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A METHOD FOR EVALUATING THE DISTRIBUTION OF POWER IN A COMMITTEE SYSTEM

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In the following paper we offer a method for the *a priori* evaluation of the division of power among the various bodies and members of a legislature or committee system. The method is based on a technique of the mathematical theory of games, applied to what are known there as "simple games" and "weighted majority games."[1] We apply it here to a number of illustrative cases, including the United States Congress, and discuss some of its formal properties.

The designing of the size and type of a legislative body is a process that may continue for many years, with frequent revisions and modifications aimed at reflecting changes in the social structure of the country; we may cite the role of the House of Lords in England as an example. The effect of a revision usually cannot be gauged in advance except in the roughest terms; it can easily happen that the mathematical structure of a voting system conceals a bias in power distribution unsuspected and unintended by the authors of the revision. How, for example, is one to predict the degree of protection which a proposed system affords to minority interests? Can a consistent criterion for "fair representation" be found?[2] It is difficult even to *describe* the net effect of a double representation system such as is found in the U. S. Congress (i.e., by states and by population), without attempting to deduce it *a priori*. The method of measuring "power" which we present in this paper is intended as a first step in the attack on these problems.

Our definition of the power of an individual member depends on the chance he has of being critical to the success of a winning coalition. It is easy to see, for example, that the chairman of a board consisting of an even number of members (including himself) has no power if he is allowed to vote only to break ties. Of course he may have prestige and moral influence and will even probably get to vote when someone is not present. However, in the narrow and abstract model of the board he is without power. If the board consists of an odd number of members, then he has exactly as much power as any ordinary member because his vote is "pivotal"—i.e., turns a possible defeat into a success—as often as the vote of any other member. Admittedly he may not cast his vote as often as the others, but much of the voting done by them is not necessary to ensure victory (though perhaps useful for publicity or other purposes). If a coalition has a majority, then extra votes do not change the outcome. For any vote, only a minimal winning coalition is necessary.

Put in crude economic terms, the above implies that if votes of senators

[1] See J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior* (Princeton, 1944, 1947, 1953), pp. 420 ff.

[2] See K. J. Arrow, *Social Choice and Individual Values* (New York, 1951), p. 7.

were for sale, it might be worthwhile buying forty-nine of them, but the market value of the fiftieth (to the same customer) would be zero. It is possible to buy votes in most corporations by purchasing common stock. If their policies are entirely controlled by simple majority votes, then there is no more power to be gained after one share more than 50% has been acquired.[3]

Let us consider the following scheme: There is a group of individuals all willing to vote for some bill. They vote in order. As soon as a majority[4] has voted for it, it is declared passed, and the member who voted last is given credit for having passed it. Let us choose the voting order of the members randomly. Then we may compute the frequency with which an individual belongs to the group whose votes are used and, of more importance, we may compute how often he is *pivotal*. This latter number serves to give us our index. It measures the number of times that the action of the individual actually changes the state of affairs. A simple consequence of this formal scheme is that where all voters have the same number of votes, they will each be credited with $1/n$th of the power, there being n participants. If they have different numbers of votes (as in the case of stockholders of a corporation), the result is more complicated; more votes mean more power, as measured by our index, but not in direct proportion (see below).

Of course, the actual balloting procedure used will in all probability be quite different from the above. The "voting" of the formal scheme might better be thought of as declarations of support for the bill, and the randomly chosen order of voting as an indication of the relative degrees of support by the different members, with the most enthusiastic members "voting" first, etc. The *pivot* is then the last member whose support is needed in order for passage of the bill to be assured.

Analyzing a committee chairman's tie-breaking function in this light, we see that in an *odd* committee he is pivotal as often as an ordinary member, but in an *even* committee he is never pivotal. However, when the number of members is large, it may sometimes be better to modify the strict interpretation of the formal system, and say that the number of members in attendance is about as likely to be even as odd. The chairman's index would then be just half that of an ordinary member. Thus, in the U.S. Senate the power index of the presiding officer is—strictly—equal to 1/97. Under the modified scheme it is 1/193. (But it is zero under either interpretation when we are considering decisions requiring a two-thirds majority, since ties cannot occur on such votes.) Recent history shows that the "strict" model may sometimes be the more realistic: in the present Senate (1953–54) the tie-breaking power of the Vice President, stemming from the fact that 96 is an even number, has been a very significant factor. However, in the passage of ordinary legislation, where perfect attendance is unlikely even for important issues, the modified scheme is probably more appropriate.

[3] For a brief discussion of some of the factors in stock voting see H. G. Gothman and H. E. Dougall, *Corporate Financial Policy* (New York, 1948), pp. 56–61.

[4] More generally, a minimal winning coalition.

For Congress as a whole we have to consider three separate bodies which influence the fate of legislation. It takes majorities of Senate and House, with the President, or two-thirds majorities of Senate and House without the President, to enact a bill. We take all the members of the three bodies and consider them voting[5] for the bill in every possible order. In each order we observe the relative positions of the straight-majority pivotal men in the House and Senate, the President, and also the 2/3-majority pivotal men in House and Senate. One of these five individuals will be the pivot for the whole vote, depending on the order in which they appear. For example, if the President comes after the two straight-majority pivots, but before one or both of the 2/3-majority pivots, then he gets the credit for the passage of the bill. The frequency of this case, if we consider all possible orders (of the 533 individuals involved), turns out to be very nearly 1/6. This is the President's power index. (The calculation of this value and the following is quite complicated, and we shall not give it here.) The values for the House as a whole and for the Senate as a whole are both equal to 5/12, approximately. The individual members of each chamber share these amounts equally, with the exception of the presiding officers. Under our "modified" scheme they each get about 30% of the power of an ordinary member; under the "strict" scheme, about 60%. In brief, then, the power indices for the three bodies are in the proportion 5:5:2. The indices for a *single* congressman, a *single* senator, and the President are in the proportion 2:9:350.

In a multicameral system such as we have just investigated, it is obviously easier to defeat a measure than to pass it.[6] A coalition of senators, sufficiently numerous, can block passage of any bill. But they cannot push through a bill of their own without help from the other chamber. This suggests that our analysis so far has been incomplete—that we need an index of "blocking power" to supplement the index already defined. To this end, we could set up a formal scheme similar to the previous one, namely: arrange the individuals in all possible orders and imagine them casting *negative* votes. In each arrangement, determine the person whose vote finally defeats the measure and give him credit for the block. Then the "blocking power" index for each person would be the relative number of times that he was the "blocker."

Now it is a remarkable fact that the new index is exactly equal to the index of our original definition. We can even make a stronger assertion: *any scheme for imputing power among the members of a committee system either yields the power index defined above or leads to a logical inconsistency.* A proof, or even a precise formulation, of this assertion would involve us too deeply in mathematical symbolism for the purposes of the present paper.[7] But we can conclude

[5] In the formal sense described above.

[6] This statement can be put into numerical form without difficulty, to give a quantitative description of the "efficiency" of a legislature.

[7] The mathematical formulation and proof are given in L. S. Shapley, "A Value for N-Person Games," *Annals of Mathematics Study No. 28* (Princeton, 1953), pp. 307–17. Briefly stated, any alternative imputation scheme would conflict with either *symmetry*

that the scheme we have been using (arranging the individuals in all possible orders, etc.) is just a convenient conceptual device; the indices which emerge are not peculiar to that device but represent a basic element of the committee system itself.

We now summarize some of the general properties of the power index. In pure *bi*cameral systems using simple majority votes, each chamber gets 50% of of the power (as it turns out), regardless of the relative sizes. With more than two chambers, power varies inversely with size: the smallest body is most powerful, etc. But no chamber is completely powerless, and no chamber holds more than 50% of the power. To illustrate, take Congress without the provision for overriding the President's veto by means of two-thirds majorities. This is now a pure tricameral system with chamber sizes of 1, 97, and 435. The values come out to be slightly under 50% for the President, and approximately 25% each for the Senate and House, with the House slightly less than the Senate. The exact calculation of this case is quite difficult because of the large numbers involved. An easier example is obtained by taking the chamber sizes as 1, 3, and 5. Then the division of power is in the proportions 32:27:25. The calculation is reproduced at the end of this paper.

The power division in a multicameral system also depends on the type of majority required to pass a bill. Raising the majority in *one* chamber (say from one-half to two-thirds) increases the relative power of that chamber.[8] Raising the required majority in all chambers simultaneously weakens the smaller house or houses at the expense of the larger. In the extreme case, where unanimity is required in every house, each individual in the whole legislature has what amounts to a veto, and is just as powerful as any other individual. The power index of each chamber is therefore directly proportional to its size.

We may examine this effect further by considering a system consisting of a governor and a council. Both the governor and some specified fraction of the council have to approve a bill before it can pass. Suppose first that council approval has to be unanimous. Then (as we saw above) the governor has no more power than the typical councilman. The bicameral power division is in the ratio 1:N, if we take N to be the number of councilmen. If a simple majority rule is adopted, then the ratio becomes 1:1 between governor and council. That is, the governor has N times the power of a councilman. Now suppose that the approval of only one member of the council is required. This means that an individual councilman has very little chance of being pivotal. In fact the power division turns out to be N:1 in favor of the governor.[9] If

(equal power indices for members in equal positions under the rules) or *additivity* (power distribution in a committee system composed of two strictly independent parts the same as the power distributions obtained by evaluating the parts separately).

[8] As a general rule, if one component of a committee system (in which approval of all components is required) is made less "efficient"—i.e., more susceptible to blocking maneuvers—then its share of the total power will increase.

[9] In the general case the proportion is N−M+1:M, where M stands for the number of councilmen required for passage.

votes were for sale, we might now expect the governor's price to be N^2 times as high as the average councilman's.

Several other examples of power distribution may be given. The indices reveal the decisive nature of the veto power in the United Nations Security Council. The Council consists of eleven members, five of whom have vetoes. For a substantive resolution to pass, there must be seven affirmative votes and no vetoes. Our power evaluation gives 76/77 or 98.7% to the "Big Five" and 1/77 or 1.3% to the remaining six members. Individually, the members of the "Big Five" enjoy a better than 90 to 1 advantage over the others.

It is well known that usually only a small fraction of the stock is required to keep control of a corporation. The group in power is usually able to muster enough proxies to maintain its position. Even if this were not so, the power of stockholders is not directly proportional to their holding, but is usually biased in favor of a large interest. Consider one man holding 40% of a stock while the remaining 60% is scattered among 600 small shareholders, with 0.1% each. The power index of the large holder is 66.6%, whereas for the small holders it is less than 0.06% apiece. The 400:1 ratio in holdings produces a power advantage of better than 1000:1.[10]

The preceding was an example of a "weighted majority game." Another example is provided by a board with five members, one of whom casts two extra votes. If a simple majority (four out of seven votes) carries the day, then power is distributed 60% to the multivote member, 10% to each of the others. To see this, observe that there are five possible positions for the strong man, if we arrange the members in order at random. In three of these positions he is pivotal. Hence his index is equal to 3/5. (Similarly, in the preceding example, we may compute that the strong man is pivotal 400 times out of 601.)

* * *

The values in the examples given above do not take into account any of the sociological or political superstructure that almost invariably exists in a legislature or policy board. They were not intended to be a representation of present day "reality." It would be foolish to expect to be able to catch all the subtle shades and nuances of custom and procedure that are to be found in most real decision-making bodies. Nevertheless, the power index computations may be useful in the setting up of norms or standards, the departure from which will serve as a measure of, for example, political solidarity, or regional or sociological factionalism, in an assembly. To do this we need an empirical power index, to compare with the theoretical. One possibility is as follows: The voting record of an individual is taken. He is given no credit for being on the losing side of a vote. If he is on the winning side, when n others voted with him, then he

[10] If there are two or more large interests, the power distribution depends in a fairly complicated way on the sizes of the large interests. Generally speaking, however, the small holders are better off than in the previous case. If there are two big interests, equal in size, then the small holders actually have an advantage over the large holders, on a power per share basis. This suggests that such a situation is highly unstable.

is awarded the probability of his having been the pivot (or blocker, in the case of a defeated motion), which is $1/n+1$. His probabilities are then averaged over all votes. It can be shown that this measure gives more weight than the norm does to uncommitted members who hold the "balance of power" between extreme factions. For example, in a nine-man committee which contains two four-man factions which always oppose each other, the lone uncommitted member will always be on the winning side, and will have an observed index of 1/5, compared to the theoretical value of 1/9.

A difficulty in the application of the above measure is the problem of finding the correct weights to attach to the different issues. Obviously it would not be proper to take a uniform average over all votes, since there is bound to be a wide disparity in the importance of issues brought to a vote. Again, in a multicameral legislature (or in any more complicated system), many important issues may be decided without every member having had an opportunity to go on record with his stand. There are many other practical difficulties in the way of direct applications of the type mentioned. Yet the power index appears to offer useful information concerning the basic design of legislative assemblies and policy-making boards.

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APPENDIX

The evaluation of the power distribution for a tricameral legislature with houses of 1, 3, and 5 members is given below:

There are 504 arrangements of five X's, three O's, and one ϕ, all equally likely if the nine items are ordered at random. In the following tabulation, the numbers indicate the number of permutations of predecessors ( ) and successors [ ] of the final pivot, marked with an asterisk. The dots indicate the pivots of the three separate houses.

$$\left.\begin{array}{ccccccccc} O & \dot{O} & O & X & X & \dot{\phi} & \dot{X} & X & X \\ & & (60) & & & & * & [1] & \\ O & \dot{O} & X & X & \dot{\phi} & \dot{X} & O & X & X \\ & & (30) & & & * & & [3] & \end{array}\right\} \begin{array}{l}\text{150 pivots}\\ \text{for X}\end{array}$$

$$\left.\begin{array}{ccccccccc} O & X & X & \dot{X} & X & X & \dot{\phi} & \dot{O} & O \\ & & (42) & & & & & * & [1] \\ O & X & X & \dot{X} & X & \dot{\phi} & \dot{O} & O & X \\ & & (30) & & & & * & [2] & \\ O & X & X & \dot{X} & \dot{\phi} & \dot{O} & O & X & X \\ & & (20) & & & * & & [3] & \end{array}\right\} \begin{array}{l}\text{162 pivots}\\ \text{for O}\end{array}$$

$$\left.\begin{array}{ccccccccc} O & \dot{O} & O & X & X & \dot{X} & X & X & \dot{\phi} \\ & & & (56) & & & & & * \\ O & \dot{O} & O & X & X & \dot{X} & X & \dot{\phi} & X \\ & & & (35) & & & & * & [1] \\ O & \dot{O} & O & X & X & \dot{X} & \dot{\phi} & X & X \\ & & & (20) & & & * & [1] & \\ O & \dot{O} & X & X & \dot{X} & X & X & \dot{\phi} & O \\ & & & (21) & & & & * & [1] \\ O & \dot{O} & X & X & \dot{X} & X & \dot{\phi} & O & X \\ & & & (15) & & & * & [2] & \\ O & \dot{O} & X & X & \dot{X} & \dot{\phi} & O & X & X \\ & & & (12) & & * & & [3] & \end{array}\right\} \begin{array}{l}\text{192 pivots}\\ \text{for } \phi\end{array}$$

Power indices for the houses are 192/504, 162/504, and 150/504, and hence are in the proportion 32:27:25, with the smallest house the strongest. Powers of the individual members are as 32:9:9:9:5:5:5:5:5.