Chapter 8, Solution 1.
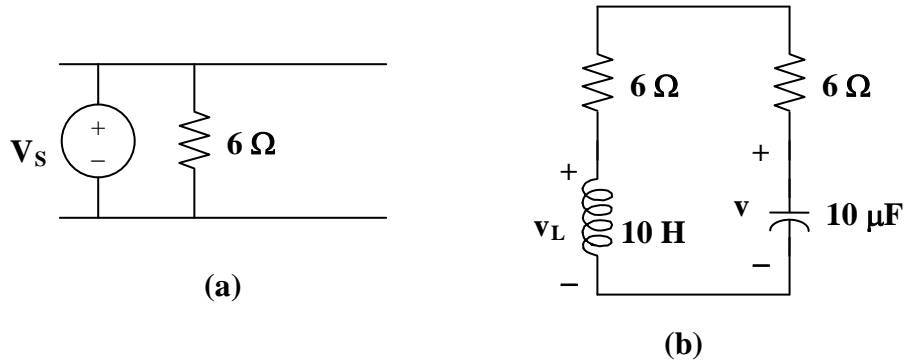
(a)   At t $=$ 0-, the circuit has reached steady state so that the equivalent circuit is shown in Figure (a).

(a)

(b)

$$i(0\text{-}) \ = \ 12/6 \ = \ 2A, \ v(0\text{-}) \ = \ 12V$$

$$\text{At } t \ = \ 0\text{+}, \ i(0\text{+}) \ = \ i(0\text{-}) \ = \ \textbf{2A}, \ v(0\text{+}) \ = \ v(0\text{-}) \ = \ \textbf{12V}$$

(b)   For t $>$ 0, we have the equivalent circuit shown in Figure (b).

$$v_L \ = \ Ldi/dt \ \text{ or } \ di/dt \ = \ v_L/L$$

Applying KVL at t $=$ 0+, we obtain,

$$v_L(0\text{+}) - v(0\text{+}) + 10i(0\text{+}) \ = \ 0$$

$$v_L(0\text{+}) - 12 + 20 \ = \ 0, \ \text{ or } \ v_L(0\text{+}) \ = \ \text{-8}$$

Hence, $di(0\text{+})/dt \ = \ \text{-8/2} \ = \ \textbf{-4 A/s}$

Similarly, $i_C \ = \ Cdv/dt, \ \text{ or } \ dv/dt \ = \ i_C/C$

$$i_C(0\text{+}) \ = \ \text{-}i(0\text{+}) \ = \ \text{-2}$$

$$dv(0\text{+})/dt \ = \ \text{-2/0.4} \ = \ \textbf{-5 V/s}$$

(c)   As t approaches infinity, the circuit reaches steady state.

$$i(\infty) \ = \ \textbf{0 A}, \ v(\infty) \ = \ \textbf{0 V}$$

Chapter 8, Solution 2.

Using Fig. 8.63, design a problem to help other students better understand finding initial and final values.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit of Fig. 8.63, determine:

(a) $i_R(0^+)$, $i_L(0^+)$, and $i_C(0^+)$,

(b) $di_R(0^+)/dt$, $di_L(0^+)/dt$, and $di_C(0^+)/dt$,
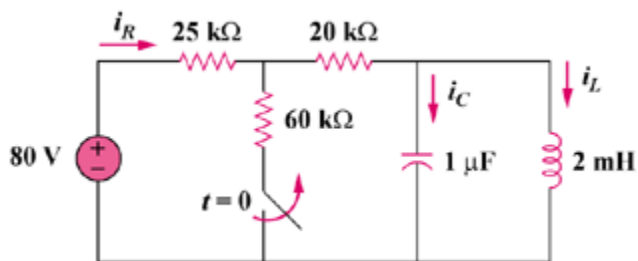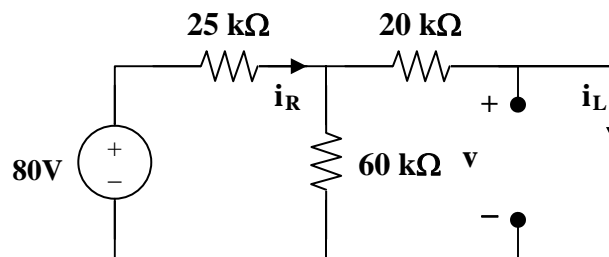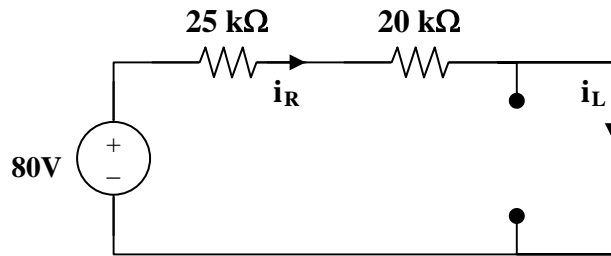
(c) $i_R(\infty)$, $i_L(\infty)$, and $i_C(\infty)$.

Figure 8.63

Solution

(a)   At t = 0-, the equivalent circuit is shown in Figure (a).

(a)

(b)

$$60\|20 = 15 \text{ kohms}, \ i_R(0\text{-}) = 80/(25 + 15) = 2\text{mA}.$$

By the current division principle,

$$i_L(0\text{-}) = 60(2\text{mA})/(60 + 20) = 1.5 \text{ mA}$$

$$v_C(0\text{-}) = 0$$

At $t = 0+$,

$$v_C(0+) = v_C(0\text{-}) = 0$$

$$i_L(0+) = i_L(0\text{-}) = \mathbf{1.5 \ mA}$$

$$80 = i_R(0+)(25 + 20) + v_C(0\text{-})$$

$$i_R(0+) = 80/45k = \mathbf{1.778 \ mA}$$

But, $\qquad\qquad\qquad i_R = i_C + i_L$

$$1.778 = i_C(0+) + 1.5 \ \text{ or } \ i_C(0+) = \mathbf{0.278 \ mA}$$

(b) $\qquad\qquad v_L(0+) = v_C(0+) = 0$

But, $\quad v_L = Ldi_L/dt \ \text{ and } \ di_L(0+)/dt = v_L(0+)/L = 0$

$$di_L(0+)/dt = \mathbf{0}$$

Again, $80 = 45i_R + v_C$

$$0 = 45di_R/dt + dv_C/dt$$

But, $\quad dv_C(0+)/dt = i_C(0+)/C = 0.278 \ \text{mamps}/1 \ \mu F = 278 \ \text{V/s}$

Hence, $\quad di_R(0+)/dt = (\text{-}1/45)dv_C(0+)/dt = \text{-}278/45$

$$\mathrm{di_R(0+)/dt} = \textbf{-6.1778 A/s}$$

$$\text{Also,} \quad \mathrm{i_R} = \mathrm{i_C} + \mathrm{i_L}$$

$$\mathrm{di_R(0+)/dt} = \mathrm{di_C(0+)/dt} + \mathrm{di_L(0+)/dt}$$

$$-6.1788 = \mathrm{di_C(0+)/dt} + 0, \quad \text{or} \quad \mathrm{di_C(0+)/dt} = \textbf{-6.1788 A/s}$$

(c)   As t approaches infinity, we have the equivalent circuit in Figure (b).

$$\mathrm{i_R(\infty)} = \mathrm{i_L(\infty)} = 80/45\mathrm{k} = \textbf{1.778 mA}$$

$$\mathrm{i_C(\infty)} = \mathrm{Cdv(\infty)/dt} = \textbf{0}.$$
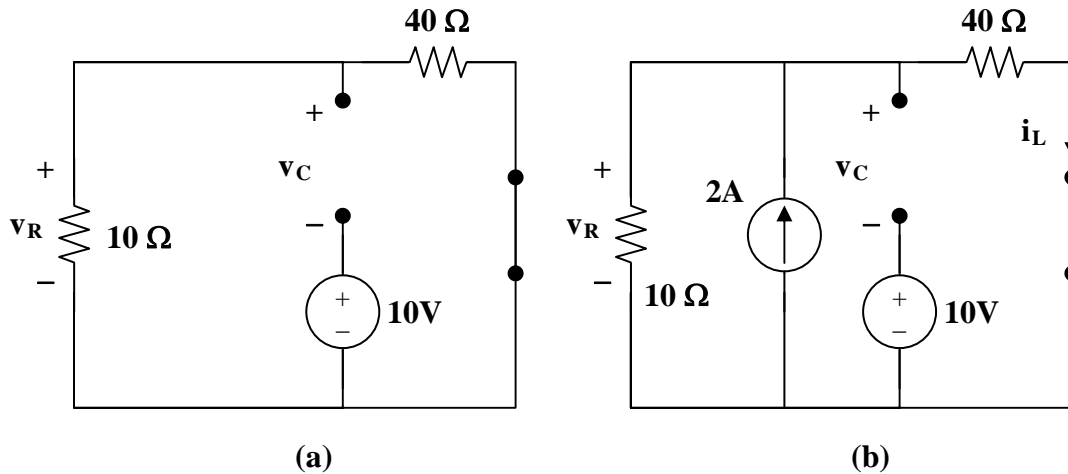
Chapter 8, Solution 3.

At $t = 0^-$, $u(t) = 0$. Consider the circuit shown in Figure (a). $i_L(0^-) = 0$, and $v_R(0^-) = 0$. But, $-v_R(0^-) + v_C(0^-) + 10 = 0$, or $v_C(0^-) = -10V$.

(a) At $t = 0^+$, since the inductor current and capacitor voltage cannot change abruptly, the inductor current must still be equal to **0A**, the capacitor has a voltage equal to **–10V**. Since it is in series with the +10V source, together they represent a direct short at $t = 0^+$. This means that the entire 2A from the current source flows through the capacitor and not the resistor. Therefore, $v_R(0^+) = \textbf{0 V}$.

(b) At $t = 0^+$, $v_L(0+) = 0$, therefore $Ldi_L(0+)/dt = v_L(0^+) = 0$, thus, $di_L/dt = \textbf{0A/s}$, $i_C(0^+) = 2$ A, this means that $dv_C(0^+)/dt = 2/C = \textbf{8 V/s}$. Now for the value of $dv_R(0^+)/dt$. Since $v_R = v_C + 10$, then $dv_R(0^+)/dt = dv_C(0^+)/dt + 0 = \textbf{8 V/s}$.

(a)   **(b)**

(c) As t approaches infinity, we end up with the equivalent circuit shown in Figure (b).

$$i_L(\infty) = 10(2)/(40 + 10) = \textbf{400 mA}$$

$$v_C(\infty) = 2[10\|40] - 10 = 16 - 10 = \textbf{6V}$$

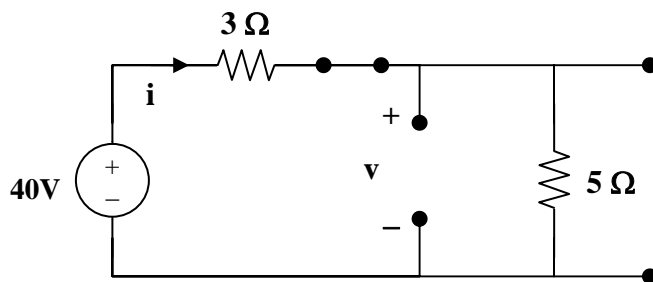$$v_R(\infty) = 2[10\|40] = \textbf{16 V}$$

Chapter 8, Solution 4.

(a)   At $t = 0^-$, $u(-t) = 1$ and $u(t) = 0$ so that the equivalent circuit is shown in Figure (a).

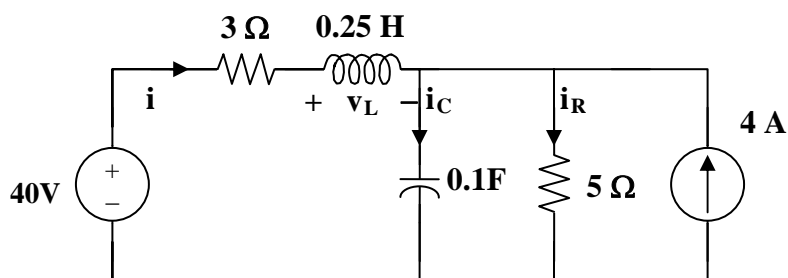$$i(0^-) = 40/(3 + 5) = 5A, \text{ and } v(0^-) = 5i(0^-) = 25V.$$

Hence, $i(0^+) = i(0^-) = \mathbf{5A}$

$$v(0^+) = v(0^-) = \mathbf{25V}$$

(a)

(b)

(b) $i_C = Cdv/dt$ or $dv(0^+)/dt = i_C(0^+)/C$

For $t = 0^+$, $4u(t) = 4$ and $4u(-t) = 0$. The equivalent circuit is shown in Figure (b). Since i and v cannot change abruptly,

$$i_R = v/5 = 25/5 = 5A, \ i(0^+) + 4 = i_C(0^+) + i_R(0^+)$$

$$5 + 4 = i_C(0^+) + 5 \text{ which leads to } i_C(0^+) = 4$$

$$dv(0^+)/dt = 4/0.1 = \mathbf{40 \ V/s}$$

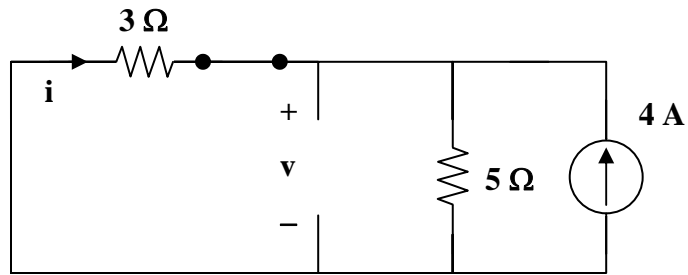Similarly, $v_L = Ldi/dt$ which leads to $di(0^+)/dt = v_L(0^+)/L$

$$3i(0^+) + v_L(0^+) + v(0^+) = 0$$

$$15 + v_L(0^+) + 25 = 0 \text{ or } v_L(0^+) = -40$$

$$di(0^+)/dt = -40/0.25 = \mathbf{-160 \text{ A/s}}$$

(c)   As t approaches infinity, we have the equivalent circuit in Figure (c).

(c)

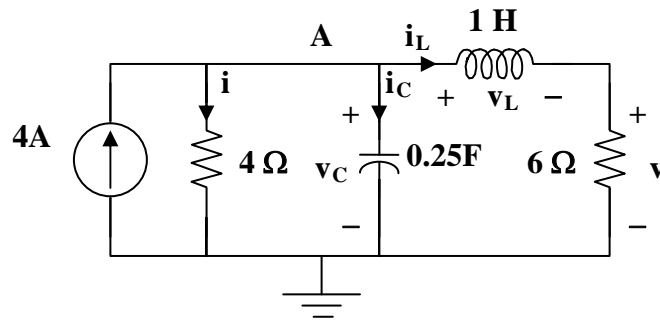$$i(\infty) = -5(4)/(3 + 5) = \mathbf{-2.5 \text{ A}}$$

$$v(\infty) = 5(4 - 2.5) = \mathbf{7.5 \text{ V}}$$

Chapter 8, Solution 5.

(a)   For $t < 0$, $4u(t) = 0$ so that the circuit is not active (all initial conditions $= 0$).

$$i_L(0-) = 0 \text{ and } v_C(0-) = 0.$$

For $t = 0+$, $4u(t) = 4$. Consider the circuit below.

Since the 4-ohm resistor is in parallel with the capacitor,

$$i(0+) = v_C(0+)/4 = 0/4 = \mathbf{0\ A}$$

Also, since the 6-ohm resistor is in series with the inductor,
$v(0+) = 6i_L(0+) = \mathbf{0V}$.

(b) $di(0+)/dt = d(v_R(0+)/R)/dt = (1/R)dv_R(0+)/dt = (1/R)dv_C(0+)/dt$

$$= (1/4)4/0.25 \text{ A/s} = \mathbf{4\ A/s}$$

$v = 6i_L$ or $dv/dt = 6di_L/dt$ and $dv(0+)/dt = 6di_L(0+)/dt = 6v_L(0+)/L = 0$

Therefore $dv(0+)/dt = \mathbf{0\ V/s}$

(c)   As t approaches infinity, the circuit is in steady-state.

$$i(\infty) = 6(4)/10 = \mathbf{2.4\ A}$$

$$v(\infty) = 6(4 - 2.4) = \mathbf{9.6\ V}$$

Chapter 8, Solution 6.

(a) Let i = the inductor current. For $t < 0$, $u(t) = 0$ so that

$$i(0) = 0 \text{ and } v(0) = 0.$$

For $t > 0$, $u(t) = 1$. Since, $v(0+) = v(0-) = 0$, and $i(0+) = i(0-) = 0$.

$$v_R(0+) = Ri(0+) = \mathbf{0\ V}$$

Also, since $v(0+) = v_R(0+) + v_L(0+) = 0 = 0 + v_L(0+)$ or $v_L(0+) = \mathbf{0\ V}$.
    (1)

(b) Since $i(0+) = 0$, $i_C(0+) = V_S/R_S$

But, $i_C = Cdv/dt$ which leads to $dv(0+)/dt = V_S/(CR_S)$
      (2)

From (1), $dv(0+)/dt = dv_R(0+)/dt + dv_L(0+)/dt$
      (3)

        $v_R = iR$ or $dv_R/dt = Rdi/dt$
(4)

But, $v_L = Ldi/dt$, $v_L(0+) = 0 = Ldi(0+)/dt$ and $di(0+)/dt = 0$ (5)

From (4) and (5), $dv_R(0+)/dt = \mathbf{0\ V/s}$

From (2) and (3), $dv_L(0+)/dt = dv(0+)/dt = \mathbf{V_s/(CR_s)}$

(c)   As t approaches infinity, the capacitor acts like an open circuit, while the inductor acts like a short circuit.

$$v_R(\infty) = \mathbf{[R/(R + R_s)]V_s}$$

$$v_L(\infty) = \mathbf{0\ V}$$

Chapter 8, Solution 7.

$\alpha = [R/(2L)] = 20 \times 10^3/(2 \times 0.2 \times 10^{-3}) = 50 \times 10^6$

$\omega_o = [1/(LC)^{0.5}] = 1/(0.2 \times 10^{-3} \times 5 \times 10^{-6})^{0.5} = 3.162 \times 10^4$

$$\alpha > \omega_o \quad \longrightarrow \quad \underline{\text{overdamped}}$$

overdamped

Chapter 8, Solution 8.

Design a problem to help other students better understand source-free *RLC* circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The branch current in an *RLC* circuit is described by the differential equation

$$\frac{d^2i}{dt^2} + 6\frac{di}{dt} + 9i = 0$$

and the initial conditions are $i(0) = 0$, $di(0)/dt = 4$. Obtain the characteristic equation and determine $i(t)$ for $t > 0$.

Solution

$s^2 + 6s + 9 = 0$, thus $s_{1,2} = \dfrac{-6 \pm \sqrt{6^2 - 36}}{2} = -3$, repeated roots.

$$i(t) = [(A + Bt)e^{-3t}], \quad i(0) = 0 = A$$

$$di/dt = [Be^{-3t}] + [-3(Bt)e^{-3t}]$$

$$di(0)/dt = 4 = B.$$

Therefore, $i(t) = \mathbf{[4te^{-3t}]\ A}$

Chapter 8, Solution 9.

$s^2 + 10s + 25 = 0$, thus $s_{1,2} = \dfrac{-10 \pm \sqrt{10-10}}{2} = -5$, repeated roots.

$$i(t) = [(A + Bt)e^{-5t}], \quad i(0) = 10 = A$$

$$di/dt = [Be^{-5t}] + [-5(A + Bt)e^{-5t}]$$

$$di(0)/dt = 0 = B - 5A = B - 50 \text{ or } B = 50.$$

Therefore, $i(t) = \mathbf{[(10 + 50t)e^{-5t}]\ A}$

Chapter 8, Solution 10.

$$s^2 + 5s + 4 = 0, \text{ thus } s_{1,2} = \frac{-5 \pm \sqrt{25-16}}{2} = -4, \ -1.$$

$$v(t) = (Ae^{-4t} + Be^{-t}), \ v(0) = 0 = A + B, \text{ or } B = -A$$

$$dv/dt = (-4Ae^{-4t} - Be^{-t})$$

$$dv(0)/dt = 10 = -4A - B = -3A \text{ or } A = -10/3 \text{ and } B = 10/3.$$

$$\text{Therefore, } v(t) = (-(10/3)e^{-4t} + (10/3)e^{-t}) \text{ V}$$

Chapter 8, Solution 11.

$$s^2 + 2s + 1 = 0, \text{ thus } s_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2} = -1, \text{ repeated roots.}$$

$$v(t) = [(A + Bt)e^{-t}], \ v(0) = 10 = A$$

$$dv/dt = [Be^{-t}] + [-(A + Bt)e^{-t}]$$

$$dv(0)/dt = 0 = B - A = B - 10 \text{ or } B = 10.$$

$$\text{Therefore, } v(t) = \mathbf{[(10 + 10t)e^{-t}] \ V}$$

Chapter 8, Solution 12.

(a)   Overdamped when $C > 4L/(R^2) = 4 \times 1.5/2500 = 2.4 \times 10^{-3}$, or

$$C > \mathbf{2.4\ mF}$$

(b)   Critically damped when $C = \mathbf{2.4\ mF}$

(c)   Underdamped when $C < \mathbf{2.4\ mF}$

Chapter 8, Solution 13.

Let $R \| 60 = R_o$. For a series RLC circuit,

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.01 \times 4}} = 5$$

For critical damping, $\omega_o = \alpha = R_o/(2L) = 5$

or $R_o = 10L = 40 = 60R/(60 + R)$

which leads to $R = $ **120 ohms**

Chapter 8, Solution 14.

When the switch is in position A, $v(0^-) = 0$ and $i_L(0) = 80/40 = 2$ A. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2 \, x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4} \, x4}} = 1$$

When the switch is in position A, $v(0^-) = 0$. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2 \, x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4} \, x4}} = 1$$

Since $\alpha > \omega_o$, we have overdamped case.

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.25 \pm \sqrt{1.56} \quad -0.5 \text{ and } -2 \qquad 0.9336$$

$$v(t) = Ae^{-2t} + Be^{-0.5t} \qquad\qquad (1)$$

$$v(0) = 0 = A + B \qquad\qquad (2)$$

$$i_C(0) = C(dv(0)/dt) = -2 \text{ or } dv(0)/dt = -2/C = -8.$$

But $\quad \dfrac{dv(t)}{dt} = -2Ae^{-2t} - 0.5Be^{-0.5t}$

$$\frac{dv(0)}{dt} = -2A - 0.5B = -8 \qquad\qquad (3)$$

Solving (2) and (3) gives A= 1.3333 and B = –1.3333

$$v(t) = \mathbf{5.333e^{-2t} - 5.333e^{-0.5t} \ V}.$$

Chapter 8, Solution 15.

Given that s_1 = -10 and s_2 = -20, we recall that

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = \text{-10, -20}$$

Clearly, $\quad s_1 + s_2 = -2\alpha = \text{-30 or } \alpha = 15 = R/(2L) \text{ or } R = 60L \qquad$ (1)

$$s_1 = -15 + \sqrt{15^2 - \omega_o^2} = \text{-10 which leads to } 15^2 - \omega_o^2 = 25$$

or $\omega_o = \sqrt{225 - 25} = \sqrt{200} = 1/\sqrt{LC}$, thus $LC = 1/200 \qquad$ (2)

Since we have a series RLC circuit, $i_L = i_C = Cdv_C/dt$ which gives,

$$i_L/C = dv_C/dt = [200e^{-20t} - 300e^{-30t}] \text{ or } i_L = 100C[2e^{-20t} - 3e^{-30t}]$$

But, $\quad$ i is also $= 20\{[2e^{-20t} - 3e^{-30t}]x10^{-3}\} = 100C[2e^{-20t} - 3e^{-30t}]$

Therefore, $\quad C = (0.02/10^2) = \textbf{200 } \mu\textbf{F}$

$$L = 1/(200C) = \textbf{25 H}$$

$$R = 30L = \textbf{750 ohms}$$

Chapter 8, Solution 16.

At $t = 0$, $i(0) = 0$, $v_C(0) = 40 \times 30/50 = 24$V

For $t > 0$, we have a source-free RLC circuit.

$$\alpha = R/(2L) = (40 + 60)/5 = 20 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 2.5}} = 20$$

$\omega_o = \alpha$ leads to critical damping

$$i(t) = [(A + Bt)e^{-20t}], \quad i(0) = 0 = A$$

$$di/dt = \{[Be^{-20t}] + [-20(Bt)e^{-20t}]\},$$

but $di(0)/dt = -(1/L)[Ri(0) + v_C(0)] = -(1/2.5)[0 + 24]$

Hence, $\quad B = -9.6 \text{ or } i(t) = [\mathbf{-9.6te^{-20t}}] \text{ A}$

Chapter 8, Solution 17.

$$i(0) = I_0 = 0, \ v(0) = V_0 = 4x5 = 20$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0 + 20) = -80$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\dfrac{1}{4}\dfrac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\dfrac{1}{4}} = 20, \ which\,is > \omega_o.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.679, -37.32$$

$$i(t) = A_1 e^{-2.679t} + A_2 e^{-37.32t}$$

$$i(0) = 0 = A_1 + A_2, \ \frac{di(0)}{dt} = -2.679A_1 - 37.32A_2 = -80$$

$$This\,leads\,to\ A_1 = -2.309 = -A_2$$

$$i(t) = 2.309\left(e^{-37.32t} - e^{-2.679t}\right)$$

$$Since, v(t) = \frac{1}{C}\int_0^t i(t)dt + 20, \ we\,get$$

$$v(t) = [21.55e^{-2.679t} - 1.55e^{-37.32t}] \ V$$

Chapter 8, Solution 18.

When the switch is off, we have a source-free parallel RLC circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2, \qquad \alpha = \frac{1}{2RC} = 0.5$$

$\alpha < \omega_o \qquad \longrightarrow \qquad$ underdamped case $\quad \omega_d = \sqrt{\omega_o^{\,2} - \alpha^2} = \sqrt{4 - 0.25} = 1.936$

$I_o(0) = i(0) =$ initial inductor current $= 100/5 = 20$ A

$V_o(0) = v(0) =$ initial capacitor voltage $= 0$ V

$$v(t) = e^{-\alpha t}(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)) = e^{-0.5\alpha t}(A_1 \cos(1.936t) + A_2 \sin(1.936t))$$

$v(0) = 0 = A_1$

$$\frac{dv}{dt} = e^{-0.5\alpha t}(-0.5)(A_1 \cos(1.936t) + A_2 \sin(1.936t)) + e^{-0.5\alpha t}(-1.936A_1 \sin(1.936t) + 1.936A_2 \cos(1.936t))$$

$$\frac{dv(0)}{dt} = -\frac{(V_o + RI_o)}{RC} = -\frac{(0+20)}{1} = -20 = -0.5A_1 + 1.936A_2 \qquad \longrightarrow \qquad A_2 = -10.333$$

Thus,

$$\underline{v(t) = [-10.333e^{-0.5t}\ \sin(1.936t)]volts}$$

Chapter 8, Solution 19.

For t < 0, the equivalent circuit is shown in Figure (a).

(a)   **(b)**
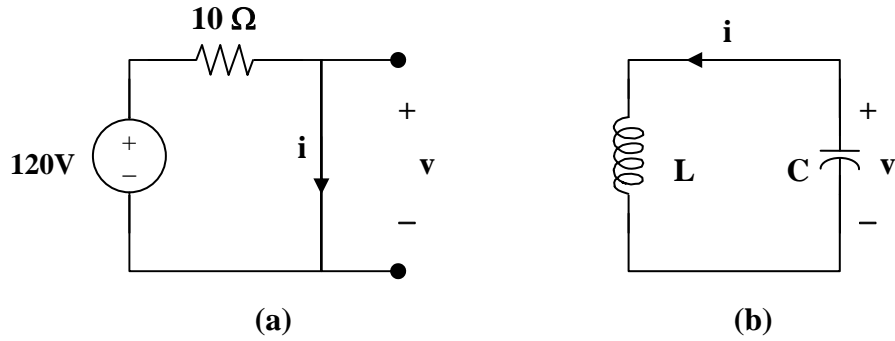
$$i(0) = 120/10 = 12, \ v(0) = 0$$

For t > 0, we have a series RLC circuit as shown in Figure (b) with R = 0 = α.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4}} = 0.5 = \omega_d$$

$$i(t) = [A\cos 0.5t + B\sin 0.5t], \ i(0) = 12 = A$$

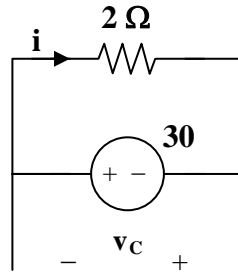$$v = -Ldi/dt, \ \text{and} \ -v/L = di/dt = 0.5[-12\sin 0.5t + B\cos 0.5t],$$

which leads to $-v(0)/L = 0 = B$

Hence, $i(t) = 12\cos 0.5t \ A \ \text{and} \ v = 0.5$

However, $v = -Ldi/dt = -4(0.5)[-12\sin 0.5t] = \textbf{24sin(0.5t) V}$

Chapter 8, Solution 20.

For $t < 0$, the equivalent circuit is as shown below.

$$v(0) = -30 \text{ V} \quad \text{and} \quad i(0) = 30/2 = 15 \text{ A}$$

For $t > 0$, we have a series RLC circuit.

$$\alpha = R/(2L) = 2/(2 \times 0.5) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.5 \times 1/4} = 2\sqrt{2}$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{8-4} = 2$$

$$i(t) = (A\cos(2t) + B\sin(2t))e^{-2t}$$

$$i(0) = 15 = A$$

$$di/dt = -2(15\cos(2t) + B\sin(2t))e^{-2t} + (-2 \times 15\sin(2t) + 2B\cos(2t))e^{-\alpha t}$$

$$di(0)/dt = -30 + 2B = -(1/L)[Ri(0) + v_C(0)] = -2[30 - 30] = 0$$

Thus, $B = 15$ and $i(t) = \mathbf{(15\cos(2t) + 15\sin(2t))e^{-2t} \ A}$

Chapter 8, Solution 21.

By combining some resistors, the circuit is equivalent to that shown below.
$60\|(15 + 25) = 24$ ohms.
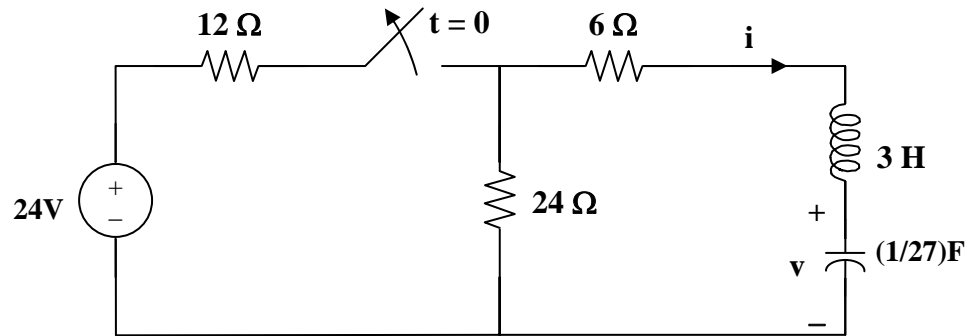
At $t = 0-$, $i(0) = 0$, $v(0) = 24 \times 24/36 = 16V$

For $t > 0$, we have a series RLC circuit. $R = 30$ ohms, $L = 3$ H, $C = (1/27)$ F

$$\alpha = R/(2L) = 30/6 = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{3 \times 1/27} = 3, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm \sqrt{5^2 - 3^2} = -9, -1$$

$$v(t) = [Ae^{-t} + Be^{-9t}], \quad v(0) = 16 = A + B \tag{1}$$

$$i = Cdv/dt = C[-Ae^{-t} - 9Be^{-9t}]$$

$$i(0) = 0 = C[-A - 9B] \text{ or } A = -9B \tag{2}$$

From (1) and (2), $B = -2$ and $A = 18$.

Hence, $v(t) = (18e^{-t} - 2e^{-9t}) V$

Chapter 8, Solution 22.

Compare the characteristic equation with eq. (8.8), i.e.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

we obtain

$$\frac{R}{L} = 100 \quad \longrightarrow \quad L = \frac{R}{100} = \frac{2000}{100} = \underline{20H}$$

$$\frac{1}{LC} = 10^6 \quad \rightarrow \quad C = \frac{1}{10^6 L} = \frac{10^{-6}}{20} = \underline{50 \text{ nF}}$$

Chapter 8, Solution 23.

Let $C_o = C + 0.01$. For a parallel RLC circuit,

$$\alpha = 1/(2RC_o), \quad \omega_o = 1/\sqrt{LC_o}$$

$\alpha = 1 = 1/(2RC_o)$, we then have $C_o = 1/(2R) = 1/20 = 50$ mF

$$\omega_o = 1/\sqrt{0.02x0.05} = 141.42 > \alpha \text{ (underdamped)}$$

$$C_o = C + 10 \text{ mF} = 50 \text{ mF or } C = \textbf{40 mF}$$

Chapter 8, Solution 24.

When the switch is in position A, the inductor acts like a short circuit so

$$i(0^-) = 4$$

When the switch is in position B, we have a source-free parallel RCL circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 10 \times 10 \times 10^{-3}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4} \times 10 \times 10^{-3}}} = 20$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$S_{1,2} = -5 + \sqrt{25 - 400} = -5 + j19.365$$

$$i(t) = e^{-5t}\left(A_1 \cos 19.365t + A_2 \sin 19.365t\right)$$

$$i(0) = 4 = A_1$$

$$v = L\frac{di}{dt} \quad \longrightarrow \quad \frac{di(0)}{dt} = \frac{v(0)}{L} = 0$$

$$\frac{di}{dt} = e^{-5t}\left(-5A_1 \cos 19.365t - 5A_2 \sin 19.365t - 19.365A_1 \sin 19.365t + 19.365A_2 \cos 19.365t\right)$$

$$0 = [di(0)/dt] = -5A_1 + 19.365A_2 \text{ or } A_2 = 20/19.365 = 1.0328$$

$$\mathbf{i(t) = e^{-5t}[4\cos(19.365t) + 1.0328\sin(19.365t)] \; A}$$

Chapter 8, Solution 25.

Using Fig. 8.78, design a problem to help other students to better understand source-free *RLC* circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

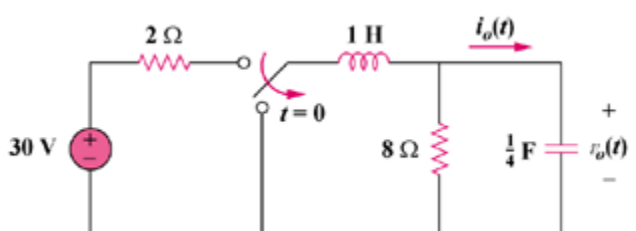In the circuit in Fig. 8.78, calculate $i_o(t)$ and $v_o(t)$ for $t > 0$.

Figure 8.78

Solution

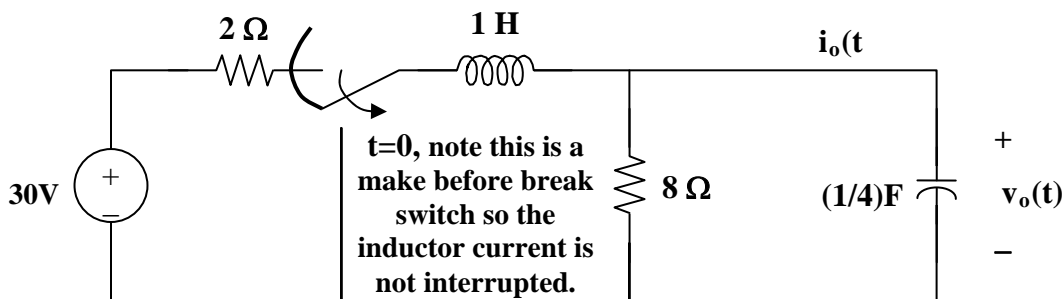In the circuit in Fig. 8.76, calculate $i_o(t)$ and $v_o(t)$ for t>0.

Figure 8.78   For Problem 8.25.

At t = 0⁻, $v_o(0) = (8/(2 + 8)(30) = 24$

For t > 0, we have a source-free parallel RLC circuit.

$$\alpha \;=\; 1/(2RC) \;=\; ¼$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1 \times 1/4} = 2$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4 - (1/16)} = 1.9843$$

$$v_o(t) = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$$

$$v_o(0) = 30(8/(2+8)) = 24 = A_1 \text{ and } i_o(t) = C(dv_o/dt) = 0 \text{ when } t = 0.$$

$$dv_o/dt = -\alpha(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t} + (-\omega_d A_1\sin\omega_d t + \omega_d A_2\cos\omega_d t)e^{-\alpha t}$$

$$\text{at } t = 0, \text{ we get } dv_o(0)/dt = 0 = -\alpha A_1 + \omega_d A_2$$

Thus, $A_2 = (\alpha/\omega_d)A_1 = (1/4)(24)/1.9843 = 3.024$

$$v_o(t) = \mathbf{(24cos1.9843t + 3.024sin1.9843t)e^{-t/4} \text{ volts}}.$$

$i_0(t) = Cdv/dt = 0.25[-24(1.9843)\sin1.9843t + 3.024(1.9843)\cos1.9843t -$
$0.25(24\cos1.9843t) - 0.25(3.024\sin1.9843t)]e^{-t/4}$
$= \mathbf{[- 12.095sin1.9843t]e^{-t/4} \text{ A}}.$

Chapter 8, Solution 26.

$$s^2 + 2s + 5 = 0, \text{ which leads to } s_{1,2} = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j4$$

These roots indicate an underdamped circuit which has the generalized solution given as:

$$i(t) = I_s + [(A_1\cos(4t) + A_2\sin(4t))e^{-t}],$$

At $t = \infty$, $(di(t)/dt) = 0$ and $(d^2i(t)/dt^2) = 0$ so that
$I_s = 10/5 = 2$ (from $(d^2i(t)/dt^2)+2(di(t)/dt)+5 = 10$)

$$i(0) = 2 = 2 + A_1, \text{ or } A_1 = 0$$

$$di/dt = [(4A_2\cos(4t))e^{-t}] + [(-A_2\sin(4t))e^{-t}] = 4 = 4A_2, \text{ or } A_2 = 1$$

$$i(t) = [2 + \sin(4te^{-t})] \text{ amps}$$

Chapter 8, Solution 27.

$$s^2 + 4s + 8 = 0 \text{ leads to s } = \frac{-4 \pm \sqrt{16 - 32}}{2} = -2 \pm j2$$

$$v(t) = V_s + (A_1 \cos 2t + A_2 \sin 2t)e^{-2t}$$

$$8V_s = 24 \text{ means that } V_s = 3$$

$$v(0) = 0 = 3 + A_1 \text{ leads to } A_1 = -3$$

$$dv/dt = -2(A_1 \cos 2t + A_2 \sin 2t)e^{-2t} + (-2A_1 \sin 2t + 2A_2 \cos 2t)e^{-2t}$$

$$0 = dv(0)/dt = -2A_1 + 2A_2 \text{ or } A_2 = A_1 = -3$$

$$v(t) = [3 - 3(\cos(2t) + \sin(2t))e^{-2t}] \text{ volts.}$$

Chapter 8, Solution 28.

The characteristic equation is

$$Ls^2 + Rs + \frac{1}{C} = 0 \quad \longrightarrow \quad \frac{1}{2}s^2 + 4s + \frac{1}{0.2} = 0 \quad \longrightarrow \quad s^2 + 8s + 10 = 0$$

$$S_{1,2} = \frac{-8 \pm \sqrt{64-40}}{2} = \quad -6.45 \text{ and } -1.5505$$

$$i(t) = i_S + Ae^{-6.45t} + Be^{-1.5505t}$$

But $[i_S/C] = 10$ or $i_S = 0.2 \times 10 = 2$

$$i(t) = 2 + Ae^{-6.45t} + Be^{-1.5505t}$$

$i(0) = 1 = 2 + A + B$ or $A + B = -1$ or $A = -1-B$ (1)

$$\frac{di(t)}{dt} = -6.45Ae^{-6.45t} - 1.5505Be^{-1.5505t}$$

$$but \ \frac{di(0)}{dt} = 0 = -6.45A - 1.5505B$$ (2)

Solving (1) and (2) gives $-6.45(-1-B) - 1.5505B = 0$ or $(6.45-1.5505)B = -6.45$
       $B = -6.45/(4.9) = -1.3163$ and $A = -1-1.3163 = -2.3163$

 A= -2.3163, B= -1.3163
Hence,
$$i(t) = [2-2.3163e^{-6.45t} - 1.3163e^{-1.5505t}] \ A.$$

Chapter 8, Solution 29.

(a) $s^2 + 4 = 0$ which leads to $s_{1,2} = \pm j2$ (an undamped circuit)

$$v(t) = V_s + A\cos 2t + B\sin 2t$$

$$4V_s = 12 \text{ or } V_s = 3$$

$$v(0) = 0 = 3 + A \text{ or } A = -3$$

$$dv/dt = -2A\sin 2t + 2B\cos 2t$$

$dv(0)/dt = 2 = 2B$ or $B = 1$, therefore $v(t) = \mathbf{(3 - 3\cos 2t + \sin 2t)\ V}$

(b) $s^2 + 5s + 4 = 0$ which leads to $s_{1,2} = -1, -4$

$$i(t) = (I_s + Ae^{-t} + Be^{-4t})$$

$$4I_s = 8 \text{ or } I_s = 2$$

$$i(0) = -1 = 2 + A + B, \text{ or } A + B = -3 \qquad (1)$$

$$di/dt = -Ae^{-t} - 4Be^{-4t}$$

$$di(0)/dt = 0 = -A - 4B, \text{ or } B = -A/4 \qquad (2)$$

From (1) and (2) we get A = -4 and B = 1

$$i(t) = \mathbf{(2 - 4e^{-t} + e^{-4t})\ A}$$

(c) $s^2 + 2s + 1 = 0$, $s_{1,2} = -1, -1$

$$v(t) = [V_s + (A + Bt)e^{-t}], \ V_s = 3.$$

$$v(0) = 5 = 3 + A \text{ or } A = 2$$

$$dv/dt = [-(A + Bt)e^{-t}] + [Be^{-t}]$$

$$dv(0)/dt = -A + B = 1 \text{ or } B = 2 + 1 = 3$$

$$v(t) = \mathbf{[3 + (2 + 3t)e^{-t}]\ V}$$

(d) $s^2 + 2s + 5 = 0$, $s_{1,2} = -1 + j2$, $-1 - j2$

$i(t) = [I_s + (A\cos 2t + B\sin 2t)e^{-t}]$, where $5I_s = 10$ or $I_s = 2$

$i(0) = 4 = 2 + A$ or $A = 2$

$di/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [(-2A\sin 2t + 2B\cos 2t)e^{-t}]$

$di(0)/dt = -2 = -A + 2B$ or $B = 0$

$i(t) = \mathbf{[2 + (2\cos 2t)e^{-t}]\ A}$

Chapter 8, Solution 30.

The step responses of a series RLC circuit are

$v_C(t) = [40–10e^{-2000t}–10e^{-4000t}]$ volts, $t > 0$ and
$i_L(t) = [3e^{-2000t}+6e^{-4000t}]$ m A, $t > 0$.

(a) Find C. (b) Determine what type of damping exhibited by the circuit.

Solution

Step 1.   For a series RLC circuit, $i_R(t) = i_L(t) = i_C(t)$.

We can determine C from $i_C(t) = i_L(t) = C(dv_C/dt)$ and we can determine that the circuit is **overdamped** since the exponent value are real and negative.

Step 2. $C(dv_C/dt) = C[20,000e^{-2000t}+40,000e^{-4000t}] = 0.003e^{-2000t}+0.006e^{-4000t}$ or

$$C = 0.003/20,000 = \textbf{150 } \boldsymbol{\eta}\textbf{F}.$$

Chapter 8, Solution 31.

For t $=$ 0-, we have the equivalent circuit in Figure (a). For t $=$ 0+, the equivalent circuit is shown in Figure (b). By KVL,

$$v(0+) \ = \ v(0-) \ = \ 40, \ i(0+) \ = \ i(0-) \ = \ 1$$

By KCL, 2 $=$ i(0+) + i_1 $=$ 1 + i_1 which leads to i_1 $=$ 1. By KVL, $-v_L + 40i_1 + v(0+) \ = \ 0$ which leads to $v_L(0+) \ = \ 40 \times 1 + 40 \ = \ 80$

$$v_L(0+) \ = \ \mathbf{80 \ V}, \quad v_C(0+) \ = \ \mathbf{40 \ V}$$

(a)   (b)
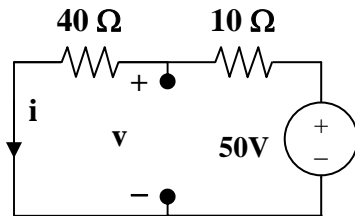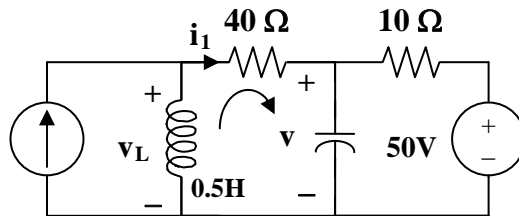
Chapter 8, Solution 32.

For t $=$ 0-, the equivalent circuit is shown below.

2 A

i

$+$ $-$

v

6 Ω
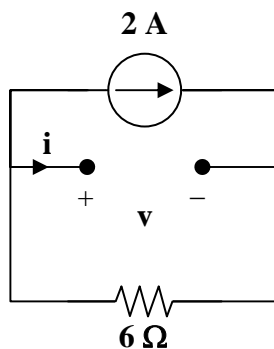
i(0-) $=$ 0, v(0-) $=$ -2x6 $=$ -12V

For t $>$ 0, we have a series RLC circuit with a step input.

α $=$ R/(2L) $=$ 6/2 $=$ 3, ω_o $=$ $1/\sqrt{LC}$ $=1/\sqrt{0.04}$

s $=$ $-3 \pm \sqrt{9-25}$ $= -3 \pm j4$

Thus, v(t) $=$ V_f + [(Acos4t + Bsin4t)e^{-3t}]

where V_f $=$ final capacitor voltage $=$ 50 V

v(t) $=$ 50 + [(Acos4t + Bsin4t)e^{-3t}]

v(0) $=$ -12 $=$ 50 + A which gives A $=$ -62

i(0) $=$ 0 $=$ Cdv(0)/dt

dv/dt $=$ [-3(Acos4t + Bsin4t)e^{-3t}] + [4(–Asin4t + Bcos4t)e^{-3t}]

0 $=$ dv(0)/dt $=$ -3A + 4B or B $=$ (3/4)A $=$ –46.5

v(t) $=$ {50 + [(–62cos4t – 46.5sin4t)e^{-3t}]} V

Chapter 8, Solution 33.

We may transform the current sources to voltage sources. For $t = 0^-$, the equivalent circuit is shown in Figure (a).

(a)   **(b)**
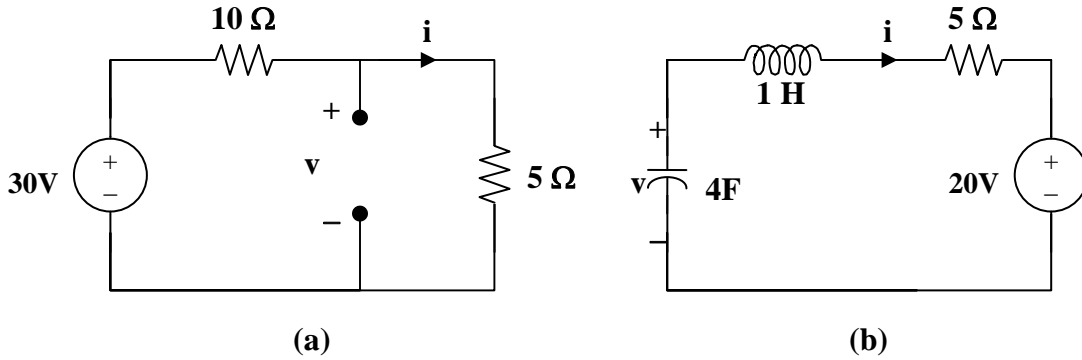
$$i(0) = 30/15 = 2 \text{ A}, \ v(0) = 5\text{x}30/15 = 10 \text{ V}$$

For $t > 0$, we have a series RLC circuit, shown in (b).

$$\alpha = R/(2L) = 5/2 = 2.5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4} = 0.5, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 0.25} = -4.949, -0.0505$$

$$v(t) = V_s + [A_1e^{-4.949t} + A_2e^{-0.0505t}], \ V_s = 20.$$

$$v(0) = 10 = 20 + A_1 + A_2 \text{ or}$$
$$A_2 = -10 - A_1$$
$$(1)$$

$$i(0) = Cdv(0)/dt \text{ or } dv(0)/dt = -2/4 = -1/2$$

Hence, $-0.5 = -4.949A_1 - 0.0505A_2$ (2)

From (1) and (2), $-0.5 = -4.949A_1 + 0.0505(10 + A_1)$ or
$$-4.898A_1 = -0.5 - 0.505 = -1.005$$

$$A_1 = 0.2052, \ A_2 = -10.205$$

$$v(t) = [20 + 0.2052e^{-4.949t} - 10.205e^{-0.05t}] \text{ V}.$$

Chapter 8, Solution 34.

Before $t = 0$, the capacitor acts like an open circuit while the inductor behaves like a short circuit.

$$i(0) = 0, \ v(0) = 50 \text{ V}$$

For $t > 0$, the LC circuit is disconnected from the voltage source as shown below.
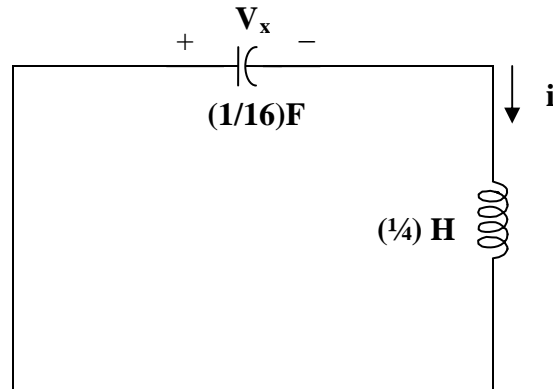
This is a lossless, source-free, series RLC circuit.

$$\alpha \ = \ R/(2L) \ = \ 0, \ \omega_o = 1/\sqrt{LC} \ = \ 1/\sqrt{\frac{1}{16} + \frac{1}{4}} \ = \ 8, \ s = \ \pm j8$$

Since α is equal to zero, we have an undamped response. Therefore,

$$i(t) = A_1\cos(8t) + A_2\sin(8t) \ \text{where} \ i(0) = 0 = A_1$$

$$di(0)/dt = (1/L)v_L(0) = -(1/L)v(0) = -4 \text{x} 50 = -200$$

However, $di/dt = 8A_2\cos(8t)$, thus, $di(0)/dt = -200 = 8A_2$ which leads to $A_2 = -25$

Now we have $i(t) \ = \ \textbf{–25sin(8t) A}$

Chapter 8, Solution 35.

Using Fig. 8.83, design a problem to help other students to better understand the step response of series *RLC* circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine $v(t)$ for $t > 0$ in the circuit in Fig. 8.83.
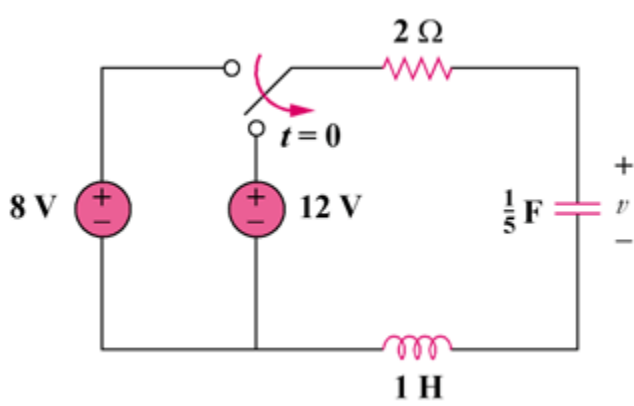
Figure 8.83

Solution

$$\text{At } t = 0\text{-}, \quad i_L(0) = 0, \quad v(0) = v_C(0) = 8 \text{ V}$$

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 2/2 = 1, \quad \omega_o = 1/\sqrt{LC} = 1/\sqrt{1/5} = \sqrt{5}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \pm j2$$

$$v(t) = V_s + [(A\cos 2t + B\sin 2t)e^{-t}], \quad V_s = 12.$$

$$v(0) = 8 = 12 + A \text{ or } A = -4, \quad i(0) = Cdv(0)/dt = 0.$$

$$\text{But } dv/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [2(-A\sin 2t + B\cos 2t)e^{-t}]$$

$$0 = dv(0)/dt = -A + 2B \text{ or } 2B = A = -4 \text{ and } B = -2$$

$$v(t) = \{12 - (4\cos 2t + 2\sin 2t)e^{-t} \text{ V}.$$

Chapter 8, Solution 36.

For t $= 0^-$, 3u(t) $= 0$. Thus, i(0) $= 0$, and v(0) $= 20$ V.

For t > 0, we have the series RLC circuit shown below.

$$\alpha = R/(2L) = (2 + 5 + 1)/(2\text{x}5) = 0.8$$
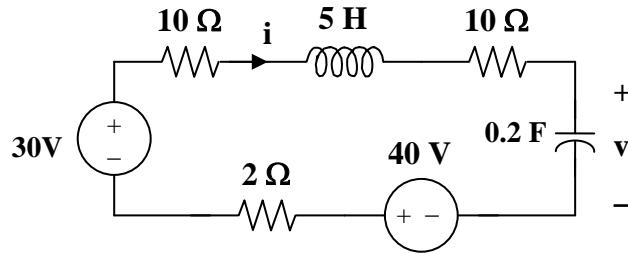
$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5\text{x}0.2} = 1$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -0.8 \pm j0.6$$

$$v(t) = V_s + [(A\cos(0.6t) + B\sin(0.6t))e^{-0.8t}]$$

$$V_s = 30 + 40 = 70 \text{ V} \text{ and } v(0) = 40 = 70 + A \text{ or } A = -30$$

$$i(0) = Cdv(0)/dt = 0$$

But $dv/dt = [-0.8(A\cos(0.6t) + B\sin(0.6t))e^{-0.8t}] + [0.6(-A\sin(0.6t) + B\cos(0.6t))e^{-0.8t}]$

$$0 = dv(0)/dt = -0.8A + 0.6B \text{ which leads to } B = 0.8\text{x}(-30)/0.6 = -40$$

$$\mathbf{v(t) = \{70 - [(30\cos(0.6t) + 40\sin(0.6t))e^{-0.8t}]\} \ V}$$

$$i = Cdv/dt$$

$$= 0.2\{[0.8(30\cos(0.6t) + 40\sin(0.6)t)e^{-0.8t}] + [0.6(30\sin(0.6t) - 40\cos(0.6t))e^{-0.8t}]\}$$

$$\mathbf{i(t) = 10\sin(0.6t)e^{-0.8t} \ A}$$

Chapter 8, Solution 37.

For $t = 0^-$, the equivalent circuit is shown below.
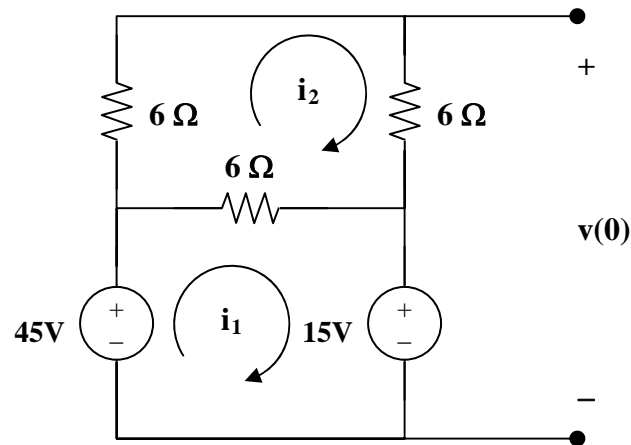
$$18i_2 - 6i_1 = 0 \text{ or } i_1 = 3i_2 \tag{1}$$

$$-45 + 6(i_1 - i_2) + 15 = 0 \text{ or } i_1 - i_2 = 30/6 \qquad = 5$$
$$\tag{2}$$

From (1) and (2), $(2/3)i_1 = 5$ or $i_1 = 7.5$ and $i_2 = i_1 - 5 = 2.5$

$$i(0) = i_1 = 7.5A$$

$$-15 - 6i_2 + v(0) = 0$$

$$v(0) = 15 + 6 \text{x} 2.5 = 30$$

For $t > 0$, we have a series RLC circuit.

$$R = 6 \| 12 = 4$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/8)} = 4$$

$$\alpha = R/(2L) = (4)/(2 \text{x}(1/2)) = 4$$

$\alpha = \omega_o$, therefore the circuit is critically damped

$$v(t) = V_s + [(A + Bt)e^{-4t}], \text{ and } V_s = 15$$

$$v(0) = 30 = 15 + A, \text{ or } A = 15$$

$$i_C = Cdv/dt = C[-4(15 + Bt)e^{-4t}] + C[(B)e^{-4t}]$$

To find $i_C(0)$ we need to look at the circuit right after the switch is opened. At this time, the current through the inductor forces that part of the circuit to act like a current source and the capacitor acts like a voltage source. This produces the circuit shown below. Clearly, $i_C(0+)$ must equal $-i_L(0) = -7.5A$.
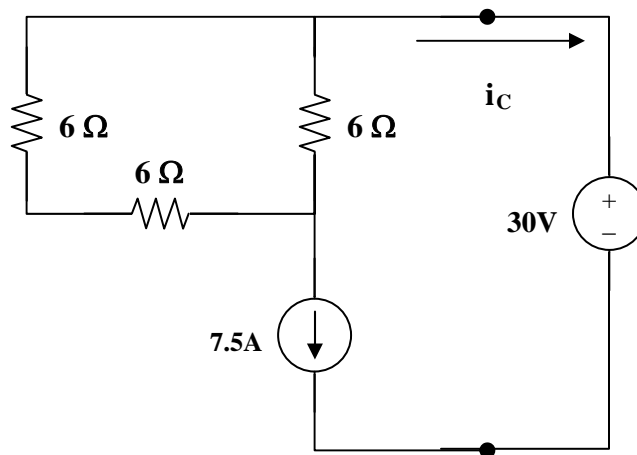
$$i_C(0) = -7.5 = C(-60 + B) \text{ which leads to } -60 = -60 + B \text{ or } B = 0$$

$$i_C = Cdv/dt = (1/8)[-4(15 + 0t)e^{-4t}] + (1/8)[(0)e^{-4t}]$$

$$i_C(t) = [-(1/2)(15)e^{-4t}]$$

$$i(t) = -i_C(t) = \mathbf{7.5e^{-4t} \ A}$$

Chapter 8, Solution 38.

At t = 0⁻, the equivalent circuit is as shown.
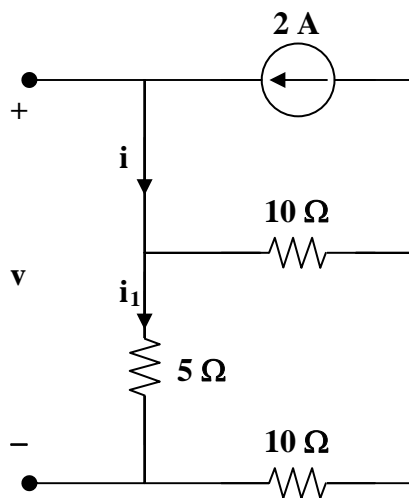
$i(0) = 2A$, $\quad i_1(0) = 10(2)/(10 + 15) = 0.8$ A

$v(0) = 5i_1(0) = 4V$

For t > 0, we have a source-free series RLC circuit.

$R = 5\|(10 + 10) = 4$ ohms

$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/3)(3/4)} = 2$

$\alpha = R/(2L) = (4)/(2x(3/4)) = 8/3$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = $ -4.431, -0.903

$i(t) = [Ae^{-4.431t} + Be^{-0.903t}]$

$$i(0) = A + B = 2 \qquad (1)$$

$di(0)/dt = (1/L)[-Ri(0) + v(0)] = (4/3)(-4x2 + 4) = -16/3 = -5.333$

$$\text{Hence, } -5.333 = -4.431A - 0.903B \qquad (2)$$

From (1) and (2), A = 1 and B = 1.

$$i(t) = [e^{-4.431t} + e^{-0.903t}] \text{ A}$$

Chapter 8, Solution 39.

For $t = 0^-$, the source voltages are equal to zero thus, the initial conditions are $v(0) = 0$ and $i_L(0) = 0$.
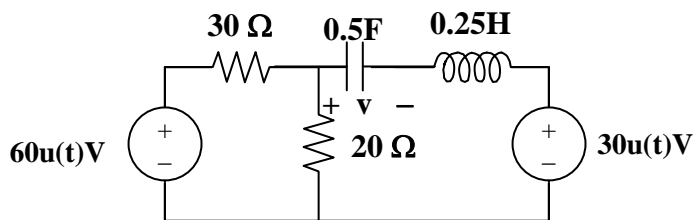
For $t > 0$, the circuit is shown above.

$$R = 20||30 = 12 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/4)} = \sqrt{8}$$

$$\alpha = R/(2L) = (12)/(0.5) = 24$$

Since $\alpha > \omega_o$, we have an overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -47.83, -0.167$$

Thus, $\quad v(t) = V_s + [Ae^{-47.83t} + Be^{-0.167t}], \quad$ where
$\quad V_s = [60/(30+20)]20 - 30 = -6 \text{ volts}.$

$$v(0) = 0 = -6 + A + B \text{ or } 6 = A + B \tag{1}$$

$$i(0) = Cdv(0)/dt = 0$$

$$\text{But,} \quad dv(0)/dt = -47.83A - 0.167B = 0 \text{ or}$$

$$B = -286.4A \tag{2}$$

From (1) and (2), $\quad A + (-286.4)A = 6 \text{ or } A = 6/(-285.4) = -0.02102 \text{ and}$
$$B = -286.4\text{x}(-0.02102) = 6.02$$

$$v(t) = [-6 + (-0.021e^{-47.83t} + 6.02e^{-0.167t})] \textbf{ volts}.$$

Chapter 8, Solution 40.

At $t = 0-$, $v_C(0) = 0$ and $i_L(0) = i(0) = (6/(6+2))4 = 3A$

For $t > 0$, we have a series RLC circuit with a step input as shown below.
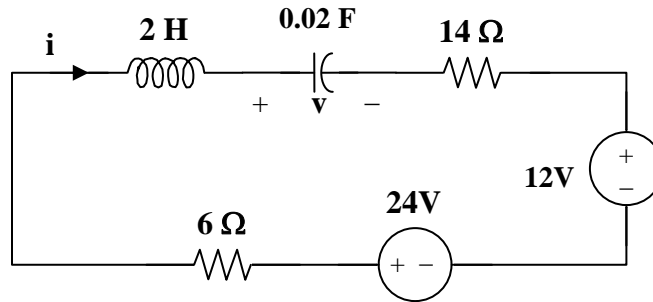
$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2 \times 0.02} = 5$$

$$\alpha = R/(2L) = (6+14)/(2 \times 2) = 5$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$v(t) = V_s + [(A + Bt)e^{-5t}], \quad V_s = 24 - 12 = 12V$$

$$v(0) = 0 = 12 + A \quad \text{or} \quad A = -12$$

$$i = Cdv/dt = C\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$$

$$i(0) = 3 = C[-5A + B] = 0.02[60 + B] \quad \text{or} \quad B = 90$$

Thus, $i(t) = 0.02\{[90e^{-5t}] + [-5(-12 + 90t)e^{-5t}]\}$

$$i(t) = \{(3 - 9t)e^{-5t}\} \text{ A}$$

Chapter 8, Solution 41.

At $t = 0^-$, the switch is open. $i(0) = 0$, and

$$v(0) = 5 \times 100/(20 + 5 + 5) = 50/3$$

For $t > 0$, we have a series RLC circuit shown in Figure (a). After source transformation, it becomes that shown in Figure (b).

10 H

(a) — 5A current source, 20 Ω, 5 Ω, 10 μF, 10 H

(b) — 4 Ω, 1 H, 20V source, i, v, 0.04F
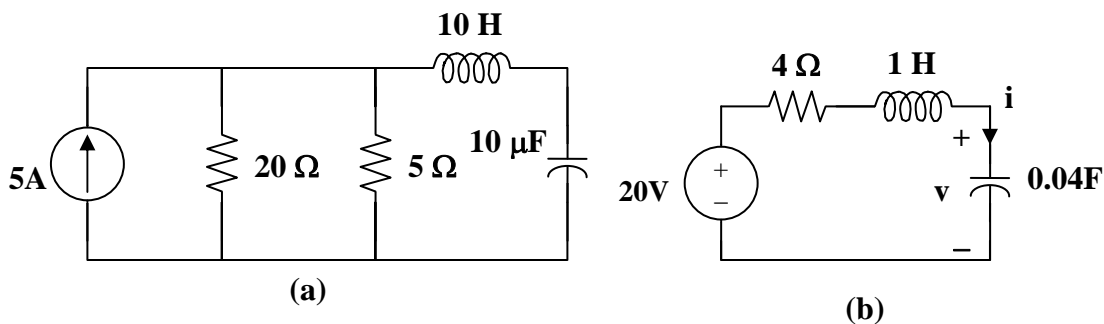
$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1 \times 1/25} = 5$$

$$\alpha = R/(2L) = (4)/(2 \times 1) = 2$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2 \pm j4.583$$

Thus,
$$v(t) = V_s + [(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}],$$

where $\omega_d = 4.583$ and $V_s = 20$

$v(0) = 50/3 = 20 + A$ or $A = -10/3$

$i(t) = Cdv/dt$
$= C(-2)\,[(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}] + C\omega_d[(-A\sin(\omega_d t) + B\cos(\omega_d t))e^{-2t}]$

$i(0) = 0 = -2A + \omega_d B$

$B = 2A/\omega_d = -20/(3 \times 4.583) = -1.455$

$i(t) = C\{[(0\cos(\omega_d t) + (-2B - \omega_d A)\sin(\omega_d t))]e^{-2t}\}$

$= (1/25)\{[(2.91 + 15.2767)\sin(\omega_d t))]e^{-2t}\}$

$$i(t) = \mathbf{727.5\sin(4.583t)e^{-2t}\ mA}$$

Chapter 8, Solution 42.

For t = 0-, we have the equivalent circuit as shown in Figure (a).

i(0) = i(0) = 0, and v(0) = 4 – 12 = -8V

(a)   (b)
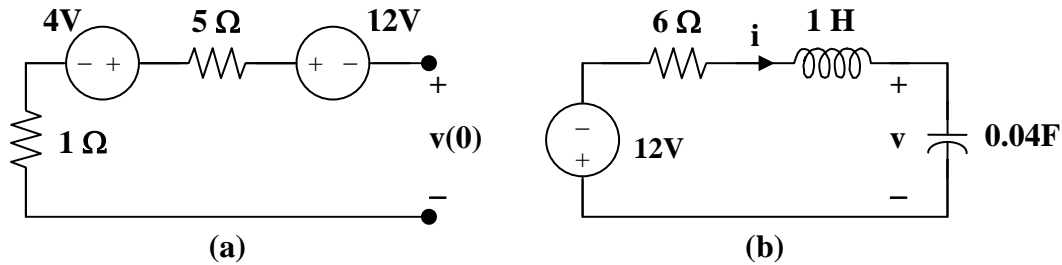
For t > 0, the circuit becomes that shown in Figure (b) after source transformation.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (6)/(2) = 3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -3 \pm j4$$

Thus, $v(t) = V_s + [(Acos4t + Bsin4t)e^{-3t}]$, $V_s = -12$

$$v(0) = -8 = -12 + A \text{ or } A = 4$$

$i = Cdv/dt$, or $i/C = dv/dt = [-3(Acos4t + Bsin4t)e^{-3t}] + [4(-Asin4t + Bcos4t)e^{-3t}]$

$$i(0) = -3A + 4B \text{ or } B = 3$$

$$v(t) = \{-12 + [(4cos4t + 3sin4t)e^{-3t}]\} \text{ A}$$

Chapter 8, Solution 43.

For t>0, we have a source-free series RLC circuit.

$$\alpha = \frac{R}{2L} \quad \longrightarrow \quad R = 2\alpha L = 2 \text{x} 8 \text{x} 0.5 = \mathbf{8\Omega}$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 30 \quad \longrightarrow \quad \omega_o = \sqrt{900 + 64} = \sqrt{964}$$

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \longrightarrow \quad C = \frac{1}{\omega_o^2 L} = \frac{1}{964 \text{x} 0.5} = \mathbf{2.075 \ mF}$$

Chapter 8, Solution 44.

$$\alpha = \frac{R}{2L} = \frac{1000}{2x1} = 500, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-9}}} = 10^4$$

$\omega_o > \alpha \qquad \longrightarrow \qquad$ **underdamped.**

Chapter 8, Solution 45.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1 \times 0.5} = \sqrt{2}$$

$$\alpha = 1/(2RC) = (1)/(2 \times 2 \times 0.5) = 0.5$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\omega_o^2 - \alpha^2} = -0.5 \pm j1.3229$$

Thus, $i(t) = I_s + [(A\cos1.3229t + B\sin1.3229t)e^{-0.5t}], \quad I_s = 4$

$$i(0) = 1 = 4 + A \text{ or } A = -3$$

$$v = v_C = v_L = Ldi(0)/dt = 0$$

$$di/dt = [1.3229(-A\sin1.3229t + B\cos1.3229t)e^{-0.5t}] + [-0.5(A\cos1.3229t + B\sin1.3229t)e^{-0.5t}]$$

$$di(0)/dt = 0 = 1.3229B - 0.5A \text{ or } B = 0.5(-3)/1.3229 = -1.1339$$

Thus, $i(t) = \{4 - [(3\cos1.3229t + 1.1339\sin1.3229t)e^{-t/2}]\}$ **A**

To find v(t) we use $v(t) = v_L(t) = Ldi(t)/dt$.

From above,
$$di/dt = [1.3229(-A\sin1.3229t + B\cos1.3229t)e^{-0.5t}] + [-0.5(A\cos1.323t + B\sin1.323t)e^{-0.5t}]$$
Thus,
$$v(t) = Ldi/dt = [1.323(-A\sin1.323t + B\cos1.323t)e^{-0.5t}] + [-0.5(A\cos1.323t + B\sin1.323t)e^{-0.5t}]$$
$$= [1.3229(3\sin1.3229t - 1.1339\cos1.3229t)e^{-0.5t}] + [(1.5\cos1.3229t + 0.5670\sin1.3229t)e^{-0.5t}]$$

$$v(t) = [(-0\cos1.323t + 4.536\sin1.323t)e^{-0.5t}] \text{ V}$$
$$= [(4.536\sin1.3229t)e^{-t/2}] \text{ V}$$

Please note that the term in front of the cos calculates out to -3.631×10^{-5} which is zero for all practical purposes when considering the rounding errors of the terms used to calculate it.

Chapter 8, Solution 46.

Using Fig. 8.93, design a problem to help other students to better understand the step response of a parallel *RLC* circuit.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

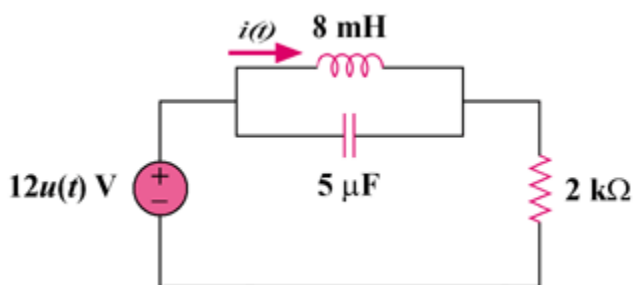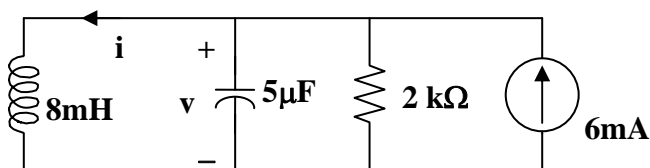Find $i(t)$ for $t > 0$ in the circuit in Fig. 8.93.

Figure 8.93

Solution

For $t = 0-$, $u(t) = 0$, so that $v(0) = 0$ and $i(0) = 0$.

For $t > 0$, we have a parallel RLC circuit with a step input, as shown below.

$$\alpha = 1/(2RC) = (1)/(2 \text{x} 2 \text{x} 10^3 \text{ x} 5 \text{x} 10^{-6}) = 50$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{8 \text{x} 10^{-3} \text{x} 5 \text{x} 10^{-6}} = 5,000$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \cong -50 \pm j5,000$$

Thus, $i(t) = I_s + [(A\cos 5,000t + B\sin 5,000t)e^{-50t}]$, $I_s = 6\text{mA}$

$$i(0) = 0 = 6 + A \text{ or } A = -6\text{mA}$$

$$v(0) \ = \ 0 \ = Ldi(0)/dt$$

$$di/dt \ = \ [5,000(-Asin5,000t + Bcos5,000t)e^{-50t}] + [-50(Acos5,000t + Bsin5,000t)e^{-50t}]$$

$$di(0)/dt \ = \ 0 \ = \ 5,000B - 50A \ \ or \ \ B \ = \ 0.01(-6) \ = \ -0.06mA$$

Thus, $\qquad\qquad$ **i(t) = {6 − [(6cos5,000t + 0.06sin5,000t)e^{-50t}]} mA**

Chapter 8, Solution 47.

At $t = 0-$, we obtain, $\qquad i_L(0) = 3 \times 5/(10 + 5) = 1A$

and $v_o(0) = 0.$

For $t > 0$, the 10-ohm resistor is short-circuited and we have a parallel RLC circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2 \times 5 \times 0.01) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1 \times 0.01} = 10$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -10$$

Thus, $\qquad i(t) = I_s + [(A + Bt)e^{-10t}], \quad I_s = 3$

$$i(0) = 1 = 3 + A \quad \text{or} \quad A = -2$$

$$v_o = Ldi/dt = [Be^{-10t}] + [-10(A + Bt)e^{-10t}]$$

$$v_o(0) = 0 = B - 10A \quad \text{or} \quad B = -20$$

Thus, $\quad v_o(t) = \mathbf{(200te^{-10t})\ V}$

Chapter 8, Solution 48.

For $t = 0^-$, we obtain $i(0) = -6/(1 + 2) = -2$ and $v(0) = 2\text{x}1 = 2$.

For $t > 0$, the voltage is short-circuited and we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = (1)/(2\text{x}1\text{x}0.25) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1\text{x}0.25} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $i(t) = [(A + Bt)e^{-2t}], \quad i(0) = -2 = A$

$$v = Ldi/dt = [Be^{-2t}] + [-2(-2 + Bt)e^{-2t}]$$

$$v_o(0) = 2 = B + 4 \text{ or } B = -2$$

Thus,
$$i(t) = [(-2 - 2t)e^{-2t}] \text{ A}$$

$$\text{and } v(t) = [(2 + 4t)e^{-2t}] \text{ V}$$

Chapter 8, Solution 49.

For $t = 0^-$, $i(0) = 3 + 12/4 = 6$ and $v(0) = 0$.

For $t > 0$, we have a parallel *RLC* circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2\text{x}5\text{x}0.05) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5\text{x}0.05} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $i(t) = I_s + [(A + Bt)e^{-2t}], \quad I_s = 3$

$$i(0) = 6 = 3 + A \text{ or } A = 3$$

$$v = Ldi/dt \text{ or } v/L = di/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

$$v(0)/L = 0 = di(0)/dt = B - 2\text{x}3 \text{ or } B = 6$$

Thus, $i(t) = \{3 + [(3 + 6t)e^{-2t}]\}$ **A**

Chapter 8, Solution 50.

For $t = 0-$, $4u(t) = 0$, $v(0) = 0$, and $i(0) = 30/10 = 3A$.

For $t > 0$, we have a parallel RLC circuit.
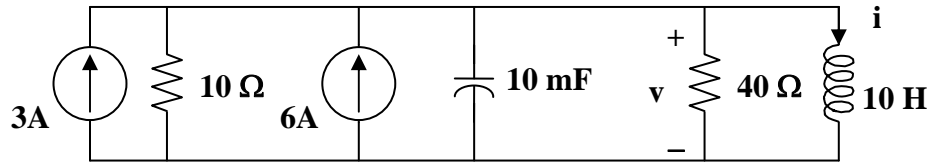
$I_s = 3 + 6 = 9A$ and $R = 10\|40 = 8$ ohms

$\alpha = 1/(2RC) = (1)/(2\times8\times0.01) = 25/4 = 6.25$

$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4\times0.01} = 5$

Since $\alpha > \omega_o$, we have a overdamped response.

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -2.5$

Thus, $i(t) = I_s + [Ae^{-10t}] + [Be^{-2.5t}]$, $I_s = 9$

$i(0) = 3 = 9 + A + B$ or $A + B = -6$

$di/dt = [-10Ae^{-10t}] + [-2.5Be^{-2.5t}]$,

$v(0) = 0 = Ldi(0)/dt$ or $di(0)/dt = 0 = -10A - 2.5B$ or $B = -4A$

Thus, $A = 2$ and $B = -8$

Clearly, $i(t) = \{ 9 + [2e^{-10t}] + [-8e^{-2.5t}]\}$ A

Chapter 8, Solution 51.

Let i = inductor current and v = capacitor voltage.

At $t = 0$, $v(0) = 0$ and $i(0) = i_o$.

For $t > 0$, we have a parallel, source-free LC circuit $(R = \infty)$.

$\alpha = 1/(2RC) = 0$ and $\omega_o = 1/\sqrt{LC}$ which leads to $s_{1,2} = \pm j\omega_o$

$v = A\cos\omega_o t + B\sin\omega_o t$, $v(0) = 0$ A

$i_C = Cdv/dt = -i$

$dv/dt = \omega_o B\sin\omega_o t = -i/C$

$dv(0)/dt = \omega_o B = -i_o/C$ therefore $B = i_o/(\omega_o C)$

$v(t) = \mathbf{-(i_o/(\omega_o C))\sin\omega_o t}$ **V where** $\omega_o = 1/\sqrt{LC}$

Chapter 8, Solution 52.

$$\alpha = 300 = \frac{1}{2RC} \tag{1}$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 400 \quad \longrightarrow \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 160{,}000 + 90{,}000 = \frac{1}{LC} \tag{2}$$

From (2),

$$C = \frac{1}{250{,}000 \times 50 \times 10^{-3}} = \textbf{80 } \boldsymbol{\mu}\textbf{F}$$

From (1),

$$R = \frac{1}{2\alpha C} = \frac{1}{2 \times 300 \times 80 \times 10^{-6}} = \textbf{20.83}\boldsymbol{\Omega}.$$

Chapter 8, Solution 53.

At $t<0$, $i(0^-)=0$, $v_c(0^-)=120V$

For $t>0$, we have the circuit as shown below.
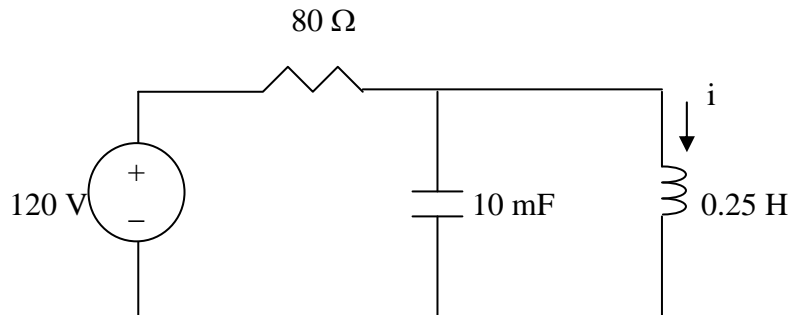
$$\frac{120-V}{R} = C\frac{dv}{dt} + i \longrightarrow 120 = V + RC\frac{dv}{dt} + iR \qquad (1)$$

But $\qquad v_L = V = L\frac{di}{dt}$ $\qquad\qquad\qquad\qquad$ (2)

Substituting (2) into (1) yields

$$120 = L\frac{di}{dt} + RCL\frac{d^2i}{dt^2} + iR \longrightarrow 120 = \frac{1}{4}\frac{di}{dt} + 80x\frac{1}{4}x10x10^{-3}\frac{d^2i}{dt^2} + 80i$$

or

$$\mathbf{(d^2i/dt^2) + 0.125(di/dt) + 400i = 600}$$

Chapter 8, Solution 54.

Using Fig. 8.100, design a problem to help other students better understand general second-order circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 8.100, let $I = 9$A, $R_1 = 40\ \Omega$, $R_2 = 20\ \Omega$, $C = 10$ mF, $R_3 = 50\ \Omega$,and $L = 20$ mH. Determine: (a) $i(0^+)$ and $v(0^+)$, (b) $di(0^+)/dt$ and $dv(0^+)/dt$, (c) $i(\infty)$ and $v(\infty)$.
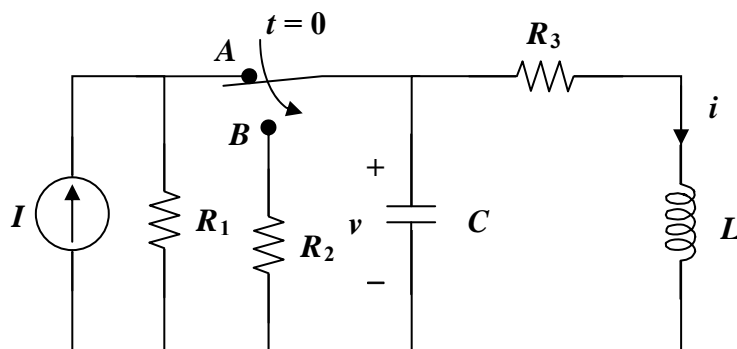
Figure 8.100
For Prob. 8.54.

Solution
(a) When the switch is at A, the circuit has reached steady state. Under this condition, the circuit is as shown below.
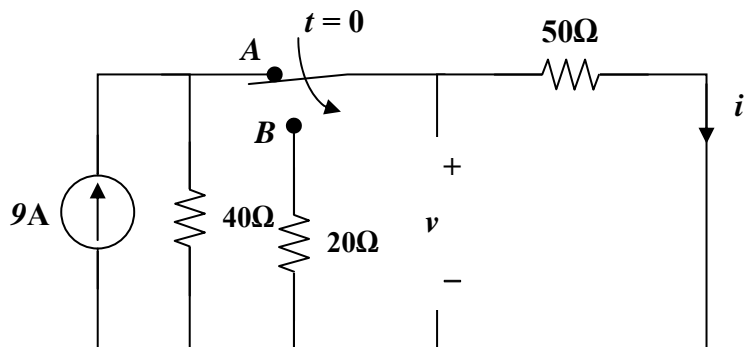
(a)   When the switch is at A, $i(0^-) = 9[(40\mathrm{x}50)/(40+50)]/50 = 4$ A and $v(0^-) = 50i(0^-) = 200$ V. Since the current flowing through the inductor cannot change in zero time, $i(0^+) = i(0^-) = 4$ **A**. Since the voltage across the capacitor cannot change in zero time, $v(0^-) = v(0^-) = 200$ **V**.

(b)   For the inductor, $v_L = L(di/dt)$ or $di(0^+)/dt = v_L(0^+)/0.02$.

At $t = 0^+$, the right hand loop becomes,

$$-200 + 50 \text{x} 4 + v_L(0^+) = 0 \text{ or } v_L(0^+) = 0 \text{ and } (di(0^+)/dt) = \underline{\mathbf{0}}.$$

For the capacitor, $i_C = C(dv/dt)$ or $dv(0^+)/dt = i_C(0^+)/0.01$.

At $t = 0^+$, and looking at the current flowing out of the node at the top of the circuit,

$$((200-0)/20) + i_C + 4 = 0 \text{ or } i_C = -14 \text{ A.}$$

Therefore,
$$dv(0^+)/dt = -14/0.01 = \mathbf{-1.4 \ kV/s.}$$

(c)   When the switch is in position B, the circuit reaches steady state. Since it is source-free, i and v decay to zero with time.

Thus,
$$i(\infty) = \mathbf{0 \ A} \text{ and } v(\infty) = \mathbf{0 \ V}.$$

Chapter 8, Solution 55.

At the top node, writing a KCL equation produces,

$$i/4 + i = C_1 dv/dt, \quad C_1 = 0.1$$

$$5i/4 = C_1 dv/dt = 0.1 dv/dt$$

$$i = 0.08 dv/dt \tag{1}$$

But, $\qquad v = -(2i + (1/C_2)\int i dt), \quad C_2 = 0.5$

$$\text{or} \qquad -dv/dt = 2di/dt + 2i \tag{2}$$

Substituting (1) into (2) gives,

$$-dv/dt = 0.16 d^2v/dt^2 + 0.16 dv/dt$$

$$0.16 d^2v/dt^2 + 0.16 dv/dt + dv/dt = 0, \quad \text{or} \quad d^2v/dt^2 + 7.25 dv/dt = 0$$

Which leads to $s^2 + 7.25s = 0 = s(s + 7.25)$ or $s_{1,2} = 0, -7.25$

$$v(t) = A + Be^{-7.25t} \tag{3}$$

$$v(0) = 4 = A + B \tag{4}$$

From (1), $\qquad i(0) = 2 = 0.08 dv(0+)/dt$ or $dv(0+)/dt = 25$

But, $\quad dv/dt = -7.25Be^{-7.25t}$, which leads to,

$dv(0)/dt = -7.25B = 25$ or $B = -3.448$ and $A = 4 - B = 4 + 3.448 = 7.448$

Thus, $v(t) = \mathbf{\{7.448 - 3.448e^{-7.25t}\}\ V}$

Chapter 8, Solution 56.

For $t < 0$, $i(0) = 0$ and $v(0) = 0$.

For $t > 0$, the circuit is as shown below.
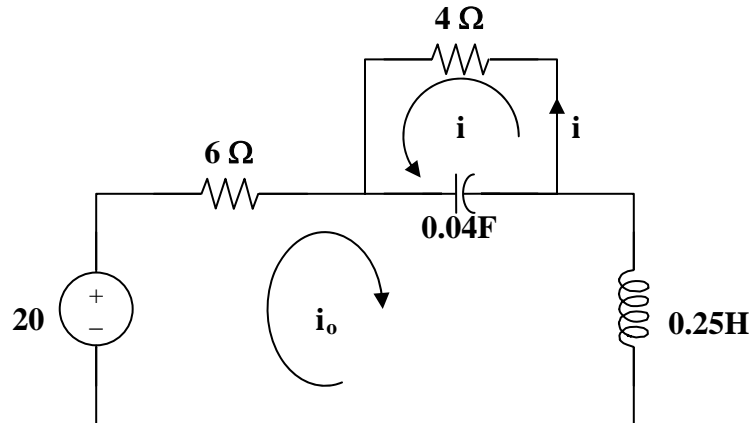
Applying KVL to the larger loop,

$$-20 + 6i_o + 0.25di_o/dt + 25\int(i_o + i)dt = 0$$

Taking the derivative,

$$6di_o/dt + 0.25d^2i_o/dt^2 + 25(i_o + i) = 0 \qquad (1)$$

For the smaller loop, $4 + 25\int(i + i_o)dt = 0$

Taking the derivative, $25(i + i_o) = 0$ or $i = -i_o$ \qquad (2)

From (1) and (2) $6di_o/dt + 0.25d^2i_o/dt^2 = 0$

This leads to, $0.25s^2 + 6s = 0$ or $s_{1,2} = 0, -24$

$\qquad i_o(t) = (A + Be^{-24t})$ and $i_o(0) = 0 = A + B$ or $B = -A$

As t approaches infinity, $i_o(\infty) = 20/10 = 2 = A$, therefore $B = -2$

Thus, $i_o(t) = (2 - 2e^{-24t}) = -i(t)$ or

$$i(t) = (-2 + 2e^{-24t}) \text{ A}$$

Chapter 8, Solution 57.

(a)   Let v = capacitor voltage and i = inductor current. At $t = 0-$, the switch is closed and the circuit has reached steady-state.

$$v(0-) = 16V \text{ and } i(0-) = 16/8 = 2A$$

At $t = 0+$, the switch is open but, $v(0+) = 16$ and $i(0+) = 2$.

We now have a source-free RLC circuit.

$$R \ 8 + 12 = 20 \text{ ohms, } L = 1H, \ C = 4mF.$$

$$\alpha = R/(2L) = (20)/(2x1) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x(1/36)} = 6$$

Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -18, -2$$

Thus, the characteristic equation is $(s + 2)(s + 18) = 0$ or $\mathbf{s^2 + 20s + 36 = 0}$.

(b) $i(t) = [Ae^{-2t} + Be^{-18t}]$ and $i(0) = 2 = A + B$ (1)

To get $di(0)/dt$, consider the circuit below at $t = 0+$.
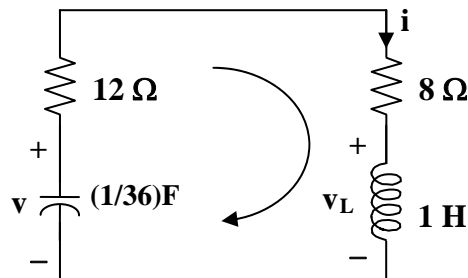
$-v(0) + 20i(0) + v_L(0) = 0$, which leads to,

$-16 + 20x2 + v_L(0) = 0$ or $v_L(0) = -24$

$Ldi(0)/dt = v_L(0)$ which gives $di(0)/dt = v_L(0)/L = -24/1 = -24$ A/s

Hence $-24 = -2A - 18B$ or $12 = A + 9B$ (2)

From (1) and (2), $B = 1.25$ and $A = 0.75$

$i(t) = [0.75e^{-2t} + 1.25e^{-18t}] = -i_x(t)$ or $i_x(t) = \mathbf{[-0.75e^{-2t} - 1.25e^{-18t}]}$ **A**

$v(t) = 8i(t) = \mathbf{[6e^{-2t} + 10e^{-18t}]}$ **A**

Chapter 8, Solution 58.

(a) Let i =inductor current, v = capacitor voltage i(0) =0, v(0) = 4

$$\frac{dv(0)}{dt} = -\frac{[v(0)+Ri(0)]}{RC} = -\frac{(4+0)}{0.5} = -8 \text{ V/s}$$

(b) For $t \geq 0$, the circuit is a source-free RLC parallel circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2x0.5x1} = 1, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2$$

$$\omega_d = \sqrt{\omega^2_o - \alpha^2} = \sqrt{4-1} = 1.732$$

Thus,

$$v(t) = e^{-t}(A_1 \cos 1.732t + A_2 \sin 1.732t)$$
$$v(0) = 4 = A_1$$

$$\frac{dv}{dt} = -e^{-t}A_1 \cos 1.732t - 1.732e^{-t}A_1 \sin 1.732t - e^{-t}A_2 \sin 1.732t + 1.732e^{-t}A_2 \cos 1.732t$$

$$\frac{dv(0)}{dt} = -8 = -A_1 + 1.732A_2 \qquad \longrightarrow \qquad A_2 = -2.309$$

$$v(t) = e^{-t}(4\cos 1.732t - 2.309\sin 1.732t) \text{ V}$$

Chapter 8, Solution 59.

Let i = inductor current and v = capacitor voltage
$v(0) = 0$, $i(0) = 40/(4+16) = 2A$
For t>0, the circuit becomes a source-free series RLC with

$$\alpha = \frac{R}{2L} = \frac{16}{2x4} = 2, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4x1/16}} = 2, \qquad \longrightarrow \qquad \alpha = \omega_o = 2$$

$$i(t) = Ae^{-2t} + Bte^{-2t}$$

$$i(0) = 2 = A$$

$$\frac{di}{dt} = -2Ae^{-2t} + Be^{-2t} - 2Bte^{-2t}$$

$$\frac{di(0)}{dt} = -2A + B = -\frac{1}{L}[Ri(0) - v(0)] \qquad \longrightarrow \qquad -2A + B = -\frac{1}{4}(32 - 0), \qquad B = -4$$

$$i(t) = 2e^{-2t} - 4te^{-2t}$$

$$v = \frac{1}{C}\int_0^t -i\,dt + v(0) = -32\int_0^t e^{-2t}dt + 64\int_0^t te^{-2t}dt = +16e^{-2t}\,\Big|_0^t + \frac{64}{4}e^{-2t}(-2t-1)\,\Big|_0^t$$

$v = \mathbf{-32te^{-2t}}$ **V**.

Checking,
$$v = Ldi/dt + Ri = 4(-4e^{-2t} - 4e^{-2t} + 8e^{-2t}) + 16(2e^{-2t} - 4te^{-2t}) = -32te^{-2t} \text{ V}.$$

Chapter 8, Solution 60.

At $t = 0-$, $4u(t) = 0$ so that $i_1(0) = 0 = i_2(0)$ (1)

Applying nodal analysis,

$$4 = 0.5di_1/dt + i_1 + i_2 \quad (2)$$

Also, $i_2 = [1di_1/dt - 1di_2/dt]/3$ or $3i_2 = di_1/dt - di_2/dt$ (3)

Taking the derivative of (2), $0 = d^2i_1/dt^2 + 2di_1/dt + 2di_2/dt$ (4)

From (2) and (3), $di_2/dt = di_1/dt - 3i_2 = di_1/dt - 3(4 - i_1 - 0.5di_1/dt)$

$$= di_1/dt - 12 + 3i_1 + 1.5di_1/dt$$

Substituting this into (4),

$d^2i_1/dt^2 + 7di_1/dt + 6i_1 = 24$ which gives $s^2 + 7s + 6 = 0 = (s + 1)(s + 6)$

Thus, $i_1(t) = I_s + [Ae^{-t} + Be^{-6t}]$, $6I_s = 24$ or $I_s = 4$

$i_1(t) = 4 + [Ae^{-t} + Be^{-6t}]$ and $i_1(0) = 4 + [A + B]$ (5)

$i_2 = 4 - i_1 - 0.5di_1/dt = i_1(t) = 4 + -4 - [Ae^{-t} + Be^{-6t}] - [-Ae^{-t} - 6Be^{-6t}]$

$= [-0.5Ae^{-t} + 2Be^{-6t}]$ and $i_2(0) = 0 = -0.5A + 2B$ (6)

From (5) and (6), $A = -3.2$ and $B = -0.8$

$$i_1(t) = \{4 + [-3.2e^{-t} - 0.8e^{-6t}]\} \text{ A}$$

$$i_2(t) = [1.6e^{-t} - 1.6e^{-6t}] \text{ A}$$

Chapter 8, Solution 61.

For $t > 0$, we obtain the natural response by considering the circuit below.
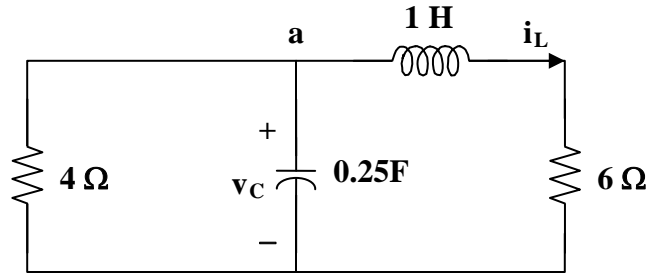
At node a, $v_C/4 + 0.25dv_C/dt + i_L = 0$ (1)

But, $v_C = 1di_L/dt + 6i_L$ (2)

Combining (1) and (2),

$$(1/4)di_L/dt + (6/4)i_L + 0.25d^2i_L/dt^2 + (6/4)di_L/dt + i_L = 0$$

$$d^2i_L/dt^2 + 7di_L/dt + 10i_L = 0$$

$$s^2 + 7s + 10 = 0 = (s + 2)(s + 5) \text{ or } s_{1,2} = -2, -5$$

Thus, $i_L(t) = i_L(\infty) + [Ae^{-2t} + Be^{-5t}]$,

where $i_L(\infty)$ represents the final inductor current $= 4(4)/(4 + 6) = 1.6$

$i_L(t) = 1.6 + [Ae^{-2t} + Be^{-5t}]$ and $i_L(0) = 1.6 + [A+B]$ or $-1.6 = A+B$ (3)

$$di_L/dt = [-2Ae^{-2t} - 5Be^{-5t}]$$

and $di_L(0)/dt = 0 = -2A - 5B$ or $A = -2.5B$ (4)

From (3) and (4), $A = -8/3$ and $B = 16/15$

$$i_L(t) = 1.6 + [-(8/3)e^{-2t} + (16/15)e^{-5t}]$$

$$v(t) = 6i_L(t) = \{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\} \text{ V}$$

$$v_C = 1di_L/dt + 6i_L = [(16/3)e^{-2t} - (16/3)e^{-5t}] + \{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\}$$

$$v_C = \{9.6 + [-(32/3)e^{-2t} + 1.0667e^{-5t}]\}$$

$$i(t) = v_C/4 = \{2.4 + [-2.667e^{-2t} + 0.2667e^{-5t}]\} \text{ A}$$

Chapter 8, Solution 62.

This is a parallel RLC circuit as evident when the voltage source is turned off.

$$\alpha \; = \; 1/(2RC) \; = \; (1)/(2 \mathrm{x} 3 \mathrm{x}(1/18)) \; = \; 3$$

$$\omega_o = 1/\sqrt{LC} \; = \; 1/\sqrt{2 \mathrm{x} 1/18} \; = \; 3$$

Since $\alpha \; = \; \omega_o$, we have a critically damped response.

$$s_{1,2} \; = \; -3$$

Let $v(t) \; = \;$ capacitor voltage

Thus, $v(t) \; = \; V_s + [(A + Bt)e^{-3t}]$ where $V_s \; = \; 0$

But $-10 + v_R + v \; = \; 0$ or $v_R \; = \; 10 - v$

Therefore $v_R \; = \; \mathbf{10 - [(A + Bt)e^{-3t}]}$ where A and B are determined from initial conditions.

Chapter 8, Solution 63.

$$\frac{v_s - 0}{R} = C\frac{d(0 - v_o)}{dt} \longrightarrow \frac{v_s}{R} = -C\frac{dv_o}{dt}$$

$$v_o = L\frac{di}{dt} \longrightarrow \frac{dv_o}{dt} = L\frac{d^2 i}{dt^2} = -\frac{v_s}{RC}$$

Thus,

$$\frac{d^2 i(t)}{dt^2} = -\frac{v_s}{RCL}$$

Chapter 8, Solution 64.

Using Fig. 8.109, design a problem to help other students to better understand second-order op amp circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain the differential equation for $v_o(t)$ in the network of Fig. 8.109.
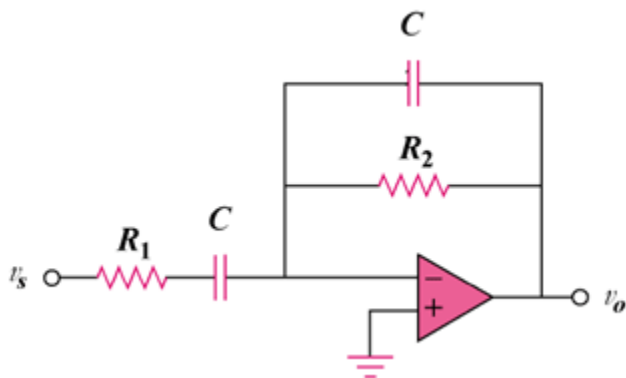
Figure 8.109

Solution
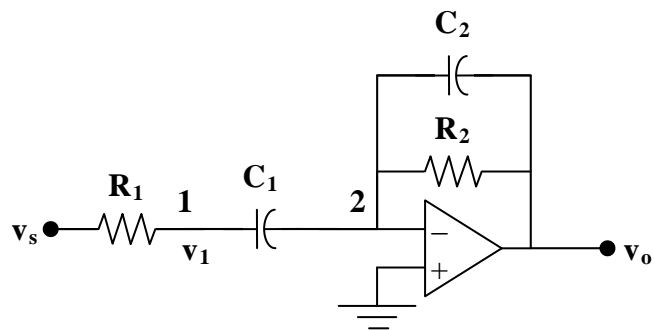
At node 1, $(v_s - v_1)/R_1 = C_1 \, d(v_1 - 0)/dt$ or $v_s = v_1 + R_1C_1dv_1/dt$ (1)

At node 2, $C_1dv_1/dt = (0 - v_o)/R_2 + C_2d(0 - v_o)/dt$

   or $-R_2C_1dv_1/dt = v_o + R_2C_2dv_o/dt$ (2)

From (1) and (2), $(v_s - v_1)/R_1 = C_1 \, dv_1/dt = -(1/R_2)(v_o + R_2C_2dv_o/dt)$

   or $v_1 = v_s + (R_1/R_2)(v_o + R_2C_2dv_o/dt)$ (3)

Substituting (3) into (1) produces,

$$v_s = v_s + (R_1/R_2)(v_o + R_2C_2dv_o/dt) + R_1C_1d\{v_s + (R_1/R_2)(v_o + R_2C_2dv_o/dt)\}/dt$$

$$= v_s + (R_1/R_2)(v_o) + (R_1C_2)dv_o/dt + R_1C_1dv_s/dt + (R_1R_1C_1/R_2)dv_o/dt$$
$$+ ((R_1)^2 C_1C_2)[d^2v_o/dt^2]$$

$$((R_1)^2 C_1C_2)[d^2v_o/dt^2] + [(R_1C_2) + (R_1R_1C_1/R_2)]dv_o/dt + (R_1/R_2)(v_o) = -R_1C_1dv_s/dt$$

Simplifying we get,

$$[d^2v_o/dt^2] + \{[(R_1C_2) + (R_1R_1C_1/R_2)]/((R_1)^2 C_1C_2)\}dv_o/dt + \{(R_1/R_2)(v_o)/((R_1)^2 C_1C_2)\} = -\{R_1C_1 /((R_1)^2 C_1C_2)\}dv_s/dt$$

$$\mathbf{d^2v_o/dt^2 + [(1/R_1C_1) + (1/(R_2C_2))]dv_o/dt + [1/(R_1R_2C_1C_2)](v_o) = -[1/(R_1C_2)]dv_s/dt}$$

Another way to successfully work this problem is to give actual values of the resistors and capacitors and determine the actual differential equation. Alternatively, one could give a differential equations and ask the other students to choose actual value of the differential equation.

Chapter 8, Solution 65.

At the input of the first op amp,

$$(v_o - 0)/R \ = \ Cd(v_1 - 0) \tag{1}$$

At the input of the second op amp,

$$(-v_1 - 0)/R \ = \ Cdv_2/dt \tag{2}$$

Let us now examine our constraints. Since the input terminals are essentially at ground, then we have the following,

$$v_o \ = \ -v_2 \ \text{ or } \ v_2 \ = \ -v_o \tag{3}$$

Combining (1), (2), and (3), eliminating v_1 and v_2 we get,

$$\frac{d^2 v_o}{dt^2} - \left(\frac{1}{R^2 C^2}\right) v_o = \frac{d^2 v_o}{dt^2} - 100 v_o = 0$$

Which leads to $s^2 - 100 \ = \ 0$

Clearly this produces roots of -10 and $+10$.

And, we obtain,

$$v_o(t) \ = \ (Ae^{+10t} + Be^{-10t})V$$

At $t \ = \ 0$, $v_o(0+) \ = \ -v_2(0+) \ = \ 0 \ = \ A + B$, thus $B \ = \ -A$

This leads to $v_o(t) \ = \ (Ae^{+10t} - Ae^{-10t})V$. Now we can use $v_1(0+) \ = \ 2V$.

From (2), $v_1 \ = \ -RCdv_2/dt \ = \ 0.1 dv_o/dt \ = \ 0.1(10Ae^{+10t} + 10Ae^{-10t})$

$$v_1(0+) \ = \ 2 \ = \ 0.1(20A) \ = \ 2A \ \text{ or } \ A \ = \ 1$$

Thus, $\ v_o(t) \ = \ (e^{+10t} - e^{-10t}) \ \mathbf{V}$

It should be noted that this circuit is unstable (clearly one of the poles lies in the right-half-plane).

Chapter 8, Solution 66.

We apply nodal analysis to the circuit as shown below.

v_2

10pF

60 kΩ
v_1

60 kΩ
v_2

20 pF

v_o

v_s

+
−

+

−

+

+

−
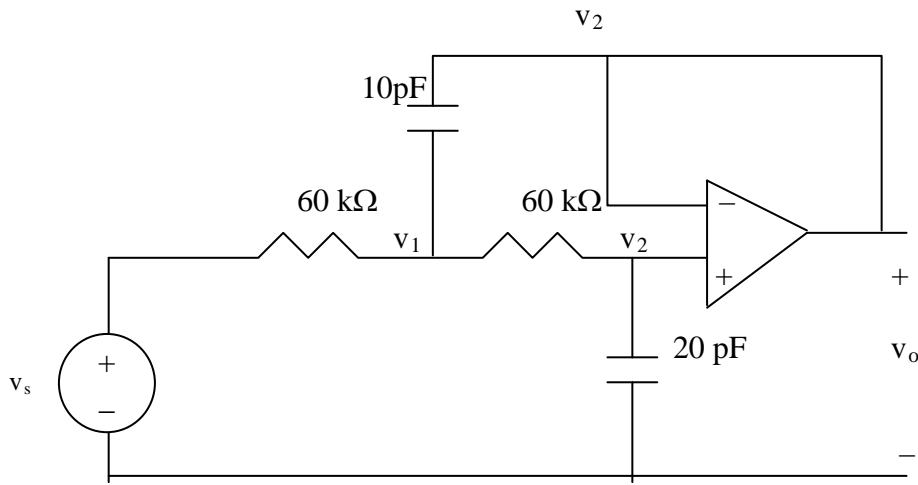
At node 1,

$$\frac{v_s - v_1}{60k} = \frac{v_1 - v_2}{60k} + 10pF\frac{d}{dt}(v_1 - v_o)$$

But $v_2 = v_o$

$$v_s = 2v_1 - v_o + 6\text{x}10^{-7}\frac{d(v_1 - v_o)}{dt} \qquad (1)$$

At node 2,

$$\frac{v_1 - v_2}{60k} = 20pF\frac{d}{dt}(v_2 - 0), \qquad v_2 = v_o$$

$$v_1 = v_o + 1.2\text{x}10^{-6}\frac{dv_o}{dt} \qquad (2)$$

Substituting (2) into (1) gives

$$v_s = 2\left(v_o + 1.2\text{x}10^{-6}\frac{dv_o}{dt}\right) - v_o + 6\text{x}10^{-7}\left(1.2\text{x}10^{-6}\frac{d^2v_o}{dt^2}\right)$$

$$\mathbf{v_s = v_o + 2.4x10^{-6}(dv_o/dt) + 7.2x10^{-13}(d^2v_o/dt^2).}$$

Chapter 8, Solution 67.

At node 1,

$$\frac{v_{in} - v_1}{R_1} = C_1 \frac{d(v_1 - v_o)}{dt} + C_2 \frac{d(v_1 - 0)}{dt} \qquad (1)$$

At node 2, $\qquad C_2 \frac{d(v_1 - 0)}{dt} = \frac{0 - v_o}{R_2}, \quad$ or $\quad \frac{dv_1}{dt} = \frac{-v_o}{C_2 R_2} \qquad (2)$

From (1) and (2),

$$v_{in} - v_1 = -\frac{R_1 C_1}{C_2 R_2} \frac{dv_o}{dt} - R_1 C_1 \frac{dv_o}{dt} - R_1 \frac{v_o}{R_2}$$

$$v_1 = v_{in} + \frac{R_1 C_1}{C_2 R_2} \frac{dv_o}{dt} + R_1 C_1 \frac{dv_o}{dt} + R_1 \frac{v_o}{R_2} \qquad (3)$$
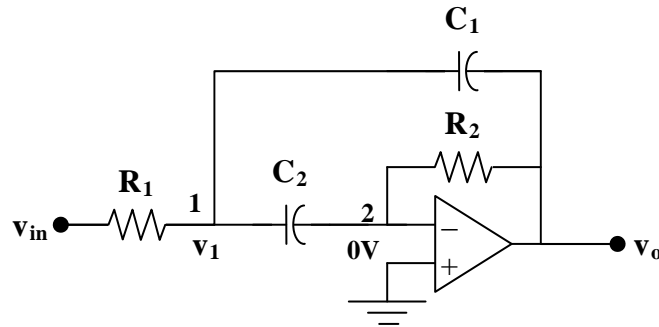
From (2) and (3),

$$-\frac{v_o}{C_2 R_2} = \frac{dv_1}{dt} = \frac{dv_{in}}{dt} + \frac{R_1 C_1}{C_2 R_2} \frac{dv_o}{dt} + R_1 C_1 \frac{d^2 v_o}{dt^2} + \frac{R_1}{R_2} \frac{dv_o}{dt}$$

$$\frac{d^2 v_o}{dt^2} + \frac{1}{R_2} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \frac{dv_o}{dt} + \frac{v_o}{C_1 C_2 R_2 R_1} = -\frac{1}{R_1 C_1} \frac{dv_{in}}{dt}$$

But $C_1 C_2 R_1 R_2 = 10^{-4} \times 10^{-4} \times 10^4 \times 10^4 = 1$

$$\frac{1}{R_2} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) = \frac{2}{R_2 C_1} = \frac{2}{10^4 \times 10^{-4}} = 2$$

$$\frac{d^2 v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = -\frac{dv_{in}}{dt}$$

Which leads to $s^2 + 2s + 1 = 0$ or $(s+1)^2 = 0$ and $s = -1, -1$

Therefore, $\quad v_o(t) = [(A + Bt)e^{-t}] + V_f$

As t approaches infinity, the capacitor acts like an open circuit so that

$$V_f = v_o(\infty) = 0$$

$v_{in} = 10u(t)$ mV and the fact that the initial voltages across each capacitor is 0

means that $v_o(0) = 0$ which leads to $A = 0$.

$$v_o(t) = [Bte^{-t}]$$

$$\frac{dv_o}{dt} = [(B - Bt)e^{-t}] \qquad\qquad (4)$$

From (2), $\qquad\qquad \frac{dv_o(0+)}{dt} = -\frac{v_o(0+)}{C_2 R_2} = 0$

From (1) at t = 0+,

$$\frac{1-0}{R_1} = -C_1 \frac{dv_o(0+)}{dt} \quad \text{which leads to} \quad \frac{dv_o(0+)}{dt} = -\frac{1}{C_1 R_1} = -1$$

Substituting this into (4) gives $B = -1$

Thus, $\qquad\qquad v(t) = -te^{-t}u(t)$ V

Chapter 8, Solution 68.

The schematic is as shown below. The unit step is modeled by VPWL as shown.   We insert a voltage marker to display V after simulation. We set Print Step = 25 ms and final step = 6s in the transient box. The output plot is shown below.
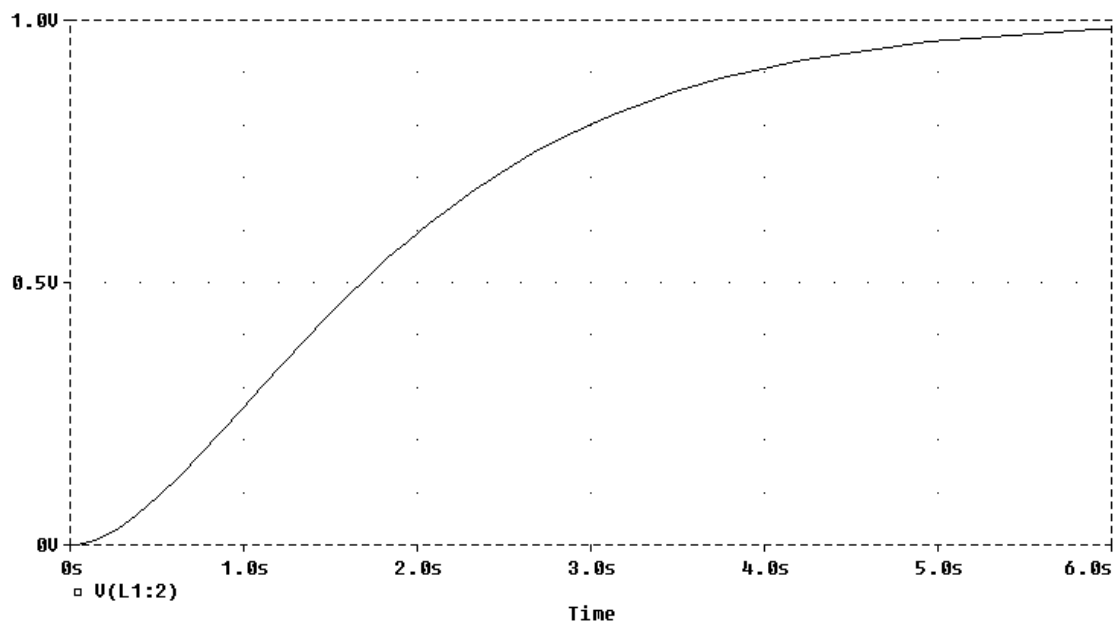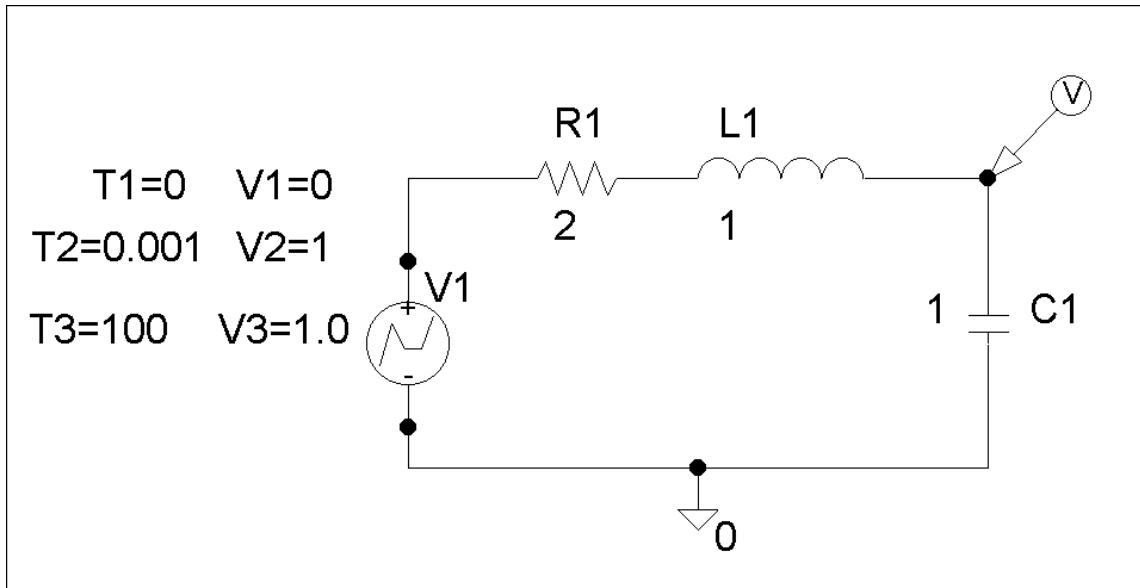
Chapter 8, Solution 69.

The schematic is shown below. The initial values are set as attributes of L1 and C1. We set Print Step to 25 ms and the Final Time to 20s in the transient box. A current marker is inserted at the terminal of L1 to automatically display i(t) after simulation. The result is shown below.
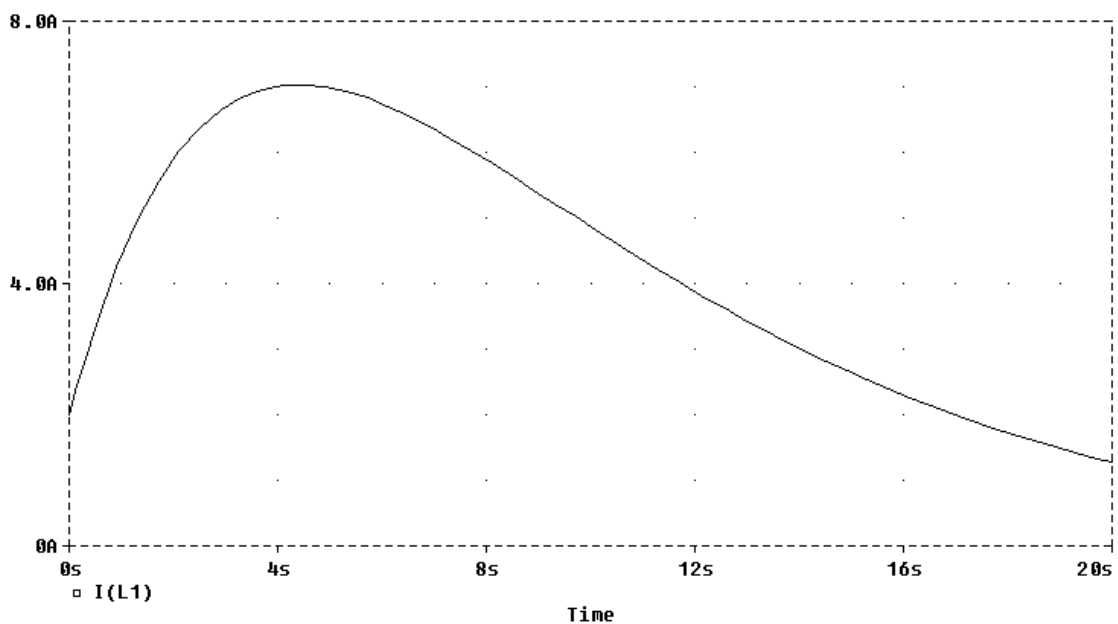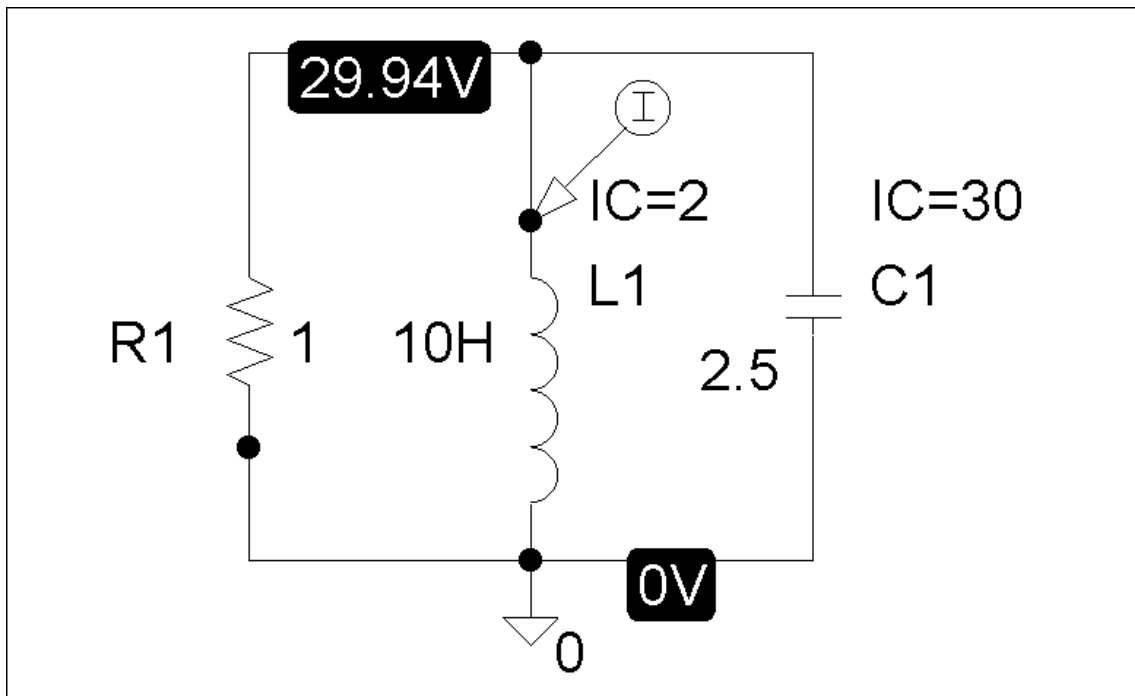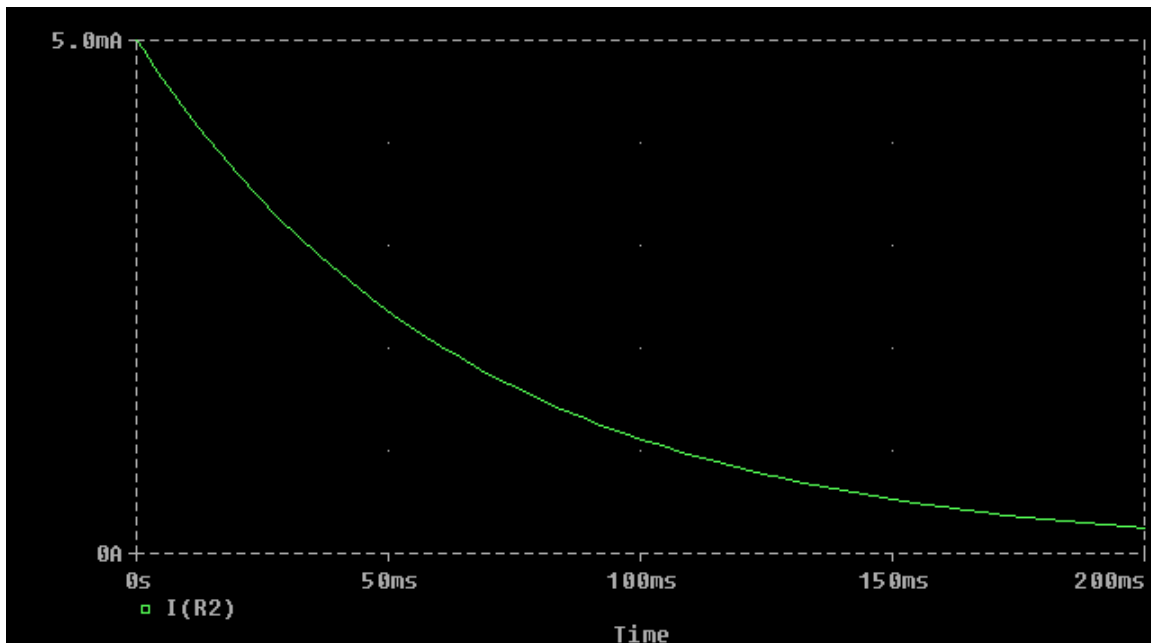
Chapter 8, Solution 70.

The schematic is shown below.
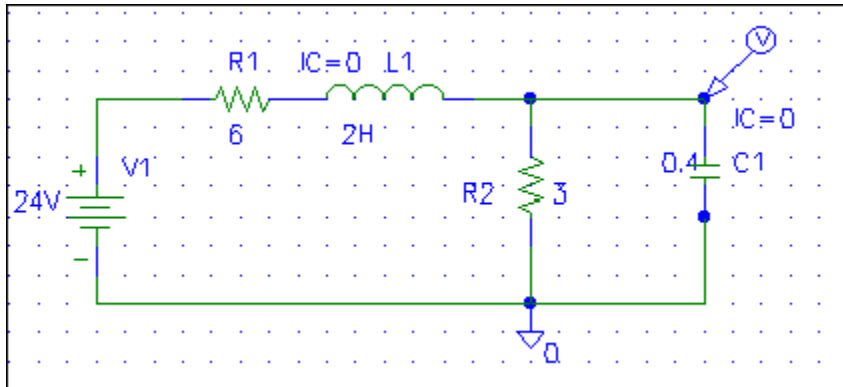
After the circuit is saved and simulated, we obtain the capacitor voltage v(t) as shown below.
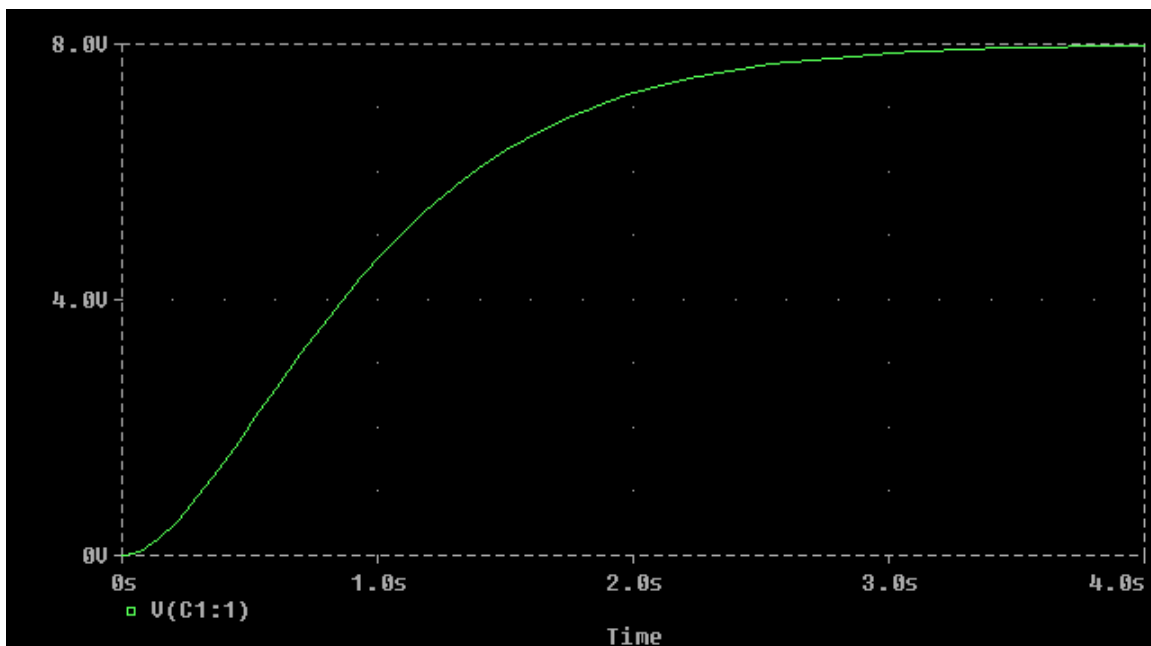
Chapter 8, Solution 71.

The schematic is shown below. We use VPWL and IPWL to model the 39 u(t) V and 13 u(t) A respectively. We set Print Step to 25 ms and Final Step to 4s in the Transient box. A voltage marker is inserted at the terminal of R2 to automatically produce the plot of v(t) after simulation. The result is shown below.
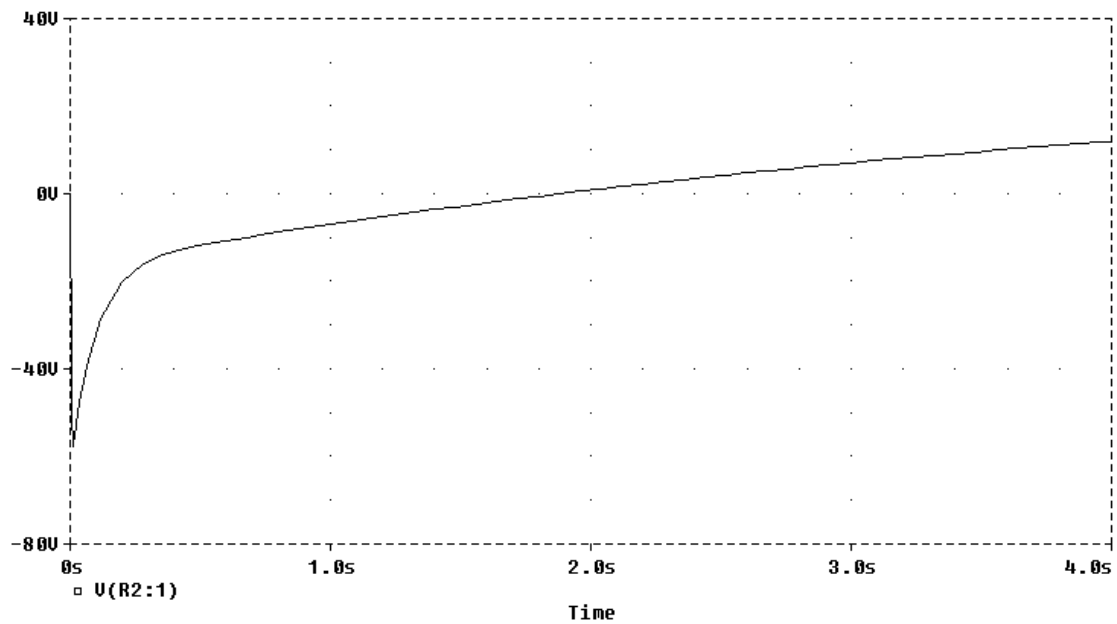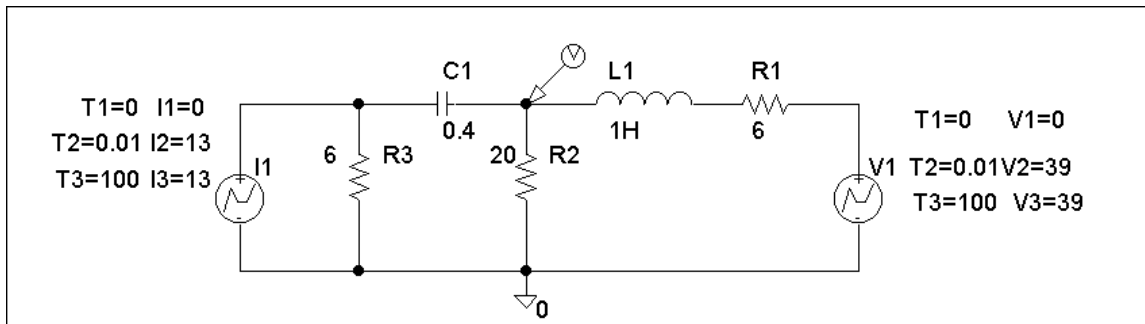
Chapter 8, Solution 72.

When the switch is in position 1, we obtain IC=10 for the capacitor and IC=0 for the inductor.   When the switch is in position 2, the schematic of the circuit is shown below.
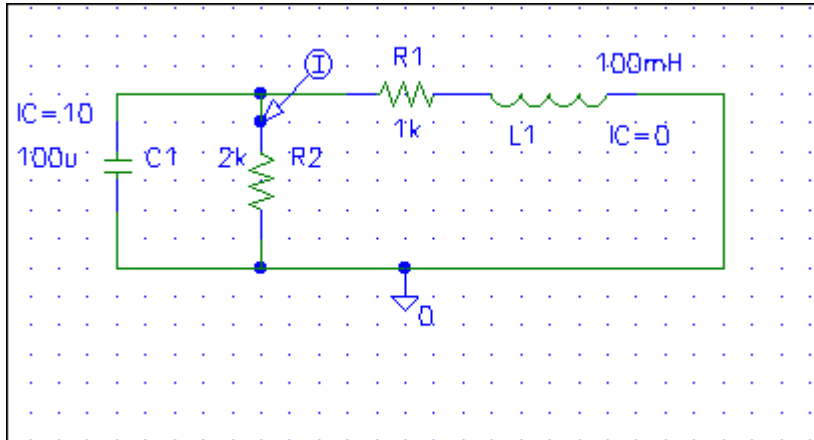
When the circuit is simulated, we obtain i(t) as shown below.

Chapter 8, Solution 73.

Design a problem, using PSpice, to help other students to better understand source-free *RLC* circuits.

Although there are many ways to work this problem, this is an example based on a somewhat similar problem worked in the third edition.

Problem

The step response of an *RLC* circuit is given by

$$\frac{d^2 i_L}{dt^2} + 0.5 \frac{d i_L}{dt} + 4 i_L = 0$$

Given that $i_L(0) = 3$ A and $v_C(0) = 24$ V, solve for $v_C(t)$ and $I_C(t)$.

Solution

(a)   For t < 0, we have the schematic below. When this is saved and simulated, we obtain the initial inductor current and capacitor voltage as

$$i_L(0) = 3 \text{ A} \quad \text{and} \quad v_c(0) = 24 \text{ V}.$$
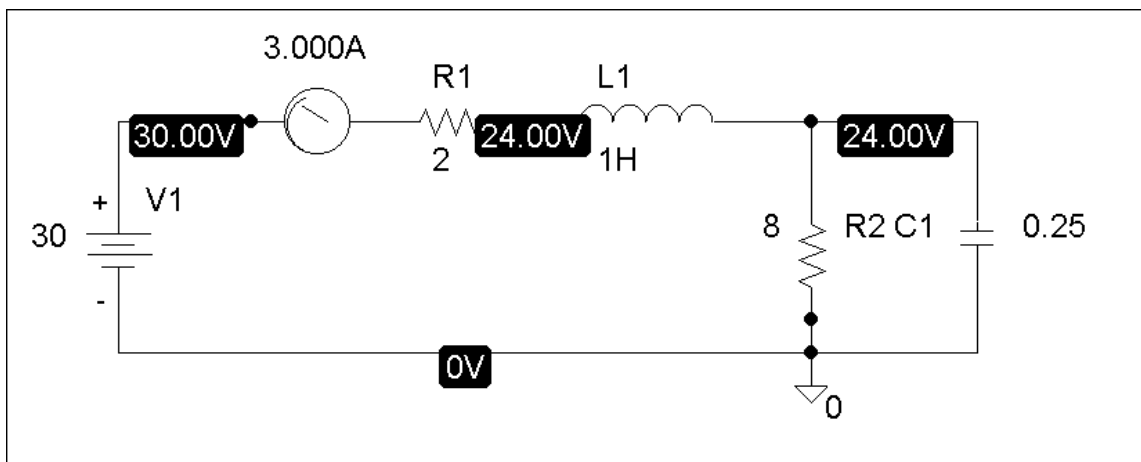
(b)   For t > 0, we have the schematic shown below. To display i(t) and v(t), we insert current and voltage markers as shown. The initial inductor current and capacitor voltage are also

incorporated. In the Transient box, we set Print Step $=$ 25 ms and the Final Time to 4s. After simulation, we automatically have $i_o(t)$ and $v_o(t)$ displayed as shown below.
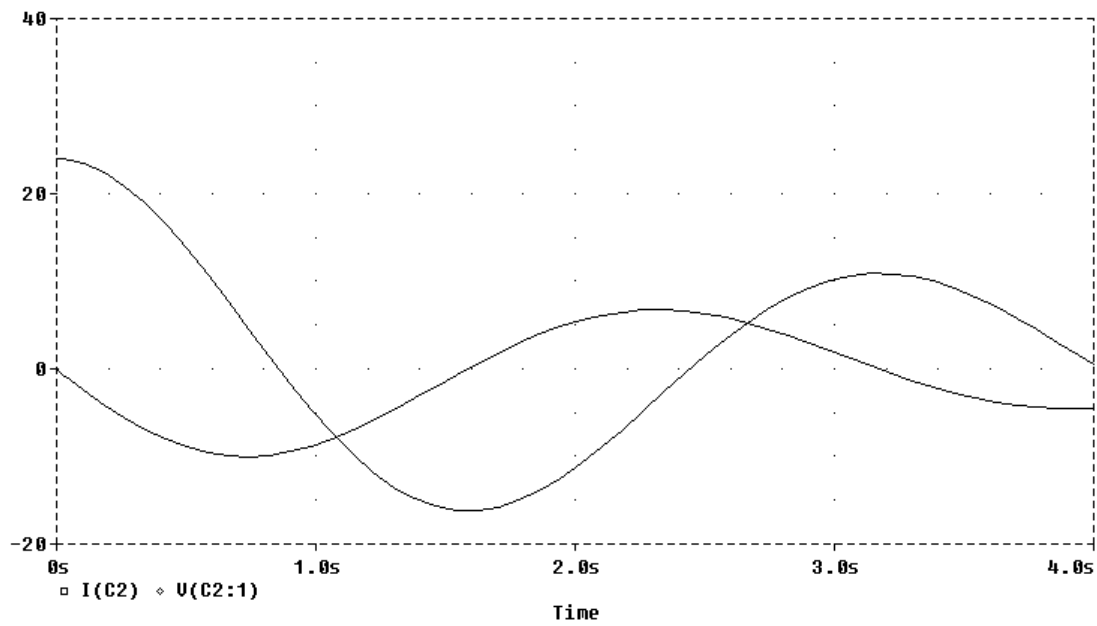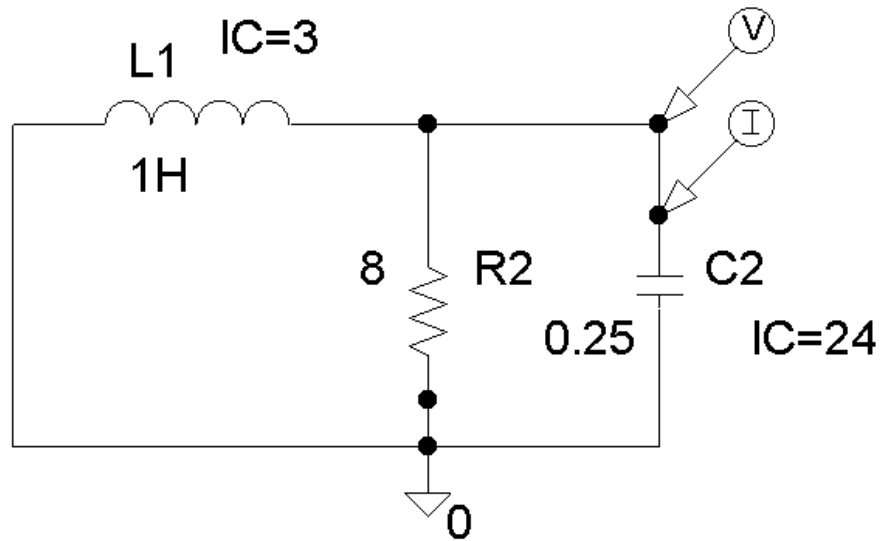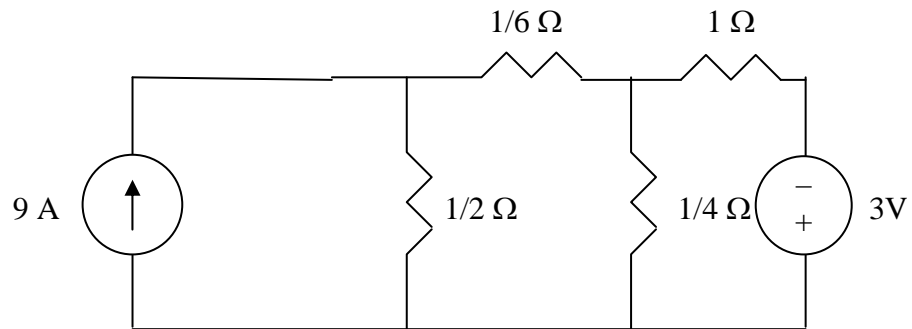
Chapter 8, Solution 74.
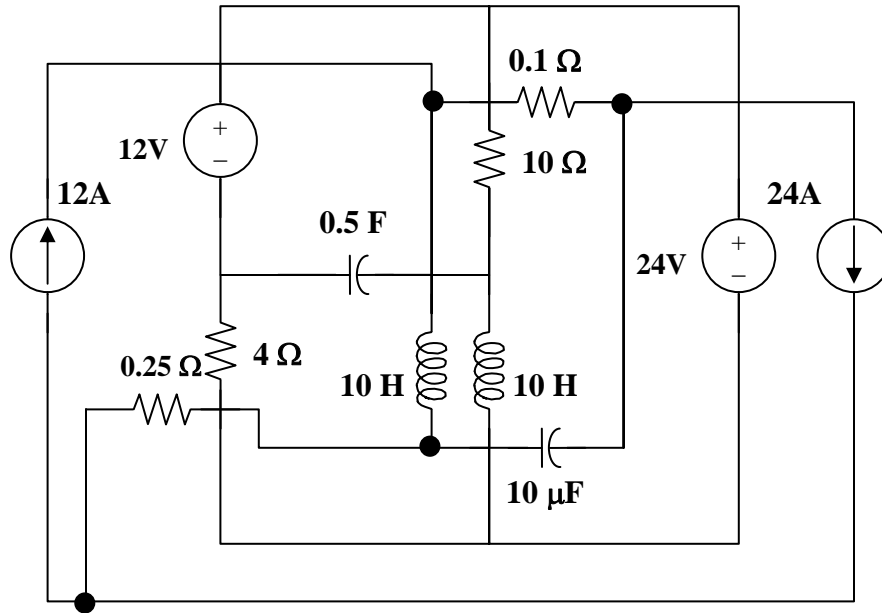
The dual is constructed as shown below.

0.5 Ω   0.25 Ω

2 Ω   4 Ω

1/6 Ω

9 V   +
      −

6 Ω   1 Ω   3A

1 Ω

9 A

3V
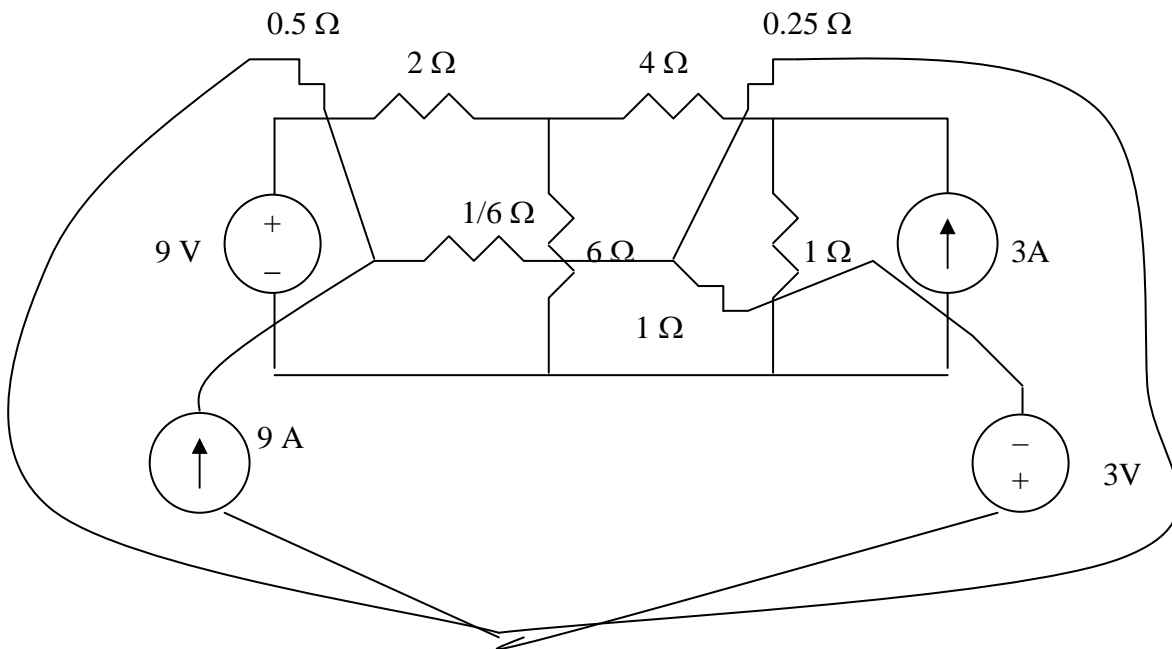
The dual is redrawn as shown below.

1/6 Ω   1 Ω

9 A   1/2 Ω   1/4 Ω   −   3V
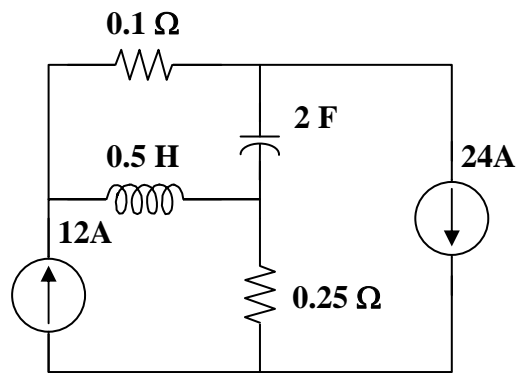   +

Chapter 8, Solution 75.

The dual circuit is connected as shown in Figure (a). It is redrawn in Figure (b).
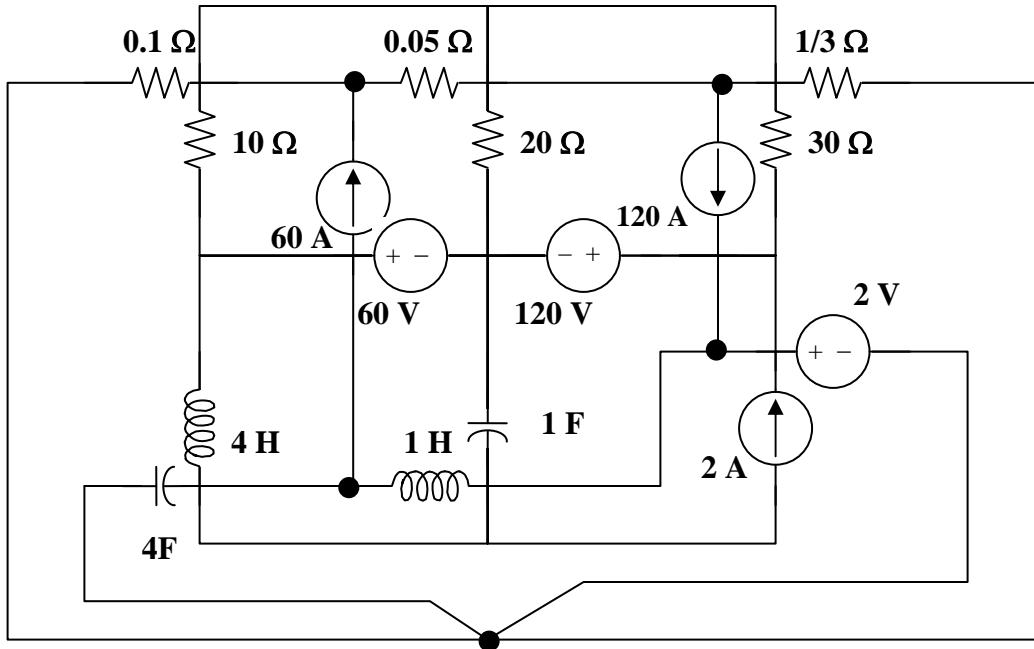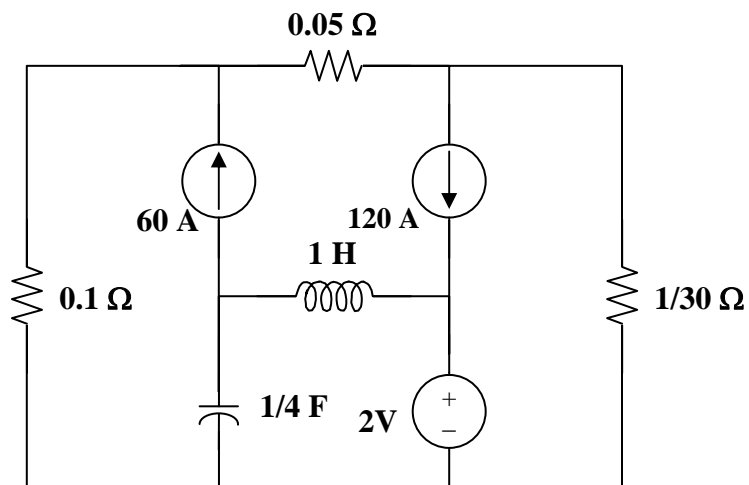

(a)

(b)

Chapter 8, Solution 77.

The dual is obtained from the original circuit as shown in Figure (a). It is redrawn in Figure (b).
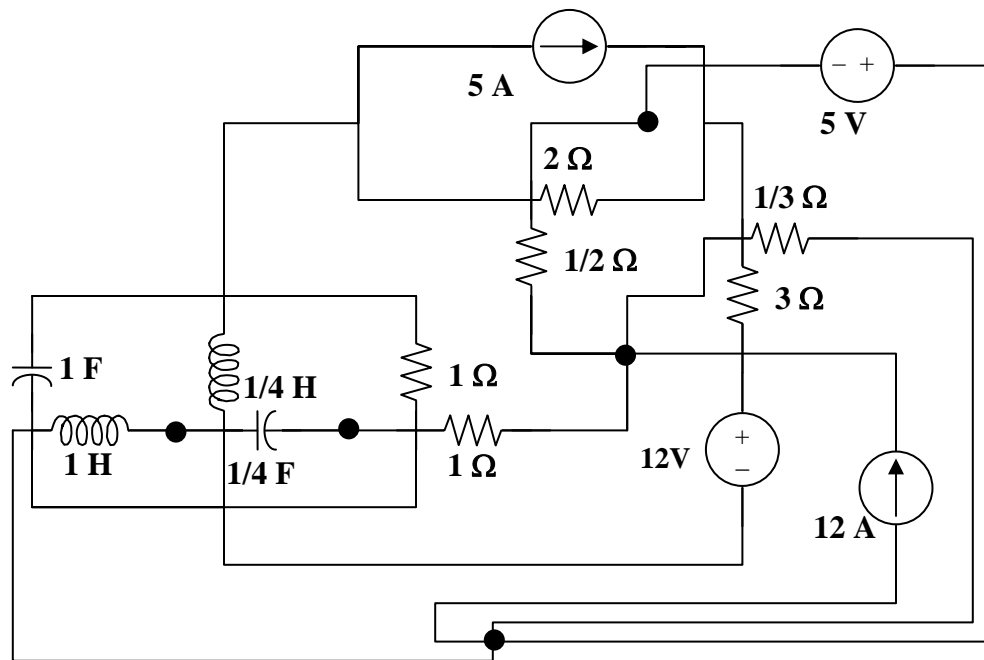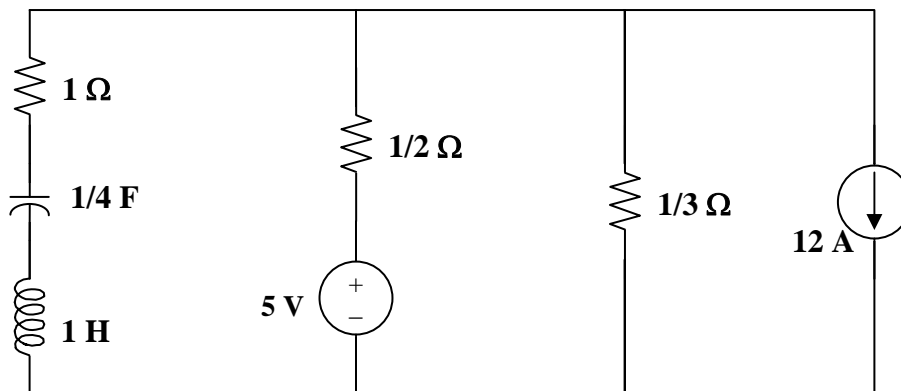

(a)

(b)

Chapter 8, Solution 77.

The dual is constructed in Figure (a) and redrawn in Figure (b).

(a)

(b)

Chapter 8, Solution 78.

The voltage across the igniter is $v_R = v_C$ since the circuit is a parallel RLC type.

$$v_C(0) = 12, \text{ and } i_L(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2\text{x}3\text{x}1/30) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{60\text{x}10^{-3}\text{x}1/30} = 22.36$$

$\alpha < \omega_o$ produces an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm j21.794$$

$$v_C(t) = e^{-5t}(A\cos21.794t + B\sin21.794t) \tag{1}$$

$$v_C(0) = 12 = A$$

$$dv_C/dt = -5[(A\cos21.794t + B\sin21.794t)e^{-5t}]$$

$$+ 21.794[(-A\sin21.794t + B\cos21.794t)e^{-5t}] \tag{2}$$

$$dv_C(0)/dt = -5A + 21.794B$$

But, $dv_C(0)/dt = -[v_C(0) + Ri_L(0)]/(RC) = -(12 + 0)/(1/10) = -120$

Hence, $-120 = -5A + 21.794B$, leads to B $(5\text{x}12 - 120)/21.794 = -2.753$

At the peak value, $dv_C(t_o)/dt = 0$, i.e.,

$$0 = A + B\tan21.794t_o + (A21.794/5)\tan21.794t_o - 21.794B/5$$

$$(B + A21.794/5)\tan21.794t_o = (21.794B/5) - A$$

$$\tan21.794t_o = [(21.794B/5) - A]/(B + A21.794/5) = -24/49.55 = -0.484$$

Therefore, $21.7945t_o = |-0.451|$

$$t_o = |-0.451|/21.794 = \textbf{20.68 ms}$$

Chapter 8, Solution 79.

For critical damping of a parallel RLC circuit,

$$\alpha = \omega_o \qquad \longrightarrow \qquad \frac{1}{2RC} = \frac{1}{\sqrt{LC}}$$

Hence,

$$C = \frac{L}{4R^2} = \frac{0.25}{4x144} = \underline{\underline{434\mu F}}$$

Chapter 8, Solution 80.

$$t_1 = 1/|s_1| = 0.1 \times 10^{-3} \text{ leads to } s_1 = -1000/0.1 = -10,000$$

$$t_2 = 1/|s_2| = 0.5 \times 10^{-3} \text{ leads to } s_1 = -2,000$$

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}$$

$$s_1 + s_2 = -2\alpha = -12,000, \text{ therefore } \alpha = 6,000 = R/(2L)$$

$$L = R/12,000 = 50,000/12,000 = \textbf{4.167H}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -2,000$$

$$\alpha - \sqrt{\alpha^2 - \omega_o^2} = 2,000$$

$$6,000 - \sqrt{\alpha^2 - \omega_o^2} = 2,000$$

$$\sqrt{\alpha^2 - \omega_o^2} = 4,000$$

$$\alpha^2 - \omega_o^2 = 16 \times 10^6$$

$$\omega_o^2 = \alpha^2 - 16 \times 10^6 = 36 \times 10^6 - 16 \times 10^6$$

$$\omega_o = 10^3 \sqrt{20} = 1/\sqrt{LC}$$

$$C = 1/(20 \times 10^6 \times 4.167) = \textbf{12 nF}$$

Chapter 8, Solution 81.

$$t = 1/\alpha = 0.25 \text{ leads to } \alpha = 4$$

But, $\alpha \ 1/(2RC)$ or, $C = 1/(2\alpha R) = 1/(2 \times 4 \times 200) = \textbf{625 } \boldsymbol{\mu}\textbf{F}$

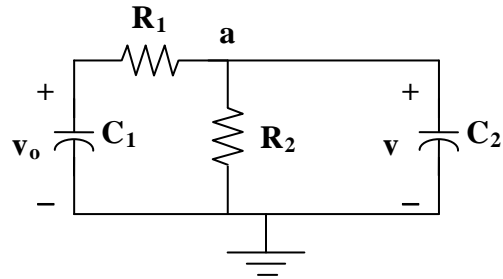$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = (2\pi 4 \times 10^3)^2 + 16 \cong (2\pi 4 \times 10^3 0^2 = 1/(LC)$$

This results in $L = 1/(64\pi^2 \times 10^6 \times 625 \times 10^{-6}) = \textbf{2.533 } \boldsymbol{\mu}\textbf{H}$

Chapter 8, Solution 82.

For t = 0-, v(0) = 0.

For t > 0, the circuit is as shown below.

At node a,

$$(v_o - v/R_1 = (v/R_2) + C_2 dv/dt$$

$$v_o = v(1 + R_1/R_2) + R_1 C_2\ dv/dt$$

$$60 = (1 + 5/2.5) + (5x10^6\ x5x10^{-6})dv/dt$$

$$60 = 3v + 25dv/dt$$

$$v(t) = V_s + [Ae^{-3t/25}]$$

where $3V_s = 60$ yields $V_s = 20$

$$v(0) = 0 = 20 + A\ \text{or}\ A = -20$$

$$v(t) = \mathbf{20(1 - e^{-3t/25})V}$$

Chapter 8, Solution 83.

$$i = i_D + Cdv/dt \qquad\qquad (1)$$

$$-v_s + iR + Ldi/dt + v = 0 \qquad\qquad (2)$$

Substituting (1) into (2),

$$v_s = Ri_D + RCdv/dt + Ldi_D/dt + LCd^2v/dt^2 + v = 0$$

$$LCd^2v/dt^2 + RCdv/dt + Ri_D + Ldi_D/dt = v_s$$

$$\mathbf{d^2v/dt^2 + (R/L)dv/dt + (R/LC)i_D + (1/C)di_D/dt = v_s/LC}$$