

**Fallacies in Statistically-Based Claims about Massive
Election Fraud in 2020:
A Compendium**

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5 **Election Fraud in 2020:**
6 **A Compendium***
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ABSTRACT

An immense amount has been written about alleged massive electoral fraud in the 2020 presidential election: In 2022, tens of millions of voters, and many Republican officials, still believe the 2020 election was stolen. Here, we examine claims of fraud in the presidential election of 2020 that are based on aggregate election data, and we further limit ourselves to claims in which the data itself is essentially undisputed. We show that the implications of that data for the presence of massive fraud have been largely or entirely misinterpreted through invalid statistical or logical reasoning. Our goal is not to provide new insights, since virtually all the points we make have been made by others, but rather to put together in one place a compendium of recent glaring misuses of statistical inference in a form that we believe will be useful as a teaching tool about the dangers of sloppy use of statistics. We discuss the fallacies in a non-technical way to make our critiques broadly accessible to non-specialist audiences.

I. Introduction

After the 2020 presidential election, the losing candidate, Donald Trump, claimed that he had been the victim of massive voter fraud that denied him the election. At the time, many Republican congress members, state attorneys general, and other Republican office holders endorsed this claim -- a claim which his supporters continue to reiterate. Tens of millions of voters, including a clear majority of Republicans and non-trivial numbers of independents and Democrats currently believe that there was massive fraud in 2020 (Gardner, 2021; Solender, 2020; UMass Amherst, 2021). Others believe that most elections are rigged.¹ In the 2022 primaries, some Republican candidates made the assertion of massive election fraud in 2020 a fundamental part of their election platform. Many of those candidates won their primary, and several will be able to affect future election administration in their states if they win in November (Medina et al., 2022). Thus, not only have claims about 2020 election fraud remained politically salient, but the persistence of voter beliefs in these claims, and the deep partisan divide about them leading to affective polarization, have frightening implications for the prospects of democratic breakdown in the U.S. (Grofman, 2022; Homans & Peterson, 2022; Leonhardt, 2022).²

The supposed evidence supporting massive election fraud comes in many forms, including personal affidavits alleging fraud in particular precincts (McClallen, 2021), to

¹ For fraud claims about earlier elections see e.g., Zorn (2017).

² For example, post-election audits that take place outside the regular legal framework for election certification have been demanded in contexts where they make no sense (Bump, 2022a).

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3 videos allegedly showing direct evidence of vote tampering by poll workers (Gray, 2020),
4 to how-to-videos showing the supposed ease for manipulating the record of votes
5 produced by voting machines or mail ballots, to claims about a conspiracy by a particular
6 voting machine vendor (Sganga, 2022), to assertions that more voters voted than were
7 on the jurisdiction's electoral roll (Ayyadurai, 2020; Swenson, 2020). Despite the
8 implausibility of a massive multi-state conspiracy, the volume and variety of claims make
9 them like a hydra-headed monster almost impossible to successfully rebut all of them to
10 a given voter's satisfaction. Moreover, the fact that many of these claims about massive
11 fraud in 2020 (including most of those we discuss in this essay) are plausible on their
12 face, even though fallacious, make them harder to refute. Also, we have the mesmerizing
13 power of repetition. The claim of massive fraud in 2020 is stated again and again in
14 conservative media sources and by former President Trump and his allies.³ Particular
15 factual claims are often repeated even after clear contrary evidence has been presented
16 (Hsu & Thompson, 2022).

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24 There are many reasons that can be offered about why beliefs about massive
25 election fraud in 2020 persists (see e.g., (Berlinski et al., 2021; Douglas et al., 2019;
26 Edsall, 2022; Holman & Lay, 2018)⁴. But exploring why voters believe what they do
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³ We would also note that, in the four years before the election, the claim that the 2020 election would be stolen by the Democrats was also repeatedly asserted by President Trump and his supporters, thus preparing the way for the post-election claims of fraud.

⁴ One obvious key factor is the level of present-day polarization, in which partisan identities shape beliefs. Strongly embedded partisan identities mean that the public polarizes on the fraud claim based on partisanship. Relatedly, we have a siloization

(Bump, 2022b) is not the purpose of this essay.⁵ Moreover, in this essay we will not discuss the vast bulk of claims about fraud in 2020, namely those that rest on contested facts. Our concern here is a narrowly focused one. We deal solely with claims about fraud that are grounded, at least in part, on indisputable facts about statistical features of the

of communication channels along partisan and ideological lines. The fraud claims are echoed as indisputable by a multiplicity of sources that voters trust. Relatedly, Lenz (2012), among others, have shown that the public changes its policy views to match the politicians they support (e.g., attitudes toward Russia among Republicans track Trump's changing views about Putin). Also, sources supporting the claim of Trump's having won the 2020 election denigrate the reliability of the mainstream media who refute the fraud claim and insist that the mainstream media are simply partisan mouthpieces for the Democrats.

⁵ We would note, though, that media choice is not the only factor. It was impossible not to be constantly reminded about the fraud claims regardless of which media sources one followed. Virtually every claim made in the conservative press alleging election fraud was given considerable space in the mainstream media, followed by a rebuttal that often went on longer than the statement of the claim. For example, claims that a particular election had more voters than were registered were subsequently rebutted by showing that the list of eligible voters used for the comparison was incomplete (Reality Check team, 2020)). And there were follow-up pieces in the mainstream press repeating multiple claims of fraud and their rebuttals, sometimes drawing on social science analysis such as the discussion of fallacious assertions about voting machine vulnerability to fraud in Eggers et al. (2021). We might also note that, in the *Washington Post*, President Trump had previously been given tens of thousands of Pinocchio points based on other kinds of claims he made found by the *Post* to be untruthful, with points added every time the claim was repeated (Kessler, 2020). This constant drumbeat about Trump's factual misstatements and deceitfulness by a source that conservative voters did not trust may simply have reinforced the view among these voters that Trump was telling truths that the mainstream press was determined to hide. Relevant in psychological terms is the aphorism that "the lady doth protest too much," in which repeated assertions that a given allegation is untrue become taken as support for a belief that it probably is true.

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3 2020 presidential election, and comparisons of its outcomes to those of previous
4 presidential elections.⁶
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8 Our goal is not to provide new insights about these statistical fallacies. Rather, our
9 goal is put together in one place a useful compendium of many of the most glaring recent
10 misuses of statistical reasoning as applied to understanding elections,⁷ and to do so in a
11 way that is readily accessible to non-technical readers⁸ We believe strongly that a
12 discussion of statistical fallacies based on real-world examples should be part of any
13 statistics or public policy curriculum.
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16 We begin our inventory of fallacies with (a) arithmetic fallacies of a simple sort,
17 such as drawing conclusions from unweighted averages where use of weighted average
18 was required, cherry-picking the data to emphasize only those facts that lead to the
19 desired conclusion, and confusing percentages and percentage point changes. Then we
20 discuss (b) improper use of statistical significance, and then turn to (c) inaccurate
21 statistics or public policy curriculum.
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⁶ Similarly, we do not wish to argue about what state or federal courts should or should not have decided about election law in the cases brought before them in 2020.

⁷ We do, however, believe that the framework of classifying election-related statistical fallacies that we offer may prove useful to other scholars.

⁸ We would call particular attention to the excellent survey and analysis of 2020 presidential election fallacies by Eggers et al. (2021). We began writing our own essay before we were familiar with this article in its on-line form, but we have learned a lot from it and cite to it herein. Their detailed rebuttal of fallacies is more technical than ours, and we cover some fallacies that they do not. There are also various journalistic surveys of election fraud claims (Alba & Frenkel, 2021; Corasaniti et al., 2020; Feldman, 2020), but these primarily include claims that do not fall within the scope of this essay

probabilistic reasoning, such as improperly using as an indicia of fraud having voters with the same name and date of birth. Then we discuss (d) syllogistic arguments based on cross-election statistical comparisons that are either fallacious in form, or that have at least one premise that is indubitably false, and thus which give rise either to invalid or unfounded conclusions. Finally, we briefly consider similar types of statistical errors in (e) syllogistic arguments based on within-election comparisons. Though we certainly do not claim that we have identified every fallacious claim about 2020 that fits within our classification scheme, we believe we have a typology that includes the most common errors involving analysis of aggregate election data.

II. Fallacious Statistically Based Claims about Massive Fraud in the 2020 Presidential Election

A. Arithmetic Fallacies

Failing to weight units. Failing to recognize that large changes in one direction in low population subsets or low population demographic subgroups can be compensated for by small changes in the other direction in high population subsets or demographic units is a common mistake. There were various instances of this type of error in discussions of the 2020 election. We first focus on the most common instance.

It was observed that Trump won more counties in 2020 than he did in 2016, with the implication being that he must have done better in 2020 than in 2016 in terms of the popular vote (Swenson, 2021). But, of course, that is nonsense, since he could have done better in the remaining counties. These counties, though fewer in number, had more

voters in them. Indeed, Biden received over three million votes in Los Angeles County, alone. In fact, Biden netted an additional 609,000 more votes in 2020 than Clinton did in 2016, just in this one county! Out of the over 3,000 counties in the United States, the top 150 contained half of the total votes cast. Biden won 125 of those 150 (83.3%).

Errors in using unweighted averages when it is appropriate to use a weighted average can be plotted in several ways. Consider a choropleth map of election results by county (**Figure 1**). As Chief Justice Earl Warren famously quipped in *Reynolds v. Sims* (1964), “legislators represent people, not trees or acres”. Trump certainly won more acres, but he did not win more voters. If you look at map of U.S. counties showing those won by President Trump in red and those by Hillary Clinton (or Joe Biden) in blue, you will see a sea of red and only a relative handful of pockets of blues. But those pockets (mostly big cities) have lots of voters in them. When counties are instead represented on a map in proportion to their votes, such as with a cartogram -- a map that has been resized so the units’ area is equal to its population weight – it become clear that the 2020 election was close, but certainly not an overwhelmingly “red” county shown by a county-level election map.⁹ Similarly, in the context of unequally sized units, a “bubble map” can be especially useful (see **Figure 2**).

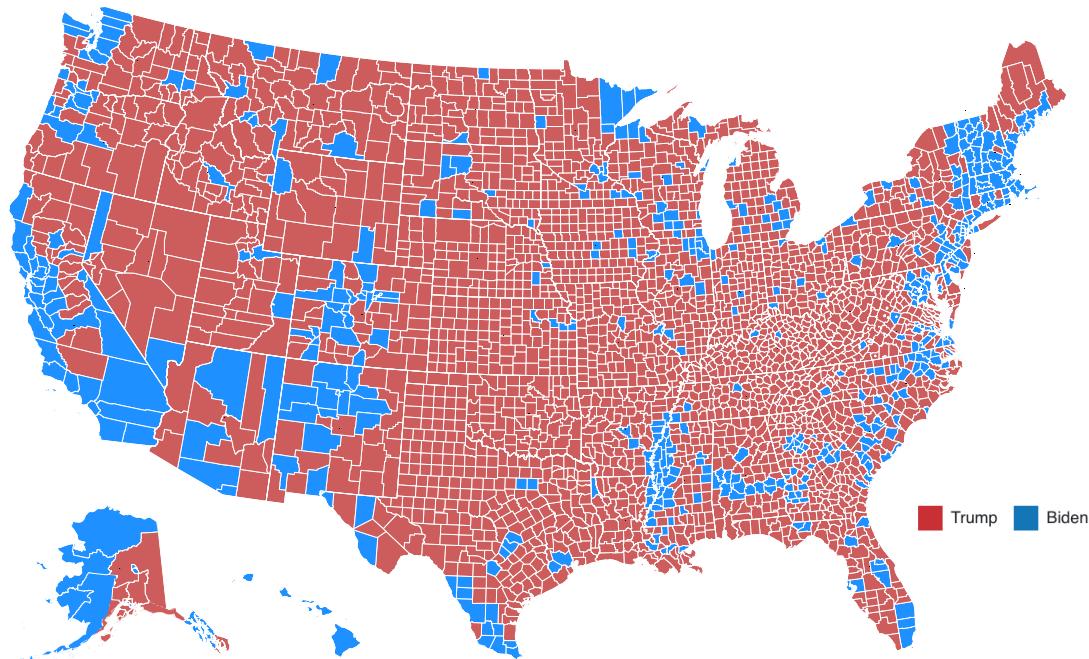
<<Figures 1 and 2 about here>>

⁹ For a nice overview of political graphics of different kinds, including a cartogram of the 2020 presidential election at the county level, see Bliss & Patino (2020). See also Cartographic Views of the 2020 US Presidential Election - Worldmapper (2020).

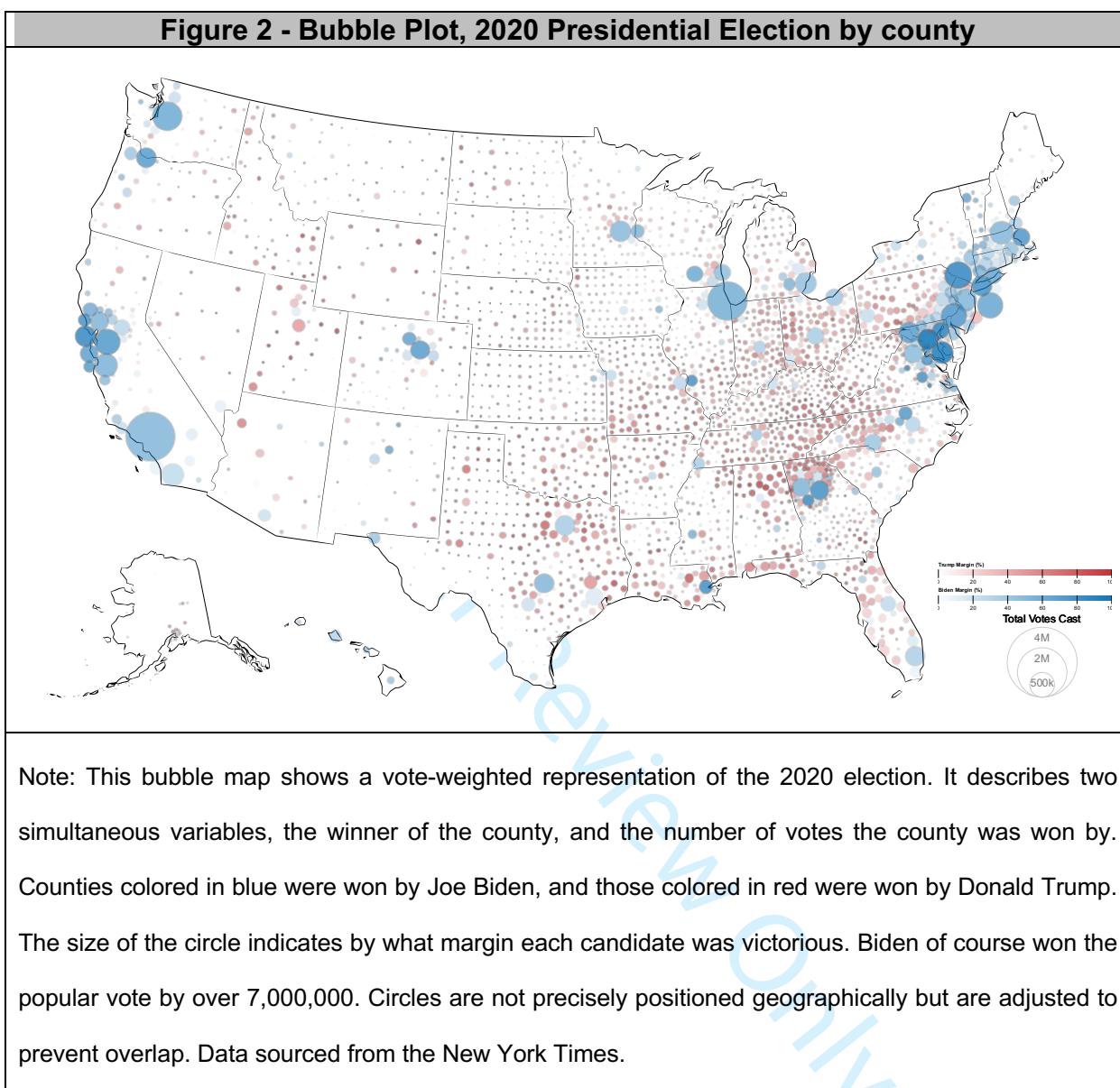
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3 But it is still virtually impossible to visually sum-total results from a cartogram or
4 bubble map to determine an election winner, especially when the number of units (say
5 counties) is large, though the bubble size gradations on a bubble map make this task
6 easier than the color variations on a cartogram that usually have a limited number of
7 victory margin categories.¹⁰ Moreover, to further confound simple calculations, it is helpful
8 to remember that the county with the most Republican votes anywhere in the USA was
9 Los Angeles County, California. Trump received 1,145,530 votes there, although that
10 number was dwarfed by Biden's 3,028,885. In fact, 8 of the 10 highest county vote totals
11 for President Trump in 2020 come in states won by Biden.

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55 ¹⁰ Though, note that the circle size in our map is scaled using the square root of the
56 vote total, so the circle size itself is not a linear scale.
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Figure 1 - Choropleth Plot, 2020 Presidential Election by county



Note: This choropleth map shows the 2020 Presidential election results by county. Each county is shown as geographically-sized using the Albers projection. This map depicts an election in which an overwhelming number of counties are colored red, indicating Trump received more votes than Biden. It does not indicate the number of votes each county was won by. Biden won 556 counties, while Trump won 2,595 (Data is estimated, as no official national election results are compiled. Brookings reports the differential to be 2,588 versus 551 (Frey, 2021). Dave Leip's Atlas of Presidential Elections, a widely reputed accumulator of election results, reports totals that do not match the certified, official federal elections results produced by the FEC).



Failure to weight units also lead some commentators to emphasize Trump's gains among minority voters. Trump did show a substantial percentage point gain among minorities as compared to 2016. He won an estimated 12% of the Black vote in 2020, compared to 8% in 2016. Likewise, he won an estimated 32% of the Latino vote in 2020, an increase of 4 percentage points from 2016. But President Trump also won 1 percentage point more of the White vote in 2020 than he did 2016.¹¹

It might seem that these facts imply that Biden must have done worse in popular vote share than Clinton did in 2016. But there is one critical data point that suggests otherwise. While Trump did 1 percentage point better among White voters, Biden did 4 percentage points better among this same demographic! In 2020, 67% of the electorate identified as White in the exit polls, so increasing the margin among that category of voters will swamp decreases among other demographics. The actual combination of percentage point losses among minority voters and percentage point gains among non-minority voters gave rise to an election in which Biden won 81,268,924 votes (51.3%) of the popular vote (i.e., total vote) in 2020 compared to Clinton's vote in 2016 of 65,853,514 (48.2%).¹² Moreover, the increased total turnout further increased Biden's margin of

¹¹ Data extracted from exit polls: <https://www.cnn.com/election/2016/results/exit-polls>; <https://www.cnn.com/election/2020/exit-polls/president/national-results>

¹² Clinton won a plurality of the vote in 2016, not a majority. Several third-party candidates combined had vote totals surpassing the margin between Trump and Clinton.

<https://www.fec.gov/resources/cms-content/documents/2020presgeresults.pdf>

<https://www.fec.gov/resources/cms-content/documents/federalelections2016.pdf>

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3 victory over Trump. Trump lost to Clinton by 2,868,686 million votes in 2016; he lost to
4 Biden by 7,052,770 million votes.¹³
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16 Of course, what really matters for Electoral College outcomes is *where* these
17 changes occur. Since presidential elections are 51 separate state-wide elections (with the
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38 ¹³ There are other types of statistical confusions related to the Electoral College that
39 we simply mention in passing because they are not directly linked to issues of fraud.
40 Many voters have trouble understanding how the Electoral College works. It can
41 most simply be thought of an example of weighted voting, where the weights are the
42 number of electors each state is allocated; namely the sum of the number of
43 representatives the state has in the U.S. House plus two, the number of U.S.
44 Senators from the state. A common fallacy is to believe that a reversal of the popular
45 vote outcome in the Electoral College occurs simply because of the two-seat “federal
46 bonus” favoring small states. But Donald Trump would have won the Electoral
47 College in 2016 even without the two-seat bonus (Cervas & Grofman, 2019). The
48 vote in an Electoral College with only 438 members (538 minus 100 for the senate
49 bonus) in 2016 would have been 248/438 (56.6%) as compared to the actual EC,
50 306/538 (56.9%); while Biden’s percentage an Electoral College of 438 members in
51 2020 would only one have gone up from 256/438 (58.4%) from the actual result of
52 306/538 (56.9%).
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55 Because the Electoral College is a state-based weighted voting game (except for the
56 two states where seats are allocated at the congressional district level as well), once
57 a unit is won by a candidate, the margin of victory is irrelevant for Electoral College
58 seat share but is relevant in terms of the popular vote share. Thus, we can have a
59 candidate winning some states by very close margins in such a fashion that there is
60 a discrepancy between the popular vote winner and Electoral College winner. Of the
four instances post-Civil War examples of such a discrepancy, two of the four, 1888
and 2016, would still have happened without the bonus. But, in 2000, without the
bonus, Al Gore would have been elected president, and in 1876 there also would
have been no inversion had the two-seat bonus been eliminated. (Cervas &
Grofman, 2019; Table 2, pp. 1328-29). Of course, we must be careful in asserting
counterfactuals. For example, had there been no two-seat bonus in 2000, Gore and
Bush might have deployed campaign resources differently.

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3 minor complication of two states awarding votes by congressional districts), winning many
4 more votes in states where the plurality is already secured creates “wasted” votes, since
5 they do not contribute to winning any new electors. Taken all together, Biden was the
6 benefactor of changes from 2016 that propelled him to the presidency, including narrow
7 victories in a number of states won very narrowly by President Trump in 2016.
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15 **Two-party vote share versus shares of total votes cast.** Reading the paragraphs
16 above, the reader may understandably have gotten confused. President Trump’s
17 percentage of support among non-minority voters increased in 2020 from what it had
18 been in 2016. But Trump also made improvements among minority voters. So, how is it
19 possible for President Trump not to have done better in 2020 than in 2016??!! The
20 answer rests on a key difference between 2016 and 2020, namely the proportion of
21 votes going to someone other than one of the major party candidates. In 2016, third-
22 party candidates Jill Stein, Gary Jacobson, and others received a significant number of
23 popular votes (but no electoral votes) totaling almost 8 million votes (almost 5% of the
24 electorate), but in 2020, no candidate besides Trump or Biden received more than
25 1.25% of the national vote, with minor party candidates totaling less than 2% of the
26 electorate collectively raw vote totals.
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44 Clinton won 37% of the non-Hispanic White vote in 2016 and Trump won 57% of
45 the non-Hispanic White vote (with 6% going to other candidates), which was 71% of the
46 electorate. This amounts to 35,903,019 votes for Clinton and 55,310,056 votes for Trump.
47 In 2020, Biden won 41% of the non-Hispanic White vote compared to Trump’s 58%
48 (Trump gained 1-percentage point, and Biden gained 4-percentage points over Clinton),
49 but the share of the electorate that was non-Hispanic White was now just 67%. Biden in
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3 2020 received 43,507,921 non-Hispanic White votes and Trump 61,547,790. That is, the
4 Democratic 2020 candidate had an increase of 7,604,902 non-Hispanic White votes
5 compared to the Republican candidate's 6,237,734. So, on balance, in terms of raw votes
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7 2020, Biden made more of a gain among non-Hispanic white voters over Clinton's 2016
8 performance than Trump did in 2020 relative to his 2016 performance among that group.¹⁴
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15 **Cherry-picking the data.** The most primitive form of failing to weigh the data properly is
16 cherry-picking the data to emphasize only those facts that lead to the desired conclusion.
17 In presenting only some facts, a claimant can appear to have been honest while
18 suppressing pertinent information that otherwise would prove their claims either false or
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31 ¹⁴ Total turnout rose in 2020 as compared to 2016 among both minority and non-
32 minority voters. The proportion of the electorate that was minority increased in 2020
33 as compared to 2016 both in raw numbers and in proportion of the total. In 2016,
34 approximately 97,035,186 voters were non-Hispanic White, compared to in 2020,
35 where approximately 106,116,880 were non-Hispanic White (a difference of 9
36 million). By comparison, the electorate was composed of 39,634,090 minority voters
37 in 2016, and 52,266,523 minority voters in 2020 (a difference of 12.6 million). In
38 2016, roughly 29% of the electorate was minority, in 2020 that percentage rose to
39 33%. However, Trump's gain from minority voters in both absolute number of voters
40 received and proportion of the group supporting him was compensated for by loss in
41 support among the larger group of non-minority voters. Moreover, even though
42 President Trump made percentage point gains among minority voters in 2020
43 compared to 2016, as well as gains in raw vote totals, nonetheless, in terms of total
44 votes from minority voters, Biden also did better than Clinton since there were more
45 minority voters in 2020 than in 2016. To understand what is happening we need to
46 look at changes in each group's share of the electorate, which is a function of the
47 change in the size of the voting populations and the change in vote propensities
48 among those populations.
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3 incomplete. Because the data being cited is accurate, cherry-picking can prove a
4 persuasive tool.¹⁵
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8 A standard way to cherry pick election data to show the potential for fraud is to
9 focus on racial or demographic groups and to highlight the situations where Biden did
10 worse than Clinton. For example, Biden won fewer counties than Clinton. You can also
11 get even more specific, e.g., Trump won a greater share of the vote in the city of
12 Philadelphia in 2020 than he did in 2016; you can find other big cities where Biden
13 underperformed Clinton. In these cherry-picked examples, Biden did less well against
14 Trump than did the previous Democratic candidate and, since Trump beat Clinton, absent
15 fraud, the implicit (and often explicit) conclusion is Trump must also have beaten Biden
16 in terms of overall total vote nationally. But, of course, while there were some urban areas
17 where support for Biden was lower than for Hillary Clinton, there were also urban areas
18 where support for Biden was higher than for Hillary Clinton. Similarly, as discussed above,
19 it is true that in terms of percentage voting for the Democratic candidate, racial minority
20 support for Biden was (marginally) lower than for Hillary Clinton, and Trump did better
21 among rural voters in 2020 than he did in 2016. However, support among categories of
22 white voters, namely the college-educated and those living in suburbs, was higher for
23 Biden than for Hillary Clinton. And there are more voters living in suburban residences
24 than in rural areas.
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54 ¹⁵ Cherry picking is thus a strategy useful to those who know that the full evidence is
55 against them.
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3 It is, of course, the combination of all the subgroup patterns of voting and the
4 (changes) in their sizes across elections that matters for the total popular vote. To
5 reiterate the obvious, looking only at some subsets of voters, or only some geographic
6 areas, e.g., only rural counties, is misleading and can lead to ridiculous claims that the
7 candidate who received more votes did not actually receive more votes.¹⁶
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16 B. Misinterpreting Statistical Significance

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18 In statistics, warnings about misinterpreting statistical significance are legion. One
19 point frequently made is that statistical significance can only be interpreted in the context
20 of the specific null hypothesis being tested. Another is a reminder that p-values are very
21 strongly linked to sample sizes. A third point is a warning not to confuse statistical
22 significance with substantive importance. A further note of caution is that high statistical
23 significance in regressions does not imply a causal relationship between variables.
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33 Our first example of misuse of statistical significance falls into the second and third
34 categories. Our second example involves the misinterpretation of causality. Both are
35 found in the expert witness testimony of Dr. Charles Cicchetti in the lawsuit brought by
36 Texas Attorney General Ken Paxton challenging election results in Georgia, Michigan,
37 Pennsylvania, and Wisconsin (*Texas v. Pennsylvania*, 592 U.S. ___, 2020).¹⁷ Dr. Charles
38 Cicchetti's calculations were picked up and widely spread on the internet.
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48 16 Moreover, in understanding Electoral College outcomes, we need also to look at the
49 geographic location of each candidate's support.

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51 17 As we noted earlier, we are providing a compendium; this and other claims of Dr.
52 Cicchetti have already been rebutted elsewhere (see e.g.,
53 <http://web.archive.org/web/20220416221931/https://reason.com/volokh/2020/12/09/>

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3 First, in comparisons between what was found in 2020 and what was found in
4 2016, Dr. Cicchetti noted that he could demonstrate beyond any possibility of error that
5 the vote share distribution for Joe Biden in 2020 differed from that of Hillary Clinton to a
6 statistically improbable degree. He is certainly right about that fact.
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13 But what that shows about election fraud is — exactly nothing! It does show that
14 the vote share distribution for Donald Trump in 2016 was not the same as in 2020. But
15 we could have picked ANY two adjacent presidential election years and showed that the
16 vote shares of the Democratic candidates in those two elections at the level of states or
17 counties or precincts differed from one another. With a large enough sample size, such
18 comparisons are virtually certain to generate statistically significant differences.
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28 Second, in a comparison of partisan voting patterns in ballots in 2020 among
29 ballots that are tallied early in the counting process and those that are tallied later, Dr.
30 Cicchetti pointed out that, in some states, President Trump's share of the vote declined
31 relative to those first reported as polls closed on election night as more ballots were
32 tabulated, while in other states he found the reverse pattern. He found the difference
33 between the early vote share for Trump and later vote share for share to be statistically
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45 more-on-statistical-stupidity-at-scotus/
46 and <http://web.archive.org/web/20220416221815/https://statmodeling.stat.columbia.edu/2020/12/08/the-p-value-is-4-76x10%E2%88%92264-1-in-a-quadrillion/>. That
47 report has been devastatingly critiqued in the expert witness report of Gary King in
48 the same case. Claims about election fraud in *Texas v. Pennsylvania*, including Dr.
49 Cicchetti's report, can be found here:
50 https://www.supremecourt.gov/DocketPDF/22/22O155/163048/20201208132827887_TX-v-State-ExpedMot%202020-12-07%20FINAL.pdf
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3 significant beyond any reasonable doubt, though of course, the directionality of the
4 difference was not uniform. And about those facts and the statistical significance of the
5 observed differences he is again quite correct. But what that shows about election fraud
6 is — once again, exactly nothing! Indeed, for those familiar with elections, it shows a
7 pattern that was predicted in advance (Foley & Stewart III, 2020).¹⁸
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15 While once it was believed that mail-in (absentee) ballots were disproportionately
16 Republican, for the last two decades or so the evidence has shifted, suggesting that mail-
17 in ballots are more Democratic-leaning than is the case for ballots cast in person.¹⁹ In
18 some states in 2020, mail-in ballots were tallied as they came in; thus, they were the first
19 ballot results to be reported; in other states, the tallying of mail-in ballots did not begin
20 until after the polls were closed; and in select states, they were the last to be tallied. Thus,
21 we would expect that there would be differences across states that would be in part
22 predictable by when mail-in ballots were tallied. This could account for why, in some
23 states, the early vote was more Democratic than the later vote, while the reverse was true
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39 18 In the hearings by the House Select Committee investigating January 6, this
40 discrepancy between the patterns in early and late votes was referred to as a “red
41 mirage.”
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45 19 Perhaps because states with more Democrats allow for more wide-spread use of
46 mail-in balloting, e.g., eight states conduct general elections entirely by mail. They
47 include California, Colorado, Hawaii, Nevada, Oregon, Utah, Vermont, and
48 Washington. See (*Voting Outside the Polling Place: Absentee, All-Mail and Other*
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50 *Voting at Home Options*, 2022). All of these, except Utah, voted their electors to both
51 Clinton and Biden.
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3 in other states. Thirty-seven states allow election officials to begin processing mail-in
4 ballots as they arrive prior to election day. Another ten states allow processing to begin
5 on election day, but prior to polls closing. Only Maryland does not permit counting mail-in
6 ballots until after polls close.²⁰ Battleground states Michigan, Wisconsin, and
7 Pennsylvania are among the ten states that do not allow counting mail-in ballots until
8 election day.

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18 But there is also another reason why late ballot tallies and early ballot tallies might
19 be expected to be different in their partisan outcomes. Ballots tallies come in from
20 counties and then get tallied statewide. When ballots get reported depends upon the
21 efficiency of the county officials doing the tallying. But it is often easier to complete in-
22 person voting tallies in small rural counties where there are few polling stations than in
23 counties with large cities. But rural counties differ from urban counties in their partisan
24 propensities. So, it is essentially guaranteed that there will be differences in partisan vote
25 share as county reports come in at different times on election night. In the end, only the
26 final count of all ballots is valid.

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34 In sum, when the null hypothesis is nonsensical, there is nothing at all surprising
35 about it being rejected at a high level of statistical significance, especially when the
36 sample size is high. Indeed, what would be surprising is if silly null hypotheses were NOT
37 rejected.

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²⁰ Two states, Connecticut and Ohio, do not specify when counting may begin. For more information, see Voting Outside the Polling Place: Absentee, All-Mail and Other Voting at Home Options (2022; Table 16).

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3 **C. Meretricious Probabilistic Reasoning**
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6 **Voters with the Same Name and Date and Year of Birth.** It has been noted that
7 Joe Frazier voted in Pittsburgh in 2020 even though he had died in 2011. This fact was
8 taken as evidence of fraud (Sadeghi, 2020). But of course, not all Joe Fraziers' are the
9 famous (and now dead) boxer. Similarly, instances, where individuals with the same
10 name and same birthday, and/or same birth year were found on voter rolls, have been
11 taken as evidence of fraud.
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14 It should be noted that some states record a default birth date (or a code for
15 missing data) if none is provided, particularly if no date was provided when this
16 information was collected on paper. This will, of course, if not corrected for, increase the
17 likelihood of the appearance of voters who appear on the voter rolls multiple times. But
18 the likelihood of occurrence of voters appearing on voting lists who have identical names
19 and even identical birth dates, and even identical years of birth is dramatically
20 underestimated by virtually everybody, except for those who have truly understood the
21 famous "Birthday Paradox." The Birthday Paradox in its basic form is about how the
22 number of people in a group affects the probability that some pair of them have the same
23 birthday.
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26 When first confronted with the question of how the number of people in a group
27 affects the probability that some pair of them have the same birthday, students in statistics
28 classes (or at least those in the undergraduate statistics classes taught by one of the
29 present authors) think that, since any given birthday has a probability of only 1/365 (or
30 366 if you take into account leap years), it would take around 183 people before you
31 would get a probability of 50% that two people in the group had the same birthday. The
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3 Birthday Paradox is called a paradox because it only takes a group of size 23 for the
4 probability of two people in the group to have the same birthday to exceed 50%.
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8 The probability that two people in a group of size n do NOT share the same
9 birthday can be written as $1 * \left(1 - \frac{1}{365}\right) * \left(1 - \frac{2}{365}\right) * \left(1 - \frac{3}{365}\right) * \dots * \left(1 - \frac{n}{365}\right)$. This
10 product goes down much faster than one might think, and thus the probability that at least
11 two people in the group share the same birthday, which is one minus this product, goes
12 up much faster than one might think.²¹ Indeed, with only 75 people, the probability of a
13 birthday match rises to 99.95%. In a similar way one can calculate the expected
14 proportion of a group of size n who share a birthday with at least one other person in the
15 group. The difficulty in appreciating how the increasing number of possible pairs that
16 could share a birthday (and other attributes) increases in a non-linear way with increasing
17 n is relevant to claims made in 2020 (and earlier) that find examples of people with the
18 same name and same date of birth on the voting rolls was evidence of “double-voting”
19 fraud.²²
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40 ²¹ This product can be rewritten in terms of combination or permutations, which gives a
41 somewhat more intuitive way to see how the probability changes with n . See
42 https://en.wikipedia.org/wiki/Birthday_problem#Calculating_the_probability. A
43 program that calculates the probability of the birthday paradox for any given n is
44 found at <https://www.statisticshowto.com/same-birthday-odds/>

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48 ²² Matching procedures are sometimes used to identify duplicate voter registration
49 entries. States are required to maintain lists of eligible voters, which are complicated
50 by people migrating between counties and states, but also within neighborhoods and
51 municipalities. But matching procedures sometimes lead to “false positives”, where it
52 appears two entries are the same people, but they are in fact not. See more in
53 (Hasen, 2020), and (Weiser et al., 2006).
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3 While, for any given name, it is obviously harder to find someone born on the
4 same day and in the same year as it is merely to find people with the same name and the
5 same birthday (but not the same year of birth). The logic of figuring out the probability of
6 such a match happening is the same as the simple product formula given above, but now
7 the divisor is no longer 365 since we need to take into account the years in which a person
8 might have been born (though we may reasonably assume that everyone who votes was
9 born at least 18 years ago). Moreover, as a further complication, we must consider the
10 degree of heterogeneity in the distribution of names.²³ But if we take name, birthday, and
11 birth year as mutually independent factors,²⁴ then we can simply multiply probabilities.²⁵
12 And we can further simplify by assuming a uniform distribution across the first two factors
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- 28 ²³ Calculating this probability from the name distribution in the population requires
29 calculating a self-weighted average based on name and surname. Names vary
30 greatly in their frequency. The U.S. Bureau of the Census publishes a list of
31 surname frequencies based on national values, but the distribution of names (and
32 their degree of heterogeneity) will vary with the racial/ethnic composition of the
33 political unit (Grofman & Garcia, 2014, 2015).
- 34 ²⁴ Birth year and name are not fully independent of one another. Changing patterns of
35 immigration affect the relative surname shares in different generations, and some
36 rather dramatic changes over time in the popularity of first names mean that first
37 name probabilities are not independent of the year of birth. Also, first and last names
38 are far from independent since both are linked to ethnicity. Indeed, we could, in
39 principle, estimate birth decade probabilities just by looking at the prevalence of first
40 and last names of those born during the decade (c.f., Grofman & Garcia, 2014,
41 2015)
- 42 ²⁵ Note that the probability of finding such a match from among a set of n voters must
43 not be confused with the probability of any specific name + birthday + birthyear
44 combination being repeated. For obvious reasons the likelihood of a given
45 combination being repeated will depend, *ceteris paribus*, on how common is the
46 name.
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3 and assess the likelihood of two randomly chosen individuals bearing the same name
4 from the name distribution in empirical data.²⁶
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8 Of course, multiplying probabilities for three different factors gives us low
9 probability values, but not as low as one might think. For example, if a randomly chosen
10 person has a 0.000074 percent chance of sharing both a first and a last name with the
11 next randomly chosen person, which was the estimate from the McDonald & Levitt (2008;
12 p. 119, fn. 26) study,²⁷ a total electorate of only 21,071 is enough to bring the probability
13 of finding two people with the same name and the same birthday and the same birth year
14 above 50%. And an n of 57, 314 brings that probability to 99.5%. Moreover, as McDonald
15 & Levitt (2008) demonstrate with their detailed analyses of New Jersey electoral rolls,
16 there are other reasons why we see individuals shown with identical data identifiers,
17 including flaws in the data such as the same individual simply being entered twice or
18 individuals with missing data birth data (assigned a particular missing data code) being
19 treated as having identical records.
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39 ²⁶ The most sophisticated study of double voting of which we are aware (McDonald &
40 Levitt, 2008) investigates complaints of election fraud in New Jersey in 2004 that
41 were based on apparent observance of thousands of instances of double voting,
42 includes a comparison of the assumption of uniform distribution of year of birth and
43 name distribution to actual data and show that assuming a uniform distribution over
44 a 64 year interval and over the names in the data set tends to underestimate the
45 prevalence of birth year matching by about 12% $\cong (487 - 433)/433$, see p. 119).
46 The more there are birth bulges (as in the post-WWII baby boom), the more likely it
47 is that two randomly chosen individuals will share the same year of birth, and some
48 names are much more prevalent than others.
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54 ²⁷ For further compatibility with the McDonald & Levitt (2008) study, in the model,
55 presented below, we also took our birth year time period as a 64-year span.
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3 **Benford's Law.** There have been many attempts to use statistical tools to detect election
4 fraud. One of these involve looking at suspicious data, e.g., vote tallies that
5 disproportionately end in 0's or 5's since, in "inventing" data, there can be a human
6 tendency to call these numbers to mind more often than would be expected from a purely
7 uniform distribution ("Russian Elections Once Again Had a Suspiciously Neat Result,"
8 2021). While it makes no sense to look at the first digits of election returns since these
9 will be obviously contingent on the mean size of the units,²⁸ in investigating fraud, the
10 frequency of digits other than the last or first digit has also been investigated. Benford's
11 Law, which is a hypothesized frequency distribution of k^{th} digits, is an example of one tool
12 that has been used to assess *prima facie* evidence of voter fraud. The political scientist
13 Walter Mebane applied this tool to elections in multiple countries (Mebane & Kalinin,
14 2009). In the 2020 presidential election, analyses based on Benford's Law were provided
15 in videos and tweets as evidence that, in various locales, elections had been rigged.²⁹
16 We will make no attempt to repeat the logic that leads to Benford's Law (see the
17 discussion of the supposed Law in Wikipedia and references therein);³⁰ we simply note
18 that almost all of those who have investigated it empirically is dubious about its application
19 to elections.³¹

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44 ²⁸ Mebane (2020) notes that "It is widely understood that the first digits of precinct vote
45 counts are not useful for trying to diagnose election frauds."
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47 ²⁹ See e.g., (Jenny, 2020).

48 ³⁰ Wikipedia entry [Benford's Law]:
49 https://en.wikipedia.org/wiki/Benford%27s_law#cite_note-37

50 ³¹ See e.g., Deckert et al. (2011). Even Mebane, who has been a repeated applicant of
51 the Law as a fraud detector in elections in multiple countries, has reiterated that
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3 According to (Mebane, 2011), *Benford's Law* implies that the mean value of the
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5 second digit of a distribution of votes at the precinct or other unit-level should equal 4.187.
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7 But a little reflection will show that the frequency of even second digits will be linked both
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9 to the size of units and their variance.
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13 Imagine, for example, that the mean Democratic votes cast in a set of precincts is
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15 1,650 voters, with a variance of 200. The second digit will have disproportionately many
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17 more 4's, 5's, 6's, 7's and 8's than it has other numbers. If we change either the mean or
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19 the standard deviation of that distribution, then the expected mean value of the second
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21 digit will also change. Thus, there cannot be some fixed value that is the expectation of
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23 the mean value of the 2nd digit in all distributions. And the same point applies to whichever
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25 digit we focus on, or whether we look at all of them since the scale of units will affect
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27 which digits are most likely to deviate from Benford's Law expectation. But perhaps even
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29 more importantly, it is quite possible for Benford's Law to work well for some candidates
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31 and badly for others since different candidates will have different means and variances in
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33 their vote distribution across units..
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39 The next two examples of meretricious probabilistic reasoning are more about
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41 human psychology than they are about statistics, *per se*.
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45 **Tip of the iceberg fallacy.** No one who studies elections thinks that the level of election
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47 fraud in 2020 was zero. In an election with over 150 million voters, there will be some who
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52 while its violation might be taken as suggesting an anomaly, a violation of the
53 supposed Law does not prove fraud. Fraud would need to be directly investigated
54 (Mebane, 2020).
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voted who were not entitled to do so, some who voted twice, some whose ballots were lost, some whose mail ballots were stolen and cast by others, and some who voted while dead (perhaps having died after casting a ballot). And there will be some election officials who (at least initially) reported inaccurate results. But while fraud in any given state may have cases in the tens, or perhaps even hundreds, this level is not enough to change presidential election outcomes (see below).

One fallacy of probabilistic inference is to infer from the indubitable fact of some electoral wrongdoing in most states that electoral fraud is massive in both magnitude and geographic spread. The careful studies of fraud, such as the one of the 2020 election undertaken by the Ohio Secretary of State, reveals fraud at a minuscule level of fraud: 27 cases out of 6 million or so ballots cast in Ohio (McDade, 2022). However, findings such as those of the Ohio Secretary of State, rather than being taken as evidence that the fraud level was trivial, can instead be interpreted as supporting evidence for a high probability of massive fraud.³² The implicit and quite wrong-headed probabilistic argument is that any examples of fraud that are found should be presumed to be "only the tip of the iceberg."³³

³² There is also a belief in conservative internet circles that, if there is fraud, it will be fraud committed by Democrat officials and found among the kinds of voters (such as racial minorities) who are most likely to vote Democratic. But is it is amusing to note that one of the first documented examples of actual fraud, so-called "voting the graveyard," was committed by a Republican "in an attempt to further President Trump's campaign" (Vella, 2020).

³³ As Gelman, 2021) points out, some people say things like "who's to say" when they hear claims that are patently implausible but respond in a way suggesting that they believe that claim might be true. Here, possibility is confounded with probability. A common aphorism also relates to this confusion, e.g., "Where there is smoke there

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3 **Straw man fallacy.** On the other hand, the claim that there was no massive fraud can
4 be rhetorically equated to the claim that there was no fraud, and the latter claim rebutted
5 as a straw man, as if successfully rebutting the claim of no fraud demonstrated the truth
6 of the claim of massive fraud. Straw man arguments can be used to perpetuate a big lie,
7 as has been the case following the 2020 presidential election.
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15 Now let us turn to a set of other claims that have as their general form: “The only
16 way these election results could have happened is if there was massive fraud.”
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22 **D. Logically Invalid Arguments with a True Premise involving Historical
23 Election Results Comparisons**

24 **Spoiled ballots.** An example the empirically accurate observation that the spoiled ballot
25 rate of mail-in ballots in 2020 was much lower (in some states remarkably lower) than in
26 2016. This fact is taken to be evidence of mail-ballot fraud by Trump supporters.
27 Presumably the theory is that the lowered spoilage rate came about either because more
28 ballots were illegally cast in 2020 than in 2016 and that those stealing unfilled-in or
29 incomplete mail ballots and submitting them knew how to properly fill out the forms, or
30 that ballot workers in polling stations where the mail ballots were likely to be Democratic
31 ones deliberately allowed improperly filled out mail ballots to pass muster in a way that
32 they had not in 2016.
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49 is fire.” Here, the size of the fire remains unspecified. In the context of election fraud,
50 confusing possibility with probability is likely to be more prevalent among those who
51 see the world in conspiratorial terms. Gelman quotes one person who accepts
52 claims such as Obama is a Muslim as saying: “We see what they want us to see, I
53 mean anything could be anything.”
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3 We can write the argument as
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6 If there is ballot fraud involving mail-in ballots (A),
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10 then the spoilage rate among mail-in ballots will be lower than in the past
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12 (B).
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15 The spoilage rate among mail-in ballots was lower than in the past (B)
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18 Therefore, there was ballot fraud involving mail-in ballots (A).
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25 Here we have the fallacy of *affirming the consequent*.
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28 There are good reasons why ballot spoilage was lower in 2020 than in 2016 that
29 have nothing to do with fraud, namely much greater effort on the part of election
30 administrators to inform voters of what they needed to do to cast a valid ballot. For
31 instance, popular late night comedy Stephen Colbert created a rather sophisticated
32 website aimed at informing those in all 50 states about the specifics for casting a ballot in
33 each of those states (*Better Know a Ballot*, 2021). His “Better Know a Ballot” also aired
34 many times in the months before the elections on his highly rated “The Late Show.” Ads
35 developed by the states themselves aired on television channels and as ads on streaming
36 services.³⁴ The Democratic National Committee also spent millions of dollars on television
37 ads with information about returning mail-in ballots(*DNC Launches New Digital Ads in PA*
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54 ³⁴ In Pennsylvania, this included pointing viewers to a website developed by the
55 Department of State. Ads featured prominent actors and athletes from the state.
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3 *Reaching Vote-By-Mail Voters: "How to Return Your Ballot!", 2020).* There was also
4 ample coverage in newspapers about properly filing out and mailing a ballot so that it
5 would not be rejected (Lai, 2020).
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11 Moreover, in some states, there were greater efforts to ensure that those who
12 submitted a mailed-in ballot with an envelope which had some correctable error that
13 would prevent the ballot inside the still unopened envelope from being counted were
14 informed of the error and given the opportunity to correct it. Eighteen states allowed voters
15 to "cure" their ballots if there is a discrepancy (*States That Permit Voters to Correct*
16 *Signature Discrepancies*, 2020). These states are disproportionately Democratic; Trump
17 won just 5 of the 18. But in our federal system, absent genuine issues of potential voting
18 rights violations, states can and do differ in the details of their election administration.
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31 E. Logically Valid Arguments with a False Premise involving Historical Election 32 Results Comparisons 33

34 Now we turn to claims about election fraud that fall into the category of valid
35 arguments with one or more false premises.
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38 **Presidential coattails.** One claim about election fraud was based on the observation that
39 winning presidential candidates have coattails that aid members of their party in the
40 House of Representatives to gain seats. The Democrats lost 12 seats in the House of
41 Representatives in 2020, which violates this expectation, and so the claim goes that the
42 implication is that there must have been massive multi-state fraud.
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53 The structure of this argument is:
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If a presidential candidate wins election (A),

then there will be a gain in the number of members of his party in the U.S. House of Representative (B).

There was no gain for the Democrats in the House in 2020 (not B)

Therefore, Biden must have lost the election (not A)

This is a valid argument. It is an example of *denying the consequent*. However, the premise on which it is built, that presidential coattails are inevitable, is false.

By *coattails*, we are referring to an increase in the number of members in the U.S. House of Representatives that share the incoming president's party (Campbell, 1986). Negative coattails are not uncommon, and in contemporary politics, have become more likely.³⁵ Since 1868, there have been thirteen elections where a president has had negative coattails (including 2016 and 2020). Negative coattails are more likely when (a) elections are close in popular vote (b) there is substantial partisan bias against the party of the presidential winner in the House, (c) a substantial portion of the votes for the winning presidential candidate are wasted in states that are won by large margins, and

³⁵ See Morris Fiorina (2017) "Unstable Majorities", and Frances Lee (2016) "Insecure Majorities". Modern elections are very likely to result in divided government, and control of any branch of government is often won or lost in the margins. Partisan bias, such as that introduced by malapportionment or gerrymandering, can also affect the ability to carry marginal House or Senate seats.

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3 (d) the winning president's party picked up a significant number of seats in the previous
4 midterm election. All four of these features are found in 2020.
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8 Biden's share of the major party vote was only 52.27%; the estimated partisan bias
9 in 2020 in the House of Representatives in 2020 was 2.7%. Congressional districts have
10 become far less competitive in recent elections, leaving fewer chances for a president to
11 provide coattails large enough to flip seats (Engstrom, 2020).³⁶ If we eliminate the states
12 that gave the widest raw margin to Biden (California and New York and Massachusetts)
13 from the calculations, Trump has a majority of the vote in the remaining states – hence,
14 we would not expect to see Biden coattails in those remaining states.³⁷ Democratic gains
15 in the House in the 2018 midterm were significant, and turnout was a level not seen before
16 universal adult franchise (Jacobson, 2019). Moreover, up through 2016 there is a time
17 trend of decreasing presidential coattails which, when projected onto 2020, would create
18 an expectation of a negative coattail in the 2020 election.³⁸ But perhaps most importantly,
19 there were 35 House constituencies carried by Trump in 2016 but with a Democratic
20 House member elected in 2018,³⁹ and only 5 House constituencies lost by Trump in 2016
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45 ³⁶ For more details on Biden's overperformance compared to U.S. House Democratic
46 candidates, see Galston (2020).

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48 ³⁷ House Democrats nationally underperformed their 2018 performance, which
49 accounts for the fourteen net seats gained by the Republicans. Relative to the 115th
50 Congress (2016-2018), the 117th Congress (2021-2023) has 28 more Democrats.
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53 ³⁸ Figure omitted for space reasons.
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55 ³⁹ An increase of 22 from 2016.
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3 but with a Republican House member elected in 2018.⁴⁰ Thus, Democrats in 2020 had
4 many more vulnerable House seats than did the Republicans.
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11 **Bellwether counties.** An argument of a similar form is that Biden lost most of the counties
12 that had been bellwether counties, and therefore since bellwether counties predict
13 presidential elections, Biden must really have lost the election.
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16 Again, we can write this argument as
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19 If a presidential candidate wins the election (A)
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22 then they can be expected to carry almost all the bellwether counties (B),
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25 Biden lost almost all of the bellwether counties (not B)
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28 Therefore, Biden must have lost the election (not A)
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31 This, too, is a valid argument -- another example of *denying the consequent*. However,
32 even though “not B” is empirically accurate, the premise on which the argument is built,
33 namely that bellwethers predict elections, is false.
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36 Many decades ago, the political scientist Edward Tufte (1974; chapter 3) wrote a
37 devastating rebuttal to work on the power of bellwethers. Tufte showed that, over the
38 period 1916-1968, there were no real state-level bellwethers and, most importantly, the
39 U.S. counties identified as presidential bellwethers at time t had no better track record at
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⁴⁰ A decrease by 19 from 2016.

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3 the next presidential election than the non-bellwether counties. (Hopkins, 2017) showed
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5 the same result for much more recent data. Yet, belief in bellwether units of geography,
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7 more particularly in the existence of bellwether counties, refuses to die.
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11 Grofman & Chen (2022) explain the predictive failures of bellwethers partly in
12 terms of classic work of Deutsch & Madow (1961). A simple binomial model shows that
13 by chance alone, in large groups, some individuals can appear to have repeated
14 (predictive) success even though, for any given event, the probability of success of any
15 actor is only 0.5. This is true when we are dealing with independent trials where past
16 success tells us exactly nothing about future success.⁴¹ But Grofman and Chen also
17 elaborate on the conditional probability model that generates the likelihood of bellwethers
18 by showing that as partisan polarization increases, and presidential politics nationally is
19 competitive, so that the Electoral College sometimes have Democrats winning and
20 sometimes Republican winning (three each in the 21st century), the likelihood of
21 bellwethers declines.
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25 The intuition here -- which they examine at the level of counties -- is a very simple
26 one: in order to be a bellwether county, a county must vote for the winner both when the
27 winner is a Democrat and when the winner is a Republican, but *ceteris paribus*, the
28 increasingly polarized patterns of voting have by now put virtually all counties firmly into
29 one partisan camp or the other. In the 21st century, Grofman and Chen show that more
30 than 70%+ of all counties vote consistently Republican, and another portion are
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34 ⁴¹ Relatedly, (Grofman & Chen, 2022) provide a conditional probability model to further
35 explain change over time in the ability to use past elections to predict future
36 elections.

dependably Democrat. These counties make great bellwethers only when their party is always winning, but awful bellwethers otherwise.⁴² Thus, bellwether performance has been falling. Moreover, if inter-election changes in vote propensities vary across types of voters (or geographic units) than bellwethers can perform particularly dismally in a subsequent election. As a graph in 538 shows, counties that previously served as bellwethers have been shifting rightward (Matsumoto, 2021). Thus, when the Republican candidate loses, they will not serve as bellwethers.

Other Cross-Election Comparisons. Other arguments with specious premises supposedly demonstrating that Biden could not have won in 2020 also make use of differences between the 2020 election and patterns found in previous elections. For example, Trump supporters such as Shurk (2020) noted that no incumbent who has won more than 75% of the primary vote has lost their election. Therefore, since Trump had won 94% of the primary vote, he must have won re-election.⁴³ Shurk also observes that “no incumbent in over 100 years who has gained votes in his reelection bid has lost his

⁴² Indeed, Eggers et al. (2021; Figure 2) show that only 2% of counties had a different party winner in 2020 than they did in 2016.

⁴³ This claim links to Twitter user David Chapman (@davidchapman141), a self-proclaimed “Author & Historian”. This “thread” is filled with statistics purportedly showing how Biden is the historic underdog going into the 2020 election., e.g., “Incumbents are 6/6 when facing re-election during civil unrest”. See more: <https://twitter.com/davidchapman141/status/1315440579485069314?s=20>

Internal links to this claim on Twitter say that the first primary was in 1912 and that Trump had received a higher percentage of the primary vote than Eisenhower, Nixon, Clinton, and Obama. And it is noted that only five incumbents have received at least 90% of their primary vote.

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3 quest for reelection."⁴⁴ On the other hand, we could just as easily to claim that incumbents
4 whose overall presidential approval rating among independents was as low as that of
5 President Trump must lose re-election. Unfortunately for this type of argument, there is
6 no guarantee that what was true in the past will be true in the future. And, of course, these
7 historical comparisons are based on small n , especially when we limit our comparisons
8 to those where there is an incumbent running for re-election.⁴⁵

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18 **F. Logically Valid Arguments with False Statistical Premises Using**
19 **Comparisons Based on Features or Components of the Same Presidential Election**

20 **Matching design (within election split ticket voting versus straight ticket voting).**

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22 One matching comparison involves comparison of elections where there are two kinds of
23 votes: straight ticket votes (where a single check casts a vote for all candidates of a given
24 party for all offices) and ballots where this box is not checked. Ayyadurai (2020)⁴⁶ claims
25 that support levels in straight ticket votes and split ticket votes should be unrelated, unless
26 there is voter fraud. He uses data from four counties in Michigan where the straight ticket
27 vote option was available. Ayyadurai asserts that a negative correlation indicates that the
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41 ⁴⁴ This claim fails to account for well-known political science theory that suggests that
42 competitive elections increase voter turnout (Downs, 1957), and the empirical fact
43 that modern elections are increasingly competitive at the national level (Lee, 2016)
44 even though less competitive for other offices, such as the U.S. House
45 (<https://alarm-redist.github.io/fifty-states/>).
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47 ⁴⁵ We might also argue that neither Trump nor Hillary Clinton could possibly have been
48 elected President in 2016 since both were so disliked. Of course, no matter how
49 unpopular one candidate may be, the other person might be even more unpopular.
50 But also, relative likeability, is only one of the factors affecting vote choice.
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53 ⁴⁶ Ayyadurai has a Ph.D. from MIT in biological engineering.
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3 split ticket votes have been manipulated to reduce the apparent Trump share of the vote,
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5 i.e., that a negative correlation is a sign of voter fraud.
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9 We can write this argument as
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12 If the difference between straight ticket vote share for the candidate of a
13 given party and split ticket vote share for the candidate of that party is
14 negatively correlated (A),
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17 then there is vote fraud in favor of the other party (B).
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20 The difference between straight ticket vote Republican share and split
21 ticket Republican vote share is four Michigan counties is negatively
22 correlated with straight ticket Republican vote share (A).
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25 Therefore, there must have been vote fraud favoring the Democrats in
26 those counties (B).
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29 This is, in principle, a valid argument. But the premise makes no sense.
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32 The notion that the two-vote shares should be uncorrelated if there is nothing
33 suspicious going on might appear on its face to be plausible, but some reflection reveals
34 that it is completely misguided. Depending upon whether we take the difference as split
35 ticket vote share minus straight-ticket vote share or as straight-ticket vote share minus
36 split ticket vote share, it should be inevitable to get a non-zero correlation. In fact, a
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negative correlation is guaranteed by the model used in Ayyadurai, since he subtracts x from y . Below we work out a toy example.

Let's take as an approximation that straight ticket voters have the same vote propensities as among split-ticket vote. Let's set s to the share of the Republican voters who cast a straight ticket vote and r_i to the Republican share of the vote cast by voters of the given type, with $i = 1$ straight ticket voters and $i = 2$, split ticket voters. $r_1 s$ is then correlated with $r_1 s - r_2(1 - s)$, or with $r_2(1 - s) - r_1 s$. Let us simplify further by positing that $r_1 = r_2$. Now, doing what Ayyadurai wishes us to require simply correlating r with either $2r - 1$ or $1 - 2r$, thus guaranteeing either a positive or a negative correlation depending upon which way we do the difference. But there is NO reason to expect is a zero correlation.

But, if the argument above did not convince you that Ayyadurai syllogism is misguided, just think about the failure of symmetry. Under our simplifying assumptions, the exact same argument he makes for Republicans goes through for Democrat votes as well, so that his argument leads us, on the one hand, to the conclusion that there was fraud favoring Democrats (when we look at Republican vote shares) and on the other hand, to the contradictory conclusion that there was fraud favoring Republicans (when we look at Democratic vote shares).⁴⁷

⁴⁷ This rebuttal to Ayyadurai (2020) is presented in (Eggers et al., 2021), acknowledging its previous elucidation by Kabir (2020) and Parker (2020), each of whom show empirical evidence of the party-independence of results. However, the Eggers, Garro, and Grimmer main rebuttal to the Ayyadurai (2020) claim uses the logic of latent variable analysis by demonstrating how regression to the mean effects lead to negatively sloped regression lines in the situation posited by Ayyadurai.

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3 **Comparison of differences involving samples with different means.** There's a
4 second type of fallacy involved with the examples shown in Ayyadurai. It is claimed that
5 in Wayne County the pattern of Trump drop-off is different, and therefore the algorithm
6 that transferred votes to Biden was not used. However, one need only look at the x-axis
7 to see that in Wayne County the most Republican precinct has only about 30% Trump
8 support. In the other counties, some precincts have 80% Trump support. In the range of
9 the Wayne County plot, the pattern is like the other counties. One must be careful when
10 presenting or consuming information from graphs that may mislead, even if done in ways
11 that are not intentional. In this case, the data simply does not exist such that there are
12 heavily Republican precincts in Wayne County to compare to the other counties.
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Matching design (within-election comparisons of areas with and without fraud claims). Lott (2020) uses an apparently sophisticated attempt to prove election fraud via statistical analysis, which is now forthcoming in a peer-reviewed journal (Lott, n.d.). While Lott offers various forms of analysis, the most intriguing one, which appears to be quite plausible,⁴⁸ is a matched pairs analysis which compares the difference in support for Trump in adjacent precincts in a pair of counties in Georgia and pair of counties in Pennsylvania, such that one of each pair is in a county which voted for Biden and one is in a county which voted for Trump.⁴⁹ The counties are distinguished by their expected

48 Indeed, we regard Lott's analysis as sufficiently plausible, on its face, to explain why, despite being either completely wrong or at best much overstated, it could nonetheless pass peer review.

49 Lott (2020) also offers a similar kind of argument about why turnout differences in adjacent pro-Biden versus pro-Trump counties could also be indicators of fraud. But that argument also is fundamentally flawed because it assigns to fraud differences

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3 partisan outcome. Lott takes a major urban county (which voted heavily for Biden) and
4 treats it as one in which fraud might be present; the adjacent counties (which are either
5 suburban or rural) are usually Republican dominated (and voted for Trump) are treated
6 as a baseline in which no fraud would be expected. Lott attempts to explain the difference
7 in absentee votes for Trump between adjacent pairs of precincts in the two types of
8 counties by comparing them to the difference in in-person votes for Trump in the same
9 pairs of precincts. The regression model includes a dummy variable for precincts located
10 in the county which voted for Biden. He claims that a positive sign on this dummy variable
11 indicated the presence of fraud. He obtains positive but not statistically significant results
12 for both Georgia precinct pairs and Pennsylvania precinct pairs. He then combines the
13 data for both states and finds a positive and statistically significant result on his dummy
14 variable for the pooled data.
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Eggers et al. (2021) provide a variety of arguments, both theoretical and empirical, to show that the econometric designs in the (Lott, 2020) study are so flawed that its conclusions are completely vitiated. That study has the advantage of making use of the data provided by Lott for replication purposes.⁵⁰ Here we simply make two points about conceptual flaws in the Lott analyses.

across units that have other very plausible non-fraudulent explanations. See Eggers et al. (2021).

⁵⁰ One point made by Eggers et al., (2021) is that the results for Democratic vs. Republican County comparisons depend upon the order in which we select the units; another point they make is that the exactly which precincts are paired can reverse

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3 On the one hand, it is a huge leap to believe that a positive sign on the dummy
4 variable for precincts in pro-Biden counties in his analyses can be taken as evidence of
5 fraud. There can be many reasons that have nothing to do with fraud for why Trump vote
6 shares among mail-in voters differ from in-person voters across precincts. For example,
7 while we do expect the area on the border between the two paired counties are similar,
8 in Pennsylvania where we are most familiar with the data, there are key differences, such
9 as income levels, homeownership levels and tax rates between the two paired counties
10 that can be expected to affect voting choices.⁵¹ These differences are unlikely to be
11 captured by control variables such as age and gender, and can lead to differences in
12 mean levels of Trump voting that can have major consequences for how the expected
13 shape of in-person versus mail-in ballots across the two counties. Such
14 matching/experimental design assumes that the units (and the individuals who make up
15 that unit) are identical on every meaningful characteristic. The premise is the same as a
16 laboratory experiment when a scientist wants to determine if a treatment has an effect. If
17 the control group varies from the treated group, we cannot know whether the treatment
18 has caused the effect, or if some other difference has a causal impact. Counties, which
19 are the administrators of elections, vary on many aspects of election administration that
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45 the sign of the results and that Lott's specification is one of those that results in a
46 higher level of statistical significance for the dummy variable.

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50 ⁵¹ One of the present authors grew up in Pennsylvania in one of the precincts in Lott's
51 data set and is familiar not just with the county in which that precinct is located but
52 also familiar with the adjacent county whose precincts are used for comparison
53 purposes.
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3 make comparisons across counties potential fraught. For instance, the location and
4 number of polling places might differ. The availability and location (accessibility) of drop-
5 boxes for mail-in ballots might have an impact on the propensity of voters to utilize those
6 resources. County administrators might also have different procedures for curing ballots.⁵²
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8 Most importantly, the likelihood that a person chooses to vote by mail is correlated highly
9 with vote choice. Even in matched precincts in adjacent counties, there might be
10 significant problems with an independence assumption.

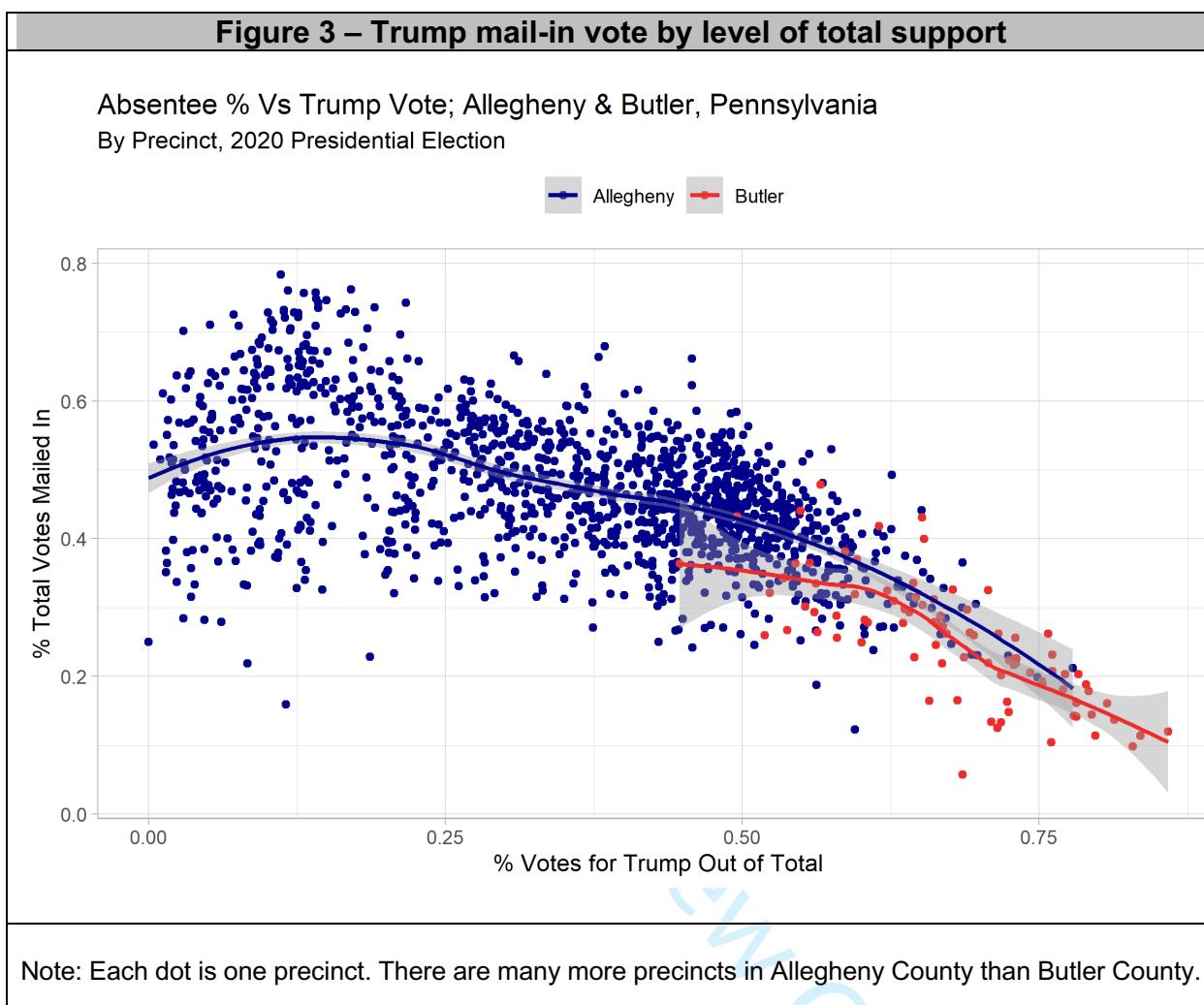
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15 More particularly, implicit in the Lott regression model is that in all precincts, both
16 in counties where Trump wins and in counties where Trump loses, the share of the Trump
17 vote that is cast in-person is independent (at least on average) of the level of Trump
18 support in the precinct. That is, the share of the Trump vote that is cast in person (as
19 opposed to mail) is uncorrelated with Trump's share of the overall vote. Since the total
20 vote in a precinct, V_i , is the sum of the two types of ballots, P_i or M_i , and with $0 \leq P_i + M_i \leq 1$. We can posit that, on average, one (say, in-person, \hat{P}_i), is not correlated with \hat{V}_i
21 (and therefore \hat{M}_i is not correlated with \hat{V}_i).
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38 We show in Figure 3 that the assumption of independence is meritless.
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52 Counties following different procedures for allowing the correction of what would be invalid ballots in Pennsylvania was challenged and led to a lawsuit. White House Press Secretary Kayleigh McEnany said that the PA Secretary of State allowed ballot curing to “tip the scales of an election to functionally favor the Democrat Party.” But all counties were provided the opportunity to cure ballots, even if some refused to. For more information in the differential treatment of invalid ballots across PA counties, see (Farley, 2020).

If this assumption, which is critical to Lott's argument were to hold, then we can write the in-person vote share difference between the precincts (weighed by the population in the precincts) in a pro-Trump county and those in a pro-Biden county as $\hat{P} * (V_1 - V_2)$, where V is the Trump share of each county's vote. We can likewise write the mail-in vote difference between the precincts in the pro-Trump county and those in the pro-Biden county as $\hat{M} * (V_1 - V_2)$. Hence, we can form a regression with Trump mail-in vote share in County 1 minus Trump mail-in vote share in County 2 as the dependent and Trump in-person vote share in County 1 minus Trump in-person vote share in County 2 as the independent variable. But, in accord with the homogeneity assumption above, this leads us to the regression $y = [(\hat{P}/\hat{M})]x$.

In this regression, because of the posited linearity, we should find an intercept of zero. Lott interprets a non-zero intercept as evidence of fraud. But any non-uniformity in the posited linkages can produce a non-zero intercept. Below we demonstrate this non-uniformity for precincts in Allegheny and Butler Counties in Pennsylvania, two of the counties analyzed by Lott. Indeed, what we find is that the higher the Republican vote share in the precinct, the higher the proportion of voters who cast in-person votes. Moreover, the two counties have similar levels of in-person voting at similar precinct Trump support. We interpret this as a result of the more highly Republican areas having been more effectively cautioned by party leaders against the use of mail-in ballots.



On the other hand, even if we reject the various rebuttals of the appropriateness of Lott's matched pairs design and take as reasonable that the sign of the dummy variable tells us something about fraud, the evidence that the dummy variable he uses as an indicator of fraud is non-zero is not that strong. As noted earlier, Lott obtains statistically significant results for the dummy variable only for data pooled across two very different states. But that pooling can in principle create a fallacy akin to *Simpson's paradox*. If the two states differ, say, in their partisan propensities in the chosen counties, then an effect

(e.g., a positive sign on the dummy variable) that is due to differences between the states can be attributed to within-county differences in each state.⁵³

III. Conclusions

To paraphrase Jeremy Bentham, claims of massive fraud based on aggregate level statistical features of the 2020 election are not just nonsense, but “nonsense on stilts” (Bentham, 2002). While it is impossible to address all the misleading claims and specious arguments made on the internet or even by President Trump himself, we believe the compendium of statistical fallacies given above can be useful to those interested in the misuse of statistics. And, as noted earlier, this essay is deliberately written in a non-technical way to be comprehensible even to beginning students in statistics.

⁵³ Grofman (1989) looks at an analogous issue. Pooling data across nations he finds that minimum-winning cabinet coalitions are considerably more durable, on average than non-minimum-winning cabinet coalitions, i.e., it takes longer until the coalition is dissolved. And yet, within each country, he finds that minimum winning coalitions are no more durable than non-minimum winning coalitions. Rather, he finds that countries with more durable coalitions also have a higher-than-average proportion of minimal winning coalitions.

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