AML 253: Introduction to Mathematical Tools and Modeling for the Life & Social Sciences

26.09.2023

Mock Exam, Fall 2023
Intro to Math Tools & Modeling
(Duration: 60 minutes)

Last Name: _____ First Name: _____

I confirm that I have read the following notes and that I have checked the completeness of this exam (pages 1-6).

_____

Signature of the above-named exam attendee

Notes:

1. No additional materials are allowed.
2. Unreadable answers or answers written with a pencil can be disqualified from the evaluation.
3. Please ensure that your responses directly address the questions posed.
4. You have 60 minutes to complete the exam.

Only for the examiner:

1	2	3	-	-	-	-	-	-	total
(5)	(10)	(10)							(25)

Question 1. (5 Points)

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Select True or False accordingly and justify your answer.

	True	**False**
The value of a certain Cryptocurrency increases in value at the rate of 8.5% annually. If the purchase price of the coin this year is \$1,950, the value to the nearest dollar in 15 years is greater than 10,000.00	☐	☐
There are 500 rabbits in Lancaster on February 1st. The amount of rabbits triples every month. There will beare less than 350,000 rabbits in Lancaster on August 1st.	☐	☐
The exponential model applies only for small populations and short timescales.	☐	☐
The solution for the matrix model of structure population $n(t+1) = M \cdot n(t)$ is given by $n(t) = M^t n(0)$ with $n(0)$ and M initial condition and the transference matrix, respectively.	☐	☐
If $\lambda_{max} > 1$, the larger eigenvalue associated with the transfer matrix, then the population will collapse.	☐	☐

Question 2. (10 Points)

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In the 1930's, before industrial-scale whale hunting, a researcher studied the blue whale population and estimated average birth rates and survival rates for females as a function of their ages to construct a Leslie Matrix. Blue whale females can produce a calf at most once in two years, so age classes where divided by two years. Thus, the youngest age class contained individuals < 2 years, the next class, 2 and 3 year-olds, the next 4 and 5 year-olds and so on. Turns out, the average survival probability from one age class to the next was almost the same for all age classes (blue whales have no predators except humans), but the fecundity of females was age-dependent. However, once females were 12 or older, fecundities were the same until death. Because whales can become very old, instead of keeping track of all age classes, whales of 12 or older were lumped into one class. Here s is the average bi-annual survival rate, and the f_i are the age-specific average bi-annual female birth rates. Draw the diagram associated with the model. Write the dynamics in system of equations and in matricial representation.

Question 3. (10 Points)

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Consider a population of some plant species. We know that each individual plant spreads, on average, 100 seeds each year. Also, that each seed has 25% chance to become an adult plant next year; otherwise, the seed is wiped out and never grows. Finally, the 25% of the adult plants die each year.

(a) Draw the diagram for the described model before.

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(b) Write the dynamics as a set of two equations and in a matrix representation.

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(c) Compute the eigenvalues and the corresponding eigenvectors associated with the transfer matrix.

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(d) Find λ_{max} and determine whether the population grows or is prone to extinction

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(e) Find the normalized eigenvector associated with the largest eigenvalue and determine what it tells us about the proportion of healthy people in the population.

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