Chapter 13, Solution 1.

For coil 1, $L_1 - M_{12} + M_{13} = 12 - 8 + 4 = 8$

For coil 2, $L_2 - M_{21} - M_{23} = 16 - 8 - 10 = -2$

For coil 3, $L_3 + M_{31} - M_{32} = 20 + 4 - 10 = 14$

$$L_T = 8 - 2 + 14 = 20\text{H}$$

or $$L_T = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{13}$$

$$L_T = 12 + 16 + 20 - 2\text{x}8 - 2\text{x}10 + 2\text{x}4 = 48 - 16 - 20 + 8$$

$$= \mathbf{20H}$$

Chapter 13, Solution 2.

Using Fig. 13.73, design a problem to help other students to better understand mutual inductance.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the inductance of the three series-connected inductors of Fig. 13.73.

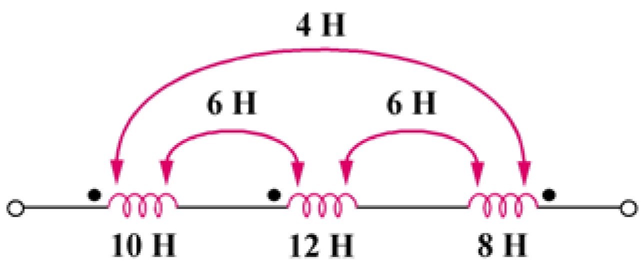


Figure 13.73

Solution

$L = L_1 + L_2 + L_3 + 2M_{12} – 2M_{23} –2M_{31}$

$= 10 + 12 +8 + 2x6 – 2x6 –2x4$

$=$ **22H**

Chapter 13, Solution 3.

$L_1 + L_2 + 2M = 500$ mH (1)

$L_1 + L_2 - 2M = 300$ mH (2)

Adding (1) and (2),

$$2L_1 + 2L_2 = 800 \text{ mH}$$

But, $L_1 = 3L_2$,, or $8L_2 + 400$, and $L_2 =$ **100 mH**

$$L_1 = 3L_2 = \mathbf{300\ mH}$$

From (2), $150 + 50 - 2M = 150$ leads to $M =$ **50 mH**

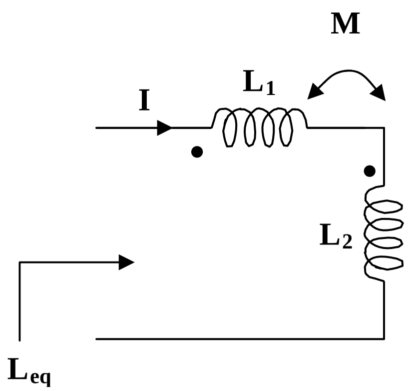
$$k = M/\sqrt{L_1 L_2} = 50/\sqrt{100x300} = \mathbf{0.2887}$$

300 mH, 100 mH, 50 mH, 0.2887

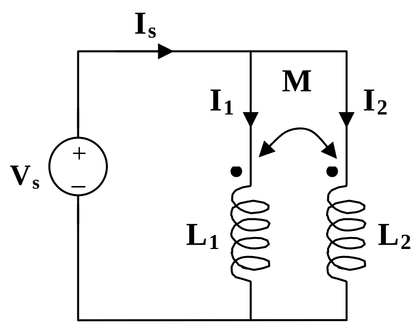
Chapter 13, Solution 4.

(a) For the series connection shown in Figure (a), the current I enters each coil from its dotted terminal. Therefore, the mutually induced voltages have the same sign as the self-induced voltages. Thus,

$$L_{eq} = \mathbf{L_1 + L_2 + 2M}$$



(a)



(b)

(b) For the parallel coil, consider Figure (b).

$$I_s = I_1 + I_2 \quad \text{and} \quad Z_{eq} = V_s/I_s$$

Applying KVL to each branch gives,

$$V_s = j\omega L_1 I_1 + j\omega M I_2 \tag{1}$$

$$V_s = j\omega M I_1 + j\omega\, L_2 I_2 \tag{2}$$

or
$$\begin{bmatrix} V_s \\ V_s \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = -\omega^2 L_1 L_2 + \omega^2 M^2, \quad \Delta_1 = j\omega V_s(L_2 - M), \quad \Delta_2 = j\omega V_s(L_1 - M)$$

$$I_1 = \Delta_1/\Delta, \quad \text{and} \quad I_2 = \Delta_2/\Delta$$

$$I_s = I_1 + I_2 = (\Delta_1 + \Delta_2)/\Delta = j\omega(L_1 + L_2 - 2M)V_s/(-\omega^2(L_1L_2 - M^2))$$
$$= (L_1 + L_2 - 2M)V_s/(j\omega(L_1L_2 - M^2))$$

$$Z_{eq} = V_s/I_s = j\omega(L_1L_2 - M^2)/(L_1 + L_2 - 2M) = j\omega L_{eq}$$

i.e., $$L_{eq} = \mathbf{(L_1L_2 - M^2)/(L_1 + L_2 - 2M)}$$

Chapter 13, Solution 5.

(a) If the coils are connected in series,

$$L = L_1 + L_2 + 2M = 50 + 120 + 2(0.5)\sqrt{50x120} = \mathbf{247.4\ mH}$$

(b) If they are connected in parallel,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{50x120 - 38.72^2}{50 + 120 - 2x38.72}\ \text{mH} = \mathbf{48.62\ mH}$$

(a) 247.4 mH, (b) 48.62 mH

Chapter 13, Solution 6.

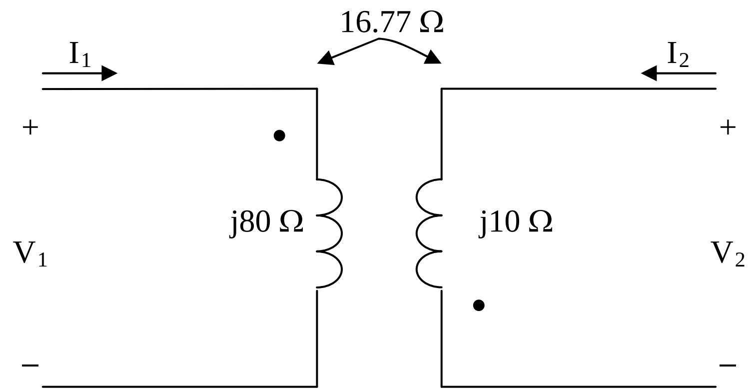
$M = k\sqrt{L_1 L_2} = 0.6\sqrt{40x5} = 8.4853$ mH

$40mH \quad \longrightarrow \quad j\omega L = j2000x40x10^{-3} = j80$

$5mH \quad \longrightarrow \quad j\omega L = j2000x5x10^{-3} = j10$

$8.4853mH \quad \longrightarrow \quad j\omega M = j2000x8.4853x10^{-3} = j16.97$

We analyze the circuit below.



$$V_1 = j80I_1 - j16.97I_2 \quad (1)$$
$$V_2 = -16.97I_1 + j10I_2 \quad (2)$$

But, $V_1 = 20\angle 0^\circ$ V and $I_2 = 4\angle -90^\circ$ A. Substituting these into (1) produces

$I_1 = [(V_1+j16.97I_2)/j80] = [(20+j16.97(-j4))/j80] = 1.0986\angle -90^\circ$ A or

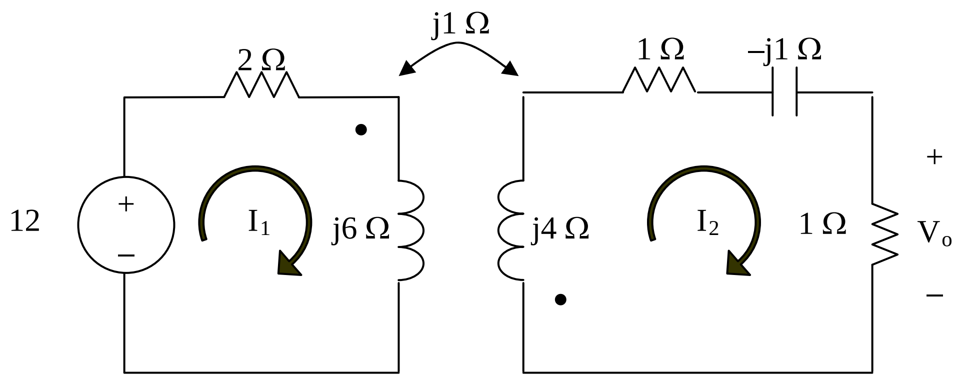
i_1 = **1.0986sin(ωt) A**

From (2), $V_2 = -16.97x(-j1.0986) + j10(-j4) = 40 + j18.643 = 44.13\angle 25^\circ$ or

v_2 = **44.13cos(ωt+25˚) V**.

Chapter 13, Solution 7.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$(2+j6)I_1 + jI_2 = 24$

For mesh 2,

$jI_1 + (2–j+j4)I_2 = jI_1 + (2+j3)I_2 = 0$ or $I_1 = (–3+j2)I_2$

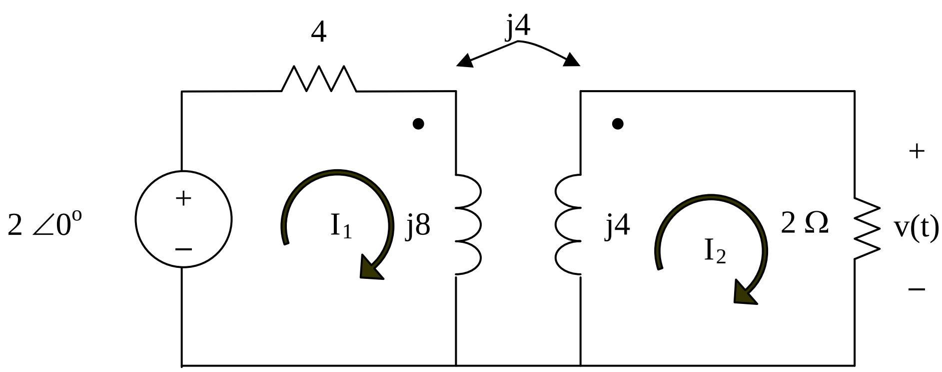
Substituting into the first equation results in $I_2 = (–0.8762+j0.6328)$ A.

$V_o = I_2 x1 =$ **$1.081\angle 144.16^\circ$ V**.

Chapter 13, Solution 8.

$$2H \longrightarrow j\omega L = j4x2 = j8$$
$$1H \longrightarrow j\omega L = j4x1 = j4$$

Consider the circuit below.



$$2 = (4 + j8)I_1 - j4I_2 \qquad (1)$$
$$0 = -j4I_1 + (2 + j4)I_2 \qquad (2)$$

In matrix form, these equations become

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j8 & -j4 \\ -j4 & 2 + j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this leads to

$I_2 = 0.2353 – j0.0588$

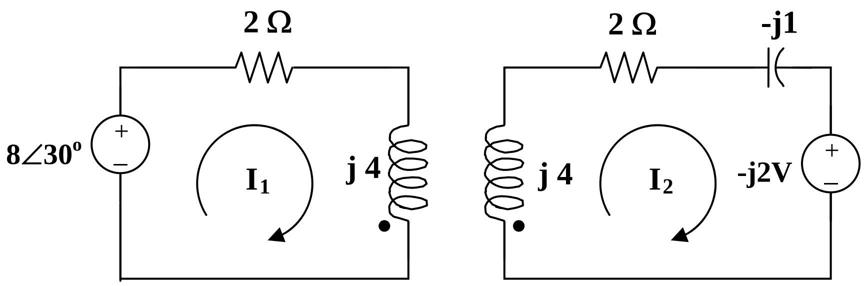
$V = 2I_2 = 0.4851 \angle –14.04^o$

Thus,

v(t) = **485.1cos(4t–14.04°) mV**

Chapter 13, Solution 9.

Consider the circuit below.



For loop 1,

$$8\angle 30° = (2 + j4)I_1 - jI_2 \qquad (1)$$

For loop 2, $(j4 + 2 - j)I_2 - jI_1 + (-j2) = 0$

or $I_1 = (3 - j2)i_2 - 2 \qquad (2)$

Substituting (2) into (1), $8\angle 30° + (2 + j4)2 = (14 + j7)I_2$

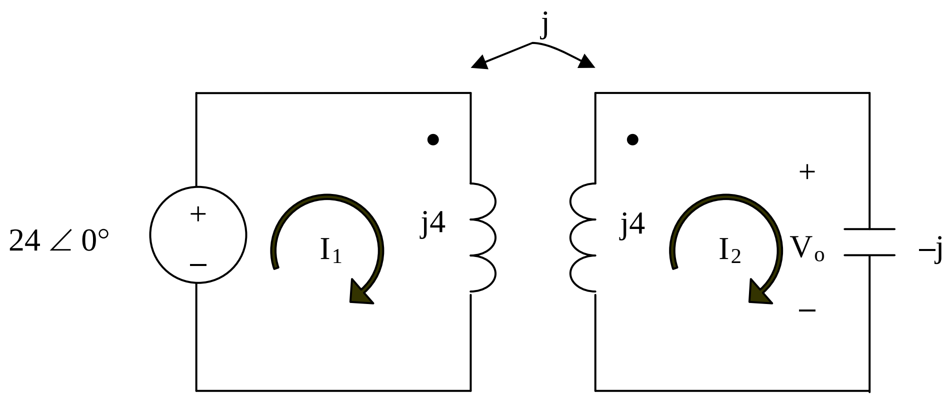
$$I_2 = (10.928 + j12)/(14 + j7) = 1.037\angle 21.12°$$

$$V_x = 2I_2 = \mathbf{2.074\angle 21.12° \ V}$$

Chapter 13, Solution 10.

$$2H \longrightarrow j\omega L = j2x2 = j4$$
$$0.5H \longrightarrow j\omega L = j2x0.5 = j$$
$$\frac{1}{2}F \longrightarrow 1/j\omega C = \frac{1}{j2x1/2} = -j$$

Consider the circuit below.



$$24 = j4I_1 - jI_2 \qquad (1)$$
$$0 = -jI_1 + (j4 - j)I_2 \longrightarrow 0 = -I_1 + 3I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 24 \\ 0 \end{bmatrix} = \begin{bmatrix} j4 & -j \\ -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this,

$$I_2 = -j2.1818, \qquad V_o = -jI_2 = -2.1818$$

$v_o(t)$ = **–2.1818cos2t V**

Chapter 13, Solution 11.

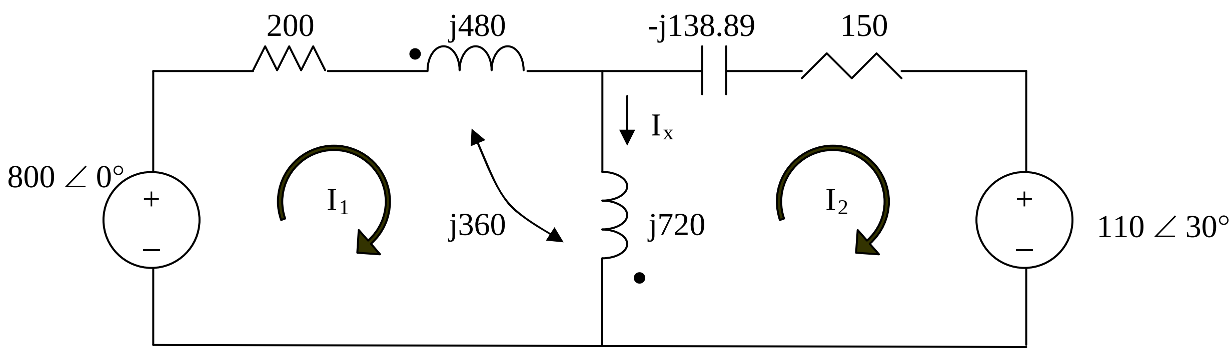
$$800mH \longrightarrow j\omega L = j600x800x10^{-3} = j480$$

$$600mH \longrightarrow j\omega L = j600x600x10^{-3} = j360$$

$$1200mH \longrightarrow j\omega L = j600x1200x10^{-3} = j720$$

$$12\mu F \rightarrow \frac{1}{j\omega C} = \frac{-j}{600x12x10^{-6}} = -j138.89$$

After transforming the current source to a voltage source, we get the circuit shown below.



For mesh 1,

$$800 = (200 + j480 + j720)I_1 + j360I_2 - j720I_2 \text{ or}$$

$$800 = (200 + j1200)I_1 - j360I_2 \qquad (1)$$

For mesh 2,

$$110\angle 30^\circ + 150 - j138.89 + j720)I_2 + j360I_1 = 0 \text{ or}$$

$$-95.2628 - j55 = -j360I_1 + (150 + j581.1)I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 800 \\ -95.2628 - j55 \end{bmatrix} = \begin{bmatrix} 200 + j1200 & -j360 \\ -j360 & 150 + j581.1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this using MATLAB leads to:

```
>> Z = [(200+1200i),-360i;-360i,(150+581.1i)]
Z =
  1.0e+003 *
   0.2000 + 1.2000i        0 - 0.3600i
```

```
      0 - 0.3600i   0.1500 + 0.5811i
>> V = [800;(-95.26-55i)]
V =
  1.0e+002 *
   8.0000
  -0.9526 - 0.5500i
>> I = inv(Z)*V
I =
   0.1390 - 0.7242i
   0.0609 - 0.2690i
```

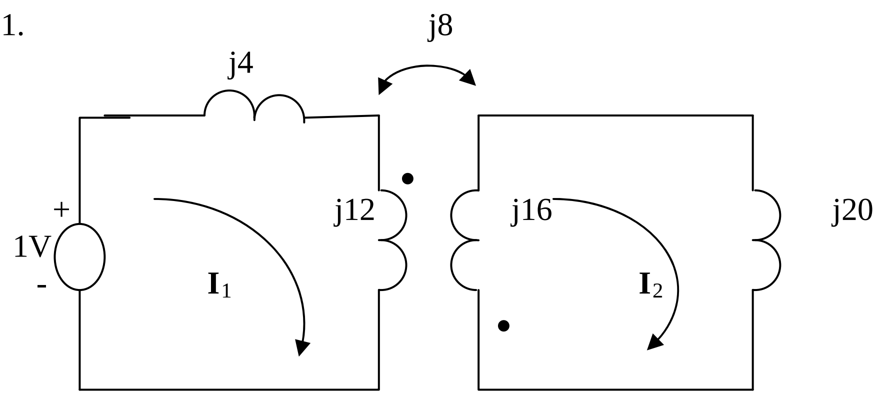
$I_x = I_1 - I_2 = 0.0781 - j0.4552 = 0.4619\angle -80.26^\circ$.

Hence,

$$i_x(t) = \mathbf{461.9cos(600t{-}80.26^\circ)\ mA}.$$

Chapter 13, Solution 12.

Let $\omega = 1$.



Applying KVL to the loops,

$$1 = j16I_1 + j8I_2 \qquad (1)$$

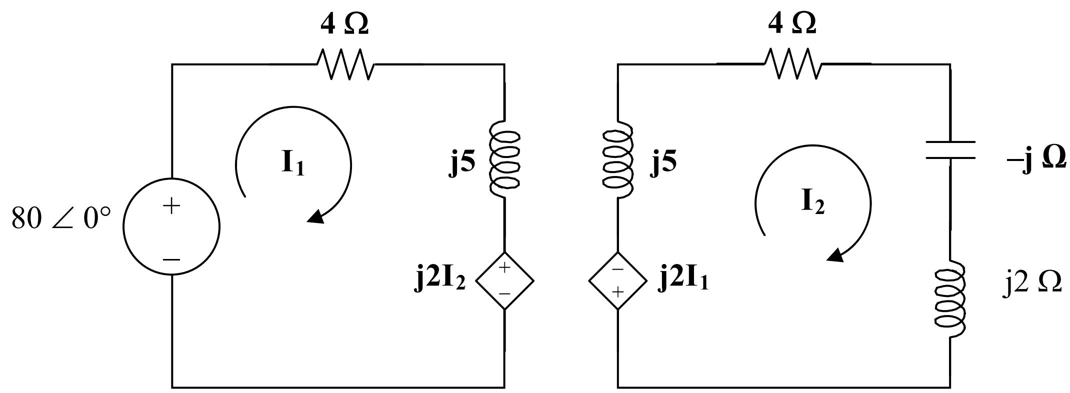
$$0 = j8I_1 + j36I_2 \qquad (2)$$

Solving (1) and (2) gives $\mathbf{I}_1$ = –j0.0703. Thus

$$Z = \frac{1}{I_1} = jL_{eq} \quad \longrightarrow \quad L_{eq} = \frac{1}{jI_1} = \mathbf{14.225\ H}.$$

We can also use the equivalent T-section for the transform to find the equivalent inductance.

Chapter 13, Solution 13.



$-80 + (4+j5)\mathbf{I_1} + j2\mathbf{I_2} = 0$ or $(4+j5)\mathbf{I_1} + j2\mathbf{I_2} = 80$

$j2\mathbf{I_1} + (4+j6)\mathbf{I_2} = 0$ or $\mathbf{I_2} = [-j2/(7.2111\angle 56.31°)]\mathbf{I_1} = (0.27735\angle -146.31°)\mathbf{J_1}$

$[4+j5 + j2(-0.230769-j0.153846)]I_1 = [4+j5+0.307692-j0.461538]I_1 = 80$

$[4.307692+j4.538462]I_1 = 80$ or $I_1 = 80/(6.2573\angle 46.494°)$

$= 12.78507\angle -46.494°$ A.

$$Z_{in} = 80/I_1 = 6.2573\angle 46.494°\ \Omega = \mathbf{(4.308 + j4.538)\ \Omega}$$

An alternate approach would be to use the equation,

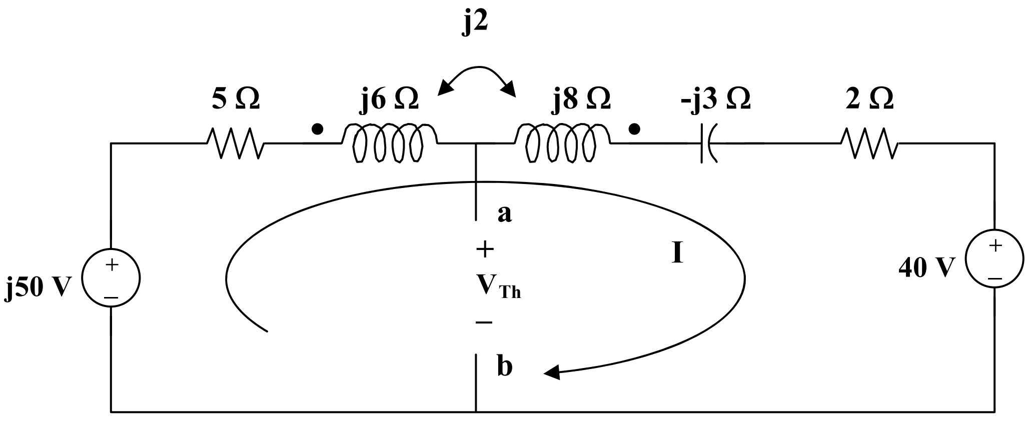
$$\mathbf{Z_{in}} = 4 + j(5) + \frac{4}{j5 + 4 - j + j2} = 4 + j5 + \frac{4}{7.2111\angle 56.31°}$$

$= 4+j5+0.5547\angle{-56.31°} = 4+0.30769+j(5-0.46154)$

$= \mathbf{[4.308+j4.538]\ \Omega}$.

Chapter 13, Solution 14.

To obtain V_{Th}, convert the current source to a voltage source as shown below.



Note that the two coils are connected series aiding.

$$\omega L = \omega L_1 + \omega L_2 - 2\omega M$$

$$j\omega L = j6 + j8 - j4 = j10$$

Thus, $$-j50 + (5 + j10 - j3 + 2)I + 40 = 0$$

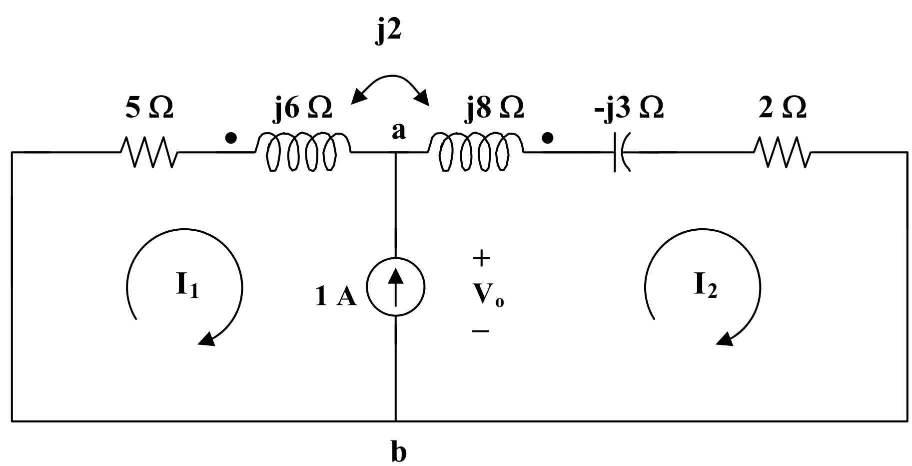
$$I = (-40 + j50)/(7 + j7)$$

But, $$-j50 + (5 + j6)I - j2I + V_{Th} = 0$$

$$V_{Th} = j50 - (5 + j4)I = j50 - (5 + j4)(-40 + j50)/(7 + j7)$$

$$V_{Th} = \mathbf{26.74\angle 34.11° \ V}$$

To obtain Z_{Th}, we set all the sources to zero and insert a 1-A current source at the terminals a–b as shown below.



Clearly, we now have only a super mesh to analyze.

$$(5 + j6)I_1 - j2I_2 + (2 + j8 - j3)I_2 - j2I_1 = 0$$

$$(5 + j4)I_1 + (2 + j3)I_2 = 0 \qquad (1)$$

But, $$I_2 - I_1 = 1 \text{ or } I_2 = I_1 - 1 \qquad (2)$$

Substituting (2) into (1), $$(5 + j4)I_1 + (2 + j3)(1 + I_1) = 0$$

$$I_1 = -(2 + j3)/(7 + j7)$$

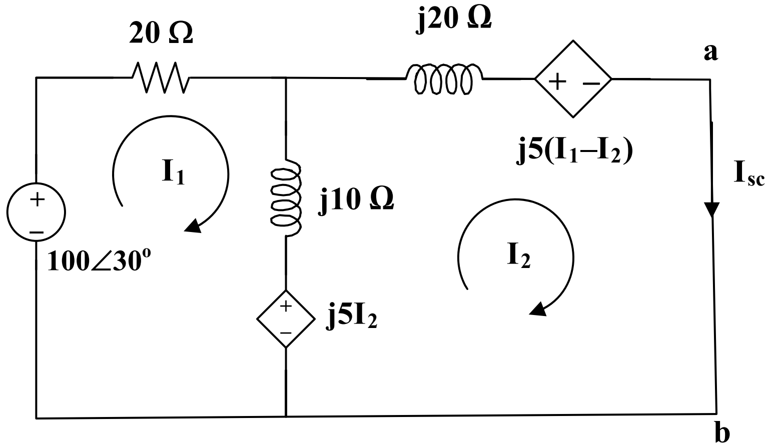
Now, $$((5 + j6)I_1 - j2I_1 + V_o = 0$$

$$V_o = -(5 + j4)I_1 = (5 + j4)(2 + j3)/(7 + j7) = (-2 + j23)/(7 + j7) = 2.332\angle 50°$$

$$Z_{Th} = V_o/1 = \mathbf{2.332\angle 50°\ \Omega}.$$

Chapter 13, Solution 15.

The first step is to replace the mutually coupled circuits with the equivalent circuits using dependent sources. To obtain I_N, short-circuit a–b as shown in Figure (a) and solve for I_{sc}.



(a)

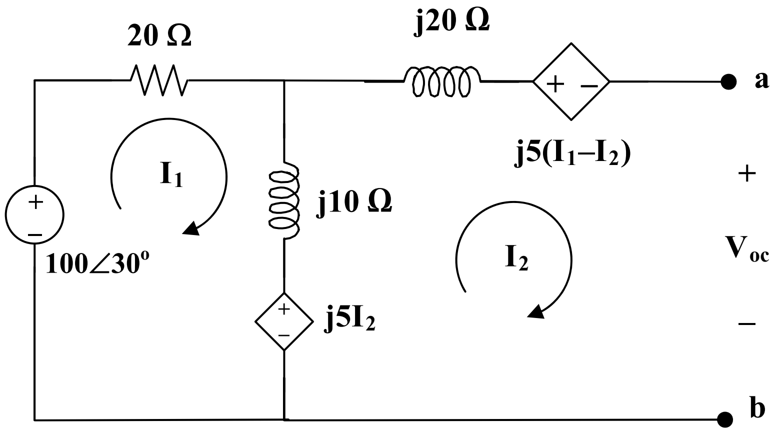
Now all we need to do is to write our two mesh equations.

Loop 1. $-100\angle 60^\circ + 20\mathbf{I_1} + j10(\mathbf{I_1}-\mathbf{I_2}) + j5\mathbf{I_2} = 0$ or $(20+j10)\mathbf{I_1} - j5\mathbf{I_2} = 100\angle 60^\circ$
or $(4+j2)\mathbf{I_1} - j\mathbf{I_2} = 20\angle 30^\circ$

Loop 2. $-j5\mathbf{I_2} + j10(\mathbf{I_2}-\mathbf{I_1}) + j20\mathbf{I_2} + j5(\mathbf{I_1}-\mathbf{I_2}) = 0$ or $-j5\mathbf{I_1} + j20\mathbf{I_2} = 0$ or $\mathbf{I_1} = 4\mathbf{I_2}$

Substituting back into the first equation, we get, $(4+j2)4\mathbf{I_2} - j\mathbf{I_2} = 20\angle 30^\circ$ or $(16+j7)\mathbf{I_2} = 20\angle 30^\circ$.

Now to solve for $\mathbf{I_2} = \mathbf{I_{sc}} = \mathbf{I_N} = (20\angle 30^\circ)/(16+j7) = (20\angle 30^\circ)/(17.464\angle 23.63^\circ) = \mathbf{1.1452\angle 6.37^\circ}$ **A**.



(b)

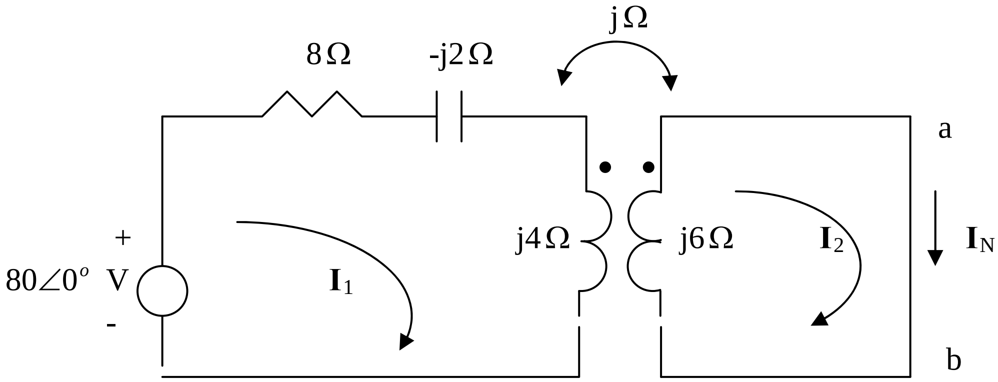
To solve for $\mathbf{Z_N} = \mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}}$, all we need to do is to solve for $\mathbf{V_{oc}}$. In circuit (b) we note that $\mathbf{I_2} = 0$ and we get the mesh equation, $-100\angle 30^\circ + (20+j10)\mathbf{I_1} = 0$ or $\mathbf{I_1} = (100\angle 30^\circ)/(22.36\angle 26.57^\circ) = 4.472\angle 3.43^\circ$ A.
$\mathbf{V_{oc}} = j10\mathbf{I_1} - j5\mathbf{I_1}$ (induced voltage due to the mutual coupling) $= j5\mathbf{I_1} = 22.36\angle 93.43^\circ$ V.

$$\mathbf{Z_{eq}} = \mathbf{Z_N} = (22.36\angle 93.43^\circ)/(1.1452\angle 6.37^\circ) = \mathbf{19.525\angle 87.06^\circ\ \Omega}.$$

or **[1.0014+j19.498] Ω**.

Chapter 13, Solution 16.

To find $\mathbf{I}_N$, we short-circuit a-b.



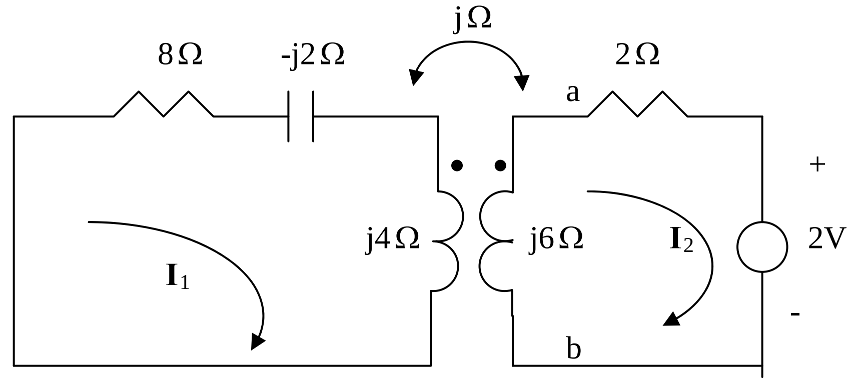
$$-80 + (8 - j2 + j4)I_1 - jI_2 = 0 \longrightarrow (8 + j2)I_1 - jI_2 = 80 \qquad (1)$$

$$j6I_2 - jI_1 = 0 \longrightarrow I_1 = 6I_2 \qquad (2)$$

Solving (1) and (2) leads to

$$I_N = I_2 = \frac{80}{48 + j11} = 1.584 - j0.362 = \mathbf{1.6246\angle{-12.91^\circ}\ A}$$

To find Z_N, insert a 1-A current source at terminals a-b. Transforming the current source to voltage source gives the circuit below.



$$0 = (8 + j2)I_1 - jI_2 \longrightarrow I_1 = \frac{jI_2}{8 + j2}$$

$= [j/(8.24621\angle 14.036°)]\mathbf{I_2} = 0.121268\angle 75.964°\mathbf{I_2}$

$= (0.0294113+j0.117647)\mathbf{I_2}$ (3)

$2 + (2 + j6)I_2 - jI_1 = 0$ (4)

Solving (3) and (4) leads to $(2+j6)\mathbf{I_2} - j(0.0294113+j0.117647)\mathbf{I_2} = -2$ or

$(2.117647+j5.882353)\mathbf{I_2} = -2$ or $\mathbf{I_2} = -2/(6.25192\angle 70.201°) = 0.319902\angle 109.8°$.

$V_{ab} = 2(1+ \mathbf{I_2}) = 2(1-0.1083629+j0.30099) = (1.78327+j0.601979)$ V $= 1\mathbf{Z_{eq}}$ or

$$\mathbf{Z_{eq}} = (1.78327+j0.601979) = \mathbf{1.8821\angle 18.65°\ \Omega}$$

An alternate approach would be to calculate the open circuit voltage.

$-80 + (8+j2)\mathbf{I_1} - j\mathbf{I_2} = 0$ or $(8+j2)\mathbf{I_1} - j\mathbf{I_2} = 80$ (5)

$(2+j6)\mathbf{I_2} - j\mathbf{I_1} = 0$ or $\mathbf{I_1} = (2+j6)\mathbf{I_2}/j = (6-j2)\mathbf{I_2}$ (6)

Substituting (6) into (5) we get,

$(8.24621\angle 14.036°)(6.32456\angle -18.435°)\mathbf{I_2} - jI_2 = 80$ or

$[(52.1536\angle -4.399°)-j]I_2 = [52-j5]I_2 = (52.2398\angle -5.492°)I_2 = 80$ or

$I_2 = 1.5314\angle 5.492°$ A and $V_{oc} = 2I_2 = 3.0628\angle 5.492°$ V which leads to,

$Z_{eq} = V_{oc}/I_{sc} = (3.0628\angle 5.492°)/(1.6246\angle{-12.91}°) =$ **$1.8853\angle 18.4°\ \Omega$**

This is in good agreement with what we determined before.

Chapter 13, Solution 17.

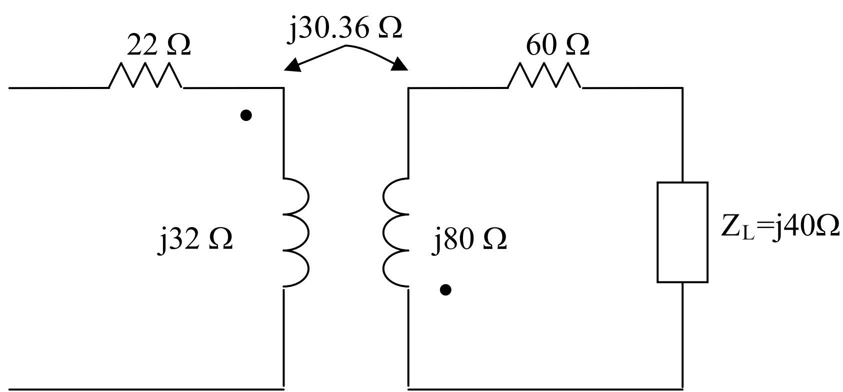
$$j\omega L = j40 \quad \longrightarrow \quad \omega = \frac{40}{L} = \frac{40}{15x10^{-3}} = \quad 2667 \text{ rad/s}$$

$$M = k\sqrt{L_1L_2} = 0.6\sqrt{12x10^{-3}x30x10^{-3}} = \quad 11.384 \text{ mH}$$

If 15 mH ⟶ 40 Ω

Then 12 mH ⟶ 32 Ω
30 mH ⟶ 80 Ω
11.384 mH ⟶ 30.36 Ω

The circuit becomes that shown below.



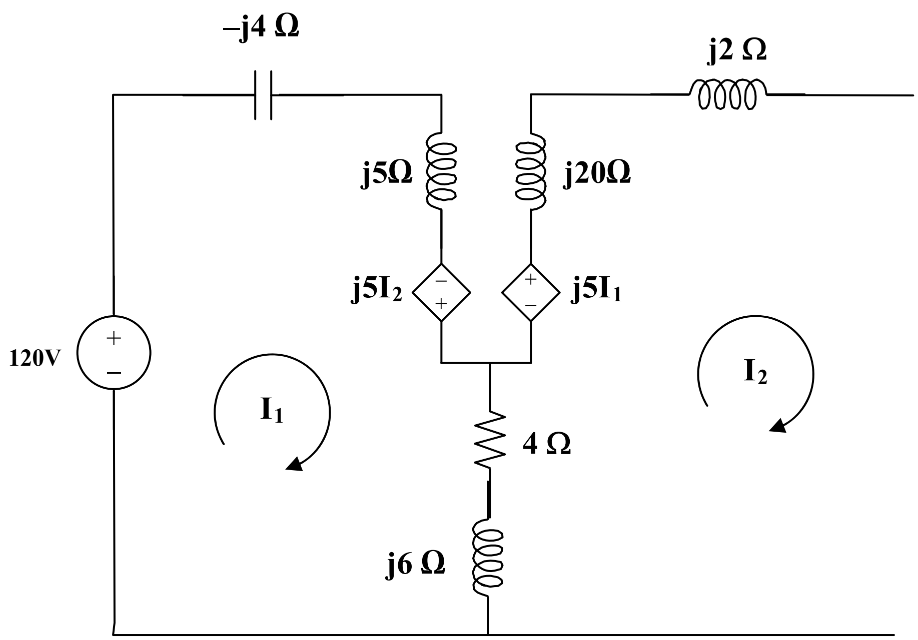
$$Z_{in} = 22 + j32 + \frac{\omega^2 M^2}{j80 + 60 + j40} = 22 + j32 + \frac{(30.36)^2}{60 + j120}$$

$$= 22 + j32 + \frac{921.7}{134.16\angle 63.43^\circ} = 22 + j32 + 6.87\angle -63.43^\circ = 22 + j32 + 3.073 - j6.144$$

= **[25.07 + j25.86] Ω**.

Chapter 13, Solution 18.

Replacing the mutually coupled circuit with the dependent source equivalent we get,



Now all we need to do is to find $\mathbf{V_{oc}}$ and $\mathbf{I_{sc}}$. To calculate the open circuit voltage, we note that I_2 is equal to zero. Thus,

$$-120 + (4 + j(-4+5+6))\mathbf{I_1} = 0 \text{ or } \mathbf{I_1} = 120/(4+j7) = 120/(8.06226\angle 60.255°)$$

$$= 14.8842\angle -60.255°.$$

$$\mathbf{V_{oc}} = \mathbf{V_{Thev}} = j5\mathbf{I_1} + (4+j6)\mathbf{I_1} = (4+j11)\mathbf{I_1}$$

$$= (11.7047\angle 70.017°)(14.8842\angle -60.255°) = \mathbf{174.22\angle 9.76°\ V}$$

To find the short circuit current ($\mathbf{I_{sc} = I_2}$), we need to solve the following mesh equations, Mesh 1

$$-120 + (-j4+j5)\mathbf{I_1} - j5\mathbf{I_2} + (4+j6)(\mathbf{I_1}-\mathbf{I_2}) = 0 \text{ or}$$
$$(4+j7)\mathbf{I_1} - (4+j11)\mathbf{I_2} = 120 \qquad (1)$$

Mesh 2

$$(4+j6)(\mathbf{I_2}-\mathbf{I_1}) - j5\mathbf{I_1} + j22\mathbf{I_2} = 0 \text{ or } -(4+j11)\mathbf{I_1} + (4+j28)\mathbf{I_2} = 0 \text{ or}$$

$$\mathbf{I_1} = (28.2843\angle 81.87°)\mathbf{I_2}/(11.7047\angle 70.0169°) = (2.4165\angle 11.853°)\mathbf{I_2}$$

Substituting this into equation (1) we get,

$$(8.06226\angle 60.255°)(2.4165\angle 11.853°)\mathbf{I_2} - (4+j11)\mathbf{I_2} = 120 \text{ or}$$

$$[(19.4825\angle 72.108°) - 4 - j11]I_2 = 120 \text{ and}$$

$$[5.9855+j18.5403-4-j11]I_2 = (1.9855+j7.5403)I_2 = 120 \text{ or}$$

$$\mathbf{I_2} = \mathbf{I_{sc}} = 120/(7.79733\angle 75.248°) = 15.3899\angle -75.248° \text{ A}$$

Checking using MATLAB we get,

```
>> Z = [(4+7j) (-4-11j);(-4-11j) (4+28j)]

Z =

   4.0000 + 7.0000i  -4.0000 -11.0000i
  -4.0000 -11.0000i   4.0000 +28.0000i

>> V = [120;0]

V =

   120
     0
```

```
>> I = inv(Z)*V

I =

  16.6551 -33.2525i

   3.9188 -14.8829i
```

(I_1)

$(I_2 = I_{sc}) = 15.3902\angle{-75.248°}$ (answer checks)

Finally,

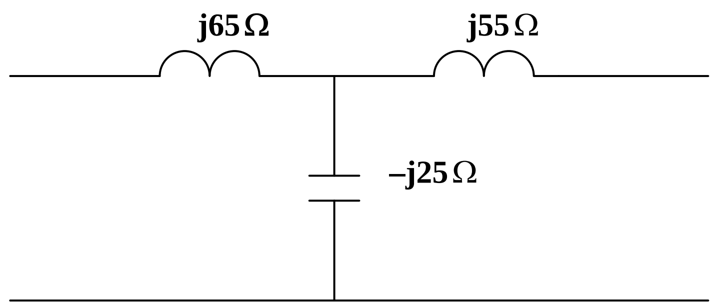
$$Z_{eq} = V_{Thev}/I_{sc} = (174.22\angle 9.76°)/(15.3899\angle{-75.248°})$$

$$= \mathbf{(11.32\angle 85.01°)\ \Omega}$$

Chapter 13, Solution 19.

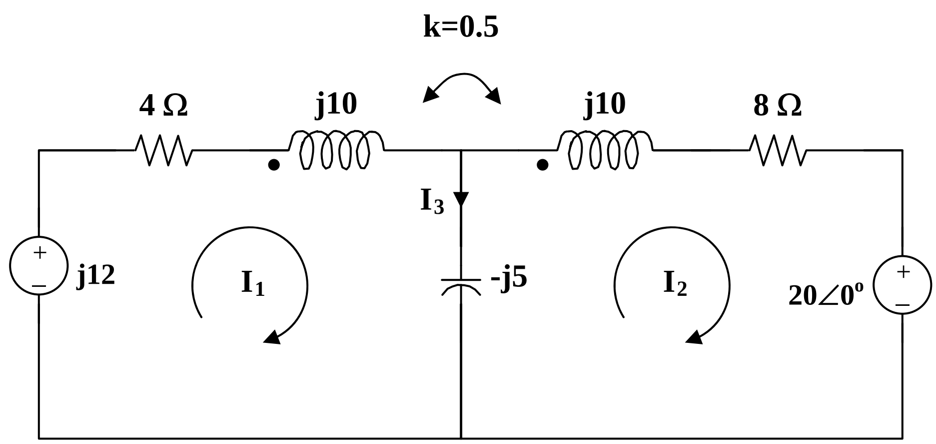
$X_{La} = X_{L1} - (-X_M) = 40 + 25 = 65\ \Omega$ and $X_{Lb} = X_{L2} - (-X_M) = 40 + 25 = 55\ \Omega$.

Finally, $X_C = X_M$ thus, the T-section is as shown below.



Chapter 13, Solution 20.

Transform the current source to a voltage source as shown below.



$$k = M/\sqrt{L_1L_2} \quad \text{or} \quad M = k\sqrt{L_1L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.5(10) = 5$$

For mesh 1, $\quad j12 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2 \qquad (1)$

For mesh 2, $\quad 0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$

$$-20 = +j10I_1 + (8 + j5)I_2 \qquad (2)$$

From (1) and (2), $\begin{bmatrix} j12 \\ 20 \end{bmatrix} = \begin{bmatrix} 4+j5 & +j10 \\ +j10 & 8+j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 107 + j60, \quad \Delta_1 = -60 - j296, \quad \Delta_2 = 40 - j100$$

$$I_1 = \Delta_1/\Delta = \mathbf{2.462\angle 72.18^\circ\ A}$$

$$I_2 = \Delta_2/\Delta = \mathbf{878\angle{-97.48^\circ}\ mA}$$

$$I_3 = I_1 - I_2 = \mathbf{3.329\angle 74.89^\circ\ A}$$

$$i_1 = 2.462\cos(1000t + 72.18^\circ)\ A$$

$$i_2 = 0.878\cos(1000t - 97.48^\circ)\ A$$

At $t = 2$ ms, $\quad 1000t = 2$ rad $= 114.6°$

$$i_1(0.002) = 2.462\cos(114.6^\circ + 72.18^\circ) = -2.445A$$

$$-2.445$$

$$i_2 = 0.878\cos(114.6° - 97.48°) = -0.8391$$

The total energy stored in the coupled coils is

$$w = 0.5L_1 i_1^2 + 0.5L_2 i_2^2 + Mi_1 i_2$$

Since $\omega L_1 = 10$ and $\omega = 1000$, $L_1 = L_2 = 10$ mH, $M = 0.5L_1 = 5$mH

$$w = 0.5(0.01)(-2.445)^2 + 0.5(0.01)(-0.8391)^2 + 0.05(-2.445)(-0.8391)$$

$$\mathbf{w = 43.67\ mJ}$$

Chapter 13, Solution 21.

Using Fig. 13.90, design a problem to help other students to better understand energy in a coupled circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.90. Calculate the power absorbed by the 4-Ω resistor.

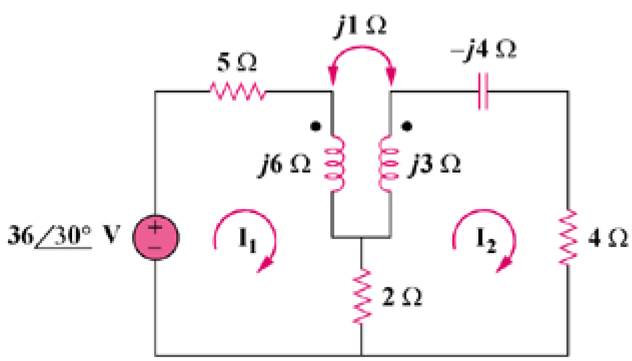


Figure 13.90

Solution

For mesh 1, $36\angle 30° = (7 + j6)I_1 - (2 + j)I_2$ (1)

For mesh 2, $0 = (6 + j3 - j4)I_2 - 2I_1 - jI_1 = -(2 + j)I_1 + (6 - j)I_2$ (2)

Placing (1) and (2) into matrix form, $\begin{bmatrix} 36\angle 30° \\ 0 \end{bmatrix} = \begin{bmatrix} 7 + j6 & -2 - j \\ -2 - j & 6 - j \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 45 + j25 = 51.48\angle 29.05°, \quad \Delta_1 = (6 - j)36\angle 30° = 219\angle 20.54°$$

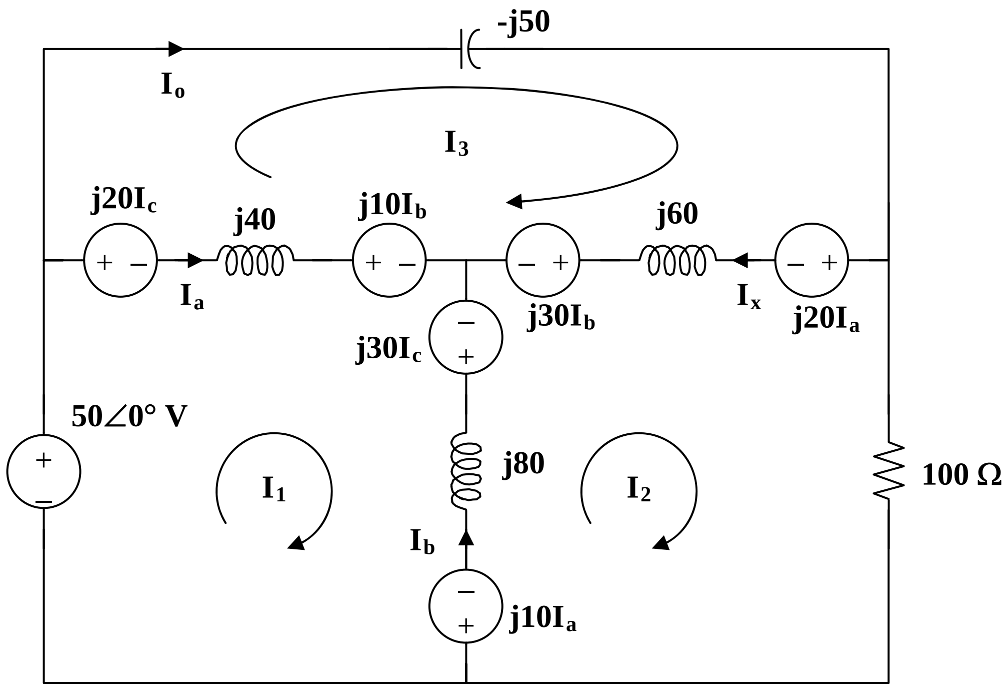
$\Delta_2 = (2 + j)36\angle 30° = 80.5\angle 56.57°$, $I_1 = \Delta_1/\Delta = \mathbf{4.254\angle{-8.51}°\ A}$, $I_2 = \Delta_2/\Delta = \mathbf{1.5637\angle 27.52°\ A}$

Power absorbed by the 4-ohm resistor,

$$= 0.5(I_2)^2 4 = 2(1.5637)^2 = \mathbf{4.89\ watts}$$

Chapter 13, Solution 22.

With more complex mutually coupled circuits, it may be easier to show the effects of the coupling as sources in terms of currents that enter or leave the dot side of the coil. Figure 13.85 then becomes,



Note the following,

$$I_a = I_1 - I_3$$
$$I_b = I_2 - I_1$$
$$I_c = I_3 - I_2$$

$$\text{and} \quad I_o = I_3$$

Now all we need to do is to write the mesh equations and to solve for I_o.

Loop # 1,

$-50 + j20(I_3 - I_2)\ j\ 40(I_1 - I_3) + j10(I_2 - I_1) - j30(I_3 - I_2) + j80(I_1 - I_2) - j10(I_1 - I_3)$
$= 0$

$$j100I_1 - j60I_2 - j40I_3 = 50$$

Multiplying everything by (1/j10) yields $10I_1 - 6I_2 - 4I_3 = -j5$ (1)

Loop # 2,

$j10(I_1 - I_3) + j80(I_2 – I_1) + j30(I_3 – I_2) - j30(I_2 - I_1) + j60(I_2 - I_3) - j20(I_1 - I_3) + 100I_2 = 0$

$$-j60I_1 + (100 + j80)I_2 - j20I_3 = 0 \qquad (2)$$

Loop # 3,

$-j50I_3 + j20(I_1 - I_3) + j60(I_3 - I_2) + j30(I_2 - I_1) - j10(I_2 - I_1) + j40(I_3 - I_1) - j20(I_3 - I_2) = 0$

$$-j40I_1 - j20I_2 + j10I_3 = 0$$

Multiplying by (1/j10) yields, $-4I_1 - 2I_2 + I_3 = 0$ (3)

Multiplying (2) by (1/j20) yields $-3I_1 + (4 - j5)I_2 - I_3 = 0$ (4)

Multiplying (3) by (1/4) yields $-I_1 - 0.5I_2 - 0.25I_3 = 0$ (5)

Multiplying (4) by (-1/3) yields $I_1 - ((4/3) - j(5/3))I_2 + (1/3)I_3 = -j0.5$ (7)

Multiplying [(6)+(5)] by 12 yields $(-22 + j20)I_2 + 7I_3 = 0$ (8)

Multiplying [(5)+(7)] by 20 yields $-22I_2 - 3I_3 = -j10$ (9)

(8) leads to $I_2 = -7I_3/(-22 + j20) = 0.2355\angle 42.3^o = (0.17418+j0.15849)I_3$ (10)

(9) leads to $I_3 = (j10 - 22I_2)/3$, substituting (1) into this equation produces,

$$I_3 = j3.333 + (-1.2273 - j1.1623)I_3$$

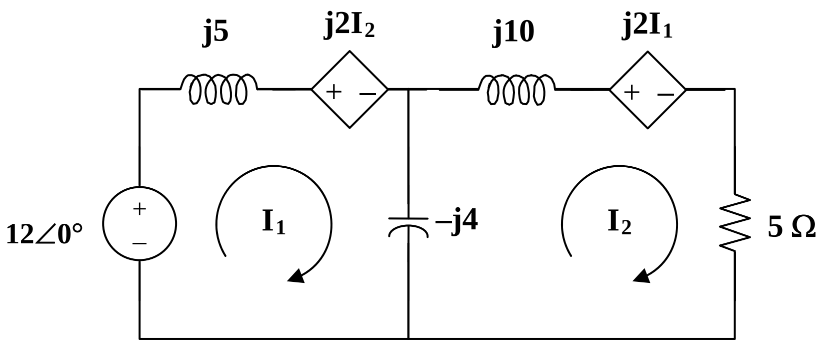
or $I_3 = I_o =$ **$1.3040\angle 63^o$ amp.**

Chapter 13, Solution 23.

$\omega = 10$

0.5 H converts to $j\omega L_1 = j5$ ohms
1 H converts to $j\omega L_2 = j10$ ohms
0.2 H converts to $j\omega M = j2$ ohms
25 mF converts to $1/(j\omega C) = 1/(10\text{x}25\text{x}10^{-3}) = -j4$ ohms

The frequency-domain equivalent circuit is shown below.



For mesh 1, $\quad -12 + j5I_1 + j2I_2 + (-j4)(I_1 - I_2) = 0$ or $jI_1 + j6I_2 = 12$ or
$I_1 + 6I_2 = -j12$ (1)

For mesh 2, $\quad (-j4)(I_2 - I_1) + j10I_2 + j2I_1 + 5I_2 = 0$ or
$j6I_1 + (5+j6)I_2 = 0$ (2)

From (1), $\quad I_1 = -j12 - 6I_2$

Substituting this into (2) produces,

$j6(-j12 - 6I_2) + (5+j6)I_2 = 0 = 72 + (5+j6-j36)I_2$ or

$(5-j30)I_2 = (30.414\angle{-80.54°})I_2 = -72$ or $I_2 = 2.367\angle{-99.46°}$ A

$I_1 = -j12 - 6(-0.38909 - j2.3351) = 2.33454 + j(-12+14.0106)$

$= 2.33454 + j2.0106) = 3.081\angle 40.74°$ A

Checking using matrices,

$$\begin{bmatrix} 1 & 6 \\ j6 & (5+j6) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -j12 \\ 0 \end{bmatrix} \text{ which leads to } \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} (5+j6) & -6 \\ -j6 & 1 \end{bmatrix}}{5+j6-j36}\begin{bmatrix} -j12 \\ 0 \end{bmatrix}$$

$I_1 = [(72–j60)/(5–j30)] = (93.723\angle–39.806°)/(30.414\angle–80.538°) = 3.082\angle40.73°$ A and

$I_2 = [–72/(30.414\angle–80.54)] = 2.367\angle–99.46°$ A

Thus,

$i_1(t)$ = **3.081cos(10t + 40.74°) A**, $i_2(t)$ = **2.367cos(10t – 99.46°) A**.

At t = 15 ms, $10t = 10x15x10^{-3} = 0.15$ rad = 8.59°

$i_1 = 3.081\cos(49.33°)$ = **2.00789 A**

$i_2 = 2.367\cos(–90.87°)$ = **–0.03594 A**

$$w = 0.5(5)(2.00789)^2 + 0.5(1)(–0.03594)^2 – (0.2)(2.00789)(–0.03594)$$
$$= 10.079056 + 0.0006458 + 0.0144327 = \mathbf{10.094\ J}.$$

3.081cos(10t + 40.74°) A, 2.367cos(10t – 99.46°) A, 10.094 J.

Chapter 13, Solution 24.

(a) $k = M/\sqrt{L_1 L_2} = 1/\sqrt{4x2} = \mathbf{0.3535}$

(b) $\omega = 4$

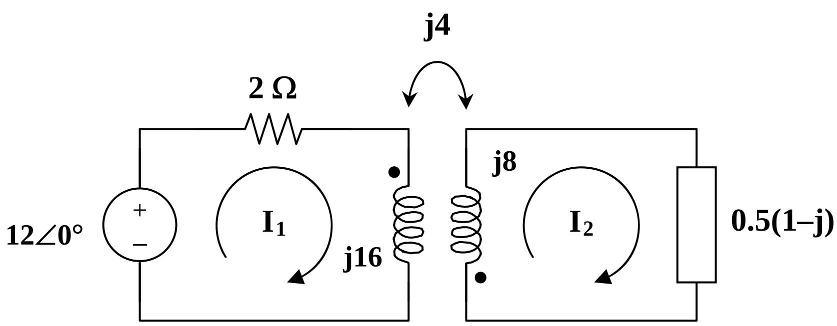
1/4 F leads to $1/(j\omega C) = -j/(4x0.25) = -j$

$1||(-j) = -j/(1 - j) = 0.5(1 - j)$

1 H produces $j\omega M = j4$

4 H produces j16

2 H becomes j8



$$12 = (2 + j16)I_1 + j4I_2$$

or $\quad 6 = (1 + j8)I_1 + j2I_2 \qquad (1)$

$$0 = (j8 + 0.5 - j0.5)I_2 + j4I_1 \text{ or } I_1 = (0.5 + j7.5)I_2/(-j4) \qquad (2)$$

Substituting (2) into (1),

$$24 = (-11.5 - j51.5)I_2 \text{ or } I_2 = -24/(11.5 + j51.5) = -0.455\angle{-77.41°}$$

$$V_o = I_2(0.5)(1 - j) = 0.3217\angle 57.59°$$

$$v_o = \mathbf{321.7\cos(4t + 57.6°)\ mV}$$

(c) From (2), $I_1 = (0.5 + j7.5)I_2/(-j4) = 0.855\angle{-81.21°}$

$$i_1 = 0.885\cos(4t - 81.21°) \text{ A}, \ i_2 = -0.455\cos(4t - 77.41°) \text{ A}$$

At t = 2s,

$$4t = 8 \text{ rad} = 98.37°$$

$$i_1 = 0.885\cos(98.37° - 81.21°) = 0.8169$$

$$i_2 = -0.455\cos(98.37° - 77.41°) = -0.4249$$

$$w = 0.5L_1 i_1^2 + 0.5L_2 i_2^2 + M i_1 i_2$$

$$= 0.5(4)(0.8169)^2 + 0.5(2)(-.4249)^2 + (1)(0.1869)(-0.4249) = \mathbf{1.168\ J}$$

Chapter 13, Solution 25.

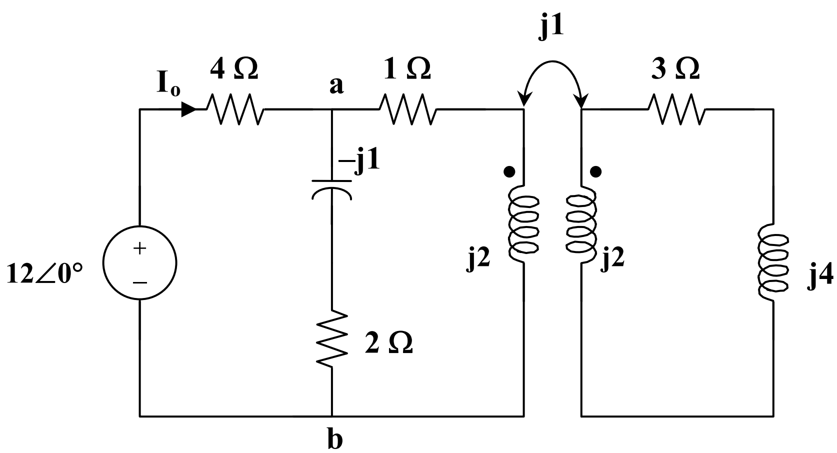
$$m = k\sqrt{L_1 L_2} = 0.5 \text{ H}$$

We transform the circuit to frequency domain as shown below.

$$12\sin 2t \text{ converts to } 12\angle 0°, \ \omega = 2$$

$$0.5 \text{ F converts to } 1/(j\omega C) = -j$$

$$2 \text{ H becomes } j\omega L = j4$$



Applying the concept of reflected impedance,

$$Z_{ab} = (2 - j)||(1 + j2 + (1)^2/(j2 + 3 + j4))$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1.0667 + j1.8667)$$

$$=(2 - j)(1.0667 + j1.8667)/(3.0667 + j0.8667) = \mathbf{1.5085\angle 17.9° \ \Omega}$$

$$I_o = 12\angle 0°/(Z_{ab} + 4) = 12/(5.4355 + j0.4636) = 2.2\angle -4.88°$$

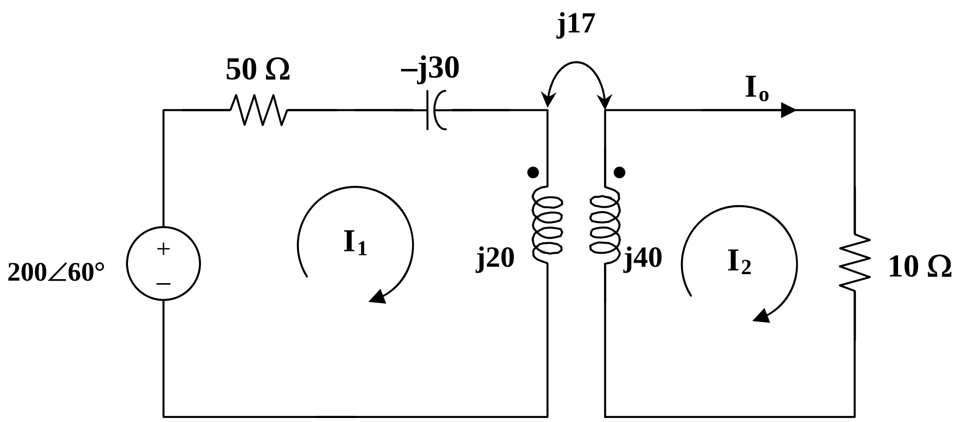
$$i_o = \mathbf{2.2\sin(2t - 4.88°) \ A}$$

Chapter 13, Solution 26.

$$M = k\sqrt{L_1 L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.601\sqrt{20x40} = 17$$

The frequency-domain equivalent circuit is shown below.



For mesh 1, $-200\angle 60° + (50-j30+j20)I_1 - j17I_2 = 0$ or

$$(50-j10)I_1 - j17I_2 = 200\angle 60° \qquad (1)$$

For mesh 2, $(10 + j40)I_2 - j17I_1 = 0$ or $-j17I_1 + (10+j40)I_2 = 0 \qquad (2)$

In matrix form,

$$\begin{bmatrix} 200\angle 60° \\ 0 \end{bmatrix} = \begin{bmatrix} 50-j10 & -j17 \\ -j17 & 10+j40 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{or}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 10+j40 & j17 \\ j17 & 50-j10 \end{bmatrix}}{500+400+289-j100+j2{,}000} \begin{bmatrix} 200\angle 60° \\ 0 \end{bmatrix}$$

$I_1 = (10+j40)(200\angle 60°)/(1{,}189+j1{,}900)$

$= (41.231\angle 75.964°)(200\angle 60°)/(2{,}241.4\angle 57.962°) = 3.679\angle 78°$ A and

$I_2 = j17(200\angle 60°)/(2{,}241.4\angle 57.962°) = 1.5169\angle 92.04°$ A

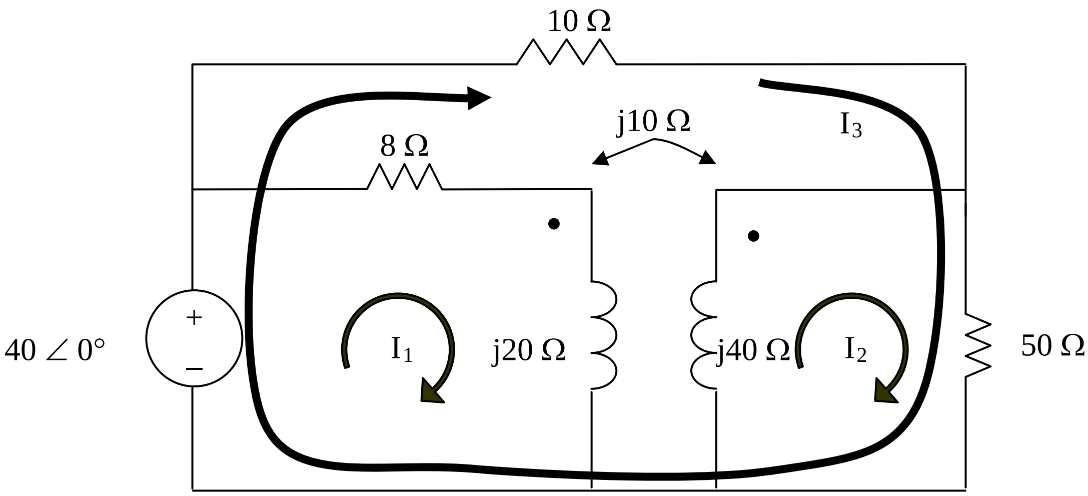
$$\mathbf{I_o = I_2 = 1.5169\angle 92.04°\ A}$$

It should be noted that switching the dot on the winding on the right only reverses the direction of $\mathbf{I_o}$.

Chapter 13, Solution 27.

$$1H \longrightarrow j\omega L = j20$$
$$2H \longrightarrow j\omega L = j40$$
$$0.5H \longrightarrow j\omega L = j10$$

We apply mesh analysis to the circuit as shown below.



To make the problem easier to solve, let us have I_3 flow around the outside loop as shown.

For mesh 1,

$$-40 + 8I_1 + j20I_1 - j10I_2 = 0 \text{ or } (8+j20)I_1 - j10I_2 = 40 \qquad (1)$$

For mesh 2,

$$j40I_2 - j10I_1 + 50(I_2 + I_3) = 0 \text{ or } -j10I_1 + (50+j40)I_2 + 50I_3 = 0 \qquad (2)$$

For mesh 3,

$$-40 + 10I_3 + 50(I_3 + I_2) = 0 \text{ or } 50I_2 + 60I_3 = 40 \qquad (3)$$

In matrix form, (1) to (3) become

$$\begin{bmatrix} 8+j20 & -j10 & 0 \\ -j10 & 50+j40 & 50 \\ 0 & 50 & 60 \end{bmatrix} I = \begin{bmatrix} 40 \\ 0 \\ 40 \end{bmatrix}$$

```
>> Z=[(8+20i),-10i,0;-10i,(50+40i),50;0,50,60]
Z =
   8.0000 +20.0000i        0 -10.0000i        0
```

```
        0 -10.0000i  50.0000 +40.0000i  50.0000
        0            50.0000            60.0000

>> V=[40;0;40]
V =
    40
     0
    40

>> I=inv(Z)*V
I =
   0.6354 - 1.5118i
   0.0613 + 0.4682i
   0.6156 - 0.3901i
```

Solving this leads to $\mathbf{I_{50}} = \mathbf{I_2} + \mathbf{I_3}$ = 0.0613+0.6156 + j(0.4682–0.3901) =

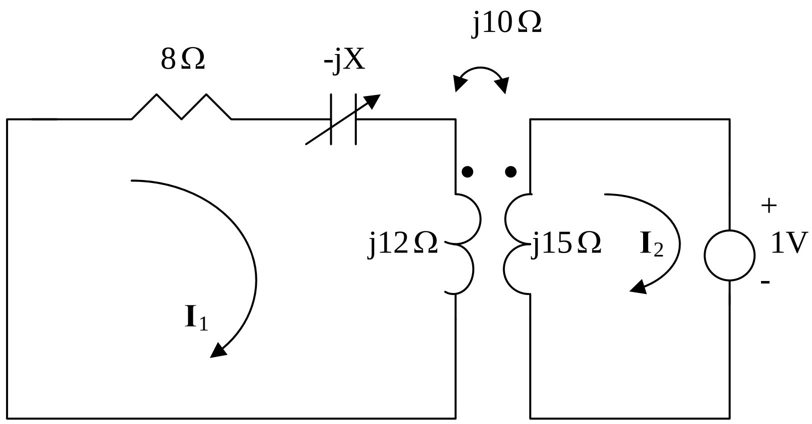
0.6769+j0.0781 = 0.6814∠6.58° A or I_{50rms} $|\mathbf{I_{50rms}}|$ = 0.6814/1.4142 = 481.8 mA.

The power delivered to the 50-Ω resistor is

$$P = (I_{50rms})^2R = (0.4818)^2 50 = \mathbf{11.608\ W}.$$

Chapter 13, Solution 28.

We find Z_{Th} by replacing the 20-ohm load with a unit source as shown below.



For mesh 1, $$0 = (8 - jX + j12)I_1 - j10I_2 \qquad (1)$$

For mesh 2,

$$1 + j15I_2 - j10I_1 = 0 \longrightarrow I_1 = 1.5I_2 - 0.1j \qquad (2)$$

Substituting (2) into (1) leads to

$$I_2 = \frac{-1.2 + j0.8 + 0.1X}{12 + j8 - j1.5X}$$

$$Z_{Th} = \frac{1}{-I_2} = \frac{12 + j8 - j1.5X}{1.2 - j0.8 - 0.1X}$$

$$|Z_{Th}| = 20 = \frac{\sqrt{12^2 + (8 - 1.5X)^2}}{\sqrt{(1.2 - 0.1X)^2 + 0.8^2}} \longrightarrow 0 = 1.75X^2 + 72X - 624$$

Solving the quadratic equation yields X = **6.425 Ω**

Chapter 13, Solution 29.

$$30\text{ mH becomes } j\omega L = j30\text{x}10^{-3}\text{x}10^{3} = j30$$

$$50\text{ mH becomes } j50$$

$$\text{Let } X = \omega M$$

Using the concept of reflected impedance,

$$Z_{in} = 10 + j30 + X^2/(20 + j50)$$

$$I_1 = V/Z_{in} = 165/(10 + j30 + X^2/(20 + j50))$$

$$p = 0.5|I_1|^2(10) = 320 \text{ leads to } |I_1|^2 = 64 \text{ or } |I_1| = 8$$

$$8 = |165(20 + j50)/(X^2 + (10 + j30)(20 + j50))|$$

$$= |165(20 + j50)/(X^2 - 1300 + j1100)|$$

or $$64 = 27225(400 + 2500)/((X^2 - 1300)^2 + 1{,}210{,}000)$$

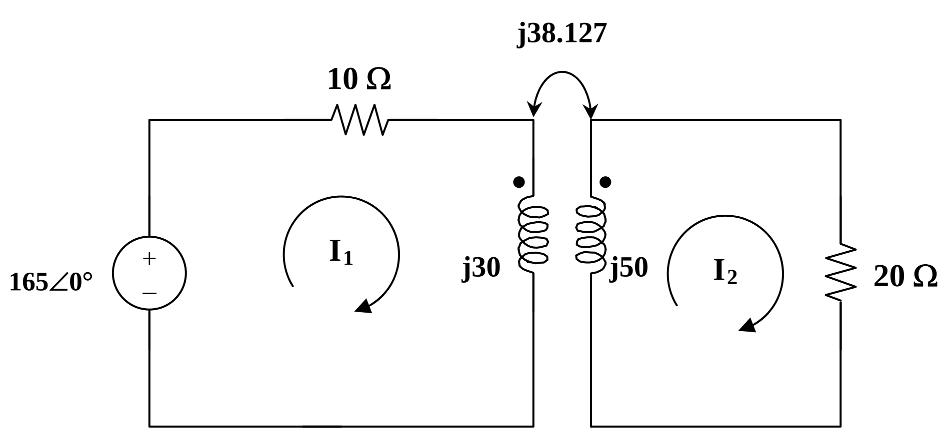
$$(X^2 - 1300)^2 + 1{,}210{,}000 = 1{,}233{,}633$$

$$X = 33.86 \text{ or } 38.13$$

$$\text{If } X = 38.127 = \omega M$$

$$M = 38.127\text{ mH}$$

$$k = M/\sqrt{L_1L_2} = 38.127/\sqrt{30\text{x}50} = \mathbf{0.984}$$



$$165 = (10 + j30)I_1 - j38.127I_2 \qquad (1)$$

$$0 = (20 + j50)I_2 - j38.127I_1 \qquad (2)$$

In matrix form, $$\begin{bmatrix} 165 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 + j30 & -j38.127 \\ -j38.127 & 20 + j50 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 154 + j1100 = 1110.73\angle 82.03°, \ \Delta_1 = 888.5\angle 68.2°, \ \Delta_2 = j6291$$

$$I_1 = \Delta_1/\Delta = 8\angle -13.81°, \ I_2 = \Delta_2/\Delta = 5.664\angle 7.97°$$

$$i_1 = 8\cos(1000t - 13.83°), \ i_2 = 5.664\cos(1000t + 7.97°)$$

$$\text{At } t = 1.5 \text{ ms}, \ 1000t = 1.5 \text{ rad} = 85.94°$$

$$i_1 = 8\cos(85.94° - 13.83°) = 2.457$$

$$i_2 = 5.664\cos(85.94° + 7.97°) = -0.3862$$

$$w = 0.5L_1 i_1^2 + 0.5L_2 i_2^2 + Mi_1 i_2$$

$$= 0.5(30)(2.547)^2 + 0.5(50)(-0.3862)^2 - 38.127(2.547)(-0.3862)$$

$$= \mathbf{130.51 \ mJ}$$

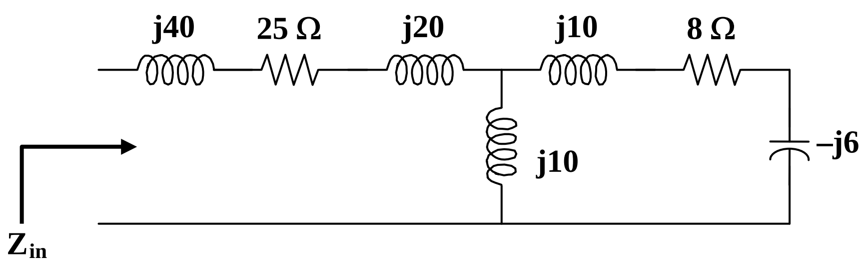
Chapter 13, Solution 30.

(a) $Z_{in} = j40 + 25 + j30 + (10)^2/(8 + j20 - j6)$

$= 25 + j70 + 100/(8 + j14) =$ **(28.08 + j64.62) ohms**

(b) $j\omega L_a = j30 - j10 = j20$, $j\omega L_b = j20 - j10 = j10$, $j\omega L_c = j10$

Thus the Thevenin Equivalent of the linear transformer is shown below.



$Z_{in} = j40 + 25 + j20 + j10||(8 + j4) = 25 + j60 + j10(8 + j4)/(8 + j14)$

$=$ **(28.08 + j64.62) ohms**

Chapter 13, Solution 31.

Using Fig. 13.100, design a problem to help other students to better understand linear transformers and how to find T-equivalent and Π–equivalent circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 13.99, find:

(a) the *T*-equivalent circuit,

(b) the Π-equivalent circuit.

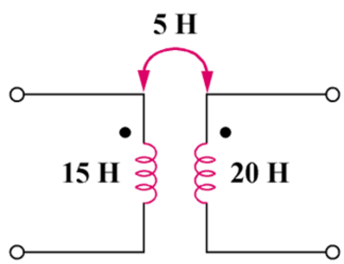


Figure 13.99

Solution

(a) $L_a = L_1 - M = \mathbf{10\ H}$

$L_b = L_2 - M = \mathbf{15\ H}$

$L_c = M = \mathbf{5\ H}$

(b) $L_1L_2 - M^2 = 300 - 25 = 275$

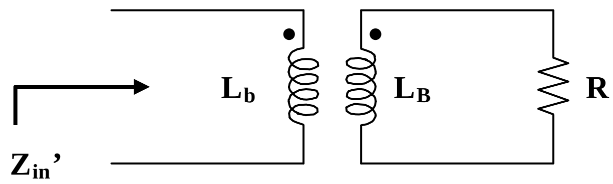
$L_A = (L_1L_2 - M^2)/(L_1 - M) = 275/15 = \mathbf{18.33\ H}$

$L_B = (L_1L_2 - M^2)/(L_1 - M) = 275/10 = \mathbf{27.5\ H}$

$L_C = (L_1L_2 - M^2)/M = 275/5 = \mathbf{55\ H}$

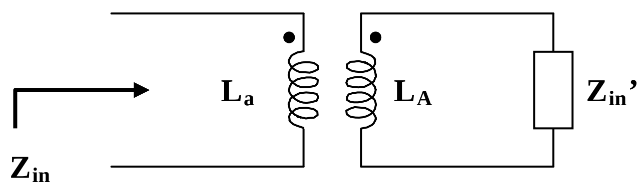
Chapter 13, Solution 32.

We first find Z_{in} for the second stage using the concept of reflected impedance.



$Z_{in}' = j\omega L_b + \omega^2 M_b^2/(R + j\omega L_b) = (j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)/(R + j\omega L_b)$
(1)

For the first stage, we have the circuit below.



$$Z_{in} = j\omega L_a + \omega^2 M_a^2/(j\omega L_a + Z_{in})$$

$$= (-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a Z_{in})/(j\omega L_a + Z_{in}) \qquad (2)$$

Substituting (1) into (2) gives,

$$= \frac{-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a \dfrac{(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{R + j\omega L_b}}{j\omega L_a + \dfrac{j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2}{R + j\omega L_b}}$$

$$= \frac{-R\omega^2 L_a^2 + \omega^2 M_a^2 R - j\omega^3 L_b L_a + j\omega^3 L_b M_a^2 + j\omega L_a(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{j\omega R La - \omega^2 L_a L_b + j\omega L_b R - \omega^2 L_a^2 + \omega^2 M_b^2}$$

$$Z_{in} = \frac{\boldsymbol{\omega^2 R(L_a^2 + L_a L_b - M_a^2) + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}}{\boldsymbol{\omega^2(L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}}$$

Chapter 13, Solution 33.

$$Z_{in} = 10 + j12 + (15)^2/(20 + j\,40 - j5) = 10 + j12 + 225/(20 + j35)$$

$$= 10 + j12 + 225(20 - j35)/(400 + 1225)$$

$$\mathbf{= (12.769 + j7.154)\ \Omega}$$

Chapter 13, Solution 34.

Using Fig. 13.103, design a problem to help other students to better understand how to find the input impedance of circuits with transformers.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the input impedance of the circuit in Fig. 13.102.

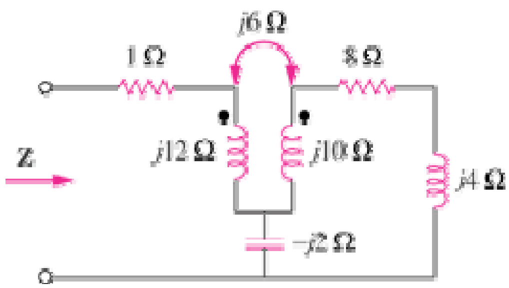
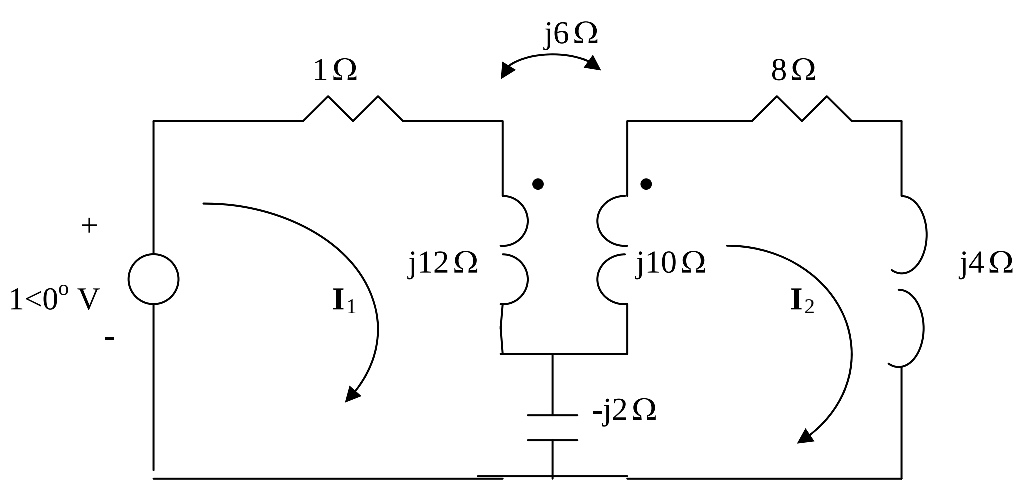


Figure 13.102

Solution

Insert a 1-V voltage source at the input as shown below.



For loop 1,

$$1 = (1 + j10)I_1 - j4I_2 \qquad (1)$$

For loop 2,

$$0 = (8 + j4 + j10 - j2)I_2 + j2I_1 - j6I_1 \quad \longrightarrow \quad 0 = -jI_1 + (2 + j3)I_2 \qquad (2)$$

Solving (1) and (2) leads to $\mathbf{I}_1$=0.019 –j0.1068

$$\boldsymbol{Z = \frac{1}{I_1} = 1.6154 + j9.077 = \underline{9.219\angle 79.91^o\,\Omega}}$$

Alternatively, an easier way to obtain **Z** is to replace the transformer with its equivalent T circuit and use series/parallel impedance combinations. This leads to exactly the same result.

Chapter 13, Solution 35.

For mesh 1,

$$16 = (10 + j4)I_1 + j2I_2 \qquad (1)$$

For mesh 2, $$0 = j2I_1 + (30 + j26)I_2 - j12I_3 \qquad (2)$$

For mesh 3, $$0 = -j12I_2 + (5 + j11)I_3 \qquad (3)$$

We may use MATLAB to solve (1) to (3) and obtain

$$\mathbf{I_1} = 1.3736 - \text{j}0.5385 = \mathbf{1.4754\angle{-21.41^\circ}\ A}$$
$$\mathbf{I_2} = -0.0547 - \text{j}0.0549 = \mathbf{77.5\angle{-134.85^\circ}\ mA}$$
$$\mathbf{I_3} = -0.0268 - \text{j}0.0721 = \mathbf{77\angle{-110.41^\circ}\ mA}$$

1.4754∠–21.41° A, 77.5∠–134.85° mA, 77∠–110.41° mA

Chapter 13, Solution 36.

Following the two rules in section 13.5, we obtain the following:

(a) $V_2/V_1 = \mathbf{-n}$, $I_2/I_1 = \mathbf{-1/n}$ $(n = V_2/V_1)$

(b) $V_2/V_1 = \mathbf{-n}$, $I_2/I_1 = \mathbf{-1/n}$

(c) $V_2/V_1 = \mathbf{n}$, $I_2/I_1 = \mathbf{1/n}$

(d) $V_2/V_1 = \mathbf{n}$, $I_2/I_1 = \mathbf{-1/n}$

Chapter 13, Solution 37.

(a) $\boldsymbol{n = \frac{V_2}{V_1} = \frac{2400}{480} = \underline{5}}$

(b) $\boldsymbol{S_1 = I_1 V_1 = S_2 = I_2 V_2 = 50{,}000 \quad \longrightarrow \quad I_1 = \frac{50{,}000}{480} = \underline{104.17\ \text{A}}}$

(c) $\boldsymbol{I_2 = \frac{50{,}000}{2400} = \underline{20.83\ \text{A}}}$

Chapter 13, Solution 38.

Design a problem to help other students to better understand ideal transformers.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A 4-kVA, 2300/230-V rms transformer has an equivalent impedance of $2\angle 10°\Omega$ on the primary side. If the transformer is connected to a load with 0.6 power factor leading, calculate the input impedance.

Solution

$Z_{in} = Z_p + Z_L/n^2, \quad n = v_2/v_1 = 230/2300 = 0.1$

$v_2 = 230 \text{ V}, \; s_2 = v_2 I_2^*$

$I_2^* = s_2/v_2 = 17.391\angle{-53.13°}$ or $I_2 = 17.391\angle 53.13°$ A

$Z_L = v_2/I_2 = 230\angle 0°/17.391\angle 53.13° = 13.235\angle{-53.13°}$

$Z_{in} = 2\angle 10° + 1323.5\angle{-53.13°}$

$= 1.97 + j0.3473 + 794.1 - j1058.8$

$Z_{in} = \mathbf{1.324\angle{-53.05°}\ k\Omega}$

Chapter 13, Solution 39.

Referred to the high-voltage side,

$$Z_L = (1200/240)^2(0.8\angle 10°) = 20\angle 10°$$

$$Z_{in} = 60\angle{-30°} + 20\angle 10° = 76.4122\angle{-20.31°}$$

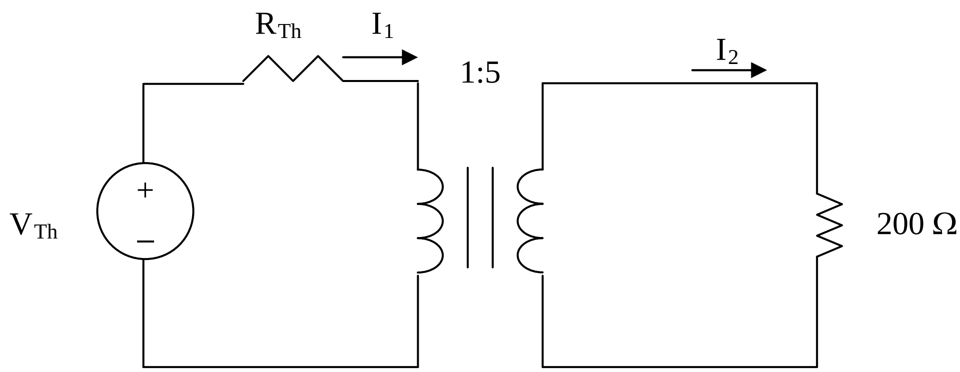
$$I_1 = 1200/Z_{in} = 1200/76.4122\angle{-20.31°} = \mathbf{15.7\angle 20.31°\ A}$$

Since $S = I_1 v_1 = I_2 v_2$, $I_2 = I_1 v_1/v_2$

$$= (1200/240)(15.7\angle 20.31°) = \mathbf{78.5\angle 20.31°\ A}$$

Chapter 13, Solution 40.

Consider the circuit as shown below.



We reflect the 200-Ω load to the primary side.

$$Z_p = 100 + \frac{200}{5^2} = 108$$

$$I_1 = \frac{10}{108}, \qquad I_2 = \frac{I_1}{n} = 2/108$$

$$P = \frac{1}{2}|I_2|^2 R_L = \frac{1}{2}\left(\frac{2}{108}\right)^2(200) = \underline{34.3\text{ mW}}$$

Chapter 13, Solution 41.

We reflect the 2-ohm resistor to the primary side.

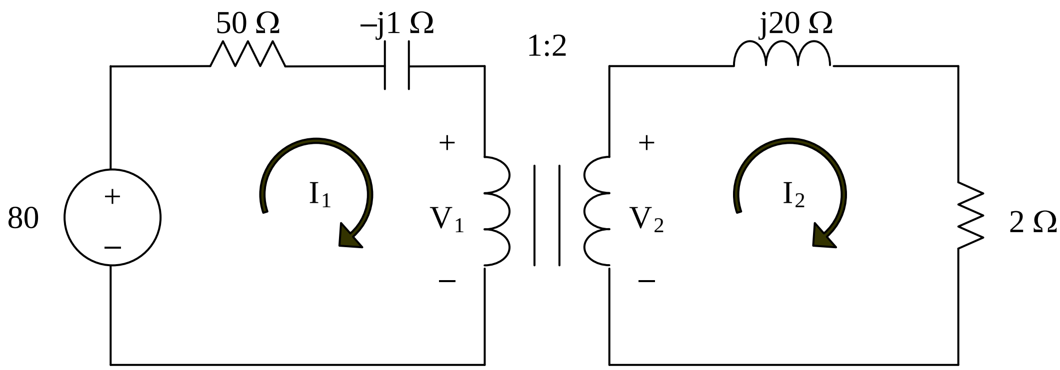
$$\mathbf{Z_{in}} = 10 + 2/n^2, \quad n = -1/3$$

Since both $\mathbf{I_1}$ and $\mathbf{I_2}$ enter the dotted terminals, $\quad \mathbf{Z_{in}} = 10 + 18 = 28$ ohms

$$\mathbf{I_1} = 14\angle 0°/28 = \mathbf{500\ mA} \text{ and } \mathbf{I_2} = \mathbf{I_1}/n = 0.5/(-1/3) = \mathbf{-1.5\ A}$$

Chapter 13, Solution 42.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-80 + (50-j)I_1 + V_1 = 0 \qquad (1)$$

For mesh 2,

$$-V_2 + (2+j20)I_2 = 0 \qquad (2)$$

At the transformer terminals,

$$V_2 = 2V_1 \text{ or } 2V_1 - V_2 = 0 \qquad (3)$$

$$I_1 = 2I_2 \text{ or } I_1 - 2I_2 = 0 \qquad (4)$$

From (1) to (4),

$$\begin{bmatrix} 50-j & 0 & 1 & 0 \\ 0 & 2+j20 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this with MATLAB,

```
>> A = [(50-j) 0 1 0;0 (2+20j) 0 -1;0 0 2 -1;1 -2 0 0]

A =

  Columns 1 through 3

  50.0000 - 1.0000i        0              1.0000
        0             2.0000 +20.0000i        0
        0                  0              2.0000
   1.0000            -2.0000                  0

  Column 4

        0
  -1.0000
  -1.0000
```

0

```
>> B = [80;0;0;0]

B =

    80
     0
     0
     0

>> C = inv(A)*B

C =
```

1.5743 - 0.1247i (I$_1$)
0.7871 - 0.0623i (I$_2$)
1.4106 + 7.8091i (V$_1$)
2.8212 +15.6181i (V$_2$)

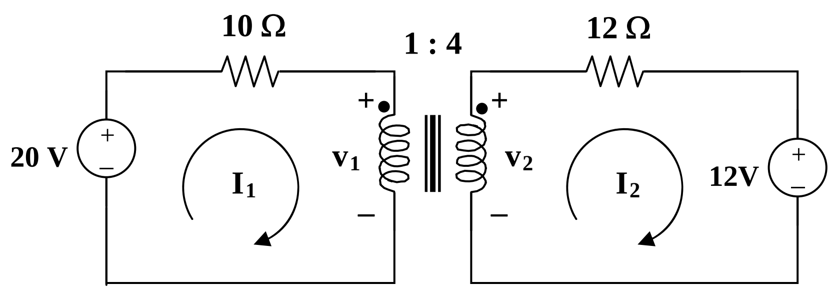
I_2 = (787.1–j62.3) mA or 789.6∠–4.53° mA

The power absorbed by the 2-Ω resistor is

$$P = |I_2|^2R = (0.7896)^2 2 = \mathbf{1.2469\ W}.$$

Chapter 13, Solution 43.

Transform the two current sources to voltage sources, as shown below.



Using mesh analysis, $-20 + 10I_1 + v_1 = 0$

$$20 = v_1 + 10I_1 \qquad (1)$$

$$12 + 12I_2 - v_2 = 0 \text{ or } 12 = v_2 - 12I_2 \qquad (2)$$

At the transformer terminal, $v_2 = nv_1 = 4v_1 \qquad (3)$

$$I_1 = nI_2 = 4I_2 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we get,

$$20 = v_1 + 40I_2 \qquad (5)$$

$$12 = 4v_1 - 12I_2 \qquad (6)$$

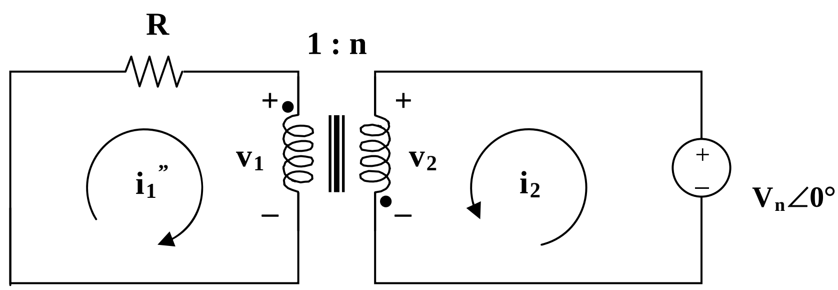
Solving (5) and (6) gives $v_1 = \mathbf{4.186\ V}$ and $v_2 = 4v = \mathbf{16.744\ V}$

Chapter 13, Solution 44.

We can apply the superposition theorem. Let $i_1 = i_1' + i_1''$ and $i_2 = i_2' + i_2''$ where the single prime is due to the DC source and the double prime is due to the AC source. Since we are looking for the steady-state values of i_1 and i_2,

$$i_1' = i_2' = 0.$$

For the AC source, consider the circuit below.



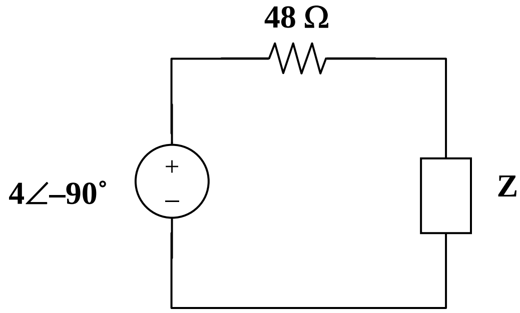
$$v_2/v_1 = -n, \quad I_2''/I_1'' = -1/n$$

But $v_2 = v_m$, $v_1 = -v_m/n$ or $I_1'' = v_m/(Rn)$

$$I_2'' = -I_1''/n = -v_m/(Rn^2)$$

Hence, $i_1(t) = \mathbf{(v_m/Rn)\cos\omega t\ A}$, and $i_2(t) = \mathbf{(-v_m/(n^2R))\cos\omega t\ A}$

Chapter 13, Solution 45.



$$Z_L = 8 - \frac{j}{\omega C} = 8 - j4\,, \;\; n = 1/3$$

$$Z = \frac{Z_L}{n^2} = 9Z_L = 72 - j36$$

$$I = \frac{4\angle -90^\circ}{48 + 72 - j36} = \frac{4\angle -90^\circ}{125.28\angle -16.7^\circ} = 0.03193\angle -73.3^\circ$$

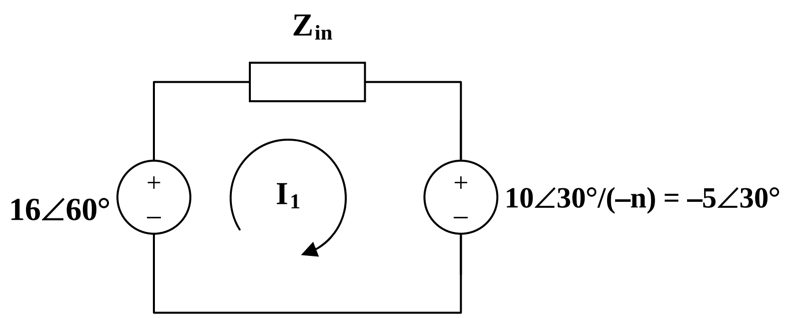
We now have some choices, we can go ahead and calculate the current in the second loop and calculate the power delivered to the 8-ohm resistor directly or we can merely say that the power delivered to the equivalent resistor in the primary side must be the same as the power delivered to the 8-ohm resistor. Therefore,

$$P_{8\Omega} = \left|\frac{I}{2}\right|^2 72 = 0.5098\text{x}10^{-3}72 = \mathbf{36.71\ mW}$$

The student is encouraged to calculate the current in the secondary and calculate the power delivered to the 8-ohm resistor to verify that the above is correct.

Chapter 13, Solution 46.

(a) Reflecting the secondary circuit to the primary, we have the circuit shown below.



$\mathbf{Z_{in}} = 10 + j16 + (1/4)(12 - j8) = 13 + j14$

$-16\angle 60° + \mathbf{Z_{in}}\mathbf{I_1} - 5\angle 30° = 0$ or $\mathbf{I_1} = (16\angle 60° + 5\angle 30°)/(13 + j14)$

Hence, $\mathbf{I_1} = \mathbf{1.072\angle 5.88°\ A}$, and $\mathbf{I_2} = -0.5\mathbf{I_1} = \mathbf{0.536\angle 185.88°\ A}$

(b) Switching a dot will not affect Z_{in} but will affect $\mathbf{I_1}$ and $\mathbf{I_2}$.

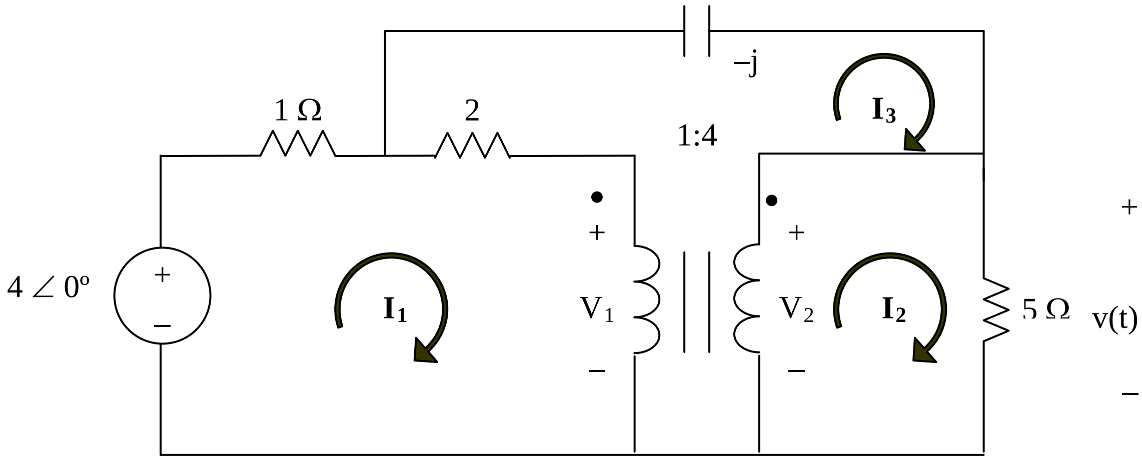
$\mathbf{I_1} = (16\angle 60° - 5\angle 30°)/(13 + j14) = \mathbf{0.625\ \angle 25\ A}$

and $\mathbf{I_2} = 0.5\mathbf{I_1} = \mathbf{0.3125\angle 25°\ A}$

Chapter 13, Solution 47.

$$(1/3)\text{ F } 1F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j3x1/3} = -j1$$

Consider the circuit shown below.



For mesh 1,

$3I_1 - 2I_3 + V_1 = 4$ (1)

For mesh 2,

$5I_2 - V_2 = 0$ (2)

For mesh 3,

$-2I_1\ (2-j)I_3 - V_1 + V_2 = 0$ (3)

At the terminals of the transformer,

$V_2 = nV_1 = 4V_1$ (4)

$I_1 - I_3 = 4(I_2 - I_3)$ (5)

In matrix form,

$$\begin{bmatrix} 3 & 0 & -2 & 1 & 0 \\ 0 & 5 & 0 & 0 & -1 \\ -2 & 0 & 2-j & -1 & 1 \\ 0 & 0 & 0 & -4 & 1 \\ 1 & -4 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this using MATLAB yields

```
>>A = [3,0,-2,1,0; 0,5,0,0,-1; -2,0,(2-j),-1,1 ;0,0,0,-4,1; 1,-4,3,0,0]
```

A =

3.0000	0	-2.0000	1.0000	0
0	5.0000	0	0	-1.0000
-2.0000	0	2.0000 - 1.0000i	-1.0000	1.0000
0	0	0	-4.0000	1.0000
1.0000	-4.0000	3.0000	0	0

```
>>U = [4;0;0;0;0]

>>X = inv(A)*U
```

X =

1.2952 + 0.0196i
0.5287 + 0.0507i
0.2733 + 0.0611i
0.6609 + 0.0634i
2.6437 + 0.2537i

$\mathbf{V} = 5\mathbf{I_2} = V_2 = 2.6437+j0.2537 = 2.656\angle 5.48° \text{ V}$, therefore,

v(t) = **2.656cos(3t+5.48˚) V**

Chapter 13, Solution 48.

Using Fig. 13.113, design a problem to help other students to better understand how ideal transformers work.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find $\mathbf{I}_x$ in the ideal transformer circuit of Fig. 13.112.

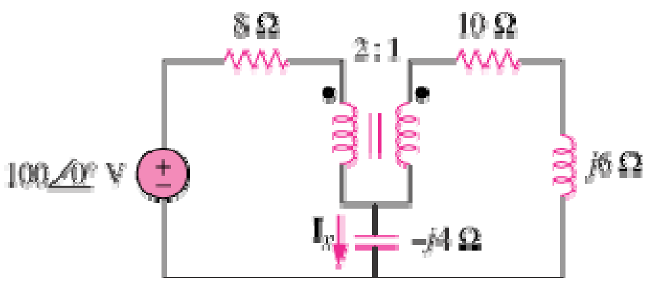
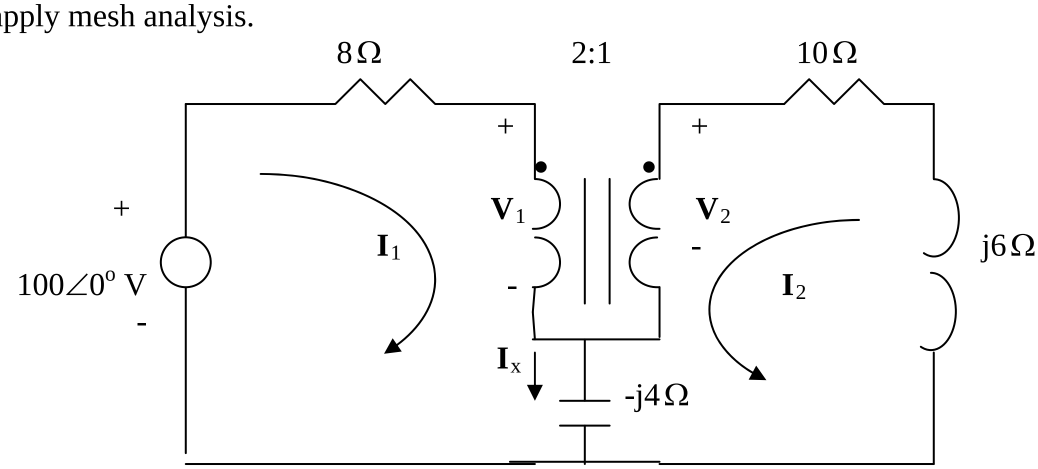


Figure 13.112

Solution

We apply mesh analysis.



$$100 = (8 - j4)I_1 - j4I_2 + V_1 \qquad (1)$$

$$0 = (10 + j2)I_2 - j4I_1 + V_2 \qquad (2)$$

But

$$\frac{V_2}{V_1} = n = \frac{1}{2} \longrightarrow V_1 = 2V_2 \qquad (3)$$

$$\frac{I_2}{I_1} = -\frac{1}{n} = -2 \quad \longrightarrow \quad I_1 = -0.5I_2 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we obtain

$$100 = (-4 - j2)I_2 + 2V_2 \qquad (1)a$$

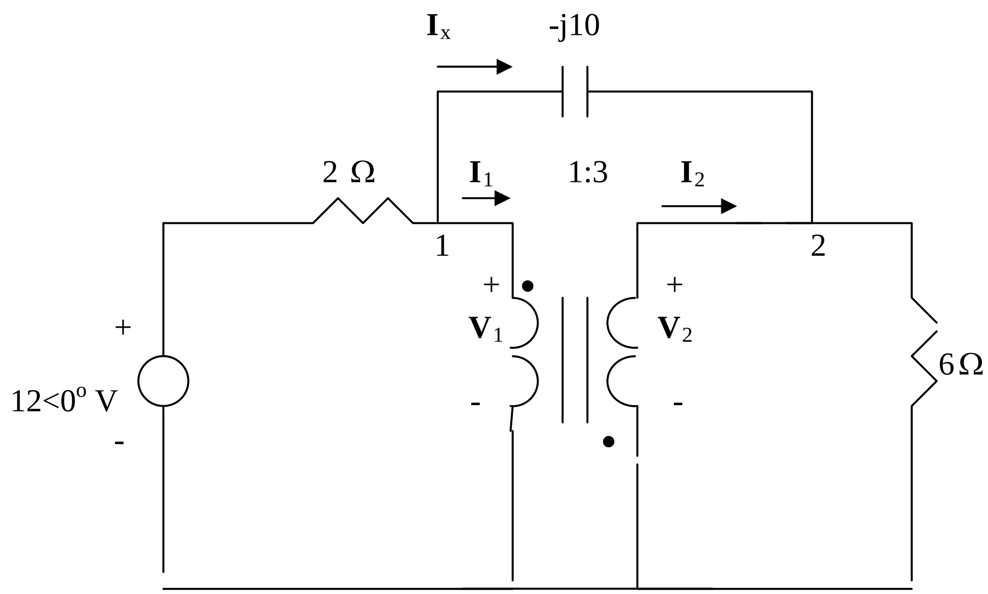
$$0 = (10 + j4)I_2 + V_2 \qquad (2)a$$

Solving (1)a and (2)a leads to **I**$_2$ = -3.5503 +j1.4793

$$\boldsymbol{I_x = I_1 + I_2 = 0.5I_2 = \underline{1.923\angle 157.4^o \ \ A}}$$

Chapter 13, Solution 49.

$$\omega = 2, \qquad \frac{1}{20}\text{F} \quad \longrightarrow \quad \frac{1}{j\omega C} = -j10$$



At node 1,

$$\frac{12 - V_1}{2} = \frac{V_1 - V_2}{-j10} + I_1 \quad \longrightarrow \quad 12 = 2I_1 + V_1(1 + j0.2) - j0.2V_2 \qquad (1)$$

At node 2,

$$I_2 + \frac{V_1 - V_2}{-j10} = \frac{V_2}{6} \quad \longrightarrow \quad 0 = 6I_2 + j0.6V_1 - (1 + j0.6)V_2 \qquad (2)$$

At the terminals of the transformer, $V_2 = -3V_1, \quad I_2 = -\frac{1}{3}I_1$

Substituting these in (1) and (2),

$$12 = -6I_2 + V_1(1 + j0.8), \qquad 0 = 6I_2 + V_1(3 + j2.4)$$

Adding these gives $\mathbf{V}_1$=1.829 –j1.463 and

$$I_x = \frac{V_1 - V_2}{-j10} = \frac{4V_1}{-j10} = 0.937\angle 51.34^o$$

$i_x(t)$ = **937cos(2t+51.34˚) mA**.

Chapter 13, Solution 50.

The value of Z_{in} is not effected by the location of the dots since n^2 is involved.

$$Z_{in}' = (6 - j10)/(n')^2, \quad n' = 1/4$$

$$Z_{in}' = 16(6 - j10) = 96 - j160$$

$$Z_{in} = 8 + j12 + (Z_{in}' + 24)/n^2, \quad n = 5$$

$$Z_{in} = 8 + j12 + (120 - j160)/25 = 8 + j12 + 4.8 - j6.4$$

$$Z_{in} = \mathbf{(12.8 + j5.6)\ \Omega}$$

Chapter 13, Solution 51.

Let $\mathbf{Z}_3 = 36 + j18$, where $\mathbf{Z}_3$ is reflected to the middle circuit.

$\mathbf{Z}_R' = \mathbf{Z}_L/n^2 = (12 + j2)/4 = 3 + j0.5$

$$\mathbf{Z}_{in} = 5 - j2 + \mathbf{Z}_R' = \mathbf{[8 - j1.5]\ \Omega}$$

$$\mathbf{I}_1 = 24\angle 0°/\mathbf{Z}_{eq} = 24\angle 0°/(8 - j1.5) = 24\angle 0°/8.14\angle -10.62° = \mathbf{8.95\angle 10.62°\ A}$$

[8 – j1.5] Ω, 8.95∠10.62° A

Chapter 13, Solution 52.

For maximum power transfer,

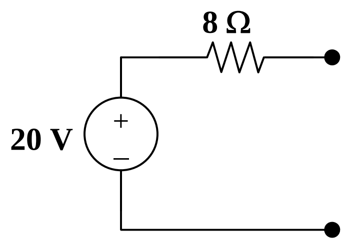
$40 = Z_L/n^2 = 10/n^2$ or $n^2 = 10/40$ which yields $n = 1/2 = 0.5$

$I = 120/(40 + 40) = 3/2$

$p = I^2R = (9/4)x40 =$ **90 watts**.

Chapter 13, Solution 53.

(a) The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is $Z_L' = Z_L/n^2 = 200/n^2$.

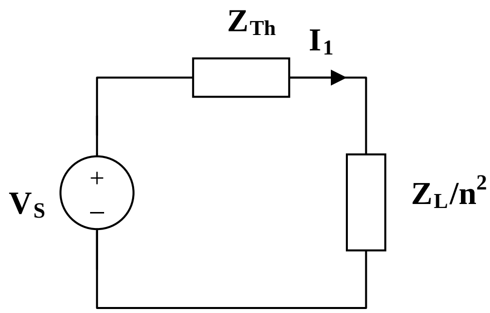
For maximum power transfer, $8 = 200/n^2$ produces $n = \mathbf{5}$.

(b) If $n = 10$, $Z_L' = 200/10 = 2$ and $I = 20/(8 + 2) = 2$

$p = I^2 Z_L' = (2)^2(2) =$ **8 watts**.

Chapter 13, Solution 54.

(a)



For maximum power transfer,

$$Z_{Th} = Z_L/n^2, \text{ or } n^2 = Z_L/Z_{Th} = 8/128$$

$$n = \mathbf{0.25}$$

(b) $I_1 = V_{Th}/(Z_{Th} + Z_L/n^2) = 10/(128 + 128) = \mathbf{39.06\ mA}$

(c) $v_2 = I_2 Z_L = 156.24\text{x}8 \text{ mV} = 1.25 \text{ V}$

But $v_2 = nv_1$ therefore $v_1 = v_2/n = 4(1.25) = \mathbf{5\ V}$

Chapter 13, Solution 55.

We first reflect the 60-Ω resistance to the middle circuit.

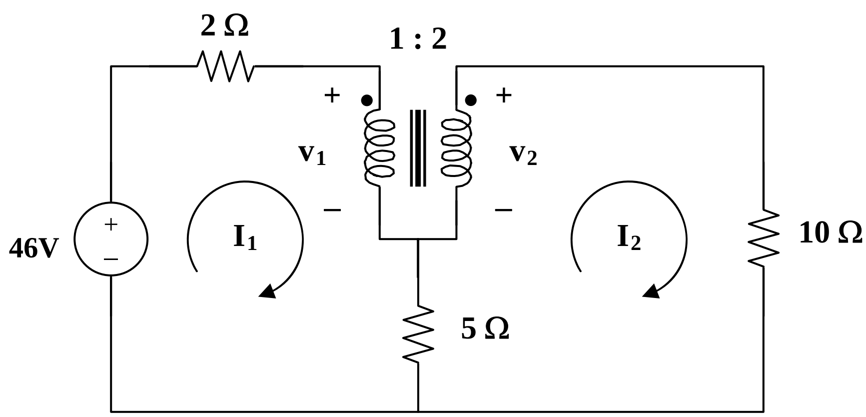
$$Z_L' = 20 + \frac{60}{3^2} = 26.67\Omega$$

We now reflect this to the primary side.

$$Z_L = \frac{Z_L'}{4^2} = \frac{26.67}{16} = \mathbf{1.6669\ \Omega}$$

Chapter 13, Solution 56.

We apply mesh analysis to the circuit as shown below.



For mesh 1, $46 = 7\mathbf{I_1} - 5\mathbf{I_2} + \mathbf{v_1}$ (1)

For mesh 2, $\mathbf{v_2} = 15\mathbf{I_2} - 5\mathbf{I_1}$ (2)

At the terminals of the transformer,

$$\mathbf{v_2} = n\mathbf{v_1} = 2\mathbf{v_1} \quad (3)$$

$$\mathbf{I_1} = n\mathbf{I_2} = 2\mathbf{I_2} \quad (4)$$

Substituting (3) and (4) into (1) and (2),

$$46 = 9\mathbf{I_2} + \mathbf{v_1} \quad (5)$$

$$\mathbf{v_1} = 2.5\mathbf{I_2} \quad (6)$$

Combining (5) and (6), $46 = 11.5\mathbf{I_2}$ or $I_2 = 4$

$$P_{10} = 0.5|\mathbf{I_2}|^2(10) = \mathbf{80\ watts}.$$

Chapter 13, Solution 57.

(a) $Z_L = j3||(12 - j6) = j3(12 - j6)/(12 - j3) = (12 + j54)/17$

Reflecting this to the primary side gives

$Z_{in} = 2 + Z_L/n^2 = 2 + (3 + j13.5)/17 = 2.3168\angle 20.04°$

$I_1 = v_s/Z_{in} = 60\angle 90°/2.3168\angle 20.04° =$ **$25.9\angle 69.96°$ A(rms)**

$I_2 = I_1/n =$ **$12.95\angle 69.96°$ A(rms)**

(b) $60\angle 90° = 2I_1 + v_1$ or $v_1 = j60 - 2I_1 = j60 - 51.8\angle 69.96°$

$v_1 =$ **$21.06\angle 147.44°$ V(rms)**

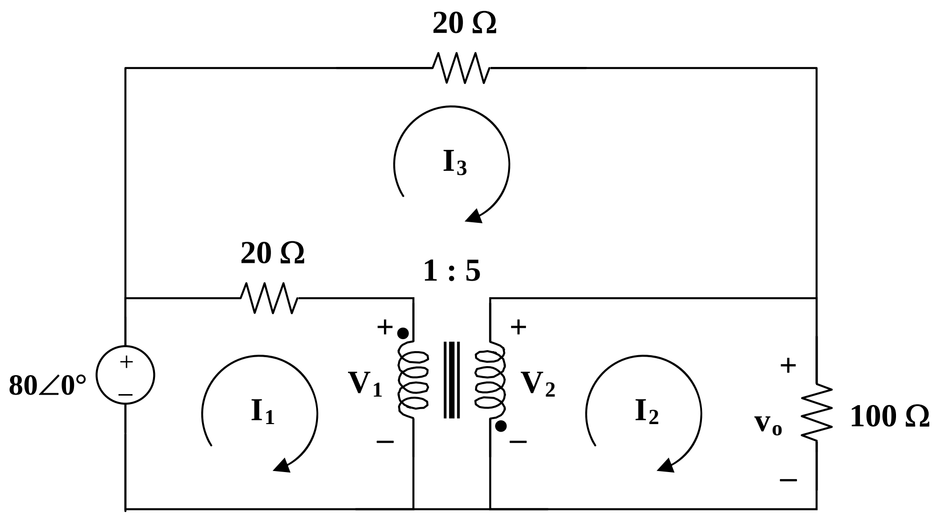
$v_2 = nv_1 =$ **$42.12\angle 147.44°$ V(rms)**

$v_o = v_2 =$ **$42.12\angle 147.44°$ V(rms)**

(c) $S = v_s I_1^* = (60\angle 90°)(25.9\angle -69.96°) =$ **$1.554\angle 20.04°$ kVA**

Chapter 13, Solution 58.

Consider the circuit below.



For mesh1, $-80 + 20I_1 - 20I_3 + V_1 = 0$ or

$20I_1 - 20I_3 + V_1 = 80$ (1)

For mesh 2, $V_2 = 100I_2$ or $100I_2 - V_2 = 0$

(2)

For mesh 3, $40I_3 - 20I_1 + V_2 - V_1 = 0$ which leads to

$-20 I_1 + 40I_3 - V_1 + V_2 = 0$ (3)

At the transformer terminals, $V_2 = -nV_1 = -5V_1$ or $5V_1 + V_2$ (4)

$I_1 - I_3 = -n(I_2 - I_3) = -5(I_2 - I_3)$ or

$I_1 + 5I_2 - 6I_3 = 0$ (5)

Solving using MATLAB,

```
>>A = [ 20  0  -20  1  0 ; 0  100  0  0  -1; -20  0  40  -1  1; 0  0  0  5  1;  1  5  -6  0  0 ]

A =

    20     0   -20     1     0
     0   100     0     0    -1
   -20     0    40    -1     1
     0     0     0     5     1
     1     5    -6     0     0
```

```
>> B = [ 80  0  0  0  0 ]'

B =

    80
     0
     0
     0
     0

>> Y = inv(A)*B

Y =

    5.9355
    0.5161
    1.4194
  -10.3226
   51.6129
```

P20,1 = 0.5*(I1 – I3)^2*20 = 0.5*(5.9355– 1.4194)^2*20 = 203.95

p_{20}(the one between 1 and 3) = $0.5(20)(I_1 - I_3)^2 = 10(5.9355 - 1.4194)^2$

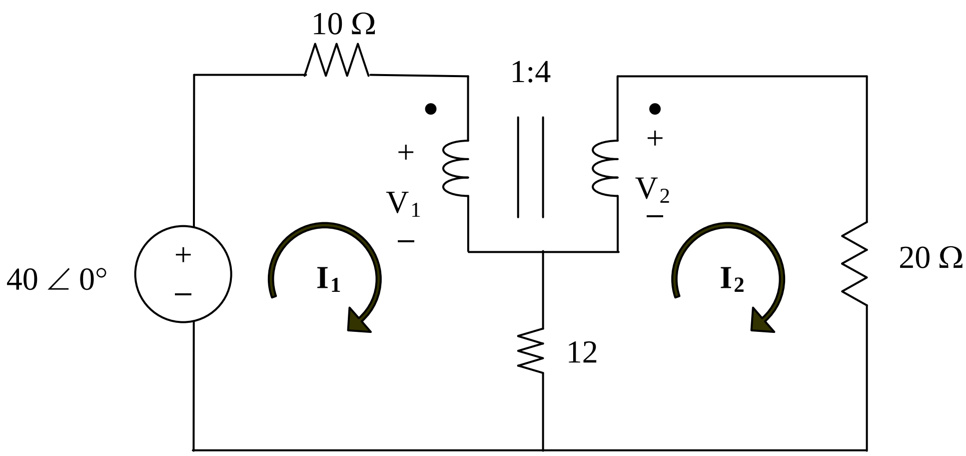
= **203.95 watts**

p_{20}(at the top of the circuit) = $0.5(20)I_3^2$ = **20.15 watts**

$p_{100} = 0.5(100)I_2^2$ = **13.318 watts**

Chapter 13, Solution 59.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-40 + 22\mathbf{I_1} - 12\mathbf{I_2} + \mathbf{V_1} = 0 \qquad (1)$$

For mesh 2,

$$-12\mathbf{I_1} + 32\mathbf{I_2} - \mathbf{V_2} = 0 \qquad (2)$$

At the transformer terminals,

$$-4\mathbf{V_1} + \mathbf{V_2} = 0 \qquad (3)$$

$$\mathbf{I_1} - 4\mathbf{I_2} = 0 \qquad (4)$$

Putting (1), (2), (3), and (4) in matrix form, we obtain

$$\begin{bmatrix} 22 & -12 & 1 & 0 \\ -12 & 32 & 0 & -1 \\ 0 & 0 & -4 & 1 \\ 1 & -4 & 0 & 0 \end{bmatrix} I = \begin{bmatrix} 40 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
>> A=[22,-12,1,0;-12,32,0,-1;0,0,-4,1;1,-4,0,0]
A =
   22  -12    1    0
  -12   32    0   -1
    0    0   -4    1
    1   -4    0    0
```

```
>> U=[40;0;0;0]
U =
    40
     0
     0
     0
>> X=inv(A)*U
X =
    2.2222
    0.5556
   -2.2222
   -8.8889
```

For 10-Ω resistor,

$$P_{10} = [(2.222)^2/2]10 = \mathbf{24.69\ W}$$

For 12-Ω resistor,

$$P_{12} = [(2.222–0.5556)^2/2]12 = \mathbf{16.661\ W}$$

For 20-Ω resistor,

$$P_{20} = [(0.5556)^2/2]20 = \mathbf{3.087\ W}.$$

24.69 W, 16.661 W, 3.087 W

Chapter 13, Solution 60.

(a) Transferring the 40-ohm load to the middle circuit,

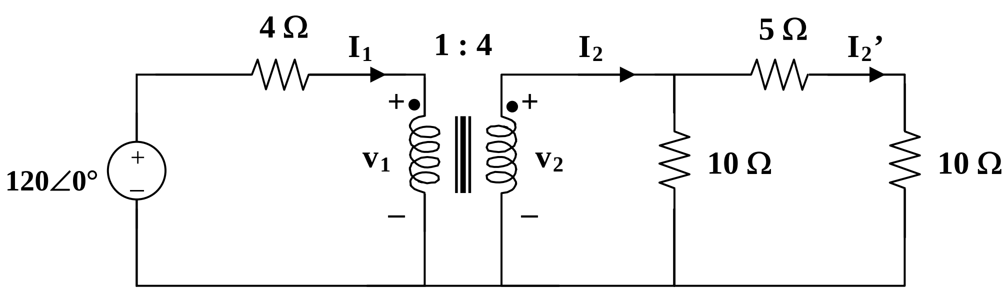
$Z_L' = 40/(n')^2 = 10$ ohms where $n' = 2$

$10||(5 + 10) = 6$ ohms

We transfer this to the primary side.

$Z_{in} = 4 + 6/n^2 = 4 + 0.375 = 4.375$ ohms, where $n = 4$

$I_1 = 120/4.375 =$ **27.43 A** and $I_2 = I_1/n =$ **6.857 A**



Using current division, $I_2' = (10/25)I_2 = 2.7429$ and

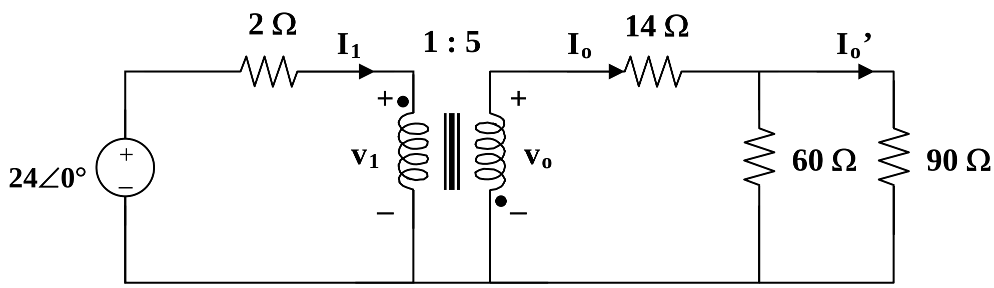
$I_3 = I_2'/n' =$ **1.3714 A**

(b) $p = 0.5(I_3)^2(40) =$ **37.62 watts**

Chapter 13, Solution 61.

We reflect the 160-ohm load to the middle circuit.

$$Z_R = Z_L/n^2 = 160/(4/3)^2 = 90 \text{ ohms, where } n = 4/3$$



$$14 + 60||90 = 14 + 36 = 50 \text{ ohms}$$

We reflect this to the primary side.

$Z_R' = Z_L'/(n')^2 = 50/5^2 = 2$ ohms when $n' = 5$

$I_1 = 24/(2 + 2) =$ **6A**

$24 = 2I_1 + v_1$ or $v_1 = 24 – 2I_1 = 12$ V

$v_o = –nv_1 =$ **–60 V**, $I_o = –I_1 /n_1 = –6/5 = –1.2$

$I_o' = [60/(60 + 90)]I_o = –0.48$A

$I_2 = –I_o'/n = 0.48/(4/3) =$ **360 mA**

Chapter 13, Solution 62.

(a) Reflect the load to the middle circuit.

$\mathbf{Z_L}' = 8 - j20 + (18 + j45)/3^2 = 10 - j15$

We now reflect this to the primary circuit so that

$\mathbf{Z_{in}} = 6 + j4 + (10 - j15)/n^2 = 7.6 + j1.6 = 7.767\angle 11.89°$, where $n = 5/2 = 2.5$

$\mathbf{I_1} = 40/\mathbf{Z_{in}} = 40/7.767\angle 11.89° = 5.15\angle -11.89°$

$$\mathbf{S} = \mathbf{v_s I_1^*} = (40\angle 0°)(5.15\angle 11.89°) = \mathbf{206\angle 11.89°\ VA}$$

(b) $\mathbf{I_2} = -\mathbf{I_1}/n, \quad n = 2.5$

$\mathbf{I_3} = -\mathbf{I_2}/n', \quad n = 3$

$\mathbf{I_3} = \mathbf{I_1}/(nn') = 5.15\angle -11.89°/(2.5x3) = 0.6867\angle -11.89°$

$$p = |\mathbf{I_2}|^2(18) = 18(0.6867)^2 = \mathbf{8.488\ watts}$$

Chapter 13, Solution 63.

Reflecting the (9 + j18)-ohm load to the middle circuit gives,

$Z_{in}' = 7 - j6 + (9 + j18)/(n')^2 = 7 - j6 + 1 + j2 = 8 - j4$ when $n' = 3$

Reflecting this to the primary side,

$Z_{in} = 1 + Z_{in}'/n^2 = 1 + 2 - j = 3 - j$, where $n = 2$

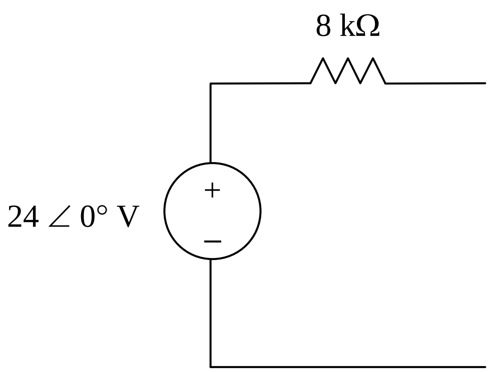
$I_1 = 12\angle 0°/(3 - j) = 12/3.162\angle -18.43° =$ $\mathbf{3.795\angle 18.43A}$

$I_2 = I_1/n =$ $\mathbf{1.8975\angle 18.43°\ A}$

$I_3 = -I_2/n^2 =$ $\mathbf{632.5\angle 161.57°\ mA}$

Chapter 13, Solution 64.

The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is

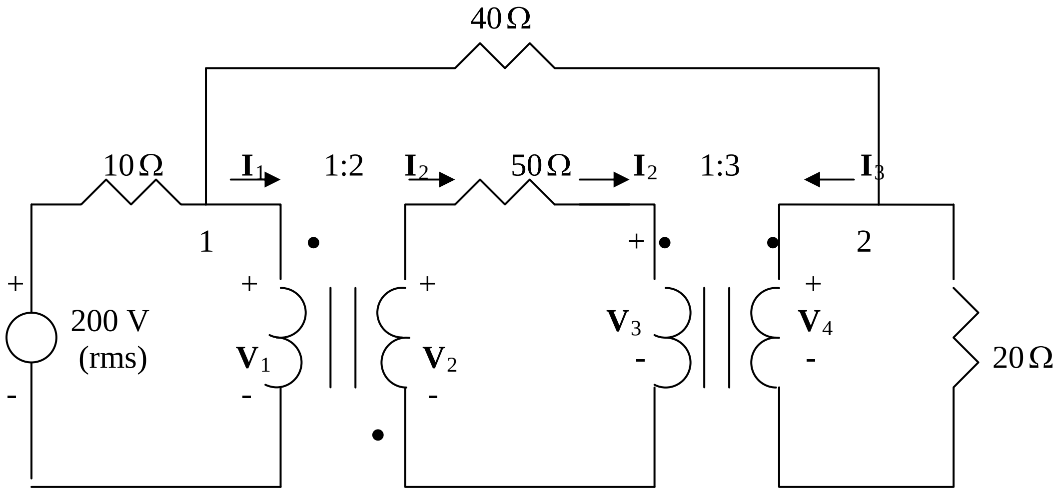
$$Z_L^{'} = \frac{Z_L}{n^2} = \frac{30k}{n^2}$$

For maximum power transfer,

$$8k\Omega = \frac{30k\Omega}{n^2} \quad \longrightarrow \quad n^2 = 30/8 = 3.75$$

n =**1.9365**

Chapter 13, Solution 65.



At node 1,

$$\frac{200 - V_1}{10} = \frac{V_1 - V_4}{40} + I_1 \quad \longrightarrow \quad 200 = 1.25V_1 - 0.25V_4 + 10I_1 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_4}{40} = \frac{V_4}{20} + I_3 \quad \longrightarrow \quad V_1 = 3V_4 + 40I_3 \qquad (2)$$

At the terminals of the first transformer,

$$\frac{V_2}{V_1} = -2 \quad \longrightarrow \quad V_2 = -2V_1 \qquad (3)$$

$$\frac{I_2}{I_1} = -1/2 \quad \longrightarrow \quad I_1 = -2I_2 \qquad (4)$$

For the middle loop,

$$-V_2 + 50I_2 + V_3 = 0 \quad \longrightarrow \quad V_3 = V_2 - 50I_2 \qquad (5)$$

At the terminals of the second transformer,

$$\frac{V_4}{V_3} = 3 \quad \longrightarrow \quad V_4 = 3V_3 \qquad (6)$$

$$\frac{I_3}{I_2} = -1/3 \quad \longrightarrow \quad I_2 = -3I_3 \tag{7}$$

We have seven equations and seven unknowns. Combining (1) and (2) leads to

$$200 = 3.5V_4 + 10I_1 + 50I_3$$

But from (4) and (7), $I_1 = -2I_2 = -2(-3I_3) = 6I_3$. Hence

$$200 = 3.5V_4 + 110I_3 \tag{8}$$

From (5), (6), (3), and (7),

$$V_4 = 3(V_2 - 50I_2) = 3V_2 - 150I_2 = -6V_1 + 450I_3$$

Substituting for V_1 in (2) gives

$$V_4 = -6(3V_4 + 40I_3) + 450I_3 \quad \longrightarrow \quad I_3 = \frac{19}{210}V_4 \tag{9}$$

Substituting (9) into (8) yields

$$200 = 13.452V_4 \quad \longrightarrow \quad V_4 = 14.87$$

$$\boldsymbol{P = \frac{V^2{}_4}{20} = \underline{11.05\ \mathrm{W}}}$$

Chapter 13, Solution 66.

Design a problem to help other students to better understand how the ideal autotransformer works.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

An ideal autotransformer with a 1:4 step-up turns ratio has its secondary connected to a 120-Ω load and the primary to a 420-V source. Determine the primary current.

Solution

$$v_1 = 420 \text{ V} \tag{1}$$

$$v_2 = 120I_2 \tag{2}$$

$$v_1/v_2 = 1/4 \text{ or } v_2 = 4v_1 \tag{3}$$

$$I_1/I_2 = 4 \text{ or } I_1 = 4 I_2 \tag{4}$$

Combining (2) and (4),

$$v_2 = 120[(1/4)I_1] = 30 I_1$$

$$4v_1 = 30I_1$$

$$4(420) = 1680 = 30I_1 \text{ or } I_1 = \mathbf{56\ A}$$

Chapter 13, Solution 67.

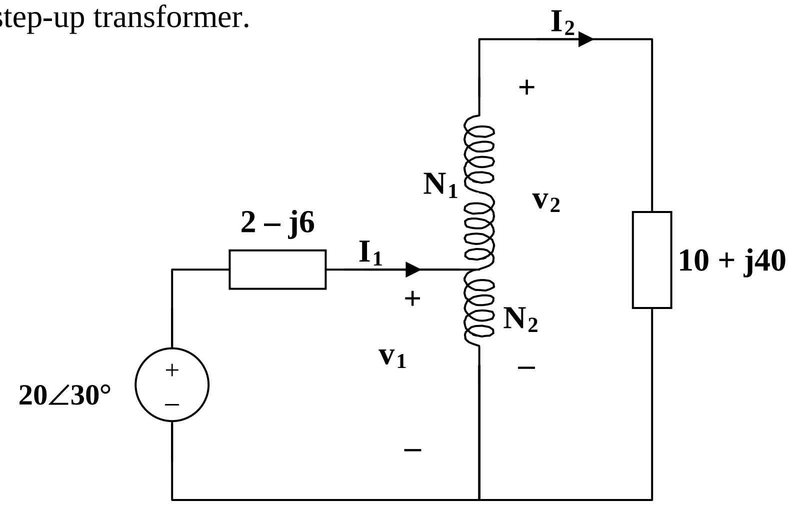
(a) $$\frac{V_1}{V_2} = \frac{N_1 + N_2}{N_2} = \frac{1}{0.4} \longrightarrow V_2 = 0.4V_1 = 0.4x400 = \underline{160\text{ V}}$$

(b) $$S_2 = I_2V_2 = 5{,}000 \longrightarrow I_2 = \frac{5000}{160} = \underline{31.25\text{ A}}$$

(c) $$S_2 = S_1 = I_1V_1 = 5{,}000 \longrightarrow I_1 = \frac{5000}{400} = \underline{12.5\text{ A}}$$

Chapter 13, Solution 68.

This is a step-up transformer.



For the primary circuit, $20\angle 30° = (2 - j6)\mathbf{I_1} + \mathbf{v_1}$ (1)

For the secondary circuit, $\mathbf{v_2} = (10 + j40)\mathbf{I_2}$ (2)

At the autotransformer terminals,

$$\mathbf{v_1}/\mathbf{v_2} = N_1/(N_1 + N_2) = 200/280 = 5/7,$$

$$\text{thus } \mathbf{v_2} = 7\mathbf{v_1}/5 \tag{3}$$

Also, $\mathbf{I_1}/\mathbf{I_2} = 7/5$ or $\mathbf{I_2} = 5\mathbf{I_1}/7$ (4)

Substituting (3) and (4) into (2), $\mathbf{v_1} = (10 + j40)25\mathbf{I_1}/49$

Substituting that into (1) gives $20\angle 30° = (7.102 + j14.408)\mathbf{I_1}$

$$\mathbf{I_1} = 20\angle 30°/16.063\angle 63.76° = \mathbf{1.245\angle{-33.76}°\ A}$$

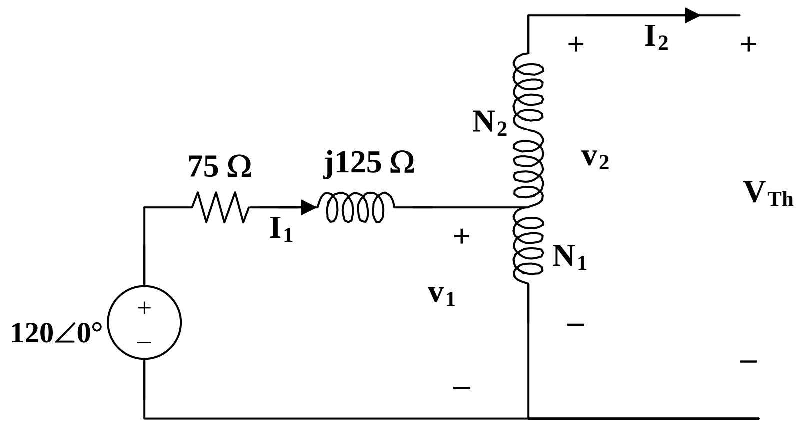
$$\mathbf{I_2} = 5\mathbf{I_1}/7 = \mathbf{889.3\angle{-33.76}°\ mA}$$

$$\mathbf{I_o} = \mathbf{I_1} - \mathbf{I_2} = [(5/7) - 1]\mathbf{I_1} = -2\mathbf{I_1}/7 = \mathbf{355.7\angle 146.2°\ mA}$$

$$p = |\mathbf{I_2}|^2 R = (0.8893)^2(10) = \mathbf{7.51\ watts}$$

Chapter 13, Solution 69.

We can find the Thevenin equivalent.

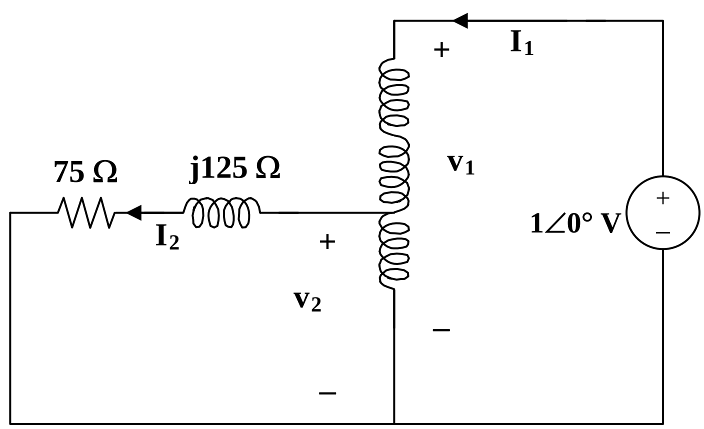


$$I_1 = I_2 = 0$$

As a step up transformer, $v_1/v_2 = N_1/(N_1 + N_2) = 600/800 = 3/4$

$$v_2 = 4v_1/3 = 4(120)/3 = 160\angle 0° \text{ rms} = V_{Th}.$$

To find Z_{Th}, connect a 1-V source at the secondary terminals. We now have a step-down transformer.



$$v_1 = 1V, \; v_2 = I_2(75 + j125)$$

But $v_1/v_2 = (N_1 + N_2)/N_1 = 800/200$ which leads to $v_1 = 4v_2 = 1$

and $v_2 = 0.25$

$$I_1/I_2 = 200/800 = 1/4 \text{ which leads to } I_2 = 4I_1$$

Hence $0.25 = 4I_1(75 + j125)$ or $I_1 = 1/[16(75 + j125)$

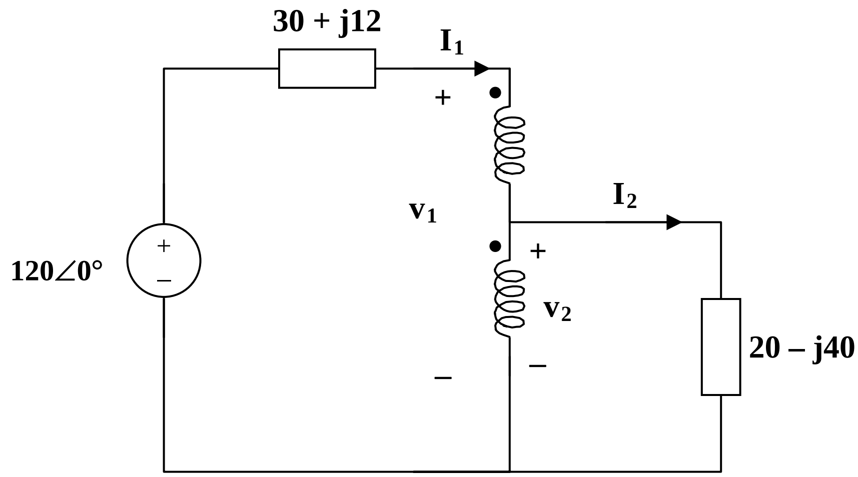
$Z_{Th} = 1/I_1 = 16(75 + j125)$

Therefore, $Z_L = Z_{Th}^* =$ **$(1.2 - j2)$ kΩ**

Since V_{Th} is rms, $p = (|V_{Th}|/2)^2/R_L = (80)^2/1200 =$ **5.333 watts**

Chapter 13, Solution 70.

This is a step-down transformer.



$$I_1/I_2 = N_2/(N_1 + N_2) = 200/1200 = 1/6, \text{ or } I_1 = I_2/6 \tag{1}$$

$$v_1/v_2 = (N_2 + N_2)/N_2 = 6, \text{ or } v_1 = 6v_2 \tag{2}$$

For the primary loop, $120 = (30 + j12)I_1 + v_1$ (3)

For the secondary loop, $v_2 = (20 - j40)I_2$ (4)

Substituting (1) and (2) into (3),

$$120 = (30 + j12)(I_2/6) + 6v_2$$

and substituting (4) into this yields

$$120 = (49 - j38)I_2 \text{ or } I_2 = 1.935\angle 37.79°$$

$p = |I_2|^2(20) =$ **74.9 watts**.

Chapter 13, Solution 71.

$$Z_{in} = V_1/I_1$$

But $\quad V_1 I_1 = V_2 I_2$, or $V_2 = I_2 Z_L$ and $I_1/I_2 = N_2/(N_1 + N_2)$

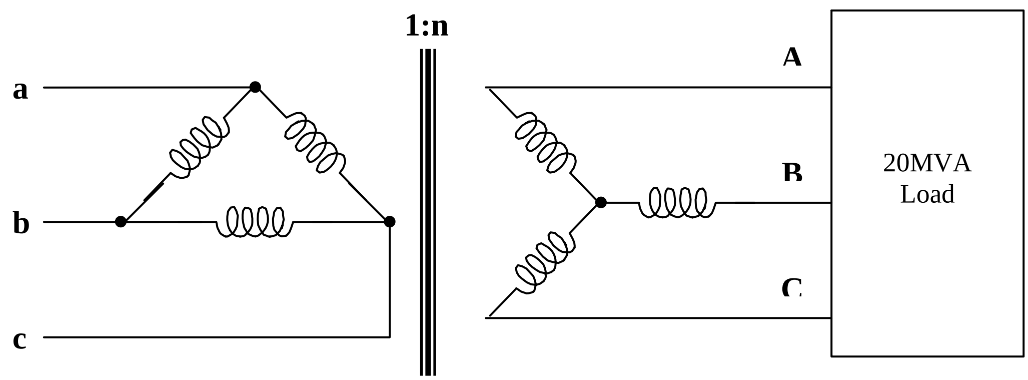
Hence $\quad V_1 = V_2 I_2/I_1 = Z_L(I_2/I_1)I_2 = Z_L(I_2/I_1)^2 I_1$

$$V_1/I_1 = Z_L[(N_1 + N_2)/N_2]^2$$

$$Z_{in} = \mathbf{[1 + (N_1/N_2)]^2 Z_L}$$

Chapter 13, Solution 72.

(a) Consider just one phase at a time.



$$n = V_L/\sqrt{3}V_{Lp} = 7200/(12470\sqrt{3}) = \mathbf{1/3}$$

(b) The load carried by each transformer is 60/3 = 20 MVA.

Hence I_{Lp} = 20 MVA/12.47 k = **1604 A**

I_{Ls} = 20 MVA/7.2 k = **2778 A**

(c) The current in incoming line a, b, c is

$$\sqrt{3}I_{Lp} = \sqrt{3}\text{x}1603.85 = \mathbf{2778\ A}$$

Current in each outgoing line A, B, C is

$$2778/(n\sqrt{3}) = \mathbf{4812\ A}$$

Chapter 13, Solution 73.

(a) This is a **three-phase Δ-Y transformer**.

(b) $V_{Ls} = nv_{Lp}/\sqrt{3} = 450/(3\sqrt{3}) = 86.6$ V, where $n = 1/3$

As a Y-Y system, we can use per phase equivalent circuit.

$I_a = V_{an}/Z_Y = 86.6\angle 0°/(8 - j6) = 8.66\angle 36.87°$

$I_c = I_a\angle 120° =$ $\mathbf{8.66\angle 156.87°\ A}$

$I_{Lp} = n\sqrt{3}\,I_{Ls}$

$I_1 = (1/3)\sqrt{3}\,(8.66\angle 36.87°) = 5\angle 36.87°$

$I_2 = I_1\angle -120° =$ $\mathbf{5\angle -83.13°\ A}$

(c) $p = 3|I_a|^2(8) = 3(8.66)^2(8) =$ **1.8 kw**.

Chapter 13, Solution 74.

(a) This is a **Δ-Δ connection**.

(b) The easy way is to consider just one phase.

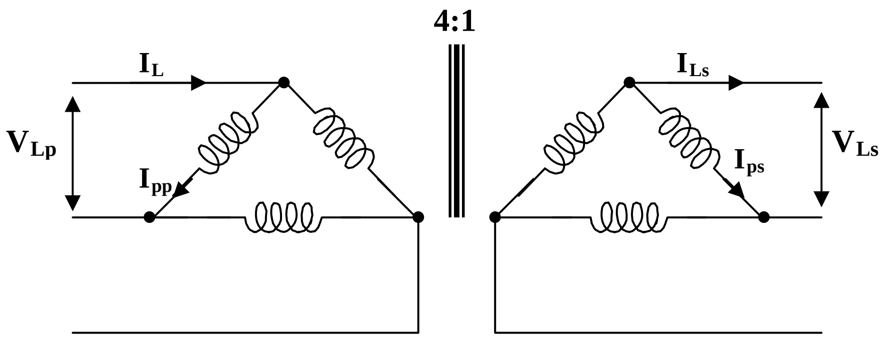
$$1{:}n = 4{:}1 \text{ or } n = 1/4$$

$n = V_2/V_1$ which leads to $V_2 = nV_1 = 0.25(2400) = 600$

$$\text{i.e. } V_{Lp} = 2400 \text{ V and } V_{Ls} = 600 \text{ V}$$

$S = p/\cos\theta = 120/0.8 \text{ kVA} = 150 \text{ kVA}$

$p_L = p/3 = 120/3 = 40 \text{ kw}$



But $p_{Ls} = V_{ps}I_{ps}$

For the Δ-load, $I_L = \sqrt{3}\, I_p$ and $V_L = V_p$

Hence, $I_{ps} = 40{,}000/600 = 66.67 \text{ A}$

$$I_{Ls} = \sqrt{3}\, I_{ps} = \sqrt{3}\text{x}66.67 = \mathbf{115.48 \text{ A}}$$

(c) Similarly, for the primary side

$$p_{pp} = V_{pp}I_{pp} = p_{ps} \text{ or } I_{pp} = 40{,}000/2400 = \mathbf{16.667 \text{ A}}$$

$$\text{and} \quad I_{Lp} = \sqrt{3}\, I_p = \mathbf{28.87 \text{ A}}$$

(d) Since $S = 150$ kVA therefore $S_p = S/3 = $ **50 kVA**

Chapter 13, Solution 75.

(a) $n = V_{Ls}/(\sqrt{3}\,V_{Lp}) = 900/(4500\sqrt{3}) = \mathbf{0.11547}$

(b) $S = \sqrt{3}\,V_{Ls}I_{Ls}$ or $I_{Ls} = 120{,}000/(900\sqrt{3}) = \mathbf{76.98\ A}$

$I_{Ls} = I_{Lp}/(n\sqrt{3}) = 76.98/(2.887\sqrt{3}) = \mathbf{15.395\ A}$

Chapter 13, Solution 76.

Using Fig. 13.138, design a problem to help other students to better understand a wye-delta, three-phase transformer and how they work.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A Y-Δ three-phase transformer is connected to a 60-kVA load with 0.85 power factor (leading) through a feeder whose impedance is 0.05 + j0.1Ω per phase, as shown in Fig. 13.137 below. Find the magnitude of:

(a) the line current at the load,

(b) the line voltage at the secondary side of the transformer,

(c) the line current at the primary side of the transformer.

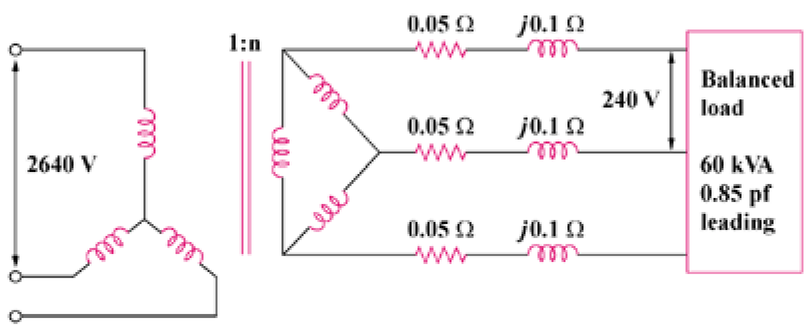


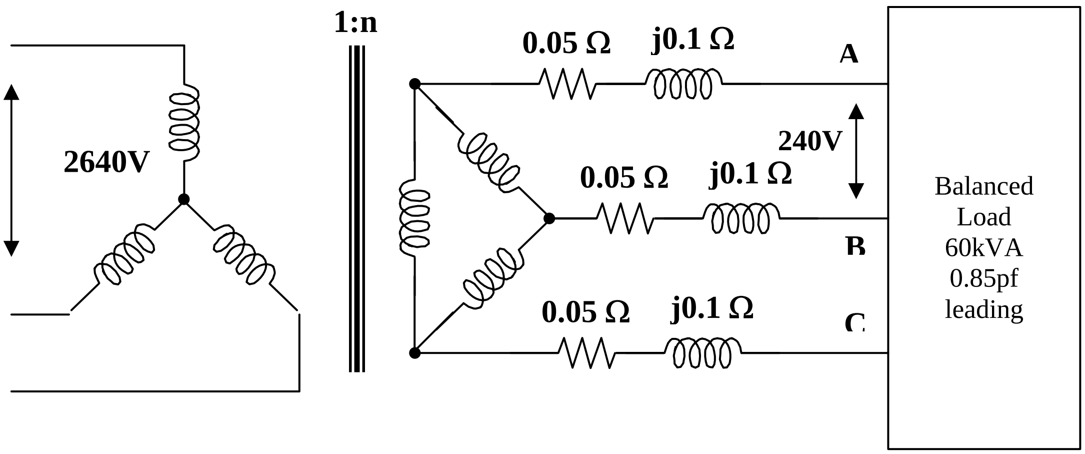
Figure 13.137

Solution

(a) At the load, $V_L = 240 \text{ V} = V_{AB}$

$$V_{AN} = V_L/\sqrt{3} = 138.56 \text{ V}$$

Since $S = \sqrt{3}V_L I_L$ then $I_L = 60{,}000/(240\sqrt{3}) =$ **144.34 A**



(b) Let $V_{AN} = |V_{AN}|\angle 0° = 138.56\angle 0°$

$\cos\theta = pf = 0.85$ or $\theta = 31.79°$

$I_{AA'} = I_L\angle\theta = 144.34\angle 31.79°$

$V_{A'N'} = ZI_{AA'} + V_{AN}$

$= 138.56\angle 0° + (0.05 + j0.1)(144.34\angle 31.79°)$

$= 138.03\angle 6.69°$

$V_{Ls} = V_{A'N'}\sqrt{3} = 138.03\sqrt{3} = \mathbf{239.1\ V}$

(c) For Y-Δ connections,

$n = \sqrt{3}\,V_{Ls}/V_{ps} = \sqrt{3}\,x238.7/2640 = 0.1569$

$f_{Lp} = nI_{Ls}/\sqrt{3} = 0.1569x144.34/\sqrt{3} = \mathbf{13.05\ A}$

Chapter 13, Solution 77.

(a) This is a single phase transformer. $V_1 = 13.2$ kV, $V_2 = 120$ V

$$n = V_2/V_1 = 120/13{,}200 = 1/110, \text{ therefore } n = \mathbf{1/110}$$

or 110 turns on the primary to every turn on the secondary.

(b) $P = VI$ or $I = P/V = 100/120 = 0.8333$ A

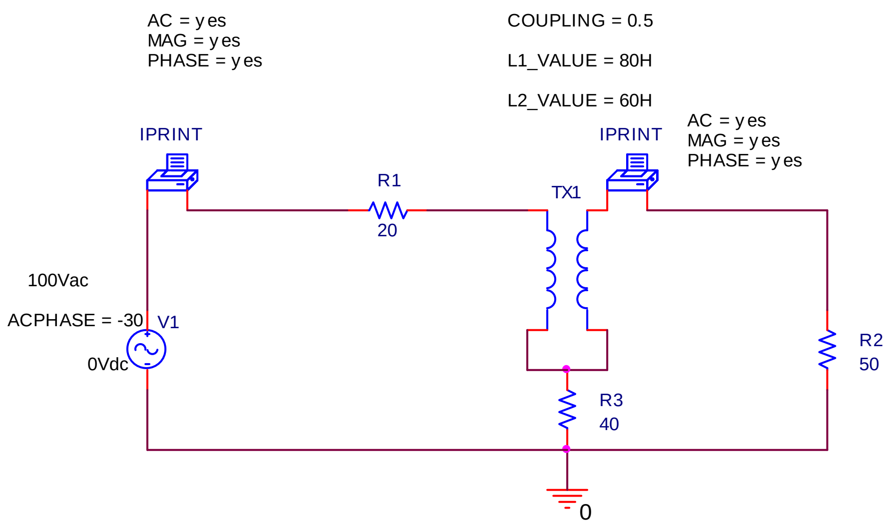
$$I_1 = nI_2 = 0.8333/110 = \mathbf{7.576\ mA}$$

Chapter 13, Solution 78.

We convert the reactances to their inductive values.

$$X = \omega L \quad \longrightarrow \quad L = \frac{X}{\omega}$$

The schematic is as shown below.



FREQ IM(V_PRINT1)IP(V_PRINT1)

1.592E-01 1.347E+00 -8.489E+01

FREQ IM(V_PRINT2)IP(V_PRINT2)

1.592E-01 6.588E-01 -7.769E+01

Thus,

$$\mathbf{I_1 = 1.347\angle{-84.89^\circ}\ amps} \text{ and } \mathbf{I_2 = 658.8\angle{-77.69^\circ}\ mA}$$

Chapter 13, Solution 79.

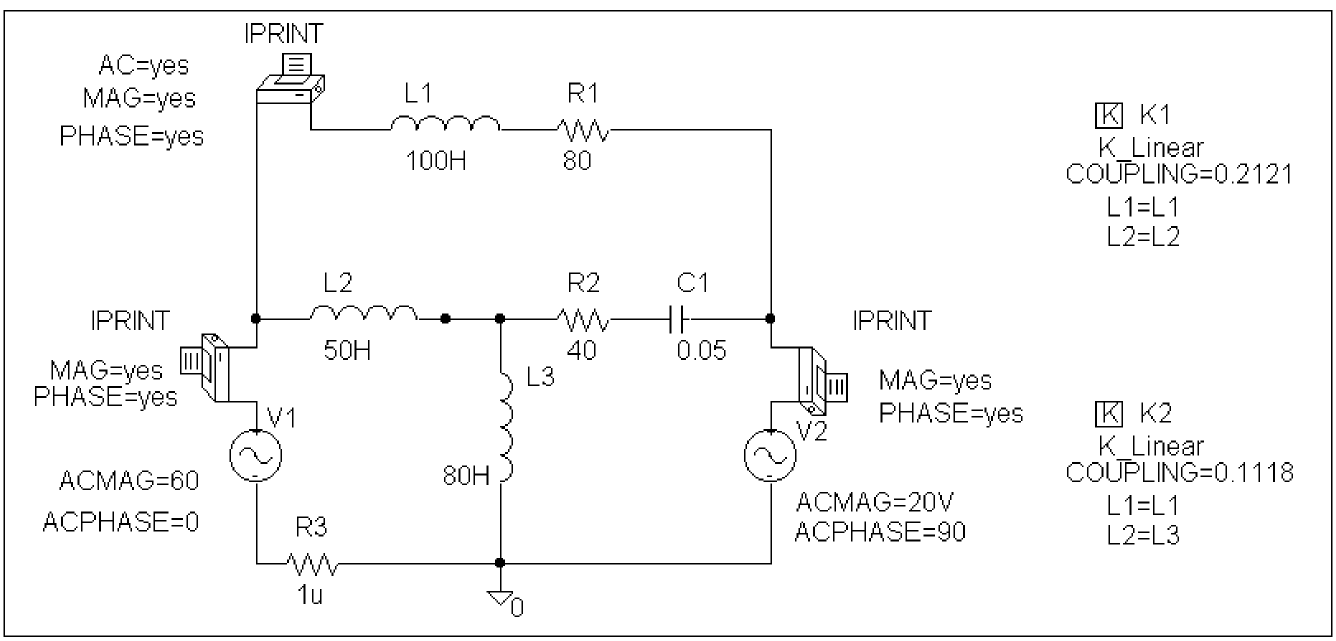
The schematic is shown below.

$$k_1 = 15/\sqrt{5000} = 0.2121, \quad k_2 = 10/\sqrt{8000} = 0.1118$$

In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the circuit is saved and simulated, the output includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.068 E–01	–7.786 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.306 E+00	–6.801 E+01
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.336 E+00	–5.492 E+01

Thus, $\mathbf{I_1 = 1.306\angle{-68.01}° \text{ A}}$, $\mathbf{I_2 = 406.8\angle{-77.86}° \text{ mA}}$, $\mathbf{I_3 = 1.336\angle{-54.92}° \text{ A}}$



Chapter 13, Solution 80.

The schematic is shown below.

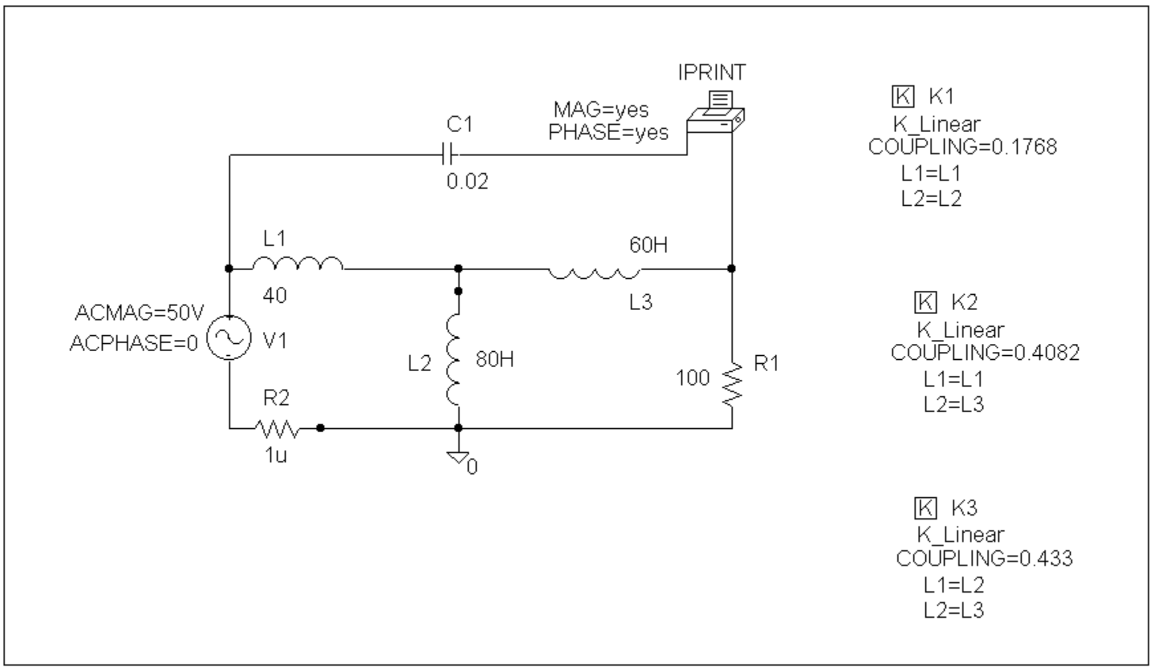
$$k_1 = 10/\sqrt{40x80} = 0.1768,\ \ k_2 = 20/\sqrt{40x60} = 0.4082$$

$$k_3 = 30/\sqrt{80x60} = 0.433$$

In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.304 E+00	6.292 E+01

i.e. $I_o = \mathbf{1.304\angle 62.92°\ A}$



Chapter 13, Solution 81.

The schematic is shown below.

$$k_1 = 2/\sqrt{4x8} = 0.3535, \quad k_2 = 1/\sqrt{2x8} = 0.25$$

In the AC Sweep box, we let Total Pts = 1, Start Freq = 100, and End Freq = 100. After simulation, the output file includes

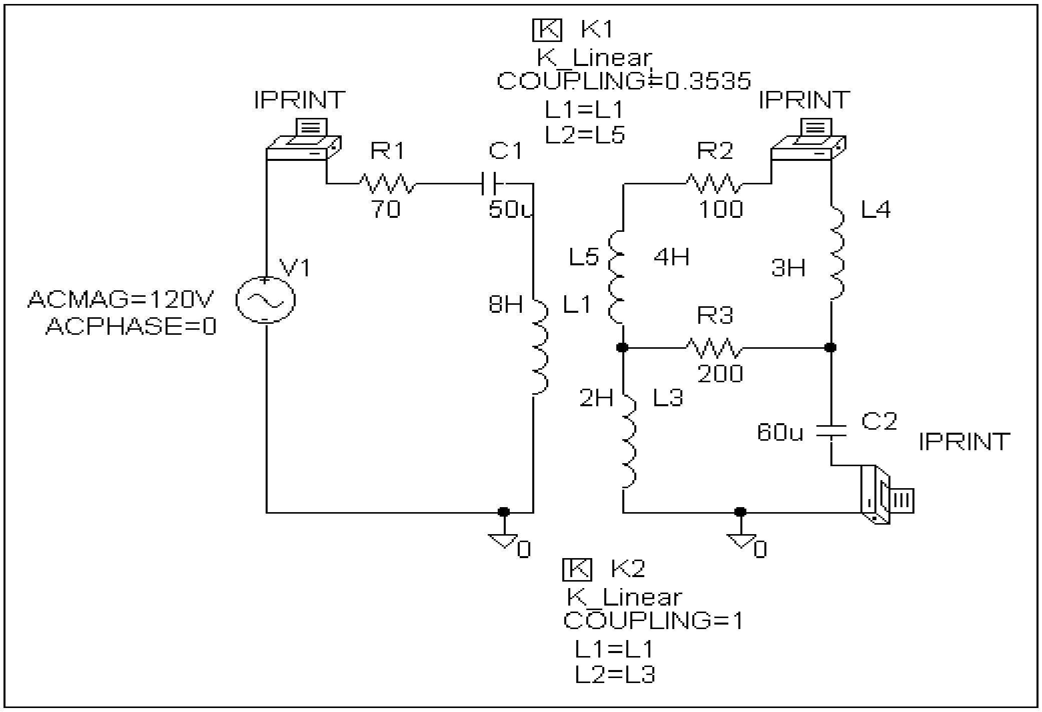
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.000 E+02	1.0448 E–01	1.396 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.000 E+02	2.954 E–02	–1.438 E+02

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.000 E+02	2.088 E–01	2.440 E+01

i.e. I_1 = **104.5∠13.96° mA**, I_2 = **29.54∠–143.8° mA**,

I_3 = **208.8∠24.4° mA**.



Chapter 13, Solution 82.

The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.955 E+01	8.332 E+01

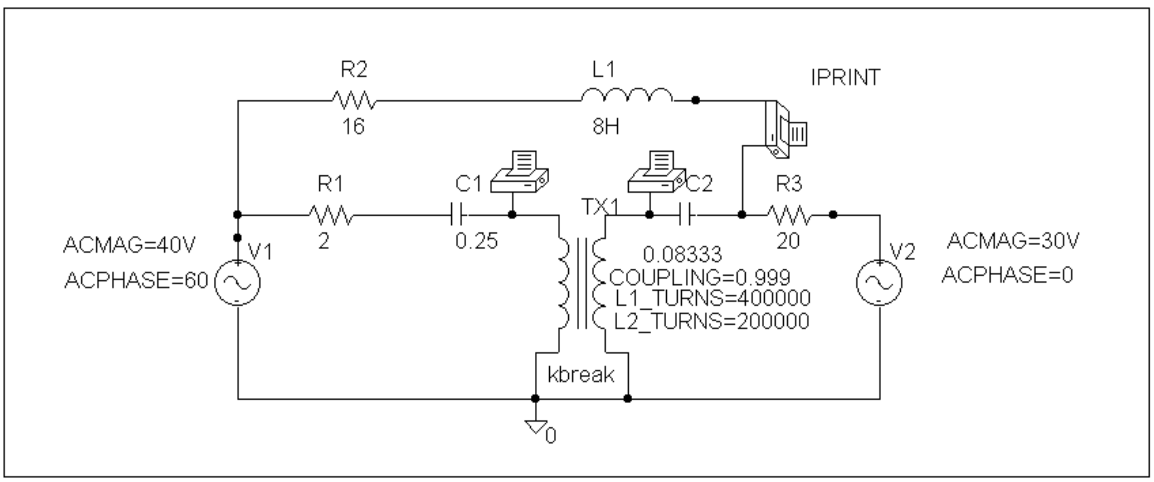
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.847 E+01	4.640 E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	4.434 E–01	–9.260 E+01

i.e. $V_1 = \mathbf{19.55\angle 83.32^\circ\ V}$, $V_2 = \mathbf{68.47\angle 46.4^\circ\ V}$,

$I_o = \mathbf{443.4\angle{-92.6^\circ}\ mA}$.

These answers are incorrect, we need to adjust the magnitude of the inductances.



Chapter 13, Solution 83.

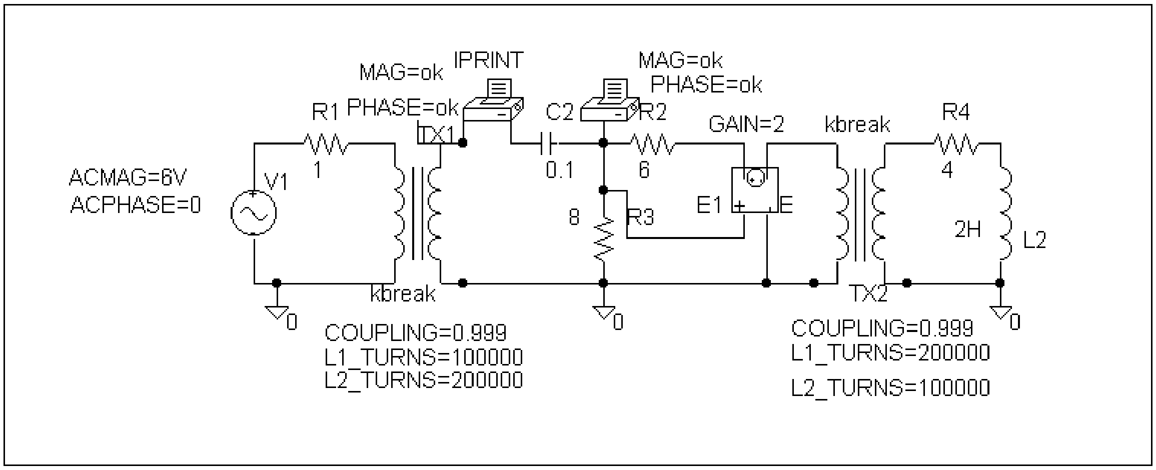
The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.080 E+00	3.391 E+01

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	1.514 E+01	–3.421 E+01

i.e. $\mathbf{i_X = 1.08\angle 33.91° \text{ A}, V_x = 15.14\angle{-34.21°} \text{ V}}$.

This is most likely incorrect and needs to have the values of turns changed.



Checking with hand calculations.

Loop 1. $-6 + 1I_1 + V_1 = 0$ or $I_1 + V_1 = 6$ (1)

Loop 2. $-V_2 - j10I_2 + 8(I_2 - I_3) = 0$ or $(8 - j10)I_2 - 8I_3 - V_2 = 0$ (2)

Loop 3. $8(I_3 - I_2) + 6I_3 + 2V_x + V_3 = 0$ or $-8I_2 + 14I_3 + V_3 + 2V_x = 0$ but
$V_x = 8(I_2 - I_3)$, therefore we get $8I_2 - 2I_3 + V_3 = 0$ (3)

Loop 4. $-V_4 + (4+j2)I_4 = 0$ or $(4+j2)I_4 - V_4 = 0$ (4)

We also need the constraint equations, $V_2 = 2V_1$, $I_1 = 2I_2$, $V_3 = 2V_4$, and $I_4 = 2I_3$. Finally,
$I_x = I_2$ and $V_x = 8(I_2 - I_3)$.

We can eliminate the voltages from the equations (we only need I_2 and I_3 to obtain the required answers) by,

$(1)+0.5(2) = I_1 + (4–j5)I_2 – 4I_3 = 6$ and

$0.5(3) + (4) = 4I_2 – I_3 + (4+j2)I_4 = 0$.

Next we use $I_1 = 2I_2$ and $I_4 = 2I_3$ to end up with the following equations,

$(6–j5)I_2 – 4I_3 = 6$ and $4I_2 + (7+j4)I_3 = 0$ or $I_2 = –[(7+j4)I_3]/4 = (–1.75–j)I_3$

$= (2.01556\angle–150.255°)I_3$

This leads to $(6–j5)(–1.75–j)I_3 – 4I_3 = (–10.5–5–4+j(8.75–6))I_3 = (–19.5+j2.75)I_3 = 6$ or

$I_3 = 6/(19.69296\angle171.973°) = 0.304677\angle–171.973°$ amps

$= –0.301692–j0.042545$.

$I_2 = (–1.75–j)(0.304677\angle–171.973°)$

$= (2.01556\angle–150.255°)(0.304677\angle–171.973°)$

$= 614.096\angle37.772°$ mA $= 0.48541+j0.37615$

and $I_2 – I_3 = 0.7871+j0.4187 = 0.89154\angle28.01°$.

Therefore,

$$V_x = 8(0.854876\angle 22.97°) = \mathbf{7.132\angle 28.01°\ V}$$

$$I_x = I_2 = \mathbf{614.1\angle 37.77°\ mA}.$$

Checking with MATLAB we get A and X from equations (1) – (4) and the four constraint equations.

```
>> A = [1 0 0 0 1 0 0 0;0 (8-10j) -8 0 0 -1 0 0;0 8 -2 0 0 0 1 0;0 0 0 (4+2j) 0 0 0 -1;0 0 0 0 -2 1 0 0;1 -2 0 0 0 0 0 0;0 0 0 0 0 0 1 -2;0 0 -2 1 0 0 0 0]

A =

   1.0000              0              0              0         1.0000         0         0         0
        0        8.0000 -10.0000i  -8.0000              0              0   -1.0000         0         0
        0        8.0000        -2.0000              0              0         0    1.0000         0
        0              0              0    4.0000 + 2.0000i        0         0         0   -1.0000
        0              0              0              0        -2.0000    1.0000         0         0
   1.0000        -2.0000              0              0              0         0         0         0
        0              0              0              0              0         0    1.0000   -2.0000
        0              0        -2.0000         1.0000              0         0         0         0

>> X = [6;0;0;0;0;0;0;0]

X =

     6
     0
     0
     0
```

0
0
0
0

```
>> Y = inv(A)*X

Y =
```

0.9708 + 0.7523i = I_1 = 1.2817∠37.773° amps

0.4854 + 0.3761i = I_2 = 614.056∠37.769° mA = I_x

-0.3017 - 0.0425i = I_3 = 0.30468∠–171.982° amps

-0.6034 - 0.0851i = I_4
5.0292 - 0.7523i = V_1
10.0583 - 1.5046i = V_2
-4.4867 - 3.0943i = V_3
-2.2434 - 1.5471i = V_4

I_x = 614.1∠37.77° mA

Finally, $V_x = 8(I_2 - I_3)$ = 8(0.7871+j0.4186) = 8(0.891489∠28.01°)

= **7.132∠28.01° volts**

Chapter 13, Solution 84.

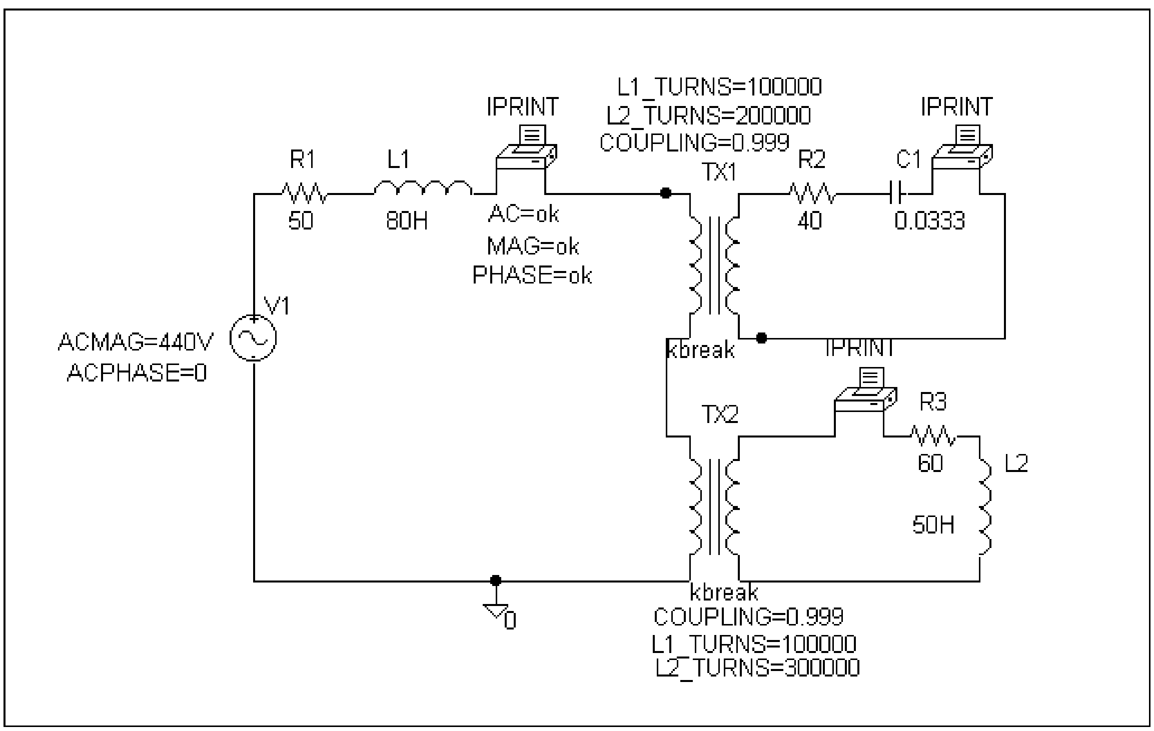
The schematic is shown below. we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.028 E+00	–5.238 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	2.019 E+00	–5.211 E+01
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.338 E+00	–5.220 E+01

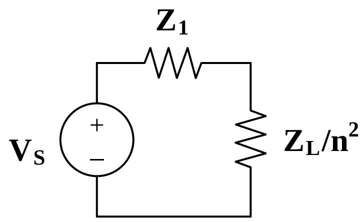
i.e. I_1 = $\mathbf{4.028\angle{-52.38^\circ}}$ **A**, I_2 = $\mathbf{2.019\angle{-52.11^\circ}}$ **A**,

I_3 = $\mathbf{1.338\angle{-52.2^\circ}}$ **A**.

Dot convention is wrong.



Chapter 13, Solution 85.



For maximum power transfer,

$$Z_1 = Z_L/n^2 \text{ or } n^2 = Z_L/Z_1 = 8/7200 = 1/900$$

$n = 1/30 = N_2/N_1$. Thus $N_2 = N_1/30 = 3000/30 =$ **100 turns**.

Chapter 13, Solution 86.

$$n = N_2/N_1 = 48/2400 = 1/50$$

$$Z_{Th} = Z_L/n^2 = 3/(1/50)^2 = \mathbf{7.5\ k\Omega}$$

Chapter 13, Solution 87.

$$Z_{Th} = Z_L/n^2 \text{ or } n = \sqrt{Z_L / Z_{Th}} = \sqrt{75/300} = \mathbf{0.5}$$

Chapter 13, Solution 88.

$n = V_2/V_1 = I_1/I_2$ or $I_2 = I_1/n = 2.5/0.1 = 25$ A

$p = IV = 25x12.6 =$ **315 watts**

Chapter 13, Solution 89.

$n = V_2/V_1 = 120/240 =$ **0.5**

$S = I_1V_1$ or $I_1 = S/V_1 = 10x10^3/240 =$ **41.67 A**

$S = I_2V_2$ or $I_2 = S/V_2 = 10^4/120 =$ **83.33 A**

Chapter 13, Solution 90.

(a) $n = V_2/V_1 = 240/2400 =$ **0.1**

(b) $n = N_2/N_1$ or $N_2 = nN_1 = 0.1(250) =$ **25 turns**

(c) $S = I_1V_1$ or $I_1 = S/V_1 = 4x10^3/2400 =$ **1.6667 A**

$S = I_2V_2$ or $I_2 = S/V_2 = 4x10^4/240 =$ **16.667 A**

Chapter 13, Solution 91.

(a) The kVA rating is $S = VI = 25{,}000 \text{x} 75 =$ **1.875 MVA**

(b) Since $S_1 = S_2 = V_2 I_2$ and $I_2 = 1875 \text{x} 10^3/240 =$ **7.812 kA**

Chapter 13, Solution 92.

(a) $V_2/V_1 = N_2/N_1 = n$, $V_2 = (N_2/N_1)V_1 = (28/1200)4800 =$ **112 V**

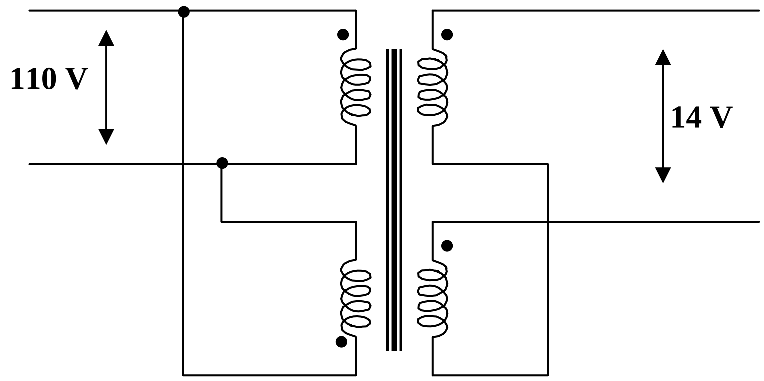
(b) $I_2 = V_2/R = 112/10 =$ **11.2 A** and $I_1 = nI_2$, $n = 28/1200$

$I_1 = (28/1200)11.2 =$ **261.3 mA**

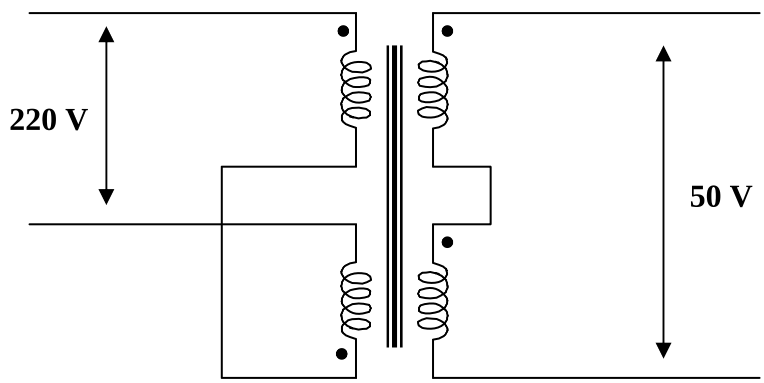
(c) $p = |I_2|^2R = (11.2)^2(10) =$ **1254 watts**.

Chapter 13, Solution 93.

(a) For an input of 110 V, the primary winding must be connected in parallel, with series aiding on the secondary. The coils must be series opposing to give 14 V. Thus, the connections are shown below.



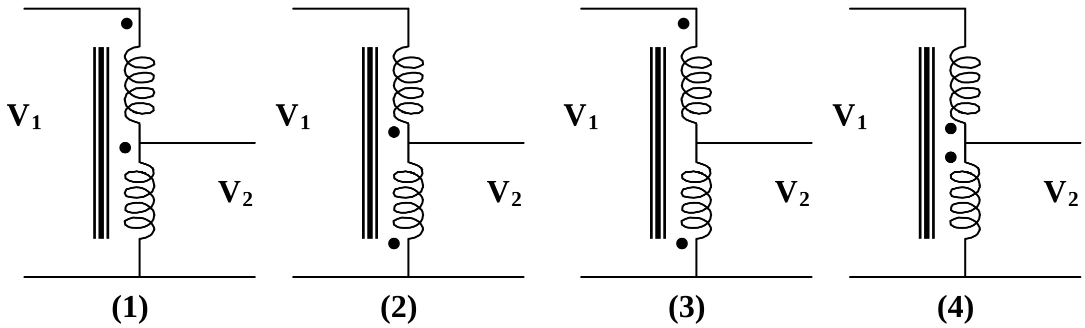
(b) To get 220 V on the primary side, the coils are connected in series, with series aiding on the secondary side. The coils must be connected series aiding to give 50 V. Thus, the connections are shown below.



Chapter 13, Solution 94.

$$V_2/V_1 = 110/440 = 1/4 = I_1/I_2$$

There are four ways of hooking up the transformer as an auto-transformer. However it is clear that there are only two outcomes.



(1) and (2) produce the same results and (3) and (4) also produce the same results. Therefore, we will only consider Figure (1) and (3).

(a) For Figure (3), $V_1/V_2 = 550/V_2 = (440 - 110)/440 = 330/440$

Thus, $V_2 = 550x440/330 =$ **733.4 V (not the desired result)**

(b) For Figure (1), $V_1/V_2 = 550/V_2 = (440 + 110)/440 = 550/440$

Thus, $V_2 = 550x440/550 =$ **440 V (the desired result)**

Chapter 13, Solution 95.

(a) $n = V_s/V_p = 120/7200 = \mathbf{1/60}$

(b) $I_s = 10x120/144 = 1200/144$

$$S = V_p I_p = V_s I_s$$

$$I_p = V_s I_s/V_p = (1/60)x1200/144 = \mathbf{139\ mA}$$

***Chapter 13, Solution 96.**

Problem,

Some modern power transmission systems now have major, high voltage DC transmission segments. There are a lot of good reasons for doing this but we will not go into them here. To go from the AC to DC, power electronics are used. We start with three-phase AC and then rectify it (using a full-wave rectifier). It was found that using a delta to wye and delta combination connected secondary would give us a much smaller ripple after the full-wave rectifier. How is this accomplished? Remember that these are real devices and are wound on common cores. Hint, using Figures 13.47 and 13.49, and the fact that each coil of the wye connected secondary and each coil of the delta connected secondary are wound around the same core of each coil of the delta connected primary so the voltage of each of the corresponding coils are in phase. When the output leads of both secondaries are connected through full-wave rectifiers with the same load, you will see that the ripple is now greatly reduced. Please consult the instructor for more help if necessary.

Solution,

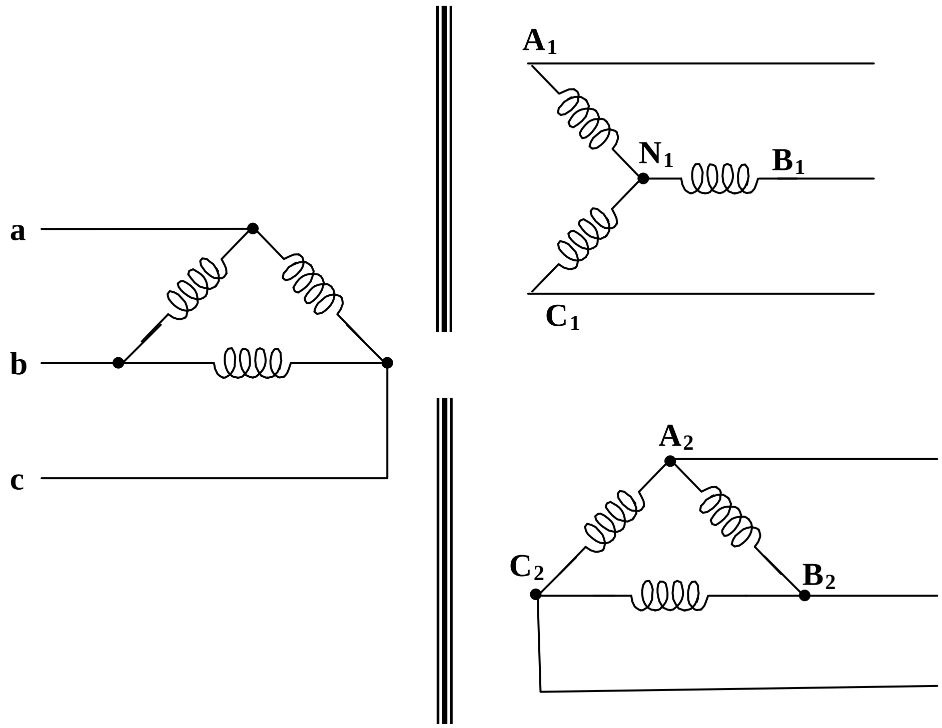
This is a most interesting and very practical problem. The solution is actually quite easy, you are creating a second set of sine waves to send through the full-wave rectifier, 30˚ out of phase with the first set. We will look at this graphically in a minute. We begin by showing the transformer components.

The key to making this work is to wind the secondary coils with each phase of the primary. Thus, a-b is wound around the same core as A_1-N_1 and A_2-B_2. The next thing we need to do is to make sure the voltages come out equal. We need to work the number of turns of each secondary so that the peak of $V_{A1} – V_{B1}$ is equal to $V_{A2}–V_{B2}$. Now, let us look at some of the equations involved.

If we let $v_{ab}(t) = 100\sin(t)$ V, assume that we have an ideal transformer, and the turns ratios are such that we get $v_{A1\text{-}N1}(t) = 57.74\sin(t)$ V and $V_{A2\text{-}B2}(t) = 100\sin(t)$ V. Next, let us look at $V_{bc}(t) = 100\sin(t+120˚)$ V. This leads to $V_{B1\text{-}N1}(t) = 57.74\sin(t+120˚)$ V. We now need to determine $V_{A1\text{-}B1}(t)$.

$$V_{A1\text{-}B1}(t) = 57.74\sin(t) – 57.74\sin(t+120˚) = 100\sin(t–30˚)\text{ V.}$$

This then leads to the output per phase voltage being equal to $v_{out}(t) = [100\sin(t) + 100\sin(t–30˚)]$ V. We can do this for each phase and end up with the output being sent to the full-wave rectifier. This looks like $v_{out}(t) = [|100\sin(t)| + |100\sin(t–30˚)| + |\sin(t+120˚)|\ |100\sin(t+90˚)| + |100\sin(t–120˚)| + |100\sin(t–150˚)|]$ V. The end result will be more obvious if we look at plots of the rectified output.



In the plot below we see the normalized (1 corresponds to 100 volts) ripple with only one of the secondary sets of windings and then the plot with both. Clearly the ripple is greatly reduced!