

Chapter 6, Solution 1.

$$i = C \frac{dv}{dt} = 7.5(2e^{-3t} - 6te^{-3t}) = \mathbf{15(1 - 3t)e^{-3t} \text{ A}}$$

$$p = vi = 15(1-3t)e^{-3t} \cdot 2te^{-3t} = \mathbf{30t(1 - 3t)e^{-6t} \text{ W.}}$$

$$\mathbf{15(1 - 3t)e^{-3t} \text{ A}, 30t(1 - 3t)e^{-6t} \text{ W}}$$

Chapter 6, Solution 2.

$$\begin{aligned}w(t) &= (1/2)C(v(t))^2 \text{ or } (v(t))^2 = 2w(t)/C = (20\cos^2(377t))/(50 \times 10^{-6}) = \\&0.4 \times 10^6 \cos^2(377t) \text{ or } v(t) = \pm 632.5 \cos(377t) \text{ V. Let us assume that } v(t) = \\&632.5 \cos(377t) \text{ V, which leads to } i(t) = C(dv/dt) = 50 \times 10^{-6}(632.5)(-377 \sin(377t)) \\&= -11.923 \sin(377t) \text{ A.}\end{aligned}$$

*Please note that if we had chosen the negative value for v ,
then $i(t)$ would have been positive.*

Chapter 6, Solution 3.

Design a problem to help other students to better understand how capacitors work.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In 5 s, the voltage across a 40-mF capacitor changes from 160 V to 220 V. Calculate the average current through the capacitor.

Solution

$$i = C \frac{dv}{dt} = 40 \times 10^{-3} \frac{220 - 160}{5} = \mathbf{480 \text{ mA}}$$

Chapter 6, Solution 4.

$$v = \frac{1}{C} \int_0^t i dt + v(0)$$

$$= \frac{1}{5} \int_0^t 4 \sin(4t) dt + 1 = \left(-\frac{0.8}{4} \cos(4t) \right) \bigg|_0^t + 1 = -0.2 \cos(4t) + 0.2 + 1$$

$$= [1.2 - 0.2 \cos(4t)] \text{ V.}$$

Chapter 6, Solution 5.

$$v = \begin{cases} 5000t, & 0 < t < 2\text{ms} \\ 20 - 5000t, & 2 < t < 6\text{ms} \\ -40 + 5000t, & 6 < t < 8\text{ms} \end{cases}$$

$$i = C \frac{dv}{dt} = \frac{4 \times 10^{-6}}{10^{-3}} \begin{cases} 5, & 0 < t < 2\text{ms} \\ -5, & 2 < t < 6\text{ms} \\ 5, & 6 < t < 8\text{ms} \end{cases} = \underline{\begin{cases} 20 \text{ mA}, & 0 < t < 2\text{ms} \\ -20 \text{ mA}, & 2 < t < 6\text{ms} \\ 20 \text{ mA}, & 6 < t < 8\text{ms} \end{cases}}$$

Chapter 6, Solution 6.

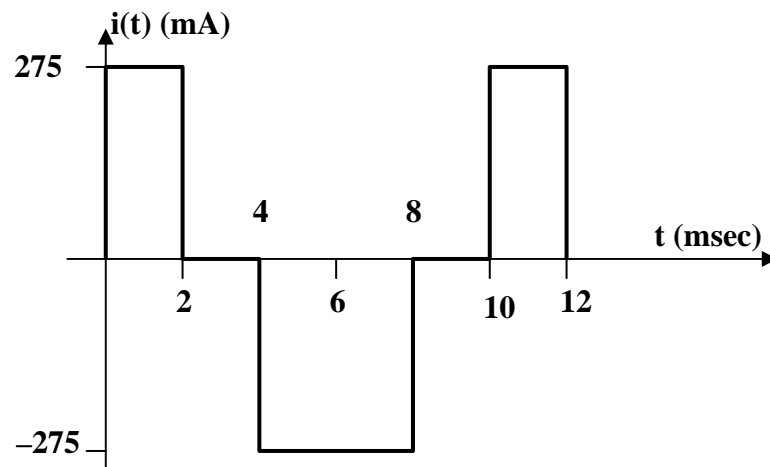
$$i = C \frac{dv}{dt} = 55 \times 10^{-6} \text{ times the slope of the waveform.}$$

For example, for $0 < t < 2$,

$$\frac{dv}{dt} = \frac{10}{2 \times 10^{-3}}$$

$$i = C \frac{dv}{dt} = (55 \times 10^{-6}) \frac{10}{2 \times 10^{-3}} = 275 \text{ mA}$$

Thus the current $i(t)$ is sketched below.



Chapter 6, Solution 7.

$$v = \frac{1}{C} \int i dt + v(t_o) = \frac{1}{25 \times 10^{-3}} \int_o^t 5tx10^{-3} dt + 10$$

$$= \frac{2.5t^2}{25} + 10 = \mathbf{[0.1t^2 + 10] \text{ V.}}$$

Chapter 6, Solution 8.

$$(a) \quad i = C \frac{dv}{dt} = -100ACe^{-100t} - 600BCe^{-600t} \quad (1)$$

$$i(0) = 2 = -100AC - 600BC \quad \longrightarrow \quad 5 = -A - 6B \quad (2)$$

$$v(0^+) = v(0^-) \quad \longrightarrow \quad 50 = A + B \quad (3)$$

Solving (2) and (3) leads to

$$\underline{A=61, \quad B=-11}$$

$$(b) \quad \text{Energy} = \frac{1}{2} C v^2(0) = \frac{1}{2} \times 4 \times 10^{-3} \times 2500 = \underline{5 \text{ J}}$$

(c) From (1),

$$i = -100 \times 61 \times 4 \times 10^{-3} e^{-100t} - 600 \times 11 \times 4 \times 10^{-3} e^{-600t} = \underline{-24.4e^{-100t} - 26.4e^{-600t} \text{ A}}$$

Chapter 6, Solution 9.

$$v(t) = \frac{1}{1/2} \int_0^t 6(1 - e^{-t}) dt + 0 = 12 \left(t + e^{-t} \right) \Big|_0^t \text{ V} = 12(t + e^{-t}) - 12$$

$$v(2) = 12(2 + e^{-2}) - 12 = \mathbf{13.624 \text{ V}}$$

$$p = iv = [12(t + e^{-t}) - 12]6(1 - e^{-t})$$

$$p(2) = [12(2 + e^{-2}) - 12]6(1 - e^{-2}) = \mathbf{70.66 \text{ W}}$$

Chapter 6, Solution 10

$$i = C \frac{dv}{dt} = 5 \times 10^{-3} \frac{dv}{dt}$$

$$v = \begin{cases} 16t, & 0 < t < 1\mu s \\ 16, & 1 < t < 3\mu s \\ 64 - 16t, & 3 < t < 4\mu s \end{cases}$$

$$\frac{dv}{dt} = \begin{cases} 16 \times 10^6, & 0 < t < 1\mu s \\ 0, & 1 < t < 3\mu s \\ -16 \times 10^6, & 3 < t < 4\mu s \end{cases}$$

$$i(t) = \begin{cases} 80 \text{ kA}, & 0 < t < 1\mu s \\ 0, & 1 < t < 3\mu s \\ -80 \text{ kA}, & 3 < t < 4\mu s \end{cases}$$

Chapter 6, Solution 11.

$$v = \frac{1}{C} \int_0^t i dt + v(0) = 10 + \frac{1}{4 \times 10^{-3}} \int_0^t i(t) dt$$

For $0 < t < 2$, $i(t) = 15 \text{ mA}$, $V(t) = 10 + \frac{10^3}{4 \times 10^{-3}} \int_0^t 15 dt = 10 + 3.75t$

$$v(2) = 10 + 7.5 = 17.5$$

For $2 < t < 4$, $i(t) = -10 \text{ mA}$

$$v(t) = \frac{1}{4 \times 10^{-3}} \int_2^t i(t) dt + v(2) = -\frac{10 \times 10^{-3}}{4 \times 10^{-3}} \int_2^t dt + 17.5 = 22.5 - 2.5t$$

$$v(4) = 22.5 - 2.5 \times 4 = 12.5$$

For $4 < t < 6$, $i(t) = 0$, $v(t) = \frac{1}{4 \times 10^{-3}} \int_4^t 0 dt + v(4) = 12.5$

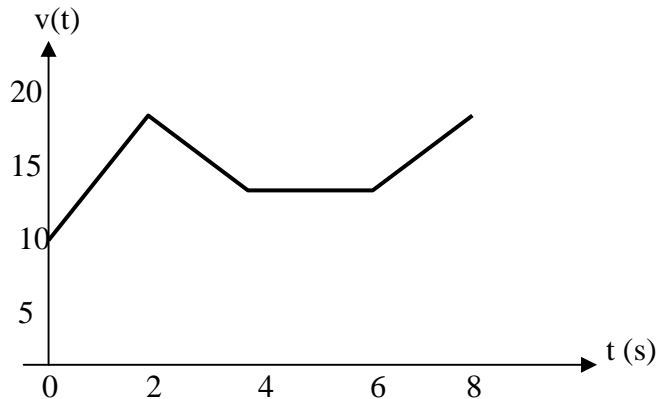
For $6 < t < 8$, $i(t) = 10 \text{ mA}$

$$v(t) = \frac{10 \times 10^{-3}}{4 \times 10^{-3}} \int_6^t dt + v(6) = 2.5(t - 6) + 12.5 = 2.5t - 2.5$$

Hence,

$$v(t) = \begin{cases} 10 + 3.75t \text{ V}, & 0 < t < 2\text{s} \\ 22.5 - 2.5t \text{ V}, & 2 < t < 4\text{s} \\ 12.5 \text{ V}, & 4 < t < 6\text{s} \\ 2.5t - 2.5 \text{ V}, & 6 < t < 8\text{s} \end{cases}$$

which is sketched below.



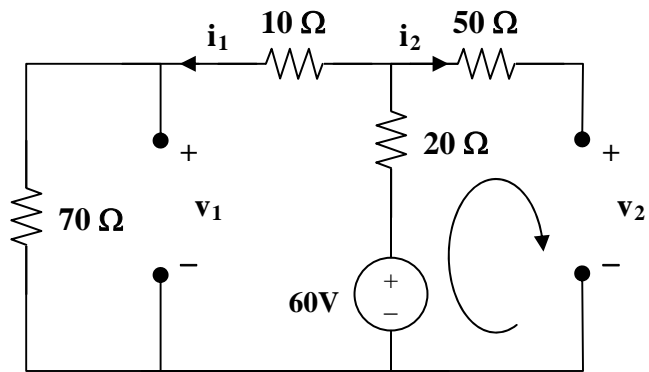
Chapter 6, Solution 12.

$i_R = V/R = (30/12)e^{-2000t} = 2.5 e^{-2000t}$ and $i_C = C(dv/dt) = 0.1 \times 30(-2000) e^{-2000t}$
 $= -6000 e^{-2000t}$ A. Thus, $i = i_R + i_C = -5,997.5 e^{-2000t}$. The power is equal to:

$$p_i = -179.925 e^{-4000t} \text{ W.}$$

Chapter 6, Solution 13.

Under dc conditions, the circuit becomes that shown below:



$$i_2 = 0, i_1 = 60/(70+10+20) = 0.6 \text{ A}$$

$$v_1 = 70i_1 = 42 \text{ V}, v_2 = 60 - 20i_1 = 48 \text{ V}$$

Thus, $v_1 = 42 \text{ V}$, $v_2 = 48 \text{ V}$.

Chapter 6, Solution 14.

20 pF is in series with 60pF = $20 \times 60 / 80 = 15$ pF

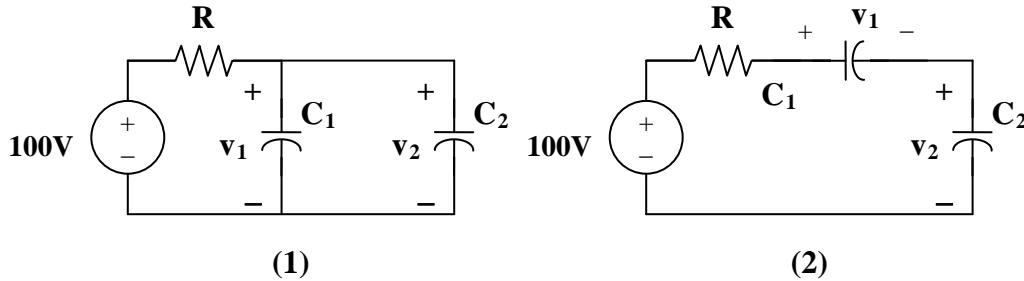
30-pF is in series with 70pF = $30 \times 70 / 100 = 21$ pF

15pF is in parallel with 21pF = $15 + 21 = \mathbf{36 \text{ pF}}$

Chapter 6, Solution 15.

Arranging the capacitors in parallel results in circuit shown in Fig. (1) (It should be noted that the resistors are in the circuits only to limit the current surge as the capacitors charge. Once the capacitors are charged the current through the resistors are obviously equal to zero.):

$$v_1 = v_2 = 100$$



$$w_{20} = \frac{1}{2} C v^2 = \frac{1}{2} \times 25 \times 10^{-6} \times 100^2 = \mathbf{125 \text{ mJ}}$$

$$w_{30} = \frac{1}{2} \times 75 \times 10^{-6} \times 100^2 = \mathbf{375 \text{ mJ}}$$

(b) Arranging the capacitors in series results in the circuit shown in Fig. (2):

$$v_1 = \frac{C_2}{C_1 + C_2} V = \frac{75}{100} \times 100 = 75 \text{ V}, v_2 = 25 \text{ V}$$

$$w_{25} = \frac{1}{2} \times 25 \times 10^{-6} \times 75^2 = \mathbf{70.31 \text{ mJ}}$$

$$w_{75} = \frac{1}{2} \times 75 \times 10^{-6} \times 25^2 = \mathbf{23.44 \text{ mJ.}}$$

(a) **125 mJ, 375 mJ** (b) **70.31 mJ, 23.44 mJ**

Chapter 6, Solution 16

$$C_{eq} = 14 + \frac{Cx80}{C + 80} = 30 \quad \longrightarrow \quad \underline{C = 20 \mu F}$$

Chapter 6, Solution 17.

- (a) $4F$ in series with $12F = 4 \times 12/(16) = 3F$
 $3F$ in parallel with $6F$ and $3F = 3+6+3 = 12F$
 $4F$ in series with $12F = 3F$
i.e. $C_{eq} = \mathbf{3F}$
- (b) $C_{eq} = 5 + [6 \times (4 + 2)/(6+4+2)] = 5 + (36/12) = 5 + 3 = \mathbf{8F}$
- (c) $3F$ in series with $6F = (3 \times 6)/9 = 2F$
$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{6} + \frac{1}{3} = 1$$

 $C_{eq} = \mathbf{1F}$

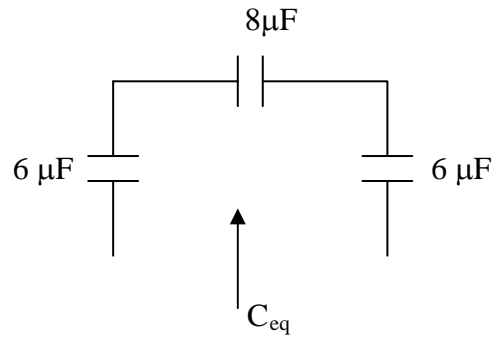
Chapter 6, Solution 18.

$4\ \mu\text{F}$ in parallel with $4\ \mu\text{F} = 8\ \mu\text{F}$

$4\ \mu\text{F}$ in series with $4\ \mu\text{F} = 2\ \mu\text{F}$

$2\ \mu\text{F}$ in parallel with $4\ \mu\text{F} = 6\ \mu\text{F}$

Hence, the circuit is reduced to that shown below.



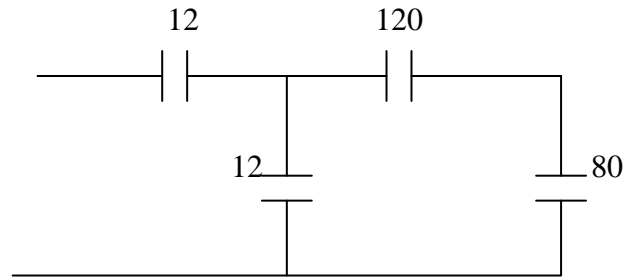
$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{6} + \frac{1}{8} = 0.4583 \quad \longrightarrow \quad C_{eq} = \underline{2.1818\ \mu\text{F}}$$

Chapter 6, Solution 19.

We combine 10-, 20-, and 30- μF capacitors in parallel to get 60 μF . The 60 - μF capacitor in series with another 60- μF capacitor gives 30 μF .

$$30 + 50 = 80 \mu\text{F}, \quad 80 + 40 = 120 \mu\text{F}$$

The circuit is reduced to that shown below.



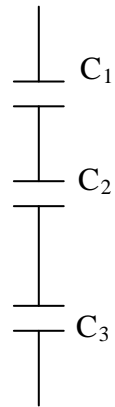
120- μF capacitor in series with 80 μF gives $(80 \times 120)/200 = 48$

$$48 + 12 = 60$$

60- μF capacitor in series with 12 μF gives $(60 \times 12)/72 = \mathbf{10 \mu\text{F}}$

Chapter 6, Solution 20.

Consider the circuit shown below.



$$C_1 = 1 + 1 = 2\mu F$$

$$C_2 = 2 + 2 + 2 = 6\mu F$$

$$C_3 = 4 \times 3 = 12\mu F$$

$$1/C_{eq} = (1/C_1) + (1/C_2) + (1/C_3) = 0.5 + 0.16667 + 0.08333 = 0.75 \times 10^{-6}$$

$$C_{eq} = \mathbf{1.3333\ \mu F}.$$

Chapter 6, Solution 21.

$$4\mu\text{F in series with } 12\mu\text{F} = (4 \times 12)/16 = 3\mu\text{F}$$

$$3\mu\text{F in parallel with } 3\mu\text{F} = 6\mu\text{F}$$

$$6\mu\text{F in series with } 6\mu\text{F} = 3\mu\text{F}$$

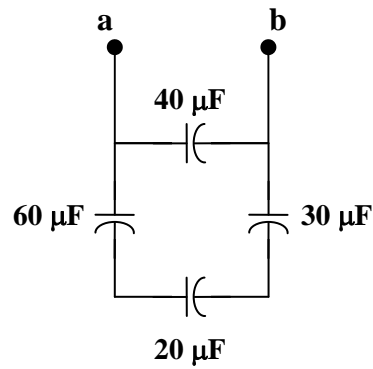
$$3\mu\text{F in parallel with } 2\mu\text{F} = 5\mu\text{F}$$

$$5\mu\text{F in series with } 5\mu\text{F} = 2.5\mu\text{F}$$

$$\text{Hence } C_{\text{eq}} = \mathbf{2.5\mu\text{F}}$$

Chapter 6, Solution 22.

Combining the capacitors in parallel, we obtain the equivalent circuit shown below:



Combining the capacitors in series gives C_{eq}^1 , where

$$\frac{1}{C_{\text{eq}}^1} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \longrightarrow C_{\text{eq}}^1 = 10\ \mu\text{F}$$

Thus

$$C_{\text{eq}} = 10 + 40 = \mathbf{50\ \mu\text{F}}$$

Chapter 6, Solution 23.

Using Fig. 6.57, design a problem to help other students better understand how capacitors work together when connected in series and parallel.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 6.57, determine:

- (a) the voltage across each capacitor,
- (b) the energy stored in each capacitor.

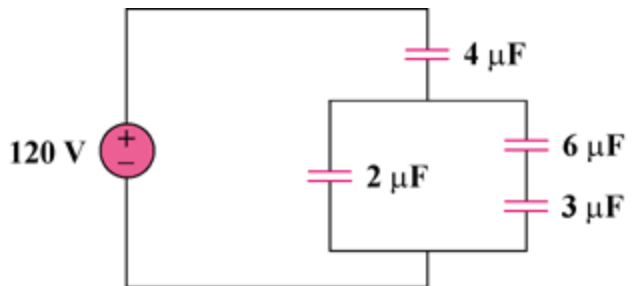


Figure 6.57

Solution

- (a) $3\mu\text{F}$ is in series with $6\mu\text{F}$ $3 \times 6 / (9) = 2\mu\text{F}$
 $v_{4\mu\text{F}} = 1/2 \times 120 = \mathbf{60\text{V}}$
 $v_{2\mu\text{F}} = \mathbf{60\text{V}}$
 $v_{6\mu\text{F}} = \frac{3}{6+3} (60) = \mathbf{20\text{V}}$
 $v_{3\mu\text{F}} = 60 - 20 = \mathbf{40\text{V}}$
- (b) Hence $w = 1/2 C v^2$
 $w_{4\mu\text{F}} = 1/2 \times 4 \times 10^{-6} \times 3600 = \mathbf{7.2\text{mJ}}$
 $w_{2\mu\text{F}} = 1/2 \times 2 \times 10^{-6} \times 3600 = \mathbf{3.6\text{mJ}}$
 $w_{6\mu\text{F}} = 1/2 \times 6 \times 10^{-6} \times 400 = \mathbf{1.2\text{mJ}}$
 $w_{3\mu\text{F}} = 1/2 \times 3 \times 10^{-6} \times 1600 = \mathbf{2.4\text{mJ}}$

Chapter 6, Solution 24.

20 μ F is series with 80 μ F = $20 \times 80 / (100) = 16\mu$ F

14 μ F is parallel with 16 μ F = 30 μ F

(a) $V_{30\mu\text{F}} = \mathbf{90V}$

$$V_{60\mu\text{F}} = \mathbf{30V}$$

$$V_{14\mu\text{F}} = \mathbf{60V}$$

$$V_{20\mu\text{F}} = \frac{80}{20 + 80} \times 60 = \mathbf{48V}$$

$$V_{80\mu\text{F}} = 60 - 48 = \mathbf{12V}$$

(b) Since $w = \frac{1}{2} C v^2$

$$W_{30\mu\text{F}} = 1/2 \times 30 \times 10^{-6} \times 8100 = \mathbf{121.5mJ}$$

$$W_{60\mu\text{F}} = 1/2 \times 60 \times 10^{-6} \times 900 = \mathbf{27mJ}$$

$$W_{14\mu\text{F}} = 1/2 \times 14 \times 10^{-6} \times 3600 = \mathbf{25.2mJ}$$

$$W_{20\mu\text{F}} = 1/2 \times 20 \times 10^{-6} \times (48)^2 = \mathbf{23.04mJ}$$

$$W_{80\mu\text{F}} = 1/2 \times 80 \times 10^{-6} \times 144 = \mathbf{5.76mJ}$$

Chapter 6, Solution 25.

(a) For the capacitors in series,

$$Q_1 = Q_2 \longrightarrow C_1 v_1 = C_2 v_2 \longrightarrow \frac{v_1}{v_2} = \frac{C_2}{C_1}$$
$$v_s = v_1 + v_2 = \frac{C_2}{C_1} v_2 + v_2 = \frac{C_1 + C_2}{C_1} v_2 \longrightarrow v_2 = \frac{C_1}{C_1 + C_2} v_s$$

Similarly, $v_1 = \frac{C_2}{C_1 + C_2} v_s$

(b) For capacitors in parallel

$$v_1 = v_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$
$$Q_s = Q_1 + Q_2 = \frac{C_1}{C_2} Q_2 + Q_2 = \frac{C_1 + C_2}{C_2} Q_2$$

or

$$Q_2 = \frac{C_2}{C_1 + C_2} Q_s$$
$$Q_1 = \frac{C_1}{C_1 + C_2} Q_s$$

$$i = \frac{dQ}{dt} \longrightarrow i_1 = \frac{C_1}{C_1 + C_2} i_s, \quad i_2 = \frac{C_2}{C_1 + C_2} i_s$$

Chapter 6, Solution 26.

(a) $C_{\text{eq}} = C_1 + C_2 + C_3 = \mathbf{35\mu F}$

(b) $Q_1 = C_1 v = 5 \times 150\mu\text{C} = \mathbf{0.75\text{mC}}$
 $Q_2 = C_2 v = 10 \times 150\mu\text{C} = \mathbf{1.5\text{mC}}$
 $Q_3 = C_3 v = 20 \times 150 = \mathbf{3\text{mC}}$

(c) $w = \frac{1}{2} C_{\text{eq}} v^2 = \frac{1}{2} \times 35 \times 150^2 \mu\text{J} = \mathbf{393.8\text{mJ}}$

Chapter 6, Solution 27.

If they are all connected in parallel, we get $C_T = 4 \times 4 \mu F = 16 \mu F$

If they are all connected in series, we get

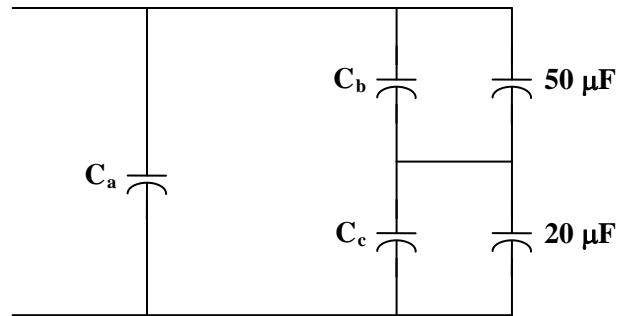
$$\frac{1}{C_T} = \frac{4}{4 \mu F} \longrightarrow C_T = 1 \mu F$$

All other combinations fall within these two extreme cases. Hence,

$$C_{\min} = \mathbf{1 \mu F}, C_{\max} = \mathbf{16 \mu F}$$

Chapter 6, Solution 28.

We may treat this like a resistive circuit and apply delta-wye transformation, except that R is replaced by $1/C$.



$$\frac{1}{C_a} = \frac{\left(\frac{1}{10}\right)\left(\frac{1}{40}\right) + \left(\frac{1}{10}\right)\left(\frac{1}{30}\right) + \left(\frac{1}{30}\right)\left(\frac{1}{40}\right)}{\frac{1}{30}}$$
$$= \frac{3}{40} + \frac{1}{10} + \frac{1}{40} = \frac{2}{10}$$

$$C_a = 5\mu\text{F}$$

$$\frac{1}{C_b} = \frac{\frac{1}{400} + \frac{1}{300} + \frac{1}{1200}}{\frac{1}{10}} = \frac{2}{30}$$

$$C_b = 15\mu\text{F}$$

$$\frac{1}{C_c} = \frac{\frac{1}{400} + \frac{1}{300} + \frac{1}{1200}}{\frac{1}{40}} = \frac{4}{15}$$

$$C_c = 3.75\mu\text{F}$$

$$C_b \text{ in parallel with } 50\mu\text{F} = 50 + 15 = 65\mu\text{F}$$

$$C_c \text{ in series with } 20\mu\text{F} = 23.75\mu\text{F}$$

$$65\mu\text{F} \text{ in series with } 23.75\mu\text{F} = \frac{65 \times 23.75}{88.75} = 17.39\mu\text{F}$$

$$17.39\mu\text{F} \text{ in parallel with } C_a = 17.39 + 5 = 22.39\mu\text{F}$$

$$\text{Hence } C_{eq} = \mathbf{22.39\mu\text{F}}$$

Chapter 6, Solution 29.

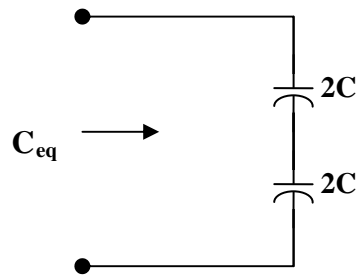
(a) C in series with $C = C/(2)$

$C/2$ in parallel with $C = 3C/2$

$$\frac{3C}{2} \text{ in series with } C = \frac{C \times \frac{3C}{2}}{5 \frac{C}{2}} = \frac{3C}{5}$$

$$3 \frac{C}{5} \text{ in parallel with } C = C + 3 \frac{C}{5} = \mathbf{1.6 C}$$

(b)



$$\frac{1}{C_{eq}} = \frac{1}{2C} + \frac{1}{2C} = \frac{1}{C}$$

$$C_{eq} = \mathbf{1 C}$$

Chapter 6, Solution 30.

$$v_o = \frac{1}{C} \int_0^t i dt + v_o(0)$$

For $0 < t < 1$, $i = 90t$ mA,

$$v_o = \frac{10^{-3}}{3 \times 10^{-6}} \int_0^t 90t dt + 0 = 15t^2 \text{ kV}$$

$$v_o(1) = 15 \text{ kV}$$

For $1 < t < 2$, $i = (180 - 90t)$ mA,

$$\begin{aligned} v_o &= \frac{10^{-3}}{3 \times 10^{-6}} \int_1^t (180 - 90t) dt + v_o(1) \\ &= [60t - 15t^2]_1^t + 15 \text{ kV} \\ &= [60t - 15t^2 - (60 - 15) + 15] \text{ kV} = [60t - 15t^2 - 30] \text{ kV} \end{aligned}$$

$$v_o(t) = \begin{cases} 15t^2 \text{ kV}, & 0 < t < 1 \\ [60t - 15t^2 - 30] \text{ kV}, & 1 < t < 2 \end{cases}$$

Chapter 6, Solution 31.

$$i_s(t) = \begin{cases} 30tmA, & 0 < t < 1 \\ 30mA, & 1 < t < 3 \\ -75 + 15t, & 3 < t < 5 \end{cases}$$

$$C_{eq} = 4 + 6 = 10\mu F$$

$$v = \frac{1}{C_{eq}} \int_0^t i dt + v(0)$$

For $0 < t < 1$,

$$v = \frac{10^{-3}}{10 \times 10^{-6}} \int_0^t 30t dt + 0 = 1.5t^2 \text{ kV}$$

For $1 < t < 3$,

$$\begin{aligned} v &= \frac{10^3}{10} \int_1^t 20dt + v(1) = [3(t-1) + 1.5]kV \\ &= [3t - 1.5]kV \end{aligned}$$

For $3 < t < 5$,

$$\begin{aligned} v &= \frac{10^3}{10} \int_3^t 15(t-5)dt + v(3) \\ &= \left[1.5 \frac{t^2}{2} - 7.5t \right]_3^t + 7.5kV = [0.75t^2 - 7.5t + 23.25]kV \end{aligned}$$

$$v(t) = \begin{cases} 1.5t^2kV, & 0 < t < 1s \\ [3t - 1.5]kV, & 1 < t < 3s \\ [0.75t^2 - 7.5t + 23.25]kV, & 3 < t < 5s \end{cases}$$

$$i_1 = C_1 \frac{dv}{dt} = 6 \times 10^{-6} \frac{dv}{dt}$$

$$i_1 = \begin{cases} 18tmA, & 0 < t < 1s \\ 18mA, & 1 < t < 3s \\ [9t - 45]mA, & 3 < t < 5s \end{cases}$$

$$i_2 = C_2 \frac{dv}{dt} = 4 \times 10^{-6} \frac{dv}{dt}$$

$$i_2 = \begin{cases} 12t mA, & 0 < t < 1s \\ 12 mA, & 1 < t < 3s \\ [6t - 30] mA, & 3 < t < 5s \end{cases}$$

Chapter 6, Solution 32.

(a) $C_{eq} = (12 \times 60)/72 = 10 \mu F$

$$v_1 = \frac{10^{-3}}{12 \times 10^{-6}} \int_0^t 50e^{-2t} dt + v_1(0) = \underline{-2083e^{-2t}} \Big|_0^t + 50 = \underline{-2083e^{-2t} + 2133V}$$

$$v_2 = \frac{10^{-3}}{60 \times 10^{-6}} \int_0^t 50e^{-2t} dt + v_2(0) = \underline{-416.7e^{-2t}} \Big|_0^t + 20 = \underline{-416.7e^{-2t} + 436.7V}$$

(b) At $t=0.5s$,

$$v_1 = -2083e^{-1} + 2133 = 1366.7, \quad v_2 = -416.7e^{-1} + 436.7 = 283.4$$

$$w_{12\mu F} = \frac{1}{2} \times 12 \times 10^{-6} \times (1366.7)^2 = \underline{11.207 J}$$

$$w_{20\mu F} = \frac{1}{2} \times 20 \times 10^{-6} \times (283.4)^2 = \underline{803.2 mJ}$$

$$w_{40\mu F} = \frac{1}{2} \times 40 \times 10^{-6} \times (283.4)^2 = \underline{1.6063 J}$$

Chapter 6, Solution 33

Because this is a totally capacitive circuit, we can combine all the capacitors using the property that capacitors in parallel can be combined by just adding their values and we combine capacitors in series by adding their reciprocals. However, for this circuit we only have the three capacitors in parallel.

$$3 \text{ F} + 2 \text{ F} = 5 \text{ F} \text{ (we need this to be able to calculate the voltage)}$$

$$C_{\text{Th}} = C_{\text{eq}} = 5 + 3 + 2 = 10 \text{ F}$$

The voltage will divide equally across the two 5 F capacitors. Therefore, we get:

$$V_{\text{Th}} = \mathbf{15 \text{ V}}, \quad C_{\text{Th}} = \mathbf{10 \text{ F}}.$$

15 V, 10 F

Chapter 6, Solution 34.

$$i = 10e^{-t/2}$$

$$v = L \frac{di}{dt} = 10 \times 10^{-3} (10) \left(\frac{1}{2} \right) e^{-t/2}$$
$$= -50e^{-t/2} \text{ mV}$$

$$v(3) = -50e^{-3/2} \text{ mV} = \mathbf{-11.157 \text{ mV}}$$

$$p = vi = -500e^{-t} \text{ mW}$$

$$p(3) = -500e^{-3} \text{ mW} = \mathbf{-24.89 \text{ mW}}.$$

Chapter 6, Solution 35.

$$v = L \frac{di}{dt} \longrightarrow L = \frac{v}{di/dt} = \frac{160 \times 10^{-3}}{\frac{(100 - 50) \times 10^{-3}}{2 \times 10^{-3}}} = \underline{6.4 \text{ mH}}$$

Chapter 6, Solution 36.

Design a problem to help other students to better understand how inductors work.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The current through a 12-mH inductor is $i(t) = 30te^{-2t}$ A, $t \geq 0$. Determine: (a) the voltage across the inductor, (b) the power being delivered to the inductor at $t = 1$ s, (c) the energy stored in the inductor at $t = 1$ s.

Solution

$$(a) \quad v = L \frac{di}{dt} = 12 \times 10^{-3} (30e^{-2t} - 60te^{-2t}) = \underline{(0.36 - 0.72t)e^{-2t} \text{ V}}$$

$$(b) \quad p = vi = (0.36 - 0.72 \times 1)e^{-2} \times 30 \times 1e^{-2} = 0.36 \times 30e^{-4} = \underline{-0.1978 \text{ W}}$$

$$(c) \quad w = \frac{1}{2} Li^2 = 0.5 \times 12 \times 10^{-3} (30 \times 1e^{-2})^2 = \underline{\mathbf{98.9 \text{ mJ}}}.$$

Chapter 6, Solution 37.

$$\begin{aligned}v &= L \frac{di}{dt} = 12 \times 10^{-3} \times 4(100) \cos 100t \\&= \mathbf{4.8 \cos (100t) \text{ V}}\end{aligned}$$

$$p = v i = 4.8 \times 4 \sin 100t \cos 100t = 9.6 \sin 200t$$

$$\begin{aligned}w &= \int_0^t p dt = \int_0^{11/200} 9.6 \sin 200t \\&= -\frac{9.6}{200} \cos 200t \Big|_0^{11/200} \text{ J} \\&= -48(\cos \pi - 1) \text{ mJ} = \mathbf{96 \text{ mJ}}\end{aligned}$$

Please note that this problem could have also been done by using $(1/2)Li^2$.

Chapter 6, Solution 38.

$$v = L \frac{di}{dt} = 40 \times 10^{-3} (e^{-2t} - 2te^{-2t}) dt$$

$$= 40(1 - 2t)e^{-2t} mV, t > 0$$

Chapter 6, Solution 39

$$v = L \frac{di}{dt} \longrightarrow i = \frac{1}{L} \int_0^t i dt + i(0)$$

$$i = \frac{1}{200 \times 10^{-3}} \int_0^t (3t^2 + 2t + 4) dt + 1$$

$$= 5(t^3 + t^2 + 4t) \Big|_0^t + 1$$

$$i(t) = [5t^3 + 5t^2 + 20t + 1] \text{ A}$$

Chapter 6, Solution 40.

$$i = \begin{cases} 5t, & 0 < t < 2\text{ ms} \\ 10, & 2 < t < 4\text{ ms} \\ 30 - 5t, & 4 < t < 6\text{ ms} \end{cases}$$

$$v = L \frac{di}{dt} = \frac{5 \times 10^{-3}}{10^{-3}} \begin{cases} 5, & 0 < t < 2\text{ ms} \\ 0, & 2 < t < 4\text{ ms} \\ -5, & 4 < t < 6\text{ ms} \end{cases} = \begin{cases} 25, & 0 < t < 2\text{ ms} \\ 0, & 2 < t < 4\text{ ms} \\ -25, & 4 < t < 6\text{ ms} \end{cases}$$

At $t = 1\text{ ms}$, $v = \mathbf{25\text{ V}}$

At $t = 3\text{ ms}$, $v = \mathbf{0\text{ V}}$

At $t = 5\text{ ms}$, $v = \mathbf{-25\text{ V}}$

Chapter 6, Solution 41.

$$\begin{aligned} i &= \frac{1}{L} \int_0^t v dt + C = \left(\frac{1}{2} \right) \int_0^t 20(1 - e^{-2t}) dt + C \\ &= 10 \left(t + \frac{1}{2} e^{-2t} \right) \Big|_0^t + C = 10t + 5e^{-2t} - 4.7 \text{ A} \end{aligned}$$

Note, we get $C = -4.7$ from the initial condition for i needing to be 0.3 A.

We can check our results by solving for $v = L di/dt$.

$$v = 2(10 - 10e^{-2t}) \text{ V which is what we started with.}$$

$$\text{At } t = 1 \text{ s, } i = 10 + 5e^{-2} - 4.7 = 10 + 0.6767 - 4.7 = \mathbf{5.977 \text{ A}}$$

$$w = \frac{1}{2} L i^2 = \mathbf{35.72 \text{ J}}$$

Chapter 6, Solution 42.

$$i = \frac{1}{L} \int_0^t v dt + i(0) = \frac{1}{5} \int_0^t v(t) dt - 1$$

$$\text{For } 0 < t < 1, \quad i = \frac{10}{5} \int_0^t dt - 1 = 2t - 1 \text{ A}$$

$$\text{For } 1 < t < 2, \quad i = 0 + i(1) = 1 \text{ A}$$

$$\begin{aligned} \text{For } 2 < t < 3, \quad i &= \frac{1}{5} \int 10 dt + i(2) = 2t \Big|_2^t + 1 \\ &= 2t - 3 \text{ A} \end{aligned}$$

$$\text{For } 3 < t < 4, \quad i = 0 + i(3) = 3 \text{ A}$$

$$\begin{aligned} \text{For } 4 < t < 5, \quad i &= \frac{1}{5} \int_4^t 10 dt + i(4) = 2t \Big|_4^t + 3 \\ &= 2t - 5 \text{ A} \end{aligned}$$

Thus,

$$i(t) = \begin{cases} 2t - 1 \text{ A}, & 0 < t < 1 \\ 1 \text{ A}, & 1 < t < 2 \\ 2t - 3 \text{ A}, & 2 < t < 3 \\ 3 \text{ A}, & 3 < t < 4 \\ 2t - 5, & 4 < t < 5 \end{cases}$$

Chapter 6, Solution 43.

$$\begin{aligned}w &= L \int_{-\infty}^t i \, dt = \frac{1}{2} Li^2(t) - \frac{1}{2} Li^2(-\infty) \\&= \frac{1}{2} \times 80 \times 10^{-3} \times (60 \times 10^{-3})^2 - 0 \\&= \mathbf{144 \, \mu J}.\end{aligned}$$

Chapter 6, Solution 44.

$$(a) \quad v_L = L \frac{di}{dt} = 100 \times 10^{-3} (-400) \times 50 \times 10^{-3} e^{-400t} = \underline{-2e^{-400t} \text{ V}}$$

$$(b) \text{ Since R and L are in parallel, } v_R = v_L = \underline{-2e^{-400t} \text{ V}}$$

(c) **No**

$$(d) \quad w = \frac{1}{2} Li^2 = 0.5 \times 100 \times 10^{-3} (0.05)^2 = \mathbf{125 \text{ }\mu\text{J.}}$$

Chapter 6, Solution 45.

$$i(t) = \frac{1}{L} \int_0^t v(t) dt + i(0)$$

For $0 < t < 1$, $v = 5t$

$$\begin{aligned} i &= \frac{1}{10 \times 10^{-3}} \int_0^t 5t dt + 0 \\ &= 250t^2 \text{ A} \end{aligned}$$

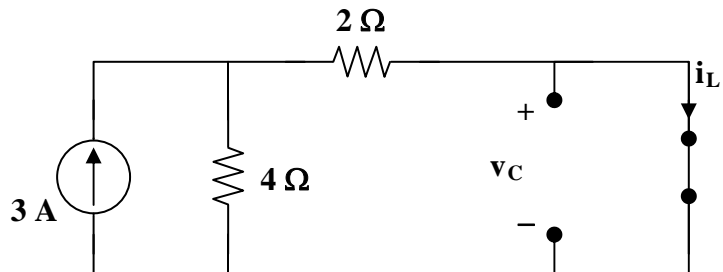
For $1 < t < 2$, $v = -10 + 5t$

$$\begin{aligned} i &= \frac{1}{10 \times 10^{-3}} \int_1^t (-10 + 5t) dt + i(1) \\ &= \int_1^t (0.5t - 1) dt + 0.25 \text{ kA} \\ &= [1 - t + 0.25t^2] \text{ kA} \end{aligned}$$

$$i(t) = \begin{cases} 250t^2 \text{ A}, & 0 < t < 1 \text{ s} \\ [1 - t + 0.25t^2] \text{ kA}, & 1 < t < 2 \text{ s} \end{cases}$$

Chapter 6, Solution 46.

Under dc conditions, the circuit is as shown below:



By current division,

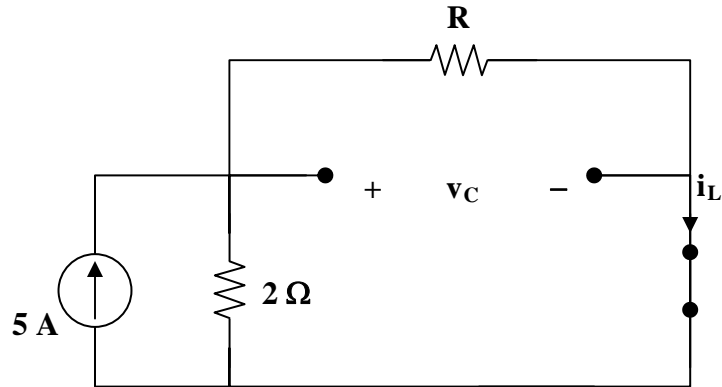
$$i_L = \frac{4}{4+2}(3) = \mathbf{2A}, \quad v_C = \mathbf{0V}$$

$$w_L = \frac{1}{2}L i_L^2 = \frac{1}{2}\left(\frac{1}{2}\right)(2)^2 = \mathbf{1J}$$

$$w_C = \frac{1}{2}C v_C^2 = \frac{1}{2}(2)(0) = \mathbf{0J}$$

Chapter 6, Solution 47.

Under dc conditions, the circuit is equivalent to that shown below:



$$i_L = \frac{2}{R+2}(5) = \frac{10}{R+2}, \quad v_C = Ri_L = \frac{10R}{R+2}$$

$$w_C = \frac{1}{2}Cv_C^2 = 80 \times 10^{-6} \times \frac{100R^2}{(R+2)^2}$$

$$w_L = \frac{1}{2}Li_L^2 = 2 \times 10^{-3} \times \frac{100}{(R+2)^2}$$

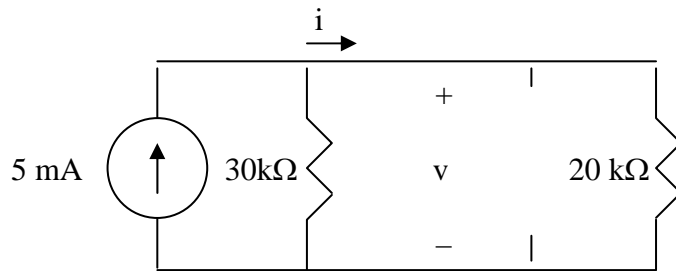
If $w_C = w_L$,

$$80 \times 10^{-6} \times \frac{100R^2}{(R+2)^2} = \frac{2 \times 10^{-3} \times 100}{(R+2)^2} \rightarrow 80 \times 10^{-3} R^2 = 2$$

$$R = 5\Omega$$

Chapter 6, Solution 48.

Under steady-state, the inductor acts like a short-circuit, while the capacitor acts like an open circuit as shown below.



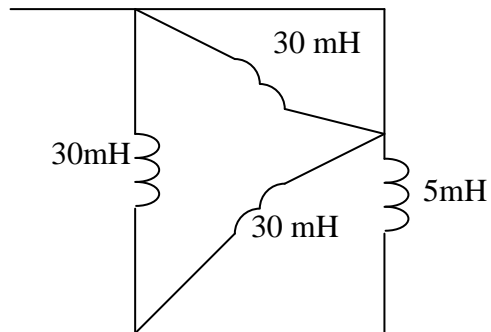
Using current division,

$$i = (30\text{k}/(30\text{k}+20\text{k}))(5\text{mA}) = \mathbf{3\text{ mA}}$$

$$v = 20\text{k}i = \mathbf{60\text{ V}}$$

Chapter 6, Solution 49.

Converting the wye-subnetwork to its equivalent delta gives the circuit below.



$$30//0 = 0, \quad 30//5 = 30 \times 5 / 35 = 4.286$$

$$L_{eq} = 30 // 4.286 = \frac{30 \times 4.286}{34.286} = \underline{3.75 \text{ mH}}$$

Chapter 6, Solution 50.

16mH in series with 14 mH = $16+14=30$ mH

24 mH in series with 36 mH = $24+36=60$ mH

30mH in parallel with 60 mH = $30 \times 60 / 90 = \mathbf{20\ mH}$

Chapter 6, Solution 51.

$$\frac{1}{L} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \quad L = 10 \text{ mH}$$

$$L_{\text{eq}} = 10 \parallel (25 + 10) = \frac{10 \times 35}{45} \\ = \mathbf{7.778 \text{ mH}}$$

Chapter 6, Solution 52.

Using Fig. 6.74, design a problem to help other students better understand how inductors behave when connected in series and when connected in parallel.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find L_{eq} in the circuit of Fig. 6.74.

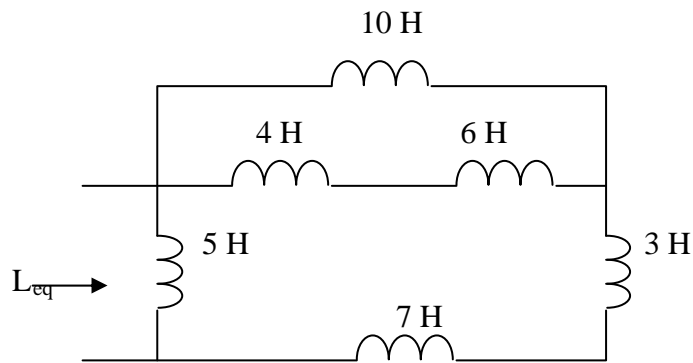


Figure 6.74 For Prob. 6.52.

Solution

$$L_{eq} = 5 // (7 + 3 + 10 // (4 + 6)) = 5 // (7 + 3 + 5) = \frac{5 \times 15}{20} = \underline{3.75 \text{ H}}$$

Chapter 6, Solution 53.

$$L_{\text{eq}} = 6 + 10 + 8 \parallel [5 \parallel (8 + 12) + 6 \parallel (8 + 4)]$$

$$= 16 + 8 \parallel (4 + 4) = 16 + 4$$

$$L_{\text{eq}} = \mathbf{20 \text{ mH}}$$

Chapter 6, Solution 54.

$$L_{\text{eq}} = 4 + (9 + 3) \parallel (10 \parallel 0 + 6 \parallel 12)$$

$$= 4 + 12 \parallel (0 + 4) = 4 + 3$$

$$L_{\text{eq}} = \mathbf{7H}$$

Chapter 6, Solution 55.

(a) $L//L = 0.5L$, $L + L = 2L$

$$L_{eq} = L + 2L // 0.5L = L + \frac{2L \times 0.5L}{2L + 0.5L} = \underline{1.4L} = \mathbf{1.4\ L}.$$

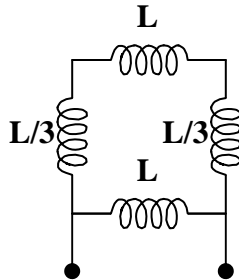
(b) $L//L = 0.5L$, $L//L + L//L = L$

$$L_{eq} = L//L = \mathbf{500\ mL}$$

Chapter 6, Solution 56.

$$L \parallel L \parallel L = \frac{1}{\frac{1}{L} + \frac{1}{L} + \frac{1}{L}} = \frac{L}{3}$$

Hence the given circuit is equivalent to that shown below:



$$L_{eq} = L \parallel \left(L + \frac{2}{3}L \right) = \frac{L \times \frac{5}{3}L}{L + \frac{5}{3}L} = \frac{5}{8}L$$

Chapter 6, Solution 57.

$$\text{Let } v = L_{\text{eq}} \frac{di}{dt} \quad (1)$$

$$v = v_1 + v_2 = 4 \frac{di}{dt} + v_2 \quad (2)$$

$$i = i_1 + i_2 \longrightarrow i_2 = i - i_1 \quad (3)$$

$$v_2 = 3 \frac{di_1}{dt} \text{ or } \frac{di_1}{dt} = \frac{v_2}{3} \quad (4)$$

and

$$\begin{aligned} -v_2 + 2 \frac{di}{dt} + 5 \frac{di_2}{dt} &= 0 \\ v_2 &= 2 \frac{di}{dt} + 5 \frac{di_2}{dt} \end{aligned} \quad (5)$$

Incorporating (3) and (4) into (5),

$$v_2 = 2 \frac{di}{dt} + 5 \frac{di}{dt} - 5 \frac{di_1}{dt} = 7 \frac{di}{dt} - 5 \frac{v_2}{3}$$

$$v_2 \left(1 + \frac{5}{3} \right) = 7 \frac{di}{dt}$$

$$v_2 = \frac{21}{8} \frac{di}{dt}$$

Substituting this into (2) gives

$$v = 4 \frac{di}{dt} + \frac{21}{8} \frac{di}{dt}$$

$$= \frac{53}{8} \frac{di}{dt}$$

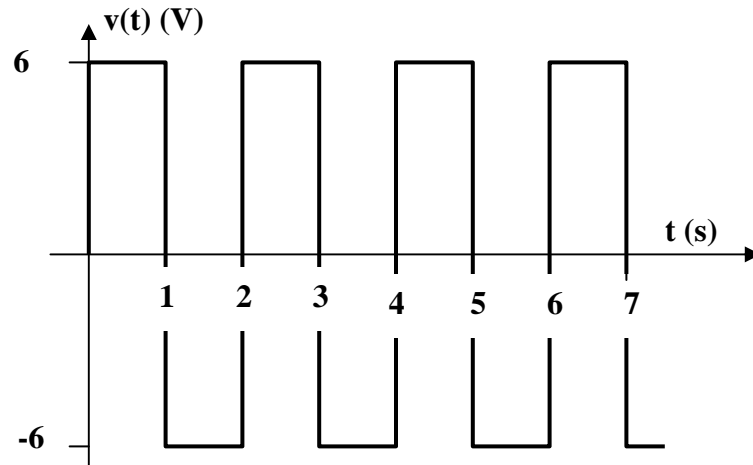
Comparing this with (1),

$$L_{\text{eq}} = \frac{53}{8} = \mathbf{6.625 \text{ H}}$$

Chapter 6, Solution 58.

$$v = L \frac{di}{dt} = 3 \frac{di}{dt} = 3 \times \text{slope of } i(t).$$

Thus v is sketched below:



Chapter 6, Solution 59.

$$(a) \quad v_s = (L_1 + L_2) \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{v_s}{L_1 + L_2}$$

$$v_1 = L_1 \frac{di}{dt}, \quad v_2 = L_2 \frac{di}{dt}$$

$$v_1 = \frac{L_1}{L_1 + L_2} v_s, \quad v_2 = \frac{L_2}{L_1 + L_2} v_s$$

$$(b) \quad v_1 = v_2 = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$$

$$i_s = i_1 + i_2$$

$$\frac{di_s}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{v}{L_1} + \frac{v}{L_2} = v \frac{(L_1 + L_2)}{L_1 L_2}$$

$$i_1 = \frac{1}{L_1} \int v dt = \frac{1}{L_1} \int \frac{L_1 L_2}{L_1 + L_2} \frac{di_s}{dt} dt = \frac{L_2}{L_1 + L_2} i_s$$

$$i_2 = \frac{1}{L_2} \int v dt = \frac{1}{L_2} \int \frac{L_1 L_2}{L_1 + L_2} \frac{di_s}{dt} dt = \frac{L_1}{L_1 + L_2} i_s$$

Chapter 6, Solution 60

$$L_{eq} = 3 // 5 = \frac{15}{8}$$

$$v_o = L_{eq} \frac{di}{dt} = \frac{15}{8} \frac{d}{dt} (4e^{-2t}) = \underline{-15e^{-2t}}$$

$$i_o = \frac{1}{L} \int_0^t v_o(t) dt + i_o(0) = 2 + \frac{1}{5} \int_0^t (-15)e^{-2t} dt = 2 + 1.5e^{-2t} \Big|_0^t$$

$$i_o = \mathbf{(0.5 + 1.5e^{-2t}) \text{ A}}$$

Chapter 6, Solution 61.

$$(a) \quad L_{eq} = 20 // (4 + 6) = 20 \times 10 / 30 = \underline{6.667 \text{ mH}}$$

Using current division,

$$i_1(t) = \frac{10}{10 + 20} i_s = \underline{e^{-t} \text{ mA}}$$

$$i_2(t) = \underline{2e^{-t} \text{ mA}}$$

$$(b) \quad v_o = L_{eq} \frac{di_s}{dt} = \frac{20}{3} \times 10^{-3} (-3e^{-t} \times 10^{-3}) = \underline{-20e^{-t} \mu\text{V}}$$

$$(c) \quad w = \frac{1}{2} L i_1^2 = \frac{1}{2} \times 20 \times 10^{-3} \times e^{-2} \times 10^{-6} = \underline{1.3534 \text{ nJ}}$$

Chapter 6, Solution 62.

$$(a) \quad L_{eq} = 25 + 20 // 60 = 25 + \frac{20 \times 60}{80} = 40 \text{ mH}$$

$$v = L_{eq} \frac{di}{dt} \quad \longrightarrow \quad i = \frac{1}{L_{eq}} \int v(t) dt + i(0) = \frac{10^{-3}}{40 \times 10^{-3}} \int_0^t 12e^{-3t} dt + i(0) = -0.1(e^{-3t} - 1) + i(0)$$

Using current division and the fact that all the currents were zero when the circuit was put together, we get,

$$i_1 = \frac{60}{80} i = \frac{3}{4} i, \quad i_2 = \frac{1}{4} i$$

$$i_1(0) = \frac{3}{4} i(0) \quad \longrightarrow \quad 0.75i(0) = -0.01 \quad \longrightarrow \quad i(0) = -0.01333$$

$$i_2 = \frac{1}{4} (-0.1e^{-3t} + 0.08667) \text{ A} = -25e^{-3t} + 21.67 \text{ mA}$$

$$i_2(0) = -25 + 21.67 = \underline{\underline{-3.33 \text{ mA}}}$$

$$(b) \quad i_1 = \frac{3}{4} (-0.1e^{-3t} + 0.08667) \text{ A} = \underline{\underline{-75e^{-3t} + 65 \text{ mA}}}$$

$$i_2 = \underline{\underline{-25e^{-3t} + 21.67 \text{ mA}}}$$

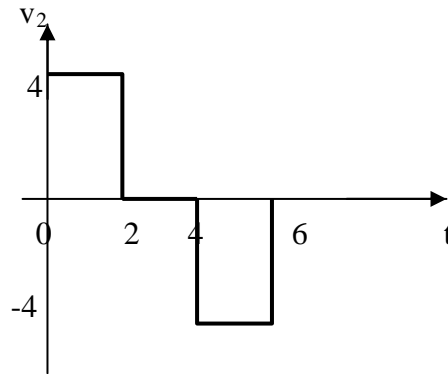
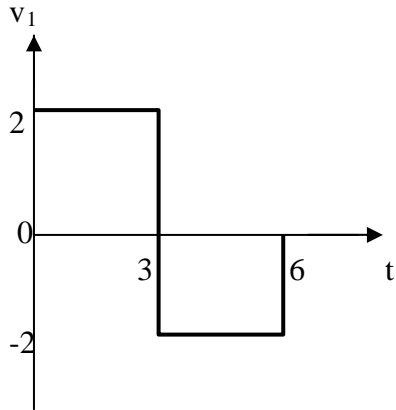
Chapter 6, Solution 63.

We apply superposition principle and let

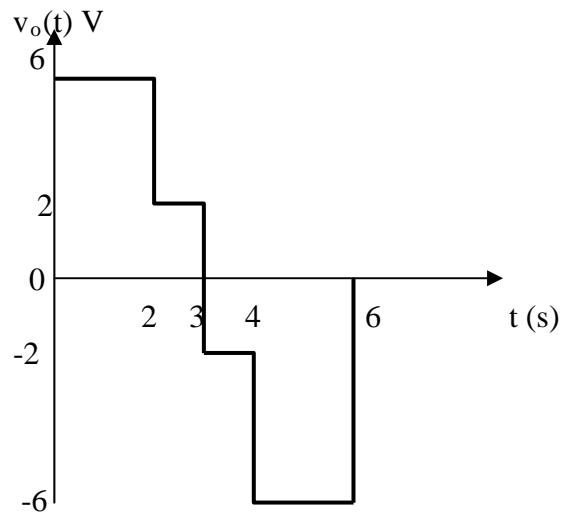
$$v_o = v_1 + v_2$$

where v_1 and v_2 are due to i_1 and i_2 respectively.

$$v_1 = L \frac{di_1}{dt} = 2 \frac{di_1}{dt} = \begin{cases} 2, & 0 < t < 3 \\ -2, & 3 < t < 6 \end{cases}$$
$$v_2 = L \frac{di_2}{dt} = 2 \frac{di_2}{dt} = \begin{cases} 4, & 0 < t < 2 \\ 0, & 2 < t < 4 \\ -4, & 4 < t < 6 \end{cases}$$



Adding v_1 and v_2 gives v_o , which is shown below.



Chapter 6, Solution 64.

(a) When the switch is in position A,

$$i = -6 = i(0)$$

When the switch is in position B,

$$i(\infty) = 12/4 = 3, \quad \tau = L/R = 1/8$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$i(t) = (3 - 9e^{-8t}) \text{ A}$$

(b) $-12 + 4i(0) + v = 0$, i.e. $v = 12 - 4i(0) = \mathbf{36 \text{ V}}$

(c) At steady state, the inductor becomes a short circuit so that

$$v = \mathbf{0 \text{ V}}$$

Chapter 6, Solution 65.

$$(a) \quad w_5 = \frac{1}{2} L_1 i_1^2 = \frac{1}{2} \times 5 \times (4)^2 = \mathbf{40 \text{ J}}$$

$$w_{20} = \frac{1}{2} (20)(-2)^2 = \mathbf{40 \text{ J}}$$

$$(b) \quad w = w_5 + w_{20} = \mathbf{80 \text{ J}}$$

$$(c) \quad i_1 = \frac{1}{L_1} \int_0^t -50e^{-200t} dt + i_1(0) = \frac{1}{5} \left(\frac{1}{200} \right) \left(50e^{-200t} \times 10^{-3} \right) \Big|_0^t + 4 \\ = \mathbf{[5 \times 10^{-5} (e^{-200t} - 1) + 4] \text{ A}}$$

$$i_2 = \frac{1}{L_2} \int_0^t -50e^{-200t} dt + i_2(0) = \frac{1}{20} \left(\frac{1}{200} \right) \left(50e^{-200t} \times 10^{-3} \right) \Big|_0^t - 2 \\ = \mathbf{[1.25 \times 10^{-5} (e^{-200t} - 1) - 2] \text{ A}}$$

$$(d) \quad i = i_1 + i_2 = \mathbf{[6.25 \times 10^{-5} (e^{-200t} - 1) + 2] \text{ A}}$$

Chapter 6, Solution 66.

If $v=i$, then

$$i = L \frac{di}{dt} \longrightarrow \frac{dt}{L} = \frac{di}{i}$$

Integrating this gives

$$\frac{t}{L} = \ln(i) - \ln(C_o) = \ln\left(\frac{i}{C_o}\right) \rightarrow i = C_o e^{t/L}$$

$$i(0) = 2 = C_o$$

$$i(t) = 2e^{t/0.02} = \mathbf{2e^{50t} \text{ A.}}$$

Chapter 6, Solution 67.

$$v_o = -\frac{1}{RC} \int v_i dt, RC = 50 \times 10^3 \times 0.04 \times 10^{-6} = 2 \times 10^{-3}$$

$$v_o = \frac{-10^3}{2} \int 10 \sin 50t dt$$

$$v_o = \mathbf{100\cos(50t) \text{ mV}}$$

Chapter 6, Solution 68.

$$v_o = -\frac{1}{RC} \int v_i dt + v(0), RC = 50 \times 10^3 \times 100 \times 10^{-6} = 5$$

$$v_o = -\frac{1}{5} \int_0^t 10 dt + 0 = -2t$$

The op amp will saturate at $v_o = \pm 12$

$$-12 = -2t \longrightarrow \mathbf{t = 6s}$$

Chapter 6, Solution 69.

$$RC = 4 \times 10^6 \times 1 \times 10^{-6} = 4$$

$$v_o = -\frac{1}{RC} \int v_i dt = -\frac{1}{4} \int v_i dt$$

$$\text{For } 0 < t < 1, v_i = 20, v_o = -\frac{1}{4} \int_0^t 20 dt = -5t \text{ mV}$$

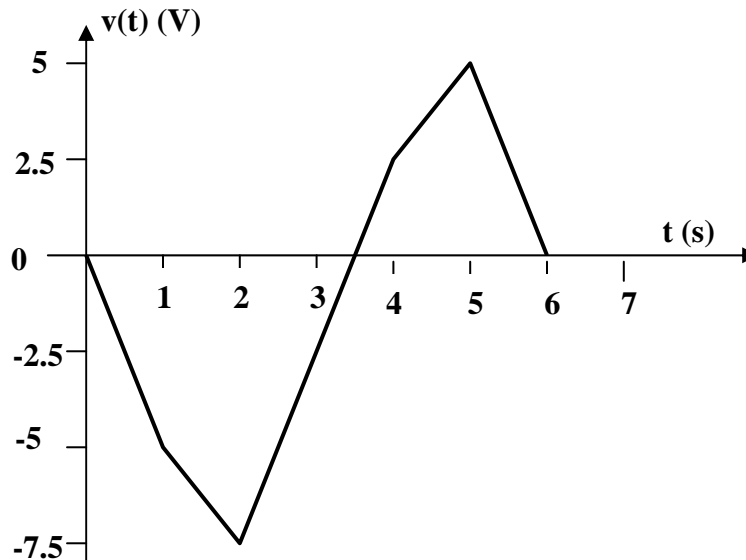
$$\begin{aligned} \text{For } 1 < t < 2, v_i = 10, v_o &= -\frac{1}{4} \int_1^t 10 dt + v(1) = -2.5(t-1) - 5 \\ &= -2.5t - 2.5 \text{ mV} \end{aligned}$$

$$\begin{aligned} \text{For } 2 < t < 4, v_i = -20, v_o &= +\frac{1}{4} \int_2^t 20 dt + v(2) = 5(t-2) - 7.5 \\ &= 5t - 17.5 \text{ mV} \end{aligned}$$

$$\begin{aligned} \text{For } 4 < t < 5, v_i = -10, v_o &= \frac{1}{4} \int_4^t 10 dt + v(4) = 2.5(t-4) + 2.5 \\ &= 2.5t - 7.5 \text{ mV} \end{aligned}$$

$$\begin{aligned} \text{For } 5 < t < 6, v_i = 20, v_o &= -\frac{1}{4} \int_5^t 20 dt + v(5) = -5(t-5) + 5 \\ &= -5t + 30 \text{ mV} \end{aligned}$$

Thus $v_o(t)$ is as shown below:



Chapter 6, Solution 70.

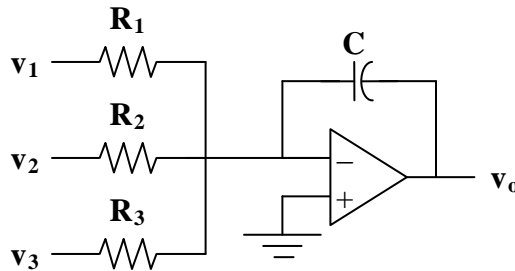
One possibility is as follows:

$$\frac{1}{RC} = 50$$

$$\text{Let } R = 100 \text{ k}\Omega, C = \frac{1}{50 \times 100 \times 10^3} = 0.2 \mu F$$

Chapter 6, Solution 71.

By combining a summer with an integrator, we have the circuit below:



$$v_o = -\frac{1}{R_1 C} \int v_1 dt - \frac{1}{R_2 C} \int v_2 dt - \frac{1}{R_3 C} \int v_3 dt$$

For the given problem, $C = 2\mu\text{F}$,

$$R_1 C = 1 \longrightarrow R_1 = 1/(C) = 10^6/(2) = \mathbf{500\text{ k}\Omega}$$

$$R_2 C = 1/(4) \longrightarrow R_2 = 1/(4C) = 500\text{ k}\Omega/(4) = \mathbf{125\text{ k}\Omega}$$

$$R_3 C = 1/(10) \longrightarrow R_3 = 1/(10C) = \mathbf{50\text{ k}\Omega}$$

Chapter 6, Solution 72.

The output of the first op amp is

$$v_i = -\frac{1}{RC} \int v_i \, dt = -\frac{1}{10 \times 10^3 \times 2 \times 10^{-6}} \int_0^t v_i \, dt = -\frac{100t}{2}$$

$$= -50t$$

$$v_o = -\frac{1}{RC} \int v_i \, dt = -\frac{1}{20 \times 10^3 \times 0.5 \times 10^{-6}} \int_0^t (-50t) \, dt$$

$$= 2500t^2$$

At $t = 1.5 \text{ ms}$,

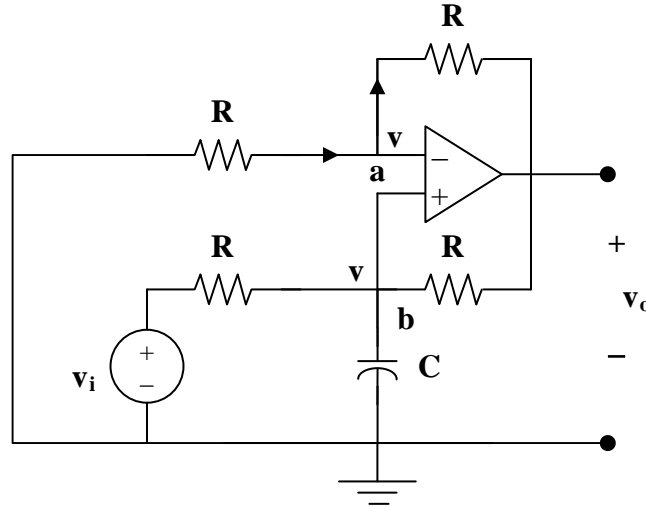
$$v_o = 2500(1.5)^2 \times 10^{-6} = \mathbf{5.625 \text{ mV}}$$

Chapter 6, Solution 73.

Consider the op amp as shown below:

Let $v_a = v_b = v$

At node a, $\frac{0 - v}{R} = \frac{v - v_o}{R} \longrightarrow 2v - v_o = 0$ (1)



At node b, $\frac{v_i - v}{R} = \frac{v - v_o}{R} + C \frac{dv}{dt}$
 $v_i = 2v - v_o + RC \frac{dv}{dt}$ (2)

Combining (1) and (2),

$$v_i = v_o - v_o + \frac{RC}{2} \frac{dv_o}{dt}$$

or

$$v_o = \frac{2}{RC} \int v_i dt$$

showing that the circuit is a noninverting integrator.

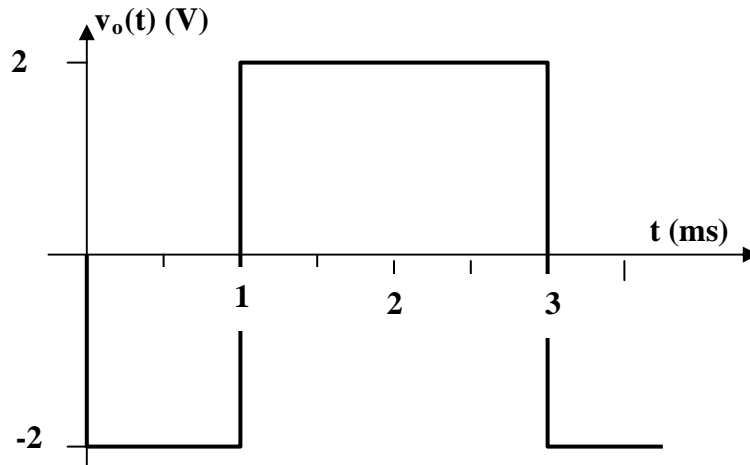
Chapter 6, Solution 74.

$$RC = 0.01 \times 20 \times 10^{-3} \text{ sec}$$

$$v_o = -RC \frac{dv_i}{dt} = -0.2 \frac{dv}{dt} \text{ m sec}$$

$$v_o = \begin{cases} -2\text{V}, & 0 < t < 1 \\ 2\text{V}, & 1 < t < 3 \\ -2\text{V}, & 3 < t < 4 \end{cases}$$

Thus $v_o(t)$ is as sketched below:



Chapter 6, Solution 75.

$$v_o = -RC \frac{dv_i}{dt}, \quad RC = 250 \times 10^3 \times 10 \times 10^{-6} = 2.5$$

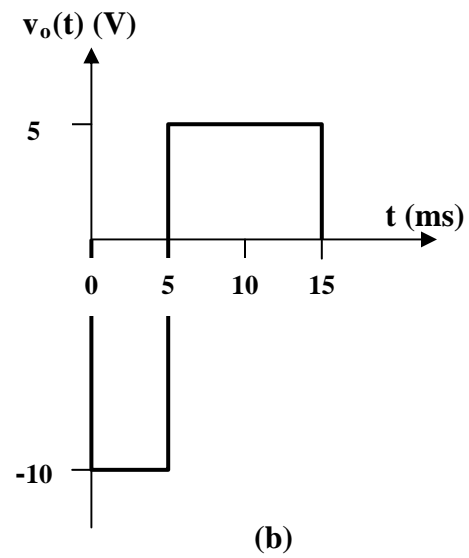
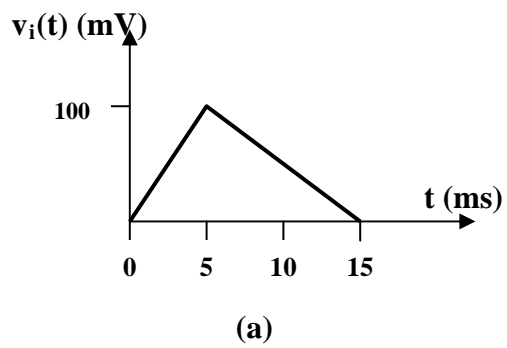
$$v_o = -2.5 \frac{d}{dt}(12t) = \mathbf{-30 \text{ mV}}$$

Chapter 6, Solution 76.

$$v_o = -RC \frac{dv_i}{dt}, \quad RC = 50 \times 10^3 \times 10 \times 10^{-6} = 0.5$$

$$v_o = -0.5 \frac{dv_i}{dt} = \begin{cases} -10, & 0 < t < 5 \\ 5, & 5 < t < 15 \end{cases}$$

The input is sketched in Fig. (a), while the output is sketched in Fig. (b).



Chapter 6, Solution 77.

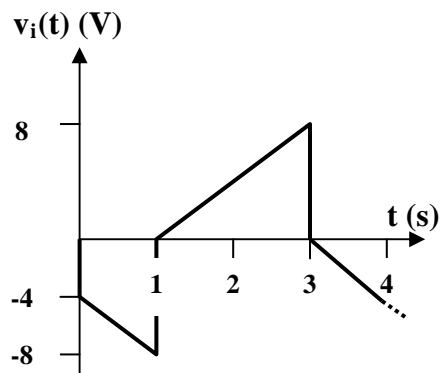
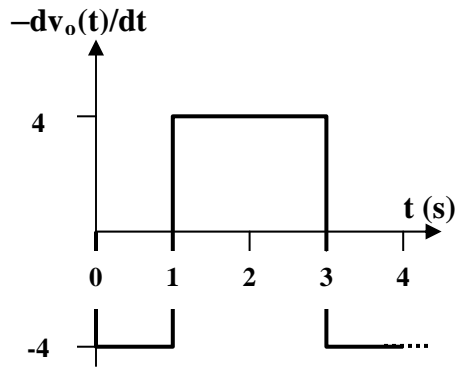
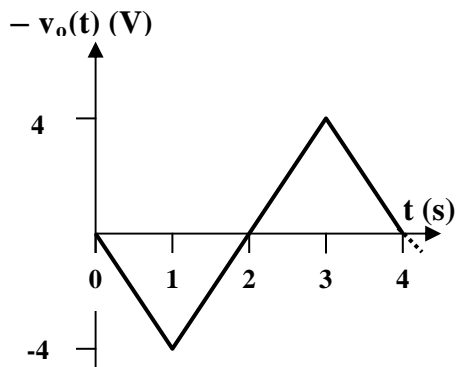
$$i = i_R + i_C$$

$$\frac{v_i - 0}{R} = \frac{0 - v_o}{R_F} + C \frac{d}{dt}(0 - v_o)$$

$$R_F C = 10^6 \times 10^{-6} = 1$$

$$\text{Hence } v_i = -\left(v_o + \frac{dv_o}{dt}\right)$$

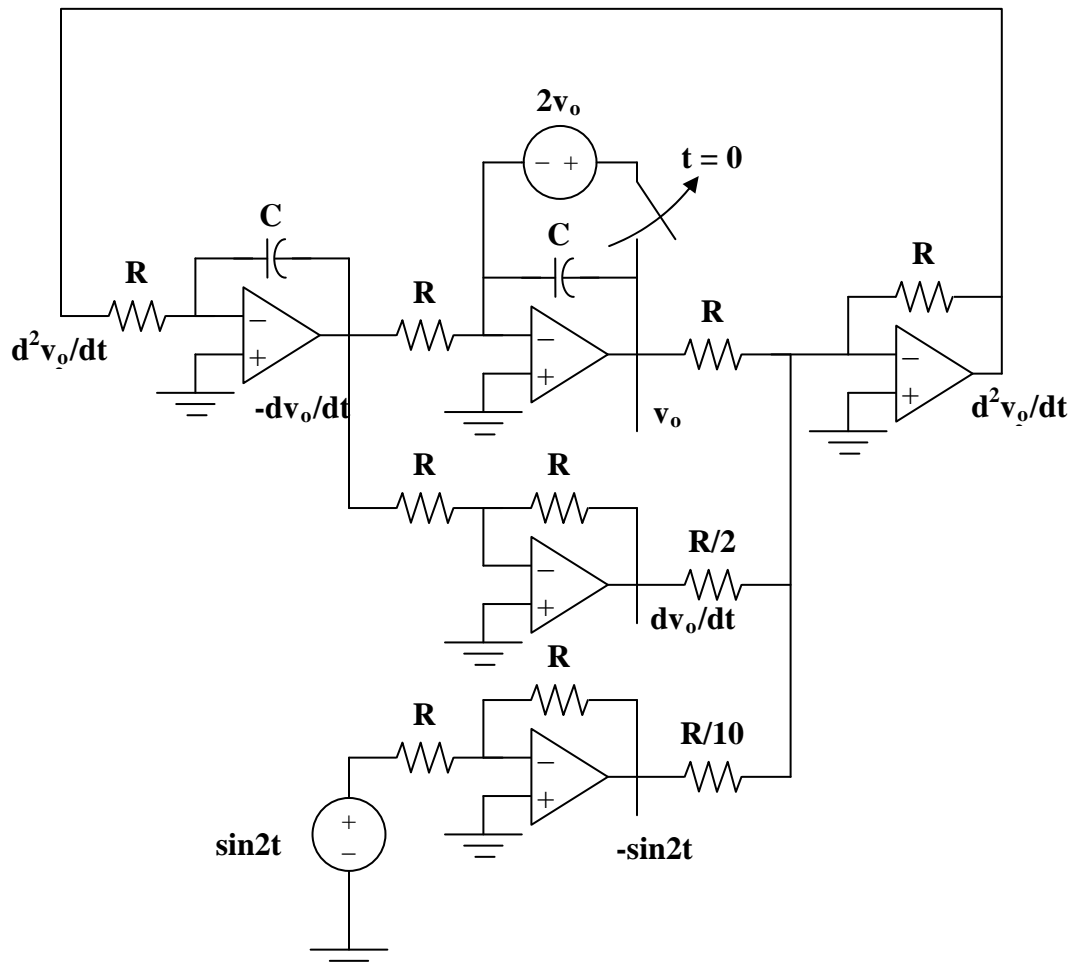
Thus v_i is obtained from v_o as shown below:



Chapter 6, Solution 78.

$$\frac{d^2 v_o}{dt} = 10 \sin 2t - \frac{2dv_o}{dt} - v_o$$

Thus, by combining integrators with a summer, we obtain the appropriate analog computer as shown below:

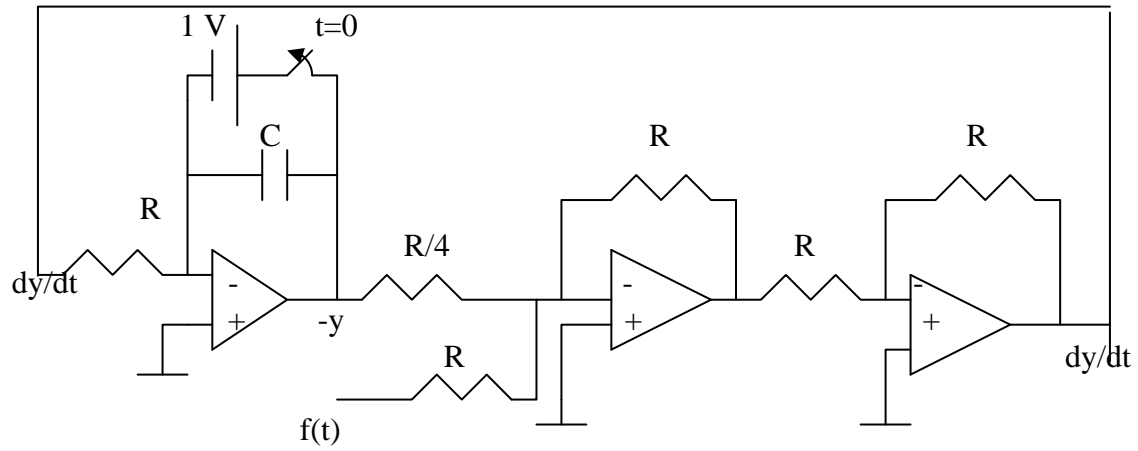


Chapter 6, Solution 79.

We can write the equation as

$$\frac{dy}{dt} = f(t) - 4y(t)$$

which is implemented by the circuit below.



Chapter 6, Solution 80.

From the given circuit,

$$\frac{d^2 v_o}{dt^2} = f(t) - \frac{1000k\Omega}{5000k\Omega} v_o - \frac{1000k\Omega}{200k\Omega} \frac{dv_o}{dt}$$

or

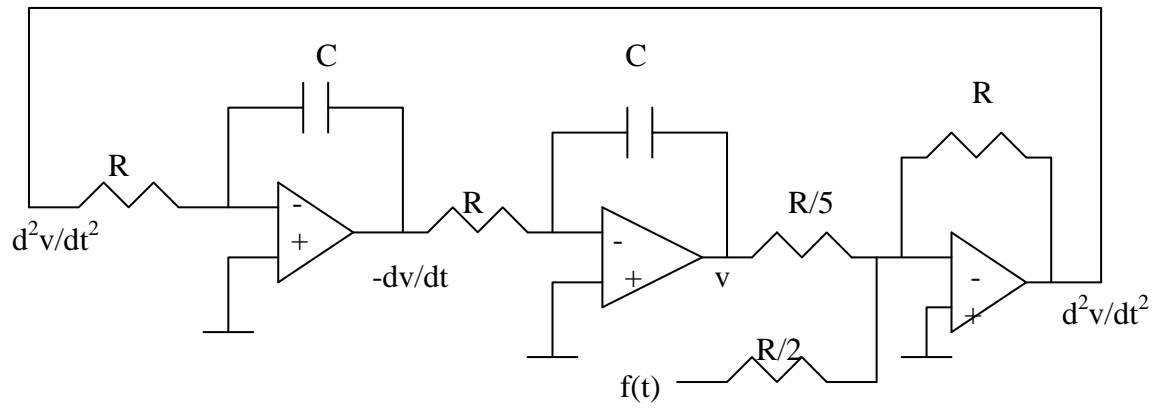
$$\frac{d^2 v_o}{dt^2} + 5 \frac{dv_o}{dt} + 2v_o = f(t)$$

Chapter 6, Solution 81

We can write the equation as

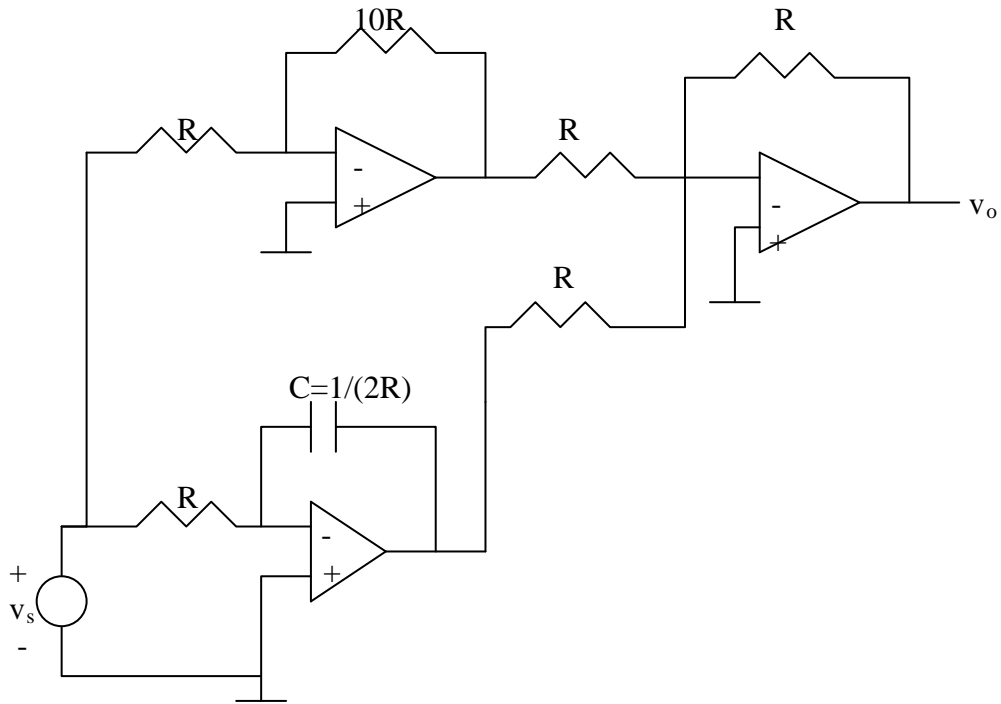
$$\frac{d^2v}{dt^2} = -5v - 2f(t)$$

which is implemented by the circuit below.



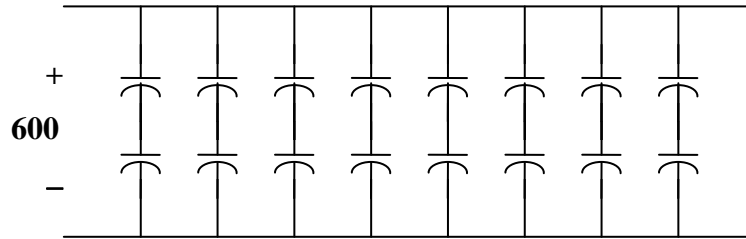
Chapter 6, Solution 82

The circuit consists of a summer, an inverter, and an integrator. Such circuit is shown below.



Chapter 6, Solution 83.

Since two $10\mu\text{F}$ capacitors in series gives $5\mu\text{F}$, rated at 600V , it requires 8 groups in parallel with each group consisting of two capacitors in series, as shown below:



Answer: 8 groups in parallel with each group made up of 2 capacitors in series.

Chapter 6, Solution 84.

$$v = L(di/dt) = 8 \times 10^{-3} \times 5 \times 2\pi \sin(\pi t) \cos(\pi t) 10^{-3} = 40\pi \sin(2\pi t) \mu\text{V}$$

$$p = vi = 40\pi \sin(2\pi t) 5 \sin^2(\pi t) 10^{-9} \text{ W, at } t=0 \text{ } p = \mathbf{0\text{W}}$$

$$w = \frac{1}{2} L i^2 = \frac{1}{2} \times 8 \times 10^{-3} \times [5 \sin^2(\pi / 2) \times 10^{-3}]^2 = 4 \times 25 \times 10^{-9} = \underline{100 \text{ nJ}}$$

$$= \mathbf{100 \text{ }\mu\text{J}}$$

Chapter 6, Solution 85.

It is evident that differentiating i will give a waveform similar to v . Hence,

$$v = L \frac{di}{dt}$$

$$i = \begin{cases} 4t, 0 < t < 1\text{ms} \\ 8 - 4t, 1 < t < 2\text{ms} \end{cases}$$

$$v = L \left[\frac{di}{dt} = \begin{cases} 4000L, 0 < t < 1\text{ms} \\ -4000L, 1 < t < 2\text{ms} \end{cases} \right]$$

But,
$$v = \begin{cases} 5\text{V}, 0 < t < 1\text{ms} \\ -5\text{V}, 1 < t < 2\text{ms} \end{cases}$$

Thus, $4000L = 5 \longrightarrow L = 1.25 \text{ mH}$ in a **1.25 mH inductor**

Chapter 6, Solution 86.

$$v = v_R + v_L = Ri + L \frac{di}{dt} = 12 \times 2te^{-10t} + 200 \times 10^{-3} \times (-20te^{-10t} + 2e^{-10t}) = \underline{(0.4 - 20t)e^{-10t} \text{ V}}$$