AML 254: Introduction to Dynamics & Control in the Biological & Social Sciences

04.11.2024

Final Exam, Spring 2024
Continuous Models for Life & Social Sciences

Last Name: _____ First Name: _____

I confirm that I have read the following notes and that I have checked the completeness of this exam (pages 1-7).

_____

Signature of the above-named exam attendee

Notes:

1. No additional materials are allowed.
2. Unreadable answers or answers written with a pencil can be disqualified from the evaluation.
3. Please ensure that your responses directly address the questions posed.

Only for the examiner:

1	2	-	-	-	-	-	-	-	total
(100)	(100)								(200)

Question 1. (100 Points)

/ 100

Eikenberry et al. proposed a compartmental model for assessing the United States, for New York and Washington States, community-wide impact of mask use by the general public during the recent COVID-19 pandemic. The authors highlight the critical participation of asymptomatically infectious in the spread of the disease.
They proposed a deterministic two-group SEAIR model. The two groups are given by whose habitually do and do not wear masks in public (subscripted by U and M, respectively). The system of differential equations of the full model, where a dot represents differentiation with respect to time, is given by

$$\begin{aligned}
\dot{S}_U &= -\beta(I_U + \eta A_U)\frac{S_U}{N} - \beta(1-\epsilon_0)(I_M + \eta A_M)\frac{S_U}{N},\\
\dot{E}_U &= \beta(I_U + \eta A_U)\frac{S_U}{N} + \beta(1-\epsilon_0)(I_M + \eta A_M)\frac{S_U}{N} - \sigma E_U,\\
\dot{I}_U &= \alpha\sigma E_U - (\phi + \gamma_I) I_U\\
\dot{A}_U &= (1-\alpha)\sigma E_U - \gamma_A A_U,\\
\dot{H}_U &= \phi I_U - \delta H_U - \gamma_H H_U,\\
\dot{R}_U &= \gamma I_U + \gamma_A A_U + \gamma_H H_U,\\
\dot{S}_M &= -\beta(1-\epsilon_i)(I_U + \eta A_U)\frac{S_M}{N} - \beta(1-\epsilon_i)(1-\epsilon_0)(I_M + \eta A_M)\frac{S_M}{N},\\
\dot{E}_M &= \beta(1-\epsilon_i)(I_U + \eta A_U)\frac{S_M}{N} + \beta(1-\epsilon_i)(1-\epsilon_0)(I_M + \eta A_M)\frac{S_M}{N} - \sigma E_M,\\
\dot{I}_M &= \alpha\sigma E_M - (\phi + \gamma_I) I_M,\\
\dot{A}_M &= (1-\alpha)\sigma E_M - \gamma_A A_M,\\
\dot{H}_M &= \phi I_M - \delta H_M - \gamma_H H_M,\\
\dot{R}_M &= \gamma I_M + \gamma_A A_M + \gamma_H H_M,
\end{aligned}$$

where

$$N = S_U + E_U + I_U + A_U + R_U + S_M + E_M + I_M + A_M + R_M,$$

and the death rate is given by

$$\dot{D} = \delta H_U + \delta H_M. \tag{1}$$

Also, the proportions ϵ_0 and ϵ_i denote the efficacy of using mask for being infected and transmitting the infection, respectively.
They mainly studied two aspects, one was the proportion of public that wear masks and the other was the effectiveness of masks (including clothes masks). The main conclusions from this work are: even the ineffective face masks may meaningfully reduce community transmission of COVID-19 and decrease peak hospitalizations and deaths. Moreover, mask use decreases the effective transmission rate. Indeed, masks are found to be useful with respect to both preventing illness in healthy persons and preventing asymptomatic transmission.

(a) What are the main assumptions in this model?

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(b) Draw the model diagram for the model described above.

/ 25

(c) Find the Disease Free Equilibrium (DFE).

/ 15

(d) Compute the $\mathcal{R}_0$ using the Next Generation Operator method.

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Question 2. (100 Points)

/ 100

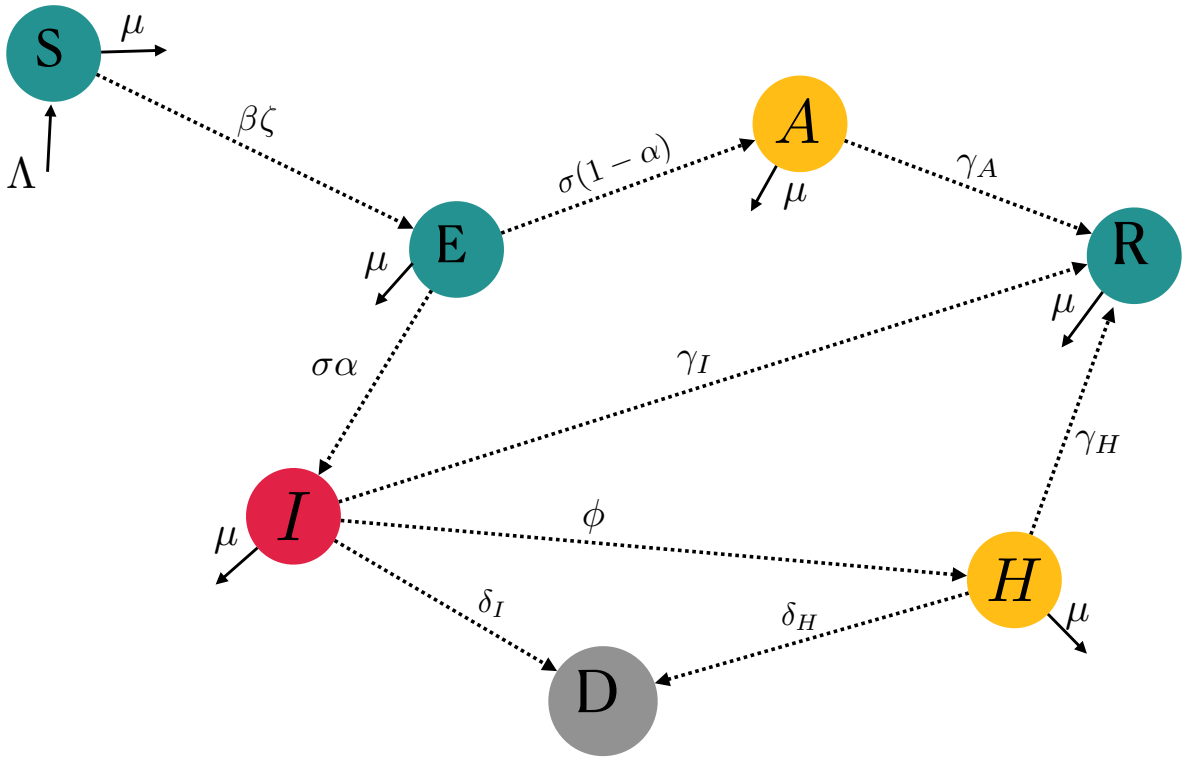

Figure 1: Schematic flow diagram for COVID-19 model. The model consists in seven subpopulations: Susceptible (S), Exposed (E), asymptomatic infected (A), symptomatic Infected (I), Recovered (R), Hospitalization (H), and Dead (D).

In the flow diagram 1 the force of infection, ζ, is a nonlinear term given by

$$\zeta = \frac{I + \eta_A A + (1-\varepsilon)\eta_H H}{S + E + A + I + R + (1-\varepsilon_H)H}, \tag{2}$$

Here we are considering isolation in hospitals is not perfect and they could mix with population though health workers, their family members, by the lack of resources, by adapting new places to hospitalize infected without effectiveness. The description of the parameters for the baseline model is given in Table 1.

Parameter	Description
β	Infectious contact rate
σ	Per capita progression rate to infectious
α	Proportion of symptomatic infectious
η_A	Infectiousness factor (asymptomatic)
η_H	Infectiousness factor (hospitalized)
ϕ	Per capita hospitalization rate
γ_A	Per capita recovery rate for Asymptomatic
γ_I	Per capita recovery rate for symptomatic
γ_H	Recovery rate for hospitalized
ε	Efficacy of the hospital isolation
δ_H	Per capita death rate (hospitalized)
δ_I	Per capita death rate (infected)

Table 1: Parameters description for the model.

(a) Write the equations of the model based on the diagram presented

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(b) Find the Disease Free Equilibrium (DFE).

/ 10

(c) Find the Endemic Equilibrium (EE) and the conditions for its existence.

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(d) Compute the $\mathcal{R}_0$ using the Next Generation Operator method.

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(e) Determine the Stability of DFE.

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(f) Determine the Stability of EE.

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