

Chapter 9, Solution 1.

(a) $V_m = \mathbf{50\text{ V}}$.

(b) Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{30} = \underline{0.2094_s} = \mathbf{209.4ms}$

(c) Frequency $f = \omega/(2\pi) = 30/(2\pi) = \mathbf{4.775\text{ Hz}}$.

(d) At $t=1ms$, $v(0.01) = 50\cos(30 \times 0.01\text{rad} + 10^\circ)$
 $= 50\cos(1.72^\circ + 10^\circ) = \mathbf{44.48\text{ V}}$ and $\omega t = \mathbf{0.3\text{ rad}}$.

Chapter 9, Solution 2.

(a) amplitude = **15 A**

(b) $\omega = 25\pi = \mathbf{78.54 \text{ rad/s}}$

(c) $f = \frac{\omega}{2\pi} = \mathbf{12.5 \text{ Hz}}$

(d) $I_s = 15\angle 25^\circ \text{ A}$
 $I_s(2 \text{ ms}) = 15 \cos((500\pi)(2 \times 10^{-3}) + 25^\circ)$
 $= 15 \cos(\pi + 25^\circ) = 15 \cos(205^\circ)$
 $= \mathbf{-13.595 \text{ A}}$

Chapter 9, Solution 3.

$$(a) \quad 10 \sin(\omega t + 30^\circ) = 10 \cos(\omega t + 30^\circ - 90^\circ) = \mathbf{10\cos(\omega t - 60^\circ)}$$

$$(b) \quad -9 \sin(8t) = \mathbf{9\cos(8t + 90^\circ)}$$

$$(c) \quad -20 \sin(\omega t + 45^\circ) = 20 \cos(\omega t + 45^\circ + 90^\circ) = \mathbf{20\cos(\omega t + 135^\circ)}$$

$$\mathbf{(a) \ 10\cos(\omega t - 60^\circ), (b) \ 9\cos(8t + 90^\circ), (c) \ 20\cos(\omega t + 135^\circ)}$$

Chapter 9, Solution 4.

Design a problem to help other students to better understand sinusoids.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

(a) Express $v = 8 \cos(7t + 15^\circ)$ in sine form.

(b) Convert $i = -10 \sin(3t - 85^\circ)$ to cosine form.

Solution

$$(a) \quad v = 8 \cos(7t + 15^\circ) = 8 \sin(7t + 15^\circ + 90^\circ) = \mathbf{8 \sin(7t + 105^\circ)}$$

$$(b) \quad i = -10 \sin(3t - 85^\circ) = 10 \cos(3t - 85^\circ + 90^\circ) = \mathbf{10 \cos(3t + 5^\circ)}$$

Chapter 9, Solution 5.

$$v_1 = 45 \sin(\omega t + 30^\circ) \text{ V} = 45 \cos(\omega t + 30^\circ - 90^\circ) = 45 \cos(\omega t - 60^\circ) \text{ V}$$

$$v_2 = 50 \cos(\omega t - 30^\circ) \text{ V}$$

This indicates that the phase angle between the two signals is **30°** and that **v_1 lags v_2** .

Chapter 9, Solution 6.

- (a) $v(t) = 10 \cos(4t - 60^\circ)$
 $i(t) = 4 \sin(4t + 50^\circ) = 4 \cos(4t + 50^\circ - 90^\circ) = 4 \cos(4t - 40^\circ)$
Thus, **$i(t)$ leads $v(t)$ by 20° .**
- (b) $v_1(t) = 4 \cos(377t + 10^\circ)$
 $v_2(t) = -20 \cos(377t) = 20 \cos(377t + 180^\circ)$
Thus, **$v_2(t)$ leads $v_1(t)$ by 170° .**
- (c) $x(t) = 13 \cos(2t) + 5 \sin(2t) = 13 \cos(2t) + 5 \cos(2t - 90^\circ)$
 $\mathbf{X} = 13\angle 0^\circ + 5\angle -90^\circ = 13 - j5 = 13.928\angle -21.04^\circ$
 $x(t) = 13.928 \cos(2t - 21.04^\circ)$
 $y(t) = 15 \cos(2t - 11.8^\circ)$
phase difference $= -11.8^\circ + 21.04^\circ = 9.24^\circ$
Thus, **$y(t)$ leads $x(t)$ by 9.24° .**

Chapter 9, Solution 7.

$$\text{If } f(\phi) = \cos\phi + j \sin\phi,$$

$$\frac{df}{d\phi} = -\sin\phi + j\cos\phi = j(\cos\phi + j\sin\phi) = jf(\phi)$$

$$\frac{df}{f} = j d\phi$$

Integrating both sides

$$\ln f = j\phi + \ln A$$

$$f = Ae^{j\phi} = \cos\phi + j \sin\phi$$

$$f(0) = A = 1$$

$$\text{i.e. } f(\phi) = e^{j\phi} = \cos\phi + j \sin\phi$$

Chapter 9, Solution 8.

$$\begin{aligned} \text{(a)} \quad \frac{60\angle 45^\circ}{7.5 - j10} + j2 &= \frac{60\angle 45^\circ}{12.5\angle -53.13^\circ} + j2 \\ &= 4.8\angle 98.13^\circ + j2 = -0.6788 + j4.752 + j2 \\ &= \mathbf{-0.6788 + j6.752} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad (6 - j8)(4 + j2) &= 24 - j32 + j12 + 16 = 40 - j20 = 44.72\angle -26.57^\circ \\ \frac{32\angle -20^\circ}{(6 - j8)(4 + j2)} + \frac{20}{-10 + j24} &= \frac{32\angle -20^\circ}{44.72\angle -26.57^\circ} + \frac{20}{26\angle 112.62^\circ} \\ &= 0.7156\angle 6.57^\circ + 0.7692\angle -112.62^\circ = 0.7109 + j0.08188 - 0.2958 - j0.71 \\ &= \mathbf{0.4151 - j0.6281} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 20 + (16\angle -50^\circ)(13\angle 67.38^\circ) &= 20 + 208\angle 17.38^\circ = 20 + 198.5 + j62.13 \\ &= \mathbf{218.5 + j62.13} \end{aligned}$$

Chapter 9, Solution 9.

$$\begin{aligned} \text{(a)} \quad (5\angle 30^\circ)(6 - j8 + 1.1197 + j0.7392) &= (5\angle 30^\circ)(7.13 - j7.261) \\ &= (5\angle 30^\circ)(10.176\angle -45.52^\circ) = \end{aligned}$$

$$\begin{aligned} &\quad \quad \quad \mathbf{50.88\angle -15.52^\circ}. \\ \text{(b)} \quad \frac{(10\angle 60^\circ)(35\angle -50^\circ)}{(-3 + j5) = (5.83\angle 120.96^\circ)} &= \mathbf{60.02\angle -110.96^\circ}. \end{aligned}$$

Chapter 9, Solution 10.

Design a problem to help other students to better understand phasors.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Given that $z_1 = 6 - j8$, $z_2 = 10\angle -30^\circ$, and $z_3 = 8e^{-j120^\circ}$, find:

(a) $z_1 + z_2 + z_3$

(b) $z_1 z_2 / z_3$

Solution

(a) $z_1 = 6 - j8$, $z_2 = 8.66 - j5$, and $z_3 = -4 - j6.9282$

$z_1 + z_2 + z_3 = \mathbf{(10.66 - j19.928)\Omega}$

(b) $\frac{z_1 z_2}{z_3} = [(10\angle -53.13^\circ)(10\angle -30^\circ)/(8\angle -120^\circ)] = 12.5\angle 36.87^\circ \Omega = \mathbf{(10 + j7.5)\Omega}$

Chapter 9, Solution 11.

(a) $V = \underline{21 \angle -15^\circ} \text{ V}$

(b) $i(t) = 8 \sin(10t + 70^\circ + 180^\circ) = 8 \cos(10t + 70^\circ + 180^\circ - 90^\circ) = 8 \cos(10t + 160^\circ)$

$$\mathbf{I = 8 \angle 160^\circ \text{ mA}}$$

(c) $v(t) = 120 \sin(10^3 t - 50^\circ) = 120 \cos(10^3 t - 50^\circ - 90^\circ)$

$$\mathbf{V = 120 \angle -140^\circ \text{ V}}$$

(d) $i(t) = -60 \cos(30t + 10^\circ) = 60 \cos(30t + 10^\circ + 180^\circ)$

$$\mathbf{I = 60 \angle -170^\circ \text{ mA}}$$

Chapter 9, Solution 12.

Let $\mathbf{X} = 4\angle 40^\circ$ and $\mathbf{Y} = 20\angle -30^\circ$. Evaluate the following quantities and express your results in polar form.

$$(\mathbf{X} + \mathbf{Y})/\mathbf{X}^*$$

$$(\mathbf{X} - \mathbf{Y})^*$$

$$(\mathbf{X} + \mathbf{Y})/\mathbf{X}$$

$$\mathbf{X} = 3.064 + j2.571; \mathbf{Y} = 17.321 - j10$$

$$\begin{aligned} \text{(a)} \quad (\mathbf{X} + \mathbf{Y})\mathbf{X}^* &= (20.38 - j7.429)(4\angle -40^\circ) \\ &= (21.69\angle -20.03^\circ)(4\angle -40^\circ) = 86.76\angle -60.03^\circ \\ &= \mathbf{86.76\angle -60.03^\circ} \end{aligned}$$

$$\text{(b)} \quad (\mathbf{X} - \mathbf{Y})^* = (-14.257 + j12.571)^* = \mathbf{19.41\angle -139.63^\circ}$$

$$\text{(c)} \quad (\mathbf{X} + \mathbf{Y})/\mathbf{X} = (21.69\angle -20.03^\circ)/(4\angle 40^\circ) = \mathbf{5.422\angle -60.03^\circ}$$

Chapter 9, Solution 13.

$$(a) \quad (-0.4324 + j0.4054) + (-0.8425 - j0.2534) = \underline{-1.2749 + j0.1520}$$

$$(b) \quad \frac{50 \angle -30^\circ}{24 \angle 150^\circ} = \underline{-2.0833} = \underline{-2.083}$$

$$(c) \quad (2+j3)(8-j5) - (-4) = \underline{35 + j14}$$

Chapter 9, Solution 14.

$$(a) \frac{3 - j14}{-7 + j17} = \frac{14.318 \angle -77.91^\circ}{18.385 \angle 112.38^\circ} = 0.7788 \angle 169.71^\circ = \underline{-0.7663 + j0.13912}$$

$$(b) \frac{(62.116 + j231.82 + 138.56 - j80)(60 - j80)}{(67 + j84)(16.96 + j10.5983)} = \frac{24186 - 6944.9}{246.06 + j2134.7} = \underline{-1.922 - j11.55}$$

$$(c) \left[\frac{10 + j20}{3 + j4} \right]^2 \sqrt{(10 + j5)(16 - j20)}$$

$$\begin{aligned} &= [(22.36 \angle 63.43^\circ)/(5 \angle 53.13^\circ)]^2 [(11.18 \angle 26.57^\circ)(25.61 \angle -51.34^\circ)]^{0.5} \\ &= [4.472 \angle 10.3^\circ]^2 [286.3 \angle -24.77^\circ]^{0.5} = (19.999 \angle 20.6^\circ)(16.921 \angle -12.38^\circ) = 338.4 \angle 8.22^\circ \end{aligned}$$

$$\text{or } \underline{334.9 + j48.38}$$

Chapter 9, Solution 15.

$$(a) \quad \begin{vmatrix} 10 + j6 & 2 - j3 \\ -5 & -1 + j \end{vmatrix} = -10 - j6 + j10 - 6 + 10 - j15 \\ = \mathbf{-6 - j11}$$

$$(b) \quad \begin{vmatrix} 20\angle -30^\circ & -4\angle -10^\circ \\ 16\angle 0^\circ & 3\angle 45^\circ \end{vmatrix} = 60\angle 15^\circ + 64\angle -10^\circ \\ = 57.96 + j15.529 + 63.03 - j11.114 \\ = \mathbf{120.99 + j4.415}$$

$$(c) \quad \begin{vmatrix} 1-j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1+j \\ 1-j & -j & 0 \\ j & 1 & -j \end{vmatrix} = 1+1+0-1-0+j^2(1-j)+j^2(1+j) \\ = 1-1(1-j+1+j) \\ = 1-2 = \mathbf{-1}$$

Chapter 9, Solution 16.

$$\begin{aligned} \text{(a)} \quad -20 \cos(4t + 135^\circ) &= 20 \cos(4t + 135^\circ - 180^\circ) \\ &= 20 \cos(4t - 45^\circ) \end{aligned}$$

The phasor form is **$20\angle-45^\circ$**

$$\begin{aligned} \text{(b)} \quad 8 \sin(20t + 30^\circ) &= 8 \cos(20t + 30^\circ - 90^\circ) \\ &= 8 \cos(20t - 60^\circ) \end{aligned}$$

The phasor form is **$8\angle-60^\circ$**

$$\text{(c)} \quad 20 \cos(2t) + 15 \sin(2t) = 20 \cos(2t) + 15 \cos(2t - 90^\circ)$$

The phasor form is $20\angle 0^\circ + 15\angle -90^\circ = 20 - j15 = \mathbf{25\angle-36.87^\circ}$

Chapter 9, Solution 17.

$$V = V_1 + V_2 = 10 \angle -60^\circ + 12 \angle 30^\circ = 5 - j8.66 + 10.392 + j6 = 15.62 \angle -9.805^\circ$$

$$v(t) = \mathbf{15.62\cos(50t-9.8^\circ) \text{ V}}$$

Chapter 9, Solution 18.

(a) $v_1(t) = \mathbf{60 \cos(t + 15^\circ)}$

(b) $\mathbf{V_2 = 6 + j8 = 10\angle 53.13^\circ}$
 $v_2(t) = \mathbf{10 \cos(40t + 53.13^\circ)}$

(c) $i_1(t) = \mathbf{2.8 \cos(377t - \pi/3)}$

(d) $\mathbf{I_2 = -0.5 - j1.2 = 1.3\angle 247.4^\circ}$
 $i_2(t) = \mathbf{1.3 \cos(10^3t + 247.4^\circ)}$

Chapter 9, Solution 19.

$$\begin{aligned} \text{(a)} \quad 3\angle 10^\circ - 5\angle -30^\circ &= 2.954 + j0.5209 - 4.33 + j2.5 \\ &= -1.376 + j3.021 \\ &= 3.32\angle 114.49^\circ \end{aligned}$$

$$\begin{aligned} \text{Therefore,} \quad 3 \cos(20t + 10^\circ) - 5 \cos(20t - 30^\circ) \\ = \mathbf{3.32 \cos(20t + 114.49^\circ)} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 40\angle -90^\circ + 30\angle -45^\circ &= -j40 + 21.21 - j21.21 \\ &= 21.21 - j61.21 \\ &= 64.78\angle -70.89^\circ \end{aligned}$$

$$\text{Therefore,} \quad 40 \sin(50t) + 30 \cos(50t - 45^\circ) = \mathbf{64.78 \cos(50t - 70.89^\circ)}$$

$$\begin{aligned} \text{(c)} \quad \text{Using } \sin\alpha &= \cos(\alpha - 90^\circ), \\ 20\angle -90^\circ + 10\angle 60^\circ - 5\angle -110^\circ &= -j20 + 5 + j8.66 + 1.7101 + j4.699 \\ &= 6.7101 - j6.641 \\ &= 9.44\angle -44.7^\circ \end{aligned}$$

$$\begin{aligned} \text{Therefore,} \quad 20 \sin(400t) + 10 \cos(400t + 60^\circ) - 5 \sin(400t - 20^\circ) \\ = \mathbf{9.44 \cos(400t - 44.7^\circ)} \end{aligned}$$

Chapter 9, Solution 20.

$7.5\cos(10t+30^\circ)$ A can be represented by $7.5\angle 30^\circ$ and $120\cos(10t+75^\circ)$ V can be represented by $120\angle 75^\circ$. Thus,

$$\mathbf{Z} = \mathbf{V/I} = (120\angle 75^\circ)/(7.5\angle 30^\circ) = 16\angle 45^\circ \text{ or } \mathbf{(11.314+j11.314) \Omega}.$$

Chapter 9, Solution 21.

$$(a) \quad F = 5\angle 15^\circ - 4\angle -30^\circ - 90^\circ = 6.8296 + j4.758 = 8.3236\angle 34.86^\circ$$

$$\underline{f(t) = 8.324\cos(30t + 34.86^\circ)}$$

$$(b) \quad G = 8\angle -90^\circ + 4\angle 50^\circ = 2.571 - j4.9358 = 5.565\angle -62.49^\circ$$

$$\underline{g(t) = 5.565\cos(t - 62.49^\circ)}$$

$$(c) \quad H = \frac{1}{j\omega} (10\angle 0^\circ + 50\angle -90^\circ), \quad \omega = 40$$

$$\text{i.e. } H = 0.25\angle -90^\circ + 1.25\angle -180^\circ = -j0.25 - 1.25 = 1.2748\angle -168.69^\circ$$

$$h(t) = 1.2748\cos(40t - 168.69^\circ)$$

Chapter 9, Solution 22.

$$\text{Let } f(t) = 10v(t) + 4 \frac{dv}{dt} - 2 \int_{-\infty}^t v(t) dt$$

$$F = 10V + j\omega 4V - \frac{2V}{j\omega}, \quad \omega = 5, \quad V = 55 \angle 45^\circ$$

$$F = 10V + j20V + j0.4V = (10 + j20.4)V = 22.72 \angle 63.89^\circ (55 \angle 45^\circ) = 1249.6 \angle 108.89^\circ$$

$$f(t) = \mathbf{1249.6 \cos(5t + 108.89^\circ)}$$

Chapter 9, Solution 23.

(a) $v = [110\sin(20t+30^\circ) + 220\cos(20t-90^\circ)]$ V leads to $\mathbf{V} = 110\angle(30^\circ-90^\circ) + 220\angle-90^\circ = 55-j95.26 - j220 = 55-j315.3 = 320.1\angle-80.11^\circ$ or

$$v = \mathbf{320.1\cos(20t-80.11^\circ) \text{ A.}}$$

(b) $i = [30\cos(5t+60^\circ)-20\sin(5t+60^\circ)]$ A leads to $\mathbf{I} = 30\angle60^\circ - 20\angle(60^\circ-90^\circ) = 15+j25.98 - (17.321-j10) = -2.321+j35.98 = 36.05\angle93.69^\circ$ or

$$i = \mathbf{36.05\cos(5t+93.69^\circ) \text{ A.}}$$

(a) $320.1\cos(20t-80.11^\circ)$ A, (b) $36.05\cos(5t+93.69^\circ)$ A

Chapter 9, Solution 24.

(a)

$$\mathbf{V} + \frac{\mathbf{V}}{j\omega} = 10\angle 0^\circ, \quad \omega = 1$$

$$\mathbf{V}(1 - j) = 10$$

$$\mathbf{V} = \frac{10}{1 - j} = 5 + j5 = 7.071\angle 45^\circ$$

Therefore,

$$v(t) = \mathbf{7.071\cos(t + 45^\circ) \text{ V}}$$

(b)

$$j\omega\mathbf{V} + 5\mathbf{V} + \frac{4\mathbf{V}}{j\omega} = 20\angle(10^\circ - 90^\circ), \quad \omega = 4$$

$$\mathbf{V}\left(j4 + 5 + \frac{4}{j4}\right) = 20\angle -80^\circ$$

$$\mathbf{V} = \frac{20\angle -80^\circ}{5 + j3} = 3.43\angle -110.96^\circ$$

Therefore,

$$v(t) = \mathbf{3.43\cos(4t - 110.96^\circ) \text{ V}}$$

Chapter 9, Solution 25.

(a)

$$2j\omega\mathbf{I} + 3\mathbf{I} = 4\angle 45^\circ, \quad \omega = 2$$

$$\mathbf{I}(3 + j4) = 4\angle 45^\circ$$

$$I = \frac{4\angle 45^\circ}{3 + j4} = \frac{4\angle 45^\circ}{5\angle 53.13^\circ} = 0.8\angle -8.13^\circ$$

$$\text{Therefore, } i(t) = \mathbf{800\cos(2t - 8.13^\circ) \text{ mA}}$$

(b)

$$10\frac{\mathbf{I}}{j\omega} + j\omega\mathbf{I} + 6\mathbf{I} = 5\angle 22^\circ, \quad \omega = 5$$

$$(-j2 + j5 + 6)\mathbf{I} = 5\angle 22^\circ$$

$$\mathbf{I} = \frac{5\angle 22^\circ}{6 + j3} = \frac{5\angle 22^\circ}{6.708\angle 26.56^\circ} = 0.745\angle -4.56^\circ$$

$$\text{Therefore, } i(t) = \mathbf{745 \cos(5t - 4.56^\circ) \text{ mA}}$$

Chapter 9, Solution 26.

$$j\omega\mathbf{I} + 2\mathbf{I} + \frac{\mathbf{I}}{j\omega} = 1\angle 0^\circ, \quad \omega = 2$$

$$\mathbf{I}\left(j2 + 2 + \frac{1}{j2}\right) = 1$$

$$\mathbf{I} = \frac{1}{2 + j1.5} = 0.4\angle -36.87^\circ$$

Therefore, $i(t) = \mathbf{0.4 \cos(2t - 36.87^\circ)}$

Chapter 9, Solution 27.

$$j\omega \mathbf{V} + 50\mathbf{V} + 100 \frac{\mathbf{V}}{j\omega} = 110 \angle -10^\circ, \quad \omega = 377$$

$$\mathbf{V} \left(j377 + 50 - \frac{j100}{377} \right) = 110 \angle -10^\circ$$

$$\mathbf{V} (380.6 \angle 82.45^\circ) = 110 \angle -10^\circ$$

$$\mathbf{V} = 0.289 \angle -92.45^\circ$$

Therefore, $v(t) = \mathbf{289 \cos(377t - 92.45^\circ) \text{ mV.}}$

Chapter 9, Solution 28.

$$i(t) = \frac{v_s(t)}{R} = \frac{156 \cos(377t + 45^\circ)}{15} = \mathbf{10.4 \cos(377t + 45^\circ) \text{ A.}}$$

Chapter 9, Solution 29.

$$\mathbf{Z} = \frac{1}{j\omega C} = \frac{1}{j(10^6)(2 \times 10^{-6})} = -j0.5$$

$$\mathbf{V} = \mathbf{IZ} = (4\angle 25^\circ)(0.5\angle -90^\circ) = 2\angle -65^\circ$$

Therefore $v(t) = 2 \sin(10^6 t - 65^\circ) \text{ V}.$

Chapter 9, Solution 30.

Since R and C are in parallel, they have the same voltage across them. For the resistor,

$$V = I_R R \quad \longrightarrow \quad I_R = V / R = \frac{100 \angle 20^\circ}{40k} = 2.5 \angle 20^\circ \text{ mA}$$

$$i_R = \underline{2.5 \cos(60t + 20^\circ) \text{ mA}}$$

For the capacitor,

$$i_C = C \frac{dv}{dt} = 50 \times 10^{-6} (-60) \times 100 \sin(60t + 20^\circ) = \underline{-300 \sin(60t + 20^\circ) \text{ mA}}$$

Chapter 9, Solution 31.

$$L = 240\text{mH} \quad \longrightarrow \quad j\omega L = j2 \times 240 \times 10^{-3} = j0.48$$

$$C = 5\text{mF} \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j2 \times 5 \times 10^{-3}} = -j100$$

$$Z = 80 + j0.48 - j100 = 80 - j99.52 =$$

$$I = \frac{V}{Z} = \frac{10 \angle 0^\circ}{80 - j99.52} = 0.0783 \angle 51.206^\circ$$

$$i(t) = \mathbf{78.3 \cos(2t + 51.21^\circ) \text{ mA}}$$

Chapter 9, Solution 32.

Using Fig. 9.40, design a problem to help other students to better understand phasor relationships for circuit elements.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Two elements are connected in series as shown in Fig. 9.40.

If $i = 12 \cos(2t - 30^\circ)$ A, find the element values.

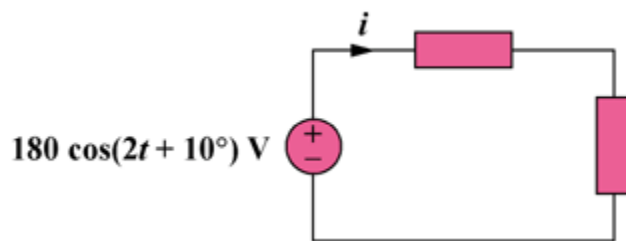


Figure 9.40

Solution

$$\mathbf{V} = 180\angle 10^\circ, \quad \mathbf{I} = 12\angle -30^\circ, \quad \omega = 2$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{180\angle 10^\circ}{12\angle -30^\circ} = 15\angle 40^\circ = 11.49 + j9.642 \, \Omega$$

One element is a resistor with $R = 11.49 \, \Omega$.

The other element is an inductor with $\omega L = 9.642$ or $L = 4.821 \, \text{H}$.

Chapter 9, Solution 33.

$$110 = \sqrt{v_R^2 + v_L^2}$$

$$v_L = \sqrt{110^2 - v_R^2}$$

$$v_L = \sqrt{110^2 - 85^2} = \mathbf{69.82 \text{ V}}$$

Chapter 9, Solution 34.

$$v_o = 0 \text{ when } jX_L - jX_C = 0 \text{ so } X_L = X_C \text{ or } \omega L = \frac{1}{\omega C} \longrightarrow \omega = \frac{1}{\sqrt{LC}}.$$

$$\omega = \frac{1}{\sqrt{(5 \times 10^{-3})(20 \times 10^{-3})}} = \mathbf{100 \text{ rad/s}}$$

Chapter 9, Solution 35.

$$v_s(t) = 50 \cos 200t \quad \longrightarrow \quad V_s = 50 \angle 0^\circ, \omega = 200$$

$$5mF \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j200 \times 5 \times 10^{-3}} = -j$$

$$20mH \quad \longrightarrow \quad j\omega L = j20 \times 10^{-3} \times 200 = j4$$

$$Z_{in} = 10 - j + j4 = 10 + j3$$

$$I = \frac{V_s}{Z_{in}} = \frac{50 \angle 0^\circ}{10 + j3} = 4.789 \angle -16.7^\circ$$

$$i(t) = \mathbf{4.789 \cos(200t - 16.7^\circ) \text{ A}}$$

Chapter 9, Solution 36.

Using Fig. 9.43, design a problem to help other students to better understand impedance.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit in Fig. 9.43, determine i . Let $v_s = 60 \cos(200t - 10^\circ)$ V.

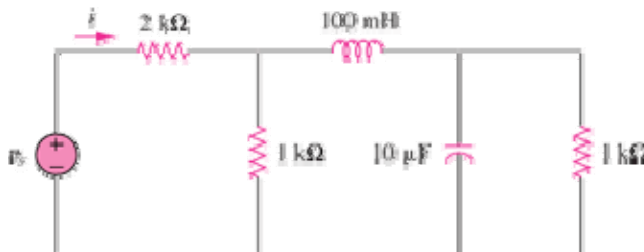


Figure 9.43

Solution

Let Z be the input impedance at the source.

$$100\text{ mH} \longrightarrow j\omega L = j200 \times 100 \times 10^{-3} = j20$$

$$10\text{ }\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10 \times 10^{-6} \times 200} = -j500$$

$$1000 // -j500 = 200 - j400$$

$$1000 // (j20 + 200 - j400) = 242.62 - j239.84$$

$$Z = 2242.62 - j239.84 = 2255 \angle -6.104^\circ$$

$$I = \frac{60 \angle -10^\circ}{2255 \angle -6.104^\circ} = 26.61 \angle -3.896^\circ \text{ mA}$$

$$\mathbf{i = 266.1 \cos(200t - 3.896^\circ) \text{ mA}}$$

Chapter 9, Solution 37.

$$\begin{aligned} Y &= (1/4) + (1/(j8)) + (1/(-j10)) = 0.25 - j0.025 \\ &= \mathbf{(250-j25) \text{ mS}} \end{aligned}$$

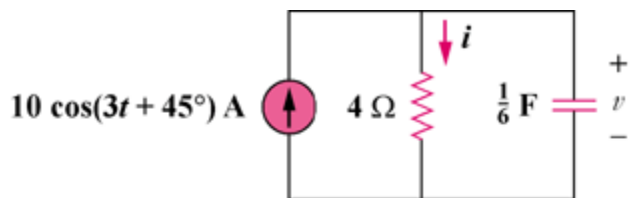
Chapter 9, Solution 38.

Using Fig. 9.45, design a problem to help other students to better understand admittance.

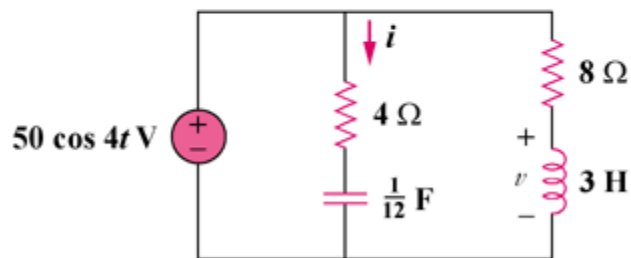
Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find $i(t)$ and $v(t)$ in each of the circuits of Fig. 9.45.



(a)



(b)

Figure 9.45

Solution

$$(a) \quad \frac{1}{6} \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = -j2$$

$$\mathbf{I} = \frac{-j2}{4 - j2} (10 \angle 45^\circ) = 4.472 \angle -18.43^\circ$$

$$\text{Hence, } i(t) = \mathbf{4.472 \cos(3t - 18.43^\circ) A}$$

$$\mathbf{V} = 4\mathbf{I} = (4)(4.472 \angle -18.43^\circ) = 17.89 \angle -18.43^\circ$$

$$\text{Hence, } v(t) = \mathbf{17.89 \cos(3t - 18.43^\circ) V}$$

$$(b) \quad \frac{1}{12} \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = -j3$$

$$3 \text{ H} \longrightarrow j\omega L = j(4)(3) = j12$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\angle 0^\circ}{4 - j3} = 10\angle 36.87^\circ$$

$$\text{Hence, } i(t) = \mathbf{10 \cos(4t + 36.87^\circ) \text{ A}}$$

$$\mathbf{V} = \frac{j12}{8 + j12}(50\angle 0^\circ) = 41.6\angle 33.69^\circ$$

$$\text{Hence, } v(t) = \mathbf{41.6 \cos(4t + 33.69^\circ) \text{ V}}$$

Chapter 9, Solution 39.

$$Z_{eq} = 4 + j20 + 10 // (-j14 + j25) = \underline{9.135 + j27.47 \text{ } \Omega}$$
$$= \mathbf{(9.135 + j27.47) \text{ } \Omega}$$

$$I = \frac{V}{Z_{eq}} = \frac{12}{9.135 + j27.47} = 0.4145 \angle -71.605^\circ$$
$$i(t) = \mathbf{414.5 \cos(10t - 71.6^\circ) \text{ mA}}$$

Chapter 9, Solution 40.

(a) For $\omega = 1$,

$$1 \text{ H} \longrightarrow j\omega L = j(1)(1) = j$$

$$0.05 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1)(0.05)} = -j20$$

$$\mathbf{Z} = j + 2 \parallel (-j20) = j + \frac{-j40}{2 - j20} = 1.98 + j0.802$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0^\circ}{1.98 + j0.802} = \frac{4\angle 0^\circ}{2.136\angle 22.05^\circ} = 1.872\angle -22.05^\circ$$

Hence,

$$i_o(t) = \mathbf{1.872 \cos(t - 22.05^\circ) \text{ A}}$$

(b) For $\omega = 5$,

$$1 \text{ H} \longrightarrow j\omega L = j(5)(1) = j5$$

$$0.05 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(5)(0.05)} = -j4$$

$$\mathbf{Z} = j5 + 2 \parallel (-j4) = j5 + \frac{-j4}{1 - j2} = 1.6 + j4.2$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0^\circ}{1.6 + j4} = \frac{4\angle 0^\circ}{4.494\angle 69.14^\circ} = 0.89\angle -69.14^\circ$$

Hence,

$$i_o(t) = \mathbf{890\cos(5t - 69.14^\circ) \text{ mA}}$$

(c) For $\omega = 10$,

$$1 \text{ H} \longrightarrow j\omega L = j(10)(1) = j10$$

$$0.05 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.05)} = -j2$$

$$\mathbf{Z} = j10 + 2 \parallel (-j2) = j10 + \frac{-j4}{2 - j2} = 1 + j9$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0^\circ}{1 + j9} = \frac{4\angle 0^\circ}{9.055\angle 83.66^\circ} = 0.4417\angle -83.66^\circ$$

Hence,

$$i_o(t) = \mathbf{441.7\cos(10t - 83.66^\circ) \text{ mA}}$$

Chapter 9, Solution 41.

$$\omega = 1,$$

$$1 \text{ H} \longrightarrow j\omega L = j(1)(1) = j$$

$$1 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1)(1)} = -j$$

$$\mathbf{Z} = 1 + (1 + j) \parallel (-j) = 1 + \frac{-j+1}{1} = 2 - j$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10}{2 - j}, \quad \mathbf{I}_c = (1 + j)\mathbf{I}$$

$$\mathbf{V} = (-j)(1 + j)\mathbf{I} = (1 - j)\mathbf{I} = \frac{(1 - j)(10)}{2 - j} = 6.325 \angle -18.43^\circ$$

Thus,

$$v(t) = \mathbf{6.325} \cos(t - 18.43^\circ) \text{ V}$$

Chapter 9, Solution 42.

$$\omega = 200$$

$$50 \text{ } \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(50 \times 10^{-6})} = -j100$$

$$0.1 \text{ H} \longrightarrow j\omega L = j(200)(0.1) = j20$$

$$50 \parallel -j100 = \frac{(50)(-j100)}{50 - j100} = \frac{-j100}{1 - j2} = 40 - j20$$

$$\mathbf{V}_o = \frac{j20}{j20 + 30 + 40 - j20} (60 \angle 0^\circ) = \frac{j20}{70} (60 \angle 0^\circ) = 17.14 \angle 90^\circ$$

Thus,

$$v_o(t) = \mathbf{17.14 \sin(200t + 90^\circ) \text{ V}}$$

or

$$v_o(t) = \mathbf{17.14 \cos(200t) \text{ V}}$$

Chapter 9, Solution 43.

$$Z_{in} = 50 + j80 \parallel (100 - j40) = 50 + \frac{j80(100 - j40)}{100 + j40} = 105.71 + j57.93$$

$$I_o = \frac{60 \angle 0^\circ}{Z_{in}} = 0.4377 - 0.2411j = \underline{0.4997 \angle -28.85^\circ} \text{ A} = \mathbf{499.7 \angle -28.85^\circ \text{ mA}}$$

Chapter 9, Solution 44.

$$\omega = 200$$

$$10 \text{ mH} \longrightarrow j\omega L = j(200)(10 \times 10^{-3}) = j2$$

$$5 \text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(5 \times 10^{-3})} = -j$$

$$\mathbf{Y} = \frac{1}{4} + \frac{1}{j2} + \frac{1}{3-j} = 0.25 - j0.5 + \frac{3+j}{10} = 0.55 - j0.4$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{0.55 - j0.4} = 1.1892 + j0.865$$

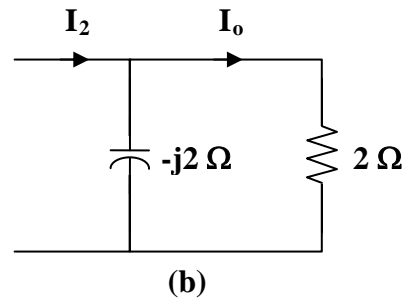
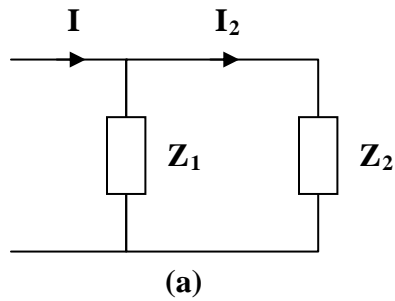
$$\mathbf{I} = \frac{6\angle 0^\circ}{5 + \mathbf{Z}} = \frac{6\angle 0^\circ}{6.1892 + j0.865} = 0.96\angle -7.956^\circ$$

Thus,

$$i(t) = \mathbf{960\cos(200t - 7.956^\circ) \text{ mA}}$$

Chapter 9, Solution 45.

We obtain I_o by applying the principle of current division twice.



$$Z_1 = -j2, \quad Z_2 = j4 + (-j2) \parallel 2 = j4 + \frac{-j4}{2 - j2} = 1 + j3$$

$$I_2 = \frac{Z_1}{Z_1 + Z_2} I = \frac{-j2}{-j2 + 1 + j3} (5 \angle 0^\circ) = \frac{-j10}{1 + j}$$

$$I_o = \frac{-j2}{2 - j2} I_2 = \left(\frac{-j}{1 - j} \right) \left(\frac{-j10}{1 + j} \right) = \frac{-10}{1 + 1} = -5\text{ A}$$

Chapter 9, Solution 46.

$$i_s = 5 \cos(10t + 40^\circ) \longrightarrow \mathbf{I}_s = 5 \angle 40^\circ$$

$$0.1 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.1)} = -j$$

$$0.2 \text{ H} \longrightarrow j\omega L = j(10)(0.2) = j2$$

$$\text{Let} \quad \mathbf{Z}_1 = 4 \parallel j2 = \frac{j8}{4 + j2} = 0.8 + j1.6, \quad \mathbf{Z}_2 = 3 - j$$

$$\mathbf{I}_o = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{I}_s = \frac{0.8 + j1.6}{3.8 + j0.6} (5 \angle 40^\circ)$$

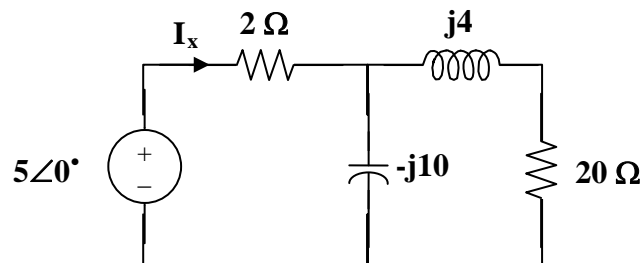
$$\mathbf{I}_o = \frac{(1.789 \angle 63.43^\circ)(5 \angle 40^\circ)}{3.847 \angle 8.97^\circ} = 2.325 \angle 94.46^\circ$$

Thus,

$$i_o(t) = \mathbf{2.325 \cos(10t + 94.46^\circ) \text{ A}}$$

Chapter 9, Solution 47.

First, we convert the circuit into the frequency domain.

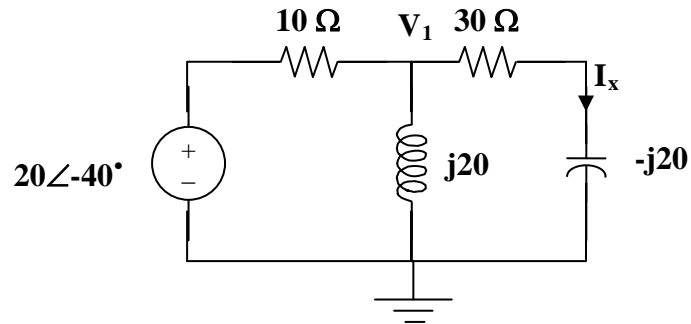


$$I_x = \frac{5}{2 + \frac{-j10(20 + j4)}{-j10 + 20 + j4}} = \frac{5}{2 + 4.588 - j8.626} = \frac{5}{10.854\angle -52.63^\circ} = 0.4607\angle 52.63^\circ$$

$$i_s(t) = 460.7\cos(2000t + 52.63^\circ) \text{ mA}$$

Chapter 9, Solution 48.

Converting the circuit to the frequency domain, we get:



We can solve this using nodal analysis.

$$\frac{V_1 - 20\angle -40^\circ}{10} + \frac{V_1 - 0}{j20} + \frac{V_1 - 0}{30 - j20} = 0$$

$$V_1(0.1 - j0.05 + 0.02307 + j0.01538) = 2\angle -40^\circ$$

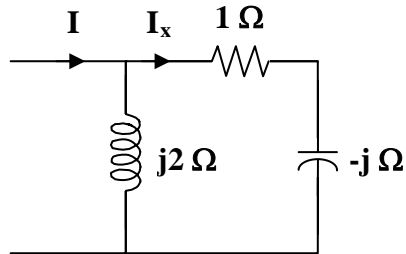
$$V_1 = \frac{2\angle 40^\circ}{0.12307 - j0.03462} = 15.643\angle -24.29^\circ$$

$$I_x = \frac{15.643\angle -24.29^\circ}{30 - j20} = 0.4338\angle 9.4^\circ$$

$$i_x = \underline{0.4338 \sin(100t + 9.4^\circ) \text{ A}}$$

Chapter 9, Solution 49.

$$\mathbf{Z}_T = 2 + j2 \parallel (1 - j) = 2 + \frac{(j2)(1 - j)}{1 + j} = 4$$



$$\mathbf{I}_x = \frac{j2}{j2 + 1 - j} \mathbf{I} = \frac{j2}{1 + j} \mathbf{I}, \quad \text{where } \mathbf{I}_x = 0.5 \angle 0^\circ = \frac{1}{2}$$

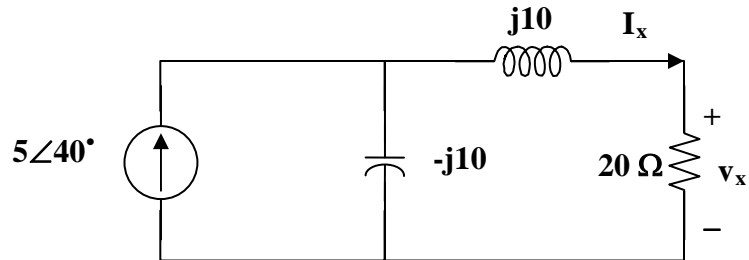
$$\mathbf{I} = \frac{1 + j}{j2} \mathbf{I}_x = \frac{1 + j}{j4}$$

$$\mathbf{V}_s = \mathbf{I} \mathbf{Z}_T = \frac{1 + j}{j4} (4) = \frac{1 + j}{j} = 1 - j = 1.414 \angle -45^\circ$$

$$v_s(t) = 1.4142 \sin(200t - 45^\circ) \text{ V}$$

Chapter 9, Solution 50.

Since $\omega = 100$, the inductor $= j100 \times 0.1 = j10 \, \Omega$ and the capacitor $= 1/(j100 \times 10^{-3}) = -j10 \, \Omega$.



Using the current dividing rule:

$$I_x = \frac{-j10}{-j10 + 20 + j10} 5\angle 40^\circ = -j2.5\angle 40^\circ = 2.5\angle -50^\circ$$

$$V_x = 20I_x = 50\angle -50^\circ$$

$$v_x(t) = 50\cos(100t - 50^\circ) \text{ V}$$

Chapter 9, Solution 51.

$$0.1 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(0.1)} = -j5$$

$$0.5 \text{ H} \longrightarrow j\omega L = j(2)(0.5) = j$$

The current \mathbf{I} through the $2\text{-}\Omega$ resistor is

$$\mathbf{I} = \frac{1}{1 - j5 + j + 2} \mathbf{I}_s = \frac{\mathbf{I}_s}{3 - j4},$$

$$\text{where } \mathbf{I} = \frac{10}{2} \angle 0^\circ = 5$$

$$\mathbf{I}_s = (5)(3 - j4) = 25 \angle -53.13^\circ$$

Therefore,

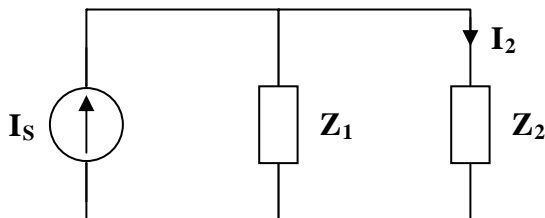
$$i_s(t) = 25 \cos(2t - 53.13^\circ) \text{ A}$$

Chapter 9, Solution 52.

We begin by simplifying the circuit. First we replace the parallel inductor and resistor with their series equivalent.

$$5 \parallel j5 = \frac{j25}{5 + j5} = \frac{j5}{1 + j} = 2.5 + j2.5$$

Next let $\mathbf{Z}_1 = 10$, and $\mathbf{Z}_2 = -j5 + 2.5 + j2.5 = 2.5 - j2.5$.



$$\text{By current division } \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{I}_s = \frac{10}{12.5 - j2.5} \mathbf{I}_s = \frac{4}{5 - j} \mathbf{I}_s.$$

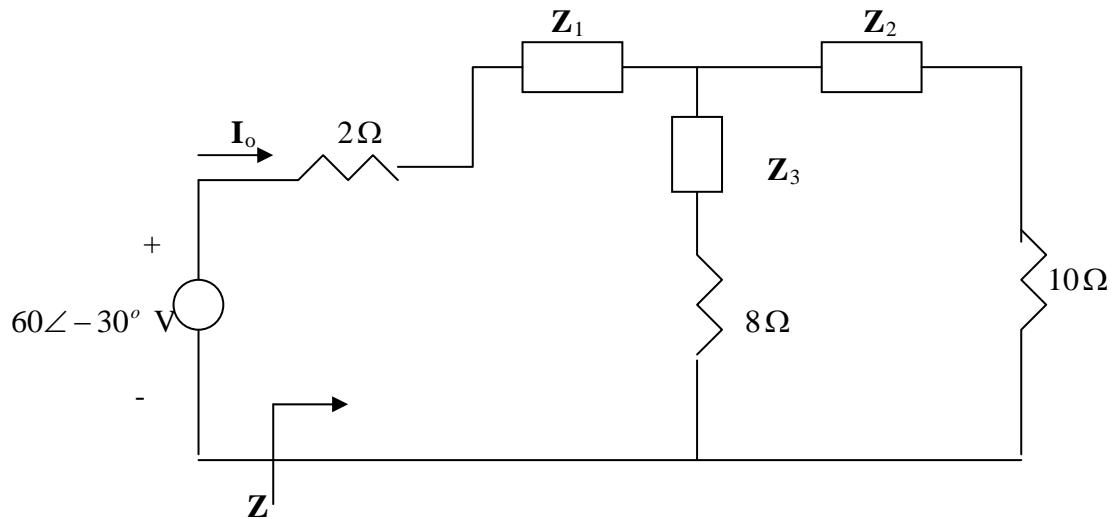
Since $\mathbf{V}_o = \mathbf{I}_2 (2.5 + j2.5)$ we can now find \mathbf{I}_s .

$$8 \angle 30^\circ = \left(\frac{4}{5 - j} \right) \mathbf{I}_s (2.5)(1 + j) = \frac{10(1 + j)}{5 - j} \mathbf{I}_s$$

$$\mathbf{I}_s = \frac{(8 \angle 30^\circ)(5 - j)}{10(1 + j)} = \mathbf{2.884 \angle -26.31^\circ \text{ A.}}$$

Chapter 9, Solution 53.

Convert the delta to wye subnetwork as shown below.



$$Z_1 = \frac{-j2 \times 4}{4 + j4} = \frac{8\angle -90^\circ}{5.6569\angle 45^\circ} = -1 - j1, \quad Z_2 = \frac{j6 \times 4}{4 + j4} = 3 + j3,$$

$$Z_3 = \frac{12}{4 + j4} = 1.5 - j1.5$$

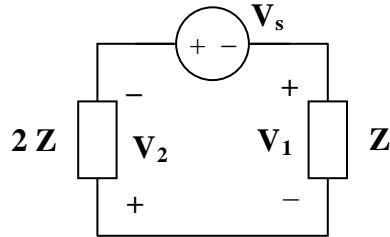
$$(Z_3 + 8) \parallel (Z_2 + 10) = (9.5 - j1.5) \parallel (13 + j3) = 5.691\angle 0.21^\circ = 5.691 + j0.02086$$

$$Z = 2 + Z_1 + 5.691 + j0.02086 = 6.691 - j0.9791$$

$$I_o = \frac{60\angle -30^\circ}{Z} = \frac{60\angle -30^\circ}{6.7623\angle -8.33^\circ} = \underline{8.873\angle -21.67^\circ \text{ A}}$$

Chapter 9, Solution 54.

Since the left portion of the circuit is twice as large as the right portion, the equivalent circuit is shown below.



$$\mathbf{V}_1 = \mathbf{I}_o(1 - j) = 2(1 - j)$$

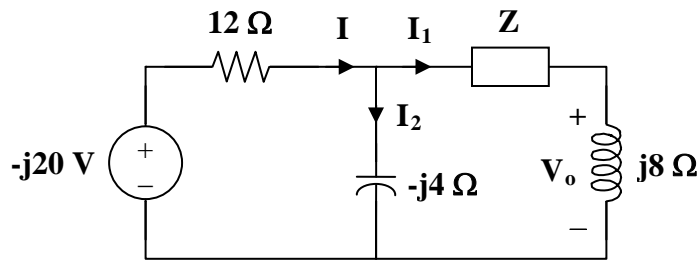
$$\mathbf{V}_2 = 2\mathbf{V}_1 = 4(1 - j)$$

$$\mathbf{V}_2 + \mathbf{V}_s + \mathbf{V}_1 = 0 \text{ or}$$

$$\mathbf{V}_s = -\mathbf{V}_1 - \mathbf{V}_2 = -6(1 - j) = (6\angle 180^\circ)(1.4142\angle -45^\circ)$$

$$\mathbf{V}_s = \mathbf{8.485\angle 135^\circ V}$$

Chapter 9, Solution 55.



$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{j8} = \frac{4}{j8} = -j0.5$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1(\mathbf{Z} + j8)}{-j4} = \frac{(-j0.5)(\mathbf{Z} + j8)}{-j4} = \frac{\mathbf{Z}}{8} + j$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = -j0.5 + \frac{\mathbf{Z}}{8} + j = \frac{\mathbf{Z}}{8} + j0.5$$

$$-j20 = 12\mathbf{I} + \mathbf{I}_1(\mathbf{Z} + j8)$$

$$-j20 = 12\left(\frac{\mathbf{Z}}{8} + \frac{j}{2}\right) + \frac{-j}{2}(\mathbf{Z} + j8)$$

$$-4 - j26 = \mathbf{Z}\left(\frac{3}{2} - j\frac{1}{2}\right)$$

$$\mathbf{Z} = \frac{-4 - j26}{\frac{3}{2} - j\frac{1}{2}} = \frac{26.31\angle 261.25^\circ}{1.5811\angle -18.43^\circ} = 16.64\angle 279.68^\circ$$

$$\mathbf{Z} = (2.798 - j16.403) \, \Omega$$

Chapter 9, Solution 56.

$$50\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j377 \times 50 \times 10^{-6}} = -j53.05$$

$$60mH \longrightarrow j\omega L = j377 \times 60 \times 10^{-3} = j22.62$$

$$Z_{in} = 12 - j53.05 + j22.62 // 40 = \underline{21.692 - j35.91 \Omega}$$

Chapter 9, Solution 57.

$$2\text{H} \longrightarrow j\omega L = j2$$

$$1\text{F} \longrightarrow \frac{1}{j\omega C} = -j$$

$$Z = 1 + j2 \parallel (2 - j) = 1 + \frac{j2(2 - j)}{j2 + 2 - j} = 2.6 + j1.2$$

$$Y = 1/Z = \underline{0.3171 - j0.1463 \text{ S}}$$

Chapter 9, Solution 58.

Using Fig. 9.65, design a problem to help other students to better understand impedance combinations.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

At $\omega = 50$ rad/s, determine Z_{in} for each of the circuits in Fig. 9.65.

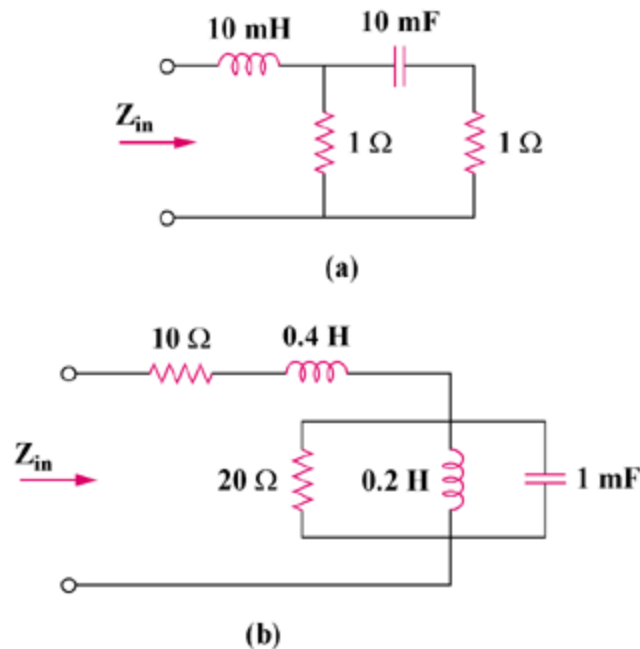


Figure 9.65

Solution

$$\begin{aligned} \text{(a)} \quad 10 \text{ mF} &\longrightarrow \frac{1}{j\omega C} = \frac{1}{j(50)(10 \times 10^{-3})} = -j2 \\ 10 \text{ mH} &\longrightarrow j\omega L = j(50)(10 \times 10^{-3}) = j0.5 \end{aligned}$$

$$Z_{in} = j0.5 + 1 \parallel (1 - j2)$$

$$Z_{in} = j0.5 + \frac{1 - j2}{2 - j2}$$

$$Z_{in} = j0.5 + 0.25(3 - j)$$

$$Z_{in} = \mathbf{0.75 + j0.25 \Omega}$$

$$\begin{aligned}
 \text{(b)} \quad 0.4 \text{ H} &\longrightarrow j\omega L = j(50)(0.4) = j20 \\
 0.2 \text{ H} &\longrightarrow j\omega L = j(50)(0.2) = j10 \\
 1 \text{ mF} &\longrightarrow \frac{1}{j\omega C} = \frac{1}{j(50)(1 \times 10^{-3})} = -j20
 \end{aligned}$$

For the parallel elements,

$$\frac{1}{\mathbf{Z}_p} = \frac{1}{20} + \frac{1}{j10} + \frac{1}{-j20}$$

$$\mathbf{Z}_p = 10 + j10$$

Then,

$$\mathbf{Z}_{in} = 10 + j20 + \mathbf{Z}_p = \mathbf{20 + j30 \, \Omega}$$

Chapter 9, Solution 59.

$$0.25F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10 \times 0.25} = -j0.4$$

$$0.5H \longrightarrow j\omega L = j10 \times 0.5 = j5$$

$$Z_{\text{in}} = j5 \parallel (5 - j0.4) = \frac{(5 \angle 90^\circ)(5.016 \angle -4.57^\circ)}{6.794 \angle 42.61^\circ} = 3.691 \angle 42.82^\circ$$

$$= \mathbf{(2.707 + j2.509) \, \Omega}.$$

Chapter 9, Solution 60.

$$Z = (25 + j15) + (20 - j50) // (30 + j10) = 25 + j15 + 26.097 - j5.122$$

$$Z = (51.1 + j9.878) \, \Omega$$

Chapter 9, Solution 61.

All of the impedances are in parallel.

$$\frac{1}{\mathbf{Z}_{\text{eq}}} = \frac{1}{1-j} + \frac{1}{1+j2} + \frac{1}{j5} + \frac{1}{1+j3}$$

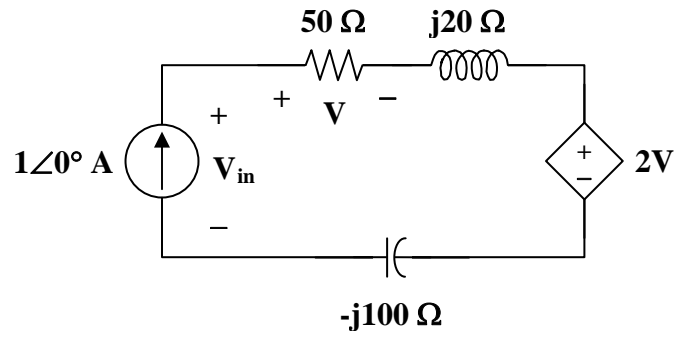
$$\frac{1}{\mathbf{Z}_{\text{eq}}} = (0.5 + j0.5) + (0.2 - j0.4) + (-j0.2) + (0.1 - j0.3) = 0.8 - j0.4$$

$$\mathbf{Z}_{\text{eq}} = \frac{1}{0.8 - j0.4} = \mathbf{(1 + j0.5) \, \Omega}$$

Chapter 9, Solution 62.

$$2 \text{ mH} \longrightarrow j\omega L = j(10 \times 10^3)(2 \times 10^{-3}) = j20$$

$$1 \text{ } \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10 \times 10^3)(1 \times 10^{-6})} = -j100$$



$$V = (1\angle 0^\circ)(50) = 50$$

$$V_{\text{in}} = (1\angle 0^\circ)(50 + j20 - j100) + (2)(50)$$

$$V_{\text{in}} = 50 - j80 + 100 = 150 - j80$$

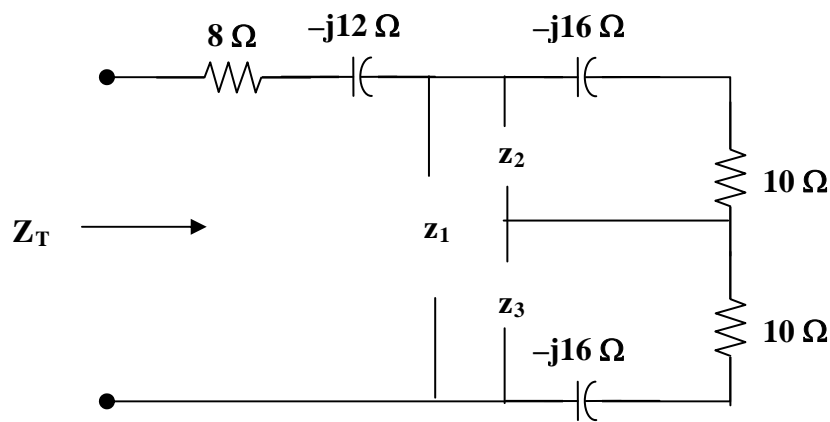
$$Z_{\text{in}} = \frac{V_{\text{in}}}{1\angle 0^\circ} = \mathbf{150 - j80 \text{ } \Omega}$$

Chapter 9, Solution 63.

First, replace the wye composed of the 20-ohm, 10-ohm, and j15-ohm impedances with the corresponding delta.

$$z_1 = \frac{200 + j150 + j300}{10} = 20 + j45$$

$$z_2 = \frac{200 + j450}{j15} = 30 - j13.333, \quad z_3 = \frac{200 + j450}{20} = 10 + j22.5$$



Now all we need to do is to combine impedances.

$$z_2 \parallel (10 - j16) = \frac{(30 - j13.333)(10 - j16)}{40 - j29.333} = 8.721 - j8.938$$

$$z_3 \parallel (10 - j16) = 21.70 - j3.821$$

$$Z_T = 8 - j12 + z_1 \parallel (8.721 - j8.938 + 21.7 - j3.821) = \underline{34.69 - j6.93\Omega}$$

Chapter 9, Solution 64.

$$Z_T = 4 + \frac{-j10(6 + j8)}{6 - j2} = \underline{19 - j5\Omega}$$

$$I = \frac{30\angle 90^\circ}{Z_T} = -0.3866 + j1.4767 = \underline{1.527\angle 104.7^\circ \text{ A}}$$

$$Z_T = \mathbf{(19-j5) \Omega}$$

$$I = \mathbf{1.527\angle 104.7^\circ \text{ A}}$$

Chapter 9, Solution 65.

$$\mathbf{Z}_T = 2 + (4 - j6) \parallel (3 + j4)$$

$$\mathbf{Z}_T = 2 + \frac{(4 - j6)(3 + j4)}{7 - j2}$$

$$\mathbf{Z}_T = \mathbf{6.83 + j1.094 \, \Omega} = 6.917 \angle 9.1^\circ \, \Omega$$

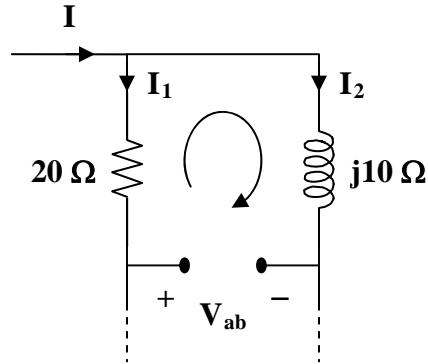
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{120 \angle 10^\circ}{6.917 \angle 9.1^\circ} = \mathbf{17.35 \angle 0.9^\circ \, A}$$

Chapter 9, Solution 66.

$$\mathbf{Z}_T = (20 - j5) \parallel (40 + j10) = \frac{(20 - j5)(40 + j10)}{60 + j5} = \frac{170}{145}(12 - j)$$

$$\mathbf{Z}_T = \mathbf{14.069} - \mathbf{j1.172} \, \Omega = 14.118 \angle -4.76^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{60 \angle 90^\circ}{14.118 \angle -4.76^\circ} = 4.25 \angle 94.76^\circ$$



$$\mathbf{I}_1 = \frac{40 + j10}{60 + j5} \mathbf{I} = \frac{8 + j2}{12 + j} \mathbf{I}$$

$$\mathbf{I}_2 = \frac{20 - j5}{60 + j5} \mathbf{I} = \frac{4 - j}{12 + j} \mathbf{I}$$

$$\mathbf{V}_{ab} = -20\mathbf{I}_1 + j10\mathbf{I}_2$$

$$\mathbf{V}_{ab} = \frac{-(160 + j40)}{12 + j} \mathbf{I} + \frac{10 + j40}{12 + j} \mathbf{I}$$

$$\mathbf{V}_{ab} = \frac{-150}{12 + j} \mathbf{I} = \frac{(-12 + j)(150)}{145} \mathbf{I}$$

$$\mathbf{V}_{ab} = (12.457 \angle 175.24^\circ)(4.25 \angle 97.76^\circ)$$

$$\mathbf{V}_{ab} = \mathbf{52.94 \angle 273^\circ} \, \mathbf{V}$$

Chapter 9, Solution 67.

$$\begin{aligned} \text{(a)} \quad 20 \text{ mH} &\longrightarrow j\omega L = j(10^3)(20 \times 10^{-3}) = j20 \\ 12.5 \text{ }\mu\text{F} &\longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(12.5 \times 10^{-6})} = -j80 \end{aligned}$$

$$\mathbf{Z_{in}} = 60 + j20 \parallel (60 - j80)$$

$$\mathbf{Z_{in}} = 60 + \frac{(j20)(60 - j80)}{60 - j60}$$

$$\mathbf{Z_{in}} = 63.33 + j23.33 = 67.494 \angle 20.22^\circ$$

$$\mathbf{Y_{in}} = \frac{1}{\mathbf{Z_{in}}} = \mathbf{14.8 \angle -20.22^\circ \text{ mS}}$$

$$\begin{aligned} \text{(b)} \quad 10 \text{ mH} &\longrightarrow j\omega L = j(10^3)(10 \times 10^{-3}) = j10 \\ 20 \text{ }\mu\text{F} &\longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(20 \times 10^{-6})} = -j50 \\ 30 \parallel 60 &= 20 \end{aligned}$$

$$\mathbf{Z_{in}} = -j50 + 20 \parallel (40 + j10)$$

$$\mathbf{Z_{in}} = -j50 + \frac{(20)(40 + j10)}{60 + j10} = -j50 + 20(41.231 \angle 14.036^\circ) / (60.828 \angle 9.462^\circ)$$

$$= -j50 + (13.5566 \angle 4.574^\circ) = -j50 + 13.51342 + j1.08109$$

$$= 13.51342 - j48.9189 = 50.751 \angle -74.56^\circ$$

$$\mathbf{Z_{in}} = 13.5 - j48.92 = 50.75 \angle -74.56^\circ$$

$$\mathbf{Y_{in}} = \frac{1}{\mathbf{Z_{in}}} = \mathbf{19.704 \angle 74.56^\circ \text{ mS}} = 5.246 + j18.993 \text{ mS}$$

Chapter 9, Solution 68.

$$\mathbf{Y}_{\text{eq}} = \frac{1}{5 - j2} + \frac{1}{3 + j} + \frac{1}{-j4}$$

$$\mathbf{Y}_{\text{eq}} = (0.1724 + j0.069) + (0.3 - j0.1) + (j0.25)$$

$$\mathbf{Y}_{\text{eq}} = \mathbf{(472.4 + j219) \text{ mS}}$$

Chapter 9, Solution 69.

$$\frac{1}{\mathbf{Y}_o} = \frac{1}{4} + \frac{1}{-j2} = \frac{1}{4}(1 + j2)$$

$$\mathbf{Y}_o = \frac{4}{1 + j2} = \frac{(4)(1 - j2)}{5} = 0.8 - j1.6$$

$$\mathbf{Y}_o + j = 0.8 - j0.6$$

$$\frac{1}{\mathbf{Y}_o'} = \frac{1}{1} + \frac{1}{-j3} + \frac{1}{0.8 - j0.6} = (1) + (j0.333) + (0.8 + j0.6)$$

$$\frac{1}{\mathbf{Y}_o'} = 1.8 + j0.933 = 2.028 \angle 27.41^\circ$$

$$\mathbf{Y}_o' = 0.4932 \angle -27.41^\circ = 0.4378 - j0.2271$$

$$\mathbf{Y}_o' + j5 = 0.4378 + j4.773$$

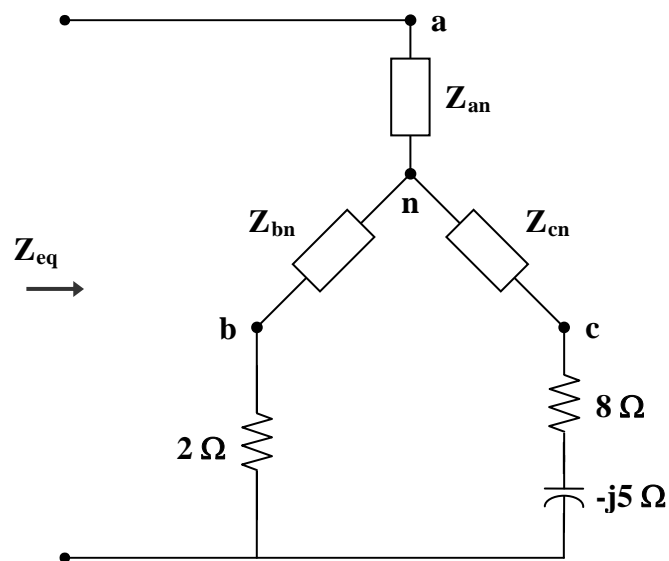
$$\frac{1}{\mathbf{Y}_{eq}} = \frac{1}{2} + \frac{1}{0.4378 + j4.773} = 0.5 + \frac{0.4378 - j4.773}{22.97}$$

$$\frac{1}{\mathbf{Y}_{eq}} = 0.5191 - j0.2078$$

$$\mathbf{Y}_{eq} = \frac{0.5191 - j0.2078}{0.3126} = \mathbf{(1.661 + j0.6647) \text{ S}}$$

Chapter 9, Solution 70.

Make a delta-to-wye transformation as shown in the figure below.



$$Z_{an} = \frac{(-j10)(10 + j15)}{5 - j10 + 10 + j15} = \frac{(10)(15 - j10)}{15 + j5} = 7 - j9$$

$$Z_{bn} = \frac{(5)(10 + j15)}{15 + j5} = 4.5 + j3.5$$

$$Z_{cn} = \frac{(5)(-j10)}{15 + j5} = -1 - j3$$

$$Z_{eq} = Z_{an} + (Z_{bn} + 2) \parallel (Z_{cn} + 8 - j5)$$

$$Z_{eq} = 7 - j9 + (6.5 + j3.5) \parallel (7 - j8)$$

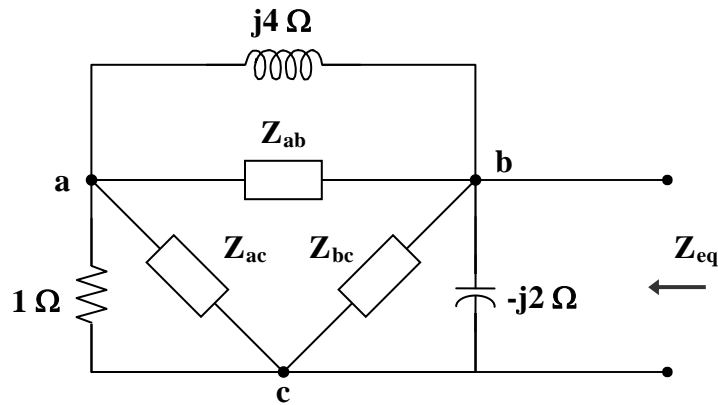
$$Z_{eq} = 7 - j9 + \frac{(6.5 + j3.5)(7 - j8)}{13.5 - j4.5}$$

$$Z_{eq} = 7 - j9 + 5.511 - j0.2$$

$$Z_{eq} = 12.51 - j9.2 = \mathbf{15.53 \angle -36.33^\circ \Omega}$$

Chapter 9, Solution 71.

We apply a wye-to-delta transformation.



$$Z_{ab} = \frac{2 - j2 + j4}{j2} = \frac{2 + j2}{j2} = 1 - j$$

$$Z_{ac} = \frac{2 + j2}{2} = 1 + j$$

$$Z_{bc} = \frac{2 + j2}{-j} = -2 + j2$$

$$j4 \parallel Z_{ab} = j4 \parallel (1 - j) = \frac{(j4)(1 - j)}{1 + j3} = 1.6 - j0.8$$

$$1 \parallel Z_{ac} = 1 \parallel (1 + j) = \frac{(1)(1 + j)}{2 + j} = 0.6 + j0.2$$

$$j4 \parallel Z_{ab} + 1 \parallel Z_{ac} = 2.2 - j0.6$$

$$\frac{1}{Z_{eq}} = \frac{1}{-j2} + \frac{1}{-2 + j2} + \frac{1}{2.2 - j0.6}$$

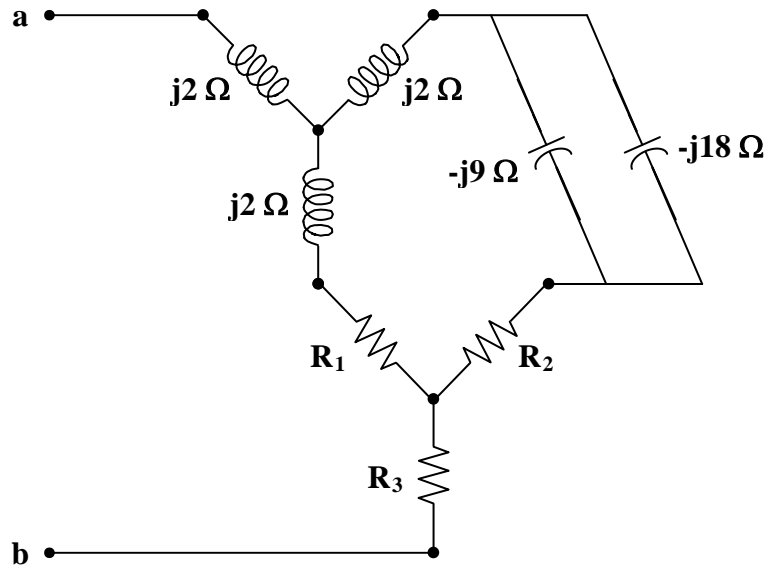
$$= j0.5 - 0.25 - j0.25 + 0.4231 + j0.1154$$

$$= 0.173 + j0.3654 = 0.4043 \angle 64.66^\circ$$

$$Z_{eq} = 2.473 \angle -64.66^\circ \Omega = (1.058 - j2.235) \Omega$$

Chapter 9, Solution 72.

Transform the delta connections to wye connections as shown below.



$$-j9 \parallel -j18 = -j6,$$

$$R_1 = \frac{(20)(20)}{20 + 20 + 10} = 8 \, \Omega,$$

$$R_2 = \frac{(20)(10)}{50} = 4 \, \Omega,$$

$$R_3 = \frac{(20)(10)}{50} = 4 \, \Omega$$

$$\mathbf{Z}_{ab} = j2 + (j2 + 8) \parallel (j2 - j6 + 4) + 4$$

$$\mathbf{Z}_{ab} = 4 + j2 + (8 + j2) \parallel (4 - j4)$$

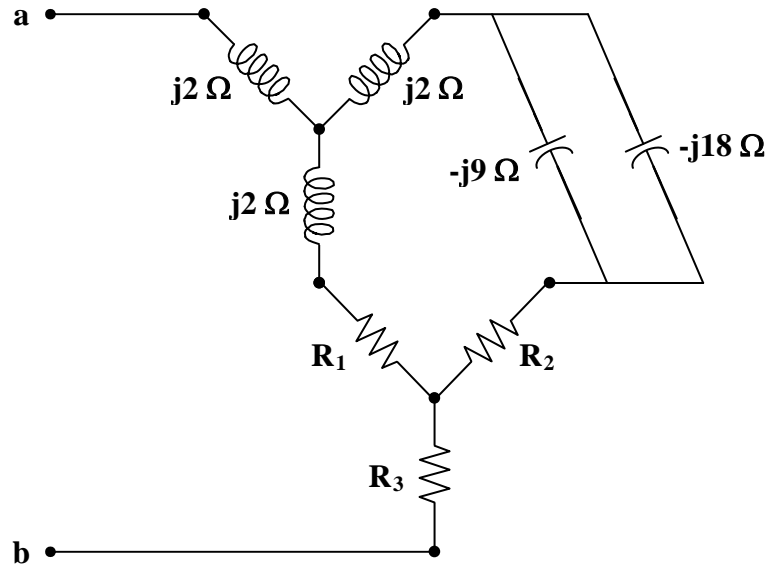
$$\mathbf{Z}_{ab} = 4 + j2 + \frac{(8 + j2)(4 - j4)}{12 - j2}$$

$$\mathbf{Z}_{ab} = 4 + j2 + 3.567 - j1.4054$$

$$\mathbf{Z}_{ab} = \mathbf{(7.567 + j0.5946) \, \Omega}$$

Chapter 9, Solution 73.

Transform the delta connection to a wye connection as in Fig. (a) and then transform the wye connection to a delta connection as in Fig. (b).



$$\mathbf{Z}_1 = \frac{(j8)(-j6)}{j8 + j8 - j6} = \frac{48}{j10} = -j4.8$$

$$\mathbf{Z}_2 = \mathbf{Z}_1 = -j4.8$$

$$\mathbf{Z}_3 = \frac{(j8)(j8)}{j10} = \frac{-64}{j10} = j6.4$$

$$(2 + \mathbf{Z}_1)(4 + \mathbf{Z}_2) + (4 + \mathbf{Z}_2)(\mathbf{Z}_3) + (2 + \mathbf{Z}_1)(\mathbf{Z}_3) =$$

$$(2 - j4.8)(4 - j4.8) + (4 - j4.8)(j6.4) + (2 - j4.8)(j6.4) = 46.4 + j9.6$$

$$\mathbf{Z}_a = \frac{46.4 + j9.6}{j6.4} = 1.5 - j7.25$$

$$\mathbf{Z}_b = \frac{46.4 + j9.6}{4 - j4.8} = 3.574 + j6.688$$

$$\mathbf{Z}_c = \frac{46.4 + j9.6}{2 - j4.8} = 1.727 + j8.945$$

$$j6 \parallel \mathbf{Z}_b = \frac{(6 \angle 90^\circ)(7.583 \angle 61.88^\circ)}{3.574 + j12.688} = 0.7407 + j3.3716$$

$$-j4 \parallel \mathbf{Z}_a = \frac{(-j4)(1.5 - j7.25)}{1.5 - j11.25} = 0.186 - j2.602$$

$$j12 \parallel \mathbf{Z}_c = \frac{(12\angle 90^\circ)(9.11\angle 79.07^\circ)}{1.727 + j20.945} = 0.5634 + j5.1693$$

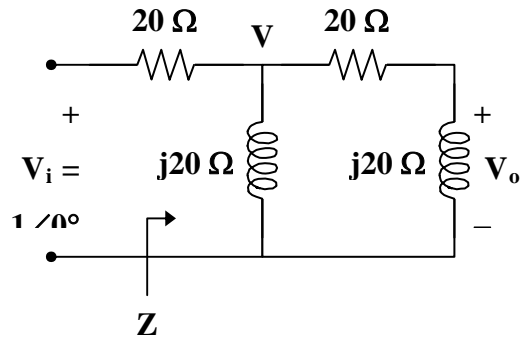
$$\mathbf{Z}_{eq} = (j6 \parallel \mathbf{Z}_b) \parallel (-j4 \parallel \mathbf{Z}_a + j12 \parallel \mathbf{Z}_c)$$

$$\mathbf{Z}_{eq} = (0.7407 + j3.3716) \parallel (0.7494 + j2.5673)$$

$$\mathbf{Z}_{eq} = 1.508\angle 75.42^\circ \Omega = \mathbf{(0.3796 + j1.46) \Omega}$$

Chapter 9, Solution 74.

One such RL circuit is shown below.



We now want to show that this circuit will produce a 90° phase shift.

$$\mathbf{Z} = j20 \parallel (20 + j20) = \frac{(j20)(20 + j20)}{20 + j40} = \frac{-20 + j20}{1 + j2} = 4(1 + j3)$$

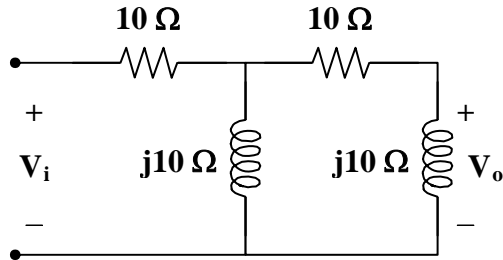
$$\mathbf{V} = \frac{\mathbf{Z}}{\mathbf{Z} + 20} \mathbf{V}_i = \frac{4 + j12}{24 + j12} (1 \angle 0^\circ) = \frac{1 + j3}{6 + j3} = \frac{1}{3}(1 + j)$$

$$\mathbf{V}_o = \frac{j20}{20 + j20} \mathbf{V} = \left(\frac{j}{1 + j} \right) \left(\frac{1}{3}(1 + j) \right) = \frac{j}{3} = 0.3333 \angle 90^\circ$$

This shows that the output leads the input by 90° .

Chapter 9, Solution 75.

Since $\cos(\omega t) = \sin(\omega t + 90^\circ)$, we need a phase shift circuit that will cause the output to lead the input by 90° . **This is achieved by the RL circuit shown below, as explained in the previous problem.**



This can also be obtained by an RC circuit.

Chapter 9, Solution 76.

(a) $v_2 = 8 \sin 5t = 8 \cos(5t - 90^\circ)$
 v_1 leads v_2 by 70° .

(b) $v_2 = 6 \sin 2t = 6 \cos(2t - 90^\circ)$
 v_1 leads v_2 by 180° .

(c) $v_1 = -4 \cos 10t = 4 \cos(10t + 180^\circ)$
 $v_2 = 15 \sin 10t = 15 \cos(10t - 90^\circ)$
 v_1 leads v_2 by 270° .

Chapter 9, Solution 77.

$$(a) \quad \mathbf{V}_o = \frac{-jX_c}{R - jX_c} \mathbf{V}_i$$

$$\text{where } X_c = \frac{1}{\omega C} = \frac{1}{(2\pi)(2 \times 10^6)(20 \times 10^{-9})} = 3.979$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-j3.979}{5 - j3.979} = \frac{3.979}{\sqrt{5^2 + 3.979^2}} \angle(-90^\circ + \tan^{-1}(3.979/5))$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{3.979}{\sqrt{25 + 15.83}} \angle(-90^\circ - 38.51^\circ)$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = 0.6227 \angle -51.49^\circ$$

Therefore, the phase shift is **51.49° lagging**

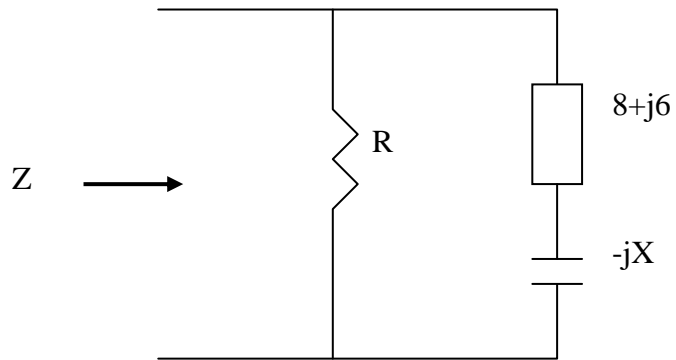
$$(b) \quad \theta = -45^\circ = -90^\circ + \tan^{-1}(X_c/R)$$

$$45^\circ = \tan^{-1}(X_c/R) \longrightarrow R = X_c = \frac{1}{\omega C}$$

$$\omega = 2\pi f = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(5)(20 \times 10^{-9})} = \mathbf{1.5915 \text{ MHz}}$$

Chapter 9, Solution 78.



$$Z = R // [8 + j(6 - X)] = \frac{R[8 + j(6 - X)]}{R + 8 + j(6 - X)} = 5$$

$$\text{i.e. } 8R + j6R - jXR = 5R + 40 + j30 - j5X$$

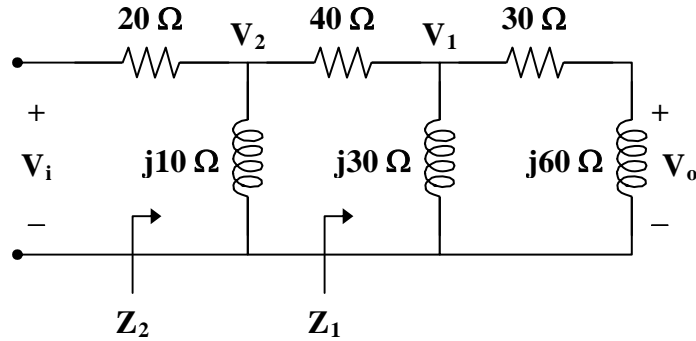
Equating real and imaginary parts:

$$8R = 5R + 40 \quad \text{which leads to} \quad \mathbf{R=13.333\Omega}$$

$$6R - XR = 30 - 5X \quad \text{which leads to} \quad \mathbf{X= 6 \Omega.}$$

Chapter 9, Solution 79.

- (a) Consider the circuit as shown.



$$Z_1 = j30 \parallel (30 + j60) = \frac{(j30)(30 + j60)}{30 + j90} = 3 + j21$$

$$Z_2 = j10 \parallel (40 + Z_1) = \frac{(j10)(43 + j21)}{43 + j31} = 1.535 + j8.896 = 9.028 \angle 80.21^\circ$$

Let $V_i = 1 \angle 0^\circ$.

$$V_2 = \frac{Z_2}{Z_2 + 20} V_i = \frac{(9.028 \angle 80.21^\circ)(1 \angle 0^\circ)}{21.535 + j8.896}$$

$$V_2 = 0.3875 \angle 57.77^\circ$$

$$V_1 = \frac{Z_1}{Z_1 + 40} V_2 = \frac{3 + j21}{43 + j21} V_2 = \frac{(21.213 \angle 81.87^\circ)(0.3875 \angle 57.77^\circ)}{47.85 \angle 26.03^\circ}$$

$$V_1 = 0.1718 \angle 113.61^\circ$$

$$V_o = \frac{j60}{30 + j60} V_1 = \frac{j2}{1 + j2} V_1 = \frac{2}{5} (2 + j) V_1$$

$$V_o = (0.8944 \angle 26.56^\circ)(0.1718 \angle 113.6^\circ)$$

$$V_o = 0.1536 \angle 140.2^\circ$$

Therefore, the phase shift is **140.2°**

- (b) The phase shift is **leading**.

- (c) If $V_i = 120\text{ V}$, then

$$V_o = (120)(0.1536 \angle 140.2^\circ) = 18.43 \angle 140.2^\circ\text{ V}$$

and the magnitude is **18.43 V**.

Chapter 9, Solution 80.

$$200 \text{ mH} \longrightarrow j\omega L = j(2\pi)(60)(200 \times 10^{-3}) = j75.4 \, \Omega$$

$$\mathbf{V_o} = \frac{j75.4}{R + 50 + j75.4} \mathbf{V_i} = \frac{j75.4}{R + 50 + j75.4} (120 \angle 0^\circ)$$

(a) When $R = 100 \, \Omega$,

$$\mathbf{V_o} = \frac{j75.4}{150 + j75.4} (120 \angle 0^\circ) = \frac{(75.4 \angle 90^\circ)(120 \angle 0^\circ)}{167.88 \angle 26.69^\circ}$$

$$\mathbf{V_o} = \mathbf{53.89 \angle 63.31^\circ \text{ V}}$$

(b) When $R = 0 \, \Omega$,

$$\mathbf{V_o} = \frac{j75.4}{50 + j75.4} (120 \angle 0^\circ) = \frac{(75.4 \angle 90^\circ)(120 \angle 0^\circ)}{90.47 \angle 56.45^\circ}$$

$$\mathbf{V_o} = \mathbf{100 \angle 33.55^\circ \text{ V}}$$

(c) To produce a phase shift of 45° , the phase of $\mathbf{V_o} = 90^\circ + 0^\circ - \alpha = 45^\circ$.

Hence, $\alpha = \text{phase of } (R + 50 + j75.4) = 45^\circ$.

For α to be 45° , $R + 50 = 75.4$

Therefore, $R = \mathbf{25.4 \, \Omega}$

Chapter 9, Solution 81.

$$\text{Let } \mathbf{Z}_1 = \mathbf{R}_1, \quad \mathbf{Z}_2 = \mathbf{R}_2 + \frac{1}{j\omega C_2}, \quad \mathbf{Z}_3 = \mathbf{R}_3, \text{ and } \mathbf{Z}_x = \mathbf{R}_x + \frac{1}{j\omega C_x}.$$

$$\mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1} \mathbf{Z}_2$$

$$\mathbf{R}_x + \frac{1}{j\omega C_x} = \frac{\mathbf{R}_3}{\mathbf{R}_1} \left(\mathbf{R}_2 + \frac{1}{j\omega C_2} \right)$$

$$\mathbf{R}_x = \frac{\mathbf{R}_3}{\mathbf{R}_1} \mathbf{R}_2 = \frac{1200}{400} (600) = \mathbf{1.8 \text{ k}\Omega}$$

$$\frac{1}{C_x} = \left(\frac{\mathbf{R}_3}{\mathbf{R}_1} \right) \left(\frac{1}{C_2} \right) \longrightarrow C_x = \frac{\mathbf{R}_1}{\mathbf{R}_3} C_2 = \left(\frac{400}{1200} \right) (0.3 \times 10^{-6}) = \mathbf{0.1 \text{ }\mu\text{F}}$$

Chapter 9, Solution 82.

$$C_x = \frac{R_1}{R_2} C_s = \left(\frac{100}{2000} \right) (40 \times 10^{-6}) = \mathbf{2 \mu F}$$

Chapter 9, Solution 83.

$$L_x = \frac{R_2}{R_1} L_s = \left(\frac{500}{1200} \right) (250 \times 10^{-3}) = \mathbf{104.17 \text{ mH}}$$

Chapter 9, Solution 84.

$$\text{Let } \mathbf{Z}_1 = R_1 \parallel \frac{1}{j\omega C_s}, \quad \mathbf{Z}_2 = R_2, \quad \mathbf{Z}_3 = R_3, \text{ and } \mathbf{Z}_x = R_x + j\omega L_x.$$

$$\mathbf{Z}_1 = \frac{\frac{R_1}{j\omega C_s}}{R_1 + \frac{1}{j\omega C_s}} = \frac{R_1}{j\omega R_1 C_s + 1}$$

$$\text{Since } \mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1} \mathbf{Z}_2,$$

$$R_x + j\omega L_x = R_2 R_3 \frac{j\omega R_1 C_s + 1}{R_1} = \frac{R_2 R_3}{R_1} (1 + j\omega R_1 C_s)$$

Equating the real and imaginary components,

$$\mathbf{R}_x = \frac{\mathbf{R}_2 \mathbf{R}_3}{\mathbf{R}_1}$$

$$\omega L_x = \frac{R_2 R_3}{R_1} (\omega R_1 C_s) \text{ implies that}$$

$$\mathbf{L}_x = \mathbf{R}_2 \mathbf{R}_3 \mathbf{C}_s$$

Given that $R_1 = 40 \text{ k}\Omega$, $R_2 = 1.6 \text{ k}\Omega$, $R_3 = 4 \text{ k}\Omega$, and $C_s = 0.45 \text{ }\mu\text{F}$

$$R_x = \frac{R_2 R_3}{R_1} = \frac{(1.6)(4)}{40} \text{ k}\Omega = 0.16 \text{ k}\Omega = \mathbf{160 \text{ }\Omega}$$

$$L_x = R_2 R_3 C_s = (1.6)(4)(0.45) = \mathbf{2.88 \text{ H}}$$

Chapter 9, Solution 85.

$$\text{Let } \mathbf{Z}_1 = R_1, \quad \mathbf{Z}_2 = R_2 + \frac{1}{j\omega C_2}, \quad \mathbf{Z}_3 = R_3, \text{ and } \mathbf{Z}_4 = R_4 \parallel \frac{1}{j\omega C_4}.$$

$$\mathbf{Z}_4 = \frac{R_4}{j\omega R_4 C_4 + 1} = \frac{-jR_4}{\omega R_4 C_4 - j}$$

$$\text{Since } \mathbf{Z}_4 = \frac{\mathbf{Z}_3}{\mathbf{Z}_1} \mathbf{Z}_2 \longrightarrow \mathbf{Z}_1 \mathbf{Z}_4 = \mathbf{Z}_2 \mathbf{Z}_3,$$

$$\begin{aligned} \frac{-jR_4 R_1}{\omega R_4 C_4 - j} &= R_3 \left(R_2 - \frac{j}{\omega C_2} \right) \\ \frac{-jR_4 R_1 (\omega R_4 C_4 + j)}{\omega^2 R_4^2 C_4^2 + 1} &= R_3 R_2 - \frac{jR_3}{\omega C_2} \end{aligned}$$

Equating the real and imaginary components,

$$\frac{R_1 R_4}{\omega^2 R_4^2 C_4^2 + 1} = R_2 R_3 \quad (1)$$

$$\frac{\omega R_1 R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1} = \frac{R_3}{\omega C_2} \quad (2)$$

Dividing (1) by (2),

$$\begin{aligned} \frac{1}{\omega R_4 C_4} &= \omega R_2 C_2 \\ \omega^2 &= \frac{1}{R_2 C_2 R_4 C_4} \\ \omega &= 2\pi f = \frac{1}{\sqrt{R_2 C_2 R_4 C_4}} \\ f &= \frac{1}{2\pi \sqrt{R_2 R_4 C_2 C_4}} \end{aligned}$$

Chapter 9, Solution 86.

$$\mathbf{Y} = \frac{1}{240} + \frac{1}{j95} + \frac{1}{-j84}$$

$$\mathbf{Y} = 4.1667 \times 10^{-3} - j0.01053 + j0.0119$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1000}{4.1667 + j1.37} = \frac{1000}{4.3861 \angle 18.2^\circ}$$

$$\mathbf{Z} = 228 \angle -18.2^\circ \Omega$$

Chapter 9, Solution 87.

$$\mathbf{Z}_1 = 50 + \frac{1}{j\omega C} = 50 + \frac{-j}{(2\pi)(2 \times 10^3)(2 \times 10^{-6})}$$

$$\mathbf{Z}_1 = 50 - j39.79$$

$$\mathbf{Z}_2 = 80 + j\omega L = 80 + j(2\pi)(2 \times 10^3)(10 \times 10^{-3})$$

$$\mathbf{Z}_2 = 80 + j125.66$$

$$\mathbf{Z}_3 = 100$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{100} + \frac{1}{50 - j39.79} + \frac{1}{80 + j125.66}$$

$$\begin{aligned}\frac{1}{\mathbf{Z}} &= 10^{-3} (10 + 12.24 + j9.745 + 3.605 - j5.663) \\ &= (25.85 + j4.082) \times 10^{-3}\end{aligned}$$

$$= 26.17 \times 10^{-3} \angle 8.97^\circ$$

$$\mathbf{Z} = 38.21 \angle -8.97^\circ \Omega$$

Chapter 9, Solution 88.

(a) $\mathbf{Z} = -j20 + j30 + 120 - j20$
 $\mathbf{Z} = (120 - j10) \, \Omega$

(b) If the frequency were halved, $\frac{1}{\omega C} = \frac{1}{2\pi f C}$ would cause the capacitive impedance to double, while $\omega L = 2\pi f L$ would cause the inductive impedance to halve. Thus,

$$\mathbf{Z} = -j40 + j15 + 120 - j40$$
$$\mathbf{Z} = (120 - j65) \, \Omega$$

Chapter 9, Solution 89.

An industrial load is modeled as a series combination of an inductor and a resistance as shown in Fig. 9.89. Calculate the value of a capacitor C across the series combination so that the net impedance is resistive at a frequency of 2 kHz.

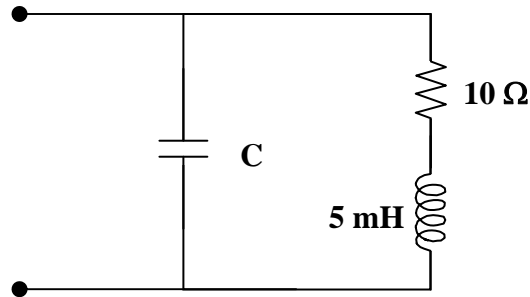


Figure 9.89
For Prob. 9.89.

Solution

Step 1.

There are different ways to solve this problem but perhaps the easiest way is to convert the series R L elements into their parallel equivalents. Then all you need to do is to make the inductance and capacitance cancel each other out to result in a purely resistive circuit.

$X_L = 2 \times 10^3 \times 5 \times 10^{-3} = 10$ which leads to $Y = 1/(10 + j10) = 0.05 - j0.05$ or a 20Ω resistor in parallel with a $j20\Omega$ inductor. $X_C = 1/(2 \times 10^3 C)$ and the parallel combination of the capacitor and inductor is equal to,

$$[(-jX_C)(j20)/(-jX_C + j20)].$$

Step 2.

Now we just need to set $X_C = 20 = 1/(2 \times 10^3 C)$ which will create an open circuit.

$$C = 1/(20 \times 2 \times 10^3) = \mathbf{25 \mu F}.$$

Chapter 9, Solution 90.

$$\text{Let } \mathbf{V}_s = 145\angle 0^\circ, \quad X = \omega L = (2\pi)(60)L = 377L$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{80 + R + jX} = \frac{145\angle 0^\circ}{80 + R + jX}$$

$$\mathbf{V}_1 = 80\mathbf{I} = \frac{(80)(145)}{80 + R + jX}$$

$$50 = \left| \frac{(80)(145)}{80 + R + jX} \right| \quad (1)$$

$$\mathbf{V}_o = (R + jX)\mathbf{I} = \frac{(R + jX)(145\angle 0^\circ)}{80 + R + jX}$$

$$110 = \left| \frac{(R + jX)(145)}{80 + R + jX} \right| \quad (2)$$

From (1) and (2),

$$\begin{aligned} \frac{50}{110} &= \frac{80}{|R + jX|} \\ |R + jX| &= (80)\left(\frac{11}{5}\right) \\ R^2 + X^2 &= 30976 \end{aligned} \quad (3)$$

From (1),

$$\begin{aligned} |80 + R + jX| &= \frac{(80)(145)}{50} = 232 \\ 6400 + 160R + R^2 + X^2 &= 53824 \\ 160R + R^2 + X^2 &= 47424 \end{aligned} \quad (4)$$

Subtracting (3) from (4),

$$160R = 16448 \longrightarrow R = \mathbf{102.8 \, \Omega}$$

From (3),

$$\begin{aligned} X^2 &= 30976 - 10568 = 20408 \\ X &= 142.86 = 377L \longrightarrow L = \mathbf{378.9 \, mH} \end{aligned}$$

Chapter 9, Solution 91.

$$\begin{aligned}\mathbf{Z}_{\text{in}} &= \frac{1}{j\omega C} + R \parallel j\omega L \\ \mathbf{Z}_{\text{in}} &= \frac{-j}{\omega C} + \frac{j\omega LR}{R + j\omega L} \\ &= \frac{-j}{\omega C} + \frac{\omega^2 L^2 R + j\omega LR^2}{R^2 + \omega^2 L^2}\end{aligned}$$

To have a resistive impedance, $\text{Im}(\mathbf{Z}_{\text{in}}) = 0$.

Hence,

$$\begin{aligned}\frac{-1}{\omega C} + \frac{\omega LR^2}{R^2 + \omega^2 L^2} &= 0 \\ \frac{1}{\omega C} &= \frac{\omega LR^2}{R^2 + \omega^2 L^2} \\ C &= \frac{R^2 + \omega^2 L^2}{\omega^2 LR^2}\end{aligned}$$

where $\omega = 2\pi f = 2\pi \times 10^7$

$$\begin{aligned}C &= \frac{9 \times 10^4 + (4\pi^2 \times 10^{14})(400 \times 10^{-12})}{(4\pi^2 \times 10^{14})(20 \times 10^{-6})(9 \times 10^4)} \\ C &= \frac{9 + 16\pi^2}{72\pi^2} \text{ nF}\end{aligned}$$

$$\mathbf{C = 235 \text{ pF}}$$

Chapter 9, Solution 92.

$$(a) \quad Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{100\angle 75^\circ}{450\angle 48^\circ \times 10^{-6}}} = \underline{471.4\angle 13.5^\circ \Omega}$$

$$(b) \quad \gamma = \sqrt{ZY} = \sqrt{100\angle 75^\circ \times 450\angle 48^\circ \times 10^{-6}} = \underline{212.1\angle 61.5^\circ mS}$$

Chapter 9, Solution 93.

$$\mathbf{Z} = \mathbf{Z}_s + 2\mathbf{Z}_\ell + \mathbf{Z}_L$$

$$\mathbf{Z} = (1 + 0.8 + 23.2) + j(0.5 + 0.6 + 18.9)$$

$$\mathbf{Z} = 25 + j20$$

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{115\angle 0^\circ}{32.02\angle 38.66^\circ}$$

$$\mathbf{I}_L = 3.592\angle -38.66^\circ \text{ A}$$