전기회로 (가, 나)

Chapter 2 : Basic Laws

2017. 1학기

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Learning Goals

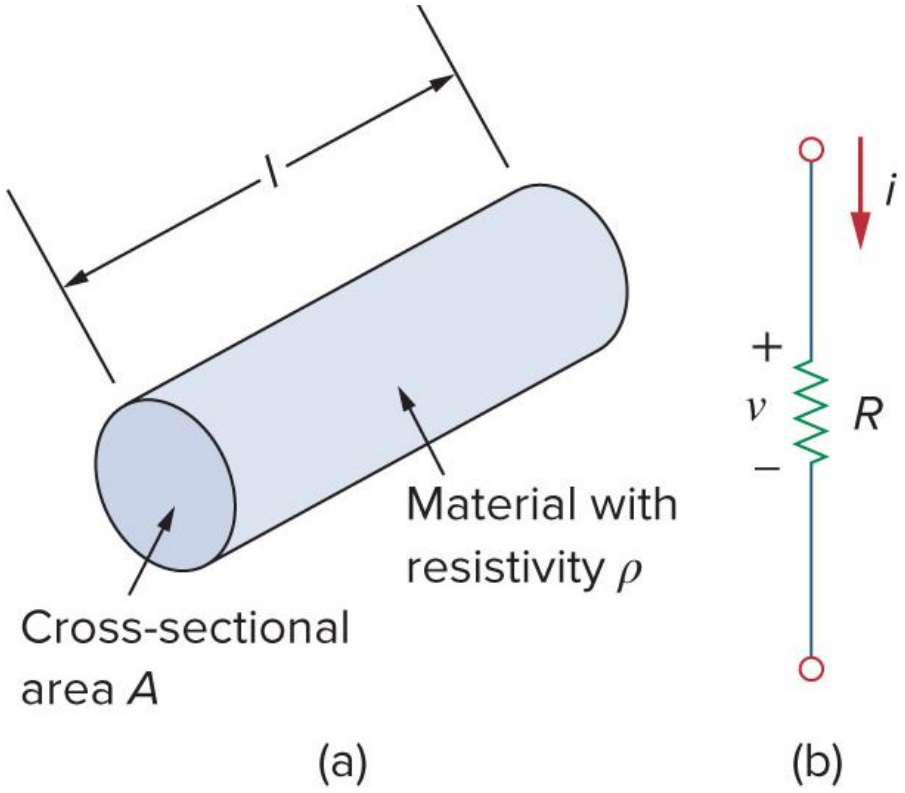
- Ohm's law
- 회로 구성 요소
- Kirchhoff's law
- 직렬연결 & 병렬연결
- 와이-델타 변환
- 응용

2.2 저항 (Resistance)

전류의 흐름을 방해하는 회로 요소를 저항 (Resistor)이라 하고,

기호 R로 표시한다.

$$R = \rho \frac{l}{A}$$



2.2 Resistivity (고유저항)

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.80×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4.00×10^{-5}	Semiconductor
Germanium	47.00×10^{-2}	Semiconductor
Silicon	6.40×10^{2}	Semiconductor
Paper	1.00×10^{10}	Insulator
Mica	5.00×10^{11}	Insulator
Glass	1.00×10^{12}	Insulator
Teflon	3.00×10^{12}	Insulator

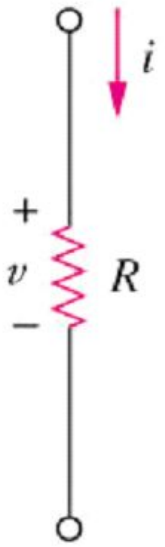
반도체

2.2 Ohm's Law

- Ohm's law (옴의 법칙) $v = iR$

- 저항(resistor, R) : 전류의 흐름을 방해하는 소자
  - 단위 : Ω(옴) = V/A

$V \propto i$

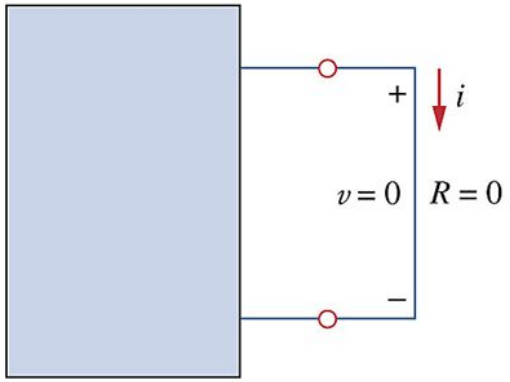


R은 비례 상수

$$R = \frac{v}{i}$$

2.2 Extreme Cases

- short circuit (단락(短絡)) : $R = 0$



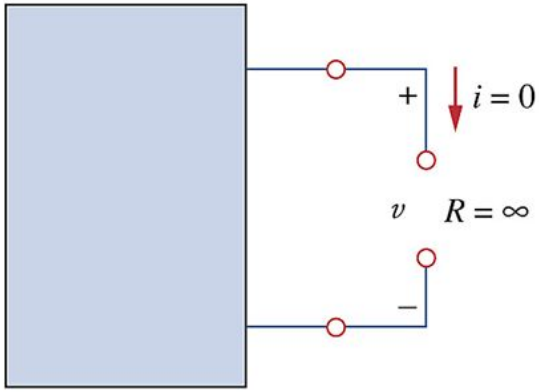
$i\ (= v/R = 0/0)$의 값은?

➔ 다른 곳에서 결정

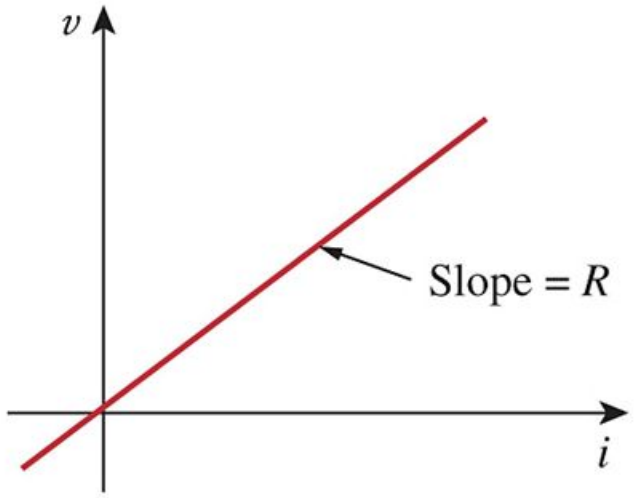
만약 $v \neq 0$ 이고 $R = 0$ 이면???

➔ 합선

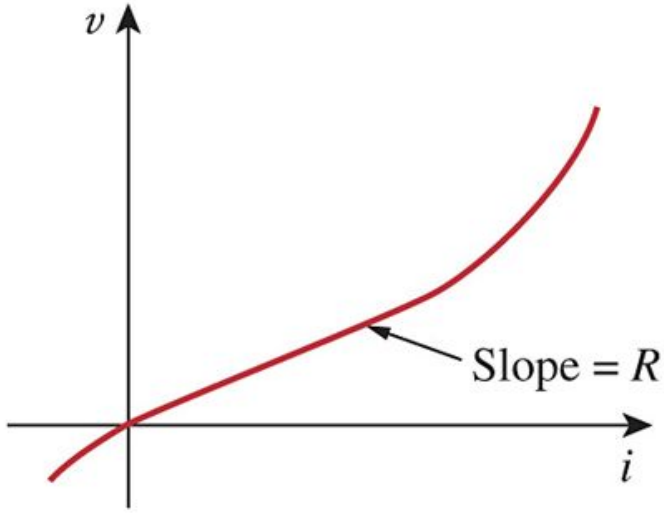
- open circuit (개방(開放)) : $R = \infty$



2.2 Ohm's Law (revisited)



[선형저항 (linear resistor)]



[비선형저항 (nonlinear resistor)]

2.2 Conductance

$$G = \frac{1}{R} = \frac{i}{v}$$

단위 : ℧ (mho 모) 또는 S (siemens 지멘스)

$$i = Gv$$

2.2 Power (revisited)

- 저항에 의하여 사용된 전력 (power)

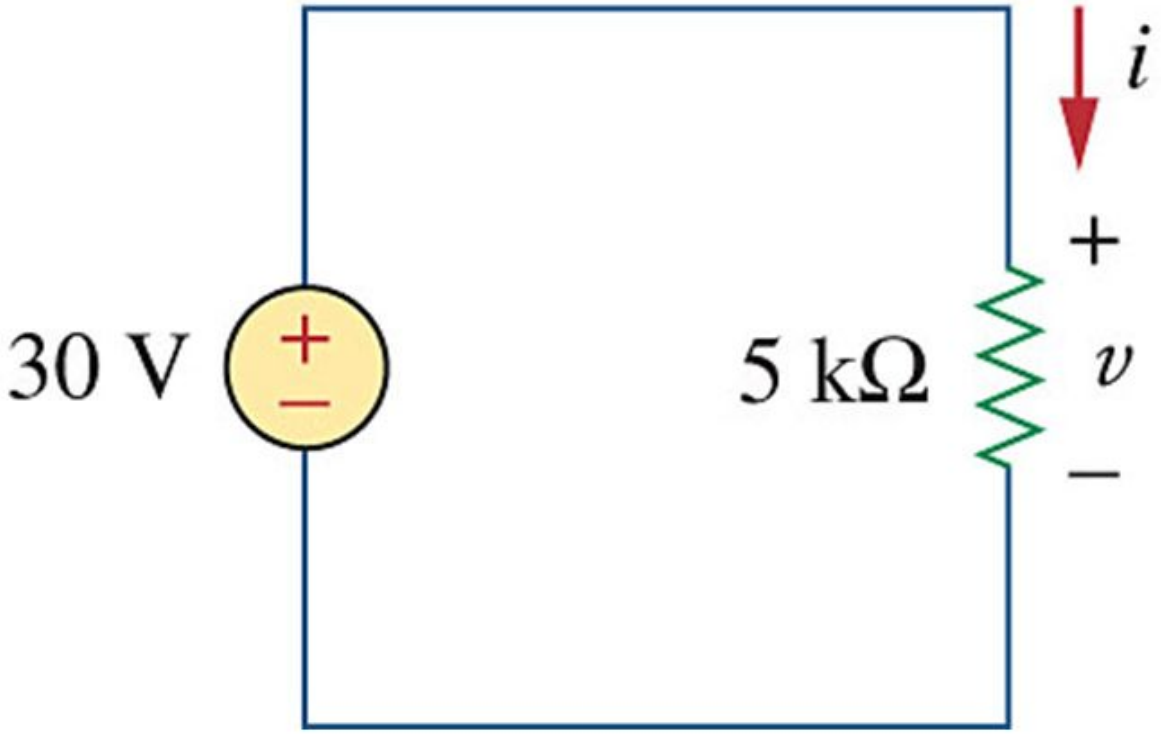
$$p = vi = i^2 R = \frac{v^2}{R}$$

$$p = vi = v^2 G = \frac{i^2}{G}$$

Quiz] power는 전류나 전압에 대하여 선형적인가?

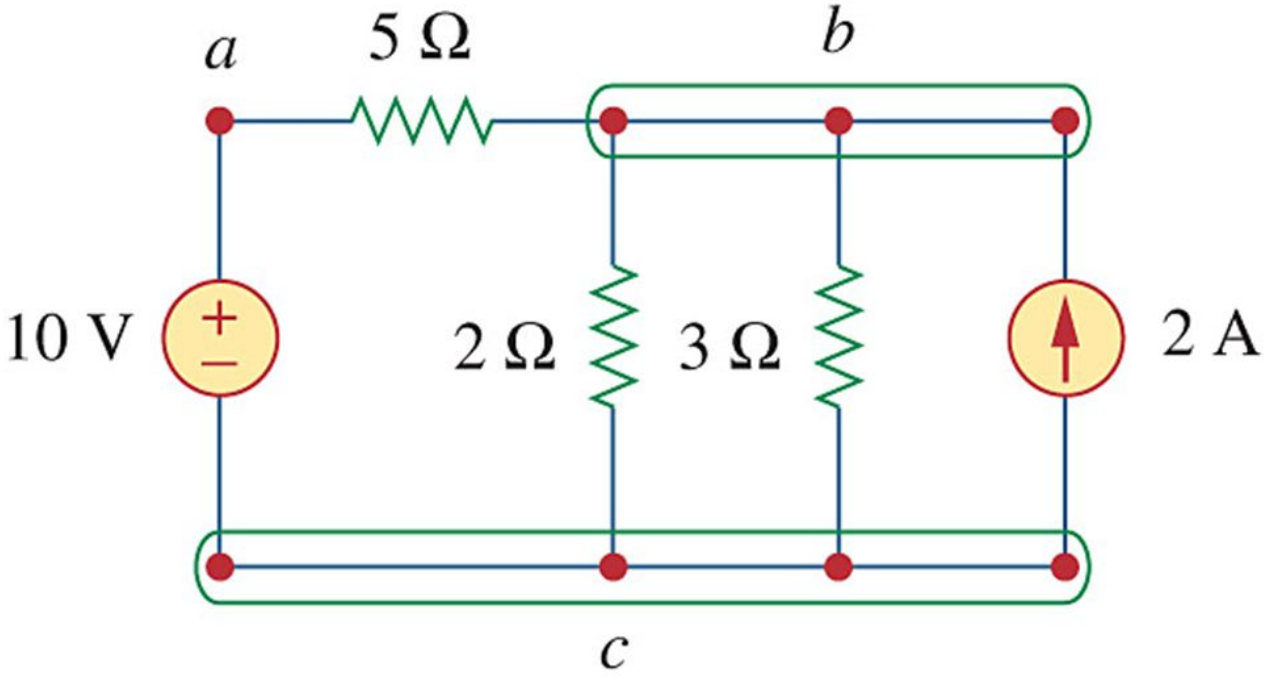
2.2 Example 2.2

- 다음 회로에서 전류 i, 컨덕턴스 G, 전력 p를 구하라



2.3 Topology

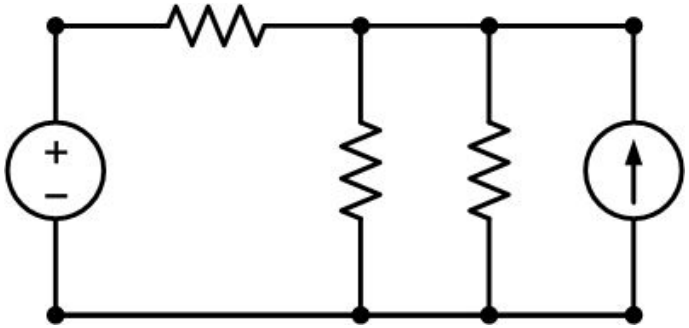
- 회로를 구성하는 요소
  - branch : 회로에 연결된 각각의 소자
  - node : branch의 연결점
  - loop : 회로내의 폐 경로



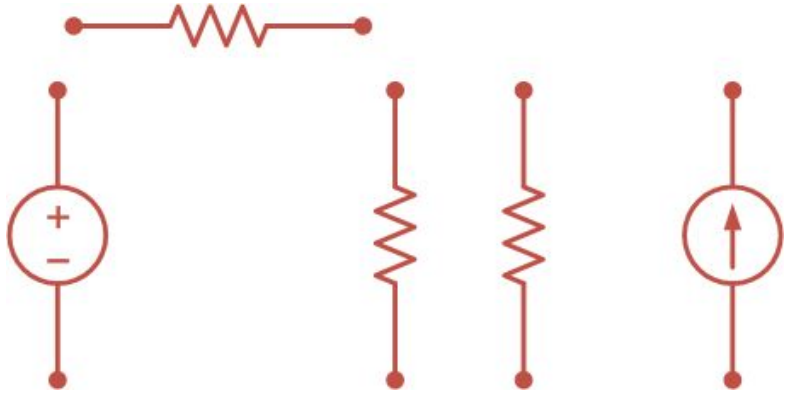
2.3 Branch

- 전압원/전류원, 저항 등의 회로를 구성하는 요소 (element)

Original Circuit



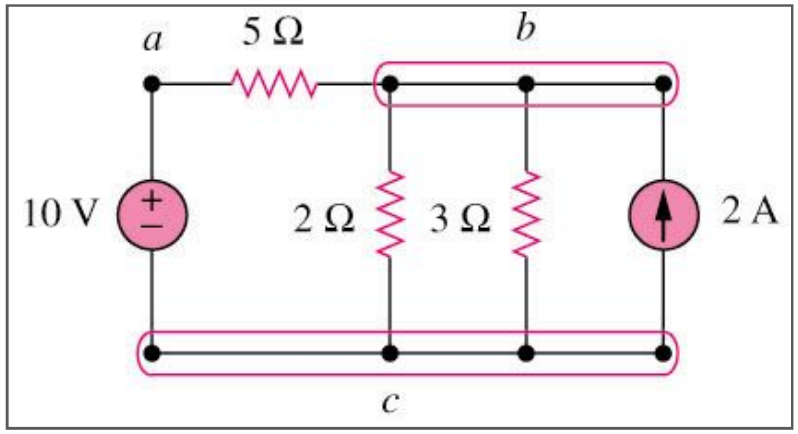
Branches



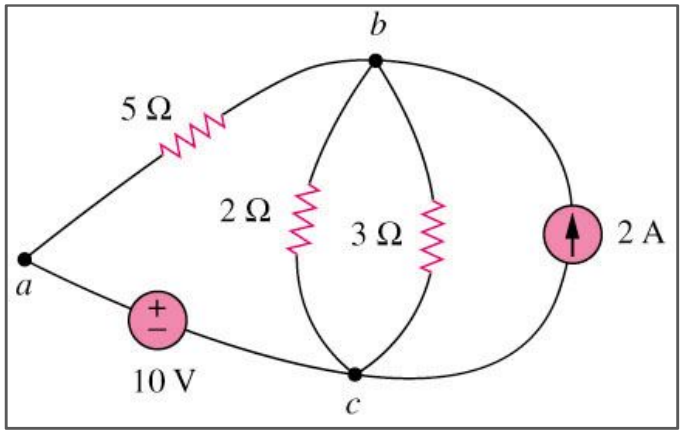
2.3 Node

- 2개 이상의 branch의 연결점

Original Circuit

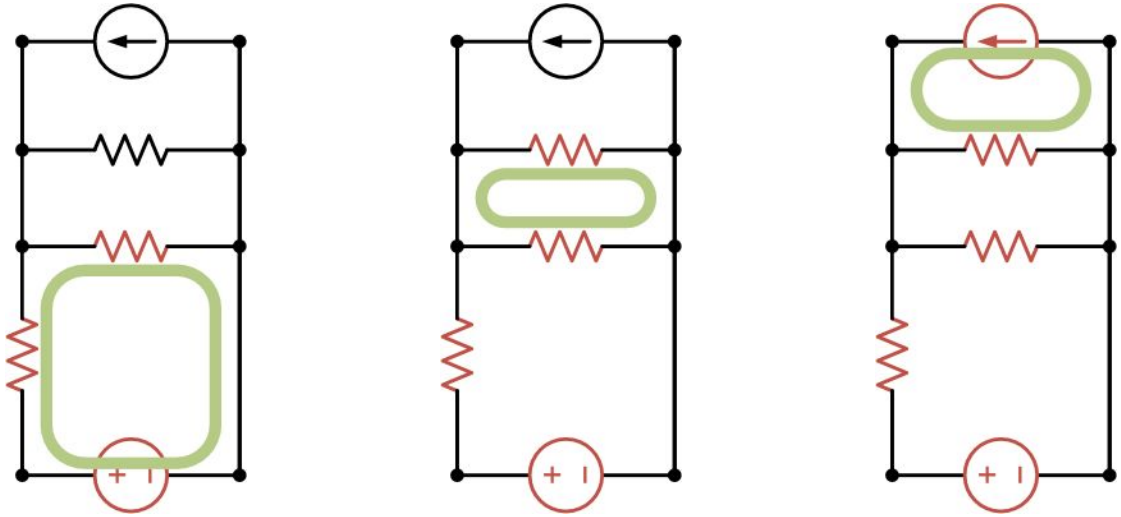


Equivalent Circuit



2.3 Loop

- 회로내의 폐 경로 (closed loop)
- 독립 loop
  - 다른 독립 loop의 일부가 아닌 branch를 가지고 있는 loop

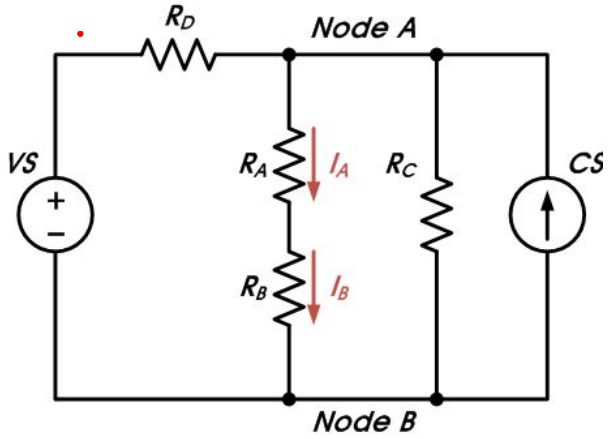


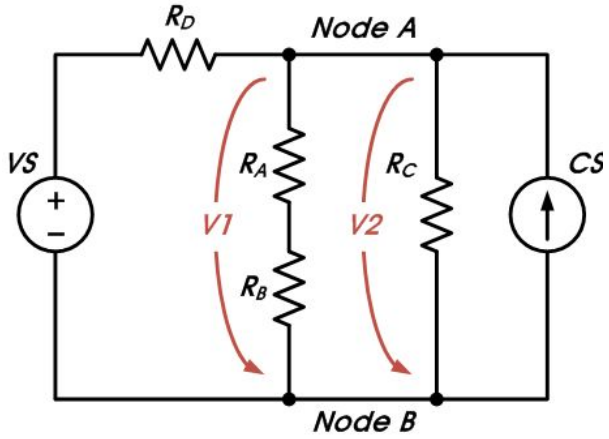
node(n), branch(b), loop(l)의 상관 관계

$$b = l + n - 1$$

2.3 Series & Parallel

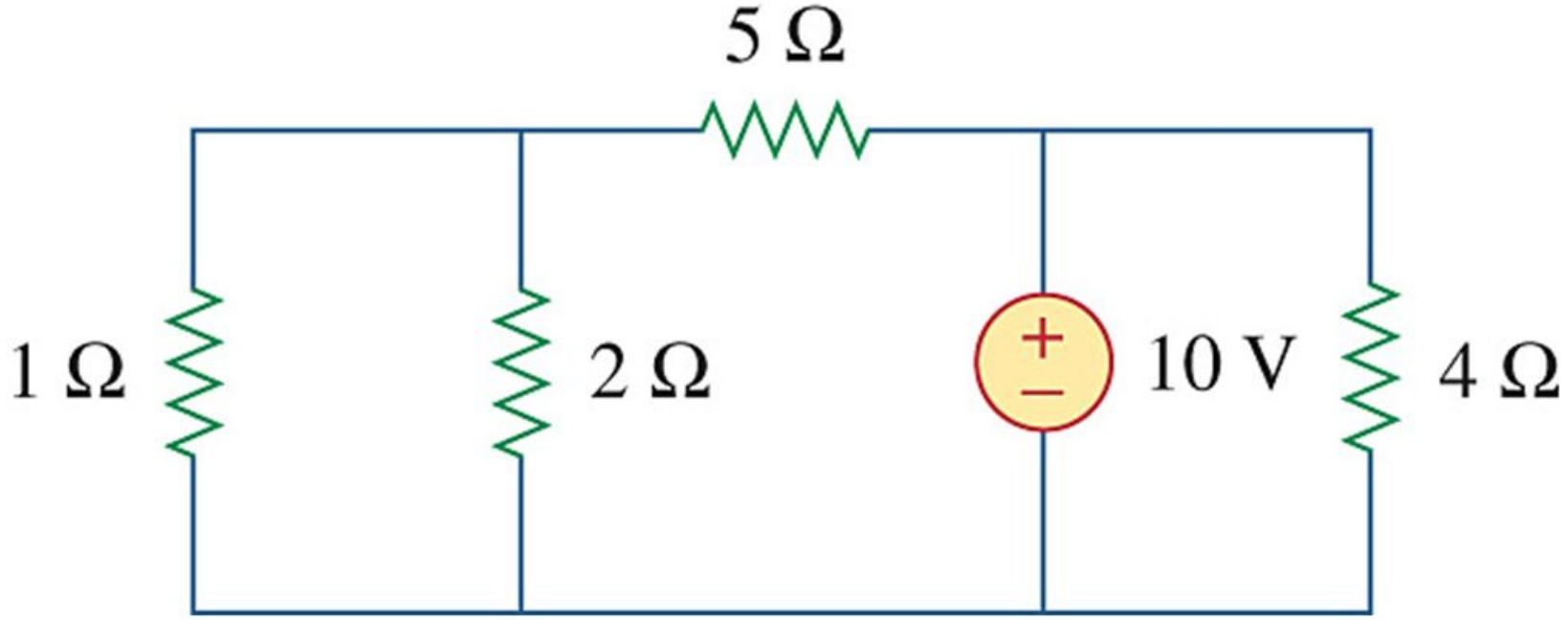
- Series (직렬연결)
  - 2개의 요소가 하나의 node를 공유
  - 각 요소에 흐르는 **전류는 동일**
- Parallel (병렬연결)
  - 2개 이상의 요소가 같은 2개의 node사이에 연결
  - 2개의 node사이의 **전압은 동일**





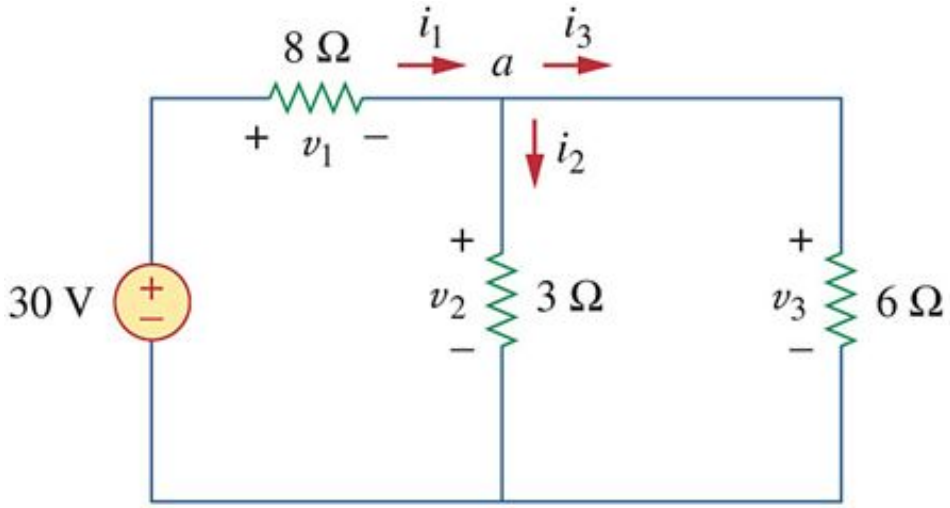
Practice (실전문제) 2.4

- 다음 회로에선 몇 개의 branch와 node가 있는가?
- 직렬 및 병렬인 회로 요소를 구별하여 나타내라.



2.4 Kirchhoff's Law (키르히호프의 법칙)

- 회로해석
  - 각 node의 전압 결정
  - 각 branch의 전류 결정



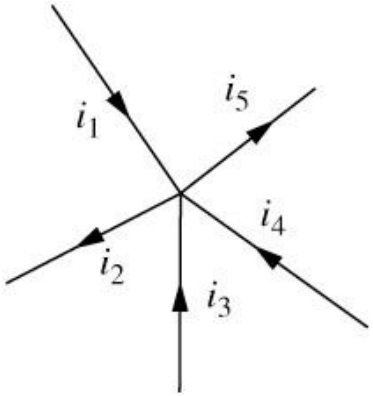
- **회로 해석 방법**
  - **Ohm's law ($v = iR$)**
  - **Kirchhoff's law**
    - **KCL** (Kirchhoff's current law)
    - **KVL** (Kirchhoff's voltage law)

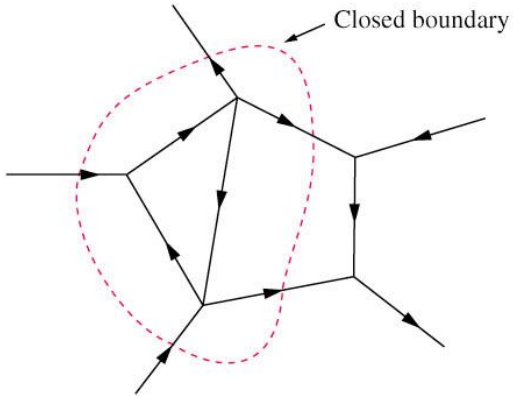
2.4 KCL

- Kirchhoff's current law (KCL)
  - 하나의 node를 통과하는 전류의 대수적 합은 "0" ←"전하 보존의 법칙"

$$\sum_{n=1}^{N} i_n = 0$$

  - 하나의 node 기준 : **들어가는 전류의 합 = 나가는 전류의 합**





$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

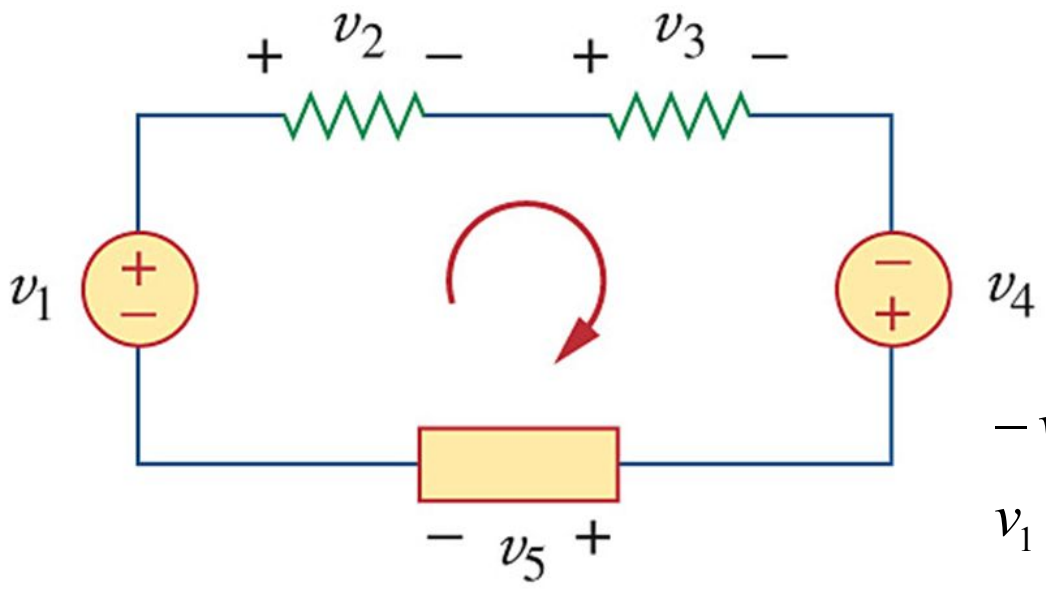
$$i_1 + i_3 + i_4 = i_2 + i_5$$

2.4 KVL

- Kirchhoff's voltage law (KVL)
  - 폐 경로에서 전압의 대수적 합은 "0"

$$\sum_{m=1}^{M} v_m = 0$$

←"에너지 보존의 법칙"

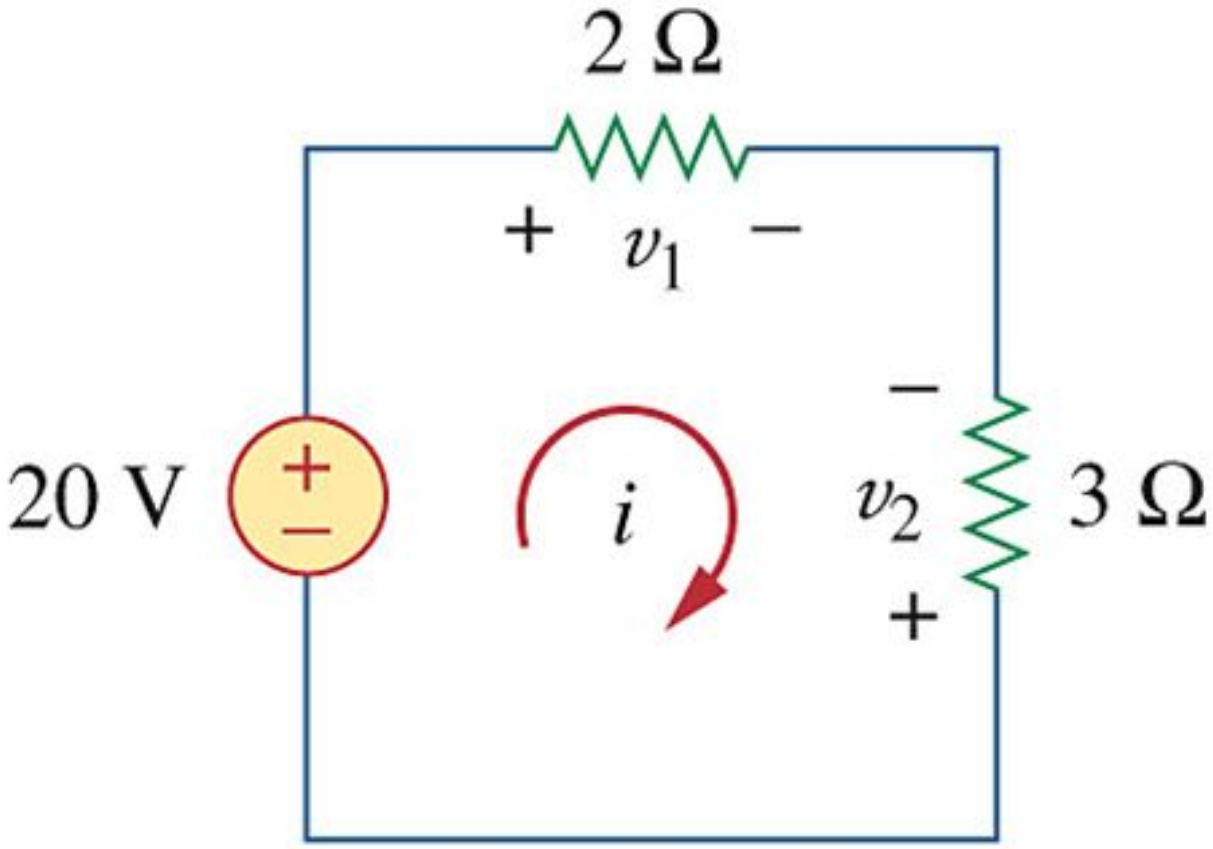


$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_1 + v_4 = v_2 + v_3 + v_5$$

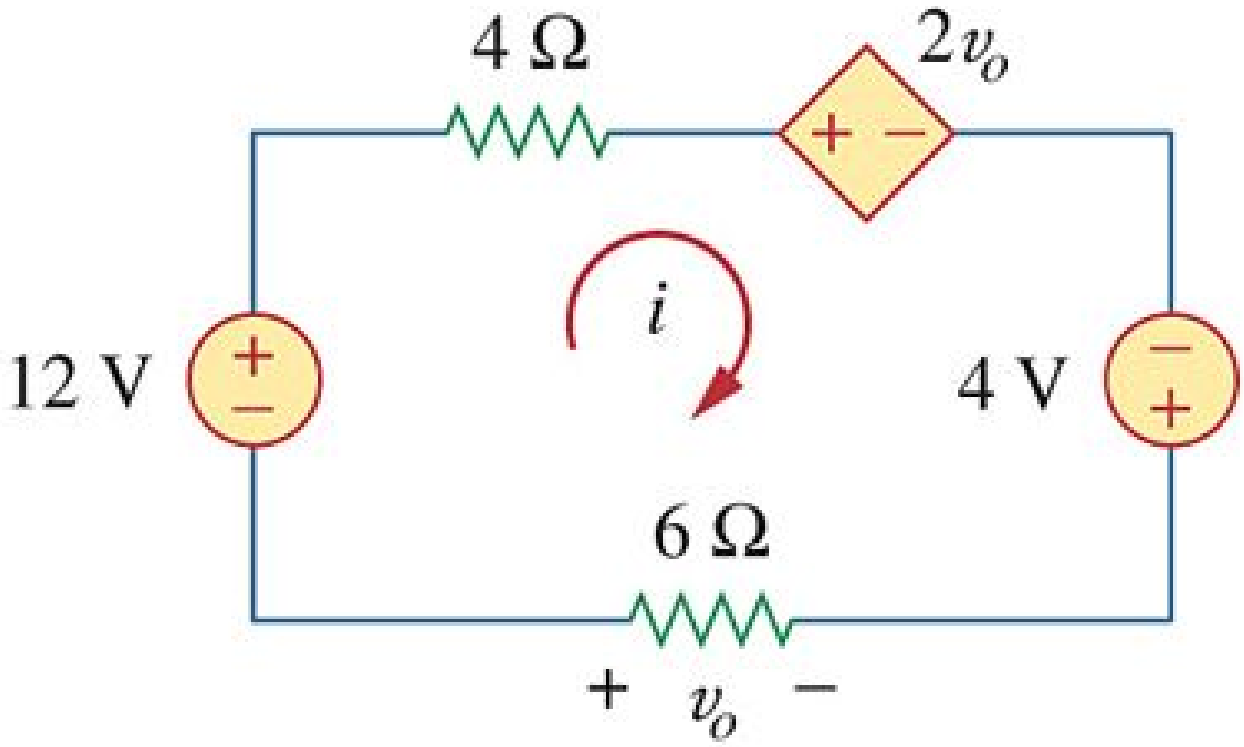
Example 2.5

- 다음 회로에서 전압 v_1과 v_2를 구하라.



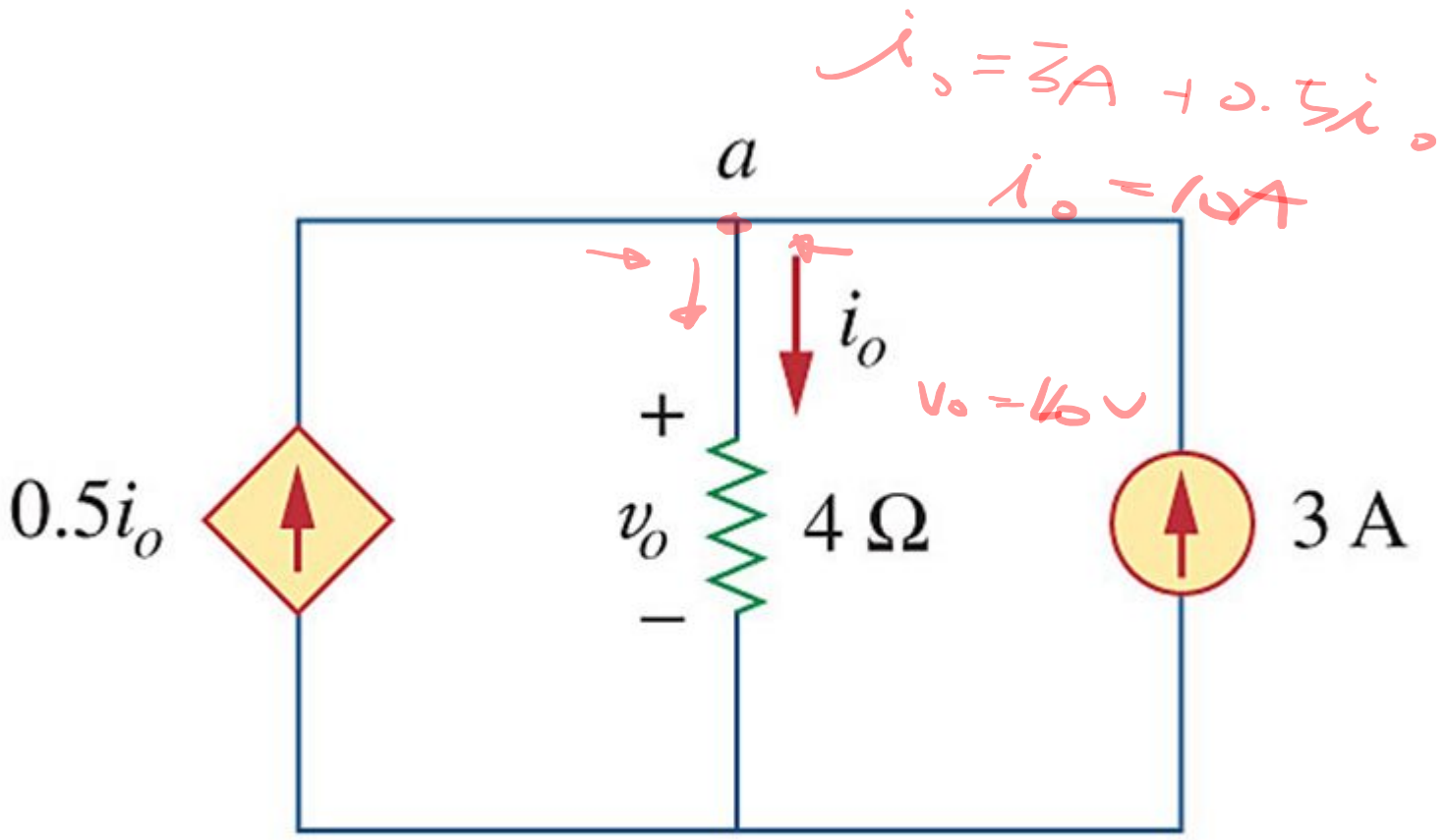
Example 2.6

- 다음 회로에서 v_0와 i를 구하라.



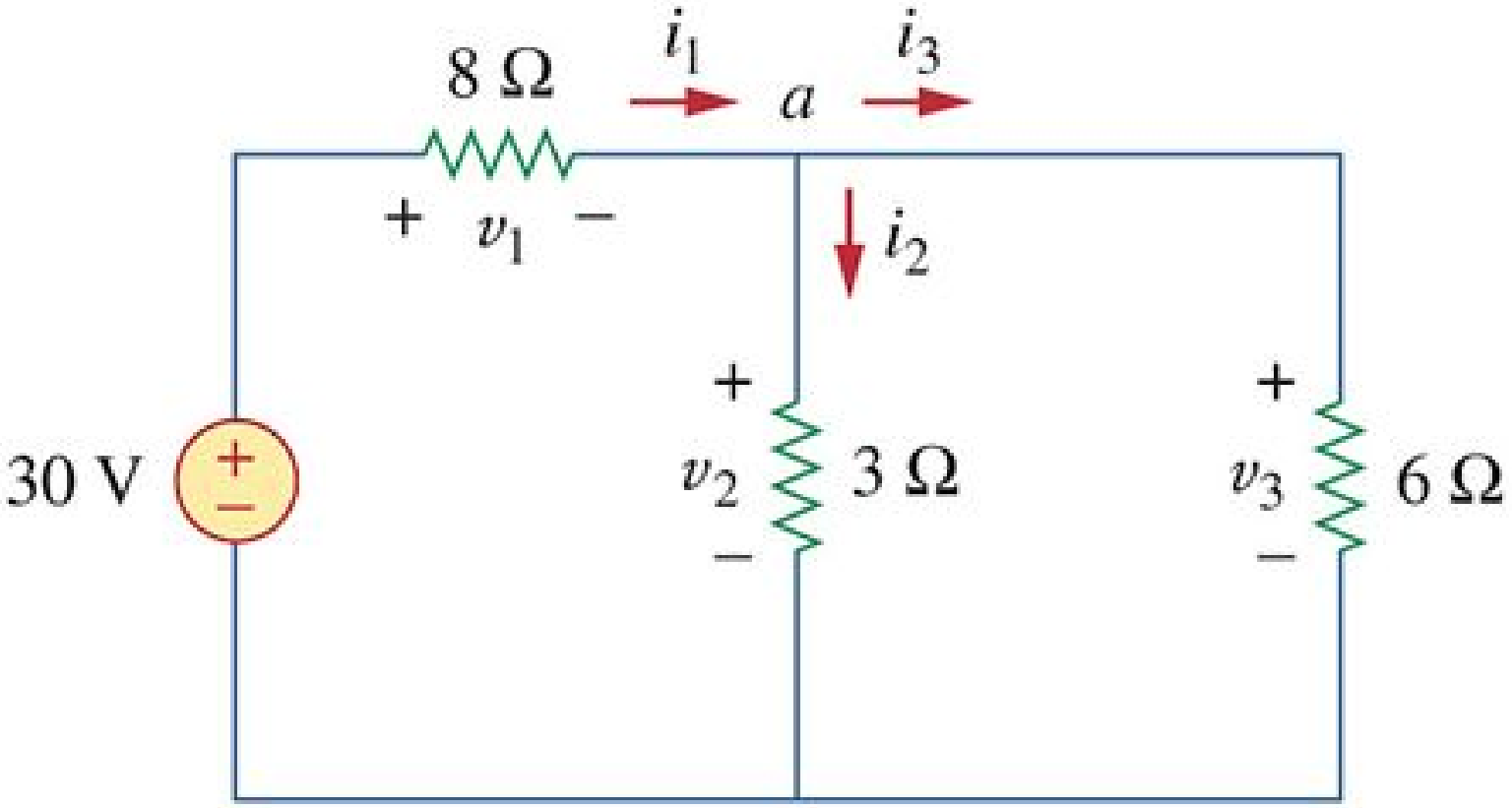
Example 2.7

- 다음 회로에서 전류 i_0와 전압 v_0를 구하라.



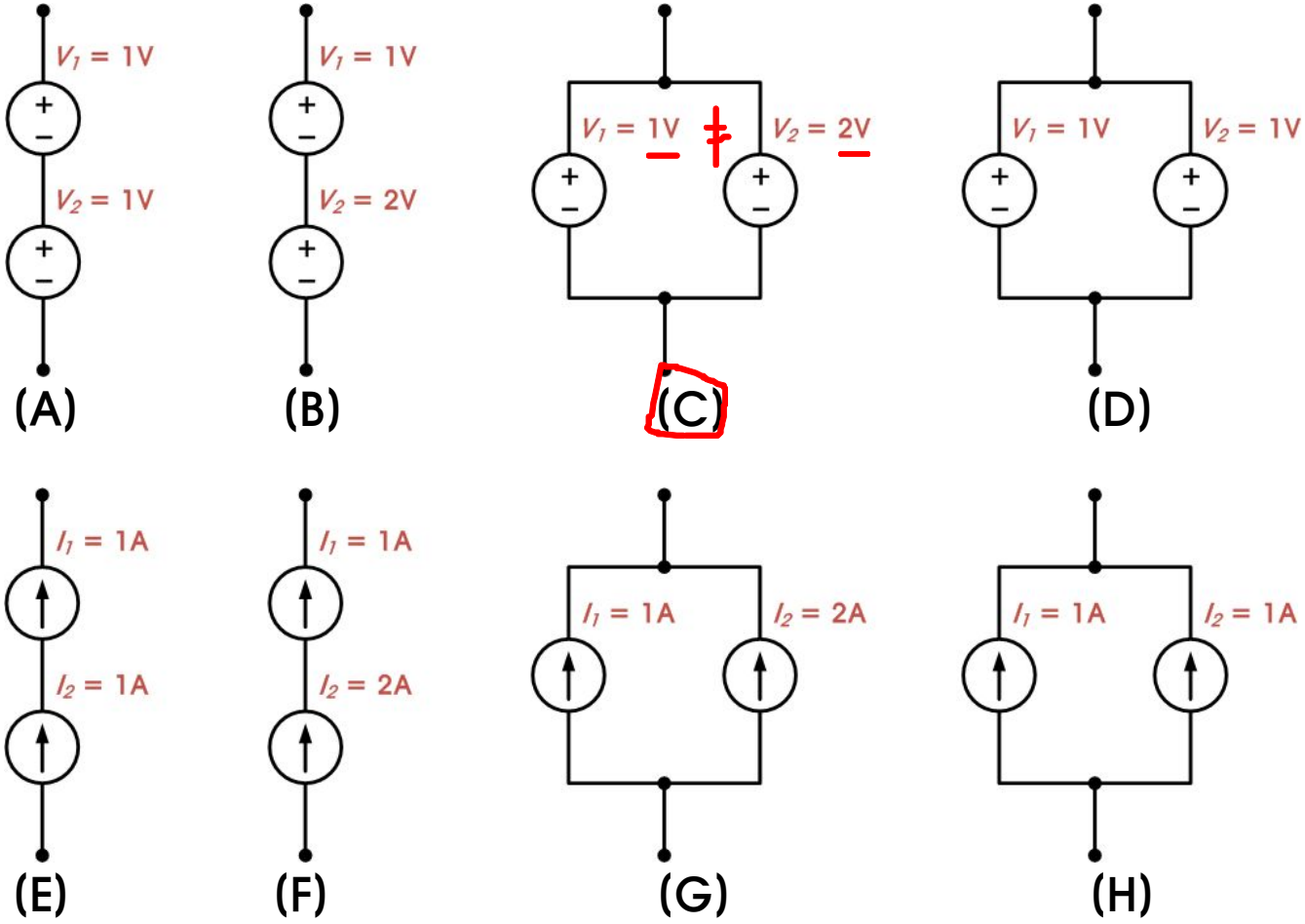
Example 2.8

- 다음 회로에서 전류와 전압을 구하라.



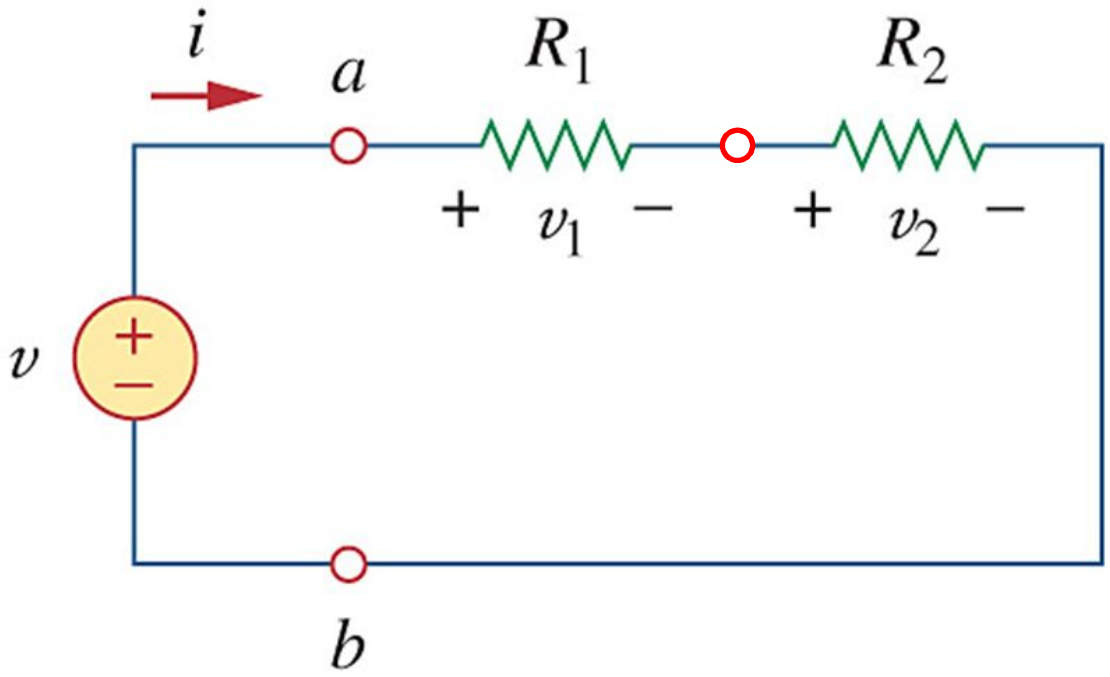
2.4 Kirchhoff's Law

- 다음 중 키르히호프 법칙에 위배되는 것은?



2.5 직렬 저항

- Series (직렬)
  - 2개 이상의 element가 순차적으로 연결
  - 2개의 element는 하나의 node를 공유
  - 동일한 전류가 흐름.

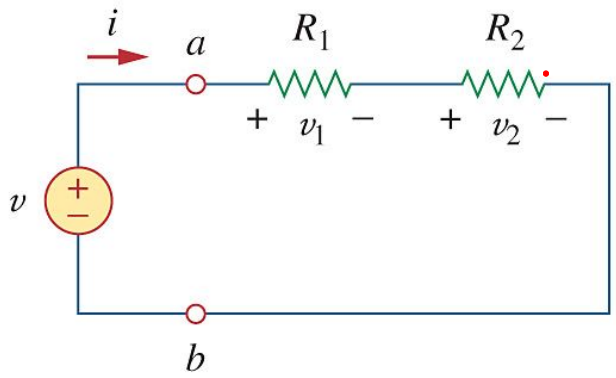


2.5 직렬에서의 등가 저항

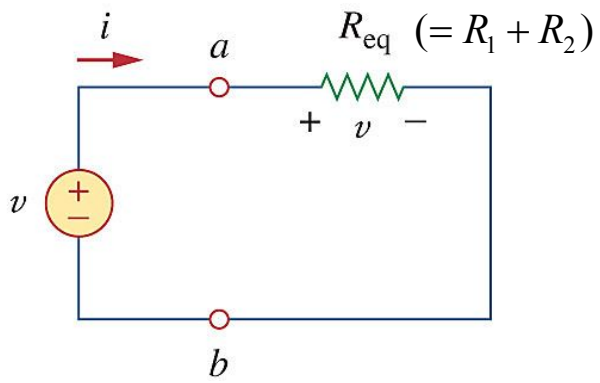
- 직렬 연결된 저항의 등가저항은 **각 저항의 합**

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

Original Circuit



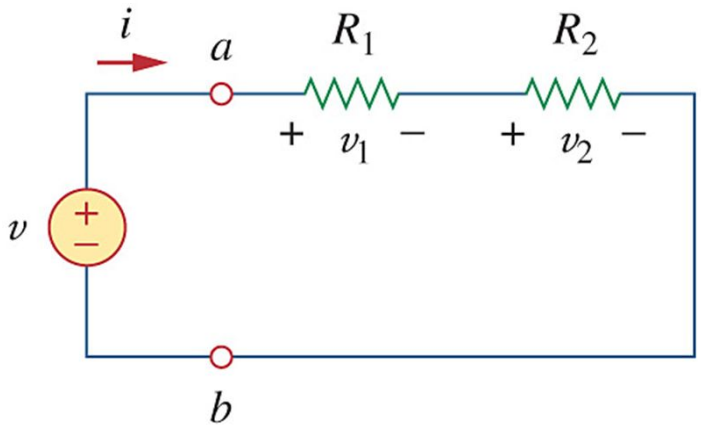
Equivalent Circuit

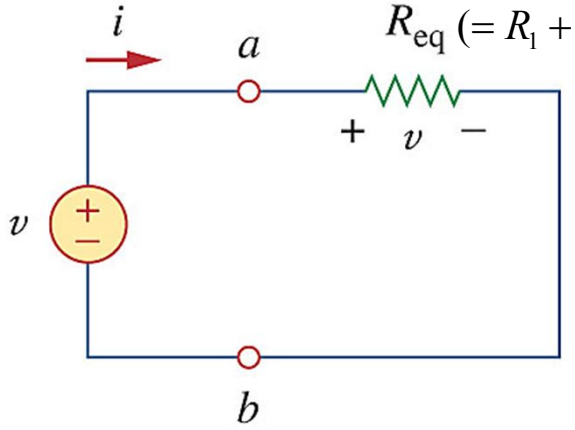


2.5 직렬에서의 전압분배

전압분배기 (Voltage divider)

- 직렬 연결된 저항에서의 voltage drop를 이용하여 원하는 전압을 생성





$$i = \frac{v}{R_1 + R_2}$$

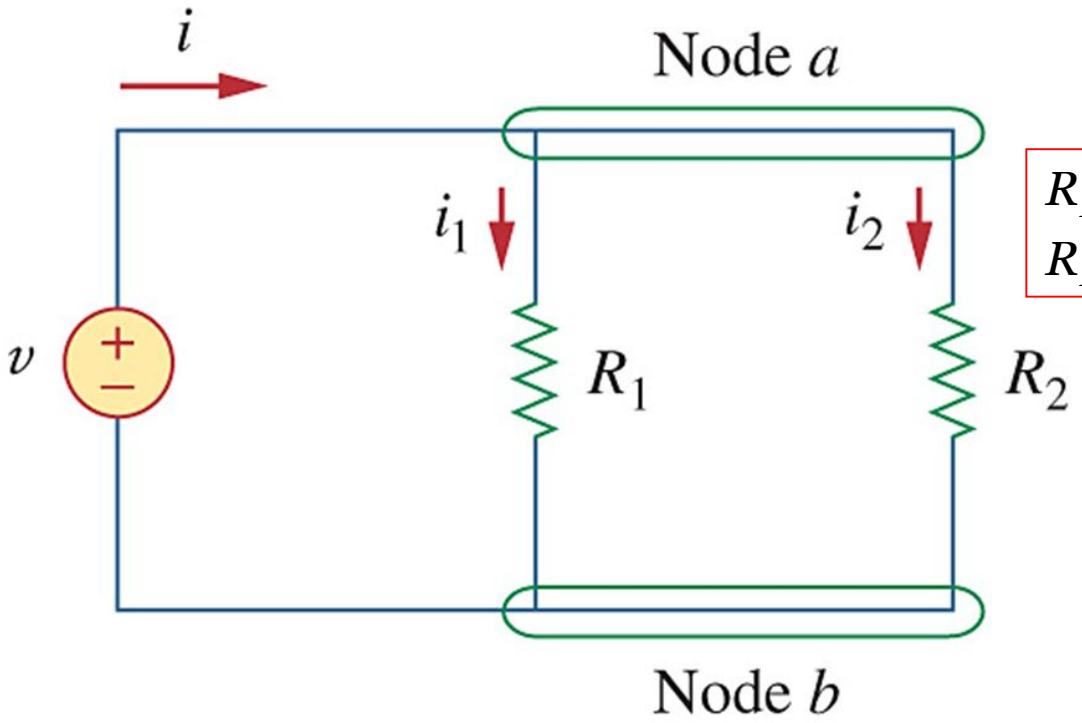
$$v_1 = \frac{R_1}{R_1 + R_2} v \ , \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

큰 저항에 전압이 더 많이 걸린다.!

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

2.6 병렬 저항

- Parallel (병렬)
  - 2개 이상의 element가 같은 2개의 node 사이에 연결
  - node사이에는 동일한 전압이 걸림.



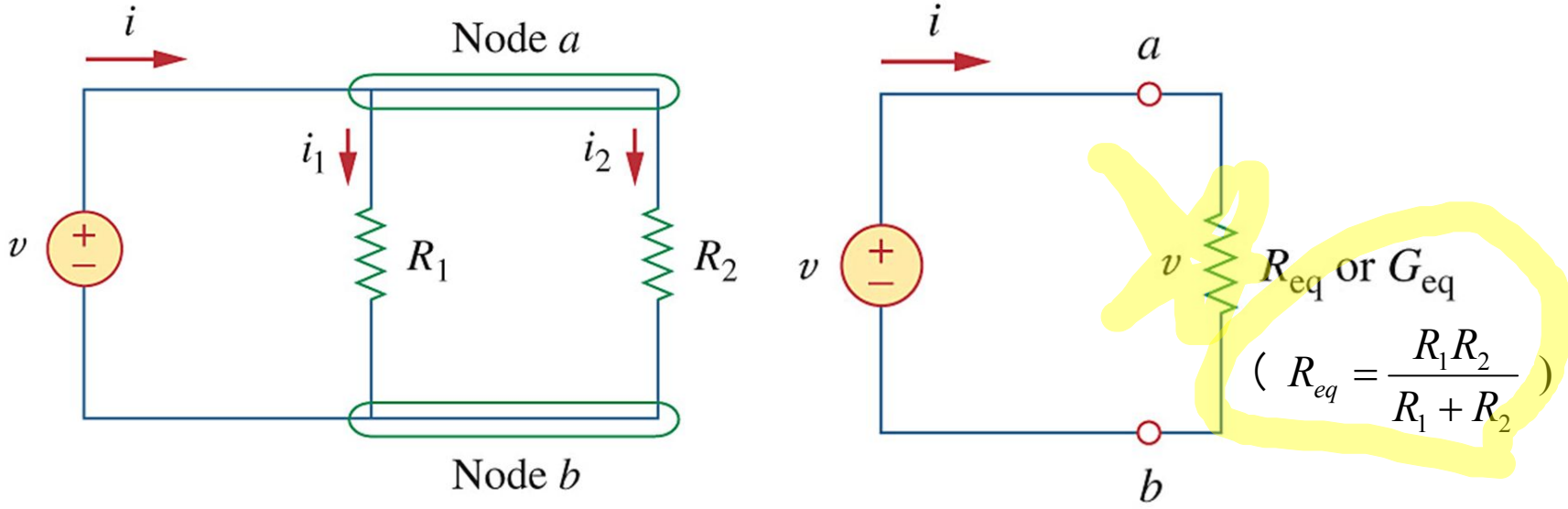
R_1 과 R_2는 **병렬**로 연결됨
R_1 과 R_2 에서는 동일한 **전압 강하**

2.5 병렬에서의 등가 저항

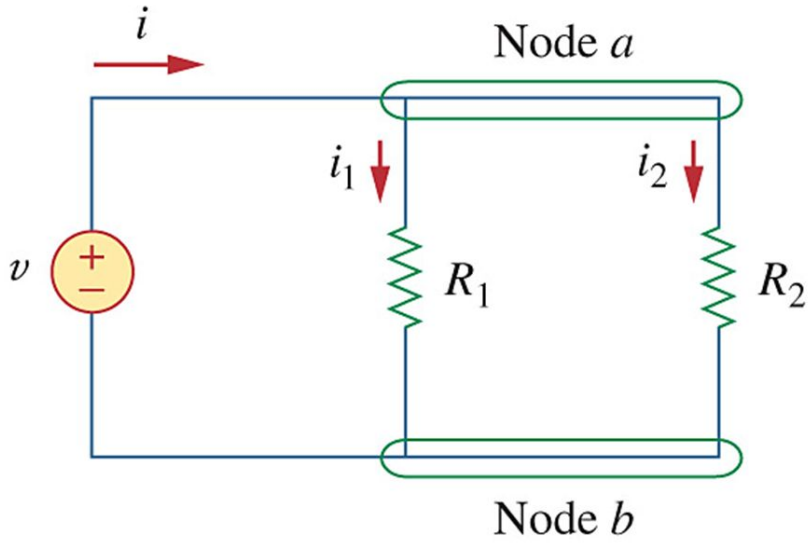
- 병렬 연결된 컨덕턴스의 등가는 **각 컨덕턴스의 합**

$$G_{eq} = G_1 + G_2 + \cdots + G_N = \sum_{n=1}^{N} G_n$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$

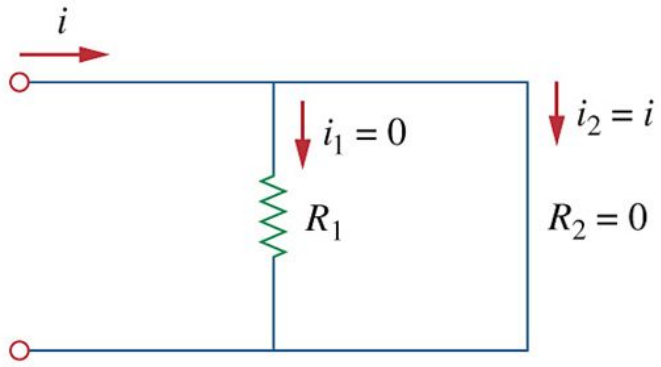


2.5 병렬에서의 전류분배

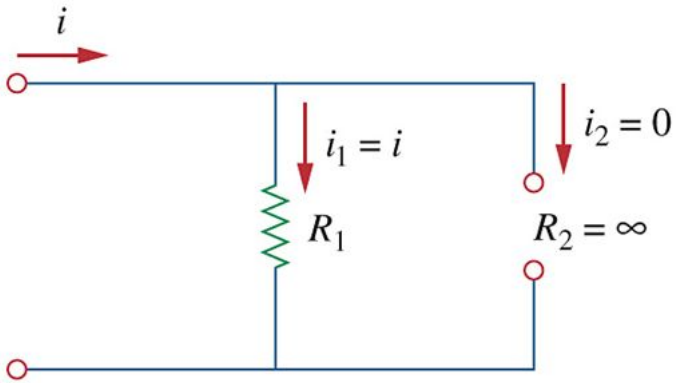


$$i_1 = \frac{R_2}{R_1 + R_2} i$$

$$i_2 = \frac{R_1}{R_1 + R_2} i$$



[단락의 경우]



[개방의 경우]

전류는 저항이 적은 곳으로 더 많이 흐른다.

2.5 직렬저항과 병렬저항

- 직렬저항

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$ ← 가장 큰 저항보다 항상 크다.

$$\text{if } R_1 = R_2 = \cdots R_N = R, \quad R_{eq} = NR$$

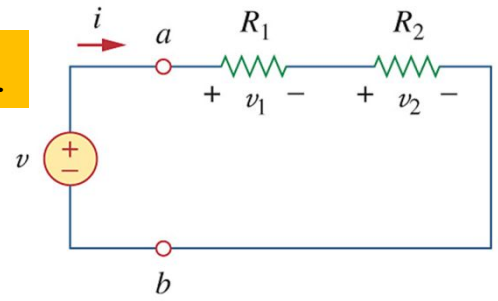
- 병렬저항

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$ ← 가장 작은 저항보다 항상 작다.

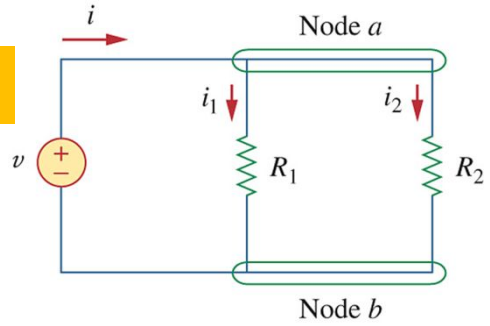
$$\text{if } R_1 = R_2 = \cdots R_N = R, \quad R_{eq} = R/N$$

Resistance vs. Conductance

직렬연결



병렬연결



$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \cdots + \frac{1}{G_N}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

$$G_{eq} = G_1 + G_2 + \cdots + G_N$$

$$v_1 = \frac{R_1 v}{R_1 + R_2}, \qquad v_2 = \frac{R_2 v}{R_1 + R_2}$$

$$i = i_1 = i_2 = \frac{v}{R_1 + R_2}$$

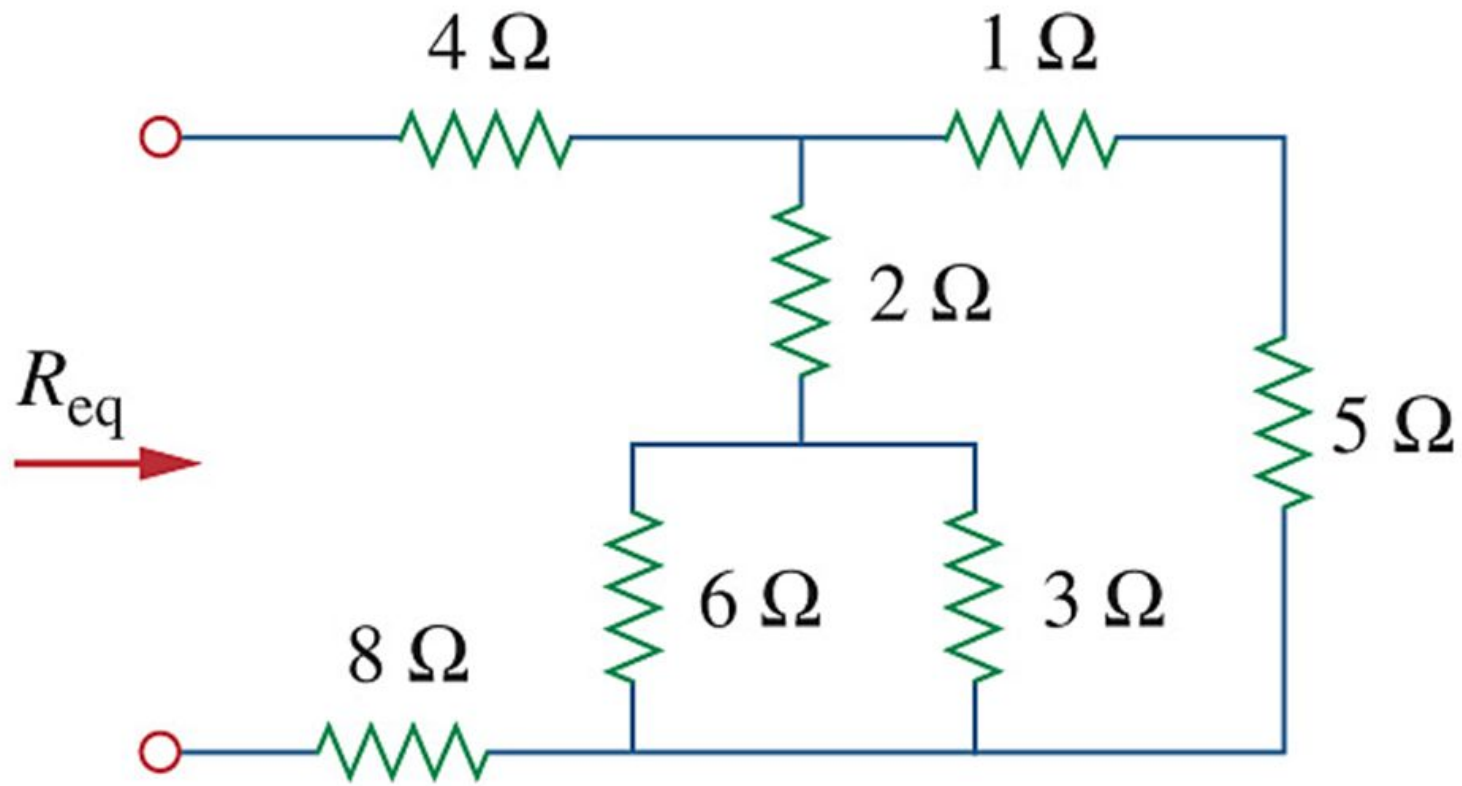
$$v = v_1 = v_2 = \frac{i}{G_1 + G_2}$$

$$i_1 = \frac{G_1 i}{G_1 + G_2}, \qquad i_2 = \frac{G_2 i}{G_1 + G_2}$$

Principle of Duality : v <-> i, R <-> G

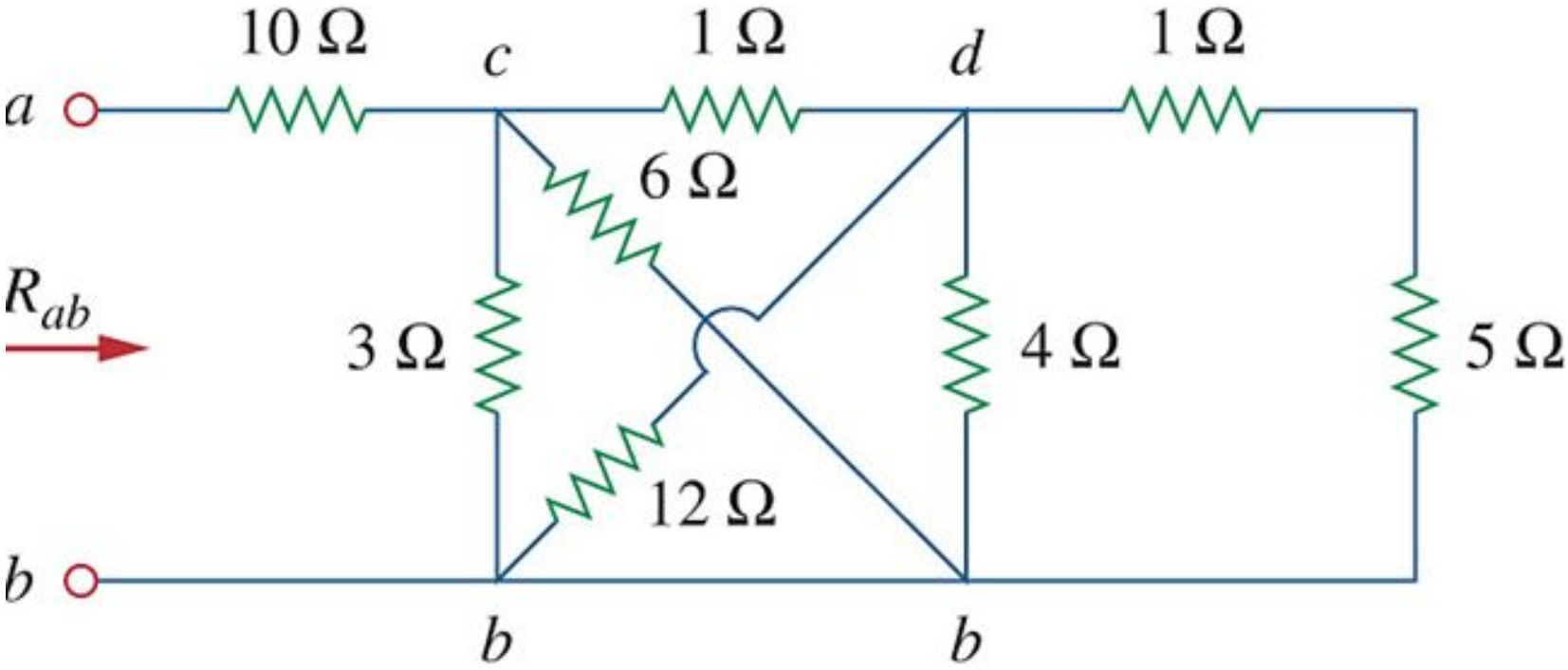
Example 2.9

- 다음 회로에서 R_{eq}를 구하라.



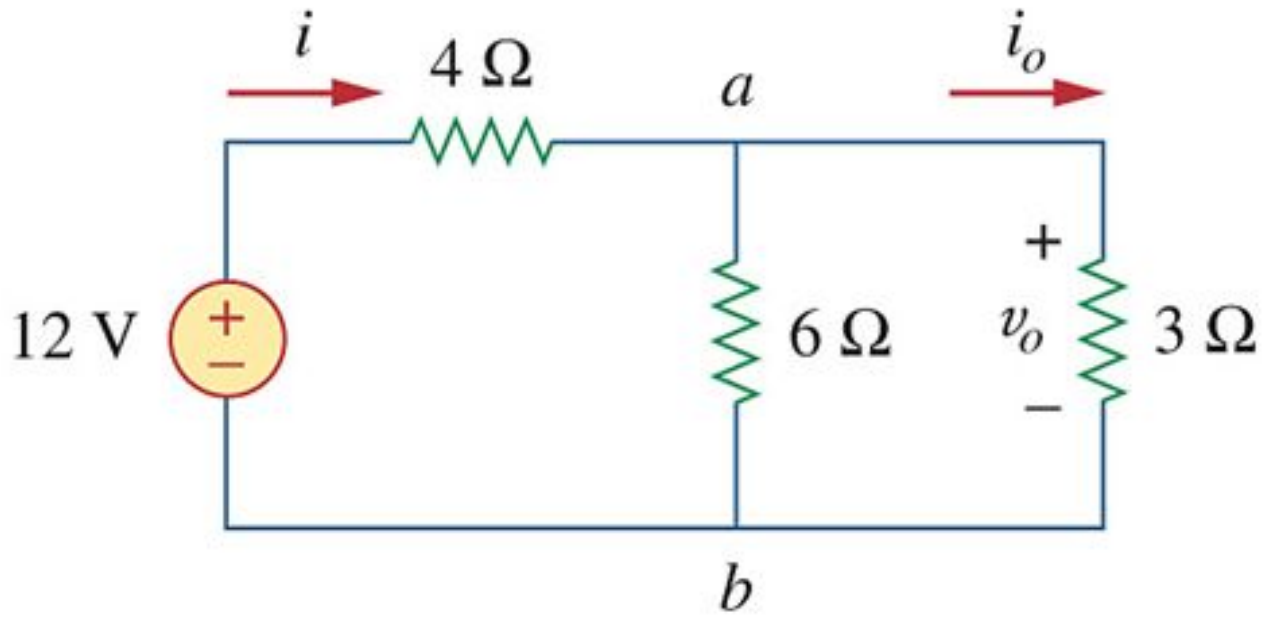
Example 2.10

- 다음 회로에서 등가 저항 R_{ab}를 구하라.



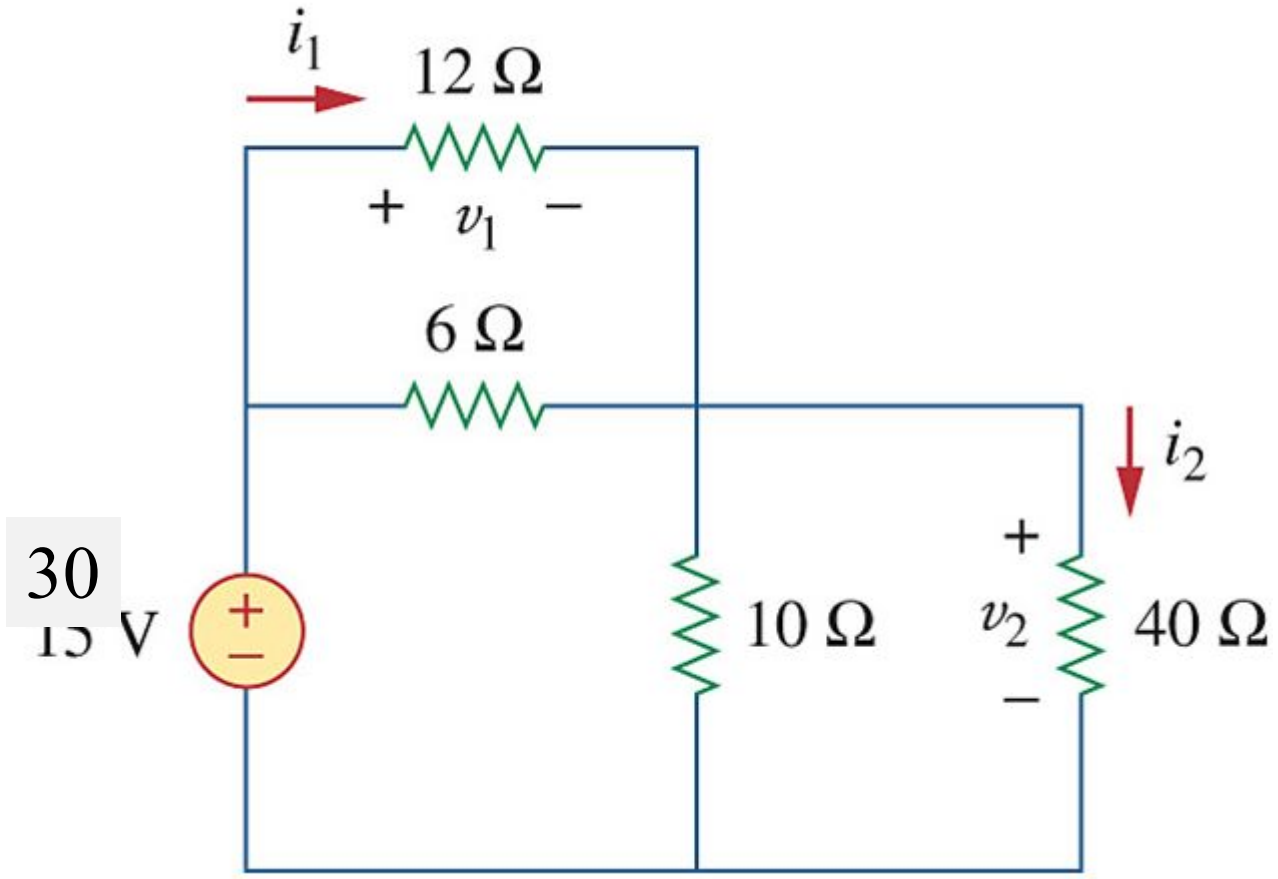
Example 2.12

- 다음 회로에서 i_0, v_0와 3Ω 저항에서 소모된 전력을 구하라.



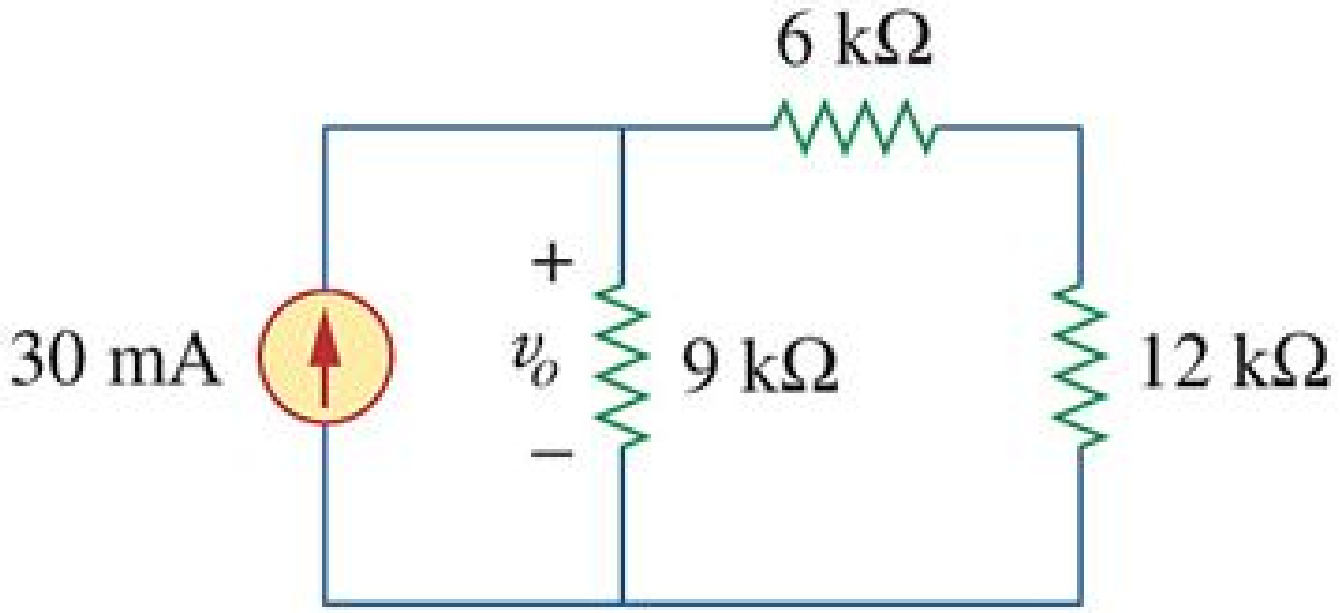
Practice 2.12

- 다음 회로에서 v_1과 v_2를 구하라. 전류 i_1과 i_2 그리고 12 Ω과 40 Ω 저항에서 소모된 전력을 구하라.



Example 2.13

- 전압 v_0를 구하라

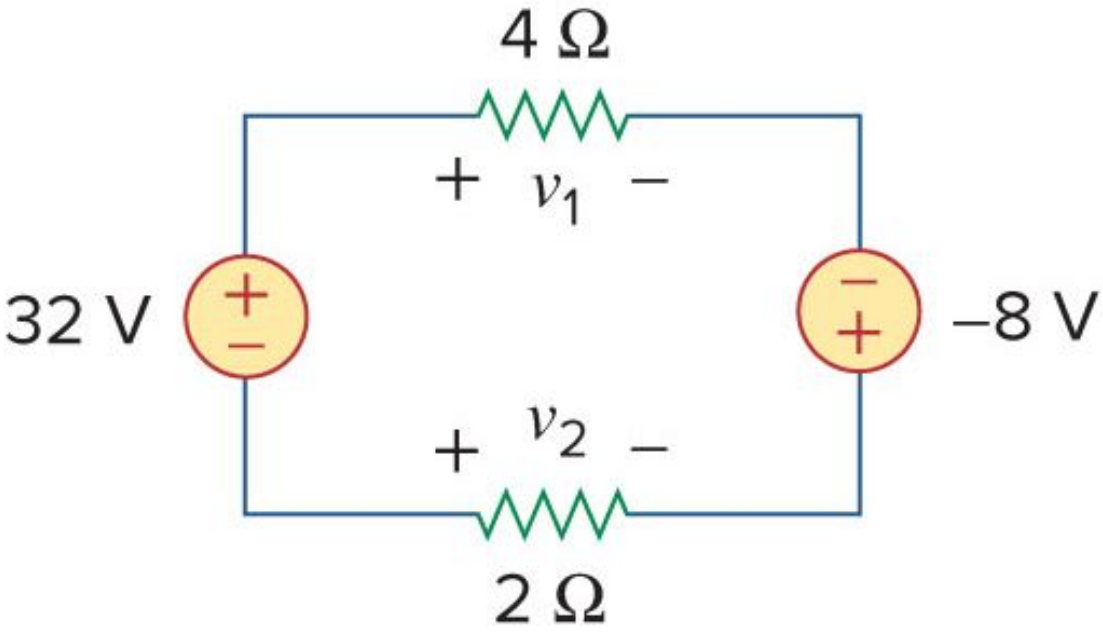


Homework #2

- Due : 다음 수업시간에 제출

#1. Practice(본문 실전문제) 2.5

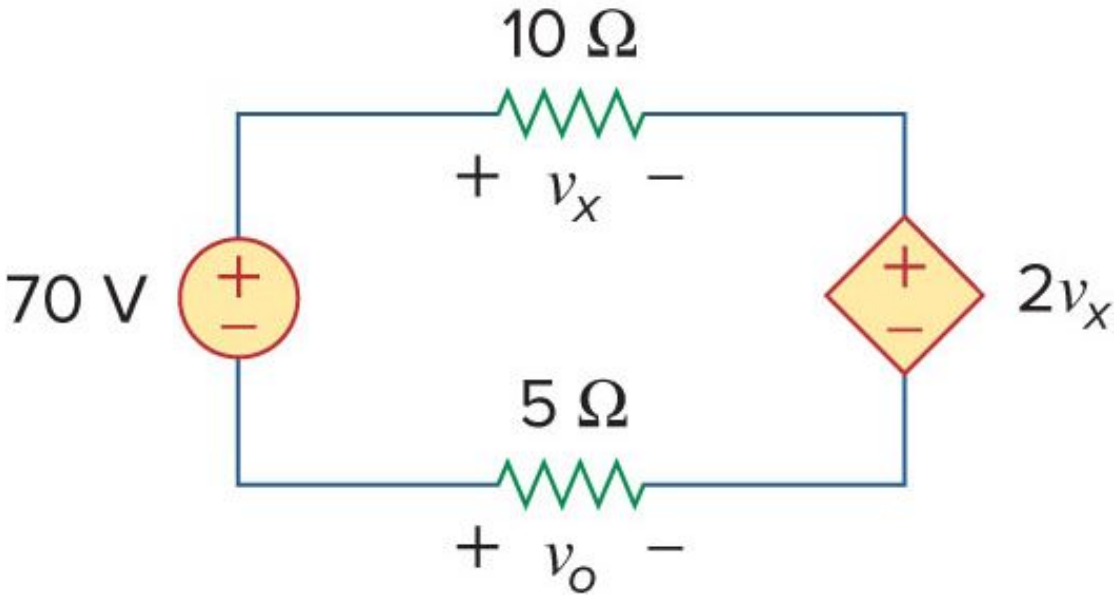
다음 회로에서 전압 v_1과 v_2를 구하라.



Homework #2

#2. Practice (본문 실전문제) 2.6

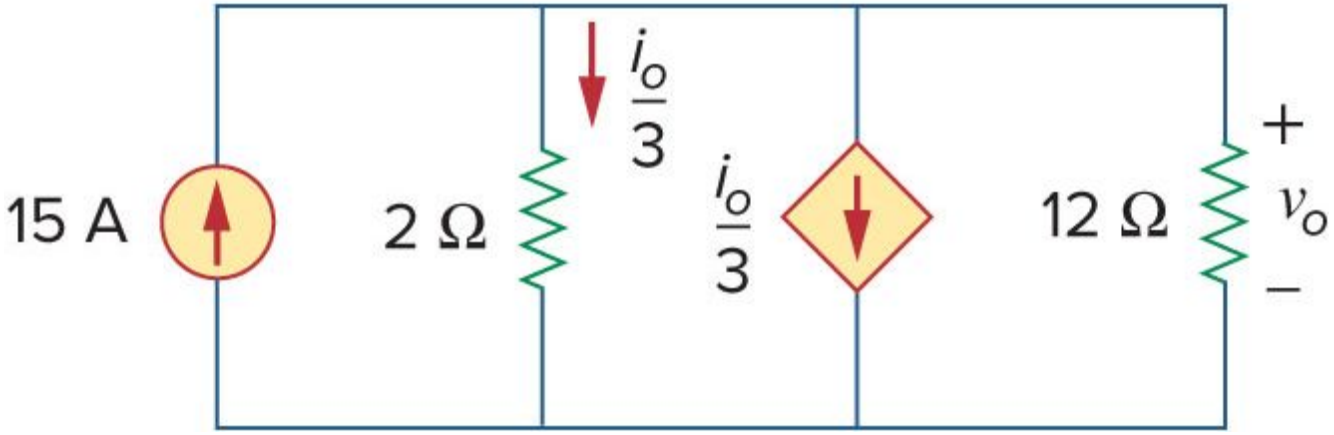
다음 회로에서 전압 v_x과 v_o를 구하라.



Homework #2

#3. Practice(본문 실전문제) 2.7

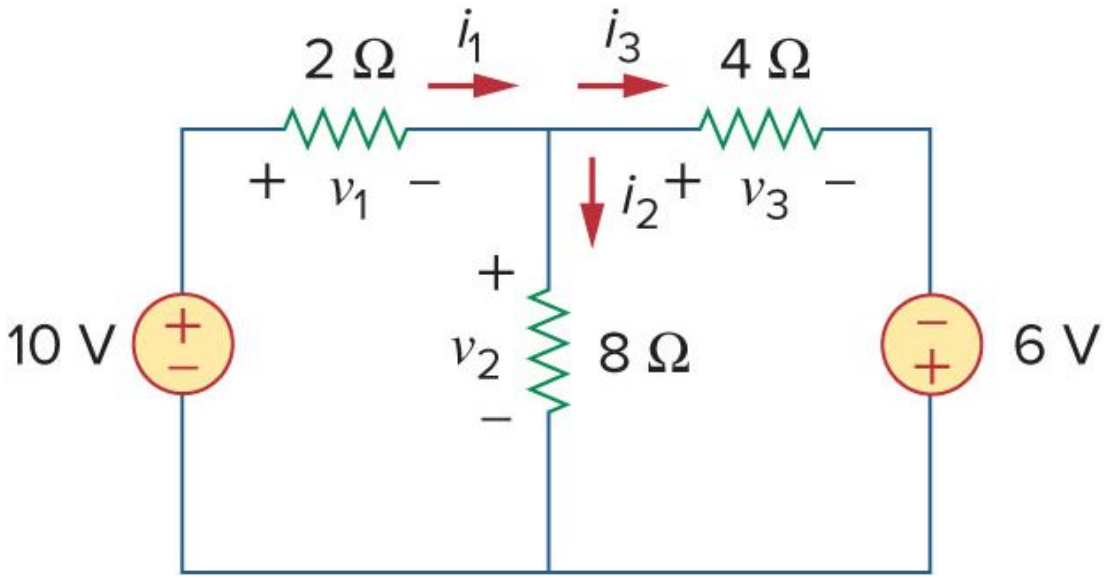
다음 회로에서 전압 v_o와 전류 i_o를 구하라.



Homework #2

#4. Practice(본문 실전문제) 2.8

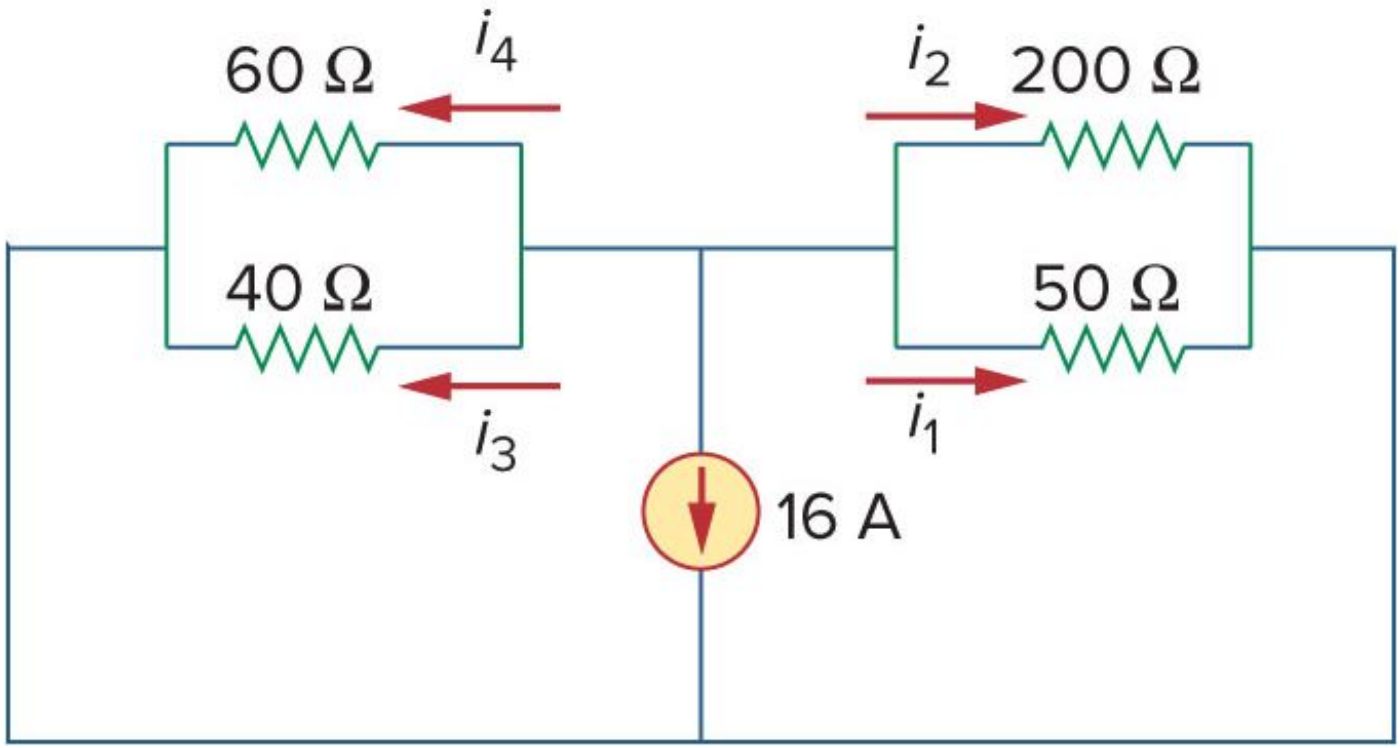
다음 회로에서 전류와 전압을 구하라.



Homework #2

#5. Problem(문제) 2.32

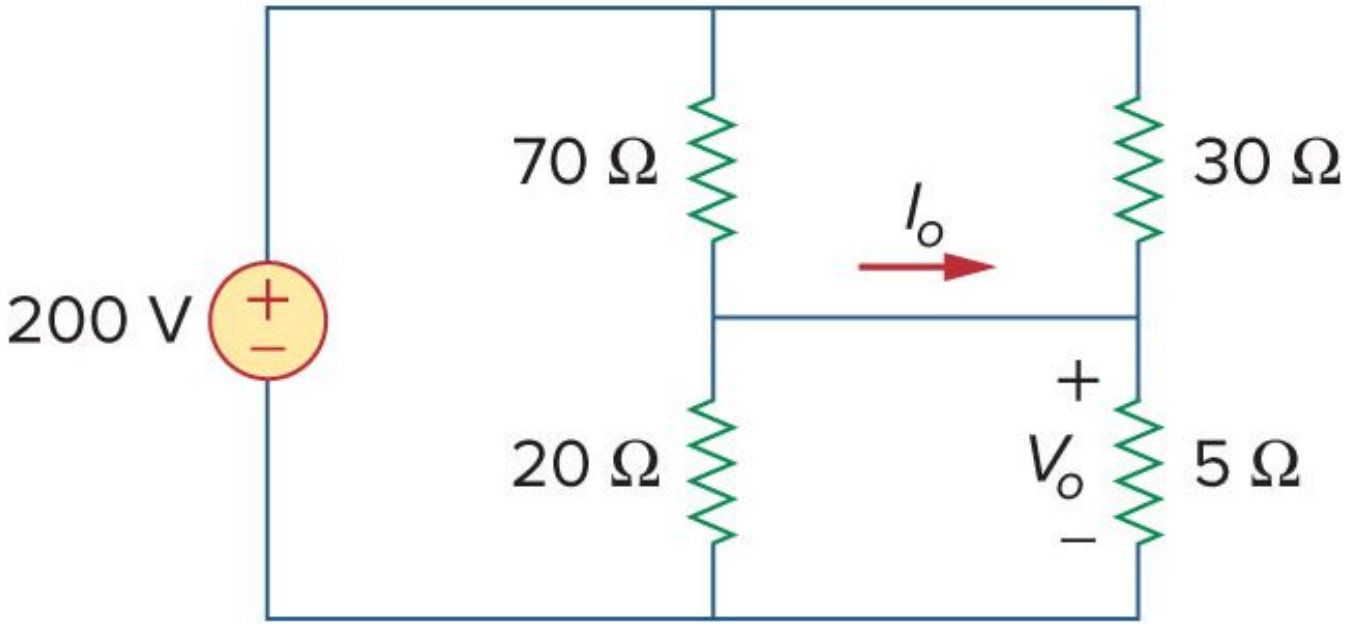
다음 회로에서 전류 $i_1 \sim i_4$를 구하라.



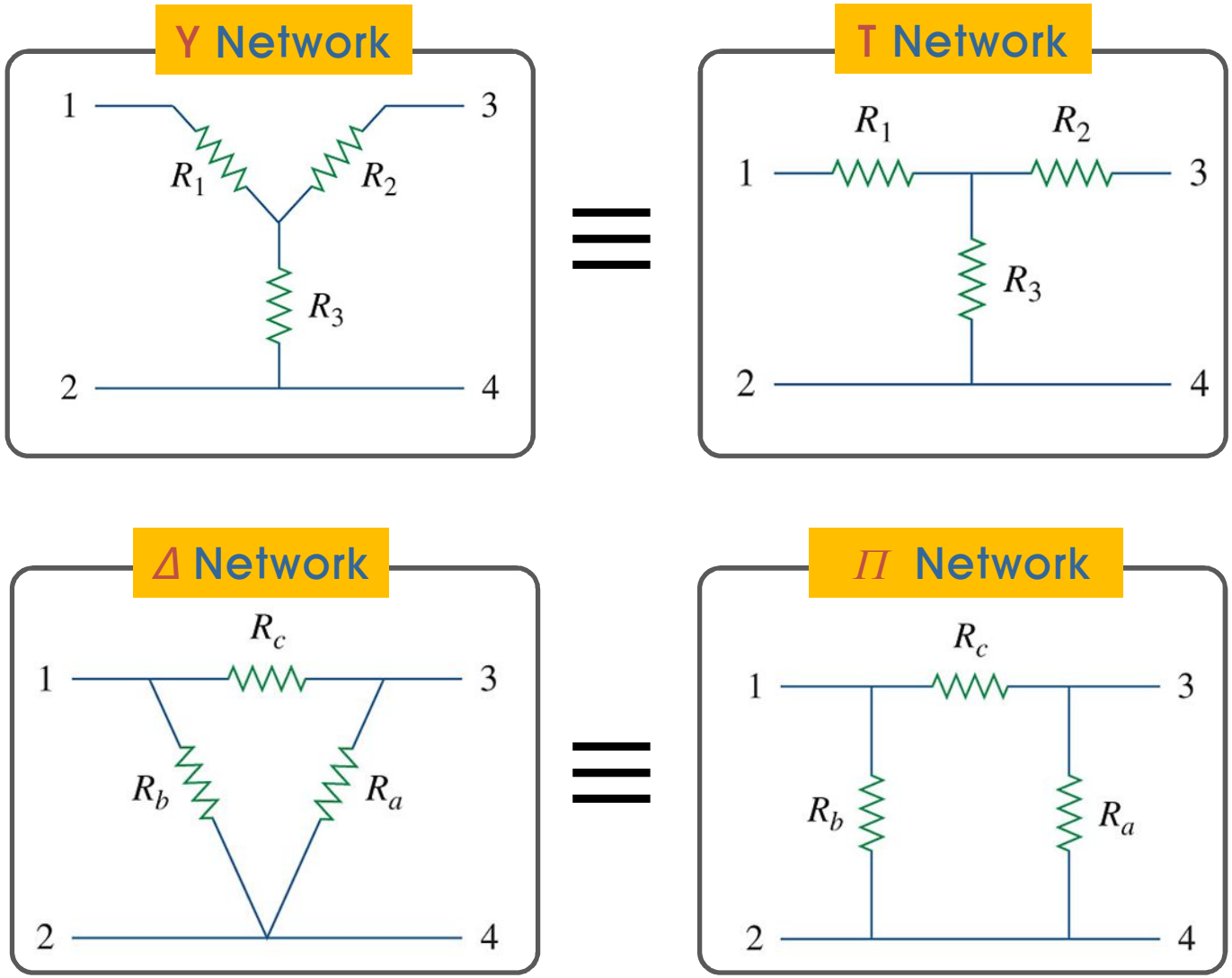
Homework #2

#6. Problem(문제) 2.35

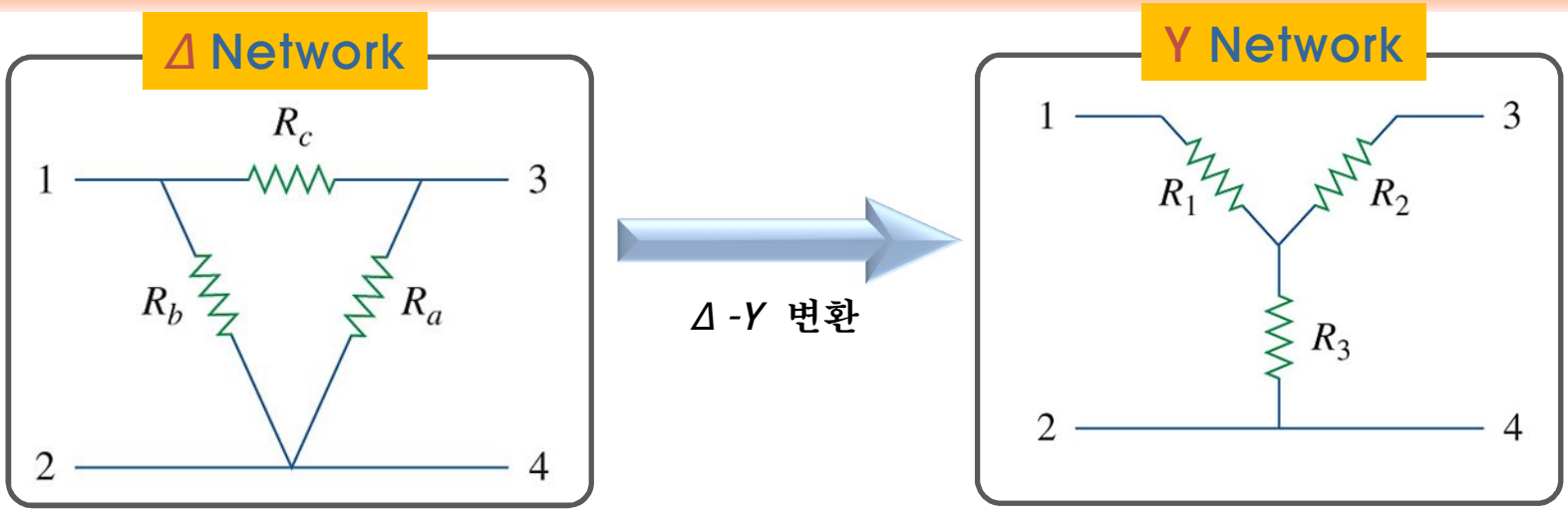
다음 회로에서 V_o와 I_o를 구하라.



2.7 와이-델타 변환


Y Network
1
3
R_1
R_2
R_3
2
4
≡
T Network
R_1
R_2
1
3
R_3
2
4
Δ Network
R_c
1
3
R_b
R_a
2
4
≡
Π Network
R_c
1
3
R_b
R_a
2
4

2.7 델타-와이 변환



$$R_{12}(\Delta) = R_b \,||\, (R_a + R_c) \qquad R_{12}(\Delta) = R_{12}(Y) \qquad R_{12}(Y) = R_1 + R_3$$

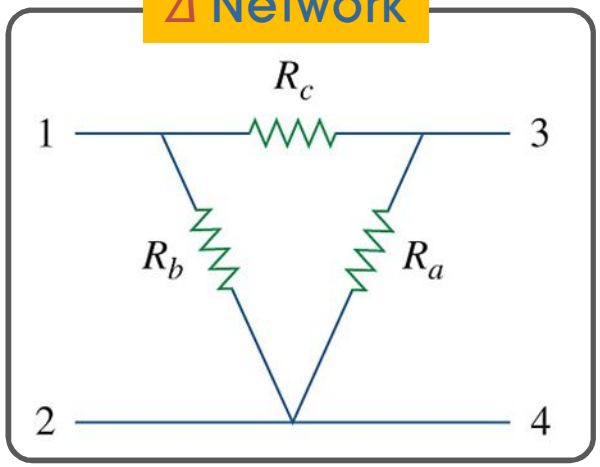
$$R_{13}(\Delta) = R_c \,||\, (R_a + R_b) \qquad R_{13}(\Delta) = R_{13}(Y) \qquad R_{13}(Y) = R_1 + R_2$$

$$R_{34}(\Delta) = R_a \,||\, (R_b + R_c) \qquad R_{34}(\Delta) = R_{34}(Y) \qquad R_{34}(Y) = R_2 + R_3$$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

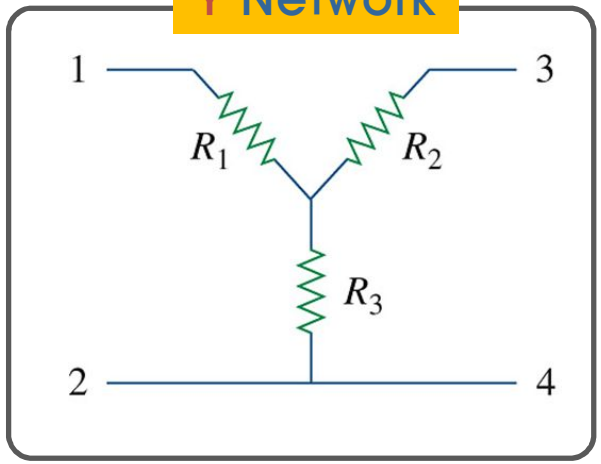
2.7 델타-와이 변환

Δ Network

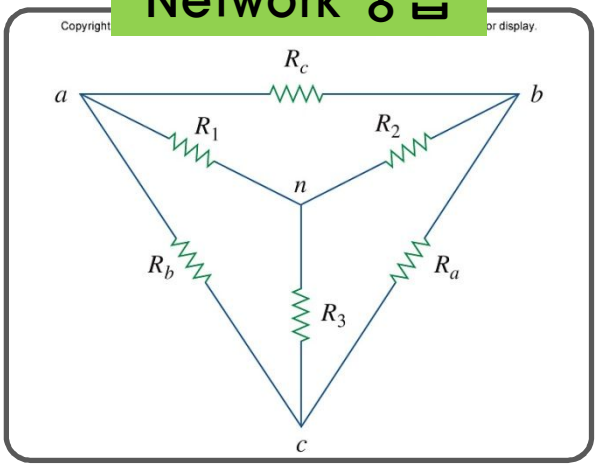


Δ -Y 변환

Y Network



Network 중첩

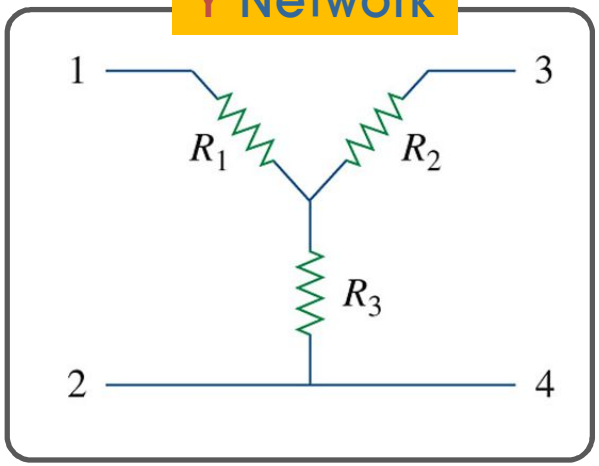


Y의 저항= $\dfrac{\text{인접하는 } \Delta \text{의 두 저항의 곱}}{\Delta \text{ 저항의 합}}$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

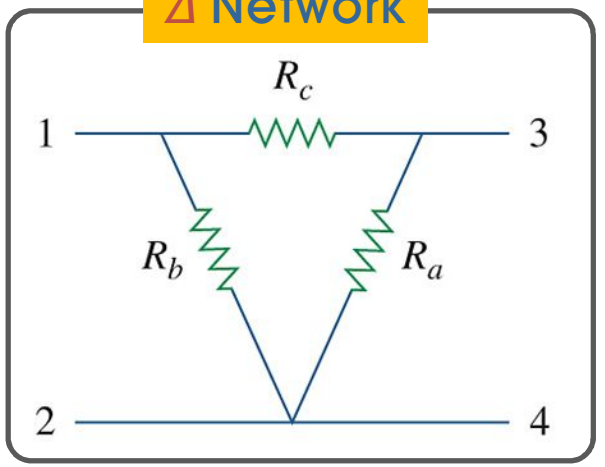
2.7 와이-델타 변환

Y Network

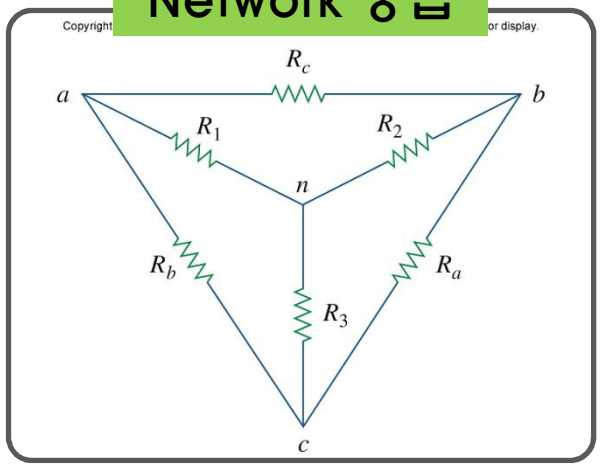


Y-Δ 변환

Δ Network



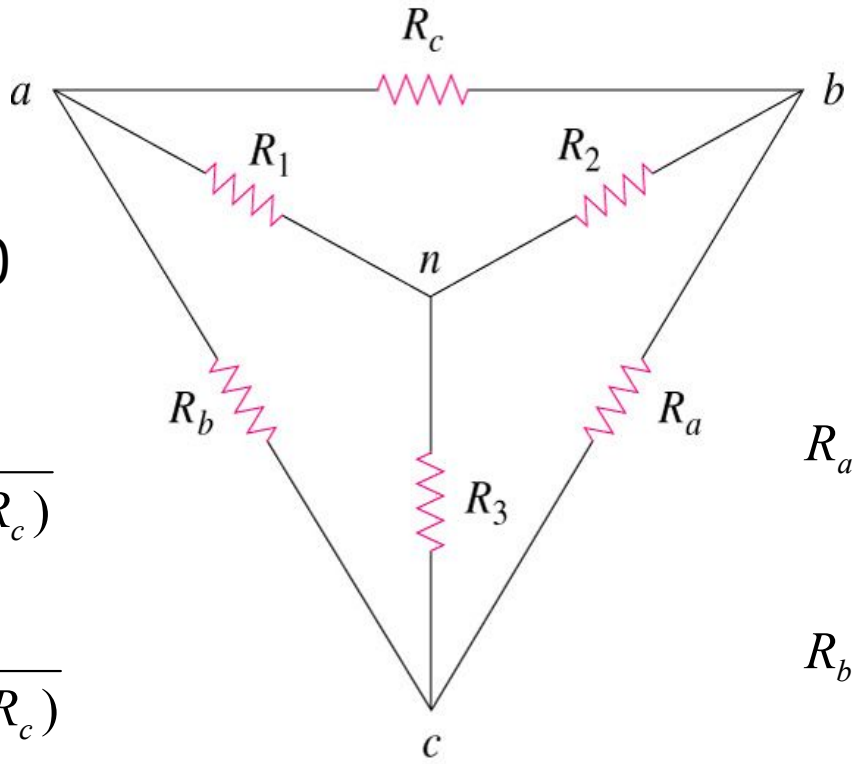
Network 중첩



$$\Delta \text{의 저항} = \frac{\text{모든 조합으로 두 저항 곱의 합}}{\text{반대편의 Y 저항}}$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

2.7 와이-델타 변환 (Summary)



Delta → Y (Star)

$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)}$$

$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)}$$

$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

Y (Star) → Delta

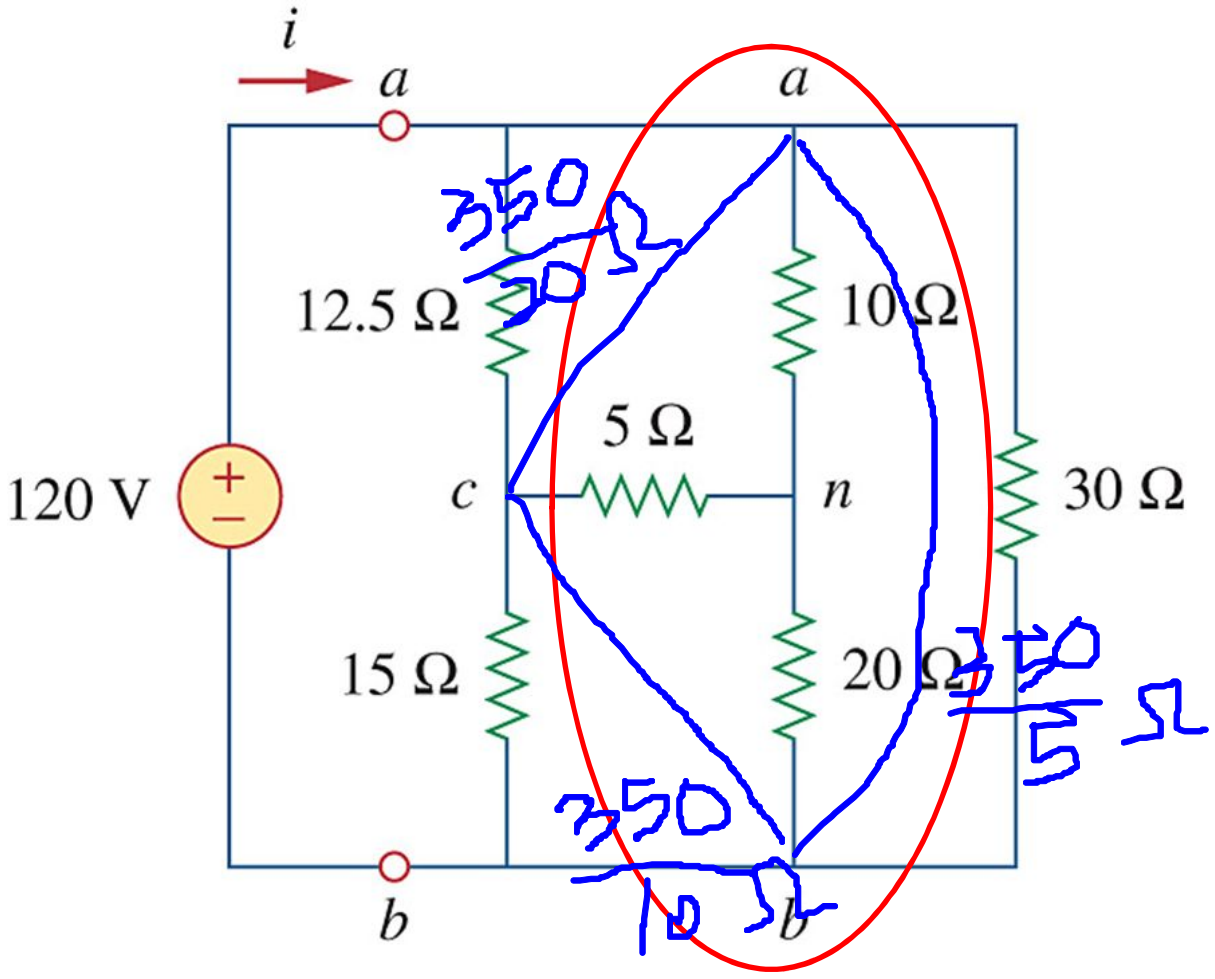
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

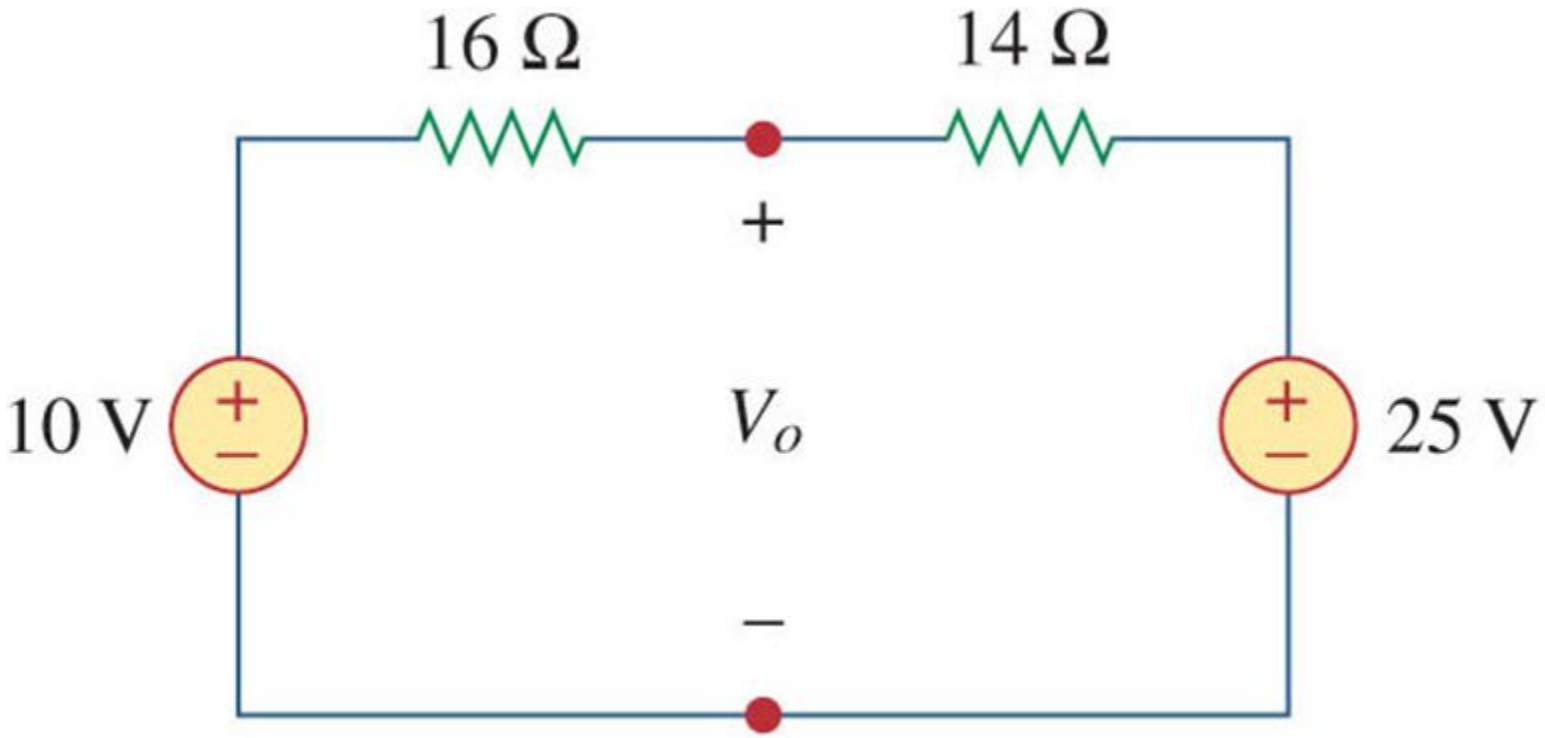
Example 2.15

- 와이-델타 변환 이용



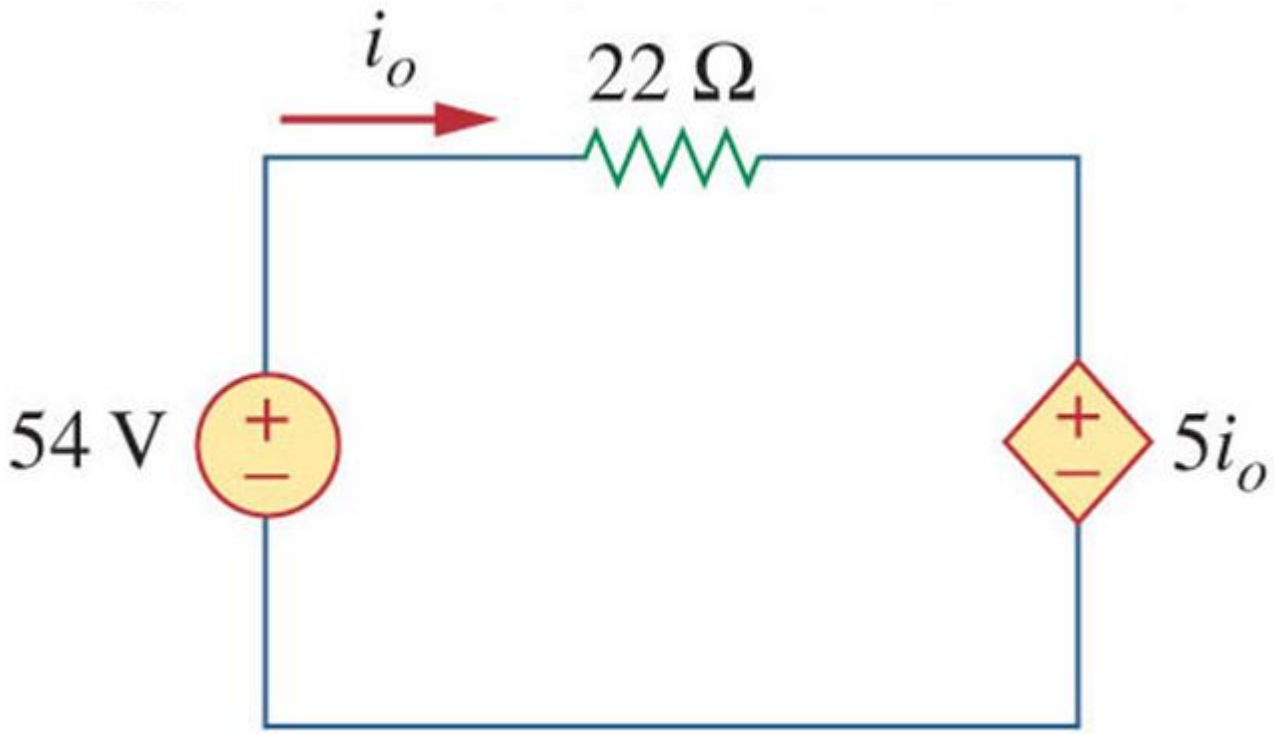
Problem 2.16

- 다음 회로에서 전압 V_o를 구하라.



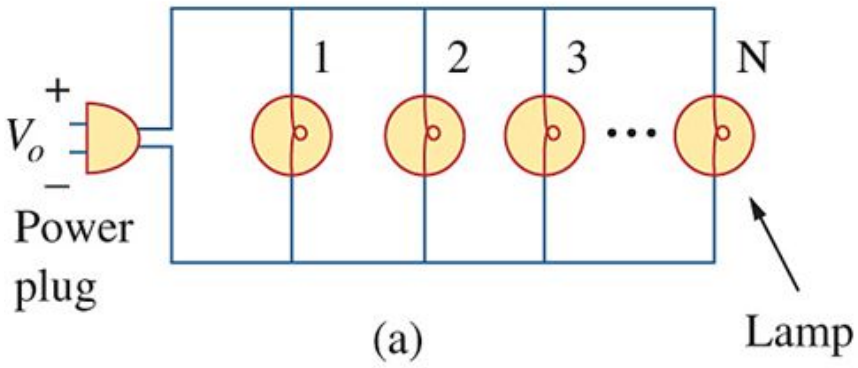
Problem 2.20

- 다음 회로에서 전류 i_o를 구하라.

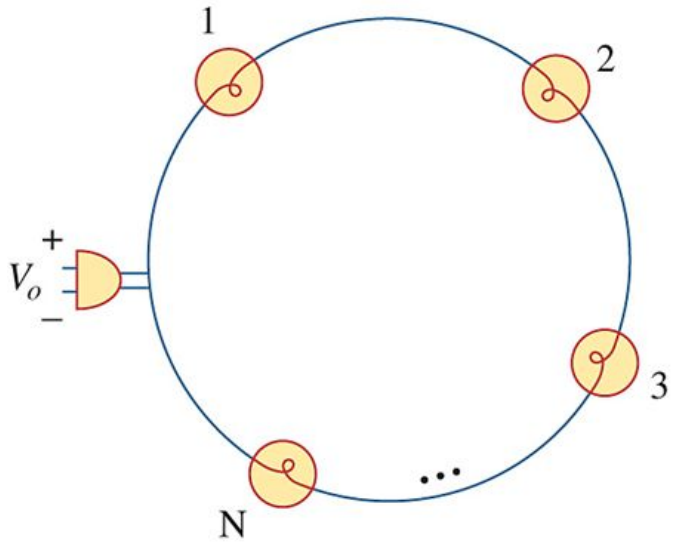


2.8 응용 : 조명 시스템

- Quiz
  - 직렬연결이 좋은가? 아니면 병렬연결이 좋은가?



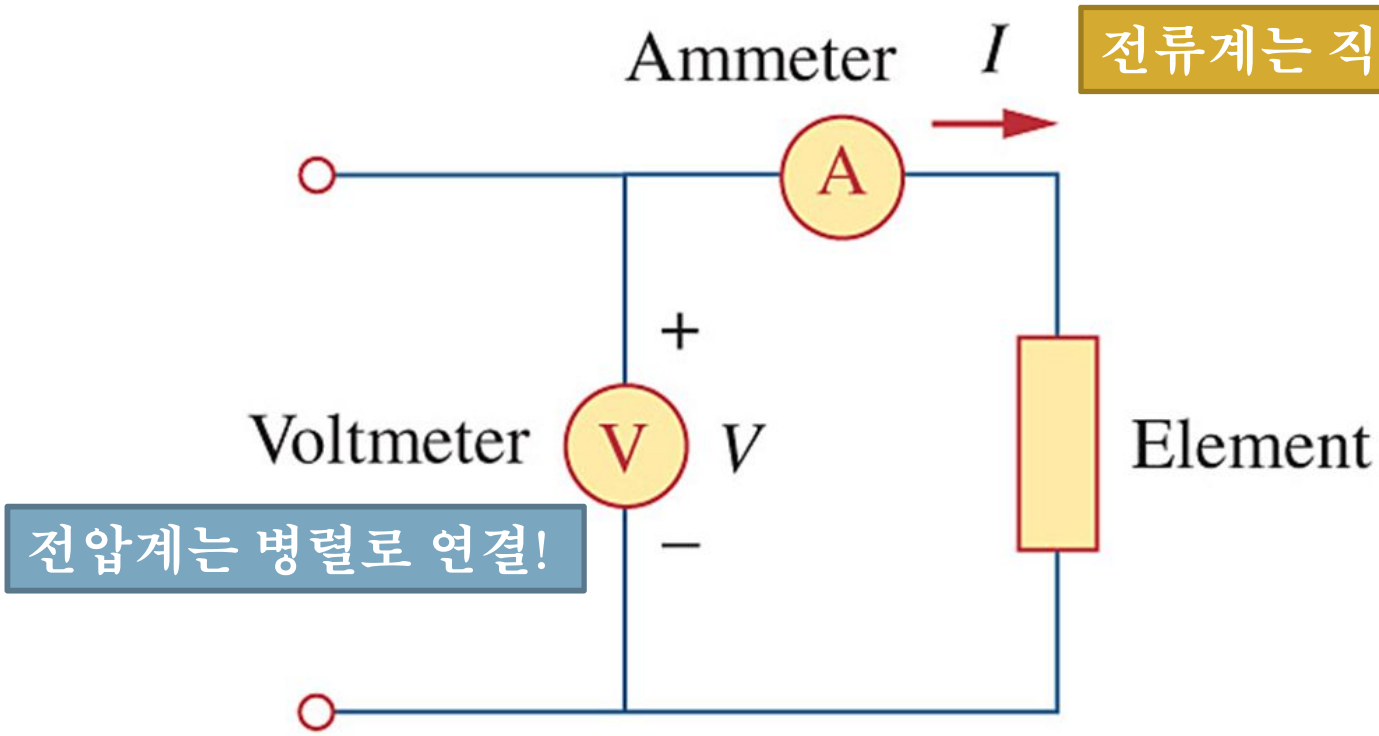
(a)



(b)

Why?
- 고장 나도 하나만 문제!
- 고장 원인 확인 쉬워!
- 물건 만들기도 쉬워!

2.8 응용 : 전압계와 전류계



전류계는 직렬로 연결!

전압계는 병렬로 연결!

- Quiz : 전압계와 전류계의 내부저항의 이상적인 값은?

전압계 : ∞
전류계 : 0

Homework #3

- Problem(문제) 2.15, 2.17, 2.21, 2,22
- Due : 다음 수업시간까지

- **Quiz #1**
  - **Homework #2, #3 중에서 출제**
  - **다음 수업시간 (10분)**

Homework #3

- Problem(문제) 2.15, 2.17, 2.21, 2,22

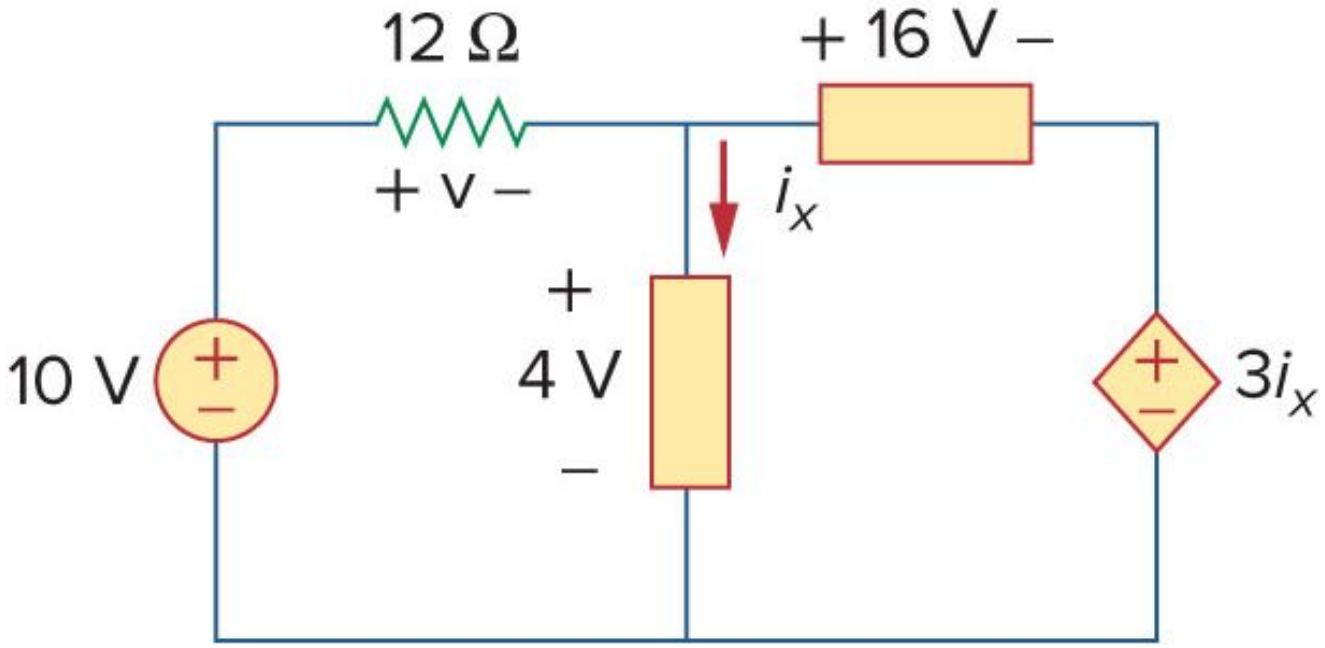
- Due : 다음 수업시간까지

- **Quiz #1**
  - **Homework #2, #3 중에서 출제**
  - **다음 수업시간 (10분)**

Homework #3

#1. Problem(문제) 2.15

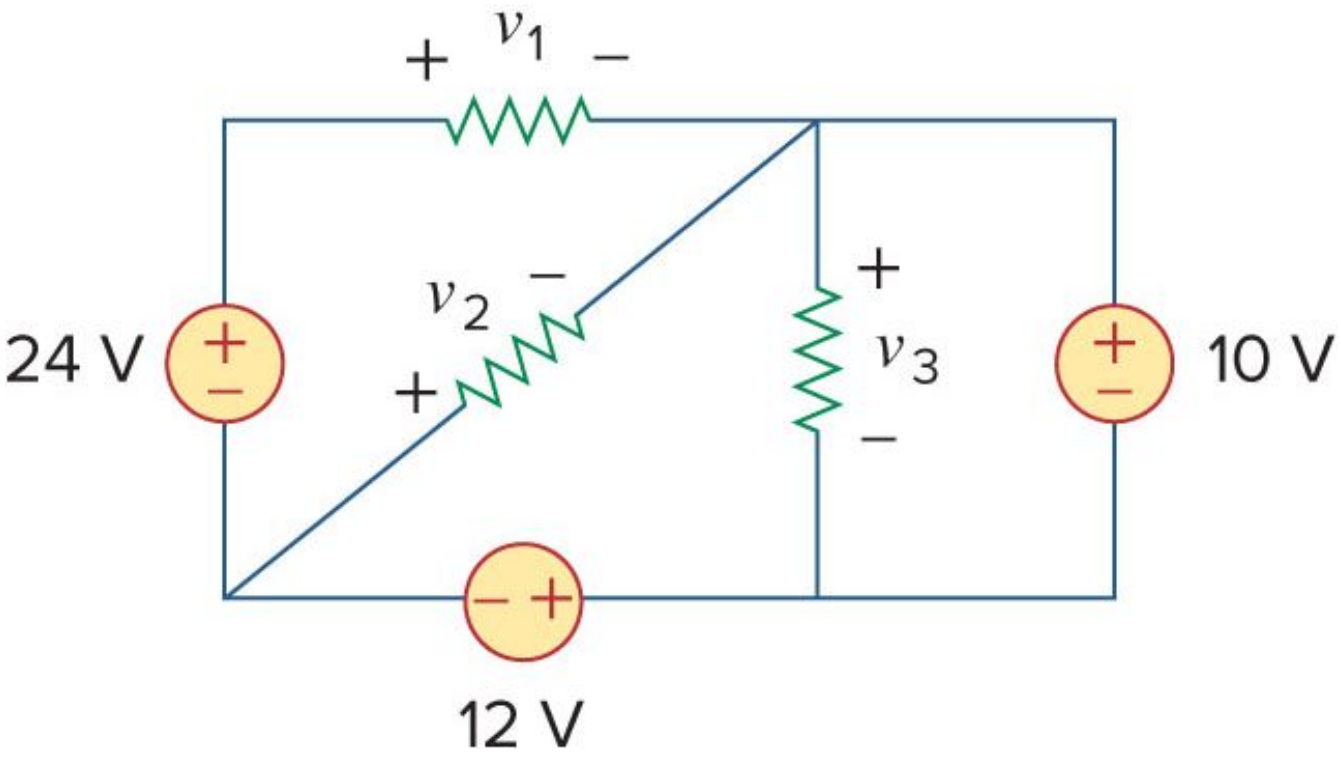
다음 회로에서 전압 v와 전류 i_x를 구하라.



Homework #3

#1. Problem(문제) 2.17

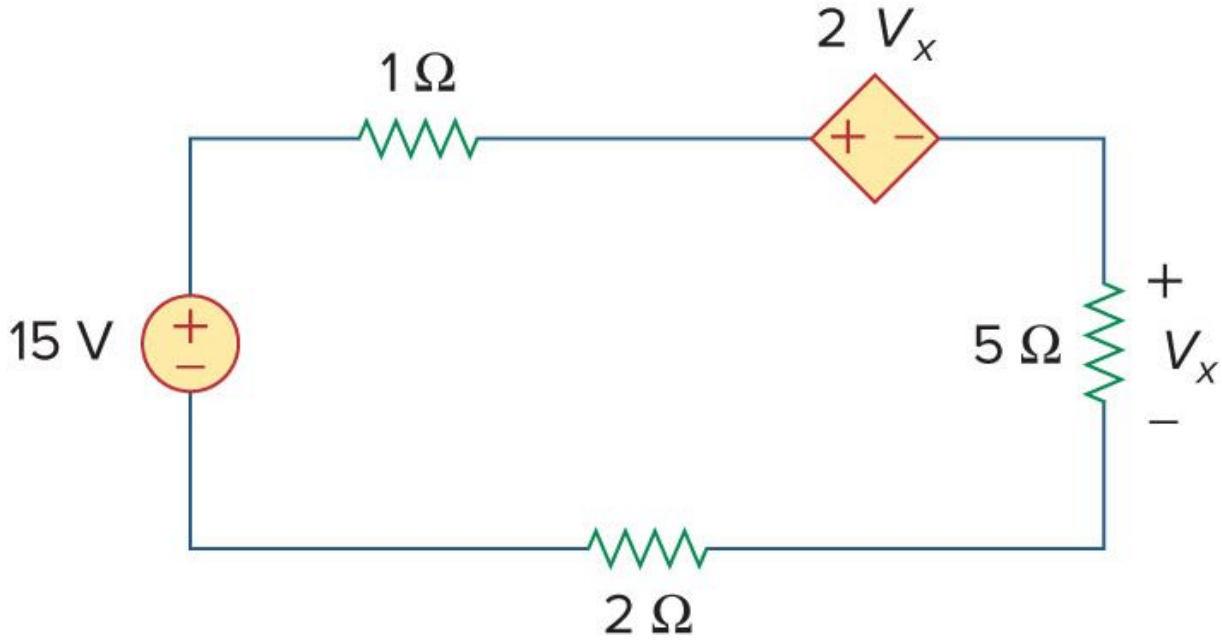
다음 회로에서 전압 v_1, v_2, v_3를 구하라.



Homework #3

#3. Problem(문제) 2.21

다음 회로에서 전압 V_x를 구하라.



Homework #3

#4. Problem(문제) 2.22

다음 회로에서 전압 V_o와 종속 전원에 의해 소모되는 전력을 구하라.