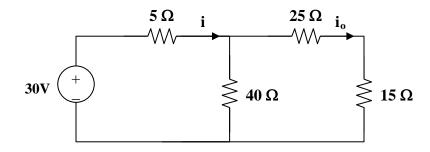
## Chapter 4, Solution 1.



$$40 \big\| (25+15) = 20 \Omega \;,\;\; i = [30/(5+20)] = 1.2 \; and \; i_o = i20/40 = \; \textbf{600 mA}.$$

Since the resistance remains the same we get can use linearity to find the new value of the voltage source = (30/0.6)5 = 250 V.

**4.2** Using Fig. 4.70, design a problem to help other students better understand linearity.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

#### **Problem**

Find  $v_o$  in the circuit of Fig. 4.70. If the source current is reduced to 1  $\mu$ A, what is  $v_o$ ?

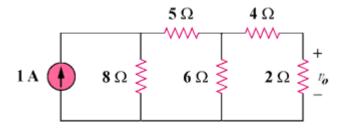


Figure 4.70

#### **Solution**

$$6 \| (4+2) = 3\Omega, \quad \mathbf{i}_1 = \mathbf{i}_2 = \frac{1}{2} \mathbf{A}$$

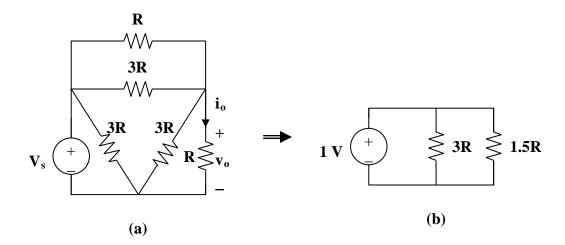
$$\mathbf{i}_o = \frac{1}{2} \mathbf{i}_1 = \frac{1}{4}, \quad \mathbf{v}_o = 2\mathbf{i}_o = \mathbf{0.5V}$$

$$\mathbf{5} \Omega \qquad \mathbf{i}_1 \qquad \mathbf{4} \Omega \qquad \mathbf{i}_0$$

$$\mathbf{i}_2 \qquad \mathbf{5} \Omega \qquad \mathbf{5} \Omega \qquad \mathbf{5} \Omega \qquad \mathbf{5} \Omega \qquad \mathbf{5} \Omega$$

If 
$$i_s = 1\mu A$$
, then  $v_o = 0.5\mu V$ 

## Chapter 4, Solution 3.



(a) We transform the Y sub-circuit to the equivalent  $\Delta$ .

$$R||3R = \frac{3R^2}{4R} = \frac{3}{4}R, \ \frac{3}{4}R + \frac{3}{4}R = \frac{3}{2}R$$

$$v_o = \frac{v_s}{2}$$
 independent of R

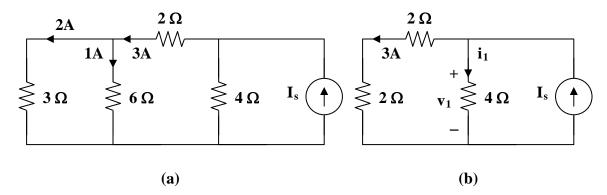
$$i_o = v_o/(R)$$

When 
$$v_s = 1V$$
,  $v_o = 0.5V$ ,  $i_o = 0.5A$ 

- (b) When  $v_s = 10V$ ,  $v_o = 5V$ ,  $i_o = 5A$
- (c) When  $v_s = 10V$  and  $R = 10\Omega$ ,  $v_o = 5V$ ,  $i_o = 10/(10) = 500mA$

## Chapter 4, Solution 4.

If  $I_o$  = 1, the voltage across the  $6\Omega$  resistor is 6V so that the current through the  $3\Omega$  resistor is 2A.

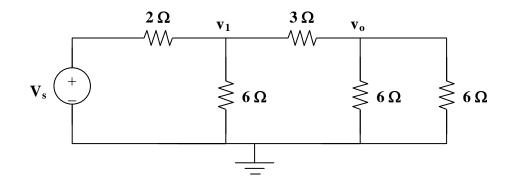


$$3\|6 = 2\Omega$$
,  $v_o = 3(4) = 12V$ ,  $i_1 = \frac{v_o}{4} = 3A$ .

Hence  $I_s = 3 + 3 = 6A$ 

If 
$$I_s = 6A \longrightarrow I_o = 1$$
  
 $I_s = 9A \longrightarrow I_o = 9/6 = 1.5A$ 

## Chapter 4, Solution 5.



If 
$$v_o = 1V$$
,  $V_1 = \left(\frac{1}{3}\right) + 1 = 2V$   
 $V_s = 2\left(\frac{2}{3}\right) + v_1 = \frac{10}{3}$ 

$$\text{If } v_s = \frac{10}{3} \quad \longrightarrow \quad v_o = 1$$

Then 
$$v_s = 15$$
 
$$v_o = \frac{3}{10} x 15 = \textbf{4.5V}$$

# Chapter 4, Solution 6.

Due to linearity, from the first experiment,

$$V_o = \frac{1}{3} V_s$$

Applying this to other experiments, we obtain:

Experiment	$V_s$	Vo
2	48	16 V
3	1 V	0.333 V
4	<u>-6 V</u>	-2V

## Chapter 4, Solution 7.

If  $V_o=1V$ , then the current through the 2- $\Omega$  and 4- $\Omega$  resistors is  $\frac{1}{2}=0.5$ . The voltage across the 3- $\Omega$  resistor is  $\frac{1}{2}(4+2)=3$  V. The total current through the 1- $\Omega$  resistor is 0.5+3/3=1.5 A. Hence the source voltage

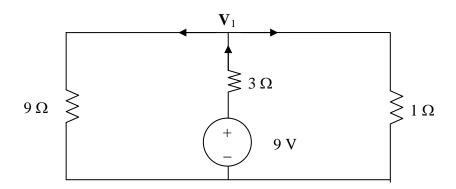
$$V_s = 1x1.5 + 3 = 4.5 \text{ V}$$

If 
$$V_s = 4.5$$
  $\longrightarrow$  1V

Then 
$$V_s = 4$$
  $\longrightarrow \frac{1}{4.5} x4 = \underline{0.8889 \ V} = 888.9 \ \text{mV}.$ 

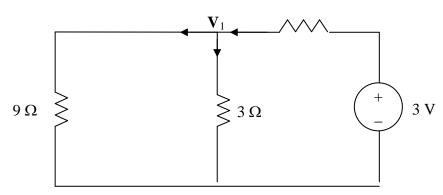
## Chapter 4, Solution 8.

Let  $V_o = V_1 + V_2$ , where  $V_1$  and  $V_2$  are due to 9-V and 3-V sources respectively. To find  $V_1$ , consider the circuit below.



$$\frac{9 - V_1}{3} = \frac{V_1}{9} + \frac{V_1}{1} \longrightarrow V_1 = 27/13 = 2.0769$$

To find V<sub>2</sub>, consider the circuit below.

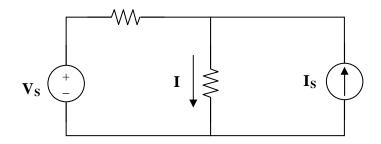


$$\frac{V_2}{9} + \frac{V_2}{3} = \frac{3 - V_2}{1}$$
  $\longrightarrow$   $V_2 = 27/13 = 2.0769$ 

$$V_0 = V_1 + V_2 = 4.1538 V$$

## Chapter 4. Solution 9.

Given that I=4 amps when  $V_s=40$  volts and  $I_s=4$  amps and I=1 amp when  $V_s=20$  volts and  $I_s=0$ , determine the value of I when  $V_s=60$  volts and  $I_s=-2$  amps.



At first this appears to be a difficult problem. However, if you take it one step at a time then it is not as hard as it seems. The important thing to keep in mind is that it is linear!

If I=1 amp when  $V_s=20$  and  $I_s=0$  then I=2 amps when  $V_s=40$  volts and  $I_s=0$  (linearity). This means that I is equal to 2 amps (4–2) when  $I_s=4$  amps and  $V_s=0$  (superposition). Thus,

$$I = (60/20)1 + (-2/4)2 = 3-1 = 2$$
 amps.

#### Chapter 4, Solution 10.

Using Fig. 4.78, design a problem to help other students better understand superposition. Note, the letter k is a gain you can specify to make the problem easier to solve but must not be zero.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

#### **Problem**

For the circuit in Fig. 4.78, find the terminal voltage  $V_{ab}$  using superposition.

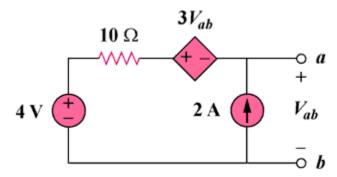
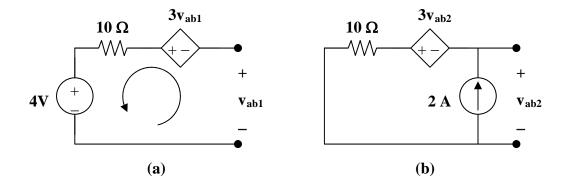


Figure 4.78 For Prob. 4.10.

#### **Solution**

Let  $v_{ab} = v_{ab1} + v_{ab2}$  where  $v_{ab1}$  and  $v_{ab2}$  are due to the 4-V and the 2-A sources respectively.



For v<sub>ab1</sub>, consider Fig. (a). Applying KVL gives,

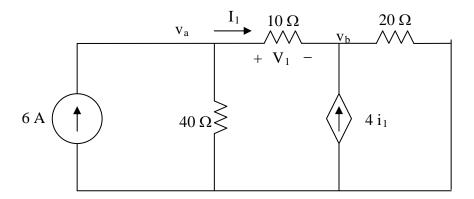
$$-v_{ab1} - 3v_{ab1} + 10x0 + 4 = 0$$
, which leads to  $v_{ab1} = 1 \text{ V}$ 

For v<sub>ab2</sub>, consider Fig. (b). Applying KVL gives,

$$-v_{ab2}-3v_{ab2}+10x2~=~0,~$$
 which leads to  $v_{ab2}~=~5$  
$$v_{ab}~=~1+5~=~\textbf{6}~\textbf{V}$$

#### Chapter 4, Solution 11.

Let  $v_0 = v_1 + v_2$ , where  $v_1$  and  $v_2$  are due to the 6-A and 80-V sources respectively. To find  $v_1$ , consider the circuit below.



At node a,

$$6 = \frac{V_a}{40} + \frac{V_a - V_b}{10} \longrightarrow 240 = 5 V_a - 4 V_b$$
 (1)

At node b,

$$-I_1 - 4I_1 + (v_b - 0)/20 = 0$$
 or  $v_b = 100I_1$ 

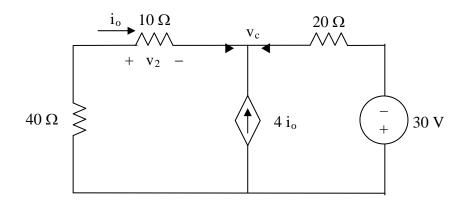
But 
$$i_1 = \frac{V_a - V_b}{10}$$
 which leads to  $100(v_a - v_b)10 = v_b$  or  $v_b = 0.9091v_a$  (2)

Substituting (2) into (1),

$$5v_a - 3.636v_a = 240$$
 or  $v_a = 175.95$  and  $v_b = 159.96$ 

However,  $v_1 = v_a - v_b = 15.99 \text{ V}.$ 

To find  $v_2$ , consider the circuit below.



$$\frac{0 - v_c}{50} + 4i_o + \frac{(-30 - v_c)}{20} = 0$$
But  $i_o = \frac{(0 - v_c)}{50}$ 

$$-\frac{5v_c}{50} - \frac{(30 + v_c)}{20} = 0 \longrightarrow v_c = -10 \text{ V}$$

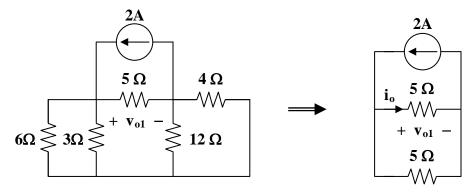
$$i_2 = \frac{0 - v_c}{50} = \frac{0 + 10}{50} = \frac{1}{5}$$

$$v_2 = 10i_2 = 2 \text{ V}$$

 $v_o = v_1 + v_2 \ = 15.99 + 2 = \textbf{17.99 V} \ \text{and} \ i_o = v_o/10 = \textbf{1.799 A}.$ 

#### Chapter 4, Solution 12.

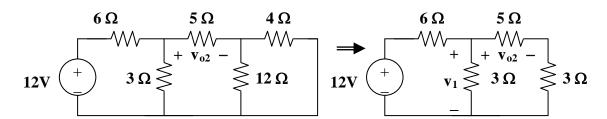
Let  $v_0 = v_{o1} + v_{o2} + v_{o3}$ , where  $v_{o1}$ ,  $v_{o2}$ , and  $v_{o3}$  are due to the 2-A, 12-V, and 19-V sources respectively. For  $v_{o1}$ , consider the circuit below.



$$6||3 = 2 \text{ ohms}, 4||12 = 3 \text{ ohms}.$$
 Hence,

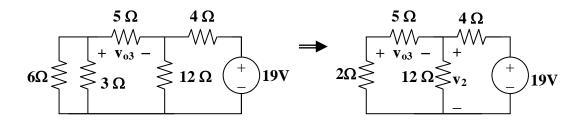
$$i_o = 2/2 = 1, v_{o1} = 5io = 5 V$$

For  $v_{o2}$ , consider the circuit below.



$$3||8 = 24/11, v_1 = [(24/11)/(6 + 24/11)]12 = 16/5$$
  
$$v_{o2} = (5/8)v_1 = (5/8)(16/5) = 2 V$$

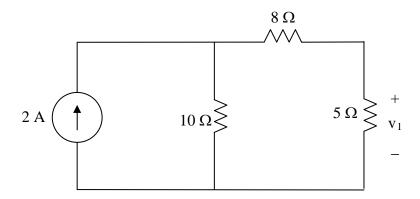
For  $v_{o3}$ , consider the circuit shown below.



$$7||12\>=\>(84/19)$$
 ohms,  $v_2\>=\>[(84/19)/(4+84/19)]19\>=\>9.975$  
$$v\>=\>(-5/7)v2\>=\>-7.125$$
 
$$v_o\>=\>5+2-7.125\>=\>\textbf{-125}\>\textbf{mV}$$

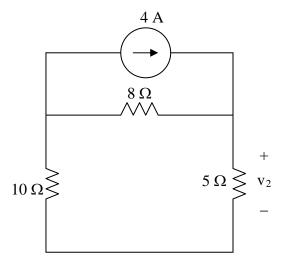
## Chapter 4, Solution 13.

Let  $V_o = V_1 + V_2 + V_3$ , where  $v_1$ ,  $v_2$ , and  $v_3$  are due to the independent sources. To find  $v_1$ , consider the circuit below.



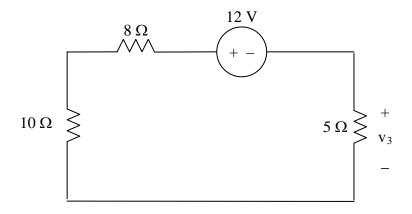
$$v_1 = 5x \frac{10}{10 + 8 + 5} x^2 = 4.3478$$

To find  $v_2$ , consider the circuit below.



$$V_2 = 5x \frac{8}{8 + 10 + 5} x4 = 6.9565$$

To find  $v_3$ , consider the circuit below.

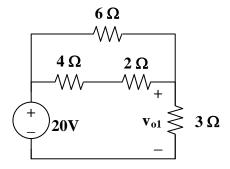


$$V_3 = -12\left(\frac{5}{5+10+8}\right) = -2.6087$$

$$V_o = V_1 + V_2 + V_3 = 8.6956 \text{ V} = 8.696 \text{V}.$$

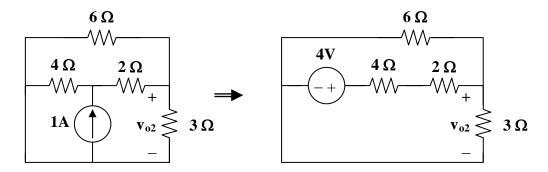
#### Chapter 4, Solution 14.

Let  $v_o = v_{o1} + v_{o2} + v_{o3}$ , where  $v_{o1}$ ,  $v_{o2}$ , and  $v_{o3}$ , are due to the 20-V, 1-A, and 2-A sources respectively. For  $v_{o1}$ , consider the circuit below.



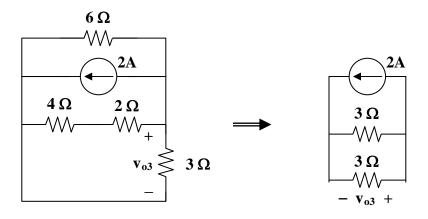
$$6||(4+2)| = 3 \text{ ohms}, v_{o1}| = (\frac{1}{2})20 = 10 \text{ V}$$

For  $v_{o2}$ , consider the circuit below.



$$3||6 = 2 \text{ ohms}, v_{o2} = [2/(4+2+2)]4 = 1 \text{ V}$$

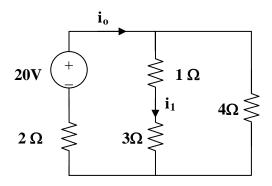
For v<sub>o3</sub>, consider the circuit below.



$$6||(4+2) = 3, v_{o3} = (-1)3 = -3$$
  
 $v_o = 10 + 1 - 3 = 8 V$ 

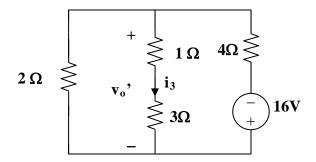
#### Chapter 4, Solution 15.

Let  $i = i_1 + i_2 + i_3$ , where  $i_1$ ,  $i_2$ , and  $i_3$  are due to the 20-V, 2-A, and 16-V sources. For  $i_1$ , consider the circuit below.



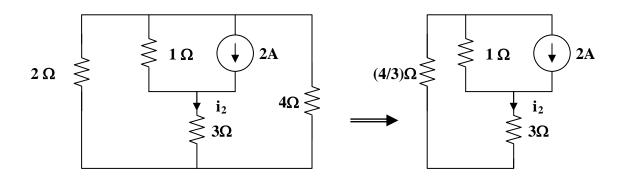
$$4||(3+1)|=2$$
 ohms, Then  $i_o=[20/(2+2)]=5$  A,  $i_1=i_o/2=2.5$  A

For  $i_3$ , consider the circuit below.



$$2||(1+3) = 4/3, v_o' = [(4/3)/((4/3) + 4)](-16) = -4$$
  
$$i_3 = v_o'/4 = -1$$

For i<sub>2</sub>, consider the circuit below.



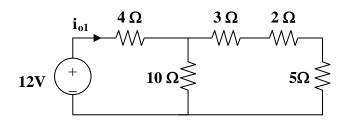
$$2||4 = 4/3, 3 + 4/3 = 13/3$$

Using the current division principle.

$$i_2 = [1/(1 + 13/2)]2 = 3/8 = 0.375$$
  
 $i = 2.5 + 0.375 - 1 = 1.875 \text{ A}$   
 $p = i^2R = (1.875)^23 = 10.55 \text{ watts}$ 

#### Chapter 4, Solution 16.

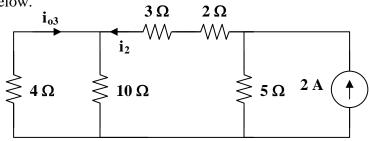
Let  $i_0 = i_{o1} + i_{o2} + i_{o3}$ , where  $i_{o1}$ ,  $i_{o2}$ , and  $i_{o3}$  are due to the 12-V, 4-A, and 2-A sources. For  $i_{o1}$ , consider the circuit below.



$$10||(3+2+5)| = 5$$
 ohms,  $i_{o1} = 12/(5+4) = (12/9)$  A

$$2+5+4||10\>=\>7+40/14\>=\>69/7\\i_1\>=\>[3/(3+69/7)]4\>=\>84/90,\;i_{o2}\>=[-10/(4+10)]i_1\>=\>-6/9$$

For i<sub>03</sub>, consider the circuit below.



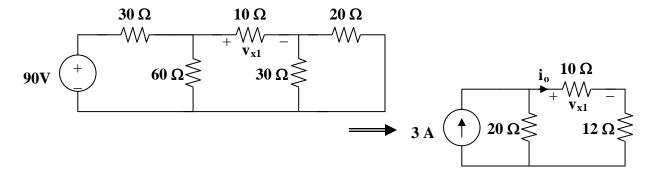
$$3 + 2 + 4||10 = 5 + 20/7 = 55/7$$

$$i_2 \ = \ [5/(5+55/7)]2 \ = \ 7/9, \ i_{o3} \ = \ [-10/(10+4)]i_2 \ = \ -5/9$$

$$i_o = (12/9) - (6/9) - (5/9) = 1/9 = 111.11 \text{ mA}$$

#### Chapter 4, Solution 17.

Let  $v_x = v_{x1} + v_{x2} + v_{x3}$ , where  $v_{x1}, v_{x2}$ , and  $v_{x3}$  are due to the 90-V, 6-A, and 40-V sources. For  $v_{x1}$ , consider the circuit below.

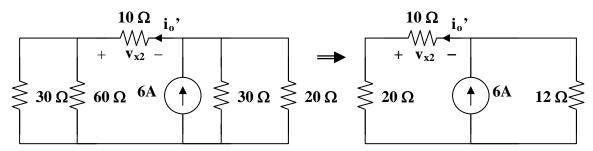


$$20||30 = 12 \text{ ohms}, 60||30 = 20 \text{ ohms}$$

By using current division,

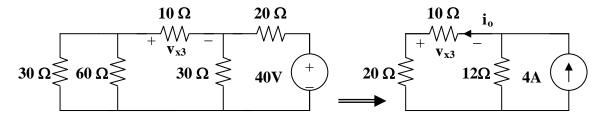
$$i_o = [20/(22 + 20)]3 = 60/42, v_{x1} = 10i_o = 600/42 = 14.286 V$$

For  $v_{x2}$ , consider the circuit below.



$$i_o$$
' =  $[12/(12+30)]6 = 72/42$ ,  $v_{x2} = -10i_o$ ' =  $-17.143$  V

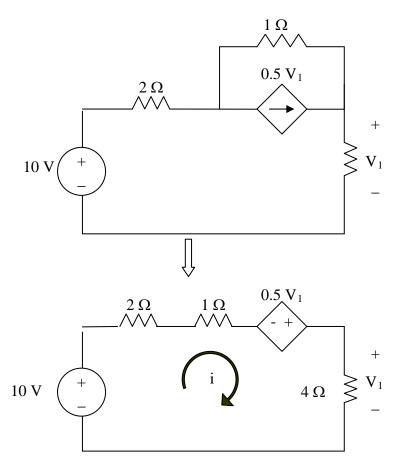
For  $v_{x3}$ , consider the circuit below.



$$\begin{split} i_o" &= [12/(12+30)]2 = 24/42, \ v_{x3} = -10i_o" = -5.714 = [12/(12+30)]2 = 24/42, \\ v_{x3} &= -10i_o" = -5.714 \\ &= [12/(12+30)]2 = 24/42, \ v_{x3} = -10i_o" = -5.714 \\ v_x &= 14.286 - 17.143 - 5.714 = \textbf{-8.571 V} \end{split}$$

## Chapter 4, Solution 18.

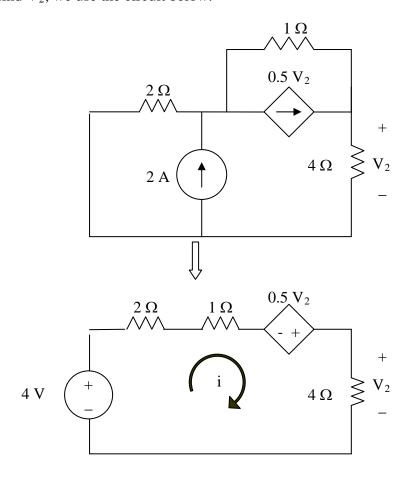
Let  $V_o = V_1 + V_2$ , where  $V_1$  and  $V_2$  are due to 10-V and 2-A sources respectively. To find  $V_1$ , we use the circuit below.



$$-10 + 7i - 0.5V_1 = 0$$
But  $V_1 = 4i$ 

$$10 = 7i - 2i = 5i \longrightarrow i = 2, V_1 = 8 \text{ V}$$

To find  $V_2$ , we use the circuit below.



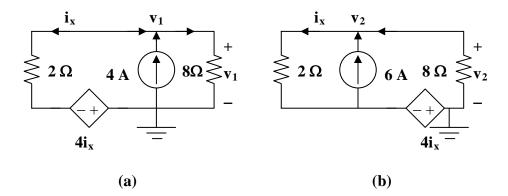
$$-4 + 7i - 0.5V_2 = 0$$
But  $V_2 = 4i$ 

$$4 = 7i - 2i = 5i \longrightarrow i = 0.8, V_2 = 4i = 3.2$$

$$V_0 = V_1 + V_2 = 8 + 3.2 = 11.2 \text{ V}$$

#### Chapter 4, Solution 19.

Let  $v_x = v_1 + v_2$ , where  $v_1$  and  $v_2$  are due to the 4-A and 6-A sources respectively.



To find  $v_1$ , consider the circuit in Fig. (a).

$$v_1/8-4+(v_1-(-4i_x))/2=0 \text{ or } (0.125+0.5)v_1=4-2i_x \text{ or } v_1=6.4-3.2i_x$$
 But, 
$$i_x=(v_1-(-4i_x))/2 \text{ or } i_x=-0.5v_1. \text{ Thus,}$$
 
$$v_1=6.4+3.2(0.5v_1), \text{ which leads to } v_1=-6.4/0.6=-10.667$$

To find  $v_2$ , consider the circuit shown in Fig. (b).

$$v_2/8 - 6 + (v_2 - (-4i_x))/2 = 0$$
 or  $v_2 + 3.2i_x = 9.6$ 

But  $i_x = -0.5v_2$ . Therefore,

$$v_2 + 3.2(-0.5v_2) = 9.6$$
 which leads to  $v_2 = -16$ 

Hence, 
$$v_x = -10.667 - 16 = -26.67V$$
.

Checking,

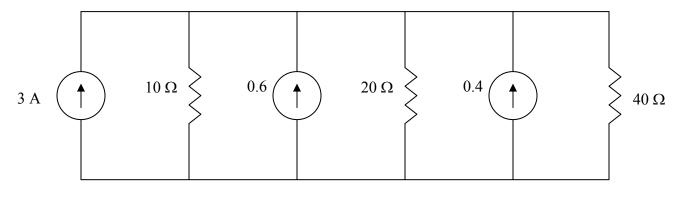
$$i_x = -0.5v_x = 13.333A$$

Now all we need to do now is sum the currents flowing out of the top node.

$$13.333 - 6 - 4 + (-26.67)/8 = 3.333 - 3.333 = 0$$

### Chapter 4, Solution 20.

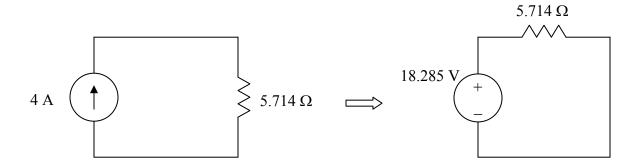
Convert the voltage sources to current sources and obtain the circuit shown below.



$$\frac{1}{R_{\text{eq}}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{40} = 0.1 + 0.05 + 0.025 = 0.175 \longrightarrow R_{\text{eq}} = 5.714 \,\Omega$$

$$I_{eq} = 3 + 0.6 + 0.4 = 4$$

Thus, the circuit is reduced as shown below. Please note, we that this is merely an exercise in combining sources and resistors. The circuit we have is an equivalent circuit which has no real purpose other than to demonstrate source transformation. In a practical situation, this would need some kind of reference and a use to an external circuit to be of real value.



**4.21** Using Fig. 4.89, design a problem to help other students to better understand source transformation.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

## Problem

Apply source transformation to determine  $v_o$  and  $i_o$  in the circuit in Fig. 4.89.

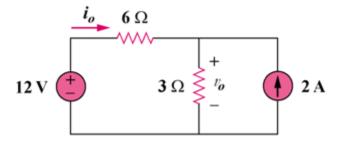
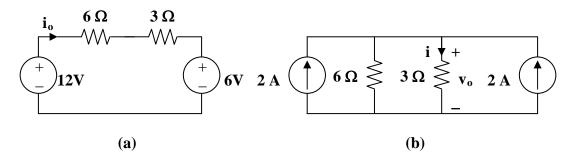


Figure 4.89

#### **Solution**

To get i<sub>0</sub>, transform the current sources as shown in Fig. (a).



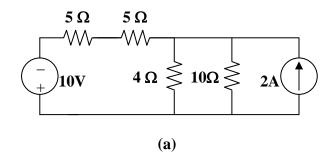
From Fig. (a), 
$$-12 + 9i_o + 6 = 0$$
, therefore  $i_o = 666.7 \text{ mA}$ 

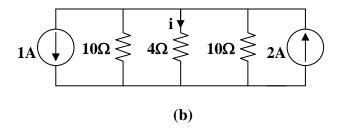
To get v<sub>o</sub>, transform the voltage sources as shown in Fig. (b).

$$i = [6/(3+6)](2+2) = 8/3$$
  
 $v_o = 3i = 8 V$ 

## Chapter 4, Solution 22.

We transform the two sources to get the circuit shown in Fig. (a).



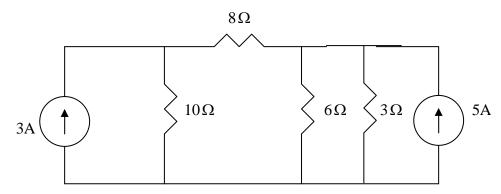


We now transform only the voltage source to obtain the circuit in Fig. (b).

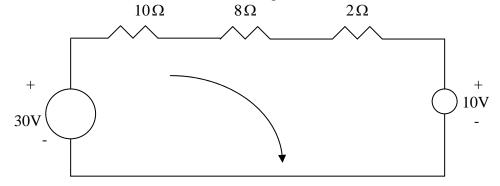
$$10||10 = 5 \text{ ohms}, i = [5/(5+4)](2-1) = 5/9 = 555.5 \text{ mA}$$

## Chapter 4, Solution 23

If we transform the voltage source, we obtain the circuit below.



3//6 = 2-ohm. Convert the current sources to voltages sources as shown below.



Applying KVL to the loop gives

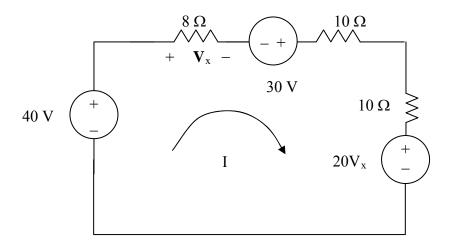
$$p = VI = I^2 R = 8 W$$

## Chapter 4, Solution 24.

Transform the two current sources in parallel with the resistors into their voltage source equivalents yield,

a 30-V source in series with a 10- $\Omega$  resistor and a  $20V_x\text{-V}$  sources in series with a 10- $\Omega$  resistor.

We now have the following circuit,



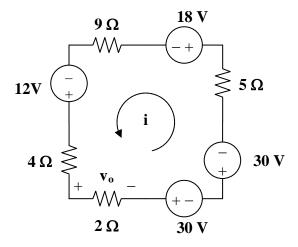
We now write the following mesh equation and constraint equation which will lead to a solution for  $V_x$ ,

$$28I - 70 + 20V_x = 0$$
 or  $28I + 20V_x = 70$ , but  $V_x = 8I$  which leads to

$$28I + 160I = 70 \text{ or } I = 0.3723 \text{ A or } V_x = 2.978 \text{ V}.$$

## Chapter 4, Solution 25.

Transforming only the current source gives the circuit below.



Applying KVL to the loop gives,

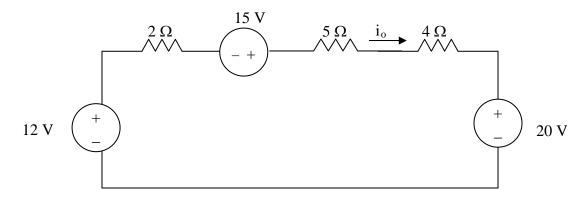
$$-(4+9+5+2)i+12-18-30-30=0$$

$$20i = -66$$
 which leads to  $i = -3.3$ 

$$v_o = 2i = -6.6 V$$

# Chapter 4, Solution 26.

Transforming the current sources gives the circuit below.



$$-12 + 11i_o - 15 + 20 = 0$$
 or  $11i_o = 7$  or  $i_o =$ **636.4 mA**.

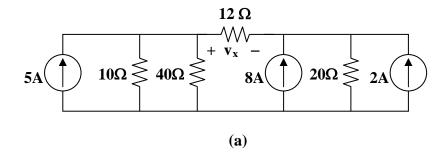
## Chapter 4, Solution 27.

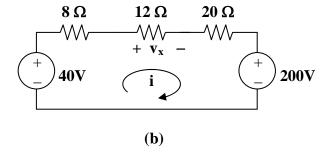
Transforming the voltage sources to current sources gives the circuit in Fig. (a).

$$10||40 = 8 \text{ ohms}$$

Transforming the current sources to voltage sources yields the circuit in Fig. (b). Applying KVL to the loop,

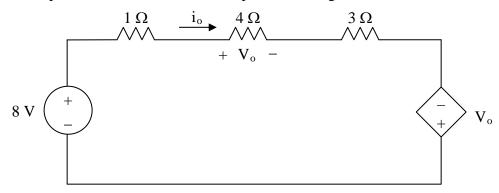
-40 + 
$$(8 + 12 + 20)i + 200 = 0$$
 leads to  $i = -4$   $v_x$   $12i = -48 V$ 





## Chapter 4, Solution 28.

Convert the dependent current source to a dependent voltage source as shown below.



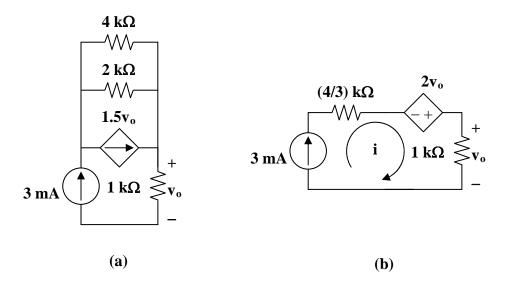
$$-8 + i_o(1+4+3) - V_o = 0$$

Applying KVL,  

$$-8 + i_o(1 + 4 + 3) - V_o = 0$$
  
But  $V_o = 4i_o$   
 $-8 + 8i_o - 4i_o = 0 \longrightarrow i_o = 2 A$ 

## Chapter 4, Solution 29.

Transform the dependent voltage source to a current source as shown in Fig. (a). 2||4 = (4/3) k ohms

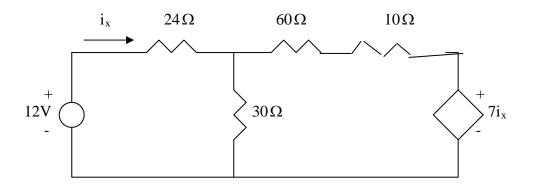


It is clear that i = 3 mA which leads to  $v_o = 1000i = 3 \text{ V}$ 

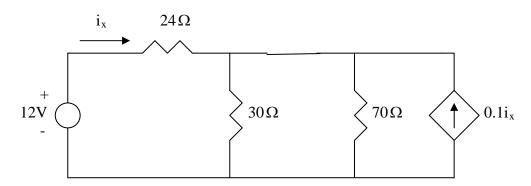
If the use of source transformations was not required for this problem, the actual answer could have been determined by inspection right away since the only current that could have flowed through the 1 k ohm resistor is 3 mA.

## Chapter 4, Solution 30

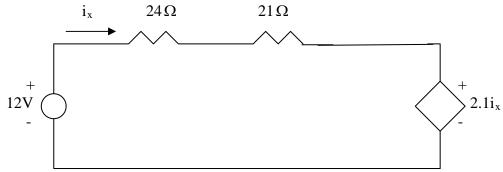
Transform the dependent current source as shown below.



Combine the 60-ohm with the 10-ohm and transform the dependent source as shown below.



Combining 30-ohm and 70-ohm gives 30//70 = 70x30/100 = 21-ohm. Transform the dependent current source as shown below.



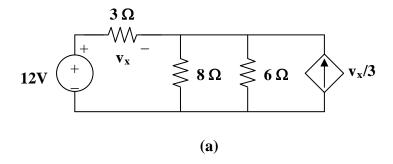
Applying KVL to the loop gives

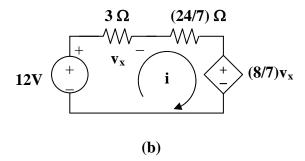
$$45i_x - 12 + 2.1i_x = 0$$
  $\longrightarrow$   $i_x = \frac{12}{47.1} = 254.8 \text{ mA}.$ 

## Chapter 4, Solution 31.

Transform the dependent source so that we have the circuit in

Fig. (a). 6||8| = (24/7) ohms. Transform the dependent source again to get the circuit in Fig. (b).





From Fig. (b),

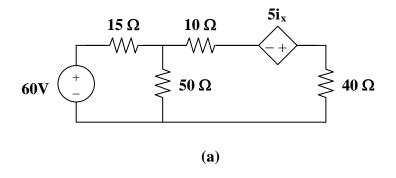
$$v_x = 3i$$
, or  $i = v_x/3$ .

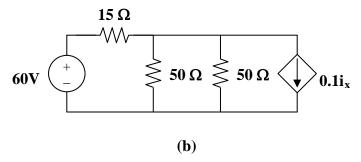
Applying KVL,

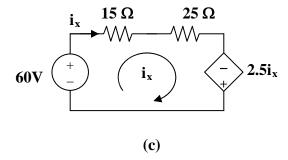
$$-12 + (3 + 24/7)i + (24/21)v_x = 0$$
 
$$12 = [(21 + 24)/7]v_x/3 + (8/7)v_x, \text{ leads to } v_x = 84/23 = \textbf{3.652 V}$$

## Chapter 4, Solution 32.

As shown in Fig. (a), we transform the dependent current source to a voltage source,







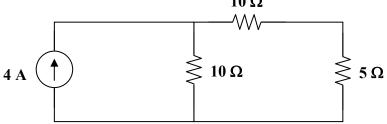
In Fig. (b), 50||50 = 25 ohms. Applying KVL in Fig. (c),

$$-60 + 40i_x - 2.5i_x = 0$$
, or  $i_x = 1.6 A$ 

### Chapter 4, Solution 33.

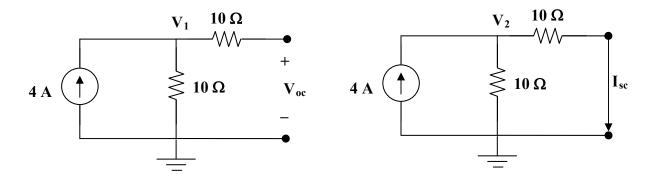
Determine the Thevenin equivalent circuit as seen by the 5-ohm resistor. Then calculate the current flowing through the 5-ohm resistor.

10  $\Omega$ 



#### **Solution**

Step 1. We need to find  $V_{oc}$  and  $I_{sc}$ . To do this, we will need two circuits, label the appropriate unknowns and solve for  $V_{oc}$ ,  $I_{sc}$ , and then  $R_{eq}$  which is equal to  $V_{oc}/I_{sc}$ .



Note, in the first case  $V_1 = V_{oc}$  and the nodal equation at 1 produces  $-4+(V_1-0)/10 = 0$ . In the second case,  $I_{sc} = (V_2-0)/10$  where the nodal equation at 2 produces,  $-4+[(V_2-0)/10]+[(V_2-0)/10] = 0$ .

Step 2.  $0.1V_1 = 4$  or  $V_1 = 40$  V =  $V_{oc} = V_{Thev}$ . Next,  $(0.1+0.1)V_2 = 4$  or  $0.2V_2 = 4$  or  $V_2 = 20$  V. Thus,  $I_{sc} = 20/10 = 2$  A. This leads to  $R_{eq} = 40/2 = 20$   $\Omega$ . We can check our results by using source transformation. The 4-amp current source in parallel with the 10-ohm resistor can be replaced by a 40-volt voltage source in series with a 10-ohm resistor which in turn is in series with the other 10-ohm resistor yielding the same Thevenin equivalent circuit. Once the 5-ohm resistor is connected to the Thevenin equivalent circuit, we now have 40 V across 25  $\Omega$  producing a current of 1.6 A.

**4.34** Using Fig. 4.102, design a problem that will help other students better understand Thevenin equivalent circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

#### **Problem**

Find the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 4.102.

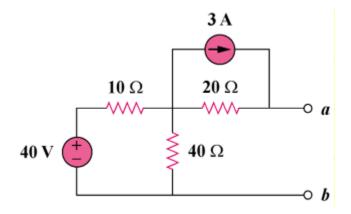
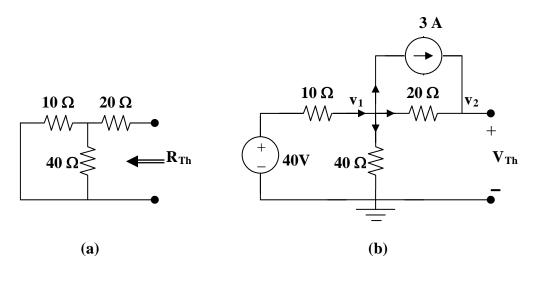


Figure 4.102

#### **Solution**

To find R<sub>Th</sub>, consider the circuit in Fig. (a).



$$R_{Th} = 20 + 10||40 = 20 + 400/50 =$$
**28 ohms**

To find  $V_{Th}$ , consider the circuit in Fig. (b).

At node 1, 
$$(40 - v_1)/10 = 3 + [(v_1 - v_2)/20] + v_1/40, \ 40 = 7v_1 - 2v_2$$
 (1)

At node 2, 
$$3 + (v1 - v2)/20 = 0$$
, or  $v1 = v2 - 60$  (2)

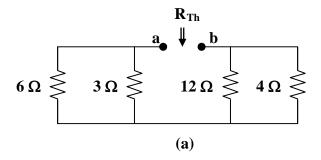
Solving (1) and (2), 
$$v_1 = 32 V$$
,  $v_2 = 92 V$ , and  $V_{Th} = v_2 = \textbf{92} V$ 

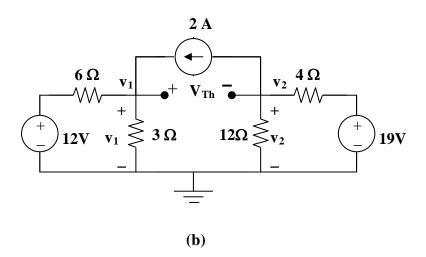
## Chapter 4, Solution 35.

To find R<sub>Th</sub>, consider the circuit in Fig. (a).

$$R_{Th} = R_{ab} = 6||3| + 12||4| = 2 + 3| = 5 \text{ ohms}$$

To find  $V_{Th}$ , consider the circuit shown in Fig. (b).

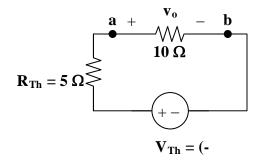




At node 1, 
$$2 + (12 - v_1)/6 = v_1/3$$
, or  $v_1 = 8$ 

At node 2, 
$$(19 - v_2)/4 = 2 + v_2/12$$
, or  $v_2 = 33/4$ 

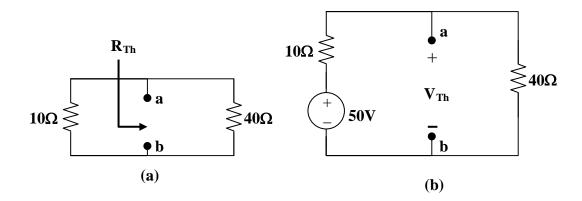
But, 
$$-v_1 + V_{Th} + v_2 = 0$$
, or  $V_{Th} = v_1 - v_2 = 8 - 33/4 = -0.25$ 



$$v_o~=~V_{Th}/2~=~\text{-}0.25/2~=~\text{-}\textbf{125}~\textbf{mV}$$

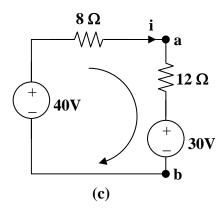
## Chapter 4, Solution 36.

Remove the 30-V voltage source and the 20-ohm resistor.



From Fig. (a), 
$$R_{Th} = 10||40 = 8 \text{ ohms}$$

From Fig. (b), 
$$V_{Th} = (40/(10 + 40))50 = 40V$$

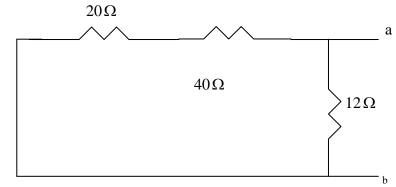


The equivalent circuit of the original circuit is shown in Fig. (c). Applying KVL,

$$30-40+(8+12)i = 0$$
, which leads to  $i = 500mA$ 

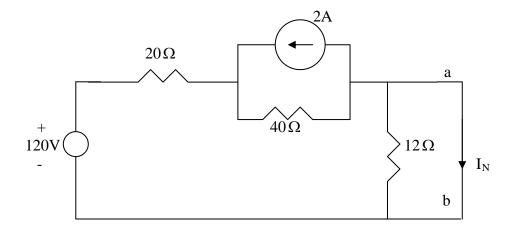
# Chapter 4, Solution 37

 $R_{\rm N}$  is found from the circuit below.

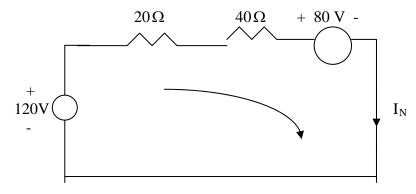


$$R_N = 12//(20 + 40) = 10 \ \Omega.$$

 $I_{\rm N}$  is found from the circuit below.



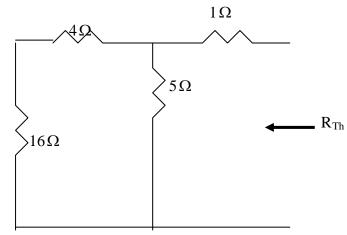
Applying source transformation to the current source yields the circuit below.



$$-120 + 80 + 60I_N = 0$$
  $\longrightarrow$   $I_N = 40/60 = 666.7 \text{ mA}.$ 

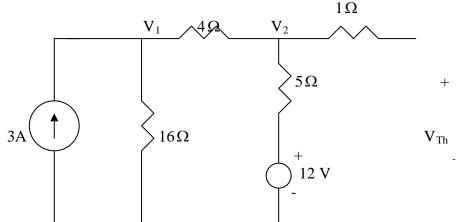
## Chapter 4, Solution 38

We find Thevenin equivalent at the terminals of the 10-ohm resistor. For  $R_{\text{Th}}$ , consider the circuit below.



$$R_{Th} = 1 + 5 //(4 + 16) = 1 + 4 = 5\Omega$$

For  $V_{Th}$ , consider the circuit below.



At node 1,

$$3 = \frac{V_1}{16} + \frac{V_1 - V_2}{4} \longrightarrow 48 = 5V_1 - 4V_2 \tag{1}$$

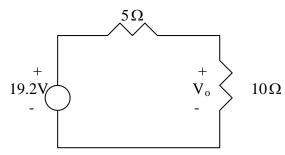
At node 2,

$$\frac{V_1 - V_2}{4} + \frac{12 - V_2}{5} = 0 \longrightarrow 48 = -5V_1 + 9V_2$$
 (2)

Solving (1) and (2) leads to

$$V_{Th} = V_2 = 19.2$$

Thus, the given circuit can be replaced as shown below.

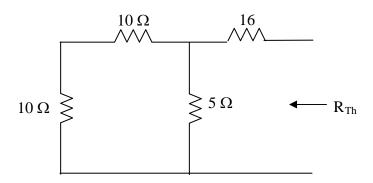


Using voltage division,

$$V_o = \frac{10}{10+5} (19.2) = 12.8 \text{ V}.$$

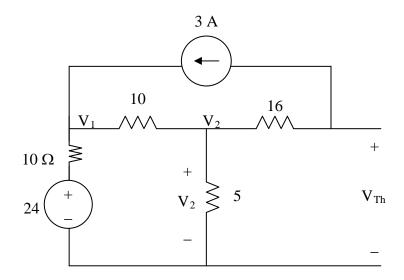
## Chapter 4, Solution 39.

We obtain  $R_{Th}$  using the circuit below.



$$R_{Thev} = 16 + (20||5) = 16 + (20x5)/(20+5) = \textbf{20} \ \Omega$$

To find  $V_{\text{Th}}$ , we use the circuit below.



At node 1,

$$\frac{24 - V_1}{10} + 3 = \frac{V_1 - V_2}{10} \longrightarrow 54 = 2V_1 - V_2 \tag{1}$$

At node 2,

$$\frac{V_1 - V_2}{10} = 3 + \frac{V_2}{5} \longrightarrow 60 = 2V_1 - 6V_2$$
 (2)

Substracting (1) from (2) gives

$$6 = -5V_2 \text{ or } V_2 = -1.2 \text{ V}$$

But

$$-V_2 + 16x3 + V_{Thev} = 0$$
 or  $V_{Thev} = -(48 + 1.2) = -49.2 V$ 

#### Chapter 4, Solution 40.

To obtain  $V_{Th}$ , we apply KVL to the loop.

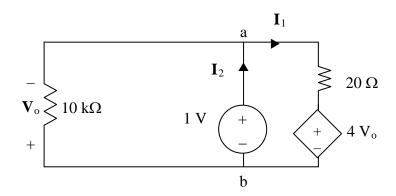
$$-70 + (10 + 20)kI + 4V_o = 0$$

But 
$$V_o = 10kI$$

$$70 = 70kI \longrightarrow I = 1mA$$

$$-70 + 10kI + V_{Th} = 0 \longrightarrow V_{Th} = \underline{60 \text{ V}}$$

To find R<sub>Th</sub>, we remove the 70-V source and apply a 1-V source at terminals a-b, as shown in the circuit below.



We notice that 
$$V_o = -1 V$$
.

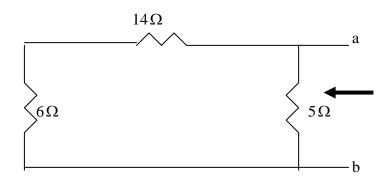
We notice that 
$$V_o = -1 \text{ V}$$
.  
 $-1 + 20 \text{ k/}_1 + 4 \text{ V}_o = 0 \longrightarrow \text{ I}_1 = 0.25 \text{ mA}$ 

$$I_2 = I_1 + \frac{1V}{10k} = 0.35 \text{ mA}$$

$$R_{Th} = \frac{1V}{l_2} = \frac{1}{0.35} k\Omega = \underline{2.857 \text{ k}\Omega}$$

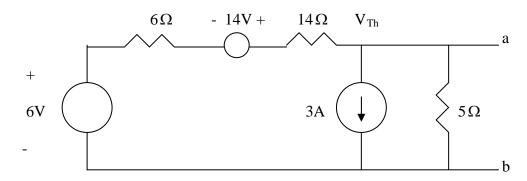
## Chapter 4, Solution 41

To find R<sub>Th</sub>, consider the circuit below



$$R_{Th} = 5//(14+6) = 4\Omega = R_N$$

Applying source transformation to the 1-A current source, we obtain the circuit below.



At node a,

$$\frac{14 + 6 - V_{Th}}{6 + 14} = 3 + \frac{V_{Th}}{5} \longrightarrow V_{Th} = -8 \text{ V}$$

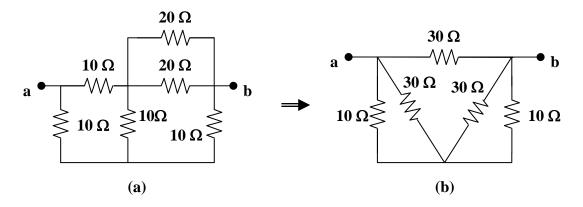
$$I_N = \frac{V_{Th}}{R_{Th}} = (-8)/4 = -2 \text{ A}$$

Thus,

$$\underline{R_{Th}} = R_N = 4\Omega, \quad V_{Th} = -8V, \quad I_N = -2 A$$

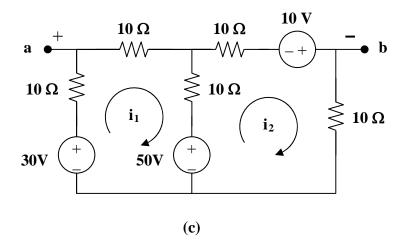
### Chapter 4, Solution 42.

To find  $R_{Th}$ , consider the circuit in Fig. (a).



20||20 = 10 ohms. Transform the wye sub-network to a delta as shown in Fig. (b). 10||30 = 7.5 ohms.  $R_{Th} = R_{ab} = 30||(7.5 + 7.5) = 10$  ohms.

To find  $V_{Th}$ , we transform the 20-V (to a current source in parallel with the 20  $\Omega$  resistor and then back into a voltage source in series with the parallel combination of the two 20  $\Omega$  resistors) and the 5-A sources. We obtain the circuit shown in Fig. (c).



For loop 1, 
$$-30 + 50 + 30i_1 - 10i_2 = 0$$
, or  $-2 = 3i_1 - i_2$  (1)

For loop 2, 
$$-50 - 10 + 30i_2 - 10i_1 = 0$$
, or  $6 = -i_1 + 3i_2$  (2)

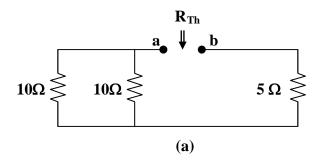
Solving (1) and (2),  $i_1 = 0$ ,  $i_2 = 2 \text{ A}$ 

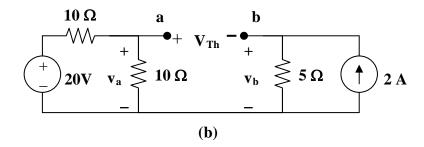
Applying KVL to the output loop,  $-v_{ab} - 10i_1 + 30 - 10i_2 = 0$ ,  $v_{ab} = 10 \text{ V}$ 

$$V_{Th} = v_{ab} = 10 \text{ volts}$$

## Chapter 4, Solution 43.

To find R<sub>Th</sub>, consider the circuit in Fig. (a).





$$R_{Th} = 10||10 + 5 = 10 \text{ ohms}$$

To find  $V_{\text{Th}}$ , consider the circuit in Fig. (b).

$$v_b = 2x5 = 10 \text{ V}, \ v_a = 20/2 = 10 \text{ V}$$

But, 
$$-v_a + V_{Th} + v_b = 0$$
, or  $V_{Th} = v_a - v_b = 0$  volts

## Chapter 4, Solution 44.

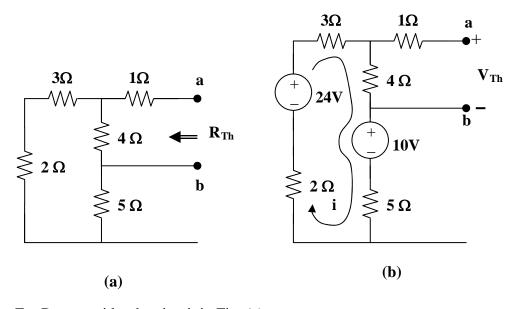
(a) For  $R_{Th}$ , consider the circuit in Fig. (a).

$$R_{Th} = 1 + 4||(3 + 2 + 5)| = 3.857$$
 ohms

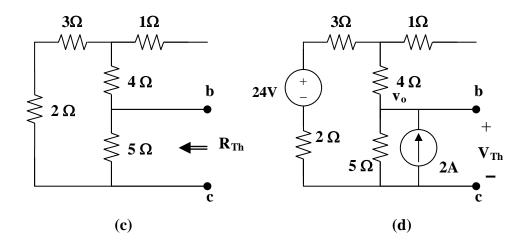
For  $V_{Th}$ , consider the circuit in Fig. (b). Applying KVL gives,

$$10 - 24 + i(3 + 4 + 5 + 2)$$
, or  $i = 1$ 

$$V_{Th}~=~4i~=~\textbf{4}~\textbf{V}$$



(b) For  $R_{Th}$ , consider the circuit in Fig. (c).



$$R_{Th} = 5||(2+3+4) = 3.214 \text{ ohms}$$

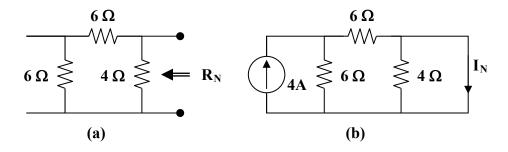
To get  $V_{\text{Th}}$ , consider the circuit in Fig. (d). At the node, KCL gives,

$$[(24 - vo)/9] + 2 = vo/5$$
, or  $vo = 15$ 

$$V_{Th} = vo = 15 V$$

## Chapter 4, Solution 45.

For R<sub>N</sub>, consider the circuit in Fig. (a).



$$R_N = (6+6)||4 = 3\Omega$$

For  $I_N$ , consider the circuit in Fig. (b). The 4-ohm resistor is shorted so that 4-A current is equally divided between the two 6-ohm resistors. Hence,

$$I_N = 4/2 = 2 A$$

## Chapter 4, Solution 46.

Using Fig. 4.113, design a problem to help other students better understand Norton equivalent circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

### Problem

Find the Norton equivalent at terminals a-b of the circuit in Fig. 4.113.

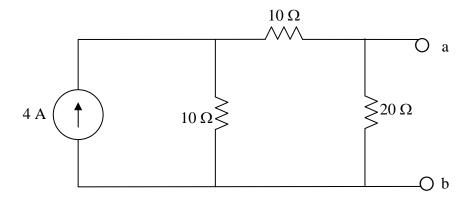
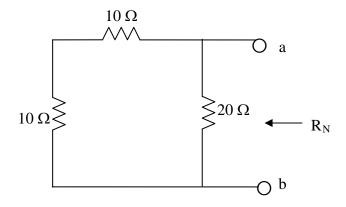


Figure 4.113 For Prob. 4.46.

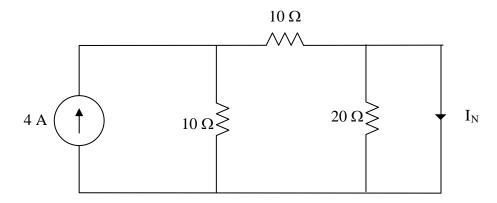
#### **Solution**

 $R_{\rm N}$  is found using the circuit below.



$$R_N = 20//(10+10) = 10 \Omega$$

$$\begin{split} R_{\rm N} &= 20 /\!/ (10 \! + \! 10) = 10 \; \Omega \\ To \; find \; I_{\rm N}, \; consider \; the \; circuit \; below. \end{split}$$



The 20- $\Omega$  resistor is short-circuited and can be ignored.

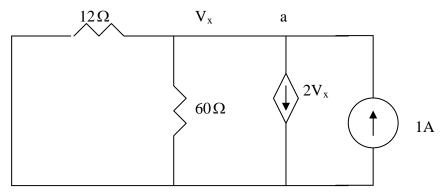
$$I_N = \frac{1}{2} \times 4 = 2 A$$

### Chapter 4, Solution 47

Since  $V_{Th} = V_{ab} = V_x$ , we apply KCL at the node a and obtain

$$\frac{30 - V_{Th}}{12} = \frac{V_{Th}}{60} + 2V_{Th} \longrightarrow V_{Th} = 150/126 = 1.1905 \text{ V}$$

To find  $R_{\text{Th}}$ , consider the circuit below.



At node a, KCL gives

$$1 = 2V_x + \frac{V_x}{60} + \frac{V_x}{12} \longrightarrow V_x = 60/126 = 0.4762$$

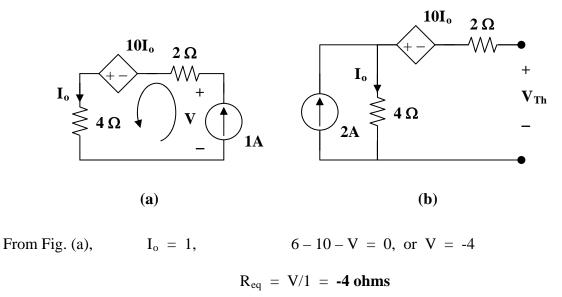
$$R_{Th} = \frac{V_x}{1} = 0.4762\Omega, \quad I_N = \frac{V_{Th}}{R_{Th}} = 1.19/0.4762 = 2.5$$

Thus,

$$V_{Thev}=$$
 1.1905 V,  $R_{eq}=$  476.2  $m\Omega,$  and  $I_{N}=$  2.5 A.

## Chapter 4, Solution 48.

To get  $R_{Th}$ , consider the circuit in Fig. (a).



Note that the negative value of  $R_{\rm eq}$  indicates that we have an active device in the circuit since we cannot have a negative resistance in a purely passive circuit.

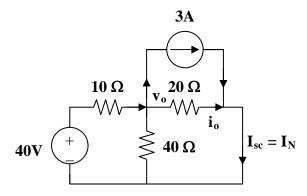
To solve for  $I_N$  we first solve for  $V_{Th}$ , consider the circuit in Fig. (b),

$$I_o \; = \; 2, \; \; V_{Th} \; = \; \text{-}10I_o + 4I_o \; = \; \text{-}12 \; V$$
 
$$I_N \; = \; V_{Th}/R_{Th} \; = \; \textbf{3A}$$

## Chapter 4, Solution 49.

$$R_N \ = \ R_{Th} \ = \ \textbf{28 ohms}$$

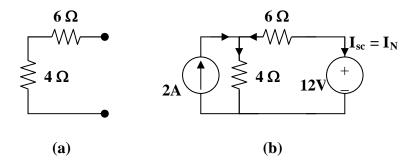
To find I<sub>N</sub>, consider the circuit below,



At the node, 
$$(40-v_o)/10 \ = \ 3+(v_o/40)+(v_o/20), \ or \ v_o \ = \ 40/7$$
 
$$i_o \ = \ v_o/20 \ = \ 2/7, \ but \ I_N \ = \ I_{sc} \ = \ i_o+3 \ = \ \textbf{3.286 A}$$

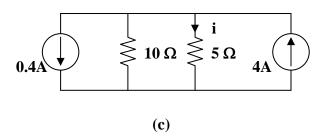
## Chapter 4, Solution 50.

From Fig. (a),  $R_N = 6 + 4 = 10$  ohms



From Fig. (b), 
$$2 + (12 - v)/6 = v/4, \text{ or } v = 9.6 \text{ V}$$
 
$$-I_N = (12 - v)/6 = 0.4, \text{ which leads to } I_N = \textbf{-0.4 A}$$

Combining the Norton equivalent with the right-hand side of the original circuit produces the circuit in Fig. (c).

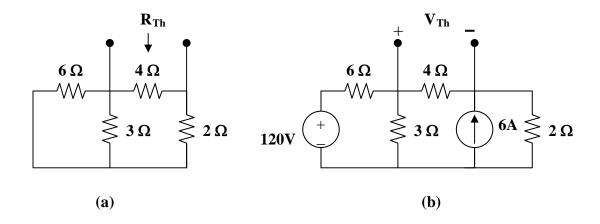


$$i = [10/(10+5)] (4-0.4) = 2.4 A$$

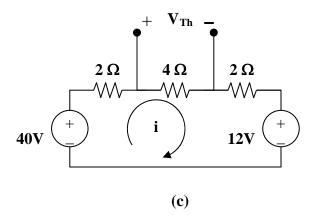
## Chapter 4, Solution 51.

(a) From the circuit in Fig. (a),

$$R_N = 4||(2+6||3) = 4||4 = 2 \text{ ohms}$$



For  $I_N$  or  $V_{Th}$ , consider the circuit in Fig. (b). After some source transformations, the circuit becomes that shown in Fig. (c).



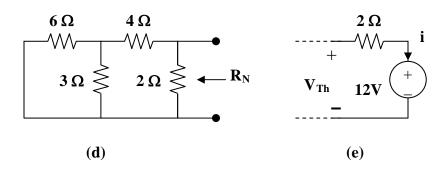
Applying KVL to the circuit in Fig. (c),

$$-40 + 8i + 12 = 0$$
 which gives  $i = 7/2$ 

$$V_{Th}~=~4i~=~14~~therefore~I_N~=~V_{Th}/R_N~=~14/2~=~\textbf{7}~\textbf{A}$$

(b) To get  $R_N$ , consider the circuit in Fig. (d).

$$R_N = 2||(4+6||3) = 2||6 = 1.5 \text{ ohms}$$



To get  $I_N$ , the circuit in Fig. (c) applies except that it needs slight modification as in Fig. (e).

$$i = 7/2$$
,  $V_{Th} = 12 + 2i = 19$ ,  $I_N = V_{Th}/R_N = 19/1.5 = 12.667 A$ 

## Chapter 4, Solution 52.

For the transistor model in Fig. 4.118, obtain the Thevenin equivalent at terminals *a-b*.

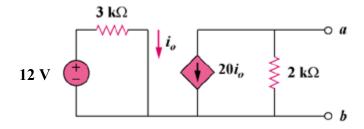
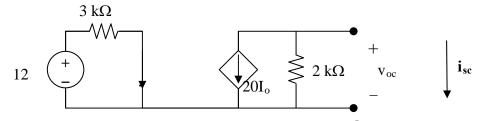


Figure 4.118 For Prob. 4.52.

### **Solution**

Step 1. To find the Thevenin equivalent for this circuit we need to find  $v_{oc}$  and  $i_{sc}$ . Then  $V_{Thev} = v_{oc}$  and  $R_{eq} = v_{oc}/i_{sc}$ .



For 
$$v_{oc},\,I_{o}$$
 = (12–0)/3k = 4 mA and  $20I_{o}$  + (v\_{oc}–0)/2k = 0.

For 
$$i_{sc}$$
,  $i_{sc} = -20I_{o}$ .

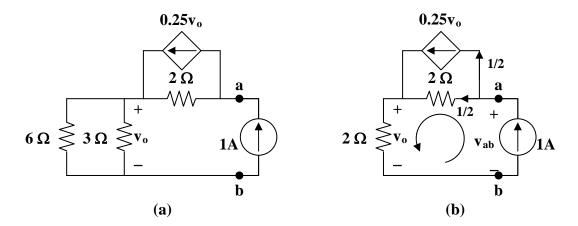
Step 2. 
$$v_{oc} = -2k(20I_o) = -40x4 = -160 \text{ volts} = V_{Thev}$$

$$i_{sc} = -20x4x10^{-3} = -80 \text{ mA or}$$

$$R_{eq} = -160/(80x10^{-3}) = 2 \text{ k}\Omega.$$

## Chapter 4, Solution 53.

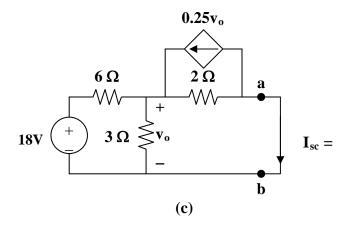
To get  $R_{Th}$ , consider the circuit in Fig. (a).



From Fig. (b),

$$v_o \,=\, 2x1 \,=\, 2V, \,\, \hbox{-}v_{ab} + 2x(1/2) \, \hbox{+}v_o \,=\, 0$$
 
$$v_{ab} \,=\, 3V$$
 
$$R_N \,=\, v_{ab}/1 \,=\, \textbf{3 ohms}$$

To get I<sub>N</sub>, consider the circuit in Fig. (c).



$$[(18-v_o)/6] + 0.25v_o = (v_o/2) + (v_o/3) \text{ or } v_o = 4V$$

But,  $(v_o/2) = 0.25v_o + I_N$ , which leads to  $I_N = 1 A$ 

### Chapter 4, Solution 54

To find  $V_{Th} = V_x$ , consider the left loop.

$$-3 + 1000i_a + 2V_x = 0 \longrightarrow 3 = 1000i_a + 2V_x$$
 (1)

For the right loop,

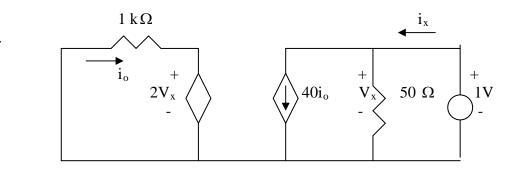
$$V_x = -50x40i_o = -2000i_o \tag{2}$$

Combining (1) and (2),

$$3 = 1000i_o - 4000i_o = -3000i_o \longrightarrow i_o = -1 \text{mA}$$

$$V_{x} = -2000i_{o} = 2$$
  $\longrightarrow$   $V_{Th} = 2$ 

To find  $R_{Th}$ , insert a 1-V source at terminals a-b and remove the 3-V independent source, as shown below.



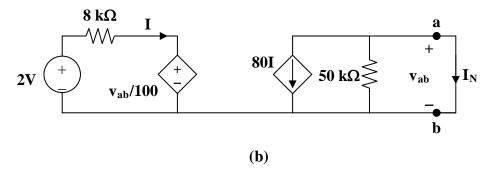
$$V_x = 1, \qquad i_o = -\frac{2V_x}{1000} = -2\text{mA}$$

$$i_x = 40i_o + \frac{V_x}{50} = -80\text{mA} + \frac{1}{50}\text{A} = -60\text{mA}$$

$$R_{Th} = \frac{1}{i_x} = -1/0.060 = -16.67\Omega$$

#### Chapter 4, Solution 55.

To get  $R_N$ , apply a 1 mA source at the terminals a and b as shown in Fig. (a).



We assume all resistances are in k ohms, all currents in mA, and all voltages in volts. At node a,

$$(v_{ab}/50) + 80I = 1$$
 (1)

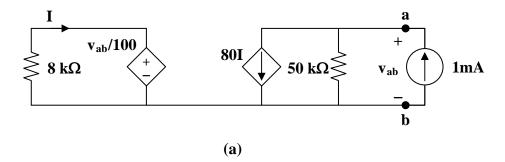
Also,

$$-8I = (v_{ab}/1000), \text{ or } I = -v_{ab}/8000$$
 (2)

From (1) and (2),  $(v_{ab}/50) - (80v_{ab}/8000) = 1$ , or  $v_{ab} = 100$ 

$$R_N = v_{ab}/1 = 100 \text{ k ohms}$$

To get I<sub>N</sub>, consider the circuit in Fig. (b).



Since the 50-k ohm resistor is shorted,

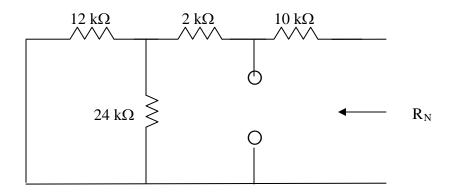
$$I_N = -80I, v_{ab} = 0$$

Hence, 8i = 2 which leads to I = (1/4) mA

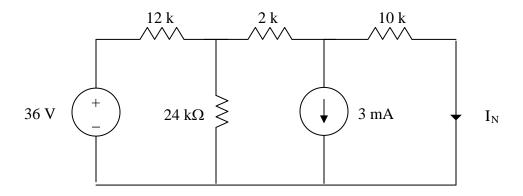
$$I_N = -20 \text{ mA}$$

## Chapter 4, Solution 56.

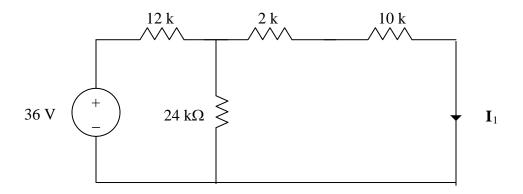
We remove the 1-k $\Omega$  resistor temporarily and find Norton equivalent across its terminals.  $R_{eq}$  is obtained from the circuit below.



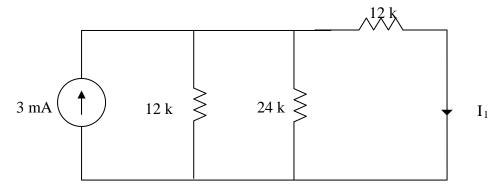
 $R_{eq} = 10 + 2 + (12/\!/24) = 12 + 8 = 20 \ k\Omega$   $I_N$  is obtained from the circuit below.



We can use superposition theorem to find  $I_N$ . Let  $I_N = I_1 + I_2$ , where  $I_1$  and  $I_2$  are due to 16-V and 3-mA sources respectively. We find  $I_1$  using the circuit below.



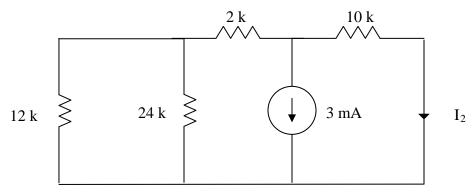
Using source transformation, we obtain the circuit below.



$$12//24 = 8 \text{ k}\Omega$$

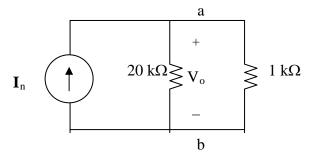
$$l_1 = \frac{8}{8+12}(3 \, mA) = 1.2 \, \text{mA}$$

To find I<sub>2</sub>, consider the circuit below.



$$\begin{array}{l} 2k + 12k/\!/24\;k &= 10\;k\Omega \\ I_2 \!\!=\!\! 0.5 (\text{-3mA}) = \text{-}1.5\;\text{mA} \\ I_N = 1.2\; \text{-}1.5 = \text{-}0.3\;\text{mA} \end{array}$$

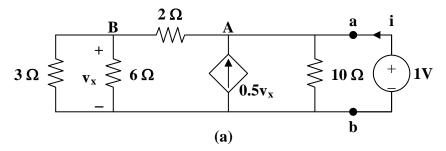
The Norton equivalent with the 1-k $\Omega$  resistor is shown below



$$V_o = 1k(20/(20+1))(-0.3 \text{ mA}) = -285.7 \text{ mV}.$$

#### Chapter 4, Solution 57.

To find  $R_{Th}$ , remove the 50V source and insert a 1-V source at a-b, as shown in Fig. (a).



We apply nodal analysis. At node A,

$$i + 0.5v_x = (1/10) + (1 - v_x)/2$$
, or  $i + v_x = 0.6$  (1)

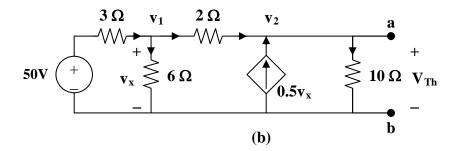
At node B,

$$(1 - v_0)/2 = (v_x/3) + (v_x/6)$$
, and  $v_x = 0.5$  (2)

From (1) and (2), i = 0.1 and

$$R_{Th} = 1/i = 10 \text{ ohms}$$

To get V<sub>Th</sub>, consider the circuit in Fig. (b).



At node 1, 
$$(50 - v_1)/3 = (v_1/6) + (v_1 - v_2)/2$$
, or  $100 = 6v_1 - 3v_2$  (3)

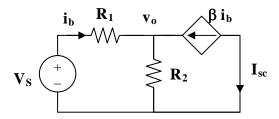
At node 2, 
$$0.5v_x + (v_1 - v_2)/2 = v_2/10$$
,  $v_x = v_1$ , and  $v_1 = 0.6v_2$  (4)

From (3) and (4),

$$\begin{aligned} v_2 \ &= \ V_{Th} \ = \ \textbf{166.67} \ \textbf{V} \\ I_N \ &= \ V_{Th}/R_{Th} = \ \textbf{16.667} \ \textbf{A} \\ R_N \ &= \ R_{Th} \ = \ \textbf{10} \ \textbf{ohms} \end{aligned}$$

#### Chapter 4, Solution 58.

This problem does not have a solution as it was originally stated. The reason for this is that the load resistor is in series with a current source which means that the only equivalent circuit that will work will be a Norton circuit where the value of  $R_{\rm N}=$  infinity.  $I_{\rm N}$  can be found by solving for  $I_{\rm sc}$ .



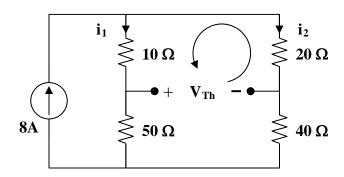
Writing the node equation at node vo,

$$\begin{split} i_b + \beta i_b &= v_o/R_2 = (1+\beta)i_b \\ i_b &= (V_s - v_o)/R_1 \\ v_o &= V_s - i_b R_1 \\ V_s - i_b R_1 &= (1+\beta)R_2 i_b, \text{ or } i_b = V_s/(R_1 + (1+\beta)R_2) \\ I_{sc} &= I_N = -\beta i_b = -\beta V_s/(R_1 + (1+\beta)R_2) \end{split}$$

## Chapter 4, Solution 59.

$$R_{Th} = (10 + 20)||(50 + 40) 30||90 =$$
**22.5 ohms**

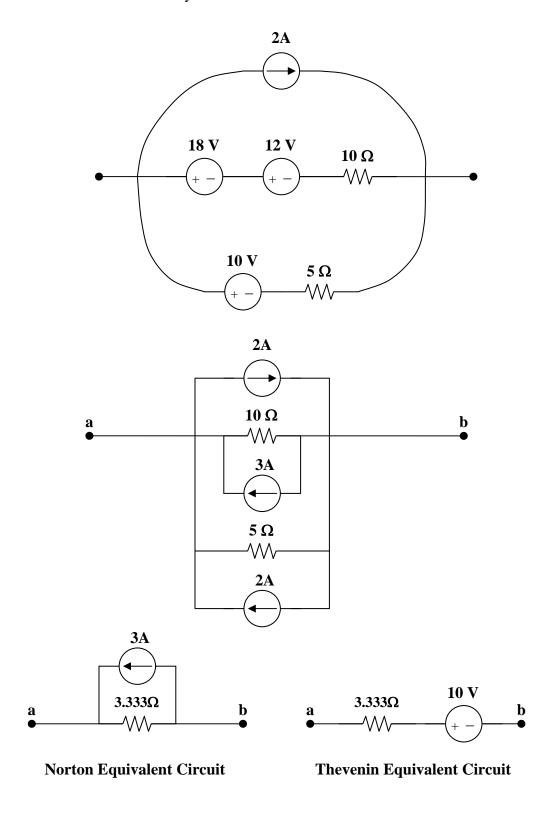
To find  $V_{Th}$ , consider the circuit below.



$$i_1=i_2=8/2=4,\ 10i_1+V_{Th}-20i_2=0,\ or\ V_{Th}=20i_2-10i_1=10i_1=10x4$$
 
$$V_{Th}=\textbf{40V},\ and\ I_N=V_{Th}/R_{Th}=40/22.5=\textbf{1.7778}\,\textbf{A}$$

## Chapter 4, Solution 60.

The circuit can be reduced by source transformations.

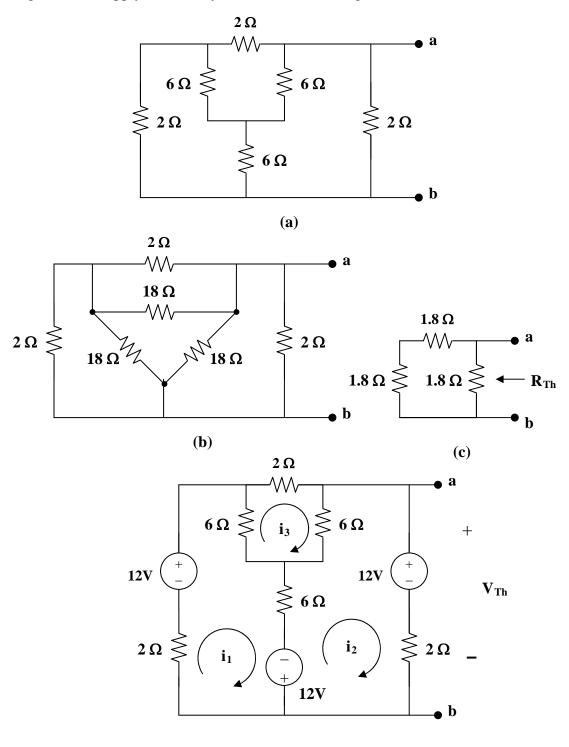


## Chapter 4, Solution 61.

To find R<sub>Th</sub>, consider the circuit in Fig. (a).

Let 
$$R = 2||18 = 1.8 \text{ ohms}, R_{Th} = 2R||R = (2/3)R = 1.2 \text{ ohms}.$$

To get  $V_{Th}$ , we apply mesh analysis to the circuit in Fig. (d).



$$-12 - 12 + 14i_1 - 6i_2 - 6i_3 = 0$$
, and  $7i_1 - 3i_2 - 3i_3 = 12$  (1)

$$12 + 12 + 14 i_2 - 6 i_1 - 6 i_3 = 0$$
, and  $-3 i_1 + 7 i_2 - 3 i_3 = -12$  (2)

$$14 i_3 - 6 i_1 - 6 i_2 = 0$$
, and  $-3 i_1 - 3 i_2 + 7 i_3 = 0$  (3)

This leads to the following matrix form for (1), (2) and (3),

$$\begin{bmatrix} 7 & -3 & -3 \\ -3 & 7 & -3 \\ -3 & -3 & 7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -3 & -3 \\ -3 & 7 & -3 \\ -3 & -3 & 7 \end{vmatrix} = 100, \qquad \Delta_2 = \begin{vmatrix} 7 & 12 & -3 \\ -3 & -12 & -3 \\ -3 & 0 & 7 \end{vmatrix} = -120$$

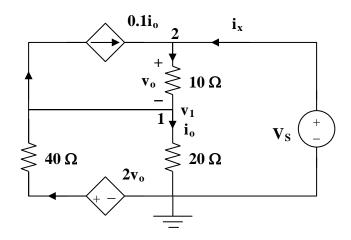
$$i_2 = \Delta/\Delta_2 = -120/100 = -1.2 A$$

$$V_{Th}~=~12+2i_2~=~\textbf{9.6}~\textbf{V},~\text{and}~I_{N}~=~V_{Th}/R_{Th}~=~\textbf{8}~\textbf{A}$$

#### Chapter 4, Solution 62.

Since there are no independent sources,  $V_{Th} = 0 V$ 

To obtain  $R_{Th}$ , consider the circuit below.



At node 2,

$$i_x + 0.1i_o = (1 - v_1)/10$$
, or  $10i_x + i_o = 1 - v_1$  (1)

At node 1,

$$(v_1/20) + 0.1i_o = [(2v_o - v_1)/40] + [(1 - v_1)/10]$$
 (2)

But  $i_o = (v_1/20)$  and  $v_o = 1 - v_1$ , then (2) becomes,

$$1.1v_{1}/20 = [(2 - 3v_{1})/40] + [(1 - v_{1})/10]$$

$$2.2v_{1} = 2 - 3v_{1} + 4 - 4v_{1} = 6 - 7v_{1}$$

$$v_{1} = 6/9.2$$
(3)

From (1) and (3),

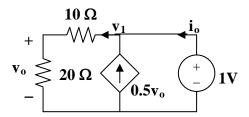
or

$$10i_x + v_1/20 = 1 - v_1$$
 
$$10i_x = 1 - v_1 - v_1/20 = 1 - (21/20)v_1 = 1 - (21/20)(6/9.2)$$
 
$$i_x = 31.52 \text{ mA}, \ R_{Th} = 1/i_x = \textbf{31.73 ohms.}$$

## Chapter 4, Solution 63.

Because there are no independent sources,  $I_N = I_{sc} = 0 A$ 

 $R_{\rm N}$  can be found using the circuit below.



Applying KCL at node 1,  $v_1 = 1$ , and  $v_0 = (20/30)v_1 = 2/3$ 

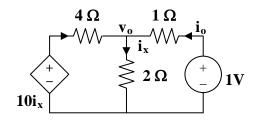
$$\begin{array}{lll} i_o = (v_1/30) - 0.5 v_o = (1/30) - 0.5 x2/3 = 0.03333 - \\ 0.33333 = -0.3 \; A. \end{array}$$

Hence,

$$R_N = 1/(-0.3) = -3.333$$
 ohms

## Chapter 4, Solution 64.

With no independent sources,  $V_{Th} = \mathbf{0} \ \mathbf{V}$ . To obtain  $R_{Th}$ , consider the circuit shown below.



$$i_x = [(1 - v_o)/1] + [(10i_x - v_o)/4], \text{ or } 5v_o = 4 + 6i_x$$
 (1)

But  $i_x = v_o/2$ . Hence,

$$5v_o = 4 + 3v_o$$
, or  $v_o = 2$ ,  $i_o = (1 - v_o)/1 = -1$ 

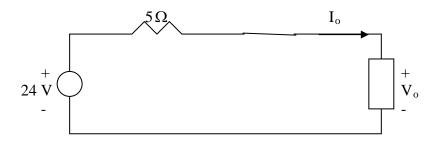
Thus, 
$$R_{Th} = 1/i_o = -1$$
 ohm

## Chapter 4, Solution 65

At the terminals of the unknown resistance, we replace the circuit by its Thevenin equivalent.

$$R_{eq} = 2 + (4 || 12) = 2 + 3 = 5\Omega,$$
  $V_{Th} = \frac{12}{12 + 4}(32) = 24 \text{ V}$ 

Thus, the circuit can be replaced by that shown below.

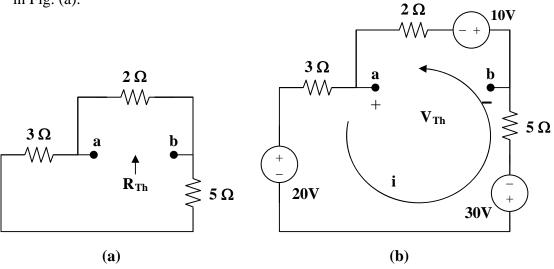


Applying KVL to the loop,

$$-24 + 5I_o + V_o = 0 \qquad \longrightarrow \quad \mathbf{V_o} = \mathbf{24} - \mathbf{5I_o}.$$

## Chapter 4, Solution 66.

We first find the Thevenin equivalent at terminals a and b. We find  $R_{Th}$  using the circuit in Fig. (a).



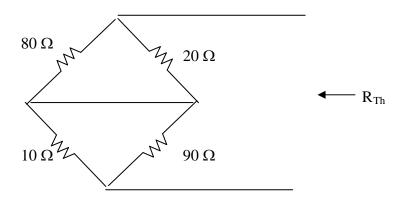
$$R_{Th} = 2||(3+5) = 2||8 = 1.6 \text{ ohms}$$

By performing source transformation on the given circuit, we obatin the circuit in (b). We now use this to find  $V_{\text{Th}}$ .

$$10i + 30 + 20 + 10 = 0$$
, or  $i = -6$   $V_{Th} + 10 + 2i = 0$ , or  $V_{Th} = 2 V$   $p = V_{Th}^2/(4R_{Th}) = (2)^2/[4(1.6)] = 625 \text{ m watts}$ 

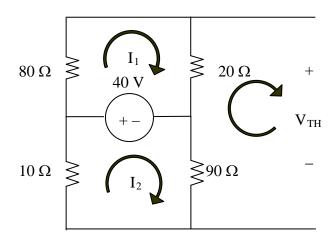
## Chapter 4, Solution 67.

We first find the Thevenin equivalent. We find  $R_{Th}$  using the circuit below.



$$R_{Th} = 20 / 80 + 90 / 10 = 16 + 9 = 25 \Omega$$

We find  $V_{\text{Th}}$  using the circuit below. We apply mesh analysis.



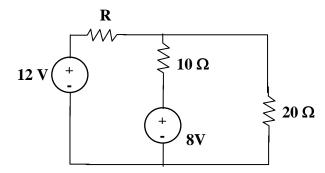
$$(80 + 20)i_1 - 40 = 0$$
  $\longrightarrow$   $i_1 = 0.4$   
 $(10 + 90)i_2 + 40 = 0$   $\longrightarrow$   $i_2 = -0.4$   
 $-90i_2 - 20i_1 + V_{Th} = 0$   $\longrightarrow$   $V_{Th} = -28 \text{ V}$ 

(a) 
$$R = R_{Th} = 25 \Omega$$

(a) 
$$R = R_{Th} = 25 \Omega$$
  
(b)  $P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(28)^2}{100} = \underline{7.84 \text{ W}}$ 

#### Chapter 4, Solution 68.

This is a challenging problem in that the load is already specified. This now becomes a "minimize losses" style problem. When a load is specified and internal losses can be adjusted, then the objective becomes, reduce  $R_{Thev}$  as much as possible, which will result in maximum power transfer to the load.



Removing the 10 ohm resistor and solving for the Thevenin Circuit results in:

$$R_{Th} = (Rx20/(R+20))$$
 and a  $V_{oc} = V_{Th} = 12x(20/(R+20)) + (-8)$ 

As R goes to zero,  $R_{Th}$  goes to zero and  $V_{Th}$  goes to 4 volts, which produces the maximum power delivered to the 10-ohm resistor.

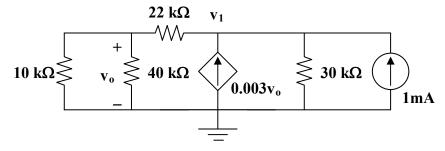
$$P = vi = v^2/R = 4x4/10 = 1.6 \text{ watts}$$

Notice that if R = 20 ohms which gives an  $R_{Th} = 10$  ohms, then  $V_{Th}$  becomes -2 volts and the power delivered to the load becomes 0.1 watts, much less that the 1.6 watts.

It is also interesting to note that the internal losses for the first case are  $12^2/20 = 7.2$  watts and for the second case are = to 12 watts. This is a significant difference.

#### Chapter 4, Solution 69.

We need the Thevenin equivalent across the resistor R. To find  $R_{Th}$ , consider the circuit below.



Assume that all resistances are in k ohms and all currents are in mA.

$$10||40 = 8$$
, and  $8 + 22 = 30$   
 $1 + 3v_0 = (v_1/30) + (v_1/30) = (v_1/15)$   
 $15 + 45v_0 = v_1$ 

But  $v_0 = (8/30)v_1$ , hence,

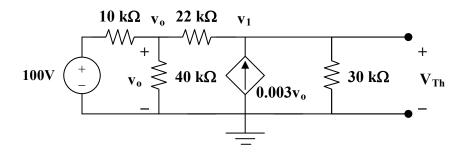
$$15 + 45x(8v_1/30)$$
 v<sub>1</sub>, which leads to v<sub>1</sub> = 1.3636

$$R_{Th} = v_1/1 = -1.3636 \text{ k ohms}$$

 $R_{\text{Th}}$  being negative indicates an active circuit and if you now make R equal to 1.3636 k ohms, then the active circuit will actually try to supply infinite power to the resistor. The correct answer is therefore:

$$p_R = \left(\frac{V_{Th}}{-1363.6 + 1363.6}\right)^2 1363.6 = \left(\frac{V_{Th}}{0}\right)^2 1363.6 = \infty$$

It may still be instructive to find V<sub>Th</sub>. Consider the circuit below.



$$(100 - v_o)/10 = (v_o/40) + (v_o - v_1)/22$$
 (1)

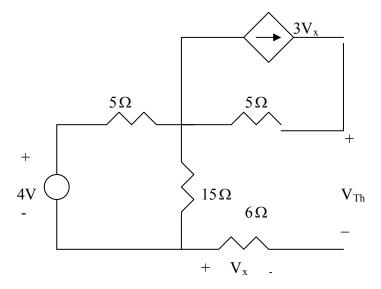
$$[(v_o - v_1)/22] + 3v_o = (v_1/30)$$
 (2)

Solving (1) and (2),

$$v_1 = V_{Th} = -243.6 \text{ volts}$$

## Chapter 4, Solution 70

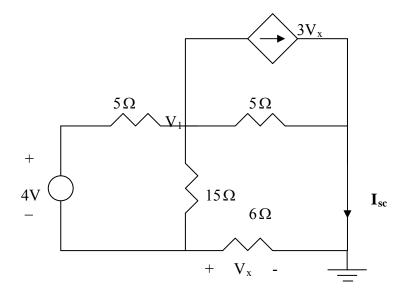
We find the Thevenin equivalent across the 10-ohm resistor. To find  $\,V_{Th}$ , consider the circuit below.



From the figure,

$$V_x = 0,$$
  $V_{Th} = \frac{15}{15 + 5}(4) = 3V$ 

To find  $R_{\text{eq.}}$  consider the circuit below:



At node 1,

$$[(V_1-V_x)/15] + [(V_1-(4+V_x))/5] + [(V_1-0)/5] + 3V_x = 0$$
 or

$$0.4667V_1 + 2.733V_x = 0.8 (1)$$

At node x,

$$[(V_x-0)/6] + [((V_x+4)-V_1)/5] + [(V_x-V_1)/15] = 0 \text{ or}$$

$$-(0.2667)V_1 + 0.4333V_x = -0.8$$
(2)

Adding (1) and (2) together lead to,

$$(0.4667-0.2667)V_1 + (2.733+0.4333)V_x = 0 \text{ or } V_1 = -(3.166/0.2)V_x = -15.83V_x$$

Now we can put this into (1) and we get,

$$0.4667(-15.83V_x) + 2.733V_x = 0.8 = (-7.388 + 2.733)V_x = -4.655V_x$$
 or  $V_x = -0.17186$  V.

$$I_{sc} = -V_x/6 = 0.02864$$
 and  $R_{eq} = 3/(0.02864) = 104.75 \Omega$ 

An alternate way to find  $R_{eq}$  is replace  $I_{sc}$  with a 1 amp current source flowing up and setting the 4 volts source to zero. We then find the voltage across the 1 amp current source which is equal to  $R_{eq}$ . First we note that  $V_x = 6$  volts;

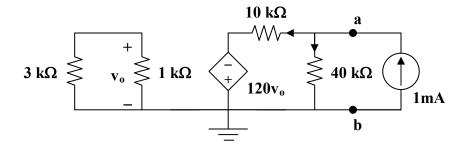
$$V_1 = 6+3.75 = -9.75$$
;  $V_2 = 19x5 + V_1 = 95+9.75 = 104.75$  or  $R_{eq} = 104.75$   $\Omega$ .

Clearly setting the load resistance to 104.75  $\Omega$  means that the circuit will deliver maximum power to it. Therefore,

$$p_{\text{max}} = [3/(2x104.75)]^2x104.75 = 21.48 \text{ mW}$$

#### Chapter 4, Solution 71.

We need  $R_{Th}$  and  $V_{Th}$  at terminals a and b. To find  $R_{Th}$ , we insert a 1-mA source at the terminals a and b as shown below.



Assume that all resistances are in k ohms, all currents are in mA, and all voltages are in volts. At node a,

$$1 = (v_a/40) + [(v_a + 120v_o)/10], \text{ or } 40 = 5v_a + 480v_o$$
 (1)

The loop on the left side has no voltage source. Hence,  $v_0 = 0$ . From (1),  $v_a = 8 \text{ V}$ .

$$R_{Th} = v_a/1 \text{ mA} = 8 \text{ kohms}$$

To get V<sub>Th</sub>, consider the original circuit. For the left loop,

$$v_0 = (1/4)8 = 2 V$$

For the right loop, 
$$v_R = V_{Th} = (40/50)(-120v_o) = -192$$

The resistance at the required resistor is

$$R = R_{Th} = 8 k\Omega$$

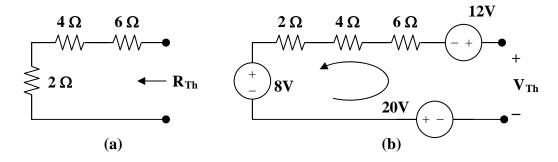
$$p = V_{Th}^2/(4R_{Th}) = (-192)^2/(4x8x10^3) = 1.152 \text{ watts}$$

## Chapter 4, Solution 72.

(a)  $R_{Th}$  and  $V_{Th}$  are calculated using the circuits shown in Fig. (a) and (b) respectively.

From Fig. (a), 
$$R_{Th} = 2 + 4 + 6 = 12$$
 ohms

From Fig. (b), 
$$-V_{Th} + 12 + 8 + 20 = 0$$
, or  $V_{Th} = 40 \text{ V}$ 



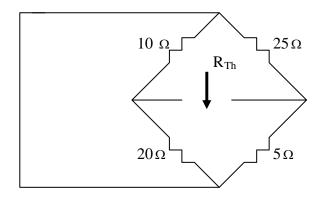
(b) 
$$i = V_{Th}/(R_{Th} + R) = 40/(12 + 8) = 2A$$

(c) For maximum power transfer, 
$$R_L = R_{Th} = 12 \text{ ohms}$$

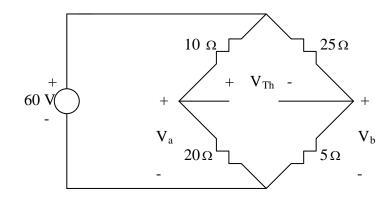
(d) 
$$p = V_{Th}^2/(4R_{Th}) = (40)^2/(4x_{12}) = 33.33 \text{ watts}.$$

## Chapter 4, Solution 73

Find the Thevenin's equivalent circuit across the terminals of R.



$$R_{Th} = 10//20 + 25//5 = 325/30 = 10.833\Omega$$



$$V_a = \frac{20}{30}(60) = 40,$$
  $V_b = \frac{5}{30}(60) = 10$   
 $-V_a + V_{Th} + V_b = 0$   $\longrightarrow$   $V_{Th} = V_a - V_b = 40 - 10 = 30 \text{ V}$ 

$$p_{\text{max}} = \frac{V_{Th}^2}{4R_{Th}} = \frac{30^2}{4x10.833} = 20.77 \text{ W}.$$

### Chapter 4, Solution 74.

When  $R_L$  is removed and  $V_s$  is short-circuited,

$$\begin{split} R_{Th} &= R_1 || R_2 + R_3 || R_4 &= [R_1 \ R_2 / (\ R_1 + R_2)] + [R_3 \ R_4 / (\ R_3 + R_4)] \\ R_L &= R_{Th} = (\textbf{R_1} \ \textbf{R_2} \ \textbf{R_3} + \textbf{R_1} \ \textbf{R_2} \ \textbf{R_4} + \textbf{R_1} \ \textbf{R_3} \ \textbf{R_4} + \textbf{R_2} \ \textbf{R_3} \ \textbf{R_4}) / [(\ \textbf{R_1} + \textbf{R_2}) (\ \textbf{R_3} + \textbf{R_4})] \end{split}$$

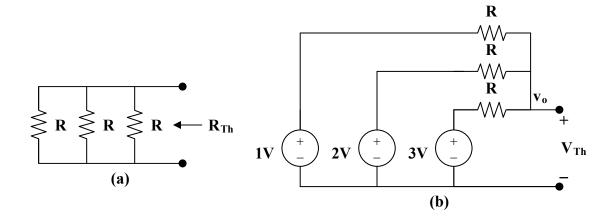
When R<sub>L</sub> is removed and we apply the voltage division principle,

$$\begin{split} V_{oc} &= V_{Th} = v_{R2} - v_{R4} \\ &= ([R_2/(R_1 + R_2)] - [R_4/(R_3 + R_4)])V_s = \{[(R_2R_3) - (R_1R_4)]/[(R_1 + R_2)(R_3 + R_4)]\}V_s \\ &\quad p_{max} = V_{Th}^2/(4R_{Th}) \\ &= \{[(R_2R_3) - (R_1R_4)]^2/[(R_1 + R_2)(R_3 + R_4)]^2\}V_s^2[(R_1 + R_2)(R_3 + R_4)]/[4(a)] \\ &\quad where \ a = (R_1\ R_2\ R_3 + R_1\ R_2\ R_4 + R_1\ R_3\ R_4 + R_2\ R_3\ R_4) \\ &\quad p_{max} = \end{split}$$

 $[(R_2R_3) - (R_1R_4)]^2V_s^2/[4(R_1 + R_2)(R_3 + R_4)(R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)]$ 

## Chapter 4, Solution 75.

We need to first find  $R_{\text{Th}}$  and  $V_{\text{Th}}$ .



Consider the circuit in Fig. (a).

$$(1/R_{eq}) = (1/R) + (1/R) + (1/R) = 3/R$$
  
 $R_{eq} = R/3$ 

From the circuit in Fig. (b),

$$((1 - v_o)/R) + ((2 - v_o)/R) + ((3 - v_o)/R) = 0$$
  
 $v_o = 2 = V_{Th}$ 

For maximum power transfer,

$$R_{L} = R_{Th} = R/3$$
 
$$P_{max} = [(V_{Th})^{2}/(4R_{Th})] = 3 \text{ mW}$$
 
$$R_{Th} = [(V_{Th})^{2}/(4P_{max})] = 4/(4xP_{max}) = 1/P_{max} = R/3$$
 
$$R = 3/(3x10^{-3}) = 1 \text{ k}\Omega$$

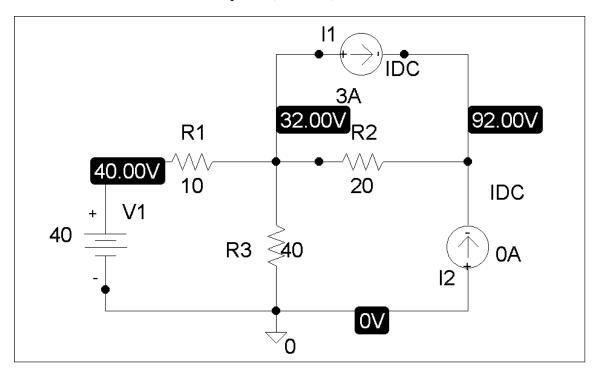
 $1 \text{ k}\Omega$ , 3 mW

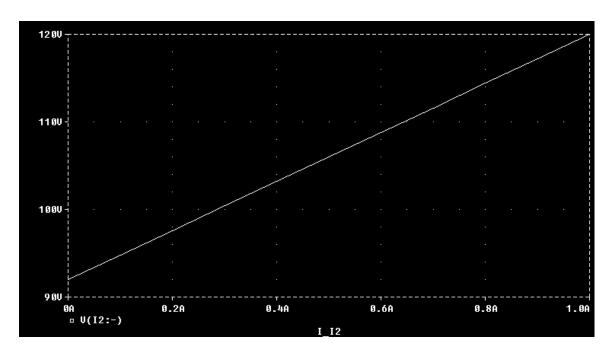
## Chapter 4, Solution 76.

Follow the steps in Example 4.14. The schematic and the output plots are shown below. From the plot, we obtain,

$$V = 92 V [i = 0, voltage axis intercept]$$

$$R = Slope = (120 - 92)/1 = 28 \text{ ohms}$$



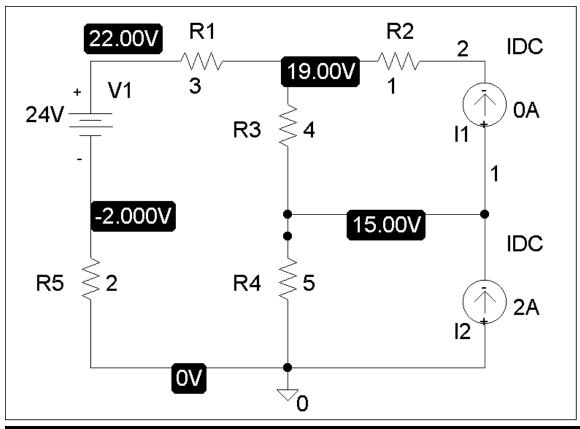


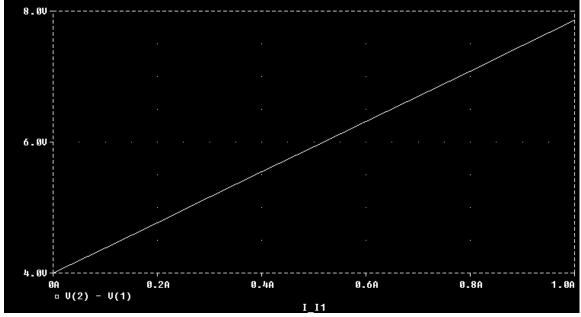
#### **Chapter 4, Solution 77.**

(a) The schematic is shown below. We perform a dc sweep on a current source, I1, connected between terminals a and b. We label the top and bottom of source I1 as 2 and 1 respectively. We plot V(2) - V(1) as shown.

$$V_{Th} = 4 V [zero intercept]$$

$$R_{Th} = (7.8 - 4)/1 = 3.8 \text{ ohms}$$

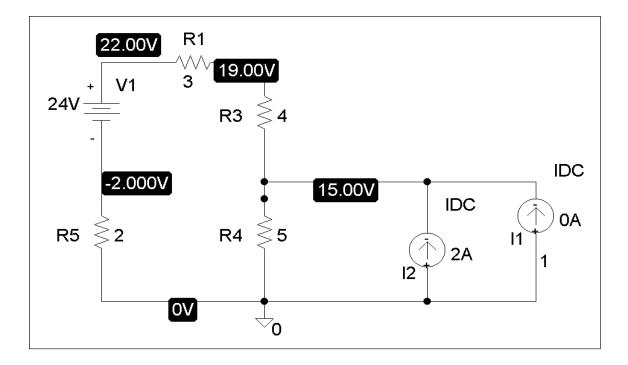


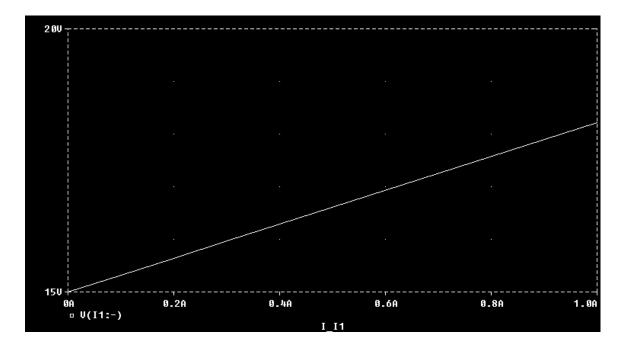


(b) Everything remains the same as in part (a) except that the current source, I1, is connected between terminals b and c as shown below. We perform a dc sweep on I1 and obtain the plot shown below. From the plot, we obtain,

$$V = 15 V [zero intercept]$$

$$R = (18.2 - 15)/1 = 3.2 \text{ ohms}$$



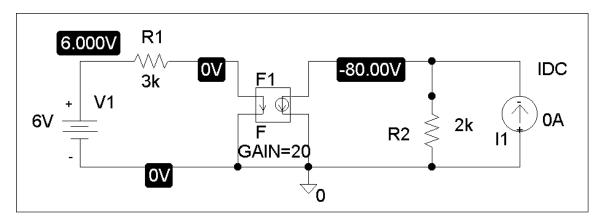


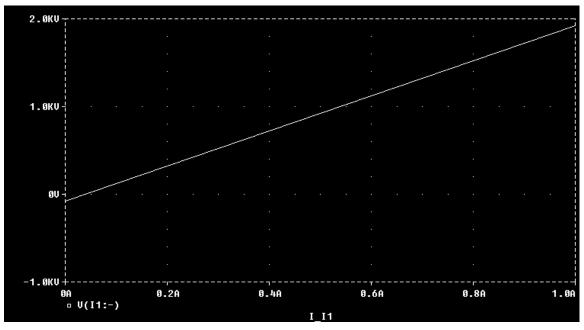
## Chapter 4, Solution 78.

The schematic is shown below. We perform a dc sweep on the current source, I1, connected between terminals a and b. The plot is shown. From the plot we obtain,

$$V_{Th} = -80 V [zero intercept]$$

$$R_{Th} = (1920 - (-80))/1 = 2 \text{ k ohms}$$



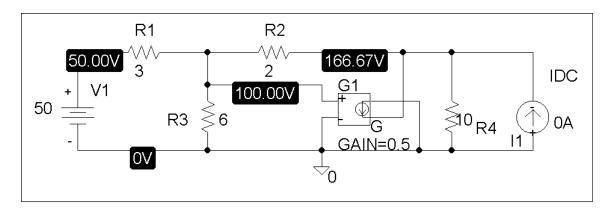


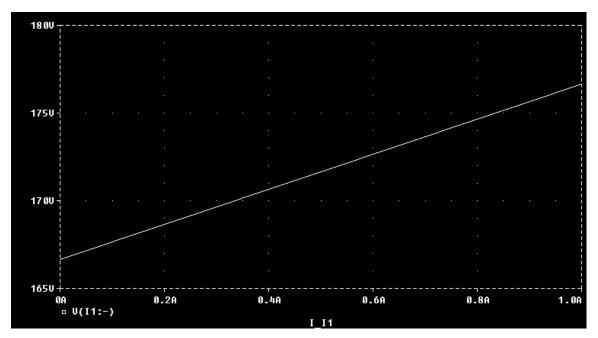
## Chapter 4, Solution 79.

After drawing and saving the schematic as shown below, we perform a dc sweep on I1 connected across a and b. The plot is shown. From the plot, we get,

$$V = 167 V [zero intercept]$$

$$R = (177 - 167)/1 = 10 \text{ ohms}$$



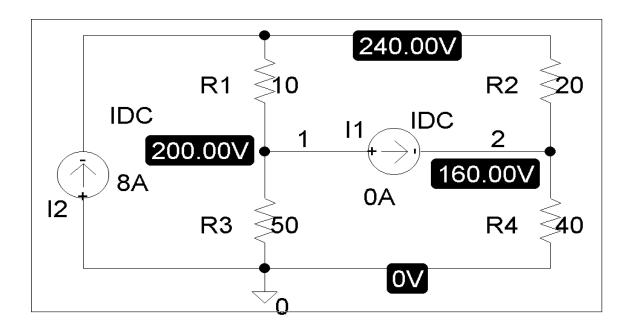


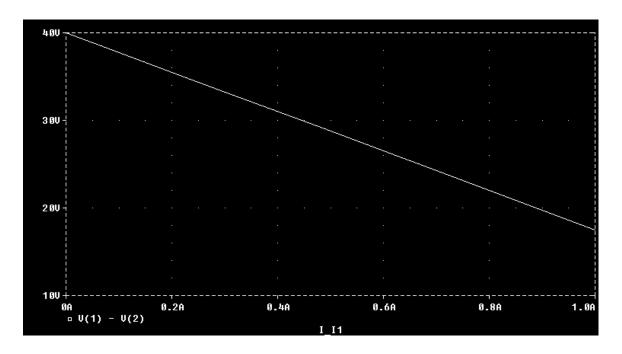
## Chapter 4, Solution 80.

The schematic in shown below. We label nodes a and b as 1 and 2 respectively. We perform dc sweep on I1. In the Trace/Add menu, type v(1) - v(2) which will result in the plot below. From the plot,

$$V_{Th} = 40 V [zero intercept]$$

$$R_{Th} = (40 - 17.5)/1 = 22.5 \text{ ohms} [slope]$$



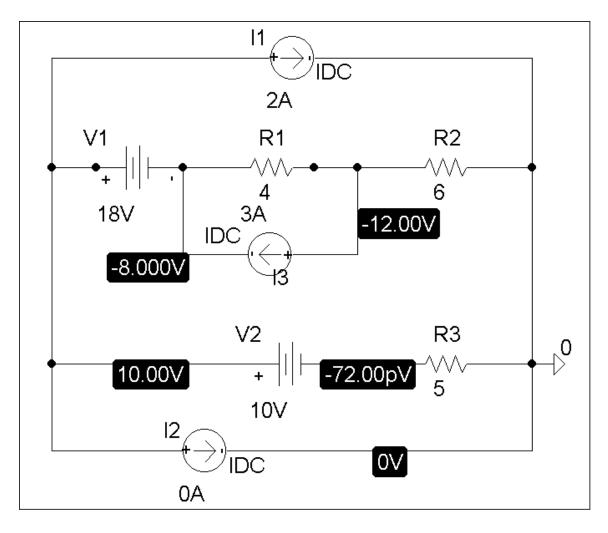


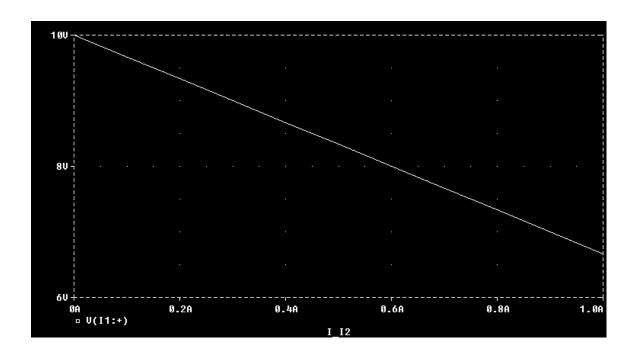
#### Chapter 4, Solution 81.

The schematic is shown below. We perform a dc sweep on the current source, I2, connected between terminals a and b. The plot of the voltage across I2 is shown below. From the plot,

$$V_{Th} = 10 V [zero intercept]$$

 $R_{Th} = (10 - 6.7)/1 =$  **3.3 ohms**. Note that this is in good agreement with the exact value of 3.333 ohms.

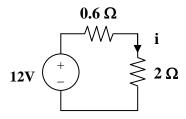




## Chapter 4, Solution 82.

$$V_{Th} \; = \; V_{oc} \; = \; 12 \; V, \; \; I_{sc} \; = \; 20 \; A$$

$$R_{Th} \ = \ V_{oc}/I_{sc} \ = \ 12/20 \ = \ 0.6 \ ohm.$$



$$i = 12/2.6$$
,  $p = i^2R = (12/2.6)^2(2) = 42.6$  watts

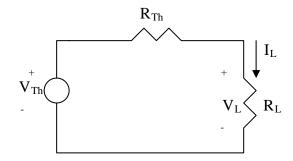
## Chapter 4, Solution 83.

$$V_{Th}~=~V_{oc}~=~12~V,~I_{sc}~=~I_{N}~=~1.5~A$$

$$R_{Th}~=~V_{Th}/I_{\rm N}~=~8~ohms,~~V_{Th}~=~\textbf{12}~\textbf{V},~~R_{Th}~=~\textbf{8}~\textbf{ohms}$$

## Chapter 4, Solution 84

Let the equivalent circuit of the battery terminated by a load be as shown below.



For open circuit,

$$R_L = \infty$$
,  $\longrightarrow$   $V_{Th} = V_{oc} = V_L = \underline{10.8 \text{ V}}$   
When  $R_L = 4 \text{ ohm}$ ,  $V_L = 10.5$ ,

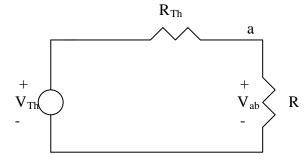
$$I_L = \frac{V_L}{R_L} = 10.8/4 = 2.7$$

But

$$V_{Th} = V_L + I_L R_{Th}$$
  $\longrightarrow$   $R_{Th} = \frac{V_{Th} - V_L}{I_L} = \frac{12 - 10.8}{2.7} = \frac{0.44440}{2.7}$   
= 444.4 m $\Omega$ .

## Chapter 4, Solution 85

(a) Consider the equivalent circuit terminated with R as shown below.



$$V_{ab} = \frac{R}{R + R_{Th}} V_{Th} \longrightarrow 6 = \frac{10}{10 + R_{Th}} V_{Th}$$

or

$$60 + 6R_{Th} = 10V_{Th}$$
 where R<sub>Th</sub> is in k-ohm.

Similarly,

$$12 = \frac{30}{30 + R_{Th}} V_{Th} \longrightarrow 360 + 12R_{Th} = 30V_{Th}$$
 (2)

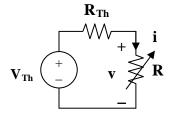
Solving (1) and (2) leads to

$$\underline{V_{Th}} = 24 \text{ V}, \ R_{Th} = 30k\Omega$$

(b) 
$$V_{ab} = \frac{20}{20 + 30} (24) = \underline{9.6 \text{ V}}$$

#### Chapter 4, Solution 86.

We replace the box with the Thevenin equivalent.



$$V_{Th}\ =\ v+iR_{Th}$$

When 
$$i = 1.5$$
,  $v = 3$ , which implies that  $V_{Th} = 3 + 1.5R_{Th}$  (1)

When 
$$i = 1$$
,  $v = 8$ , which implies that  $V_{Th} = 8 + 1xR_{Th}$  (2)

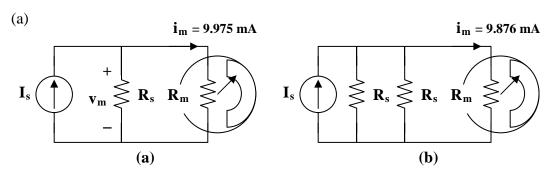
From (1) and (2),  $R_{Th} = 10$  ohms and  $V_{Th} = 18$  V.

(a) When 
$$R = 4$$
,  $i = V_{Th}/(R + R_{Th}) = 18/(4 + 10) = 1.2857 A$ 

(b) For maximum power,  $R = R_{TH}$ 

Pmax = 
$$(V_{Th})^2/4R_{Th} = 18^2/(4x10) = 8.1$$
 watts

### Chapter 4, Solution 87.



From Fig. (a),

$$v_m \; = \; R_m i_m \; = \; 9.975 \; mA \; x \; 20 \; = \; 0.1995 \; V$$

$$I_s = 9.975 \text{ mA} + (0.1995/R_s)$$
 (1)

From Fig. (b),

$$v_m \; = \; R_m i_m \; = \; 20x9.876 \; = \; 0.19752 \; V$$

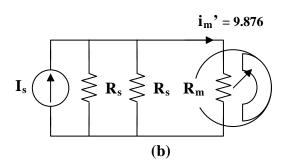
$$I_s = 9.876 \text{ mA} + (0.19752/2\text{k}) + (0.19752/R_s)$$

$$= 9.975 \text{ mA} + (0.19752/R_s) \tag{2}$$

Solving (1) and (2) gives,

$$R_s = 8 k ohms, I_s = 10 mA$$

(b)

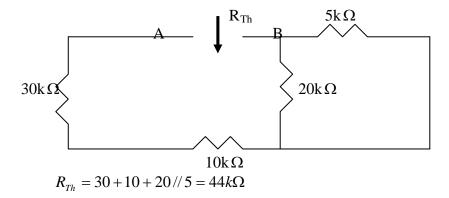


$$8k||4k = 2.667 \text{ k ohms}|$$

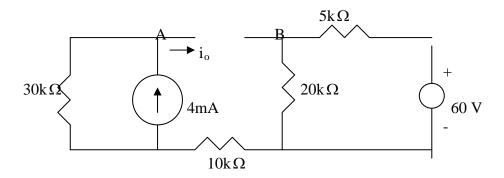
$$i_m' = [2667/(2667 + 20)](10 \text{ mA}) = 9.926 \text{ mA}$$

## Chapter 4, Solution 88

To find  $R_{\text{Th}}$ , consider the circuit below.

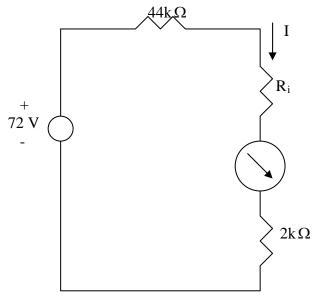


To find  $\ensuremath{V_{\text{Th}}}$  , consider the circuit below.



$$V_A = 30x4 = 120$$
,  $V_B = \frac{20}{25}(60) = 48$ ,  $V_{Th} = V_A - V_B = 72 \text{ V}$ 

The Thevenin equivalent circuit is shown below.



$$I = \frac{72}{44 + 2 + R_i} \,\mathrm{mA}$$

assuming  $R_i$  is in k-ohm.

(a) When  $R_i = 500 \Omega$ ,

$$I = \frac{72}{44 + 2 + 0.5} = \underline{1.548 \,\mathrm{mA}}$$

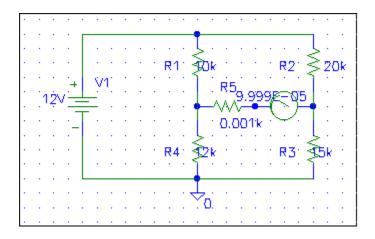
(b) When  $R_i = 0\Omega$ ,

$$I = \frac{72}{44 + 2 + 0} = \underline{1.565 \, \text{mA}}$$

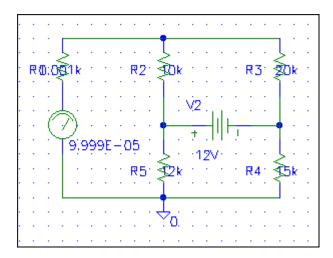
## **Chapter 4, Solution 89**

It is easy to solve this problem using Pspice.

(a) The schematic is shown below. We insert IPROBE to measure the desired ammeter reading. We insert a very small resistance in series IPROBE to avoid problem. After the circuit is saved and simulated, the current is displaced on IPROBE as  $99.99 \mu A$ .



(b) By interchanging the ammeter and the 12-V voltage source, the schematic is shown below. We obtain exactly the same result as in part (a).



# Chapter 4, Solution 90.

$$R_x \, = \, (R_3/R_1)R_2 \, = \, (4/2)R_2 \, = \, 42.6, R_2 \, = \, 21.3$$

which is (21.30hms/1000hms)% = **21.3%** 

## Chapter 4, Solution 91.

$$R_x = (R_3/R_1)R_2$$

(a) Since  $0 < R_2 < 50$  ohms, to make  $0 < R_x < 10$  ohms requires that when  $R_2 = 50$  ohms,  $R_x = 10$  ohms.

$$10 = (R_3/R_1)50 \text{ or } R_3 = R_1/5$$

so we select  $R_1 = 100 \text{ ohms}$  and  $R_3 = 20 \text{ ohms}$ 

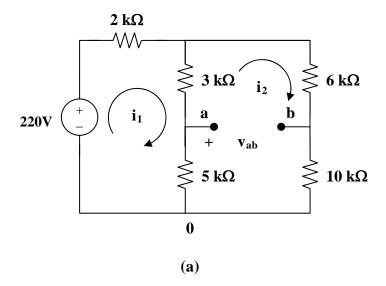
(b) For  $0 < R_x < 100 \text{ ohms}$ 

$$100 = (R_3/R_1)50$$
, or  $R_3 = 2R_1$ 

So we can select  $R_1 = 100 \text{ ohms}$  and  $R_3 = 200 \text{ ohms}$ 

#### Chapter 4, Solution 92.

For a balanced bridge,  $v_{ab}=0$ . We can use mesh analysis to find  $v_{ab}$ . Consider the circuit in Fig. (a), where  $i_1$  and  $i_2$  are assumed to be in mA.



$$220 = 2i_1 + 8(i_1 - i_2)$$
 or  $220 = 10i_1 - 8i_2$  (1)

$$0 = 24i_2 - 8i_1 \text{ or } i_2 = (1/3)i_1$$
 (2)

From (1) and (2),

$$i_1 = 30 \text{ mA} \text{ and } i_2 = 10 \text{ mA}$$

Applying KVL to loop 0ab0 gives

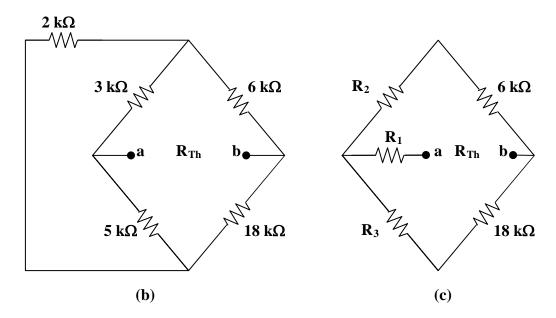
$$5(i_2 - i_1) + v_{ab} + 10i_2 = 0 \text{ V}$$

Since  $v_{ab} = 0$ , the bridge is balanced.

When the 10 k ohm resistor is replaced by the 18 k ohm resistor, the gridge becomes unbalanced. (1) remains the same but (2) becomes

Solving (1) and (3), 
$$i_1 = 27.5 \text{ mA}, \ i_2 = 6.875 \text{ mA}$$
 
$$v_{ab} = 5(i_1 - i_2) - 18i_2 = -20.625 \text{ V}$$
 
$$V_{Th} = v_{ab} = -20.625 \text{ V}$$

To obtain  $R_{Th}$ , we convert the delta connection in Fig. (b) to a wye connection shown in Fig. (c).



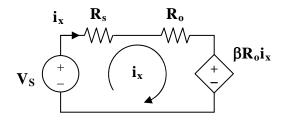
$$R_1 = 3x5/(2+3+5) = 1.5 \text{ k ohms}, R_2 = 2x3/10 = 600 \text{ ohms},$$
 
$$R_3 = 2x5/10 = 1 \text{ k ohm}.$$

$$R_{Th} = R_1 + (R_2 + 6)||(R_3 + 18)| = 1.5 + 6.6||9| = 6.398 \text{ k ohms}$$

$$R_L = R_{Th} =$$
6.398 k ohms

$$P_{max} = (V_{Th})^2/(4R_{Th}) = (20.625)^2/(4x6.398) =$$
**16.622 mWatts**

## Chapter 4, Solution 93.



$$-V_s + (R_s + R_o)i_x + \beta R_o i_x = 0$$

$$i_x = V_s/(R_s + (1+\beta)R_o)$$

#### Chapter 4, Solution 94.

(a) 
$$\begin{split} V_o/V_g &= R_p/(R_g + R_s + R_p) \\ R_{eq} &= R_p || (R_g + R_s) = R_g \\ R_g &= R_p (R_g + R_s)/(R_p + R_g + R_s) \\ R_g R_p + R_g^2 + R_g R_s &= R_p R_g + R_p R_s \\ R_p R_s &= R_g (R_g + R_s) \end{split} \tag{2}$$
 From (1), 
$$R_p/\alpha = R_g + R_s + R_p \\ R_g + R_s &= R_p ((1/\alpha) - 1) = R_p (1 - \alpha)/\alpha \tag{1a} \end{split}$$

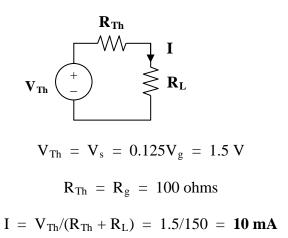
Combining (2) and (1a) gives,

$$R_s = [(1 - \alpha)/\alpha]R_{eq}$$
 (3)  
=  $(1 - 0.125)(100)/0.125 = 700$  ohms

From (3) and (1a),

$$R_p(1-\alpha)/\alpha \ = \ R_g + [(1-\alpha)/\alpha] R_g \ = \ R_g/\alpha$$
 
$$R_p \ = \ R_g/(1-\alpha) \ = \ 100/(1-0.125) \ = \ \textbf{114.29 ohms}$$

(b)



#### Chapter 4, Solution 95.

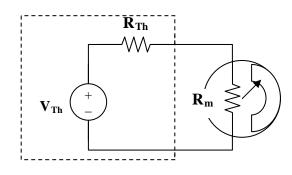
Let 1/sensitivity =  $1/(20 \text{ k ohms/volt}) = 50 \mu\text{A}$ 

For the 0 - 10 V scale,

$$R_m = V_{fs}/I_{fs} = 10/50 \ \mu A = 200 \ k \ ohms$$

For the 0 - 50 V scale,

$$R_m = 50(20 \text{ k ohms/V}) = 1 \text{ M ohm}$$



$$V_{Th} \; = \; I(R_{Th} + R_m)$$

(a) A 4V reading corresponds to

$$I = (4/10)I_{fs} = 0.4x50 \,\mu\text{A} = 20 \,\mu\text{A}$$
 
$$V_{Th} = 20 \,\mu\text{A} \,R_{Th} + 20 \,\mu\text{A} \,250 \,\text{k ohms}$$
 
$$= 4 + 20 \,\mu\text{A} \,R_{Th} \tag{1}$$

(b) A 5V reading corresponds to

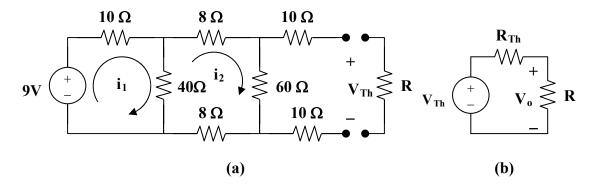
$$\begin{split} I &= (5/50)I_{fs} = 0.1 \ x \ 50 \ \mu A = 5 \ \mu A \\ V_{Th} &= 5 \ \mu A \ x \ R_{Th} + 5 \ \mu A \ x \ 1 \ M \ ohm \\ V_{Th} &= 5 + 5 \ \mu A \ R_{Th} \end{split} \tag{2}$$

From (1) and (2)

$$0 = -1 + 15~\mu A~R_{Th}~~which~leads~to~~R_{Th}~=~\textbf{66.67~k~ohms}$$
 From (1), 
$$V_{Th}~=~4 + 20x10^{-6}x(1/(15x10^{-6}))~=~\textbf{5.333~V}$$

#### Chapter 4, Solution 96.

(a) The resistance network can be redrawn as shown in Fig. (a),



$$R_{Th} = 10 + 10 + [60||(8 + 8 + [10||40])] = 20 + (60||24) = 37.14 \text{ ohms}$$

Using mesh analysis,

$$-9 + 50i_1 - 40i_2 = 0$$

$$116i_2 - 40i_1 = 0 \text{ or } i_1 = 2.9i_2$$
(1)

$$116i_2 - 40i_1 = 0 \text{ or } i_1 = 2.9i_2 \tag{2}$$

From (1) and (2),  $i_2 = 9/105 = 0.08571$ 

$$V_{Th} = 60i_2 = 5.143 \text{ V}$$

From Fig. (b),

$$V_o = [R/(R + R_{Th})]V_{Th} = 1.8 V$$

$$R/(R + 37.14) = 1.8/5.143 = 0.35$$
 or  $R = 0.35R + 13$  or  $R = (13)/(1-0.35)$ 

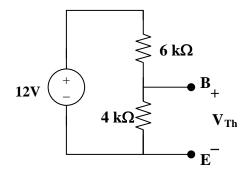
which leads to R =  $20 \Omega$  (note, this is just for the  $V_0 = 1.8 \text{ V}$ )

(b) Asking for the value of R for maximum power would lead to  $R = R_{Th} = 37.14 \Omega$ .

However, the problem asks for the value of R for maximum current. This happens when the value of resistance seen by the source is a minimum thus R = 0 is the correct value.

$$I_{max} = V_{Th}/(R_{Th}) = 5.143/(37.14) = 138.48 \text{ mA}.$$

## Chapter 4, Solution 97.

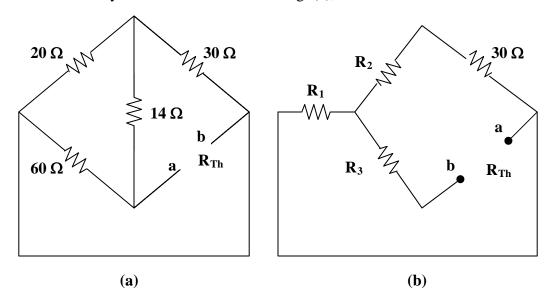


$$R_{Th} = R_1 || R_2 = 6 || 4 =$$
**2.4 k ohms**

$$V_{Th} \ = \ [R_2/(R_1+R_2)]v_s \ = \ [4/(6+4)](12) \ = \ \textbf{4.8 V}$$

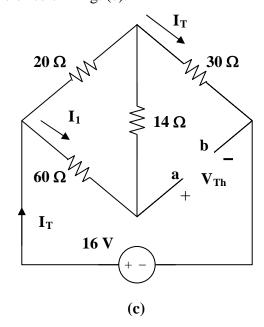
#### Chapter 4, Solution 98.

The 20-ohm, 60-ohm, and 14-ohm resistors form a delta connection which needs to be connected to the wye connection as shown in Fig. (b),



$$R_1 \,=\, 20x60/(20+60+14) \,=\, 1200/94 \,=\, 12.766 \text{ ohms}$$
 
$$R_2 \,=\, 20x14/94 \,=\, 2.979 \text{ ohms}$$
 
$$R_3 \,=\, 60x14/94 \,=\, 8.936 \text{ ohms}$$
 
$$R_{Th} \,=\, R_3 + R_1 ||(R_2+30) \,=\, 8.936 + 12.766 ||32.98 \,=\, 18.139 \text{ ohms}$$

To find V<sub>Th</sub>, consider the circuit in Fig. (c).



$$I_T \ = \ 16/(30+15.745) \ = \ 349.8 \ mA$$
 
$$I_1 \ = \ [20/(20+60+14)]I_T \ = \ 74.43 \ mA$$
 
$$V_{Th} \ = \ 14I_1 + 30I_T \ = \ 11.536 \ V$$
 
$$I_{40} \ = \ V_{Th}/(R_{Th} + 40) \ = \ 11.536/(18.139 + 40) \ = \ 198.42 \ mA$$
 
$$P_{40} \ = \ I_{40}{}^2R \ = \ \textbf{1.5748 watts}$$