CSCE 867: Computer Vision
Homework #3
Completed by Jared Gentry
3/6/19

Problems 1 and 3 are on separate scanned pages.

2. Io ⇒ [00000
00000
00100 size: 5x5
00000
00000]

Gaussian kernel ⇒ $\frac{1}{16}$ [121
242 size: 3x3
121]

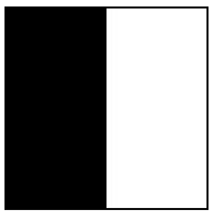
Io after filtering and zero padding ⇒ $\frac{1}{16}$ [00000
01210
02420
01210
00000]

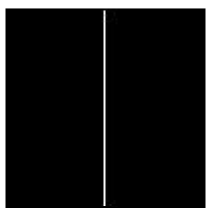
Since the Gaussian kernel is symmetric around its center, the convolution does not matter and the filtering can be thought of in terms of correlation. Since the image consists of all zeros and a single one at the center, the output becomes the kernel. The result is like a slightly spread out impulse. The circularly symmetric fuzz blob occurs due to the kernel's symmetry around its center. The center one in the image is brightest while the neighboring pixels only exhibit some brightness due to the center one pixel being farther away from the center of the Gaussian kernel (where it is strongest) when the filter pass happens. This "pulls" the brightness of the single one pixel at the center outward towards its neighbors and creates a blurring effect.

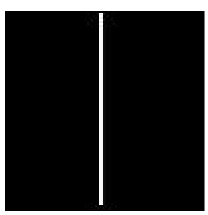
4. Typically, a larger filter or mask tends to reduce noise effects, but may result in a loss of valid edge points and an increased blurring effect. Since the Laplacian mask is symmetric about its center, it can be thought of as correlation. When passing the mask (Fig. a) about the image, the constant regions (only black pixels or only white pixels) will return a resulting pixel of 0 or a completely black pixel, since 8*0 - 8*0 = 0 and 8*255 -

8*255 = 0. Therefore the regions to the left and right of the detected edge line will be completely black regions. This is also true when filtering with the larger mask (Fig. d), since 24*0 - 24*0 = 0 and 24*255 - 24*255 = 0. And since the Laplacian mask (Fig. a) is a smaller mask, the resulting edge line is sharp and narrow, and results from the boundary between the black and white regions. The larger mask (Fig. d) will recruit more neighboring pixels in each filtered pixel calculation and will therefore have a more blurred and wider edge line in the middle of the resulting filtered picture.

Image results using Python code

Initial image:  with a border added to see white region

After Laplacian mask: 

After larger mask: 

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① $G(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$

let $K = \frac{1}{\sqrt{2\pi\sigma^2}}$ for the constant

$$\frac{\partial G(x,y)}{\partial x} = K \cdot \left(-\frac{x}{\sigma^2}\right) e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$\frac{\partial G(x,y)}{\partial y} = K\left(-\frac{y}{\sigma^2}\right) e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$\frac{\partial^2 G(x,y)}{\partial^2 x} = K\frac{x^2}{\sigma^4} e^{-\frac{(x^2+y^2)}{2\sigma^2}} - \frac{K}{\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$= K\left(\frac{x^2-\sigma^2}{\sigma^4}\right) e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

similarly,

$$\frac{\partial^2 G(x,y)}{\partial^2 y} = K\left(\frac{y^2-\sigma^2}{\sigma^4}\right) e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$\nabla^2 G(x,y) = \frac{\partial^2 G(x,y)}{\partial^2 x} + \frac{\partial^2 G(x,y)}{\partial^2 y}$$

$$= Ke^{-\frac{(x^2+y^2)}{2\sigma^2}} \left(\frac{x^2-\sigma^2}{\sigma^4} + \frac{y^2-\sigma^2}{\sigma^4}\right)$$

$$\nabla^2 G(x,y) = K\left(\frac{x^2+y^2-2\sigma^2}{\sigma^4}\right) e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

(3) if $H = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(H) = ad - cb$

λ is eigenvalue of H iff $\det(H - \lambda I) = 0$

$$\det(H - \lambda I) = \det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$= \det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)$$

$$= \det\left(\begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}\right)$$

$$= (a-\lambda)(d-\lambda) - bc$$

$$= ad - a\lambda - d\lambda + \lambda^2 - bc$$

equation 1

this polynomial function can be rewritten as

$$= \lambda^2 - \underbrace{(a+d)}_{\text{trace}(H)}\lambda + \underbrace{ad - bc}_{\det(H)}$$

$$= (\lambda - \lambda_1)(\lambda - \lambda_2),$$ where λ_1 + λ_2 are the roots/eigenvalues of H

equation 2 $\rightarrow$ $$= \lambda^2 - (\lambda_1 + \lambda_2)\lambda + \lambda_1\lambda_2$$

Comparing equation 1 with equation 2, we get:

$$\text{trace}(H) = a + d = \lambda_1 + \lambda_2$$

$$\det(H) = ad - bc = \lambda_1\lambda_2$$