7) $$\theta_{ij} \approx cE_j^{3/2}, \qquad j = 1, \cdots, 51$$

is a local pure strategy equilibrium to the election game (where c is a positive constant determined by the actual size of the candidate's budget). That is, if both candidates allocate their resources across states in proportion to each state's electoral vote, raised to the 3/2's power, neither candidate possesses any incentive to shift his strategy unilaterally, provided that candidates can make only very small shifts.[5]

Given that the 3/2's rule is but a local equilibrium strategy, it is of course tenuous to argue that candidates should adopt such strategies. Nevertheless, Brams and Davis offer empirical evidence to suggest that candidates actually conform to this rule, and it is that evidence to which we now turn.

Empirical Evidence

The data Brams and Davis use to test their model consist of the number of times a major party's presidential and vice-presidential slate visited each state during the 1960–1972 presidential campaigns.[6] While they present these data in several forms, they assert that the "most convincing case can be made for the general applicability of the 3/2's rule" from the simple correlations between actual campaign appearances and the allocations predicted by their model. In Table 1 we present those correlations as well as the correlations between appearances and the proportional rule.

Certainly, as Brams and Davis note, the correlations for the 3/2's rule are generally higher than the correlations for the proportional rule—excepting the Democratic Party's campaign in 1964. But, none of these differences appears to be significant. We cannot yet rule out the possibility that candidates allocate proportionally and that the Electoral College engenders no bias. Stated simply, we cannot discriminate between the two hypotheses with simple correlation coefficients. If we present this data differently, however, it appears strongly to support the 3/2 rule as against the proportional rule (see Brams and Davis's Tables 3 and 4). Aggregating states into "large," "medium" and "small" categories, candidates do allocate proportionately more resources in the

[4] O. Gross and R. Wagner, "A Continuous Colonel Blotto Game" (Santa Monica, Calif.: RAND Memo. #408, June 17, 1950); O. Gross, "The Symmetric Blotto Game" (Santa Monica, Calif.: RAND Memo. #424, July 19, 1950); Lawrence Friedman, "Game-Theory Models in the Allocation of Advertising Expenditures," *Operations Research,* 6 (Sept.-Oct. 1958), 699–709. See also David Sankoff and Koula Mellos, "The Swing Ratio and Game Theory," *American Political Science Review,* 66 (June, 1972), 551–554.

[5] It is important to note, however, that the 3/2's rule is vulnerable to strategies that deviate from it by *very* small amounts. That is, 3/2's is truly a local optimum in the most extreme sense. The magnitude of deviations from 3/2's that can upset it is on the order of the sensitivity of P_{ij} in expression (6) to variations in P_{ij}. Note that if n_j is large (e.g., n_j equals the population of Alaska), P_{ij} is essentially a point mass at 1 or 0 if $P_{ij} = 1/2 + \varepsilon$ where ε is a *very* small number.

[6] Throughout this analysis we omit Alaska and Hawaii because of the discontinuity in campaign costs associated with visits to them.

Table 1. Correlation Coefficients Between Campaign Appearances and the 3/2's and the Proportional Allocation Rules

Year	Republican Slate		Democratic Slate	
	3/2	Proportion	3/2	Proportion
1964	.95	.91	.96	.94
1964	.87	.86	.83	.84
1968	.94	.94	.93	.90
1972	.79	.79	.86	.85

form of trips to large states than to medium and small states (excepting 1964).

One problem with this procedure, however, is that it does not permit us to test other more complex hypotheses. Hence, let us restate the evidence by estimating the coefficient β for the following model:

$$(8) \qquad T_j = \alpha E_j^{\beta}$$

where T_j denotes the number of trips a candidate or a slate takes to state j. Assuming a multiplicative log-normal error structure, expression (8) suggests the following regression[7]

$$\log T_j = \log \alpha + \beta \log E_j + \epsilon_j$$

Presumably, the proportional rule predicts that $\hat{\beta} \approx 1.0$, whereas the 3/2's rule predicts that $\hat{\beta} \approx 1.5$.

Unfortunately, estimating β is difficult because the candidates do not visit all the states. Specifically, for $T_j = 0$, $\log T_j = -\infty$, and if only one $T_j = 0$, $\hat{\beta} = \infty$. The question, then, is how to treat these observations. Clearly, neither the simple proportional or 3/2's rules predicts $T_j = 0$. The problem lies, apparently, in the lumpiness of the dependent variable. That is, trips are not infinitely divisible, and a candidate cannot take, say, .25 trips to a state. We may observe $T_j = 0$, rather than .25, then, because the candidate rounds the optimal solution off to the nearest integer. Later we examine this lumpiness in greater detail theoretically and empirically. Presently, however, we assume that if we observe $T_j = 0$, the true value of T_j is δ_j, where $0 \leq \delta_j < .5$. Taking a median value for δ_j, .25 we set $T_j = .25$ if and only if the candidate (or slate) fails to visit state j.[8] Thus, we estimate the coefficients of the following model:

$$(9) \quad \log [T_j + \delta_j] = \log \alpha + \beta \log E_j + \epsilon_j,$$

where

$$(10) \quad \delta_j = \begin{Bmatrix} .25 \\ 0 \end{Bmatrix} \quad \text{if } T_j \begin{Bmatrix} = \\ \neq \end{Bmatrix} 0$$

Naturally, we must check the sensitivity of $\hat{\beta}$ to variations in δ.

We set forth in Table 2 the estimates of β as well as the standard errors of these estimates.[9] Clearly, all coefficients are significantly greater than zero. More important, however, strong support for the 3/2's rule is exhibited by both slates in 1960 and 1968—the two competitive elections in our study—while the proportional rule is supported "conclusively" in only three instances.[10] And, even the one "indeterminate" coefficient is nearer 1.5 than 1.0. Overall $\hat{\beta}$ averages 1.25, while for 1960 and 1968 it averages, remarkably, 1.55. Since it is reasonable, moreover, to place greater emphasis on data from 1960 and 1968—because Brams and Davis assume, essentially, that the election is competitive—Table 2 must be interpreted as strongly supporting the 3/2's rule against a simple proportional rule.

These data, of course, need to be qualified.

Table 2. Value of $\hat{\beta}$ for Expression (9)

Year	Democratic Slate	Republican Slate
1960	1.37 (.13)*	1.56 (.19)*
1964	.83 (.16)**	.81 (.20)**
1968	1.56 (.20)*	1.73 (.16)*
1972	1.45 (.25)***	.79 (.17)**

* Significantly greater than 1.0 but not significantly different from 1.5.

** Significantly less than 1.5 but not significantly different from 1.0.

*** Indeterminate.

[7] If we assume that the observed total number of trips taken by a candidate is not a random variable, the parameters of this model are overidentified. That is, $\alpha = \Sigma T_j / \Sigma E_j^{\beta}$ and $\Sigma \varepsilon_j = 1$. We do not, however, make this assumption. Rather, we let ΣT_j be a random variable and assume that the ε_j's are independently distributed.

[8] An alternative procedure is to let $\delta_j = .25$ for all T_j. This, however, decreases $\hat{\beta}$ and renders it easier to secure estimates that support the proportionate hypothesis. For example, $\hat{\beta}$ in 1960 decreases from 1.37 to 1.32 for the Democratic slate and from 1.56 to 1.48 for the Republican slate.

[9] Throughout this paper, all statements concerning significance refer to the .01 confidence level. Also, the figures in parentheses denote standard errors.

[10] Several of these estimates are sensitive to δ_j while others are not. Letting δ_j vary between .1 and .5, we find that four coefficients vary by more than 5 per cent—those for both parties for the 1968 and 1972 elections (which is to be expected, of course, since in these elections the candidates bypass a great many states whereas almost all states are visited in 1960 and 1964). The coefficient for the 1972 Democratic slate is the most sensitive: $\hat{\beta} = 1.72$ for $\delta = .1$ and $\hat{\beta} = 1.24$ for $\delta = .5$. Thus, while there is some justification for letting $\delta = .25$ rather than .1 or .5, this sensitivity should be kept in mind as a qualification of our conclusions.

First, many trips are taken for reasons other than soliciting votes. Commenting on John Kennedy's last-minute tour of Connecticut, for example, Theodore H. White writes that "[Bailey] and Ribicoff had assured Kennedy that their state, Connecticut, was going to be safe—but they wanted their one day to show, too."[11] Second, a candidate's trip to a state may be designed to influence the voters of other states as well. Visits to New York City, for example, may activate party regulars in New Jersey, or it may yield funds from wealthy contributors destined to be spent in Kansas. Third, because collection of these data relies heavily on *The New York Times*, it is not unreasonable to suppose that trips to New York are better reported than trips to other states. Finally, we should not weight all trips equally. A full day's visit is hardly equivalent to a half-hour whistle stop.

With these last two qualifications in mind, then, we consider another source of data—Stanley Kelley's estimates of the time candidates spend in states. We set forth in Table 3 a comparison between estimates of $\hat{\beta}$ for the two data sets.[12] As this table reveals, Brams and Davis's data for 1960 yield slightly higher estimates than Kelley's data, whereas the converse is true for 1964. Any inferences derived from a comparison of these two sets of estimates are, of course, tenuous, owing to δ_j. Nevertheless, this table does suggest that if we use the number of trips as an indicator of the amount of time the candidates allocate to states, Table 2 overestimates β in 1960 but underestimates β in 1964. If this is true, then 3/2's is a less valid description of strategies than Table 2 indicates. In fact, with only one possible exception, Kelley's data offers little convincing support for the 3/2's hypothesis.

As always, however, these estimates are subject to numerous qualifications, including, in 1964, Johnson's incumbent status and the landslide proportions of his victory and, in 1960, Nixon's admittedly nonoptimal pledge to visit every state. We proceed, then, using both sets of data and, in particular, we assume that the coefficients obtained from Brams and Davis's data warrant further study.

Competitiveness

Much of the descriptive and normative literature on presidential elections offers a simple rule for candidates: identify the states that you and your opponent are certain to carry, and focus your attention on the remaining (competitive) states. That is, candidates should be concerned not only with size but also with the likelihood that resources can swing a state from one candidate to another.

Simple as this rule might seem, no consideration is given *in the derivation of the 3/2's rule* to the relative competitiveness of states. Brams and Davis assume that a state's citizen's probability of voting for a candidate, p_{ij}, is an unbiased function of the resources the candidates allocate to the state (see expression 5). Thus, if both candidates allocate the same amount of resources to a state, the state is as likely to go for one candidate as the other. Suppose, however, that competitiveness is systematically related to size. It is reasonable to conjecture, then, that some rule other than 3/2's is a local equilibrium (if in fact a local pure strategy equilibrium exists). It also follows that the 3/2's rule should not be tested against a proportional rule that is unadjusted for competitiveness.

That some adjustment is necessary is apparent if we measure the competitiveness for each candidate by the absolute value of his plurality in percentage terms. Plurality, obviously, is not a wholly satisfactory measure, but it is sufficient to indicate the direction of effects. Thus, denoting this difference by $|Pl_j|$, consider

$$(11)\quad \log\ |\ Pl_j\ |\ = a + b \log E_j + u_j$$

We present the results of this regression in Table 4.[13]

Table 4 reveals the relationship that exists between competitiveness and size in 1960 and 1968. While R^2 is not high for these two elections, $\hat{b}$ is significant.[14] More importantly, however, in 1960 and 1968, larger states are more competitive on the average than smaller states. Thus, if candidates allocate according to a proportional rule that is adjusted as the literature and common sense suggest, we should find in 1960 and 1968, allocations to large states in excess of proportion and allocations to small states that are less than proportional. Conversely, we should anticipate little adjustment in 1964 and 1972. If any adjustment is warranted in 1964 (on the basis of a coefficient that is not significantly different from 0), it is in favor of the smaller states.

[11] *The Making of the President,* 1960 (New York: Atheneum, 1960), p. 338.

[12] As with Brams and Davis's data, we add a δ_j to those T_j's we observe as zero. We again choose $\delta_j = .25$ since a quarter of an hour seems a sufficiently small rounding error. None of the coefficients, however, are especially sensitive to δ_j.

[13] Because of third parties that received more votes in a state than one or both of the two major parties, we calculate two sets of coefficients in 1960 and 1968. For these elections we set Pl_j equal to the relevant slate's plurality over its strongest opponent in state *j*.

[14] The approximate adjusted R^2's for 1960 and 1968 are .10 and .22 respectively, while the adjusted R^2's for 1964 and 1972 are essentially zero.

Table 3. Comparison of $\hat{\beta}$ for Brams-Davis and Kelley Data

	Democrat		Republican	
	$B-D$	Kelley	$B-D$	Kelley
1960 President	1.49 (.21)	1.47 (.17)	1.32 (.17)	1.06 (.17)
1964 President	.75 (.17)	.89 (.17)	.97 (.19)	1.18 (.15)
1964 Slate	.83 (.16)	1.13 (.14)	.81 (.20)	.93 (.15)

The implication of Table 4, then, is that it is not reasonable to assume that the estimates of $\hat{\beta}$ in Tables 2 and 3 properly reflect the influence solely of a state's size on campaign strategies. Rather, those estimates as well as any support they lend to Brams and Davis's model might simply reflect the empirical relationship between size and competitiveness. To test this possibility more carefully we must first formulate an appropriate model of campaign allocation. This is essential because we seek some theoretical justification for the statistical models we estimate as well as additional testable propositions. In formulating a model, however, we are confronted with at least two problems. First, we are uncertain about the best conceptualization of a candidate's strategic environment—is it, for example, decision making under risk or is a game-theoretic analysis more appropriate? The second problem is that to derive explicit allocation rules (e.g., 3/2's), we must posit explicit functional forms for the relationships among the model's variables (e.g., expression (5)). But, any such form is necessarily *ad hoc*, and available data do not always permit us to reject one assumption in favor of another.

Our solution to the first problem is to proceed with a decision making under risk model. This assumption eases exposition and permits us to explore more readily the possible explanations for observed campaign strategies. Our solution to the second problem is not to seek explicit allocation rules but rather to use our model as a guide to an analysis of the data. In this way, we seek to ascertain allocation rules that are theoretically meaningful and that conform as closely as possible to the observed allocations of candidates.

Table 4. Estimates of a and b for $\log |PI_j| = a + b \log E_j$

Year	Slate	$\hat{a}$	$\hat{b}$
1960	Democrat	2.85 (.51)	−.56 (.22)
	Republican	2.90 (.52)	−.56 (.23)
1964	Democrat / Republican	2.61 (.40)	+.14 (.18)
1968	Democrat	3.54 (.38)	−.63 (.17)
	Republican	3.55 (.39)	−.65 (.17)
1972	Democrat / Republican	3.39 (.26)	−.08 (.11)

Turning then to the model, we begin with the assumption that candidates maximize their expected Electoral College vote, the objective represented by expression (3).[15] To reduce (3) to an objective function for decision making under risk, suppose that each candidate acts as if he possesses a set of prior probabilities over his opponent's strategies. When these probabilities are absorbed into the P_{ij}'s, expression (3) becomes,[16]

$$(12) \qquad \max_{\theta_1} \sum E_j q_{1j}(\theta_1)$$

where $q_{1j}(\theta_1)$ is candidate 1's derived estimate of the probability that he carries state j with strategy θ_1.

The assumption that a candidate places prior probabilities on his opponent's strategies and acts as if he is a decision maker under risk must seem strange, since most formal analyses of elections offer a game-theoretic conceptualization of competition. However, unlike spatial models that typi-

[15] Alternatively, we can assume that candidates maximize their probabilities of winning but that the variance of the random variable $E_j(\theta_1, \theta_2)$ is approximately a constant for all θ_1 and θ_2, in which case expression (1) is equivalent to expression (3). One justification for this reduction, other than the obvious way in which it facilitates analysis, is the assumption that neither candidate can manipulate his strategies so as to change appreciably (from .5 say to .25) the P_{ij}'s of a great many states. Such manipulations are plausible if candidates can advocate policies that alienate large portions of the electorate. But we are considering here a different sort of strategy—resources in general and trips to states in particular. And in the absence of any strong empirical evidence that resources such as trips affect state outcomes in an overwhelming way, the assumption of a constant standard deviation is a reasonable initial approximation.

[16] Letting $s(\theta_2)$ denote candidate 1's prior probabilities over θ_2.

$$q_{1j}(\theta_1) = \int_{P_{ij}} (\theta_1, \theta_2) s(\theta_2) d\theta_2.$$

cally yield readily identifiable solutions such as "adopt the median preference," resource allocation games are exceedingly complex and difficult to analyze. Simple solutions do not exist generally, and global equilibrium strategies are difficult or impossible to interpret substantively. Thus, even a static game-model is an heroic abstraction from theoretical possibilities; namely, it eliminates the dynamic feature of campaigns. This abstraction is less important if a global pure strategy equilibrium exists, since we can conjecture that during the campaign, the candidates converge to the equilibrium. But if such equilibria do not exist, it is not unreasonable to suppose that candidates adopt heuristics that simplify their decisions. Brams and Davis use one such heuristic: candidates act as if they believe that opponents will allocate resources as they do (hence, $\theta_{1j}=\theta_{2j}$ for all j). Our assumption is another, although less specific, heuristic.

We now denote the solution to (12), subject to the candidate's budget constraint, by $\theta_1^*=(\theta_{11}^*, \theta_{12}^* \cdots)$. If we assume that a candidate's probability of carrying a state is not a function of how he allocates resources among other states (i.e., $\partial q_{ij}/\partial\theta_{1k}=0,\ k\neq j$) and if we assume also (and temporarily) that no θ_{1j}^* is a corner solution, then θ_1^* satisfies

$$(13)\qquad E_j\frac{\partial q_{ij}}{\partial\theta_{1j}}=\lambda,\qquad \lambda\neq 0$$

Next, we decompose $\partial q_{1j}/\partial\theta_{1j}$ thus,

$$\frac{\partial q_{1j}}{\partial\theta_{1j}}=\frac{\partial q_{1j}}{\partial\mu_{1j}}\cdot\frac{\partial\mu_{1j}}{\partial\theta_{1j}}$$

(where $\mu_{1j}(\theta_{1j})$ is candidate 1's expected plurality in state j), and we rewrite condition (13) as,

$$(14)\qquad \lambda^{-1}E_j\frac{\partial q_{1j}}{\partial\mu_{1j}}=\left[\frac{\partial\mu_{1j}}{\partial\theta_{1j}}\right]^{-1}\qquad \text{for all } j.$$

Thus, a model of how candidates allocate resources among states consists essentially of assumptions about the derivatives $\partial q/\partial\mu$ and $\partial\mu/\partial\theta$. As we note earlier, however, a model that seeks to specify a particular allocation rule must substitute into (14) explicit functional forms for these derivatives. Unfortunately, we are not certain what those forms should be or what they are in the minds of the candidates. Nevertheless, we can conjecture about their general properties. Turning first to $\partial q/\partial\mu$, we see that $q_{ij}=\Pr[x_{ij}>0]$, where x_{ij} is candidate 1's realized plurality in state j. Assuming that x_{ij} is a random variable with expected value μ_{ij}, the probability q_{1j}, then, is a cumulative density that increases as μ_{1j} increases and thus possesses a functional form like the curve in Figure 1. If, for example, x_{1j} is approximately normally distributed, $\partial q_{1j}/\partial\mu_{1j}$ decreases as the absolute value of μ_{1j}, written $|\mu_{1j}|$, increases. This, of course, is simply a restatement of the intuitively reasonable proposition that if the candidate anticipates a very large or very small plurality, small changes in this plurality do not affect greatly his chances of carrying a state; but if the election is close, these small changes affect q_{1j} significantly.

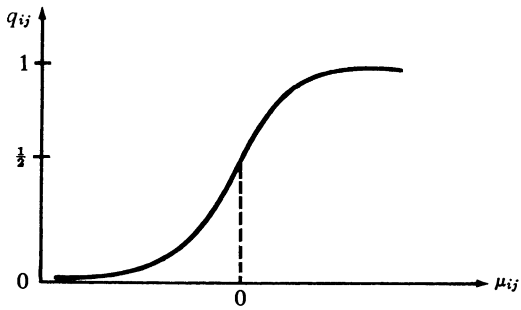

Figure 1: General Functional Form for $q(\mu)$

It is not so easy to specify the properties of $\partial\mu/\partial\theta$. First, it is entirely possible that this derivative is negative for some candidates in some states. John Kennedy, after entering the 1960 Democratic primary in West Virginia, for example, experienced an initial decline in that state's polls as Protestant citizens became aware of Kennedy's religion. Equivalently, a candidate might succeed only in increasing turnout across the board, and if registration or citizen sentiment favors his opponent, his campaign yields a net decrease in μ_{1j}.[17] Conversely, $\partial\mu_{1j}/\partial\theta_{1j}$ might be rendered positive if the candidate conducts a more sophisticated campaign designed to activate only his party's leadership which, in turn, activates only citizens who are likely to vote for him. A candidate might visit only the regions of a state that favor him. Or the candidate might use his resources to improve his image or to increase the salience of issues on which he possesses some strategic advantage.

Whatever the form of this derivative, it must be the case that $\theta_{1j}^*>0$ only if $\partial\mu_{1j}/\partial\theta_{1j}\geq 0$ at θ^*; otherwise the candidate could withdraw resources from the state and increase his expected plurality. But even if the derivative is non-negative for all θ, its form can vary considerably. The function $\mu_{1j}(\theta_{1j})$ might, for example, look like curve B in Figure 2, which is to say that the initial rate of return on θ_{1j} increases up to some point and thereafter is subject to the law of diminishing marginal

[17] For a more complete discussion of such possibilities see Gerald H. Kramer, "A Decision-Theoretic Analysis of a Problem in Political Campaigning," in *Mathematical Applications in Political Science, II*, Joseph L. Bernd, ed. (Dallas, SMU Press, 1966), 137–160.

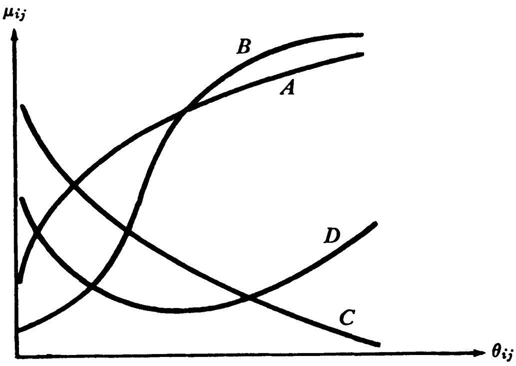

Figure 2: Alternative Functional Forms for $\mu(\theta)$

returns. Alternatively, $\mu_{1j}(\theta_{1j})$ might exhibit diminishing marginal returns for all θ_{1j} (see curve A). Curves D and C in Figure 2 illustrate two additional forms that $\mu_{1j}(\theta_{1j})$ might assume. Doubtlessly, others can be imagined and rationalized.

All of these possibilities render tenuous the exercise of deriving a simple general allocation rule for candidates. Our objective, however, is not to derive a particular rule but rather to test whether the proportional hypothesis or the 3/2's hypothesis best fits the data on candidate trips. With a few additional assumptions we can apply expression (14) to that end. (Parenthetically, however, we must warn the reader that our subsequent assumptions are not designed to yield a mathematically rigorous theory that explores all possibilities as generally as possible. Rather, they facilitate formulation of a simple and estimatable statistical model. In this way we hope to render our empirical analysis less ad hoc and its implicit assumptions more evident.)

Consider first $\partial\mu/\partial\theta$. If form C in Figure 2 applies, then $\theta^*_{1j}=0$. Since we are assuming that $\theta^*_{1j}>0$, we eliminate functions of this sort from consideration. Functions such as B and D, on the other hand, in general do not yield tractable statistical models. Consequently, we assume that $\mu_{1j}(\theta_{1j})$ looks like curve A—that is, the return from allocations is subject to the law of diminishing marginal returns. The simplest and most tractable representation of this assumption is that

$$(15) \qquad \frac{\partial\mu_{1j}}{\partial\theta_{1j}} = K/(\theta_{1j}+\xi)^A,$$

where K, A, and ξ are unknown positive constants.

Turning now to $\partial q_{1j}/\partial\mu_{1j}$, we know that, theoretically, this derivative is a function of θ_{1j} since, from Figure 1, it is a function of μ_{1j} and since μ_{1j} is a function of θ_{1j}. Suppose, however, that we approximate $\partial q_{1j}/\partial\mu_{1j}$ by its value at Pl_j—candidate 1's realized plurality in state j. In particular, suppose that:

$$(16) \qquad \frac{\partial q_{1j}}{\partial\mu_{1j}} = \frac{k}{|Pl_j|^d},$$

where k and d are non-negative constants. This expression represents, of course, a heroic assumption. First, Pl_j is the realized value of a random variable: thus, it can be a poor estimate of that variable's expected value, μ_{1j}. Second, μ_{1j} might vary considerably as the election campaign proceeds, owing, for example, to campaign blunders or to a new international crisis.[18] Thus, a candidate's estimate of his expected plurality when he initially invests resources in a state can differ substantially from his estimate when he invests the last units of his resources.

With all the necessary qualifications in mind, we substitute expressions (15) and (16) into (14) and obtain a necessary condition for a maximum. This condition, in turn, yields the following modification of expression (8):

$$(17) \qquad T_j + \delta_j = CE_j^{\beta}|Pl_j|^{-\gamma}$$

Again assuming a log-normal multiplicative error structure and letting δ_j satisfy condition (10), we present the estimates of $\hat{\beta}$ and $\hat{\gamma}$ in Table 5.

[18] Clearly, though, if Pl_j is a function of θ_{ij} in any statistically appreciable way, the appropriate methodology is simultaneous equations estimation. Our data, however, seems far too crude to apply this method and it is reasonable to asume a priori that the relationship from Pl_j to θ_{ij} is the stronger.

Table 5. Estimates of $\hat{\beta}$ and $\hat{\gamma}$ for Expression (17)

Year	Democratic Slate		Republican Slate	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
1960	1.23 (.13)**	.25 (.07)	1.40 (.19)*	.30 (.11)
1964	.87 (.15)**	.30 (.12)	.87 (.19)**	.41 (.15)
1968	1.15 (.25)***	.64 (.19)	1.54 (.17)*	.30 (.13)
1972	1.36 (.21)***	1.16 (.26)	.78 (.17)**	.12 (.22)

* Significantly greater than 1.00 but not significantly different from 1.50.
** Significantly less than 1.50 but not significantly different from 1.00.
*** Indeterminate.

Note that with one exception, $\hat{\gamma}$ is significantly greater than zero at the .01 confidence level; and even the one insignificant coefficient possesses the proper sign.[19] With Table 2 in mind, note also that the inclusion of $|Pl_j|$ affects our estimates of β in the anticipated way: If $|Pl_j|$ and E_j are negatively related as in 1960, 1968, and 1972 (see Table 3), $\hat{\beta}$ declines, but if a positive relationship exists as in 1964, $\hat{\beta}$ increases. These are highly gratifying results, and they strongly support common sense arguments about the role competitiveness plays in campaigns.

A comparison between estimates of $\hat{\beta}$ in Tables 2 and 5, however, is more interesting for our purposes. In particular, note that four coefficients in Table 2 support 3/2's, but only two do so unambiguously in Table 5. And while only three coefficients support the proportional rule in Table 2 and one coefficient is indeterminate, four coefficients in Table 5 support the (modified) proportional rule and two are indeterminate. Thus, while $\hat{\beta}$ averages 1.55 in Table 2 for 1960 and 1968, it averages 1.33 in Table 5 for these two elections.

[19] The adjusted R^2's with and without $|Pl_j|$ are as follows:

Year	Slate	Expression (9)	Expression (17)
1960	Democrat	.70	.75
	Republican	.60	.64
1964	Democrat	.36	.42
	Republican	.25	.34
1968	Democrat	.47	.57
	Republican	.71	.74
1972	Democrat	.41	.58
	Republican	.30	.29
	Average	.475	.54

Note, however, that these numbers are defined in terms of log T_j. To ascertain the variance accounted for in terms of T_j, it is necessary to retransform the variables and, given our estimates of $\hat{\beta}$, $\hat{\alpha}$ and $\hat{\gamma}$, recalculate R^2. These new values for R^2 will be higher than those we report above but the change in R^2 owing to the inclusion of $|Pl_j|$ will diminish.

Overall, $\hat{\beta}$'s average is reduced from 1.25 to 1.15, while the standard deviation of this average declines from .36 to .26. Hence, the inclusion of $|Pl_j|$ collapses the distribution of our estimates about a value that is closer to 1.0 than to 1.5.

For completeness, we conclude this section with a comparison of the Brams-Davis and Kelley data in Table 6.

Even though only one $\hat{\gamma}$ is significantly greater than zero with Kelley's data, all estimates of $\hat{\gamma}$ possess the proper sign. Thus a simple nonparametric signs test supports the proposition that candidates allocate proportionately more resources to competitive states. With Table 3 in mind, we note also that the inclusion of $|Pl_j|$ affects our estimates of $\hat{\beta}$ in the anticipated way for both data sets: In 1964, $|Pl_j|$ increases $\hat{\beta}$, whereas in 1960, $|Pl_j|$ decreases $\hat{\beta}$. And while the standard errors of these estimates preclude rejection of the 3/2's hypothesis in two cases, these coefficients vary convincingly about 1.0. Again, then, we find substantially less support for the 3/2's hypothesis with data on the amount of time candidates spend in states than we find with the data on trips.

Given Tables 5 and 6, it is reasonable to reject the 3/2's hypothesis in favor of a modified proportional rule. Of the fourteen estimates for $\hat{\beta}$, only one exceeds 1.50, which clearly indicates that $\hat{\beta}<1.5$. Nevertheless, there are several bothersome details. First, the analysis does not account presently for the failure of candidates to visit states. Second, many coefficients in Tables 5 and 6 exceed 1.0, some significantly. The inclusion of $|Pl_j|$ reduces $\hat{\beta}$, but perhaps not sufficiently to accept, without suspicion, the modified proportional hypothesis. We turn then to two additional theoretical considerations—"corner solutions" and sequential planning.

Corner Solutions and Sequential Planning

Consider the following simple constrained maximization problem: maximize the differentiable function $f(x_1, x_2)$, subject to $x_1+x_2\leq B$, where x_1 and x_2 are nonnegative. Forming the Lagrangian

Table 6. A Comparison of $\hat{\beta}$ and $\hat{\gamma}$ for the Brams-Davis and Kelley Data Using Expression (16)

	Brams-Davis		Kelley	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
1960 Democratic Presidential Candidate	1.38 (.22)	.19 (.14)	1.39 (.18)	.16 (.11)
1960 Republican Presidential Candidate	1.15 (.16)	.29 (.10)	.91 (.17)	.28 (.10)
1964 Democratic Presidential Candidate	.80 (.16)	.33 (.13)	.92 (.16)	.20 (.14)
1964 Democratic Slate	.87 (.15)	.30 (.12)	1.15 (.14)	.13 (.12)
1964 Republican Presidential Candidate	1.03 (.19)	.43 (.15)	1.20 (.15)	.16 (.13)
1964 Republican Slate	.87 (.19)	.41 (.15)	.96 (.15)	.17 (.12)

expression $f(x_1, x_2)-\lambda[x_1+x_2-B]$ and differentiating with respect to x_1, we obtain,

$$\partial f/\partial x_i \leq \lambda, \qquad i = 1, 2$$

as the necessary condition a solution, say (x_1^*, x_2^*), must satisfy. With respect to this condition, if "=" holds, then $x_i^* \geq 0$, but if "<" holds, then the "corner solution" $x_j^*=0$ prevails.

Several possibilities yield corner solutions. First, if $\partial f/\partial x_1$ is smaller than $\partial f/x_2$ everywhere in the feasible region, the entire budget should be allocated to x_2. If, for example, $\partial f/\partial x_1=0$ for all x_1 but $\partial f/\partial x_2>0$ for all x_2, then $x_1^*=0$ and $x_2^*=B$ maximizes f. A second possibility is that differentiability of either f or x_i may be only an approximate assumption. Suppose, for $B=1.0$, that $x_1^*=.01$ and $x_2^*=.99$, but that in actuality the budget can be divided between x_1 and x_2 only in units of .1. Then the solution might best be approximated by letting $x_1=0$ and $x_2=1$.

For a more relevant illustration of this latter possibility, suppose that a candidate can devote fifty days to whistle-stopping, etc., and that each day consists of sixteen useful hours (including travel, speeches, etc.). If the candidate adopts the proportional rule, he should allocate approximately 4.8 hours to a state with three electoral votes and 6.4 hours to a state with four electoral votes. Suppose, however, that, given his campaign style and the technological constraints of transportation, the candidate believes he must budget at least eight hours to any state he wishes to visit. He might approximate the proportional solution, then, by avoiding states with three and four electoral votes.[20]

Since trips, or for that matter even time, are not infinitely divisible commodities, we see from expression (13) that corner solutions are possible if $E_j q_{ij}/\partial\theta_{ij}$ is sufficiently small, or, from subsequent assumptions, if $E_j^{\beta}|Pl_j|^{-\gamma}$ is small. Consider the effects of such solutions on estimates of $\hat{\beta}$. Ignoring plurality for the present, suppose that a candidate allocates no resources to state j if $E_j \leq E_0$ and that he allocates resources proportionately among states with more than E_0 electoral votes. We illustrate this rule by the dark solid line in Figure 3, where we also graph with a thin solid curve an approximate best fit line that conforms to expression (9). Clearly, this curve corresponds to an estimate of β that exceeds 1.0 (hence its curvature).

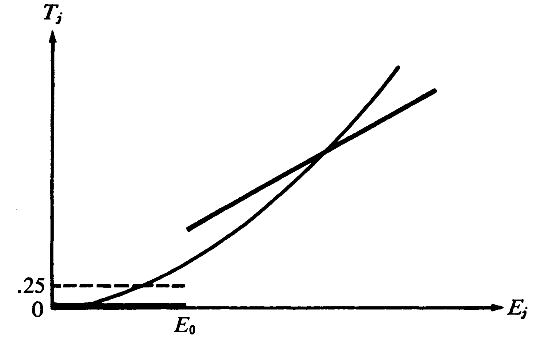

Figure 3: Effect of Corner Solutions on $\hat{\beta}$

Figure 3, then, illustrates an important possibility: corner solutions can bias our estimates of $\hat{\beta}$ upwards. An estimate of $\hat{\beta}$ that exceeds 1.0, then, can indicate simply that the candidate is allocating resources proportionately over some subset of states. The remaining states are ignored, or, if the candidate does visit them, his objective is something other than increasing his probability of carrying them (e.g., he seeks campaign funds from a wealthy potential contributor, or he is helping the party win a Senate or gubernatorial race).

The supposition that the model illustrated by Figure 1 accounts for estimates of $\hat{\beta}$ greater than 1.0 is readily rejected—or at least rendered highly suspect. Specifically, we can reestimate the coefficients of expression (17) after dropping from the sample states that the candidate fails to visit, acknowledging that β will be thereby underestimated.[21] If this estimate exceeds 1.0, then we can conclude that the concept of a corner solution by

[20] The following caveat must necessarily precede this "rounding-off" argument. Although rounding off the optimal solution of the noninteger (i.e., continuous) problem to obtain integer value solutions is often adequate, there are pitfalls in this approach. First, after it has been rounded, the optimal noninteger solution need not be feasible. Second, there is no guarantee that a feasible rounded solution is optimal to the original integer programming problem. See, F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research* (San Francisco: Holden-Day, Inc.), 1967, pp. 553–570.

[21] There is, however, a condition under which these estimates are not biased. Specifically, assume that a candidate confronts two decisions—the first being whether or not to visit a state—and let Q_j denote the probability that he visits state j. The candidate's second decision, now, is to properly allocate his trips among those he plans to visit. From all subsequent assumptions, including that error structures are multiplicative log-normal, the distribution of T_j becomes

$$f(T_j > 0 \mid \alpha, \beta, \gamma) = rQ_j e^{-S_j^2/2}$$
$$f(T_j = 0 \mid \alpha, \beta, \gamma) = 1 - Q_j$$

where

$$S = [\log T_j - \log \alpha - \beta \log E_j - \gamma \log |Pl_j|]/\sigma$$

and

$$r = (2\pi\sigma^2)^{-\frac{1}{2}}.$$

Letting

$$X_j = \begin{Bmatrix}1\\0\end{Bmatrix} \quad \text{if and only if} \quad T_j \begin{Bmatrix}>\\=\end{Bmatrix} 0,$$

Table 7. Unvisited States Deleted from Sample

	Democratic Slate			Republican Slate		
	$\hat{\beta}$	$\hat{\gamma}$	n^+	$\hat{\beta}$	$\hat{\gamma}$	n
1960	1.23 (.13)**	.25 (.08)	48	1.47 (.14)*	.18 (.09)	45
1964	.84 (.12)**	.07 (.10)	46	.89 (.14)**	.20 (.12)	45
1968	1.14 (.17)**	.26 (.14)	37	1.36 (.11)*	.04 (.08)	36
1972	1.16 (.18)***	.43 (.23)	28	.61 (.09)	.08 (.12)	29

+ n denotes the number of states in the sample.
* Significantly greater than 1.0 but not significantly different from 1.50.
** Significantly less than 1.50 but not significantly different from 1.0.
*** Indeterminate.

itself does not account for estimates that deviate from the proportional allocation hypothesis.[22] We report the results of this procedure in Table 7.

then the likelihood function for our parameters becomes

$$L = \prod_j f(T_j > 0 \mid \alpha, \beta, \gamma)^{X_j} f(T_j = 0 \mid \alpha, \beta, \gamma)^{1-X_j}$$

and the log of the likelihood function is,

$$\log L = \sum_j X_j[\log K + \log Q_j - S_j^2/2] + \sum_j (1 - X_j) \log (1 - Q_j).$$

To derive the maximum likelihood estimates of α, β, and γ, we substitute the observed values of X_j into this expression, and set the derivatives of log L with respect to α, β, and γ equal to zero. Note, however, that if Q_j is functionally independent of α, β, and γ, the maximum likelihood estimates of these parameters are exactly what we obtain if we re-estimate expression (17) as before after deleting from the sample those states the candidates fail to visit. The assumption that Q_j is functionally independent of α, β, and j, however, is inconsistent with our previous analysis, since in that analysis the decision about whether or not to visit a state depends on the magnitude of $\alpha E_j \beta |Pl_j|^{-\gamma}$. Nevertheless, if one is willing to accept the assumption that candidates adopt simplifying heuristics such as letting their decision rest on E_j or $|Pl_j|$ alone, the preceding analysis proves that the estimates in Tables 7 and 8 are unbiased.

[22] We note, however, that the derivation of the 3/2's rule also admits the possibility of corner solutions, but Brams and Davis implicitly assume that such solutions do not occur. Of course, if they are admitted into their analysis then their theory would predict estimates of β in excess of 3/2's.

Observe that these estimates for $\hat{\beta}$ differ only modestly from the estimates in Table 5—the exceptions being a slightly higher estimate for the Republican slate in 1960 and a lower estimate for the Democratic slate in 1972. Since the standard errors also change, we note a slight shifting of the number of coefficients that are different from 1.5 and 1.0. Overall, $\hat{\beta}$ averages 1.07 in Table 7 as against 1.15 in Table 5; but 25 per cent of this reduction can be attributed to a substantially smaller estimate of $\hat{\beta}$ for the 1972 Republican slate. This same, apparently inconsequential, pattern emerges also if we also estimate $\hat{\beta}$ and $\hat{\gamma}$ for Kelley's data (see Table 8).

Clearly, then, the concept of a corner solution fails to account for the values of $\hat{\beta}$ that exceed unity in Table 5. But if we employ this concept in conjunction with a particular hypothesis about how candidates plan and execute their campaigns over time, we can offer an intuitively plausible explanation about why proportional allocation rules yield $\hat{\beta} > 1.0$.

First, Kelley observes that in 1960, both Kennedy and Nixon spent 74 per cent of their time in twenty-four doubtful states during the campaign but about 88 per cent of their time in these same states during the final three weeks. Correspondingly, Kennedy spent "57 per cent of

Table 8. Comparison of $\hat{\beta}$ and $\hat{\gamma}$ for Brams-Davis and Kelley Data, with Unvisited States Deleted from Sample

	Brams-Davis			Kelley		
	$\hat{\beta}$	$\hat{\gamma}$	n	$\hat{\beta}$	$\hat{\gamma}$	n
1960 Democratic Presidential Candidate	1.39 (.15)	.07 (.09)	43	1.45 (.17)	.10 (.11)	44
1960 Republican Presidential Candidate	1.19 (.12)	.17 (.07)	44	.98 (.15)	.15 (.09)	43
1964 Democratic Presidential Candidate	.58 (.11)	.16 (.10)	40	.85 (.12)	.12 (.10)	41
1964 Democratic Slate	.84 (.12)	.07 (.10)	46	1.15 (.06)	.11 (.10)	44
1964 Republican Presidential Candidate	.99 (.13)	.28 (.17)	42	1.18 (.13)	.12 (.11)	43
1964 Republican Slate	.89 (.14)	.20 (.12)	45	.96 (.13)	.19 (.11)	45

his time in the seven largest of the doubtful states, and about 71 per cent of his time in these states in the final three weeks," while for Nixon, "the proportion of total campaign time spent in the seven largest doubtful states [was] 51 per cent [and the] proportion of campaign time spent [in these states] in the final three weeks [was] 67 per cent."[23]

Consistent with these observations is the general tendency for candidates to visit fewer states in the second half of their campaigns than in the first half. As Table 9 reveals, only Lyndon Johnson deviates from this pattern.[24]

The implication one draws from these observations and patterns is that the candidates' strategies change as the campaign proceeds, which is only reasonable since their information is changing also. They know more about their opponents' strategies, and they have better estimates about which states are close and which are foregone conclusions. Moreover, their budget constraints are changing. A constraint that might cause little concern in September grows to paramount importance in October.

Let us consider how this shrinking budget might affect campaign strategies. In particular, suppose that a candidate plans his entire campaign in September and proceeds to allocate his resources according to a proportional formula. In October, however, he decides to reassess his campaigning, make suitable adjustments to new information, and proceed from then on into November with a recalculated proportional allocation. Observe now that, aside from information effects, the principal difference between planning in the two months is that corner solutions are likely to be more prevalent in October because time is a scarcer commodity. If, for example, he reassesses his campaign two days before the election, the candidate would be wise to ignore all states except, say, California and New York.

What we wish to show now is that by thus readjusting his campaign, the least squares estimate of β in expression (8) exceeds 1.0. That is, even though the candidate makes proportional allocations in both periods, he appears overall to use a greater than proportional rule. Our proof here focuses on the case of a single replanning point although the method can be extended to show that $\hat{\beta}>1.0$ for any number of points.[25] To simplify analysis and exposition still further, we ignore competitiveness and assume that as the campaign commences the candidate plans to visit all states. For the case of a single replanning point, then, the inequality we seek to establish is,

Table 9. Number of States Visited by Presidential Candidates in First and Second Halves of Campaign

	1st Half	2nd Half
1960 Democrat	33	24
1960 Republican	36	21
1964 Democrat	20	30
1964 Republican	33	23
1968 Democrat	22	13
1968 Republican	28	14

$$\beta=\frac{\sum_j [\log E_j-\sum_j \log E_j][\log \theta_j-\sum_j \log \theta_j]}{\sum_j [\log E_j-\sum_j \log E_j]^2} > 1$$

where θ_j is the overall proportion of trips or time a candidate allocates to state j. Straightforward algebraic manipulation reduces this expression to,

$$\text{(18)} \quad \sum_j \log E_j \left\{\left(n \log \frac{\theta_j}{E_j} - \sum_j \log \frac{\theta_j}{E_j}\right)\right\} > 0.$$

Assuming now that the candidate allocates ξ per cent of his total resources in period 1, then,

$$\theta_j = \xi\overset{1}{\theta_j} + (1-\xi)\overset{2}{\theta_j},$$

where $\overset{k}{\theta_j}$ is the proportion of resources allocated in period k to state j. From the assumption that the candidate visits all states at least once in period 1 and allocates proportionally,

$$\overset{1}{\theta_j} = E_j/\sum_j E_j = E_j/E'.$$

(Note that to simplify the analysis further, we let

[23] Stanley Kelley, Jr., "The Presidential Campaign," in *The Presidential Election and Transition: 1960–1961*, Paul T. David, ed. (Washington, D.C.: Brookings, 1961), p. 71.

[24] The data in Table 9 is taken from John H. Runyon, Jennefer Verdini, and Sally S. Runyon, eds., *Source Book of American Presidential Campaign and Election Statistics: 1948–1968* (New York: Frederick Ungar, 1971), pp. 156–173. On Johnson's exception to the general pattern see Karl A. Lamb and Paul A. Smith, *Campaign Decision-Making: The Presidential Election of 1964* (Belmont: Wadsworth, 1968), pp. 202, 205–207.

[25] To forestall someone from traversing several blind alleys that we entered, we note that the conjecture that $\hat{\beta}$ increases monotonically with the number of replanning periods is true, but only under some severely restrictive conditions—conditions that seem impossible to express in any simple, substantively meaningful way.

θ_j^1 be a nonstochastic variable.) For time period 2, however,

$$\theta_j^2 = l_j E_j / \sum_j l_j E_j = l_j E_j / E'',$$

where $l_j = 0$ if $E_j < E_0$ and $l_j = 1$ if $E_j \geq E_0$. That is, if the state is "small," he does not visit it, but if it is sufficiently large, he allocates his resources to it proportionately—in proportion, that is, to its size with respect to the states he plans to visit. Substituting these expressions for θ_j^k into (18) yields, with some additional algebraic manipulation and cancellation,

$$\sum_j \log E_j \left\{ n \log\left[\xi E'' + (1-\xi) l_j E'\right] - \sum_j \log\left[\xi E'' + (1-\xi) l_j E'\right] \right\} > 0.$$

We now divide this summation into two parts—over those states for which $l_j = 0$ and over those for which $l_j = 1$. Letting v denote the total number of states for which $l_j = 1$, we get

$$v \sum_{E_j < E_0} \log E_j \{\log \xi E'' - \log[\xi E'' + (1-\xi)E']\} + (n-v) \sum_{E_j \geq E_0} \log E_j \{\log[\xi E'' + (1-\xi)E'] - \log \xi E''\} > 0$$

or, alternatively,

$$\{\log[\xi E'' + (1-\xi)E'] - \log \xi E''\} \cdot \left[(n-v) \sum_{E_j \geq E_0} \log E_j - v \sum_{E_j < E_0} \log E_j\right] > 0.$$

Since the term contained by { } is positive we can ignore it. To prove that the expression in brackets is also positive, note that if we set all $E_j = E_0$, we minimize its first term but maximize the second (subtracted) term. Thus, we obtain $(n-v)v \log E_0 - v(n-v) \log E_0 = 0$. Hence, the bracketed term is positive, which completes our proof that $\hat{\beta} > 1.0$.

Following a proof of this sort it is perhaps useful to illustrate the general idea with a simple example. In Figure 4 we graph proportion of time spent in a state against E_j, where E_j assumes the values 3, 4, 5, 6, 7, and 8. The dashed line denotes the candidate's decision rule in September and indicates that he begins his campaign by allocating time proportionately among all six states. The dotted line denotes the candidate's decision rule in October and indicates that he allocates his time proportionately among only the three largest states. The dark solid line represents the overall average proportion of time a candidate allocates to each state. The thin solid line, in turn, represents an approximate best-fit curve for expression (8) to the *overall* data. Clearly, $\hat{\beta} > 1.0$. Thus, while candidates are allocating according to a proportional rule, we estimate β's in excess of one because they are applying this rule to a smaller subset (of larger states) as the campaign progresses.

Can we assert, then, that corner solutions in conjunction with sequential replanning constitute a valid explanation for the apparent biases of the Electoral College? Unfortunately, we cannot. First, subjecting the explanation to a statistical test is a difficult task even with the best of data—and the data currently available hardly qualify as such.[26] Second, and perhaps more important, we cannot also reject the proposition that the candidates do not adjust their campaigns and that they plan *from the very beginning* to abide by a greater than proportional rule by allocating proportionally to many states initially but proportionally to only a few later. Doubtless, the truth lies somewhere between this proposition and our explanation. We cannot believe that momentum has no effect, or that candidates do not foresee the utility of concentrating their resources in big competitive states as the campaign nears its conclusion. But it is reasonable also to assume that candidates make some adjustments over time. Undoubtedly, adjustments to plans are not made in the starkly simple way we portray them. We hope only that our model captures the essential feature of these adjustments and explains why, after controlling for competitiveness, we might observe greater than proportional allocations over the course of the entire campaign.

Conclusions

Clearly, we can answer affirmatively the question "Does the Electoral College bias campaign allocations in favor of big states?"—at least for

[26] For a discussion of the relevant statistical methodology and data requirements see John U. Farley, Melvin J. Hinich, and Timothy W. McGuire, "Testing for a Shift in the Slopes of a Multivariate Linear Time Series Model," *Journal of the American Statistical Association* (forthcoming). If, however, we accept the assumptions of the statistical model outlines in footnote 21, then we can conduct a comparison of estimates of $\hat{\beta}$ for the first five weeks of the campaign as against the last three weeks (using Kelley's data). Briefly, such a comparison reveals that, as the theory predicts, $\hat{\beta}$ for either period is always less than our estimate for the overall campaign and that only one estimate of $\hat{\beta}$ is significantly different from 1.0 (.39 for Lyndon Johnson in 1964—all other coefficients vary between .76 and 1.22). We repeat, though, that the assumption that Q is functionally independent of α, β, and γ is at best tenuous and is not consistent with our central analysis.

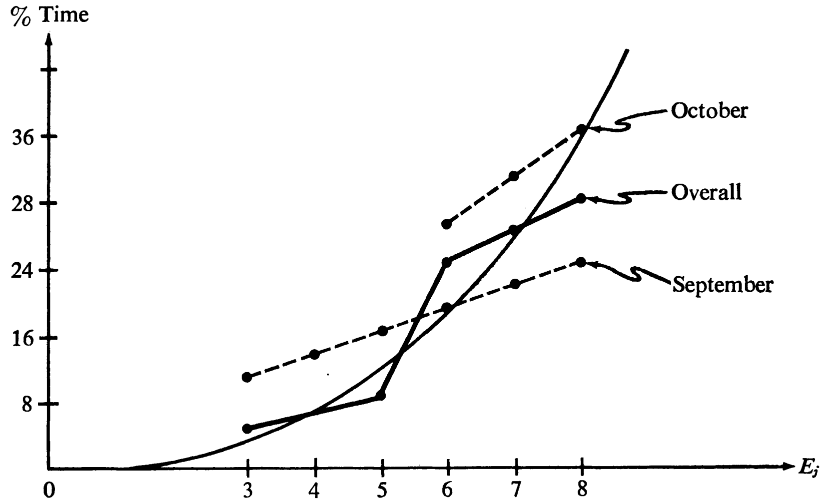

Figure 4: Effect of One Replanning Point

competitive elections in which neither candidate is an incumbent. For purposes of reform, however, we must also ask: What is the extent and source of this bias? Without an adequate answer to this question we cannot speculate meaningfully about the consequences of alternative election procedures.

First, however, we can reject the 3/2's hypothesis as an adequate explanation of bias on both theoretical and empirical grounds. Theoretically, the derivation of 3/2's rests heavily on an unreasonably limiting assumption about the relationship between voter choice and resource allocation (expression 5) and the failure to consider corner solutions to the candidates' constrained maximization problem. Justification for its use, moreover, rests on the tenuous principle that candidates adopt a local equilibrium that is readily upset by very small changes in strategy. The apparent empirical support for the 3/2's hypothesis, on the other hand, can be attributed, first, to the correlation between the size and competitiveness of states and to the failure of candidates to visit states.

The derivation of the 3/2's rule addresses the wrong issue. The issue is not whether the Electoral College induces the candidates to abide by a greater than proportional formula, but rather ascertaining the effects of the Electoral College's two principal characteristics—the unit rule and weighted voting. Together and separately these two characteristics affect strategies, and the process of isolating these effects moves us closer to an understanding of the consequences of alternative election procedures in general.

Briefly, we conclude that even without weighted voting we should anticipate some variation in resource allocations. To the extent that equally weighted states differ in competitiveness, the unit rule will admit the effects of corner solutions, sequential campaign adjustments, and differences in marginal productivity across units. Hence, reform proposals such as the district plan will not eliminate biases—and if competitiveness correlates with the distribution of policy preferences within a district, these biases may be as consequential as we presume them to be under the Electoral College.

Weighted voting, on the other hand, accentuates the effects of the unit rule: it acts in much the same way as competitiveness in that it contributes to variations in the states' marginal productivities. We cannot say whether its influence on strategies in conjunction with the unit rule follows a proportional formula, however, since our analysis in the previous section suggests that it can affect strategies in rather complex ways.

The principal conclusion of this essay is that while the consequences of the Electoral College can be studied rigorously, there is no reason to suppose that these consequences are describable by a single simple formula that is applicable to all campaigns. These consequences are very much a function of each candidate's information, his assumptions about his opponent, and his ability or willingness to reevaluate his strategy as the campaign proceeds. Doubtless, it is a function of other considerations as well—including his demand for financial support, his ability to recruit this support without visiting states, and his willingness to campaign also for candidates of lesser office. Within the context of the variables we consider, moreover, one needs only to peruse the literature on portfolio selection and sequential decision processes to recognize the primitiveness of our analysis and the opportunities for further study. For example, we disregard the candidates' attitudes towards risk and hypotheses such as: candidates who believe they are behind are risk-acceptant, while candidates who believe they hold the advantage are risk-averse. These attitudes can affect strategies since campaigning in one large state rather than several smaller states with an equivalent total of electoral votes entails less risk, *ceteris paribus*.

These facts, in conjunction with our failure to address other important issues surrounding the debate over Electoral College reform—including the viability of third parties, the probability of indeterminate outcomes and reversals, and the incentives for fraud—necessitate our resisting the temptation to render a verdict on that debate.[27]

[27] For an analysis of reversals and indeterminate outcomes see Melvin J. Hinich, Richard Mickelsen, and Peter C. Ordeshook, "The Electoral College vs. A Direct Vote: Policy Bias, Indeterminate Outcomes, and Reversals" in *Journal of Mathematical Sociology*, forthcoming and Mickelsen and Ordeshook, "The Electoral College and the Probability of Reversals" in *Modeling and Simulation,* 5 (Pittsburgh: Univ. of Pittsburgh Press, forthcoming).