AML 254: Introduction to Dynamics & Control in the Biological & Social Sciences

03.19.2024

Midterm Exam, Spring 2024
Continuous Models for Life & Social Sciences

Last Name: _____ First Name: _____

I confirm that I have read the following notes and that I have checked the completeness of this exam (pages 1-6).

_____

Signature of the above-named exam attendee

Notes:

1. No additional materials are allowed.
2. Unreadable answers or answers written with a pencil can be disqualified from the evaluation.
3. Please ensure that your responses directly address the questions posed.

Only for the examiner:

1	2	-	-	-	-	-	-	-	total
(50)	(150)								(200)

Question 1. (50 Points)

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Select True or False accordingly and **justify** your answers.
Let's define the one dimensional dynamical system:

$$N'(t) = f(N) := N(t)\left[r - (N(t) - b)^2\right]. \tag{1}$$

	True	**False**
A. Given the dynamical system (1). An equilibrium N^* of the system satisfies $f'(N^*) = 0$.	□	□
B. The system (1) is not an example of a strong Allee effect because (1) could not be rewritten as $N'(t) = rN(t)\left(1 - \frac{N(t)}{K}\right)\left(\frac{N(t)}{K'} - 1\right)$. Where K and K' are in terms of a, b and r.	□	□
C. In the system (1) the equilibrium $N^* = b + \sqrt{\frac{r}{a}}$ is a stable node if $r < ab^2$.	□	□

Question 2. (150 Points)

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Let's consider the following mathematical model that aims to mimic the interactions between lions (y) and hyenas (x).

$$\begin{aligned} \frac{dx}{dt} &:= r_x x\left(1-\frac{x}{K_x}\right)-\frac{\alpha xy}{K_x} \\ \frac{dy}{dt} &:= r_y y\left(1-\frac{y}{K_y}\right)\left(\frac{y}{K_y'}-1\right)-\frac{\beta xy}{K_y} \end{aligned} \tag{2}$$

https://infogram.com

(a) Describe the relationship between the species x and y (i.e. how do they interact? how do they affect each other?).

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(b) Compute all equilibria of the system and provide the conditions for the existence of them.

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(c) Classify all equilibria of the system as node, spiral or center.

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(d) Determine the stability of the equilibria.

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(e) Choose $K_x = K_y = 1$, and $K_0 = 1/4$. Then, pick a convenient set of parameter values (from the mathematical analysis) for r_x, r_y, α, and β to plot the phase plane of each system including the nullclines, equilibria and some curves. Repeat this task for at least two set of parameter values.

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