



Legal, political science and economics approaches to measuring malapportionment

Journal:	<i>Social Science Quarterly</i>
Manuscript ID	Draft
Wiley - Manuscript type:	Original Article
Keywords:	Malapportionment; Electoral College; Total Population Deviation; Gallagher Index; Gini Index; Elections; Representation; One per
Abstract:	<p>We compare and contrast methods for measuring malapportionment from different disciplines: law, political science, and economics. For example, in political science, the comparative politics approach to measuring malapportionment has been in terms of an adaptation of standard measures of seats-votes discrepancy, such as the \textit{Loosemore-Hanby} Index of Distortion. This approach examines malapportionment levels as a whole, taking into account the differences in each of the districts between actual population and ideal population. In contrast, U.S. courts, and courts in many other countries, including Canada, France and Germany, measure compliance with the one person, one vote standard using what is called the total population deviation, which looks only at the two extreme poles of malapportionment. In economics, the Gini index is the standard approach to measuring inequality; we adapted it to the malapportionment context. We demonstrate that which approach to measurement we take can dramatically affect some of the conclusions we reach, illustrating this point with data from the U.S. House, Senate, and Electoral College over the period 1790-2010. We demonstrate that the House and the Electoral College hardly malapportioned, regardless of which measure we use, while the level of malapportionment we observe in the Senate can depend on which measure we use.</p>
<p>Note: The following files were submitted by the author for peer review, but cannot be converted to PDF. You must view these files (e.g. movies) online.</p>	
<p>fig1ab.svg fig1cd.svg fig1ef.svg fig1g.svg fig_changeEC.svg fig_changeHouse.svg fig_changeSen.svg</p>	



Legal, political science and economics approaches to measuring malapportionment

The U.S. House, the Senate and the Electoral College 1790-2010.

Abstract: *We compare and contrast methods for measuring malapportionment from different disciplines: law, political science, and economics. For example, in political science, the comparative politics approach to measuring malapportionment has been in terms of an adaptation of standard measures of seats-votes discrepancy, such as the Loosemore-Hanby Index of Distortion. This approach examines malapportionment levels as a whole, taking into account the differences in each of the districts between actual population and ideal population. In contrast, U.S. courts, and courts in many other countries, including Canada, France and Germany, measure compliance with the one person, one vote standard using what is called the total population deviation, which looks only at the two extreme poles of malapportionment. In economics, the Gini index is the standard approach to measuring inequality; we adapted it to the malapportionment context. We demonstrate that which approach to measurement we take can dramatically affect some of the conclusions we reach, illustrating this point with data from the U.S. House, Senate, and Electoral College over the period 1790-2010. We demonstrate that the House and the Electoral College hardly malapportioned, regardless of which measure we use, while the level of malapportionment we observe in the Senate can depend on which measure we use.*

Introduction

A view supported by some distinguished political scientists (see e.g., Lee and Oppenheimer 1999; Dahl 2003) and repeated by journalists (see e.g., Badger 2016), is that the U.S. Senate is inherently undemocratic because of the equal weight given to each state in the Senate despite the vast discrepancy in population across the states. Similarly, it is part of the common wisdom that the Electoral College (EC) is currently highly malapportioned because of its two-seat bonus based on Senate seats overweights small states (see e.g., Toles 2018). In contrast, though there are some features of House apportionment that keep it from perfect proportionality, in the post-*Baker v. Carr*, 369 U.S. 186, 1962 era (Ladewig and Jasinski 2008; Ladewig and McKee 2014) the House is nonetheless regarded as providing a level of representation matching population,¹ and the same assessment is generally made for state legislative appor-

¹ Even the most proportional of allocation methods require rounding into integer values or, as is the case in the U.S. for apportioning the House, may have guaranteed seats for particular types of units regardless of their population. Such rules can create a discrepancy between apportioned seats and actual state population shares. While today's House districts are almost identical in population to one another within any given state, the combination of apportionment rounding rules (the so-called integer allocation problem, Balinski and Young 1982) and the rule that no state can be denied a seat in the House of Representatives regardless of its population, introduces malapportionment into the U.S. House when calculated nationally and not for each state separately.

tionment.² In this essay, we examine these views empirically by comparing malapportionment in the U.S. House, the U.S. Senate and the U.S. Electoral College over the period 1790-2010 by examining multiple metrics coming from law (e.g., the *total population deviation*), political science (e.g., the *Gallagher Index*, Gallagher 1991, and the *Loosemore-Hanby Index*, Loosemore and Hanby 1971), and economics (e.g., the *Gini coefficient*, Lorenz 1905). In order to make our malapportionment measures across the three institutions comparable, we use states as our units. This means that, when we examine the U.S. House of Representatives, we are not interested in questions of within-state variation in district population pre and post-*Baker v. Carr* or in the manipulation of district populations for partisan purposes (Grofman 1990; Engstrom 2013; McGann et al. 2016).

The most common metrics used by U.S. courts to measure malapportionment across individual districts looks at just two districts, the one most underrepresented and the one most over-represented. Our adaption to the malapportionment context of seats-votes measures used in the electoral systems literature look at the full distribution of population values and electoral weights, as does our adaption of the measure of inequality from economics, the *Gini coefficient*. We show that apportionment equality in the Electoral College looks far more like apportionment equality for the U.S. House of Representatives than it looks like that in the U.S. Senate, regardless of which metric we use. Indeed, when we evaluate EC malapportionment using the two metrics that are most common in the electoral systems literature to measure seats-votes disproportionality, this analysis leads us to the conclusion that the EC behaves with respect to seats to population comparisons much like a proportional representation system does with respect to seats to votes comparisons, though with much smaller deviations. Similarly, a *Gini-index* based measure of EC malapportionment suggests very little bias when we look only at seats share to population comparisons, especially as compared to the vast discrepancies we observe when we look at income distributions.

Moreover, we also show that for both the House and the EC, the historical time-series data on the magnitude of the malapportionment over the period 1790–2010 is very flat regardless of which measure we use, with some measures of the EC even showing a very minor downtrend in recent elections. In contrast, different metrics lead us to quite different perceptions of changes over time in malapportionment in the Senate. All measures, however, agree that the Senate is far more malapportioned than either the House or the EC (Ladewig and Jasinski 2008).

In the U.S., while malapportionment bias is often regarded as inherently undesirable from a normative perspective, it was the perceived effects on government policies stemming from under-representation of city dwellers within states that motivated much of the sentiment that agitated pre-*Baker v. Carr* reformers (see e.g. *Baker v. Carr*, 369 U.S. 186, 1962, *Reynolds v. Sims*, 377 U.S. 533, 1964) (Baker 1955; McCubbins and Schwartz 1988).³ And, today, while there remains concern for malapportionment in the Senate, the practical foci of current reformers are, on the one hand, on ways to

² Malapportionment across states can also occur for the U.S. House of Representatives when Congress fails to fulfill its decennial duty to reapportion the House in accord with new population data. After the 1920 Census, Congress failed to reapportion the House. (see e.g. National Archives – Pieces of History).

³ Relatedly, the failure to reapportion after the 1920 Census was brought about because of reluctance to transfer seats from more rural states whose population was falling, in relative terms, to heavily urban states with growing populations.

control partisan gerrymandering within states and, on the other hand, the perceived partisan bias in the EC that is now operating in a pro-Republican direction that leads reformers to seek to replace the EC with a popular vote mechanism for choosing the President or to find other mechanisms that will limit divergence between popular vote and EC outcome.

But malapportionment, in and of itself, may or may not have direct pernicious consequences for the treatment of particular political parties or cognizable groups of voters with distinct interests. For an international example, Singapore has high levels of parliamentary malapportionment, but that malapportionment does not appear to have effects that favor the ruling party, the PAP (Tan and Grofman 2018). In contrast, malapportionment in Japan has historically favored rural areas by over-representing rural voters, and thus been a boon to the dominant party in Japan, the LDP, whose greatest strength derived from rural voters Moriwaka (2008).⁴

Addressing the partisan consequences of malapportionment is, however, outside the scope of this research note. Here we pursue a straightforward and more limited task: assessing the levels of malapportionment in the House, Senate and Electoral College over time and under several different metrics. We regard this as a worthwhile investigation regardless of a linkage (or absence of linkage) between malapportionment and the success of Democratic and Republican candidates for the various offices. We share the view of Dahl (1971) that the “one person, one vote” principle is a necessary component of democratic governance and that, as Taagepera and Shugart (1989) have trenchantly put it, malapportionment is “a pathology”. But if we are to assess malapportionment we need to know how best to measure it, and we need to recognize when different approaches to measuring malapportionment can yield us very different conclusions about its level.

The structure of the rest of this essay is straightforward. First we introduce the definitions of the seven measures (total population deviation, ratio of largest to smallest district, proportion of population in units with enough seats to command a majority, the Gini Index, 80/20 percentile rank ratio, Loosemore-Hanby index, and Gallagher index). Then we provide graphs showing the empirical values of these seven indices over the period 1790-2010 for the U.S. House, the Senate, and the Electoral College, with some additional information about exactly how values in the various graphs were ascertained. Then we discuss the implications of our findings for both present-day malapportionment and the historical changes in malapportionment levels in the three national U.S. electorally determined institutions, including a discussion of whether malapportionment in these three institutions has move in synchrony among them.

⁴ Stewart and Weingast (1992) show that in the 19th century “Republicans had manufactured [an] advantage through the strategic admittance of sparsely populated, but strongly Republican, western states. These western “pocket” boroughs provided Republicans with a head start in the Electoral College, and an almost insurmountable lock on the Senate” (as cited in Engstrom (2013, pg. 94)). Moreover, there are other types of effects that malapportionment might produce in addition to direct effects on party representation. Samuels and Snyder (2001, pg. 653), reviewing a number of single country studies, concludes “malapportionment can have an important impact on executive-legislative relations, intra-legislative bargaining and the overall performance of democratic systems.”

Measuring Malapportionment

Regardless of whether or why we regard malapportionment as problematic, logically prior is the question: “How do we measure malapportionment?”. While there is a ‘zoo’ of potential measures (Taagepera and Grofman 2003), we focus on a select few which are preeminent in the scholarly and legal literature. It is well recognized that no single measure of disproportionality can capture every feature of interest, and each measure has some desirable properties and some flaws (Cox and Shugart 1991; Monroe 1994; Taagepera, Selb, and Grofman 2014).⁵ Our focus will be empirical, looking at the historical measurement of U.S. political institutions (the House of Representatives, the Senate, and the Electoral College) and what the different measures say about long-term trends and overall magnitudes of disproportionality in each over the period 1790-2010.⁶ Our measures will compare the most common measure used in U.S. Courts, along with other measures proposed by political scientists pre-*Baker v. Carr*, with applications to the population context of those found more recently in the comparative politics literature measuring vote-seat disparities, and of the two common measures of inequality in the economics literature. The degree of concordance among some of our measures is, we believe, rather surprising, as are the results about which measures are most in disagreement with other measures, and how the degree of disagreement among measures varies across the three institutions.

For simplicity of exposition, we present below definitions of all four measures used by courts or proposed by early reformers for use by U.S. courts for the case of single seat constituencies.⁷ Let p_i = persons in the i_{th} constituency, P = total population = $\sum_{i=1}^n p_i$, \hat{p} = ideal population, i.e., the total population divided by the total number of seats n . Constituencies are indexed by $i = 1, \dots, n$. p_{max} is the district in the constituency with the largest population, p_{min} is the district in the constituency with the smallest population.⁸ Because the central concern that motivated this paper was malapportionment in the Electoral College, as noted earlier, for all our calculations we use states as the units. This means that our measure of congressional malapportionment only captures inter-state differences in mean population per House districts.⁹

⁵ The theoretical virtues of the different measures have been extensively investigated (Monroe 1994; Taagepera and Grofman 2003; Karpov 2008; Van Puyenbroeck 2008) and we will not try seek to contribute to the literature on axiomatic comparisons of equality measures. Similarly we will not seek to discuss which is best other than to note that we believe that different measures pick up different facets of inequality. Thus we disagree with Samuels and Snyder (2001) who reject the use of *total population deviation* as completely inappropriate. While we would not say that courts in multiple countries cannot be wrong, we are not willing to dismiss a court chosen measure of malapportionment out of hand. Instead we seek to compare empirically the results we get from different measures.

⁶ The issue of malapportioned voting units can be traced at least as far back as the Roman Republic, where voting was by units based on income level, with the wealthy greatly over-represented (Manin 1997).

⁷ There are many complexities in defining malapportionment when we move from simple single-seat systems to countries with multi-seat districts and/or a mix of single and multiple seat districts, and/or a tiered system with proportional allocations or compensatory seats in the upper tier (Samuels and Snyder 2001). Because the House and Senate are single seats constituencies, such complications arise only vis-a-vis the Electoral College. There, we weight states by their EC seat share.

⁸ Issues of whether to use measures based on something other than census based population counts, such as total citizen population or eligible voters take us to issues quite distinct from those considered in this paper. See *Evenwel v. Abbott* 578 U.S. __ (2015).

⁹ This is equivalent to taking constituency populations within a state to be identical.

Legal Measures of Malapportionment and early Political Science Approaches

The aftermath of *Baker v. Carr* (1962) initially led U.S. federal courts to consider a number of different ways to measure malapportionment (NCSL 2019).¹⁰ But, rather quickly, the U.S. Supreme Court focused on a particular measure, the *total population deviation* (TPD, Equation 1, also referred to as *Relative Deviation*), which looks at the relative difference in population between the most underpopulated and the most overpopulated district to the ideal district size. Among other measures initially proposed by political scientists (see esp. Baker 1966) are the *Max/Min* (Equation 2, sometimes called *population deviation ratio*, *overall range*, or *maximum deviation*), which is the ratio of the population in the largest district to that in the smallest;¹¹ the *minimum population share*, which identifies the minimum population needed to control a majority of seats in the legislature; and the *average absolute level of deviation* (Equation 3).

The last of these measures is mathematically equivalent to *Loosemore-Hanby* (Equation 4) of malapportionment; We will reserve discussion of it until the discussion of political science approaches to malapportionment, where we refer to it under the latter title.

The TPD measure is conceptually very simple, and like the other three measures it can be used to specify a *de minimis* threshold that can serve as a “bright-line” test for courts.¹² Virtually every other democracy which imposes some form of “one person, one vote” test on its parliamentary constituencies has also adopted a TPD based measure, though with widely differing thresholds, most far higher than the ones adopted in the U.S. (e.g., 30% in Germany and 50% in Canada). See (Grofman, Handley, and Grofman 2008) for a review of legal malapportionment thresholds in many countries.¹³

Total population deviation (TPD) =

$$\frac{(p_{max} - p_{min})}{\hat{p}} \quad (1)$$

The TPD is sometimes written as

$$\frac{\hat{p} - p_{min}}{\hat{p}} + \frac{\hat{p} - p_{max}}{\hat{p}}$$

The *Max/Min ratio* is simply the ratio of the largest to the smallest persons per district, with a ratio of 1 indicating no malapportionment.

Max/Min =

$$\frac{p_{max}}{p_{min}} \quad (2)$$

¹⁰ It also took a while for there to be definitive legal standards for what levels of malapportionment would be acceptable at different levels of government (NCSL 2019).

¹¹ Note that the *total population deviation* measures the absolute difference between seats and votes while the *population deviation ratio* is based on a ratio.

¹² Though often called the “total” population deviation (or variance), it might better be called the “maximum” population deviation since it only describes the relationship between the two most extreme units and nothing about the nature of malapportionment in the other constituencies.

¹³ However, the reader must be careful in interpreting reported thresholds. For example, the threshold in Germany is stated as no more than 15% upwards or downwards from the average, and those who write about Germany may thus correctly characterize it as a 15% tolerance limit but, in our terms, this is a 30% TPD value.

Average absolute deviation =

$$\sum \frac{|p_i - \hat{p}|}{n} \quad (3)$$

Finally, to find the *minimum population share* needed to control a majority of the seats in the legislature, for the case of single seat constituencies we order the districts from smallest to largest by population per district. We find the population of the units up to and including the pivotal unit (m) and then divide by the total population to obtain the proportion we seek. To calculate it for the Electoral College, we take the population of each unit in the EC to be equal to each state's population divided by its number of EC seats.¹⁴

Adapting Political Science Measures of Seats-Votes Discrepancy to the Malapportionment Context

In contrast to the measures used in courts, when students of politics study redistricting, they utilize instead measures of malapportionment adapted from the electoral systems literature on measuring the discrepancy between party vote share and party seat share (Samuels and Snyder 2001; Sauger and Grofman 2016, pg. 654).

The *Loosemore-Hanby Index of Distortion* (Equation 4, Loosemore and Hanby 1971) along with the closely related *Gallagher Index* (Equation 5, Gallagher 1991) are the two most common metrics used for measuring seats-votes disproportionality.¹⁵ *Loosemore-Hanby* measures the summed absolute differences between seats and votes, while *Gallagher's Index*, often referred to as a "Least Squares" measure, weights each observation by the size of the deviation, i.e., it squares the deviations. Squaring the deviations puts more weight on larger deviations, while discounting smaller ones. The analogues to these two disproportionality indices in the malapportionment context are shown below.

Loosemore-Hanby Index =

$$\frac{1}{2} \sum_{i=1}^n |p_i - \hat{p}| \quad (4)$$

Gallagher Index =

$$\sqrt{\frac{1}{2} \sum_{i=1}^n (p_i - \hat{p})^2} \quad (5)$$

Adapting Economic Measures of Equality to the Study of Malapportionment

The economists use of measures of inequality is most commonly found in the study of income inequality (Yntema 1933; Foster 1985; Atkinson 1970; Bai and Lagunoff

¹⁴ This metric has also been labeled as the "electoral percentage" (Dixon 1968) and the *theoretical control index*, (Grofman and Scarrow 1981). Sometimes the resulting vote proportion is divided by two in order to indicate that only a majority of the votes in each constituency are needed to control the outcome in that constituency, i.e., a party that wins only the barest of majorities in a bare majority of seats in two-party competition can win the election. We will not make use of this normalizing divisor.

¹⁵ There are many other measures that have been proposed.

2013). A standard approach in the economic literature on inequality is to report *fractiles* or *percentile ratios*, e.g., the proportion of income held by, say, the richest 80% of the population divided by the proportion of income held by the poorest 20% of the population. Similarly, we can find the ratio of seat shares to population shares at the 20th and 80th percentiles.¹⁶ We have chosen to measure the 80th and 20th percentiles for the tables and figures presented in the empirical section of the paper, but the 80/20 ratio is only intended to be illustrative. It is but one of many ratios we might have used.

The percentile method provides just a crude understanding of malapportionment. The *Lorenz curve*, a graphical tool for displaying inequality first proposed in 1905 by Lorenz (1905) is the natural way to summarize the entire distribution. On a two-dimensional scatterplot, plot the cumulative percentages of the population, on one axis and the cumulative share of income arranged from lowest to highest on the other. This is the *Lorenz curve*. Where all points on the plot are identical, $x = y$, a straight line is drawn, often called the *line of equality*. That is, the top k% of the population holds k% of the income. To provide a single measure derived from a Lorenz curve the *Gini coefficient* is used. It is defined as the ratio of the area of the cumulative frequency distribution and the area below the *line of equality*.¹⁷ Similarly, for a legislature or for the Electoral College, we can plot cumulative population share versus cumulative seat share.¹⁸

To create a *Lorenz curve*, order districts such that $v_1 \leq v_2 \leq v_n$ where each district $i = 1, \dots, n$ gets allocated its share of V , the total vote-share. Individual shares are $v_i = p_i/V$ and $1 = \sum_{i=1}^n p_i$. The cumulative proportion of V is then plotted on the x-axis and the cumulative population share on the y-axis. The points start with (0,0) and end at (1,1).

Empirical Comparisons of Six Measures of Malapportionment

Natural questions to ask are: “How much and in what ways does the choice of malapportionment measure chosen affect the conclusions we reach about level of malapportionment?”, “How have malapportionment levels in the three institutions (the U.S. House of Representatives, the U.S. Senate, and the U.S. Electoral College) we study changed over time?” and, “Are there measures that, while appearing distinct mathematically, tend to give similar answers?”. We will address these questions with U.S. Census data over the period 1790 to 2010.

¹⁶ The ratio approach in terms of percentile ranks like the ratio approach in terms of largest and smallest district population throws away some of the information about the shape of the distribution *in toto*.

¹⁷ This area allows for meaningful comparisons among Lorenz curves which intersect. It can be found through interpolation if we have actual data or can be calculated analytically for different assumed distributional shapes.

¹⁸ In the context of economic inequality, the *Gini coefficient* been called the single best measure of inequality (Morgan 1962), but, as noted earlier, we will not attempt to judge measures normatively but rather to assess their degree of concordance when applied to important real world applications. Another approach to equality found in the economic literature is based on voting power using a game theory measure of power such as the *Shapley-Shubik* index or the *Banzhaf index* (Banzhaf 1965; Shapley and Shubik 1954). We will not consider this approach to inequality here.

We make several simplifying assumptions in order to compare across time. First, the District of Columbia is not included in either the U.S. House or Senate calculations, and its population is likewise subtracted from the national population figures we use. Only after Amendment XXIII was ratified in the 1960s giving D.C. three electoral votes (regardless of its population) is D.C. included in the Electoral College measures. After the ratification of this amendment we add D.C. population to the national population total we use for malapportionment calculations for the EC only.¹⁹ Second, even though different apportionment methods have been used in different census decades, we calculate apportionment using the Hill-Huntington method (*Method of Equal Proportions*, used for apportioning the U.S. House of Representatives and Electoral College since 1941 so as to have consistency over time.²⁰ Third, even though the basis of apportionment has changed over time with respect to the inclusion/weighting of African-Americans and Native Americans,²¹ we use total population as the basis of apportionment for the U.S. House throughout the entire time period. Lastly, because we are most interested in comparisons at the state level in order to facilitate direct comparisons between the House, the Senate, and the Electoral College, despite severe intra-state malapportionment in the U.S. House prior to *Baker v. Carr*,²² as noted earlier, for the House we treat each district within a state as the state's population divided by the number of members in that state.²³ For all three of the institutions we study, we calculate the ideal population per seat as the total U.S. population divided by the total number of seats.²⁴

Figure 1 (a-g) show the comparisons across the three institutions of single-member districts for each of our seven metrics.

¹⁹ For the purposes of this essay, we do not include the populations of U.S. territories (e.g., Puerto Rico). Though U.S. citizens, they currently do not have any voting representation in U.S. political institutions.

²⁰ After no apportionment in 1920 after a stalemate in Congress, reapportionment was resumed in 1930 and a rule was set in place that provided for automatic reapportionment after in each census in accord with a specified apportionment formula. While that formula was changed for the 1940 census, and a still different formula had been used early in the nation's history, the differences in allocation across apportionment formulae tend to be minor (see Balinski and Young 1982, cf. Janson and Linusson 2012). Though no apportionment was actually done in 1920, we provide the hypothetical 1920 apportionment from the Census population using the Hill-Huntington method.

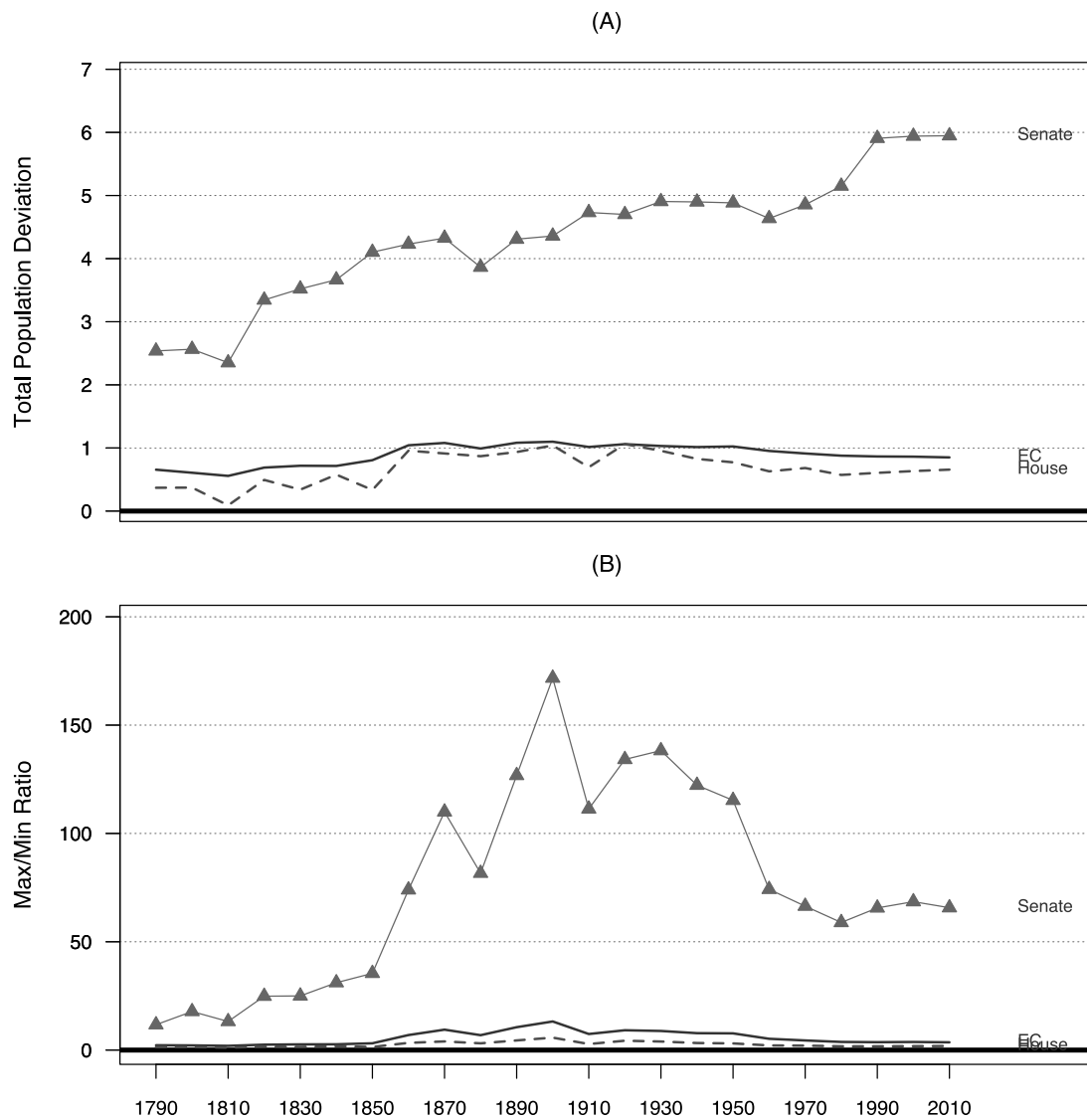
²¹ Article 1, Section 2, Clause 3 of the U.S. Constitution says "Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons." Amendment XIV repealed this provision, requiring "representative shall be apportioned . . . counting the whole number of persons in each State. . .".

²² See e.g., Altman (1998); Ladewig and Jasinski (2008); Engstrom (2013)

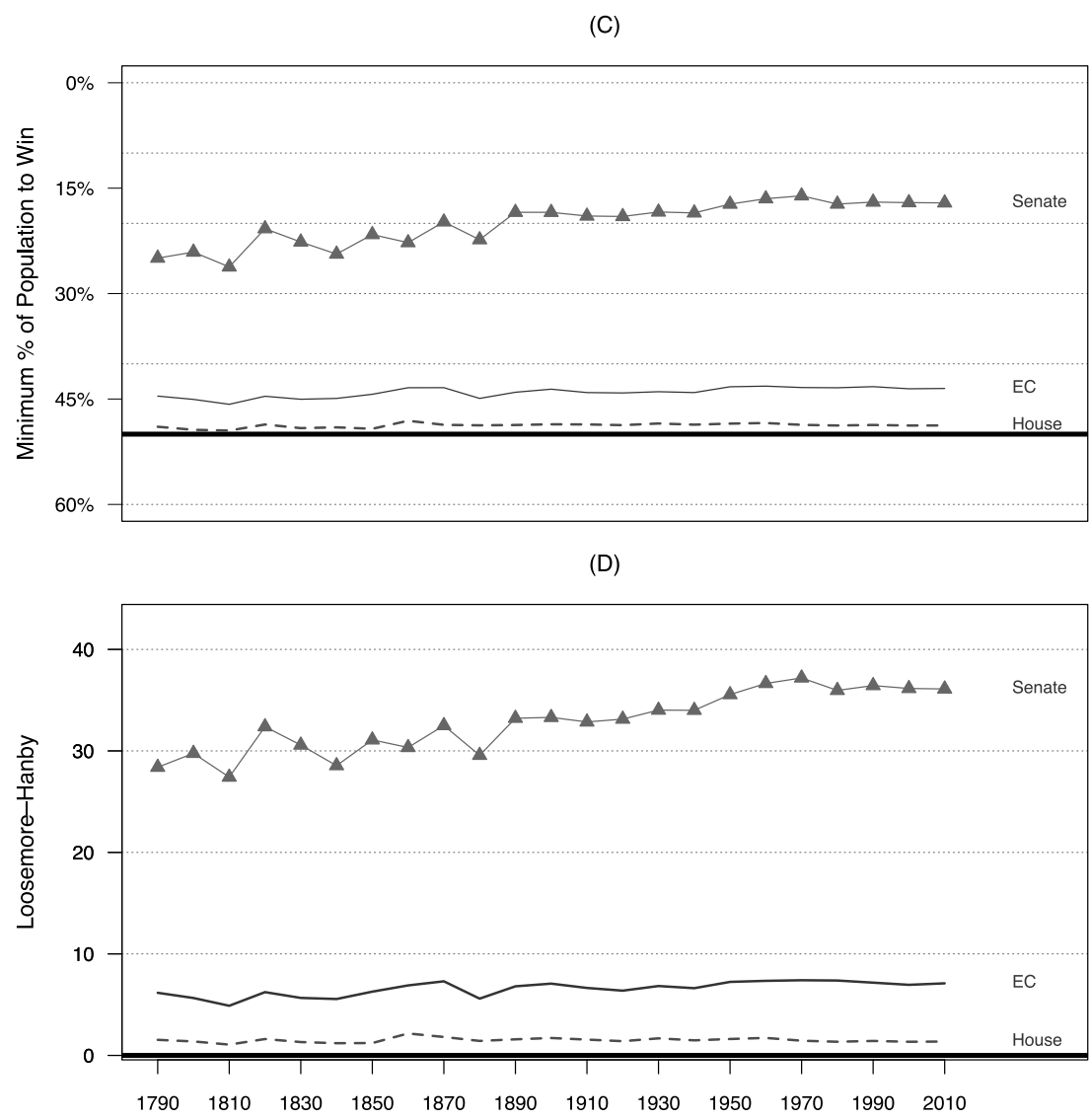
²³ With the exception of Maine and Nebraska, states currently award the state's total Electoral College votes based on the state-wide plurality winner. We use this unit-rule for all states over the entire period.

²⁴ Two for each state for the U.S. Senate, and for recent decades, 435 for the U.S. House and 538 for the EC. In effect, as noted earlier, we treat the House districts in each state as having an identical population, namely the population of the state divided by the number of House seats allocated to that state.

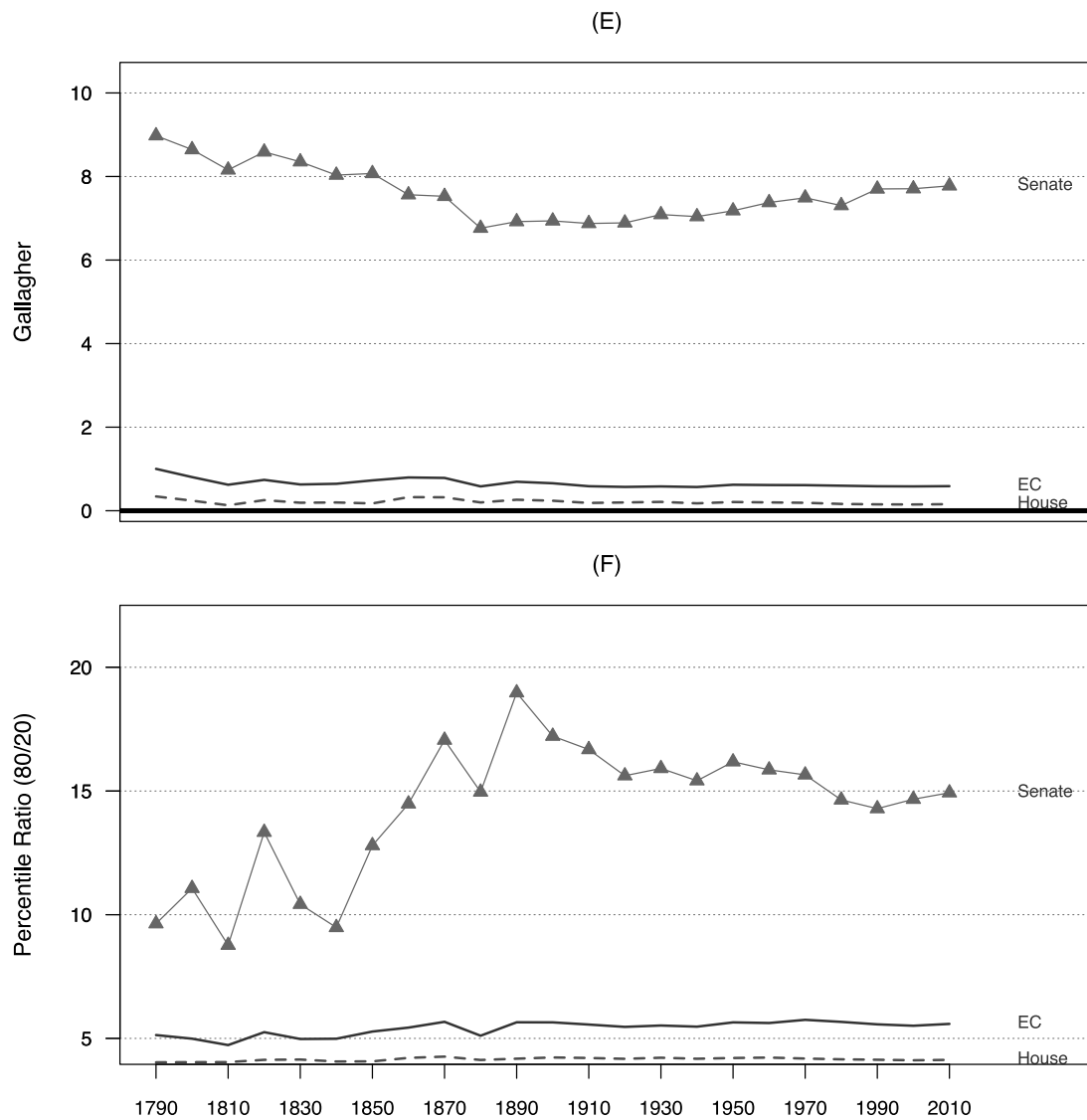
FIGURE 1 Change over time in Measures of Malapportionment: 1790-2010



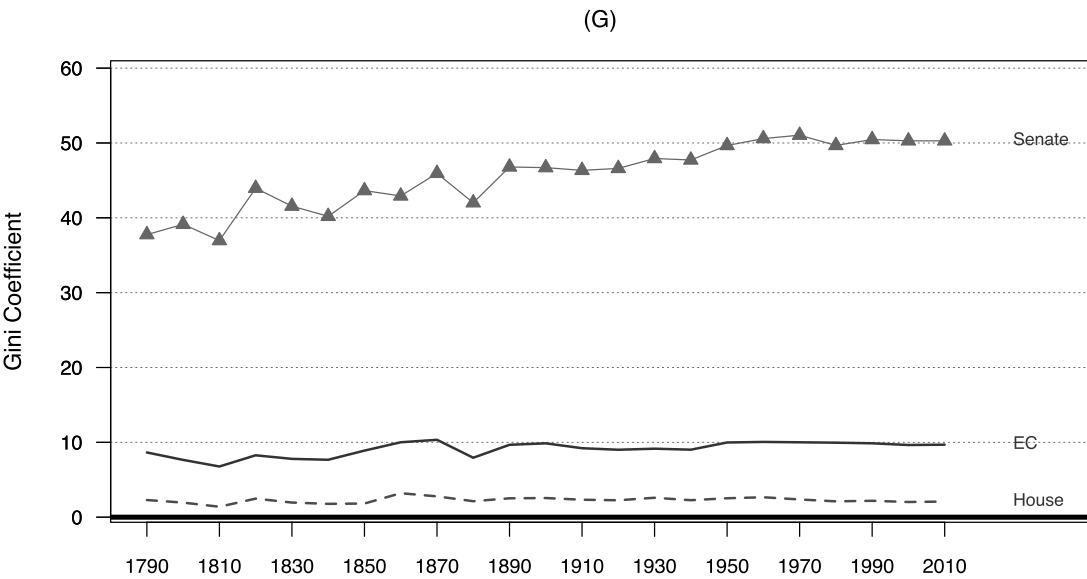
Continued... Change over time in Measures of Malapportionment: 1790-2010



Continued... Change over time in Measures of Malapportionment: 1790-2010



Continued... Change over time in Measures of Malapportionment: 1790-2010



All seven of the metrics in Figure 1 support the claim that EC malapportionment is far closer to low levels of House malapportionment than it is to the high levels of U.S. Senate malapportionment.²⁵ Indeed, if we compare the most recent values we get for those measures to their equivalents in the seats-votes disproportionality context, both the U.S. House and even the U.S. Electoral College exhibit low levels of disproportionality. The numbers shown in Figures 1 d & e, while not as small as the party-based disproportionalities reported for the most highly proportional electoral rules in use world-wide, those of Netherlands²⁶, and Israel²⁷ are comparable to the partisan disproportionalities in other western European democracies. For example, tabulating data from Döring and Manow (2017, Table 3: pg. 159) shows that proportional countries have an average *Gallagher* value of 3.89 and majoritarian countries average 12.12. The U.S. House in 2010 was 0.157, the EC was 0.588, and the Senate 7.77.

All seven measures also show a relatively flat pattern of malapportionment for the House and the EC, in general and especially over the past several decades. While, as noted above, the U.S. Senate is far and away the most disproportionate of the three institutions under all measures, unlike what we find for the House and the EC, there are substantial differences across the measures in the over time pattern of Senate malapportionment. For example, the *Total Population Deviation* metric shows the Senate rather steadily exhibiting ever higher levels of malapportionment, though recently leveling off; the *Gini coefficient* and the *Loosemore-Hanby index* show a similar upward pattern, but not as steep, as does the *minimum population share*.²⁸ But the *Max/Min ratio*, in contrast, shows a cyclical pattern, albeit one with the present values still considerably higher than those in the United States' earliest history; the same is true for the 80/20 ratio, though more extreme than it is for 80/20 percentile ratio. Finally, the *Gallagher index* shows first a gradual fall and then a more gradual rise.

We show in Table 1 correlations across our seven measures.

²⁵ For the *total population deviation*, as a matter of mathematical necessity, as long as the most over-represented and most under-represented state in the EC is the same as their counterpart in the U.S. House, malapportionment in the EC must be larger than malapportionment in the U.S. House.

²⁶ 2017 Dutch Election: *Loosemore-Hanby* – 1.3, *Gallagher* – 6. Data source: <https://www.kiesraad.nl/>

²⁷ 2015 Knesset Election: *Loosemore-Hanby* – 1.5, *Gallagher* – 7. Data source: <https://www.knesset.gov.il>

²⁸ While voting majorities are not typically determined by state size, this finding rises the prospects of significantly less than 50% of the population controlling the majority of votes in the U.S. Senate.

TABLE 1 Correlations for Seven Measures of Malapportionment

	TPD	Max/Min	Minimum Winning Population	Loosemore Hanby	Gallagher	Percentile Ratio (80/20)	Gini Index
Total Population Deviation	1						House E.C. Senate
Max/Min Ratio	0.90 0.90 0.52	1					House E.C. Senate
Min Winning Pop	0.77 0.66 0.88	0.56 0.41 0.59	1				House E.C. Senate
Loosmoore-Hanbly	0.63 0.65 0.86	0.57 0.43 0.44	0.83 0.98 0.98	1			House E.C. Senate
Gallagher	0.29 -0.33 -0.58	0.34 -0.20 -0.87	0.32 -0.19 -0.59	0.71 -0.16 -0.45	1		House E.C. Senate
Percentile Ratio (80/20)	0.77 0.77 0.65	0.71 0.58 0.85	0.80 0.93 0.79	0.72 0.97 0.67	0.20 -0.27 -0.81	1	House E.C. Senate
Gini	0.71 0.72 0.92	0.60 0.50 0.55	0.89 0.98 0.99	0.98 0.98 0.98	0.66 -0.09 -0.58	0.78 0.95 0.75	House 1 E.C. Senate

Note: The top entry is for the **House**, the middle entry is the **E.C.**, and the bottom is the **Senate**

INSERT TABLE 1 ABOUT HERE

Since the first two of our measures, *TPD* and *Max/Min*, focus on the same two states (the largest and smallest), we might expect that these two disproportionality measures should correlate highly with one another. They are quite highly correlated for both the U.S. House and the Electoral College, with a correlation of $r = 0.90$ for each institution. But, the same is not true for the U.S. Senate, as the correlation between these two measures is much lower, though still positive at $r = 0.52$. The reduced correlation in the Senate between the two measures is due, we believe, to the admission of extremely small states into the union in the mid nineteenth century (Stewart and Weingast 1992; Engstrom 2013), since the ratio measure is even more strongly dependent on extreme values than the difference measure.²⁹

Minimum winning population is strongly correlated with the *Gini coefficient* for all three institutions, as is *Loosemore-Hanby*. Similarly, *Loosemore-Hanby* and the *Minimum winning population* are also highly correlated with each other. Similarly, and rather unexpectedly, *TPD* is also highly correlated with all three of these other measures. Thus, whatever the differences in the axiomatic properties of these four metrics, in practice, at least for the historical data on the three U.S. institutions we examine, they tend to move in parallel. In factor analytic terms, these four measures scale on the same dimension. We find a similar pattern of correlation for the 80/20 ratio but, despite the high linear correlations, when we examine Figure 1 we see that, for the Senate, it yields a non-monotonic pattern and thus we should be cautious of the results of linear correlation analysis.

In contrast, *Gallagher* and *Max-Min ratio* exhibit divergent patterns with the other measures for some of our three institutions. We might have expected *Loosemore-Hanby* and *Gallagher* to be highly correlated since they are very similar in mathematical form, but empirically their correlation varies by institution. For the U.S. House, they are highly correlated (0.71); for the EC, they are only slightly negatively correlated (-0.16); for the U.S. Senate, they are more negatively correlated (-0.45). More generally, The *Gallagher* measure is negatively correlated with all the other measures for both the Senate and the Electoral College, but positively correlated for the House. We believe that this difference across institutions reflects the discounting by *Gallagher* of the contribution to malapportionment of very small jurisdictions, a factor which plays a more important role in shaping values in the Senate and the EC than it does in the House.

Discussion

The goals of this essay have (1) been to address the magnitude of bias derived from the purely mechanical effects of rules determining the relationship between seat share and state population share in historical perspective for three important U.S. institutions, and (2) to assess the degree to which measures of inequality/disproportionality common in the disciplines of law, political science, and economics, when adapted to the malapportionment context, yield different answers to determining malapportionment inequality over time for the three institutions.

Our principal finding is a clear one: in practice, Electoral College malapportionment is not very much larger than malapportionment in the U.S. House of Represen-

²⁹ For instance, Nevada entered the union in 1860 with a total population of 6,857.

tatives and EC malapportionment is far lower than the malapportionment we find in the U.S. Senate.³⁰ Indeed, as we noted earlier in the text, the numbers for EC for the *Loosemore-Hanby* and *Gallagher* indices shown in Figure 1 d & e, while not as small as the party-based disproportionalities between votes and seats reported for the most highly proportional electoral rules in use world-wide, are comparable to the partisan votes-seats disproportionalities in other western European democracies.

Moreover, the analyses we present above enable us to demonstrate that, despite the freezing of the House size and the logical presumption that malapportionment in the EC should therefore increase, the discrepancy between popular vote outcome and EC outcome that occurred in 2000 and 2016 cannot be blamed on an increasing EC malapportionment in recent decades. Contemporary levels of EC malapportionment are, by virtually all measures, presently at or slightly below historical levels.

Our results fly in the face of the common wisdom about how badly malapportioned the EC supposedly is. The explanation for the mismatch between expectations and reality is that the states with greatest seats to population advantage in the EC do not make up a large share of the EC vote.

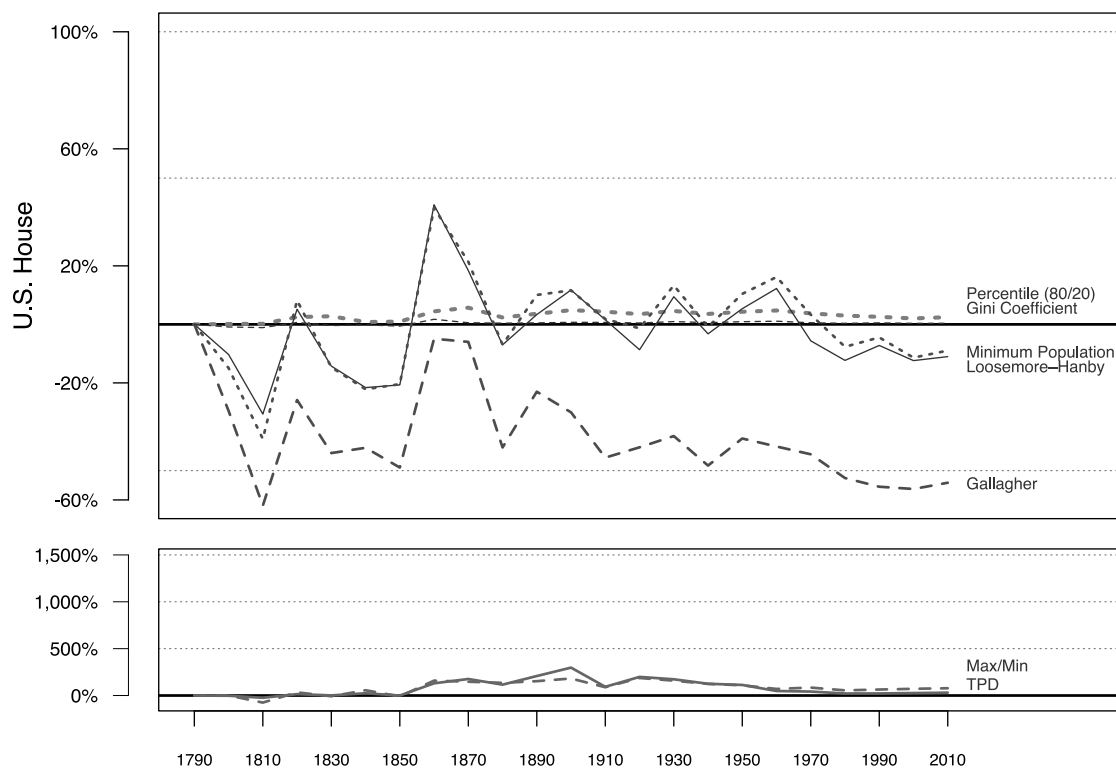
While malapportionment effects in the EC, like those for the House, seem minor, the results about population equality we get for the Senate, however, are quite different. The U.S. Senate presents a more serious challenge to our understanding of the majoritarian principle of democracy. When discussing one person, one vote; When a noted democratic theorist Dahl (2003) asks, “How Democratic Is the American Constitution?” at least with respect to malapportionment, it is the Senate rather than the Electoral College for which this question is most relevant.

But that is not to say that continued use of the Electoral College does not pose issues of political fairness. The basic reason why EC malapportionment effects are commonly overstated is the confusion between population based malapportionment and seats-votes disproportionality. To understand Electoral College effects, we need to distinguish the mechanical effects of the Electoral College that we may think of as “malapportionment related” (i.e., due to discrepancies between a state’s EC vote share and the state’s population or House delegation share), which arise simply because EC vote allocations equal the size of a state’s U.S. House delegation plus the size of the state’s U.S. Senate delegation, from effects that are tied to the *geographic* distribution of the votes across states in each election. The former applies throughout any given redistricting decade; the latter is election specific. The election specific effects can be substantial enough to generate a partisan bias that can lead to a divergence between popular vote majority winner and the winner of the Electoral College vote (Cervas and Grofman 2019).³¹

³⁰ The Electoral College may be regarded as essentially a mixture between an upper and a lower chamber, but far more closely resembling the latter. Samuels and Snyder (2001) offer analysis of malapportionment in a comparative perspective which shows that malapportionment levels in upper chambers are characteristically much greater than in lower chambers.

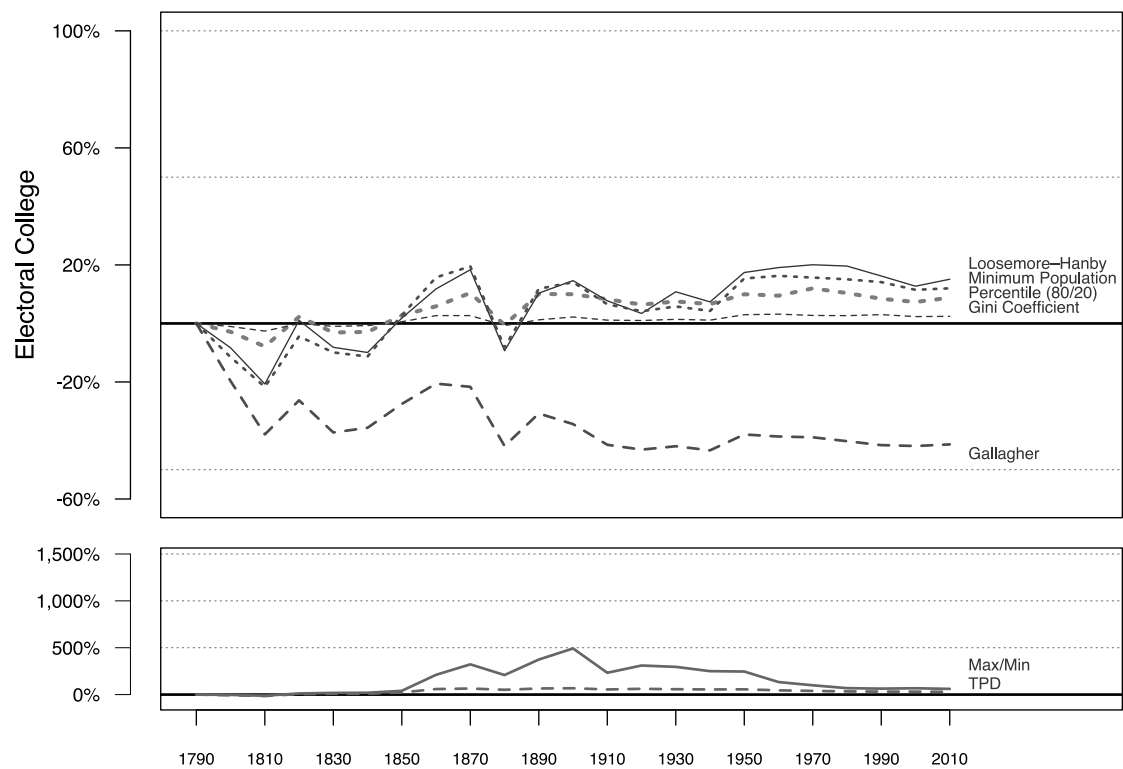
³¹ Evidence on this bias suggests that it has sometimes favored Democrats and sometimes favored Republicans (Grofman, Koetzle, and Brunell 1997; Pattie and Johnston 2014; Zingher 2016). In addition to the partisan distribution of voters across states, turnout differences among the states may also operate to bias outcome so as to create a discrepancy between the popular vote winner and the EC winner. A third factor that could matter is the size of the House. In 2000, as Neubauer and Zeitlin (2003) point out, a larger House size might have given the election to Gore but, given the magnitude of Trump’s EC victory, the House size would have to have been increased by an implausible amount in order to switch the EC outcome in 2016 (Cervas and Grofman 2019). Considering the relative importance of

FIGURE 2 Change Over Time in Measures of Malapportionment: 1790-2010 – U.S. House

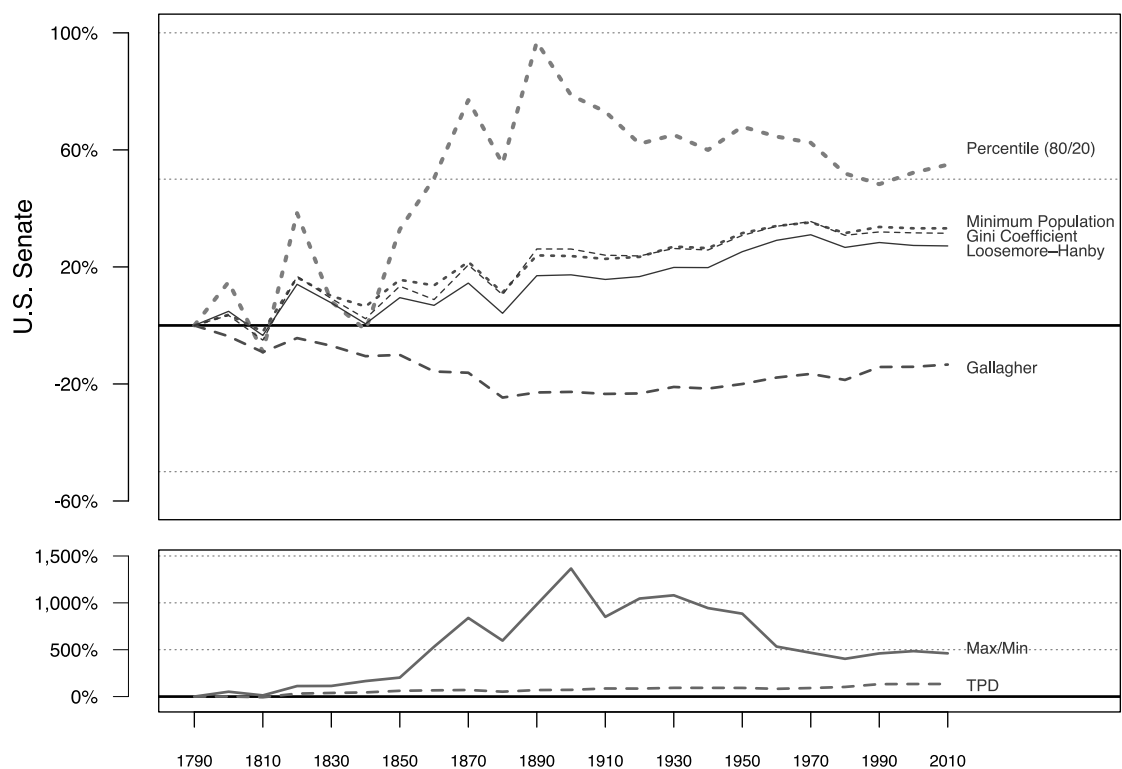


Note: Because of the wide disparities for the *Max/Min* and *TPD* measure compared to the other five, we have created separate plots on different scales.

FIGURE 2 Continued... Electoral College



Note: Because of the wide disparities for the *Max/Min* and *TPD* measure compared to the other five, we have created separate plots on different scales.

FIGURE 2 Continued... U.S. Senate

Note: Because of the wide disparities for the *Max/Min* and *TPD* measure compared to the other five, we have created separate plots on different scales.

Our second major set of findings has to do with the degree of agreement among our various malapportionment measures. We have examined this concordance in terms of graphs showing whether malapportionment has increase, decreased, or stayed flat since the country's founding under our various measures; and we have also looked at linear correlations among our seven measures. Figure 2 presents the same data as in Figure 1, but in a way that facilitates comparisons across our different metrics. From Figure 2 we see that: (a) Comparing 2010 and 1790, for the U.S. House, all of the measures show closely comparable values in the two years (with the partial exceptions of *TPD* and *80/20 ratio* which show slight increases). (b) The same is true for the seven measures of malapportionment in the Electoral College. (c) In contrast, for the U.S. Senate, six of our seven measures show higher levels of malapportionment in 2010 than in 1790, while *Gallagher* shows a downwardly sloping pattern (Figure 1 e). Furthermore, (d) unlike the monotonicity, whether positive or negative, found for the Senate in the other five measures, *Max-Min ratio* and the closely related *80/20 ratio* exhibits non-monotonic patterns for the Senate over the period 1790-2010. (e) Five of our seven measures, all but *Gallagher* and *Max/Min* correlate very strongly with one another and can, in factor analytic terms, be considered as scaling on a single dimension for all three institutions, though we place less reliance on the linear correlation for the *Percentile (80/20)* measure because of the non-monotonic pattern it exhibits for the U.S. Senate. (f) However, with respect to comparability across measures, we regard our results about the *Gallagher index* as equally important. While, in the context of seats-votes disproportionalities, *Gallagher* is a metric that has recently been given a great deal of favorable attention, its lack of strong correlations with other measures, with negative correlations for two of our three institutions, suggests a sharp note of caution vis-a-vis its use in the context of malapportionment.³²

different reasons for EC and popular vote discrepancies is beyond the scope of this study.

³² Gallagher (1991) provides comparisons of the *Gallagher index* (which he refers to as the *least squares index*) with other measures in the context of national level seats-votes relationships. In that context he finds considerable similarity in results across measures.

References

- Altman, Micah. 1998. "Traditional Districting Principles: Judicial Myths vs. Reality." *Social Science History* 22(2): 159.
- Atkinson, Anthony B. 1970. "On the measurement of inequality." *Journal of Economic Theory* 2(3): 244–263.
- Badger, Emily. 2016. "As American as Apple Pie? The Rural Vote's Disproportionate Slice of Power - The New York Times." New York Times New York Times.
- Bai, Jinhui H., and Roger Lagunoff. 2013. "Revealed Political Power." *International Economic Review* 54(4): 1085–1115.
- Baker, Gordon E. 1955. *Rural versus urban political power: The nature and consequences of unbalanced representation*. Random House.
- Baker, Gordon E. 1966. *The reapportionment revolution: Representation, political power, and the Supreme Court*. Random House.
- Balinski, Michel L., and H. Peyton Young. 1982. *Fair Representation: Meeting the Ideal of One Many One Vote*. New Haven: Yale University Press.
- Banzhaf, John F. III. 1965. "Weighted Voting Doesn't Work: A Mathematical Analysis." *Rutgers Law Review* 19(2): 317–343.
- Cervas, Jonathan R., and Bernard Grofman. 2019. "Are Presidential Inversions Inevitable? Comparing Eight Counterfactual Rules for Electing the U.S. President*." *Social Science Quarterly* 100(4): 1322–1342.
- Cox, Gary W., and Matthew Soberg Shugart. 1991. "Comment on Gallagher's 'proportionality, disproportionality and electoral systems'." *Electoral Studies* 10(4): 348–352.
- Dahl, Robert A. 1971. *Polyarchy : participation and opposition*. Yale University Press.
- Dahl, Robert A. 2003. *How democratic is the American Constitution?* Yale University Press.
- Dixon, Robert G. 1968. *Democratic Representation: Reapportionment in Law and Politics*. New York: Oxford University Press.
- Döring, Holger, and Philip Manow. 2017. "Is Proportional Representation More Favourable to the Left? Electoral Rules and Their Impact on Elections, Parliaments and the Formation of Cabinets." *British Journal of Political Science* 47(1): 149–164.
- Engstrom, Erik J. 2013. "The Partisan Impact of Malapportionment." In *Partisan Gerrymandering and the Construction of American Democracy*. University of Michigan Press, 148–164.
- Foster, James E. 1985. "Inequality measurement." *Fair allocation* 33: 31–68.
- Gallagher, Michael. 1991. "Proportionality, disproportionality and electoral systems." *Electoral Studies* 10(1): 33–51.
- Grofman, Bernard. 1990. *Political gerrymandering and the courts*. Eds. ed. Agathon Press.
- Grofman, Bernard, and Howard A. Scarrow. 1981. "The riddle of apportionment: Equality of what?" *National Civic Review* 70(5): 242–254.
- Grofman, Bernard, Lisa Handley, and Bernard Grofman. 2008. *Redistricting in Comparative Perspective*. Oxford University Press.
- Grofman, Bernard, William Koetzle, and Thomas L. Brunell. 1997. "An Integrated Perspective on the Three Potential Sources of Partisan Bias: Malapportionment, Turnout Differences, and the Geographic Distribution of Party Vote Shares." *Electoral Studies* 16(4): 457–470.
- Janson, Svante, and Svante Linusson. 2012. "The Probability of the Alabama Paradox." *Applied Probability Trust* 49(3): 773–794.
- Karpov, Alexander. 2008. "Measurement of disproportionality in proportional representation systems." *Mathematical and Computer Modelling* 48(9–10): 1421–1438.
- Ladewig, Jeffrey W., and Mathew P. Jasinski. 2008. "Remedies for Interstate Malapportionment of the U.S. House of Representatives." *Perspectives on Politics* 6(1): 89–107.
- Ladewig, Jeffrey W., and Seth C. McKee. 2014. "The Devil's in the Details: Evaluating the One Person, One Vote Principle in American Politics." *Politics and Governance* 2(1): 4.
- Lee, Frances E., and Bruce Ian. Oppenheimer. 1999. *Sizing up the Senate : the unequal consequences of equal representation*. University of Chicago Press.
- Loosemore, John, and Victor J. Hanby. 1971. "The Theoretical Limits of Maximum Distortion: Some Analytical Expressions for Electoral Systems." *British Journal Political Science* 1(1): 467–477.
- Lorenz, M O. 1905. "Methods of Measuring the Concentration of Wealth." *Publications of the American Statistical Association*, 9(70): 209–219.

- Manin, Bernard. 1997. *The Principles of Representative Government*. Cambridge: Cambridge University Press.
- McCubbins, Mathew D., and Thomas Schwartz. 1988. "Congress, the Courts, and Public Policy: Consequences of the One Man, One Vote Rule." *American Journal of Political Science* 32(2): 388.
- McGann, Anthony J., Charles Anthony Smith, Michael Latner, and Alex J. Keena. 2016. *Gerrymandering in America: The House of Representatives, the Supreme Court, and the Future of Popular Sovereignty*. Cambridge University Press.
- Monroe, Burt L. 1994. "Disproportionality and malapportionment: Measuring electoral inequity." *Electoral Studies* 13(2): 132–149.
- Morgan, James. 1962. "The Anatomy of Income Distribution." *The Review of Economics and Statistics* 44(3): 270.
- Moriwaka, Toshimasa. 2008. "The Politics of Redistricting in Japan." In *Redistricting in Comparative Perspective*, eds. Bernard Grofman, and Lisa Handley. eds ed. Oxford University Press, 107–114.
- NCSL. 2019. *Redistricting Law 2020*. Legislatures, National Conference of State.
- Neubauer, Michael G., and Joel Zeitlin. 2003. "Outcomes of Presidential Elections and the House Size." *PS: Political Science & Politics* 36(4): 721–725.
- Pattie, Charles, and Ron Johnston. 2014. "'The electors shall meet in their respective states': Bias and the US Presidential Electoral College, 1960–2012." *Political Geography* 40: 35–45.
- Samuels, David, and Richard Snyder. 2001. "The Value of a Vote: Malapportionment in Comparative Perspective." *British Journal of Political Science* 31(4): 651–671.
- Sauger, Nicolas, and Bernard Grofman. 2016. "Partisan bias and redistricting in France." *Electoral Studies* 44: 388–396.
- Shapley, Lloyd S., and Martin Shubik. 1954. "A Method for Evaluating the Distribution of Power in a Committee System." *American Political Science Review* 48(3): 787–792.
- Stewart, Charles, and Barry R. Weingast. 1992. "Stacking the Senate, Changing the Nation: Republican Rotten Boroughs, Statehood Politics, and American Political Development." *Studies in American Political Development* 6(2): 223–271.
- Taagepera, Rein, and Bernard Grofman. 2003. "Mapping the Indices of Seats–Votes Disproportionality and Inter-Election Volatility." *Party Politics* 9(6): 659–677.
- Taagepera, Rein, and Matthew Soberg Shugart. 1989. *Seats and votes: the effects and determinants of electoral systems*. Yale University Press.
- Taagepera, Rein, Peter Selb, and Bernard Grofman. 2014. "How Turnout Depends on the Number of Parties: A Logical Model." *Journal of Elections, Public Opinion & Parties* 24(4): 393–413.
- Tan, Netina, and Bernard Grofman. 2018. "Electoral rules and manufacturing legislative supermajority: evidence from Singapore." *Commonwealth & Comparative Politics* 56(3): 273–297.
- Toles, Tom. 2018. "Democrats are now forbidden by law from controlling more than one half of one branch of government - The Washington Post." *Washington Post* Washington Post.
- Van Puyenbroeck, Tom. 2008. "Proportional representation, gini coefficients, and the principle of transfers." *Journal of Theoretical Politics* 20(4): 498–526.
- Yntema, Dwight B. 1933. "Measures of the Inequality in the Personal Distribution of Wealth or Income." *Journal of the American Statistical Association* 28(184): 423.
- Zingher, Joshua N. 2016. "The relationship between bias and swing ratio in the Electoral College and the outcome of presidential elections." *Journal of Elections, Public Opinion and Parties* 26(2): 232–252.