M 327J - Differential Equations with Linear Algebra

August 29, 2022

Quiz 1

1. [**4 points**] Find the general solution to

$$\frac{dy}{dt} + \frac{1}{t}y = t.$$

Solution
We first must find the integrating factor, a function $I(t)$ satisfying

$$\frac{dI}{dt} = \frac{1}{t}I(t).$$

Dividing by $I(t)$ we find

$$\frac{d}{dt}\ln|I(t)| = \frac{1}{I(t)}\frac{dI}{dt} = \frac{1}{t}$$

and after integrating we arrive at

$$I(t) = t$$

as an acceptable integrating factor. Multiplying our equation by I we find

$$\frac{d}{dt}(ty) = t^2$$

which we can then integrate to

$$ty = \frac{1}{3}t^3 + C$$

yielding the general solution

$$y(t) = \frac{1}{3}t^2 + \frac{C}{t}.$$

2. [**3 points**] Find the particular solution to

$$\frac{dy}{dt} + ty = 0 \quad y(0) = 1.$$

Solution
Since this is a homogeneous equation we can rewrite it as

$$\frac{d}{dt}\ln(|y|) = -t$$

and integrate both sides with respect to t

$$\ln|y| = -\frac{t^2}{2} + C.$$

Solving for y we find

$$y(t) = C'e^{-\frac{t^2}{2}}$$

and using the initial condition $y(0) = 1$ we find

$$y(0) = C'e^0 = C' = 1$$

giving us the particular solution

$$y(t) = e^{-\frac{t^2}{2}}.$$

3. [3 points] Find the general solution to

$$\frac{dy}{dt} = 2ty^2.$$

Solution
We see the right hand side is the product of a function of t and a function of y. We separate variables to obtain the equality

$$\frac{1}{y^2}\frac{dy}{dt} = 2t$$

and integrating with respect to t gives

$$-\frac{1}{y} = t^2 + C.$$

We easily solve for y to obtain our general solution

$$y(t) = -\frac{1}{t^2 + C}.$$