

**Using Folded Seats-Votes Curves to Compare  
Partisan Bias in the 2020 Presidential Election  
with Partisan Bias in the Five Other  
Presidential Elections in the 21st Century\***

Jonathan Cervas

Carnegie Mellon University

Bernard Grofman,

University of California, Irvine

\* Cervas is a Post-Doctoral Fellow, Institute for Strategy and Politics, Carnegie Mellon University (CMU). Grofman is Distinguished Professor of Political Science and Jack W. Peltason Chair of Democracy Studies, the University of California, Irvine.

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## ABSTRACT

*Using a folded seats-votes curve, we examine partisan bias in the 2020 presidential election and compare it to partisan bias in the five other presidential elections in the 21<sup>st</sup> century. 2020 and 2016 are extreme outliers with respect to the absolute magnitude of partisan bias in the Electoral College. In 2016, 2020, and 2000 bias runs in a pro-Republican direction; in the other three elections in this century, the opposite is true. But partisan bias can vary with where on the seats-vote curve we look to find bias, and we identify the lowest vote share at which there is no partisan bias (if such exists).*

After the 2010 round of redistricting, partisan gerrymandering in the US House and state legislatures reached unprecedented levels, while the US Senate has record levels of malapportionment (*Cervas and Grofman, 2020*, Figure 1). The 2000 and 2016 presidential elections ended with the candidate who won fewer votes emerging victorious. With a shift of a mere 10,000 to 20,000 votes in each of a handful of close states 2020, too could have produced a popular vote winner who was an Electoral College loser, despite a winning vote margin of 7 million votes. But bias which consistently results in non-majoritarian outcomes makes a mockery of the principle of “one person, one vote” and risks the legitimacy of any system claiming to be democratic.

In this essay, rather than looking at partisan bias in legislative and congressional elections, we limit our focus to the Electoral College. Our goals are two-fold.

First, we compare the level of partisan bias (Tufte, 1973; Grofman, 1983; Gelman and King, 1994; Grofman and King, 2007) for each of the six presidential elections in the 21<sup>st</sup> Century. We contrast the results in 2020 with those in earlier years to demonstrate that 2016 and 2020 were clear outliers in the extremity of the absolute magnitude of their partisan bias. We also show that partisan bias in the Electoral College operated in a pro-Republican direction only in 2000, 2016 and 2020. In the remaining years, 2004, 2008, and 2012, it operated to give the Democrats an advantage.

Second, using a simple way to visualize seats-votes curves in the context of two-party politics -- the folded seats-votes curve (Grofman, 1983: Figure 3, p. 312), we demonstrate that the bias in an electoral system can vary at different points on the seats-votes curve, and that change in partisan bias need not be monotonic with change in Democratic vote share.<sup>1</sup> We identify bias at a

50% vote-share and at the actual vote-share that was received by the Democratic candidate in the election. We also show the Democratic vote-share needed to give a situation of zero partisan bias.

An election system that uses geographically-based single-member districts is sometimes referred to as *majoritarian*. In such a system, though voters might be treated equally in their ability to elect a member from their district, there exists the potential for aggregate outcomes (seat-shares) to not match the summary of votes casted (vote shares). Two distinct but important elements can work towards this mismatch. First, the winners often get a ‘bonus’ number of seats, since seats are awarded on a winner-take-all basis; i.e., seats are not proportional to votes. Second, unequal turnout across districts or states, or turnout differences between the parties in some states, malapportionment can create population or turnout-related bias, while one party having more concentrated geographic support can create what is often referred to as *natural bias* or *geographic bias* (Grofman et al, 1997). This is true of both Congress and the votes in the Electoral College. We can measure these biases in combination by adding up all the votes for one party and plotting them on the x-axis and finding the percentage of all the seats that party won and plotting that on the y-axis. We then create hypothetical seat shares at different vote levels by shifting the vote shares in each of the units by the same percentage point shift (either up or down) and looking at the curve that is traced in this fashion. If the distribution of vote shares is approximately bell-shaped, this trace would take the shape of an S-curve.

Like the usual seats-votes curve for a two-party contest, in a *folded seats-votes curve*, seat share is shown on the y-axis and vote share on the x-axis. However, for folded seats-votes curves, unlike the standard seats-votes curve which has values being shown for only one of the two parties (running from 0-100%, or for an included range such as 35% to 65%), a folded seats-votes curve, such as those shown in **Figure 1**, simultaneously shows values for each of the two

parties over the range 50-100%. In other words, we have arranged these plots so that both the Republican seats-votes curve and the Democratic seats-votes curve lie on the same axis. The reason we refer to this way of representing seats-votes relationship as a folded curve is because it takes the usual seats-vote curve and folds it on itself as if it were hinged at the 50% vote share.

**Figure 1** shows folded seats-votes curves generated for the set of six presidential elections held in the 21<sup>st</sup> century.<sup>2</sup> Two of these elections (2000 and 2016) were inversions, and the other four were not.<sup>3</sup> For any given vote share, the gap (divided by two) between the seat-share values at that particular vote-share for each of the two parties shows the asymmetry in the seats-votes distribution and may be used as a measure of partisan bias (and its directionality) at that point. To be clear, partisan bias is not measured as a deviation from perfect proportionality -- the equality of seat-share and vote-share -- but instead based on the asymmetry between the two parties in translating their votes into seats.<sup>4</sup> Note also that partisan bias need not be bias in favor of the winner of the election. A sufficiently strong vote performance can overcome an Electoral College bias against one's party.

Values on the x and y-axes in **Figure 1** reflect the range of likely feasible values for the dominant party: 45% to 65% vote-share. Y-axis values describe the expected percent of electors each party would win over the range of vote shares. The electoral winner's vote and seat-share is shown in the legend. The challenger is shown on the graph only when their seats and votes are within the axis's limits. Democratic candidates are shown with a blue circle and Republican candidates are shown with a red diamond.

<< **Figure 1 about here**>>

*Folded seats-votes curves* are particularly useful in visualizing how seats-votes relationship vary over the vote range. We can read out from the plots shown in **Figure 1** most of the features of interest. In particular, the figures allow to read out the partisan bias value at a 50% vote-share and the partisan bias value at the actual vote-share that was received either candidate in the election, and they also allow us to quickly find the vote-share values that result in zero partisan bias. The measure of partisan bias at a  $k\%$  vote (seat) share is simply the seat difference between the two parties at that vote (seat) share, *divided by two*. The estimated seat-share at a given vote-share is significantly different than what might be expected if seat-share were equal to vote-share. This is particularly true in the Electoral College, since most states organize their electors such that the state plurality winner captures all the state's electors. The sign tells us which party is being favored.<sup>5</sup>

Because the *folded seats-votes curve* we report are generated by a process with stochastic error, we can calculate confidence bounds around our various partisan bias estimates (not shown here). However, the seat result may not be close to the projected outcome if the actual election outcome is in the tails of the simulations for the actual vote-share, i.e., outside the 95% confidence bounds. For instance, in 2016, the size of the gap between the vote-share for Trump and that for Clinton made the Electoral College victory for Trump unlikely, but it happened nonetheless because very close states, which were determinative of the election outcome in terms of EC votes, swung disproportionately for Trump. Thus, we saw an inversion in 2016.

What we also can see from **Figure 1** is that, even though there is a considerable resemblance between the seats-votes pattern in immediately adjacent presidential elections, the resemblance is far from perfect. When we go from one election to the next there are changes in the population of eligible voters (e.g., movements from less than voting age status to voting age status, in-migrations and out-migrations, as well as deaths, and illness or incapacitation that affects who is likely to vote). But also, and arguably even more importantly, the nature of the campaign (and the candidates) may lead to differences in the relative attractiveness of the Democratic and Republican candidates to various segments of the electorate and to decisions about whether or not vote (Campbell et al, 1960). Even if aggregate level two-party vote shares do not change much or even at all, given how competitive the election is at the national level and how competitive some states are, differences in the state-specific distribution of two-party vote-share from one election to the next can decide EC election outcomes in the 21<sup>st</sup> Century.<sup>6</sup>

Comparing the various graphs in **Figure 1**, it is apparent that across a wide range of vote-share values, partisan bias levels in 2016 and 2020 are extraordinarily high as compared to the other four elections in the 21<sup>st</sup> century. The large gap between the Republican and Democratic curves both at the 50% vote-share and at the observed vote-share in the actual election indicates both substantial bias, and a high probability of an inversion. That is, Republican candidates are expected -- on average -- to receive a greater number of electors across a wide range of vote shares compared to the Democratic candidates. Moreover, while in 2016 and 2020, bias tends to diminish as the vote-share increases, it takes rather high vote shares, 56% in 2016 and 57% in 2020, before we get to a level of zero pro-Republican bias. Furthermore, in 2016 once we go past a vote-share of 56%, there is essentially no bias (perhaps a slight tilt to the Democrats) but then, once we get to vote-share values near to 64%, bias again is a pro-Republican direction. In 2020,

once we go past a vote-share of 57%, there is again virtually no bias (perhaps a slight tilt to the Democrats) until we get to vote-share of 60% or so. But at 60% we again begin to see non-trivial pro-Republican bias emerging.

The pattern in 2000 is quite different, though still exhibiting pro-Republican bias at 50% and at the actual Democratic vote share. In 2000, we see a virtually constant, but low, level of partisan bias across the entire vote-share range from 45% to 65% so, no matter how high the two-party vote share, Republican candidates are expected to win slightly more electors.

The pattern in the other three 21<sup>st</sup> century elections are ones with pro-Democratic bias at 50% and at the actual two-party vote share, but at far lower level of bias than is found in the pro-Republican direction in 2016 and 2020. Moreover, this bias was not great enough to result in inversions, though in 2004 it only did not cause an inversion because Bush was able to garner a large enough popular vote margin. In 2004, the pro-Democratic bias for vote shares near 50% reverses to become pro-Republican bias for vote shares above 54%. In 2008, the bias would have benefitted Obama until he won 56% of the two-party vote. In 2012, bias is close to zero until around 55% of the vote, at which time a pro-Republican bias strengthens.

**Table 1** summarizes some of the key pieces of information reported above.

<< **Table 1** about here >>

In sum, what we see is a complex picture: (a) with the directionality and magnitude of partisan bias varying dramatically by election despite the commonality across the six twenty-first century elections of being highly competitive nationally; (b) with the level and even directionality



of partisan bias in any given election varying with vote share; and (c) with sufficiently many competitive states so that small shifts in votes at the state level can have major impact on outcomes.

The question of which of the 21st century elections the election of 2024 is most likely to resemble is beyond the scope of this short note. But partisan bias in the Electoral College may be just as bad in 2024 as in 2020 or even 2016 and this fact does not bode well for the legitimacy of future presidential election outcomes.

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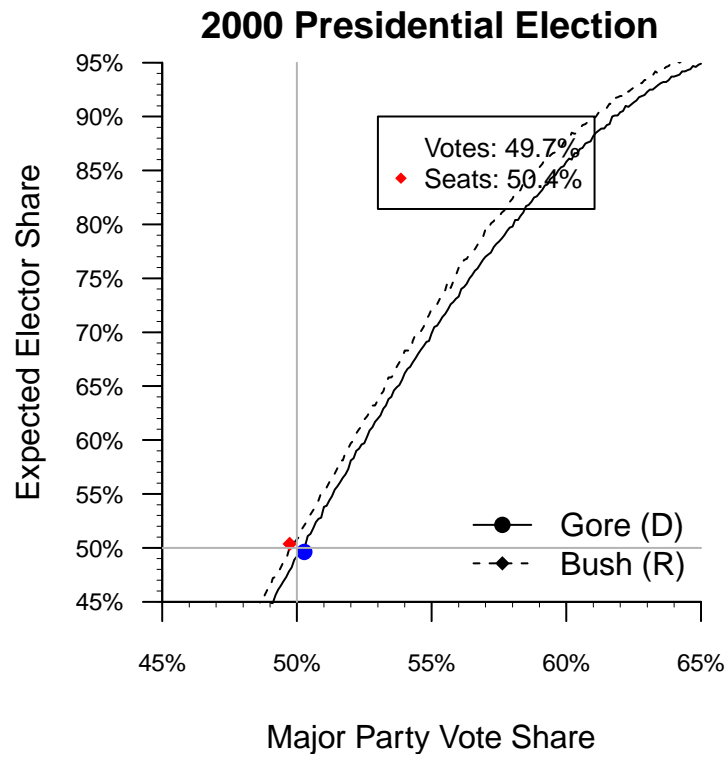
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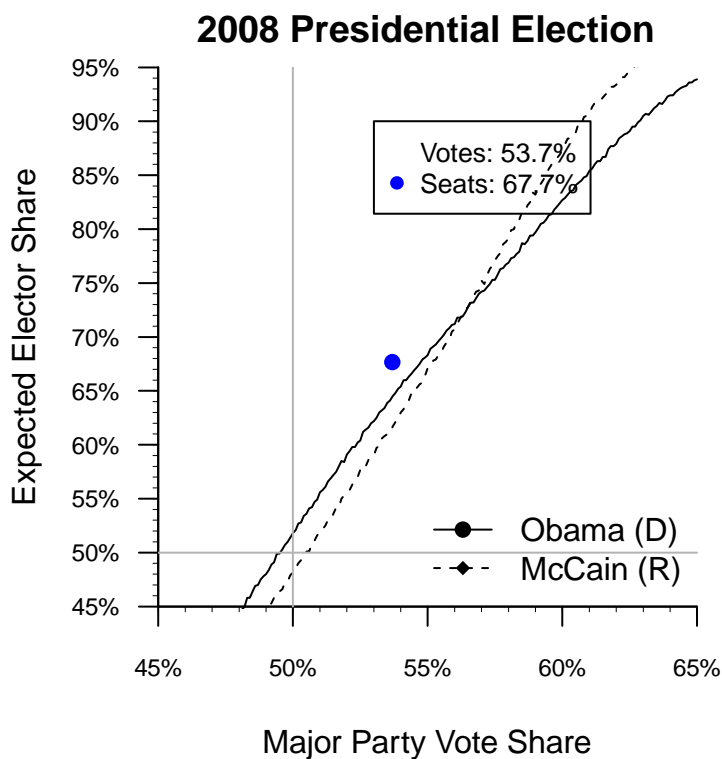
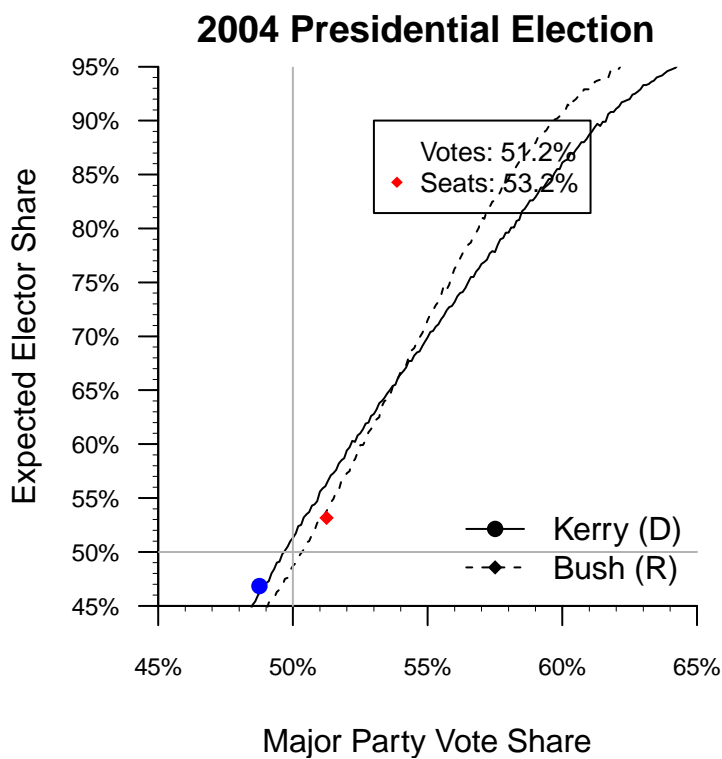
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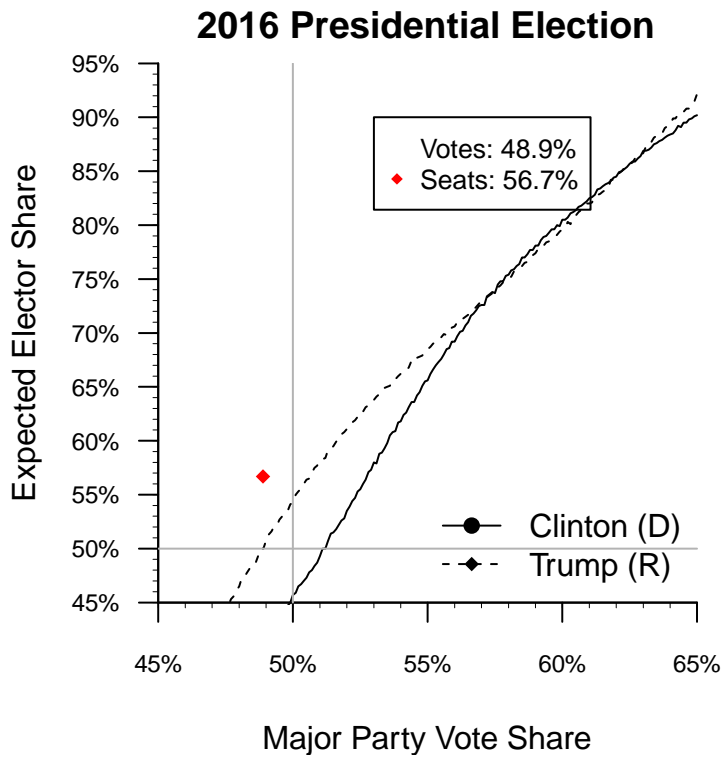
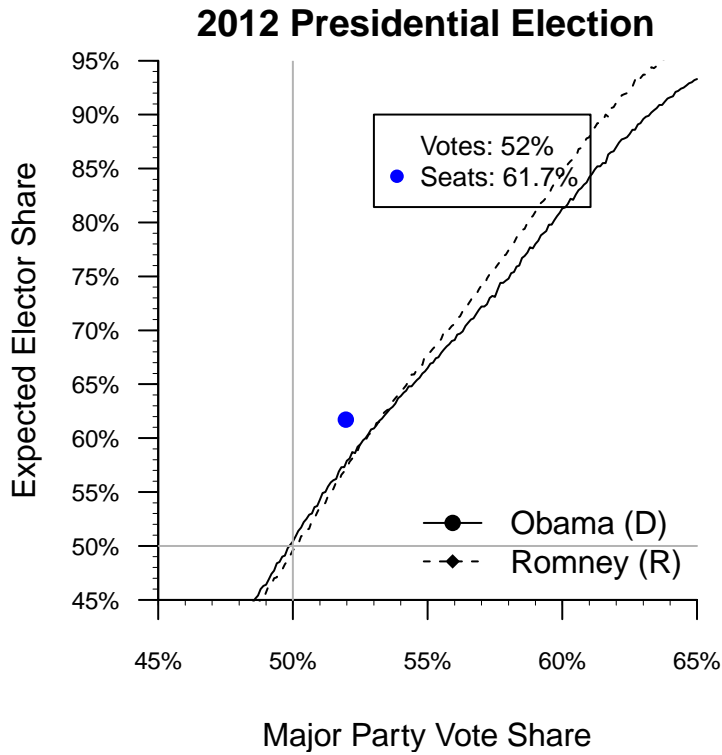
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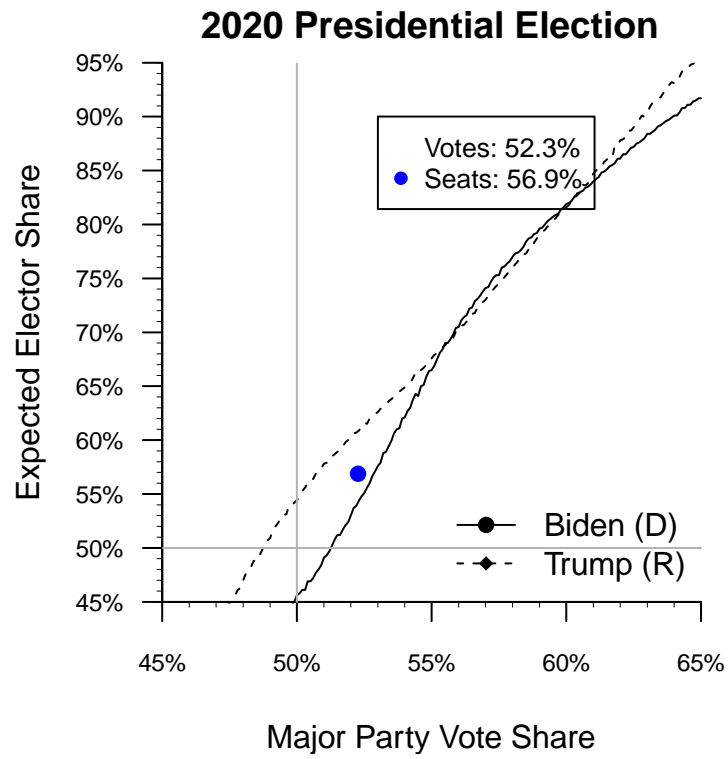
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**Figure 1. Two-Party Folded Seats-Votes Curve, U.S. Presidential Elections, 2000-2020**













**Table 1. Partisan Bias Estimates in the Six Presidential Elections of the 21<sup>st</sup> Century**

Year	Bias at 50%	Bias at Actual Vote Share	Zero Bias (vote share)
2000	0.4%	1%	NA
2004	-0.65%	-1.35 %	53.5%
2008	-0.9%	-1.4%	56%
2012	-0.2%	-0.35%	51.6%
2016	2.2%	4.2%	56.6%
2020	2.2%	3.3%	55.3%

Note: Column 2 shows the signed bias when the vote-share is tied. Column 3 shows the signed bias at the actual two-party vote-share of that year's election. Negative numbers favor the Democratic candidate in columns 2 and 3. Column 4 shows the percent of the vote-share when bias is zero; that is, when the seat-share received by each party is symmetric.

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<sup>1</sup> The folded seats-votes curve is related to the concept of *global symmetry* in Nagle and Ramsay (2021). That measure looks at total area between the two curves shown in each of the subfigures in **Figure 1**. A similar approach to plotting seats votes relationships is found in Duchin and Tenner (2018). Work in progress by the present authors allows for a non-uniform distribution over those seat-share values based on stochastic expectations derived from the past history of election popular vote outcomes and inter-election shifts.

<sup>2</sup> Following Gelman and King (1994), our simulation assumes uniform swing with stochastic error. For each election in our dataset, we find the residual standard error from a regression that predicts the current election from the previous one at the state level -- inter-election swing -- and average the previous three elections as noise and the actual election result in each state as the mean. We ran 301,000 simulations per election (1,000 at each vote share between 35-65%, with intervals of 0.1) to arrive at our curves.

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<sup>3</sup> In the terminology used in political science in the U.S., an EC *inversion* is when the candidate who wins the most overall votes, (a.k.a. the *popular vote*), fails to win a majority of the votes in the Electoral College (Miller 2012; Cervas and Grofman 2020).

<sup>4</sup> Indeed, the estimated seat-share at a given vote-share is significantly different than what might be expected if seat-share were equal to vote-share. This is particularly true in the Electoral College, since most states organize their electors such that the state plurality winner captures all the state's electors.

<sup>5</sup> By, assuming a uniform distribution of vote over some given range of vote share values, we could also find the mean partisan bias over that range as the area between the Democratic and Republican curves.

<sup>6</sup> The presidential elections in the 21<sup>st</sup> century are all remarkably similar to one another in terms of Democratic candidate share of the popular vote, since all are among the most competitive in our nation's history. We would need to go back to the elections between 1876 and 1888 to find comparable competitiveness levels (Lee 2016).