Chapter 10, Solution 1.

We first determine the input impedance.

$$1H \longrightarrow j\omega L = j1x10 = j10$$

$$1F \longrightarrow 1\!\!\diagup\!\!j\omega C = \frac{1}{j10x1} = -j0.1$$

$$Z_{in} = 1 + \left(\frac{1}{j10} + \frac{1}{-j0.1} + \frac{1}{1}\right)^{-1} = 1.0101 - j0.1 = 1.015 < -5.653^o$$

$$I = \frac{2 < 0^o}{1.015 < -5.653^o} = 1.9704 < 5.653^o$$

i(t) = **1.9704cos(10t+5.65˚) A**

Chapter 10, Solution 2.

Using Fig. 10.51, design a problem to help other students better understand nodal analysis.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve for V_o in Fig. 10.51, using nodal analysis.

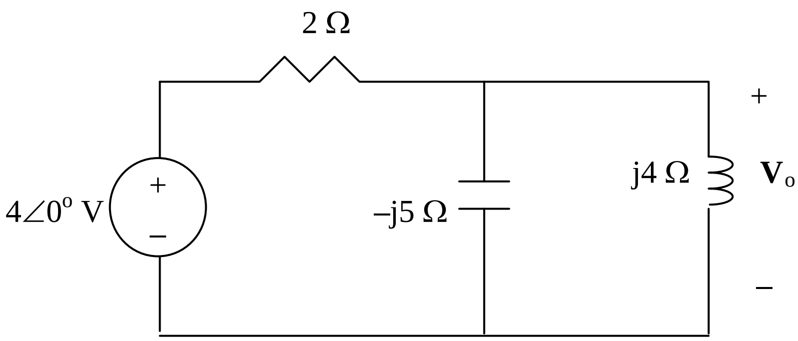
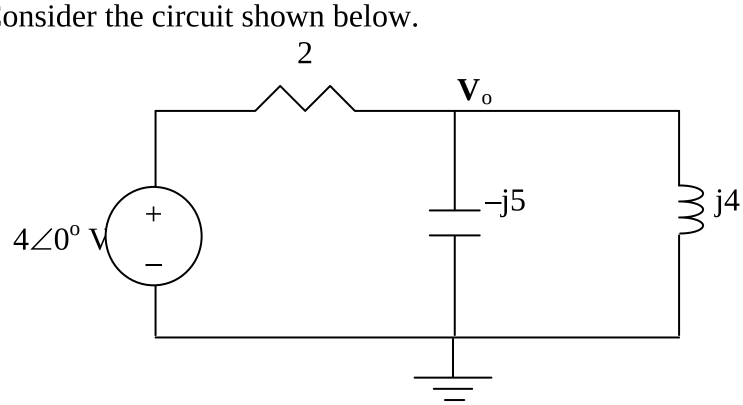


Figure 10.51 For Prob. 10.2.

Solution

Consider the circuit shown below.



At the main node,

$$\frac{4-V_o}{2}=\frac{V_o}{-j5}+\frac{V_o}{j4} \longrightarrow 40=V_o(10+j)$$

$\mathbf{V_o}$ = 40/(10–j) = (40/10.05)∠5.71˚ = **3.98∠5.71˚ V**

Chapter 10, Solution 3.

$\omega = 4$

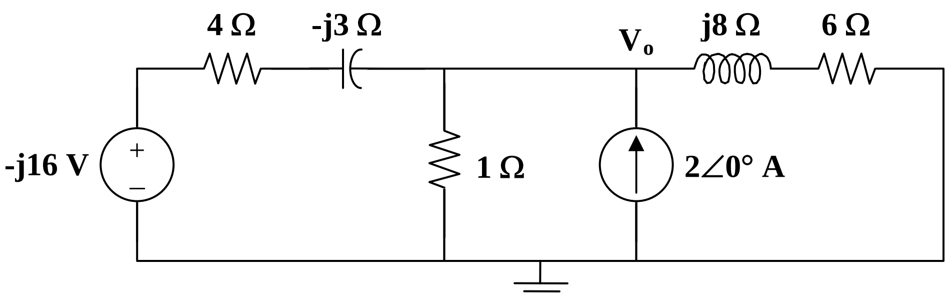
$$2\cos(4t) \longrightarrow 2\angle 0°$$

$$16\sin(4t) \longrightarrow 16\angle -90° = -j16$$

$$2 \text{ H} \longrightarrow j\omega L = j8$$

$$1/12 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = -j3$$

The circuit is shown below.



Applying nodal analysis,

$$\frac{-j16 - \mathbf{V}_o}{4 - j3} + 2 = \frac{\mathbf{V}_o}{1} + \frac{\mathbf{V}_o}{6 + j8}$$

$$\frac{-j16}{4 - j3} + 2 = \left(1 + \frac{1}{4 - j3} + \frac{1}{6 + j8}\right)\mathbf{V}_o$$

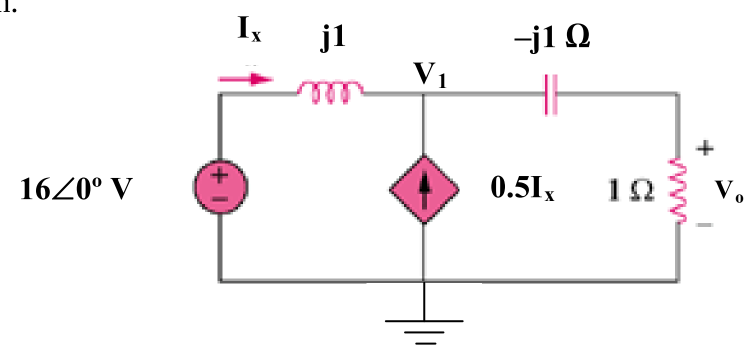
$$\mathbf{V}_o = \frac{3.92 - j2.56}{1.22 + j0.04} = \frac{4.682\angle -33.15°}{1.2207\angle 1.88°} = 3.835\angle -35.02°$$

Therefore,

$$v_o(t) = \mathbf{3.835cos(4t – 35.02°) V}$$

Chapter 10, Solution 4.

Step 1. Convert the circuit into the frequency domain and solve for the node voltage, V_1, using analysis. The find the current $I_C = V_1/[1+(1/(j4x0.25)]$ which then produces $V_o = 1xI_C$. Finally, convert the capacitor voltage back into the time domain.



Note that we represented 16sin(4t–10°) volts by 16∠0° V. That will make our calculations easier and all we have to do is to offset our answer by a –10°.

Our node equation is $[(V_1–16)/j] – (0.5I_x) + [(V_1–0)/(1–j)] = 0$. We have two unknowns, therefore we need a constraint equation. $I_x = [(16–V_1)/j] = j(V_1–16)$. Once we have V_1, we can find $I_o = V_1 /(1–j)$ and $V_o = 1xI_o$.

Step 2. Now all we need to do is to solve our equations.

$[(V_1–16)/j] – [0.5j(V_1–16] + [(V_1–0)/(1–j)] = [–j–j0.5+0.5+j0.5]V_1 +j16+j8 = 0$
or

$[0.5–j]V_1 = –j24$ or $V_1 = j24/(–0.5+j) = (24\angle 90°)/(1.118\angle 116.57°)$
$= 21.47\angle –26.57°$ V.

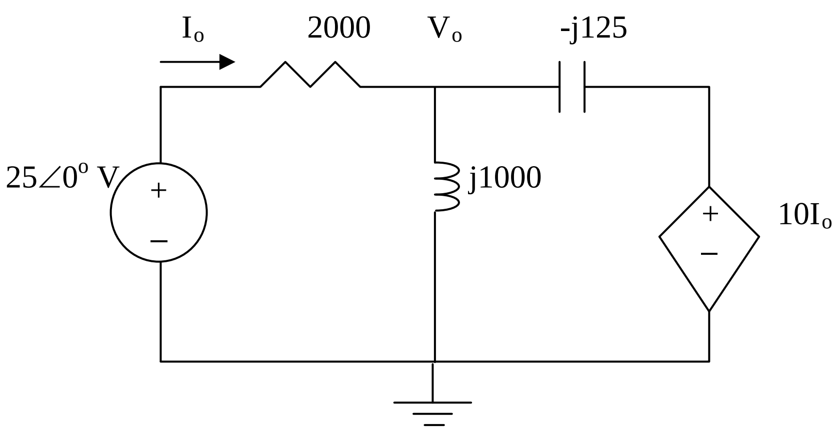
Finally, $I_x = V_1/(1–j) = (21.47\angle –26.57°) (0.7071\angle 45°) = 15.181\angle 18.43°$ A and $V_o = 1xI_o = 15.181\angle 18.43°$ V or

$v_o(t) =$ 15.181sin(4t–10°+18.43°) = **15.181sin(4t–8.43°) volts**.

Chapter 10, Solution 5.

$$0.25H \longrightarrow j\omega L = j0.25x4x10^3 = j1000$$
$$2\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x10^3 x2x10^{-6}} = -j125$$

Consider the circuit as shown below.



At node V_o,

$$\frac{V_o - 25}{2000} + \frac{V_o - 0}{j1000} + \frac{V_o - 10I_o}{-j125} = 0$$
$$V_o - 25 - j2V_o + j16V_o - j160I_o = 0$$
$$(1 + j14)V_o - j160I_o = 25$$

But $I_o = (25 – V_o)/2000$

$$(1 + j14)V_o - j2 + j0.08V_o = 25$$
$$V_o = \frac{25 + j2}{1 + j14.08} = \frac{25.08\angle 4.57^\circ}{14.115\angle 58.94^\circ} 1.7768\angle -81.37^\circ$$

Now to solve for i_o,

$$I_o = \frac{25 - V_o}{2000} = \frac{25 - 0.2666 + j1.7567}{2000} = 12.367 + j0.8784\,\text{mA}$$
$$= 12.398\angle 4.06^\circ$$

i_o = **12.398cos(4x10^{3}t + 4.06˚) mA**.

Chapter 10, Solution 6.

Let V_o be the voltage across the current source. Using nodal analysis we get:

$$\frac{V_o - 4V_x}{20} - 3 + \frac{V_o}{20 + j10} = 0 \quad \text{where} \quad V_x = \frac{20}{20 + j10} V_o$$

Combining these we get:

$$\frac{V_o}{20} - \frac{4V_o}{20 + j10} - 3 + \frac{V_o}{20 + j10} = 0 \to (1 + j0.5 - 3)V_o = 60 + j30$$

$$V_o = \frac{60 + j30}{-2 + j0.5} \text{ or } V_x = \frac{20(3)}{-2 + j0.5} =$$

$29.11\angle{-166^\circ}$ V.

Chapter 10, Solution 7.

At the main node,

$$\frac{120\angle -15^{o} - V}{40 + j20} = 6\angle 30^{o} + \frac{V}{-j30} + \frac{V}{50} \longrightarrow \frac{115.91 - j31.058}{40 + j20} - 5.196 - j3 =$$

$$V\left(\frac{1}{40 + j20} + \frac{j}{30} + \frac{1}{50}\right)$$

$$V = \frac{-3.1885 - j4.7805}{0.04 + j0.0233} = \underline{124.08\angle -154^{o}\ \ V}$$

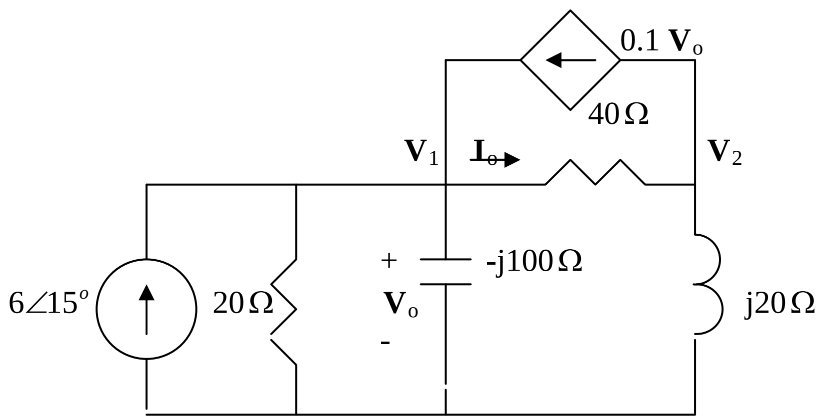
Chapter 10, Solution 8.

$\omega = 200,$

$100\text{mH} \longrightarrow \quad j\omega L = j200x0.1 = j20$

$50\mu F \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j200x50x10^{-6}} = -j100$

The frequency-domain version of the circuit is shown below.



At node 1,

$$6\angle 15^o + 0.1V_1 = \frac{V_1}{20} + \frac{V_1}{-j100} + \frac{V_1 - V_2}{40}$$

or $\quad 5.7955 + j1.5529 = (-0.025 + j0.01)V_1 - 0.025V_2 \qquad (1)$

At node 2,

$$\frac{V_1 - V_2}{40} = 0.1V_1 + \frac{V_2}{j20} \longrightarrow \quad 0 = 3V_1 + (1 - j2)V_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} (-0.025 + j0.01) & -0.025 \\ 3 & (1 - j2) \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} (5.7955 + j1.5529) \\ 0 \end{pmatrix} \quad \text{or} \quad AV = B$$

Using MATLAB,

V = inv(A)*B

leads to $V_1 = -70.63 - j127.23, \qquad V_2 = -110.3 + j161.09$

$$I_o = \frac{V_1 - V_2}{40} = 7.276\angle -82.17^o$$

Thus,

$$\underline{\boldsymbol{i_o(t) = 7.276\cos(200t - 82.17^o)\ \text{A}}}$$

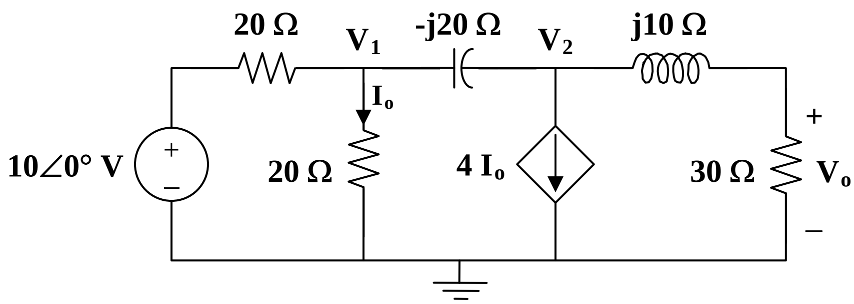
Chapter 10, Solution 9.

$$10\cos(10^3 t) \longrightarrow 10\angle 0°, \quad \omega = 10^3$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

Consider the circuit shown below.



At node 1,

$$\frac{10 - \mathbf{V}_1}{20} = \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$$

$$10 = (2 + j)\mathbf{V}_1 - j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} = (4)\frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_2}{30 + j10}, \text{ where } \mathbf{I}_o = \frac{\mathbf{V}_1}{20} \text{ has been substituted.}$$

$$(-4 + j)\mathbf{V}_1 = (0.6 + j0.8)\mathbf{V}_2$$

$$\mathbf{V}_1 = \frac{0.6 + j0.8}{-4 + j}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1)

$$10 = \frac{(2 + j)(0.6 + j0.8)}{-4 + j}\mathbf{V}_2 - j\mathbf{V}_2$$

or $$\mathbf{V}_2 = \frac{170}{0.6 - j26.2}$$

$$\mathbf{V}_o = \frac{30}{30 + j10}\mathbf{V}_2 = \frac{3}{3 + j}\cdot\frac{170}{0.6 - j26.2} = 6.154\angle 70.26°$$

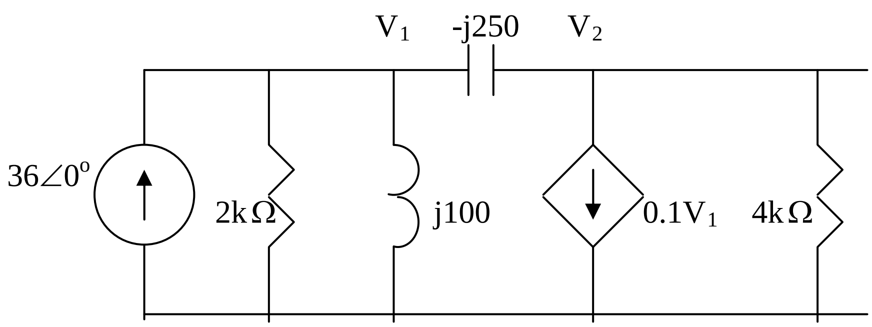
Therefore, $v_o(t) =$ $\mathbf{6.154\cos(10^3 t + 70.26°)\ V}$

Chapter 10, Solution 10.

$$50\,\text{mH} \longrightarrow j\omega L = j2000x50x10^{-3} = j100, \qquad \omega = 2000$$

$$2\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2000x2x10^{-6}} = -j250$$

Consider the frequency-domain equivalent circuit below.



At node 1,

$$36 = \frac{V_1}{2000} + \frac{V_1}{j100} + \frac{V_1 - V_2}{-j250} \longrightarrow 36 = (0.0005 - j0.006)V_1 - j0.004V_2 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{-j250} = 0.1V_1 + \frac{V_2}{4000} \longrightarrow 0 = (0.1 - j0.004)V_1 + (0.00025 + j0.004)V_2 \qquad (2)$$

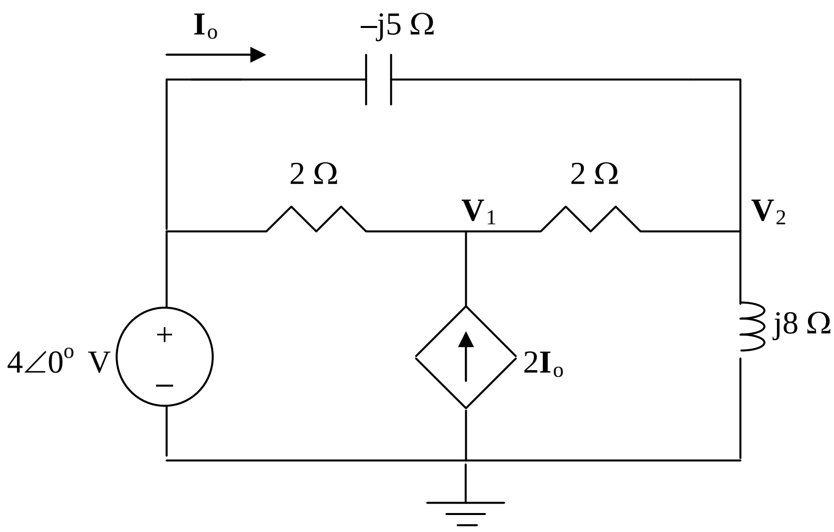
Solving (1) and (2) gives

$$V_o = V_2 = -535.6 + j893.5 = 8951.1\angle 93.43^o$$

v_o (t) = **8.951 sin(2000t +93.43°) kV**

Chapter 10, Solution 11.

Consider the circuit as shown below.



At node 1,

$$\frac{V_1 - 4}{2} - 2I_o + \frac{V_1 - V_2}{2} = 0$$
$$V_1 - 0.5V_2 - 2I_o = 2$$

But, $I_o = (4–V_2)/(–j5) = –j0.2V_2 + j0.8$

Now the first node equation becomes,

$$V_1 – 0.5V_2 + j0.4V_2 – j1.6 = 2 \text{ or}$$
$$V_1 + (–0.5+j0.4)V_2 = 2 + j1.6$$

At node 2,

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 4}{-j5} + \frac{V_2 - 0}{j8} = 0$$
$$–0.5V_1 + (0.5 + j0.075)V_2 = j0.8$$

Using MATLAB to solve this, we get,

```
>> Y=[1,(-0.5+0.4i);-0.5,(0.5+0.075i)]

Y =
```

```
   1.0000            -0.5000 + 0.4000i
  -0.5000             0.5000 + 0.0750i

>> I=[(2+1.6i);0.8i]

I =

   2.0000 + 1.6000i
        0 + 0.8000i

>> V=inv(Y)*I

V =

   4.8597 + 0.0543i
   4.9955 + 0.9050i
```

$I_o = -j0.2V_2 + j0.8 = -j0.9992 + 0.01086 + j0.8 = 0.01086 - j0.1992$

$= \mathbf{199.5\angle 86.89^\circ\ mA}$.

Chapter 10, Solution 12.

Using Fig. 10.61, design a problem to help other students to better understand Nodal analysis.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

By nodal analysis, find i_o in the circuit in Fig. 10.61.

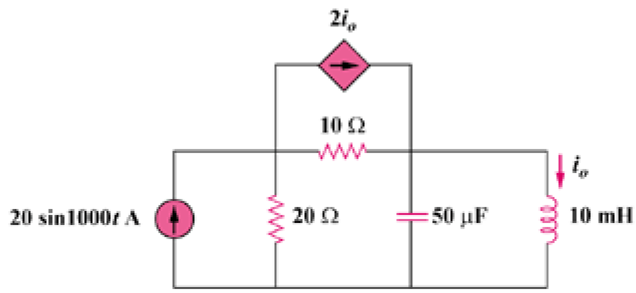


Figure 10.61

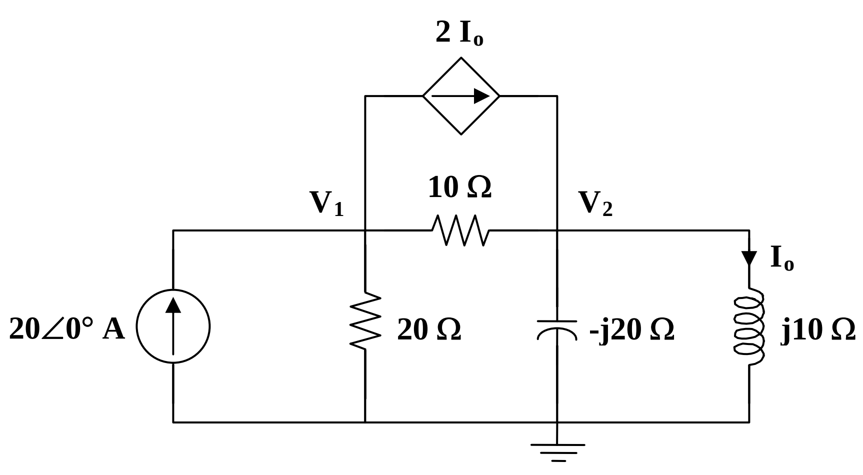
Solution

$$20\sin(1000t) \longrightarrow 20\angle 0°, \quad \omega = 1000$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

The frequency-domain equivalent circuit is shown below.



At node 1,

$$20 = 2\,\mathbf{I}_o + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10},$$ where

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10}$$

$$20 = \frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}$$

$$400 = 3\mathbf{V}_1 - (2 + j4)\,\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{\mathbf{V}_2}{-j20} + \frac{\mathbf{V}_2}{j10}$$

$$j2\,\mathbf{V}_1 = (-3 + j2)\,\mathbf{V}_2$$

or $$\mathbf{V}_1 = (1 + j1.5)\,\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$400 = (3 + j4.5)\,\mathbf{V}_2 - (2 + j4)\,\mathbf{V}_2 = (1 + j0.5)\,\mathbf{V}_2$$

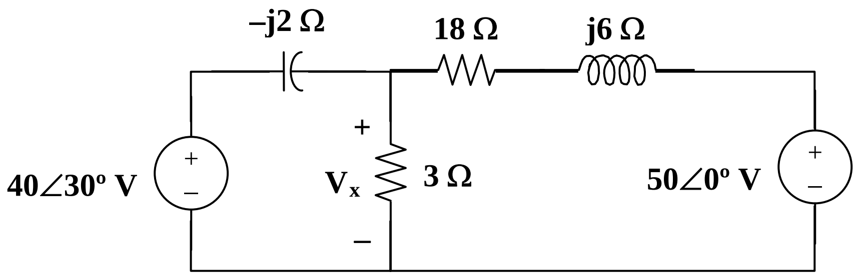
$$\mathbf{V}_2 = \frac{400}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10} = \frac{40}{j(1 + j0.5)} = 35.74\angle -116.6^\circ$$

Therefore, $i_o(t) =$ **35.74 sin(1000t – 116.6°) A**

Chapter 10, Solution 13.

Nodal analysis is the best approach to use on this problem. We can make our work easier by doing a source transformation on the right hand side of the circuit.



$$\frac{V_x - 40\angle 30^\circ}{-j2} + \frac{V_x}{3} + \frac{V_x - 50}{18 + j6} = 0$$

which leads to V_x = **29.36∠62.88° A**.

Chapter 10, Solution 14.

At node 1,

$$\frac{0-\mathbf{V}_1}{-j2} + \frac{0-\mathbf{V}_1}{10} + \frac{\mathbf{V}_2-\mathbf{V}_1}{j4} = 20\angle 30°$$

$$-(1+j2.5)\mathbf{V}_1 - j2.5\mathbf{V}_2 = 173.2 + j100 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_2}{j2} + \frac{\mathbf{V}_2}{-j5} + \frac{\mathbf{V}_2-\mathbf{V}_1}{j4} = 20\angle 30°$$

$$-j5.5\mathbf{V}_2 + j2.5\mathbf{V}_1 = 173.2 + j100 \qquad (2)$$

Equations (1) and (2) can be cast into matrix form as

$$\begin{bmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} -200\angle 30° \\ 200\angle 30° \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{vmatrix} = 20 - j5.5 = 20.74\angle -15.38°$$

$$\Delta_1 = \begin{vmatrix} -200\angle 30° & j2.5 \\ 200\angle 30° & -j5.5 \end{vmatrix} = j3(200\angle 30°) = 600\angle 120°$$

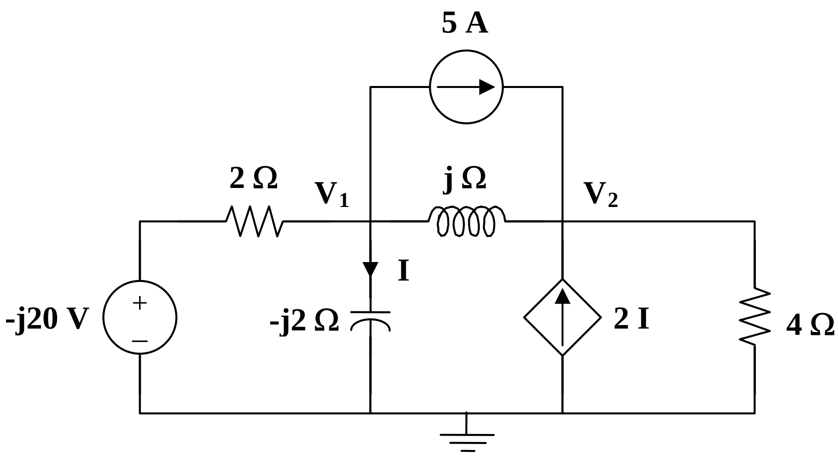
$$\Delta_2 = \begin{vmatrix} 1+j2.5 & -200\angle 30° \\ j2.5 & 200\angle 30° \end{vmatrix} = (200\angle 30°)(1+j5) = 1020\angle 108.7°$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = 28.93\angle 135.38° \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 49.18\angle 124.08° \text{ V}$$

Chapter 10, Solution 15.

We apply nodal analysis to the circuit shown below.



At node 1,

$$\frac{-\text{j}20 - \mathbf{V}_1}{2} = 5 + \frac{\mathbf{V}_1}{-\text{j}2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\text{j}}$$

$$-5 - \text{j}10 = (0.5 - \text{j}0.5)\,\mathbf{V}_1 + \text{j}\,\mathbf{V}_2 \qquad (1)$$

At node 2,

$$5 + 2\,\mathbf{I} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\text{j}} = \frac{\mathbf{V}_2}{4},$$

where $\mathbf{I} = \dfrac{\mathbf{V}_1}{-\text{j}2}$

$$\mathbf{V}_2 = \frac{5}{0.25 - \text{j}}\,\mathbf{V}_1 \qquad (2)$$

Substituting (2) into (1),

$$-5 - \text{j}10 - \frac{\text{j}5}{0.25 - \text{j}} = 0.5(1 - \text{j})\,\mathbf{V}_1$$

$$(1 - \text{j})\,\mathbf{V}_1 = -10 - \text{j}20 - \frac{\text{j}40}{1 - \text{j}4}$$

$$(\sqrt{2}\angle -45°)\,\mathbf{V}_1 = -10 - \text{j}20 + \frac{160}{17} - \frac{\text{j}40}{17}$$

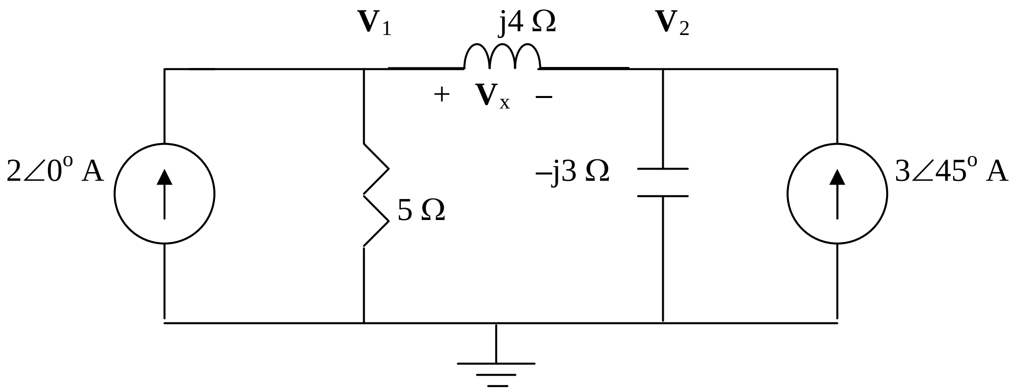
$$\mathbf{V}_1 = 15.81\angle 313.5°$$

$$\mathbf{I} = \frac{\mathbf{V}_1}{-\text{j}2} = (0.5\angle 90°)(15.81\angle 313.5°)$$

$$\mathbf{I} = \mathbf{7.906\angle 43.49°\ A}$$

Chapter 10, Solution 16.

Consider the circuit as shown in the figure below.



At node 1,

$$-2+\frac{V_1-0}{5}+\frac{V_1-V_2}{j4}=0 \qquad (1)$$

$$(0.2-j0.25)V_1+j0.25V_2=2$$

At node 2,

$$\frac{V_2-V_1}{j4}+\frac{V_2-0}{-j3}-3\angle 45^\circ=0 \qquad (2)$$

$$j0.25V_1+j0.08333V_2=2.121+j2.121$$

In matrix form, (1) and (2) become

$$\begin{bmatrix}0.2-j0.25 & j0.25\\ j0.25 & j0.08333\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}2\\2.121+j2.121\end{bmatrix}$$

Solving this using MATLAB, we get,

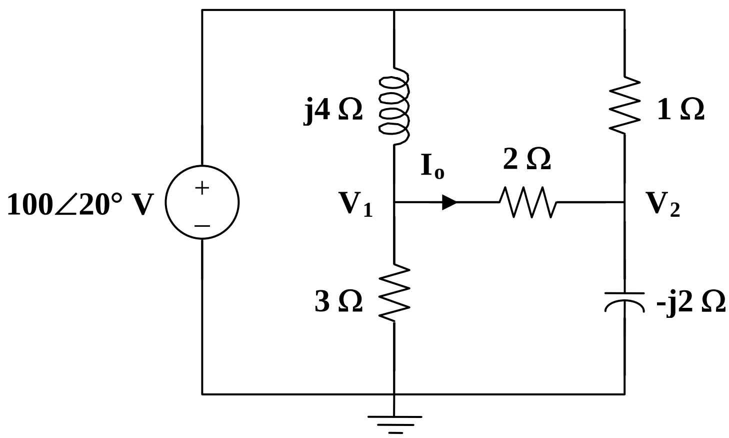
```
>> Y=[(0.2-0.25i),0.25i;0.25i,0.08333i]

Y =

   0.2000 - 0.2500i        0 + 0.2500i
        0 + 0.2500i        0 + 0.0833i

>> I=[2;(2.121+2.121i)]

I =
```

```
   2.0000
   2.1210 + 2.1210i

>> V=inv(Y)*I

V =

   5.2793 - 5.4190i
   9.6145 - 9.1955i
```

$$V_s = V_1 - V_2 = -4.335 + j3.776 = \mathbf{5.749\angle 138.94^\circ\ V}.$$

Chapter 10, Solution 17.

Consider the circuit below.



At node 1,

$$\frac{100\angle 20° - \mathbf{V}_1}{j4} = \frac{\mathbf{V}_1}{3} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2}$$

$$100\angle 20° = \frac{\mathbf{V}_1}{3}(3 + j10) - j2\,\mathbf{V}_2$$

(1)

At node 2,

$$\frac{100\angle 20° - \mathbf{V}_2}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\mathbf{V}_2}{-j2}$$

$$100\angle 20° = -0.5\,\mathbf{V}_1 + (1.5 + j0.5)\,\mathbf{V}_2$$

(2)

From (1) and (2),

$$\begin{bmatrix} 100\angle 20° \\ 100\angle 20° \end{bmatrix} = \begin{bmatrix} -0.5 & 0.5(3+j) \\ 1+j10/3 & -j2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} -0.5 & 1.5+j0.5 \\ 1+j10/3 & -j2 \end{vmatrix} = 0.1667 - j4.5$$

$$\Delta_1 = \begin{vmatrix} 100\angle 20° & 1.5+j0.5 \\ 100\angle 20° & -j2 \end{vmatrix} = -55.45 - j286.2$$

$$\Delta_2 = \begin{vmatrix} -0.5 & 100\angle 20° \\ 1+j10/3 & 100\angle 20° \end{vmatrix} = -26.95 - j364.5$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = 64.74\angle -13.08°$$

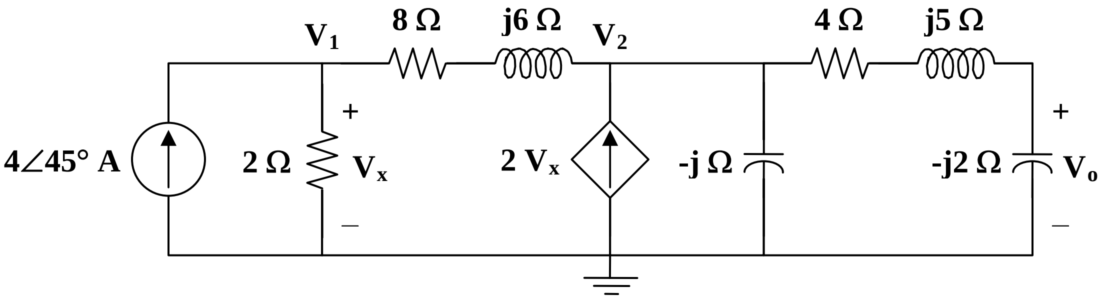
$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 81.17\angle -6.35°$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\Delta_1 - \Delta_2}{2\Delta} = \frac{-28.5 + j78.31}{0.3333 - j9}$$

$$\mathbf{I}_o = \mathbf{9.25\angle\text{-}162.12° \ A}$$

Chapter 10, Solution 18.

Consider the circuit shown below.



At node 1,

$$4\angle 45° = \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j6}$$

$$200\angle 45° = (29 - j3)\,\mathbf{V}_1 - (4 - j3)\,\mathbf{V}_2$$
(1)

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j6} + 2\mathbf{V}_x = \frac{\mathbf{V}_2}{-j} + \frac{\mathbf{V}_2}{4 + j5 - j2}, \qquad \text{where } \mathbf{V}_x = \mathbf{V}_1$$

$$(104 - j3)\,\mathbf{V}_1 = (12 + j41)\,\mathbf{V}_2$$

$$\mathbf{V}_1 = \frac{12 + j41}{104 - j3}\mathbf{V}_2$$
(2)

Substituting (2) into (1),

$$200\angle 45° = (29 - j3)\frac{(12 + j41)}{104 - j3}\mathbf{V}_2 - (4 - j3)\,\mathbf{V}_2$$

$$200\angle 45° = (14.21\angle 89.17°)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{200\angle 45°}{14.21\angle 89.17°}$$

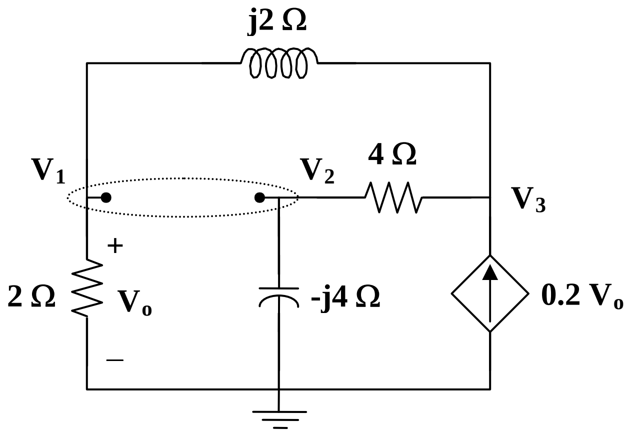
$$\mathbf{V}_o = \frac{-j2}{4 + j5 - j2}\mathbf{V}_2 = \frac{-j2}{4 + j3}\mathbf{V}_2 = \frac{-6 - j8}{25}\mathbf{V}_2$$

$$\mathbf{V}_o = \frac{10\angle 233.13°}{25} \cdot \frac{200\angle 45°}{14.21\angle 89.17°}$$

$$\mathbf{V}_o = \mathbf{5.63\angle 189°\ V}$$

Chapter 10, Solution 19.

We have a supernode as shown in the circuit below.



Notice that $\mathbf{V}_o = \mathbf{V}_1$.

At the supernode,

$$\frac{\mathbf{V}_3 - \mathbf{V}_2}{4} = \frac{\mathbf{V}_2}{-j4} + \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_3}{j2}$$

$$0 = (2 - j2)\mathbf{V}_1 + (1 + j)\mathbf{V}_2 + (-1 + j2)\mathbf{V}_3 \qquad (1)$$

At node 3,

$$0.2\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_3}{j2} = \frac{\mathbf{V}_3 - \mathbf{V}_2}{4}$$

$$(0.8 - j2)\mathbf{V}_1 + \mathbf{V}_2 + (-1 + j2)\mathbf{V}_3 = 0 \qquad (2)$$

Subtracting (2) from (1),

$$0 = 1.2\mathbf{V}_1 + j\mathbf{V}_2 \qquad (3)$$

But at the supernode,

$$\mathbf{V}_1 = 12\angle 0° + \mathbf{V}_2$$

or $$\mathbf{V}_2 = \mathbf{V}_1 - 12 \qquad (4)$$

Substituting (4) into (3),

$$0 = 1.2\mathbf{V}_1 + j(\mathbf{V}_1 - 12)$$

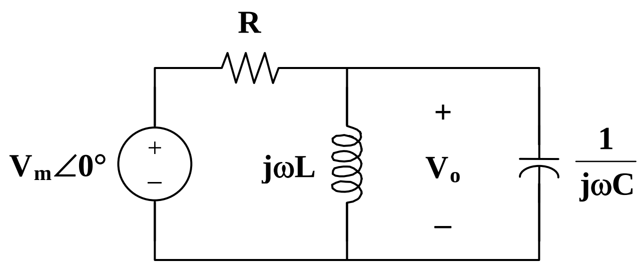
$$\mathbf{V}_1 = \frac{j12}{1.2 + j} = \mathbf{V}_o$$

$$\mathbf{V}_o = \frac{12\angle 90°}{1.562\angle 39.81°}$$

$$\mathbf{V}_o = \mathbf{7.682\angle 50.19°\ V}$$

Chapter 10, Solution 20.

The circuit is converted to its frequency-domain equivalent circuit as shown below.



Let $\mathbf{Z} = j\omega L \parallel \dfrac{1}{j\omega C} = \dfrac{\dfrac{L}{C}}{j\omega L + \dfrac{1}{j\omega C}} = \dfrac{j\omega L}{1-\omega^2 LC}$

$$\mathbf{V}_o = \frac{\mathbf{Z}}{R+\mathbf{Z}} V_m = \frac{\dfrac{j\omega L}{1-\omega^2 LC}}{R + \dfrac{j\omega L}{1-\omega^2 LC}} V_m = \frac{j\omega L}{R\,(1-\omega^2 LC) + j\omega L} V_m$$

$$\mathbf{V}_o = \frac{\omega L\, V_m}{\sqrt{R^2\,(1-\omega^2 LC)^2 + \omega^2 L^2}} \angle\left(90° - \tan^{-1}\frac{\omega L}{R\,(1-\omega^2 LC)}\right)$$

If $\mathbf{V}_o = A\angle\phi$, then

$$A = \frac{\boldsymbol{\omega L\, V_m}}{\sqrt{\mathbf{R^2\,(1-\omega^2 LC)^2 + \omega^2 L^2}}}$$

and $\phi = \mathbf{90° - tan^{-1}\dfrac{\omega L}{R\,(1-\omega^2 LC)}}$

Chapter 10, Solution 21.

(a) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{1}{1 - \omega^2 LC + j\omega RC}$$

At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \mathbf{1}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{0}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \frac{\mathbf{-j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}$$

(b) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{-\omega^2 LC}{1 - \omega^2 LC + j\omega RC}$$

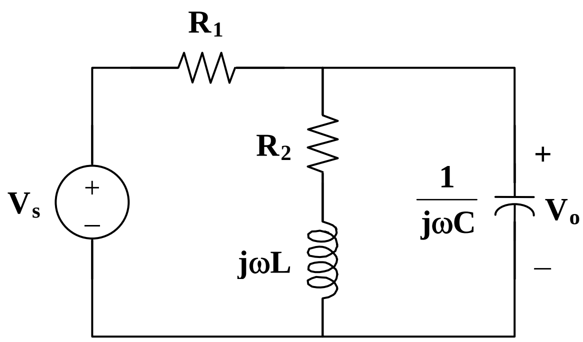
At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{0}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \mathbf{1}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \frac{\mathbf{j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}$$

Chapter 10, Solution 22.

Consider the circuit in the frequency domain as shown below.



Let $\quad \mathbf{Z} = (R_2 + j\omega L) \parallel \dfrac{1}{j\omega C}$

$$\mathbf{Z} = \frac{\dfrac{1}{j\omega C}(R_2 + j\omega L)}{R_2 + j\omega L + \dfrac{1}{j\omega C}} = \frac{R_2 + j\omega L}{1 + j\omega R_2 - \omega^2 LC}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}}{\mathbf{Z} + R_1} = \frac{\dfrac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}{R_1 + \dfrac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{R_2 + j\omega L}}{\mathbf{R_1 + R_2 - \omega^2 LCR_1 + j\omega(L + R_1 R_2 C)}}$$

Chapter 10, Solution 23.

$$\frac{V - V_s}{R} + \frac{V}{j\omega L + \dfrac{1}{j\omega C}} + j\omega CV = 0$$

$$V + \frac{j\omega RCV}{-\omega^2 LC + 1} + j\omega RCV = V_s$$

$$\left(\frac{1 - \omega^2 LC + j\omega RC + j\omega RC - j\omega^3 RLC^2}{1 - \omega^2 LC} \right) V = V_s$$

$$\boldsymbol{V = \frac{(1 - \omega^2 LC)V_s}{\underline{1 - \omega^2 LC + j\omega RC(2 - \omega^2 LC)}}}$$

Chapter 10, Solution 24.

Design a problem to help other students to better understand mesh analysis.

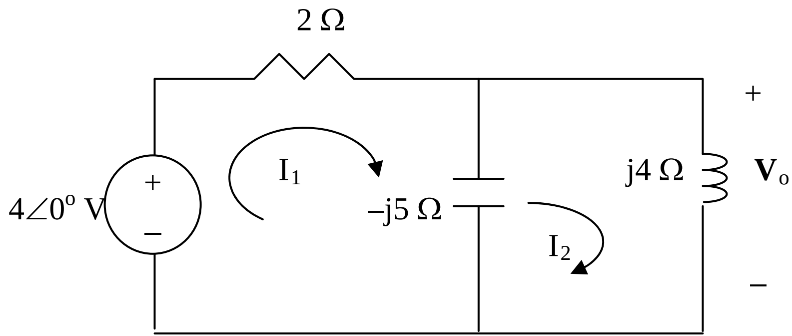
Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use mesh analysis to find $\mathbf{V}_o$ in the circuit in Prob. 10.2.

Solution

Consider the circuit as shown below.



For mesh 1,

$$4 = (2 - j5)I_1 + j5I_1 \qquad (1)$$

For mesh 2,

$$0 = j5I_1 + (j4 - j5)I_2 \quad \longrightarrow \quad I_1 = \frac{1}{5}I_2 \qquad (2)$$

Substituting (2) into (1),

$$4 = (2 - j5)\frac{1}{5}I_2 + j5I_2 \quad \longrightarrow \quad I_2 = \frac{1}{0.1 + j}$$

$\mathbf{V_o}$ = j4$\mathbf{I_2}$ = j4/(0.1+j) = j4/(1.00499∠84.29°) = **3.98∠5.71° V**

Chapter 10, Solution 25.

$\omega = 2$

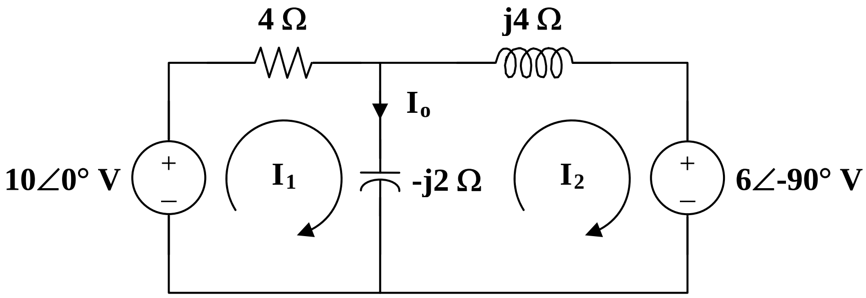
$$10\cos(2t) \longrightarrow 10\angle 0°$$

$$6\sin(2t) \longrightarrow 6\angle -90° = -j6$$

$$2\text{ H} \longrightarrow j\omega L = j4$$

$$0.25\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$

The circuit is shown below.



For loop 1,

$$-10 + (4 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

$$5 = (2 - j)\mathbf{I}_1 + j\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j2\mathbf{I}_1 + (j4 - j2)\mathbf{I}_2 + (-j6) = 0$$

$$\mathbf{I}_1 + \mathbf{I}_2 = 3 \qquad (2)$$

In matrix form (1) and (2) become

$$\begin{bmatrix} 2-j & j \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\Delta = 2(1 - j), \qquad \Delta_1 = 5 - j3, \qquad \Delta_2 = 1 - j3$$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{4}{2(1-j)} = 1 + j = 1.4142\angle 45°$$

Therefore,

$$i_o(t) = \mathbf{1.4142cos(2t + 45°)\ A}$$

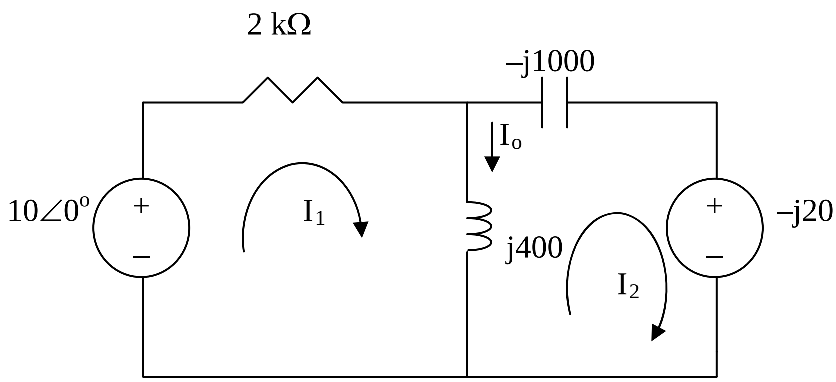
Chapter 10, Solution 26.

$$0.4H \longrightarrow j\omega L = j10^3 x0.4 = j400$$

$$1\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 x10^{-6}} = -j1000$$

$$20\sin(10^3 t) = 20\cos(10^3 t - 90^o) \textit{ which leads to } 20\angle -90^o = -j20$$

The circuit becomes that shown below.



For loop 1,

$$-10 + (12000 + j400)I_1 - j400I_2 = 0 \longrightarrow 1 = (200 + j40)I_1 - j40I_2 \quad (1)$$

For loop 2,

$$-j20 + (j400 - j1000)I_2 - j400I_1 = 0 \longrightarrow -12 = 40I_1 + 60I_2$$

(2)

In matrix form, (1) and (2) become

$$\begin{bmatrix} 1 \\ -12 \end{bmatrix} = \begin{bmatrix} 200 + j40 & -j40 \\ 40 & 60 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this leads to

I_1 =0.0025-j0.0075, I_2 = -0.035+j0.005

$I_o = I_1 - I_2$ = 0.0375 – j0.0125 = 39.5∠–18.43° mA

$i_o(t)$ = **39.5cos(10^3t–18.43°) mA**

Chapter 10, Solution 27.

For mesh 1,

$$-40\angle 30° + (j10 - j20)\mathbf{I}_1 + j20\mathbf{I}_2 = 0$$

$$4\angle 30° = -j\mathbf{I}_1 + j2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$50\angle 0° + (40 - j20)\mathbf{I}_2 + j20\mathbf{I}_1 = 0$$

$$5 = -j2\mathbf{I}_1 - (4 - j2)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 4\angle 30° \\ 5 \end{bmatrix} = \begin{bmatrix} -j & j2 \\ -j2 & -(4 - j2) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = -2 + 4j = 4.472\angle 116.56°$$

$$\Delta_1 = -(4\angle 30°)(4 - j2) - j10 = 21.01\angle 211.8°$$

$$\Delta_2 = -j5 + 8\angle 120° = 4.44\angle 154.27°$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \mathbf{4.698\angle 95.24°\ A}$$

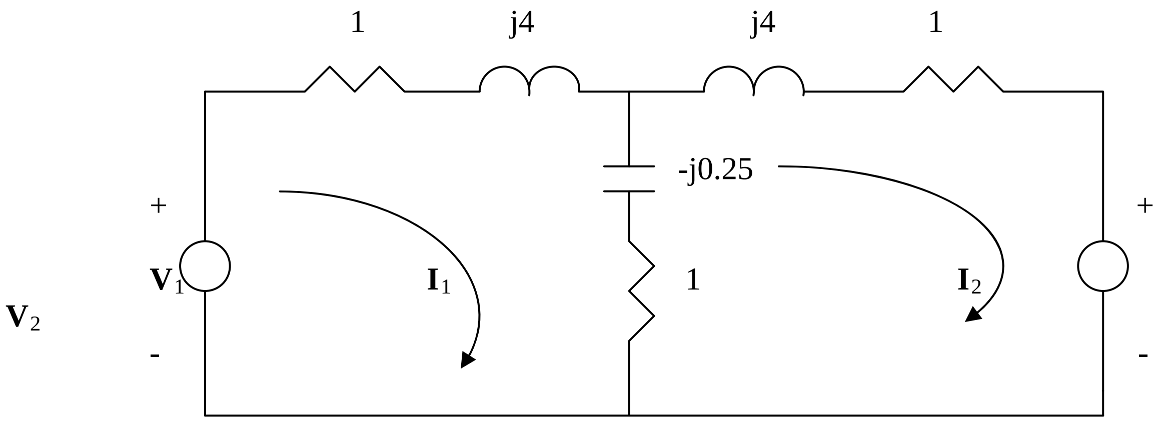
$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{992.8\angle 37.71°\ mA}$$

Chapter 10, Solution 28.

$$1\text{H} \longrightarrow j\omega L = j4, \qquad 1\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j1x4} = -j0.25$$

The frequency-domain version of the circuit is shown below, where

$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o \ .$$



$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o$$

Applying mesh analysis,

$$10 = (2 + j3.75)I_1 - (1 - j0.25)I_2 \qquad (1)$$

$$-20\angle -30^o = -(1 - j0.25)I_1 + (2 + j3.75)I_2 \qquad (2)$$

From (1) and (2), we obtain

$$\begin{pmatrix} 10 \\ -17.32 + j10 \end{pmatrix} = \begin{pmatrix} 2 + j3.75 & -1 + j0.25 \\ -1 + j0.25 & 2 + j3.75 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Solving this leads to

$$I_1 = 2.741\angle -41.07^o, \qquad I_2 = 4.114\angle 92^o$$

Hence,

$i_1(t)$ = **2.741cos(4t–41.07˚)A**, $i_2(t)$ = **4.114cos(4t+92˚)A**.

Chapter 10, Solution 29.

Using Fig. 10.77, design a problem to help other students better understand mesh analysis.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

By using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit depicted in Fig. 10.77.

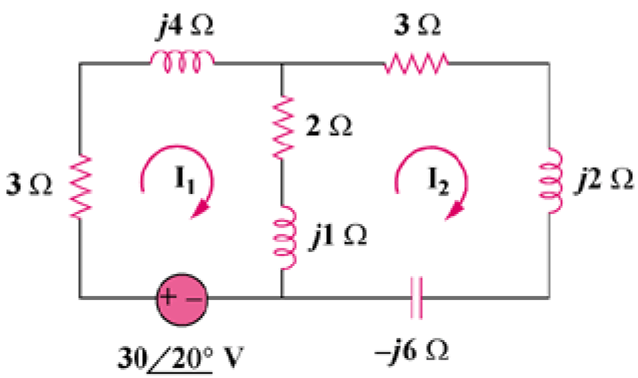


Figure 10.77

Solution

For mesh 1,

$$(5+j5)\mathbf{I}_1-(2+j)\mathbf{I}_2-30\angle 20°=0$$
$$30\angle 20°=(5+j5)\mathbf{I}_1-(2+j)\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$(5+j3-j6)\mathbf{I}_2-(2+j)\mathbf{I}_1=0$$
$$0=-(2+j)\mathbf{I}_1+(5-j3)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix}30\angle 20°\\0\end{bmatrix}=\begin{bmatrix}5+j5 & -(2+j)\\-(2+j) & 5-j3\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\\mathbf{I}_2\end{bmatrix}$$

$$\Delta=37+j6=37.48\angle 9.21°$$
$$\Delta_1=(30\angle 20°)(5.831\angle -30.96°)=175\angle -10.96°$$
$$\Delta_2=(30\angle 20°)(2.356\angle 26.56°)=67.08\angle 46.56°$$

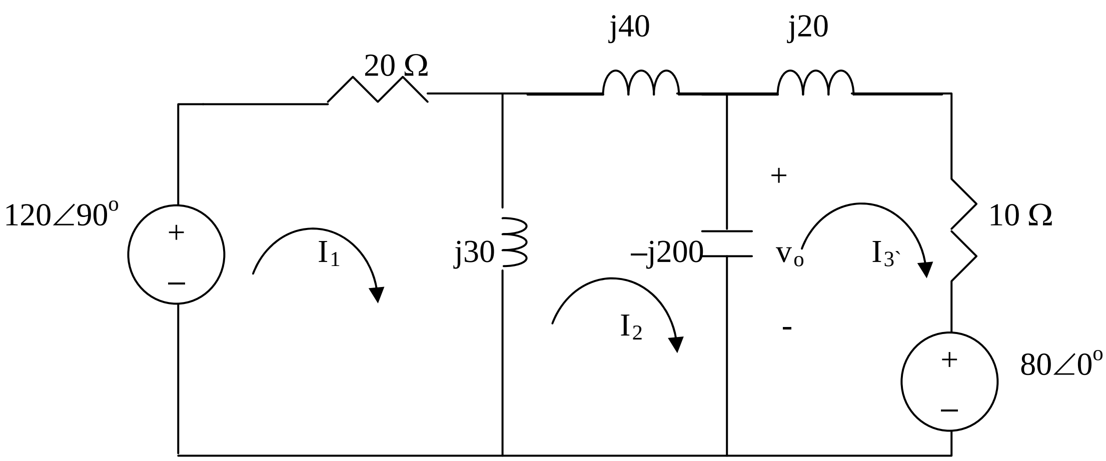
$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \mathbf{4.67\angle{-20.17}° \ A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{1.79\angle 37.35° \ A}$$

Chapter 10, Solution 30.

$$300mH \longrightarrow j\omega L = j100x300x10^{-3} = j30$$
$$200mH \longrightarrow j\omega L = j100x200x10^{-3} = j20$$
$$400mH \longrightarrow j\omega L = j100x400x10^{-3} = j40$$
$$50\mu F \longrightarrow 1/j\omega C = \frac{1}{j100x50x10^{-6}} = -j200$$

The circuit becomes that shown below.



For mesh 1,

$$-120 < 90^o + (20 + j30)I_1 - j30I_2 = 0 \longrightarrow j120 = (20 + j30)I_1 - j30I_2 \qquad (1)$$

For mesh 2,

$$-j30I_1 + (j30 + j40 - j200)I_2 + j200I_3 = 0 \longrightarrow 0 = -3I_1 - 13I_2 + 20I_3 \qquad (2)$$

For mesh 3,

$$80 + j200I_2 + (10 - j180)I_3 = 0 \rightarrow -8 = j20I_2 + (1 - j18)I_3 \qquad (3)$$

We put (1) to (3) in matrix form.

$$\begin{bmatrix} 2+j3 & -j3 & 0 \\ -3 & -13 & 20 \\ 0 & j20 & 1-j18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} j12 \\ 0 \\ -8 \end{bmatrix}$$

This is an excellent candidate for MATLAB.

```
>> Z=[(2+3i),-3i,0;-3,-13,20;0,20i,(1-18i)]

Z =

  2.0000 + 3.0000i        0 - 3.0000i        0
```

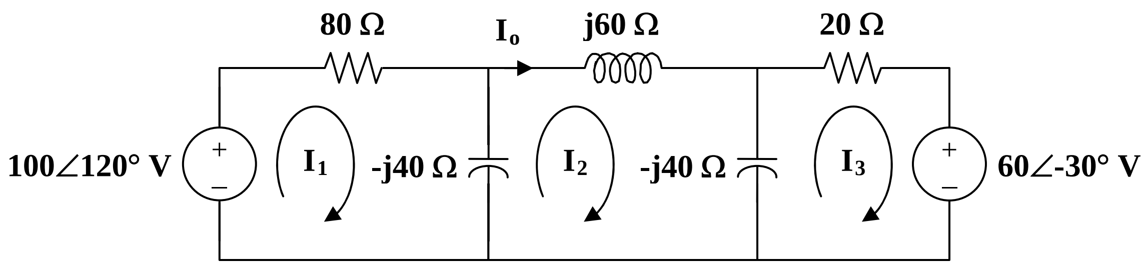
```
  -3.0000           -13.0000            20.0000
        0                  0 +20.0000i   1.0000 -18.0000i

>> V=[12i;0;-8]

V =

        0 +12.0000i
        0
  -8.0000

>> I=inv(Z)*V

I =

   2.0557 + 3.5651i
   0.4324 + 2.1946i
   0.5894 + 1.9612i
```

$V_o = -j200(I_2 - I_3) = -j200(-0.157+j0.2334) = 46.68 + j31.4 = 56.26\angle 33.93^\circ$

v_o = **56.26cos(100t + 33.93˚) V**.

Chapter 10, Solution 31.

Consider the network shown below.



For loop 1,

$$-100\angle 120° + (80 - j40)\mathbf{I}_1 + j40\mathbf{I}_2 = 0$$

$$10\angle 20° = 4(2 - j)\mathbf{I}_1 + j4\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j40\mathbf{I}_1 + (j60 - j80)\mathbf{I}_2 + j40\mathbf{I}_3 = 0$$

$$0 = 2\mathbf{I}_1 - \mathbf{I}_2 + 2\mathbf{I}_3 \qquad (2)$$

For loop 3,

$$60\angle -30° + (20 - j40)\mathbf{I}_3 + j40\mathbf{I}_2 = 0$$

$$-6\angle -30° = j4\mathbf{I}_2 + 2(1 - j2)\mathbf{I}_3 \qquad (3)$$

From (2),

$$2\mathbf{I}_3 = \mathbf{I}_2 - 2\mathbf{I}_1$$

Substituting this equation into (3),

$$-6\angle -30° = -2(1 - j2)\mathbf{I}_1 + (1 + j2)\mathbf{I}_2 \qquad (4)$$

From (1) and (4),

$$\begin{bmatrix} 10\angle 120° \\ -6\angle -30° \end{bmatrix} = \begin{bmatrix} 4(2 - j) & j4 \\ -2(1 - j2) & 1 + j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

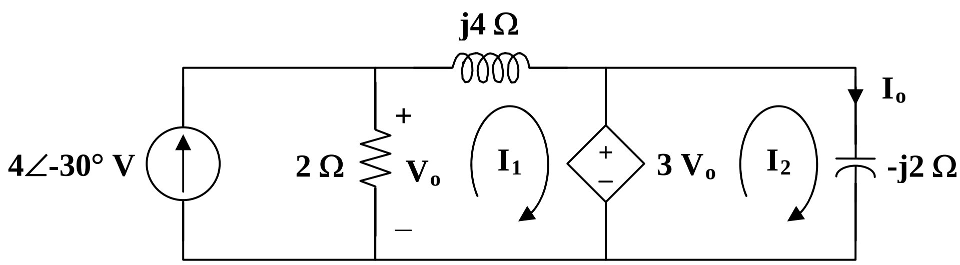
$$\Delta = \begin{vmatrix} 8 - j4 & -j4 \\ -2 + j4 & 1 + j2 \end{vmatrix} = 32 + j20 = 37.74\angle 32°$$

$$\Delta_2 = \begin{vmatrix} 8 - j4 & 10\angle 120° \\ -2 + j4 & -6\angle -30° \end{vmatrix} = -4.928 + j82.11 = 82.25\angle 93.44°$$

$$\mathbf{I}_o = \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{2.179\angle 61.44°\ A}$$

Chapter 10, Solution 32.

Consider the circuit below.



For mesh 1,

$$(2 + j4)\mathbf{I}_1 - 2(4\angle\text{-}30°) + 3\mathbf{V}_o = 0$$

where $\quad \mathbf{V}_o = 2(4\angle\text{-}30° - \mathbf{I}_1)$

Hence,

$$(2 + j4)\mathbf{I}_1 - 8\angle\text{-}30° + 6(4\angle\text{-}30° - \mathbf{I}_1) = 0$$

$$4\angle\text{-}30° = (1 - j)\mathbf{I}_1$$

or $\quad \mathbf{I}_1 = 2\sqrt{2}\angle 15°$

$$\mathbf{I}_o = \frac{3\mathbf{V}_o}{\text{-}j2} = \frac{3}{\text{-}j2}(2)(4\angle\text{-}30° - \mathbf{I}_1)$$

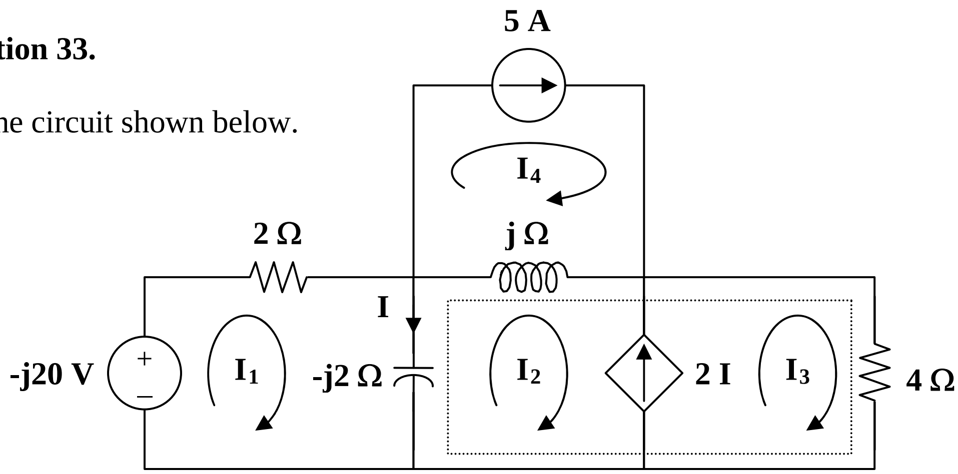
$$\mathbf{I}_o = j3(4\angle\text{-}30° - 2\sqrt{2}\angle 15°)$$

$$\mathbf{I}_o = \mathbf{8.485\angle 15°\ A}$$

$$\mathbf{V}_o = \frac{\text{-}j2\mathbf{I}_o}{3} = \mathbf{5.657\angle\text{-}75°\ V}$$

Chapter 10, Solution 33.

Consider the circuit shown below.



For mesh 1,

$$j20 + (2 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

$$(1 - j)\mathbf{I}_1 + j\mathbf{I}_2 = \text{-}j10 \qquad (1)$$

For the supermesh,

$$(j - j2)\mathbf{I}_2 + j2\mathbf{I}_1 + 4\mathbf{I}_3 - j\mathbf{I}_4 = 0 \qquad (2)$$

Also,

$$\mathbf{I}_3 - \mathbf{I}_2 = 2\mathbf{I} = 2(\mathbf{I}_1 - \mathbf{I}_2)$$

$$\mathbf{I}_3 = 2\mathbf{I}_1 - \mathbf{I}_2 \qquad (3)$$

For mesh 4,

$$\mathbf{I}_4 = 5 \qquad (4)$$

Substituting (3) and (4) into (2),

$$(8 + j2)\mathbf{I}_1 - (\text{-}4 + j)\mathbf{I}_2 = j5 \qquad (5)$$

Putting (1) and (5) in matrix form,

$$\begin{bmatrix} 1 - j & j \\ 8 + j2 & 4 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \text{-}\ j10 \\ j5 \end{bmatrix}$$

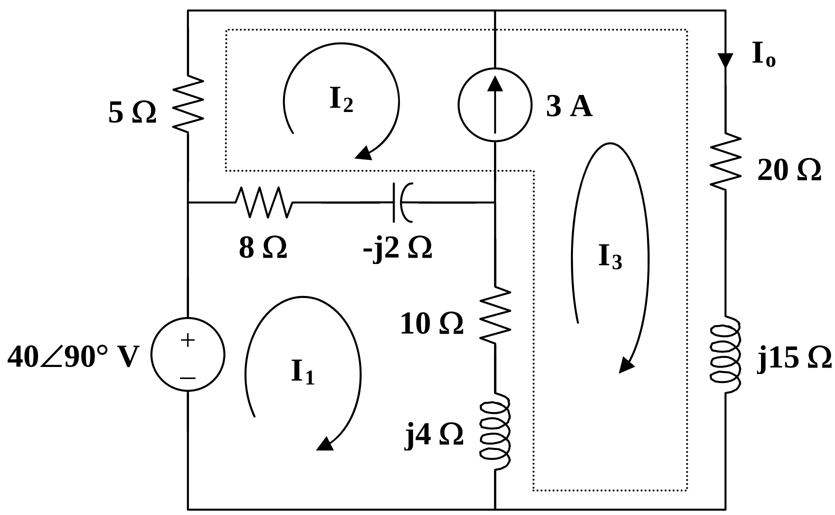
$$\Delta = \text{-}3 - j5, \qquad \Delta_1 = \text{-}5 + j40, \qquad \Delta_2 = \text{-}15 + j85$$

$$\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{10 - j45}{\text{-}3 - j5} =$$

7.906∠43.49° A

Chapter 10, Solution 34.

The circuit is shown below.



For mesh 1,

$$- j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad (1)$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (30 + j19)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad (2)$$

Also,

$$\mathbf{I}_2 = \mathbf{I}_3 - 3 \qquad (3)$$

Adding (1) and (2) and incorporating (3),

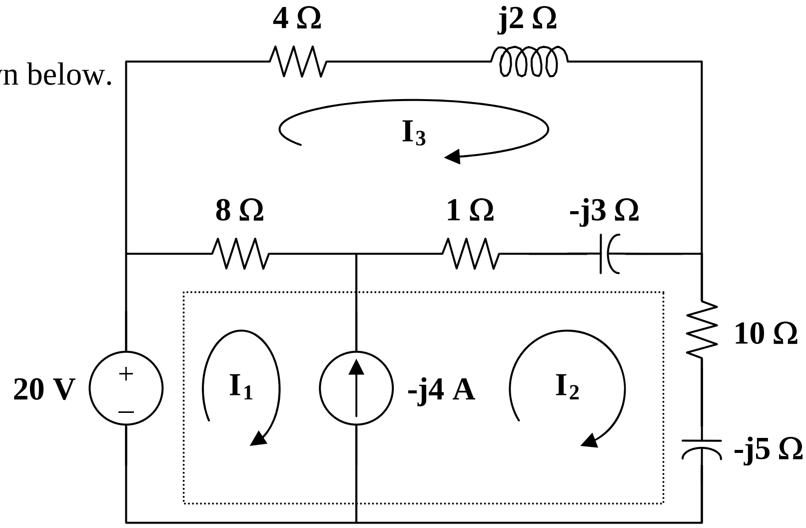
$$- j40 + 5(\mathbf{I}_3 - 3) + (20 + j15)\mathbf{I}_3 = 0$$

$$\mathbf{I}_3 = \frac{3 + j8}{5 + j3} = 1.465\angle 38.48°$$

$$\mathbf{I}_o = \mathbf{I}_3 = \mathbf{1.465\angle 38.48°\ A}$$

Chapter 10, Solution 35.

Consider the circuit shown below.



For the supermesh,

$$-20 + 8\mathbf{I}_1 + (11 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0 \qquad (1)$$

Also,

$$\mathbf{I}_1 = \mathbf{I}_2 + j4 \qquad (2)$$

For mesh 3,

$$(13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0 \qquad (3)$$

Substituting (2) into (1),

$$(19 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 20 - j32 \qquad (4)$$

Substituting (2) into (3),

$$-(9 - j3)\mathbf{I}_2 + (13 - j)\mathbf{I}_3 = j32 \qquad (5)$$

From (4) and (5),

$$\begin{bmatrix} 19 - j8 & -(9 - j3) \\ -(9 - j3) & 13 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 20 - j32 \\ j32 \end{bmatrix}$$

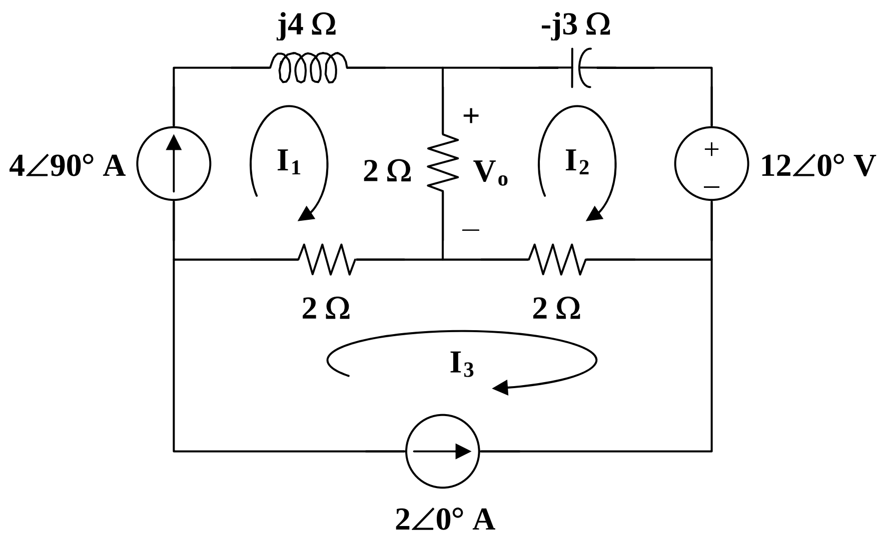
$$\Delta = 167 - j69, \qquad \Delta_2 = 324 - j148$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{324 - j148}{167 - j69} = \frac{356.2\angle -24.55^\circ}{180.69\angle -22.45^\circ}$$

$$\mathbf{I}_2 = \mathbf{1.971\angle{-2.1}^\circ\ A}$$

Chapter 10, Solution 36.

Consider the circuit below.



Clearly,

$$\mathbf{I}_1 = 4\angle 90° = j4 \qquad \text{and} \qquad \mathbf{I}_3 = -2$$

For mesh 2,

$$(4 - j3)\mathbf{I}_2 - 2\mathbf{I}_1 - 2\mathbf{I}_3 + 12 = 0$$

$$(4 - j3)\mathbf{I}_2 - j8 + 4 + 12 = 0$$

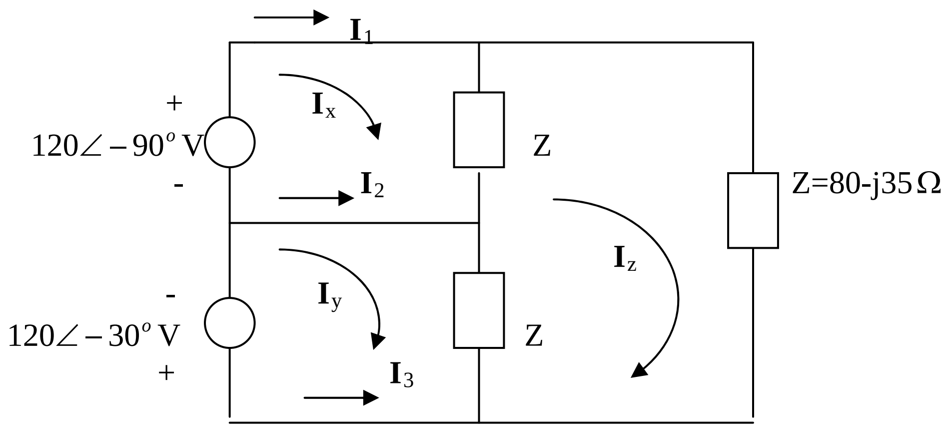
$$\mathbf{I}_2 = \frac{-16 + j8}{4 - j3} = -3.52 - j0.64$$

Thus,

$$\mathbf{V}_o = 2(\mathbf{I}_1 - \mathbf{I}_2) = (2)(3.52 + j4.64) = 7.04 + j9.28$$

$$\mathbf{V}_o = \mathbf{11.648\angle 52.82° \; V}$$

Chapter 10, Solution 37.



For mesh x,

$$ZI_x - ZI_z = -j120 \qquad (1)$$

For mesh y,

$$ZI_y - ZI_z = -120\angle 30^o = -103.92 + j60 \qquad (2)$$

For mesh z,

$$-ZI_x - ZI_y + 3ZI_z = 0 \qquad (3)$$

Putting (1) to (3) together leads to the following matrix equation:

$$\begin{pmatrix} (80-j35) & 0 & (-80+j35) \\ 0 & (80-j35) & (-80+j35) \\ (-80+j35) & (-80+j35) & (240-j105) \end{pmatrix}\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} -j120 \\ -103.92+j60 \\ 0 \end{pmatrix} \longrightarrow AI = B$$

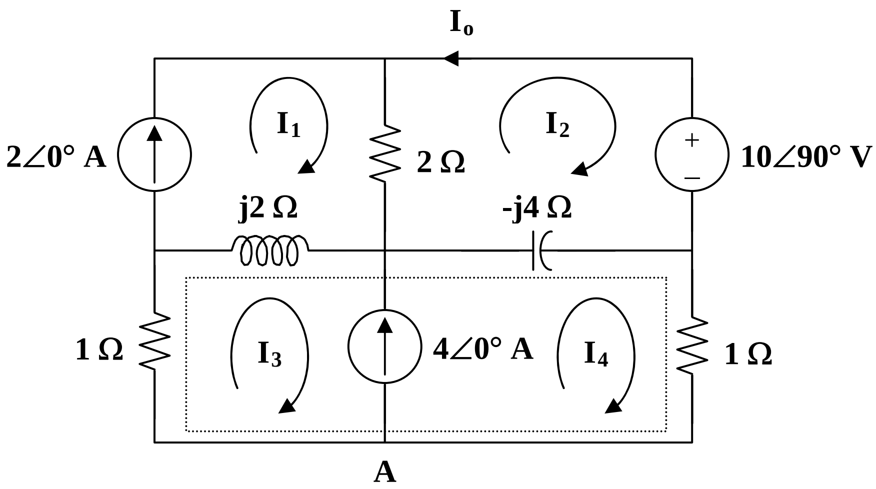
Using MATLAB, we obtain

$$I = inv(A) * B = \begin{pmatrix} -0.2641 - j2.366 \\ -2.181 - j0.954 \\ -0.815 - j1.1066 \end{pmatrix}$$

$$\boldsymbol{I_1 = I_x = -0.2641 - j2.366 = \underline{2.38\angle -96.37^o} \text{ A}}$$
$$\boldsymbol{I_2 = I_y - I_x = -1.9167 + j1.4116 = \underline{2.38\angle 143.63^o} \text{ A}}$$
$$\boldsymbol{I_3 = -I_y = 2.181 + j0.954 = \underline{2.38\angle 23.63^o \text{ A}}}$$

Chapter 10, Solution 38.

Consider the circuit below.



Clearly,

$$\mathbf{I}_1 = 2 \tag{1}$$

For mesh 2,

$$(2 - j4)\mathbf{I}_2 - 2\mathbf{I}_1 + j4\mathbf{I}_4 + 10\angle 90° = 0 \tag{2}$$

Substitute (1) into (2) to get

$$(1 - j2)\mathbf{I}_2 + j2\mathbf{I}_4 = 2 - j5$$

For the supermesh,

$$(1 + j2)\mathbf{I}_3 - j2\mathbf{I}_1 + (1 - j4)\mathbf{I}_4 + j4\mathbf{I}_2 = 0$$

$$j4\mathbf{I}_2 + (1 + j2)\mathbf{I}_3 + (1 - j4)\mathbf{I}_4 = j4 \tag{3}$$

At node A,

$$\mathbf{I}_3 = \mathbf{I}_4 - 4 \tag{4}$$

Substituting (4) into (3) gives

$$j2\mathbf{I}_2 + (1 - j)\mathbf{I}_4 = 2(1 + j3) \tag{5}$$

From (2) and (5),

$$\begin{bmatrix} 1 - j2 & j2 \\ j2 & 1 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} 2 - j5 \\ 2 + j6 \end{bmatrix}$$

$$\Delta = 3 - j3, \qquad \Delta_1 = 9 - j11$$

$$\mathbf{I}_o = \text{-}\mathbf{I}_2 = \frac{\text{-}\Delta_1}{\Delta} = \frac{\text{-}(9 - j11)}{3 - j3} = \frac{1}{3}(\text{-}10 + j)$$

$$\mathbf{I}_o = \mathbf{3.35\angle 174.3° \text{ A}}$$

Chapter 10, Solution 39.

For mesh 1,

$$(28 - j15)I_1 - 8I_2 + j15I_3 = 12\angle 64^o \qquad (1)$$

For mesh 2,

$$-8I_1 + (8 - j9)I_2 - j16I_3 = 0 \qquad (2)$$

For mesh 3,

$$j15I_1 - j16I_2 + (10 + j)I_3 = 0 \qquad (3)$$

In matrix form, (1) to (3) can be cast as

$$\begin{pmatrix} (28 - j15) & -8 & j15 \\ -8 & (8 - j9) & -j16 \\ j15 & -j16 & (10 + j) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 12\angle 64^o \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \quad AI = B$$

Using MATLAB,

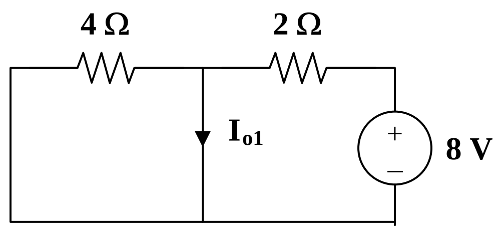
I = inv(A)*B

$$I_1 = -0.128 + j0.3593 = \mathbf{381.4\angle 109.6° \ mA}$$
$$I_2 = -0.1946 + j0.2841 = \mathbf{344.3\angle 124.4° \ mA}$$
$$I_3 = 0.0718 - j0.1265 = \mathbf{145.5\angle{-60.42}° \ mA}$$
$$I_x = I_1 - I_2 = 0.0666 + j0.0752 = \mathbf{100.5\angle 48.5° \ mA}$$

381.4∠109.6° mA, 344.3∠124.4° mA, 145.5∠–60.42° mA, 100.5∠48.5° mA

Chapter 10, Solution 40.

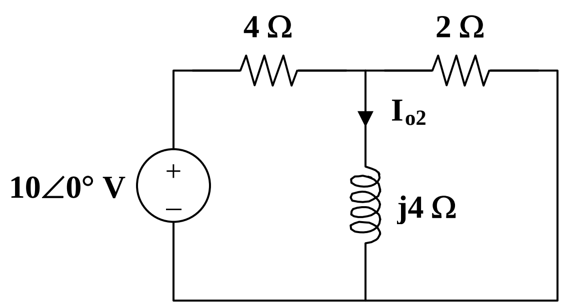
Let $I_o = I_{o1} + I_{o2}$, where I_{o1} is due to the dc source and I_{o2} is due to the ac source. For I_{o1}, consider the circuit in Fig. (a).
Clearly,



(a)

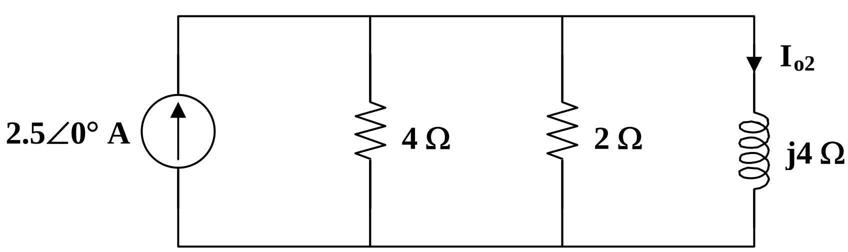
$$I_{o1} = 8/2 = 4 \text{ A}$$

For I_{o2}, consider the circuit in Fig. (b).



(b)

If we transform the voltage source, we have the circuit in Fig. (c), where $4 \parallel 2 = 4/3\,\Omega$.



(c)

By the current division principle,

$$\mathbf{I}_{o2} = \frac{4/3}{4/3 + j4}(2.5\angle 0°)$$

$$\mathbf{I}_{o2} = 0.25 - j0.75 = 0.79\angle\text{-}71.56°$$

Thus, $$I_{o2} = 0.79\cos(4t - 71.56°)\ A$$

Therefore,

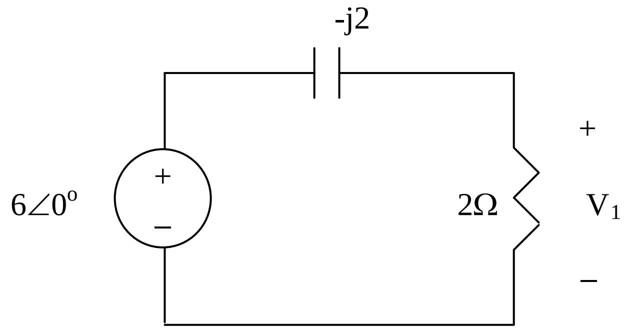
$$I_o = I_{o1} + I_{o2} = \mathbf{[4 + 0.79cos(4t–71.56°)]\ A}$$

Chapter 10, Solution 41.

We apply superposition principle. We let

$$v_o = v_1 + v_2$$

where v_1 and v_2 are due to the sources 6cos2t and 4sin4t respectively. To find v_1, consider the circuit below.



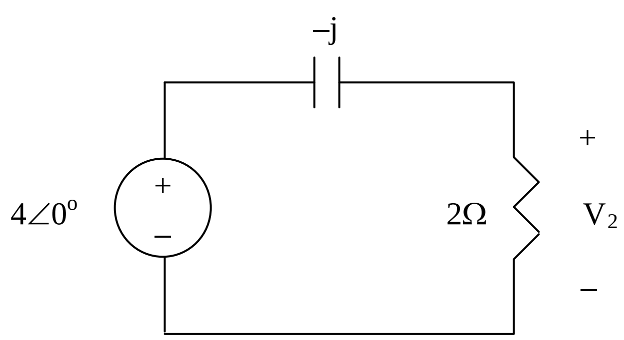
$$1/4F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2x1/4} = -j2$$

$$V_1 = \frac{2}{2-j2} V\,(6) = 3+j3 = 4.243\angle 45°$$

Thus,

$v_1(t) =$ **4.243cos(2t+45°) volts**.

To get $v_2(t)$, consider the circuit below,



$$1/4F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x1/4} = -j1$$

$$\mathbf{V_2} = \frac{2}{2-j}(4) = 3.2 + j11.6 = 3.578\angle 26.56^\circ$$ or

$v_2(t) = 3.578\sin(4t+25.56°)$ volts.

Hence,

v_o = **[4.243cos(2t+45˚) + 3.578sin(4t+25.56˚)] volts**.

Chapter 10, Solution 42.

Using Fig. 10.87, design a problem to help other students to better understand the superposition theorem.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve for I_o in the circuit of Fig. 10.87.

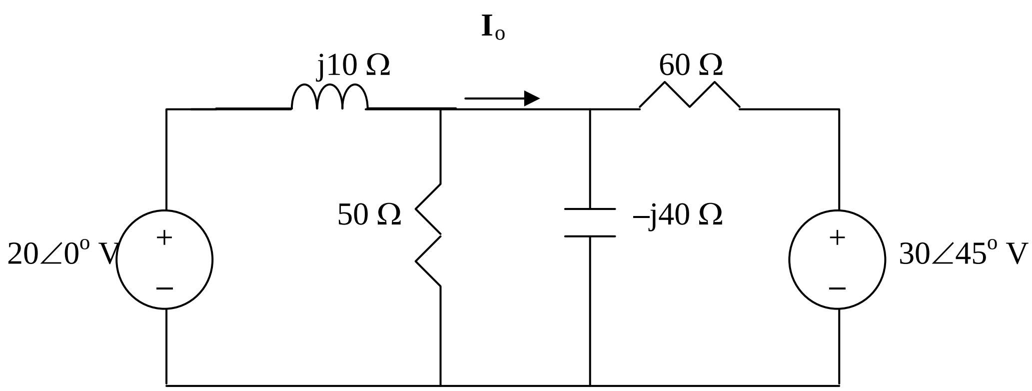
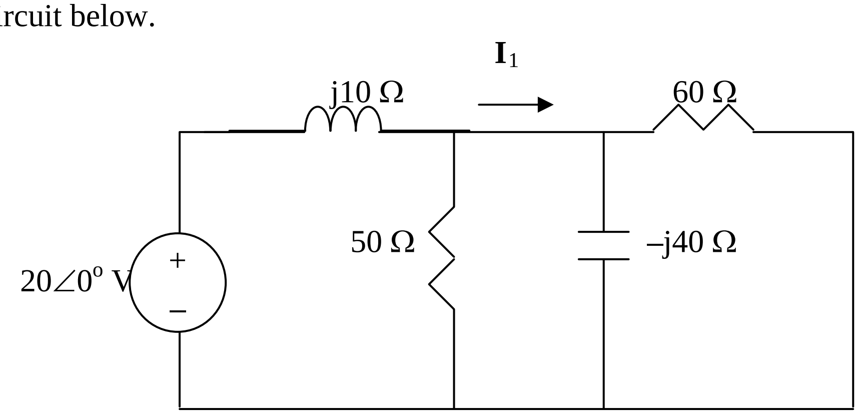


Figure 10.87 For Prob. 10.42.

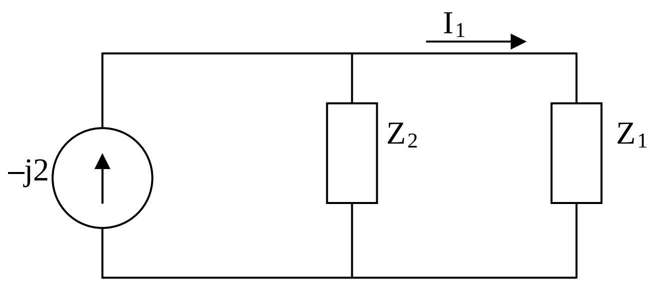
Solution

Let $I_o = I_1 + I_2$

where I_1 and I_2 are due to $20<0^o$ and $30<45^o$ sources respectively. To get I_1, we use the circuit below.



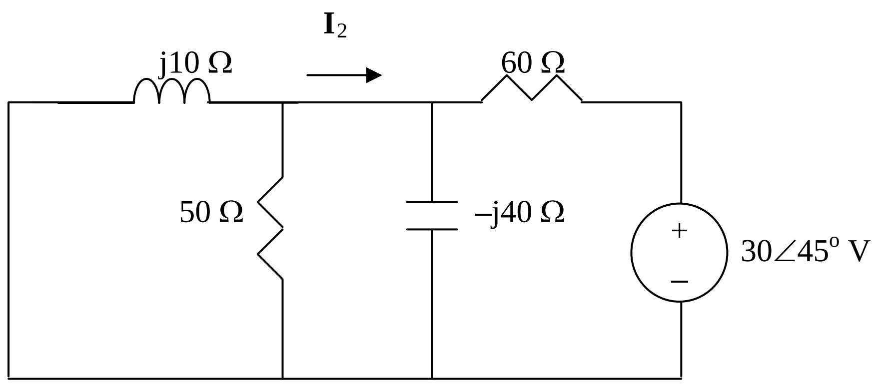
Let Z_1 = -j40//60 = 18.4615 –j27.6927, Z_2 = j10//50=1.9231 + j9.615
Transforming the voltage source to a current source leads to the circuit below.



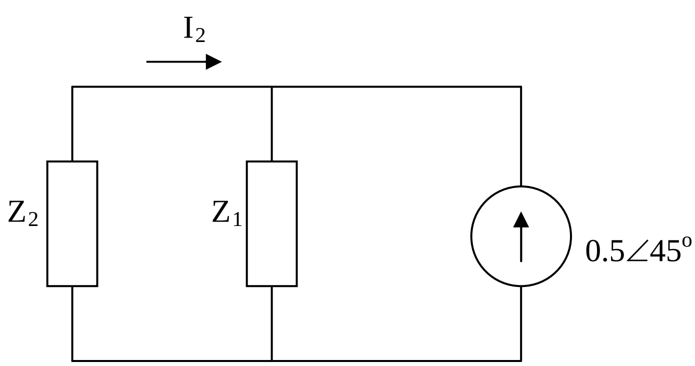
Using current division,

$$I_1 = \frac{Z_2}{Z_1 + Z_2}(-j2) = 0.6217 + j0.3626$$

To get I_2, we use the circuit below.



After transforming the voltage source, we obtain the circuit below.



Using current division,

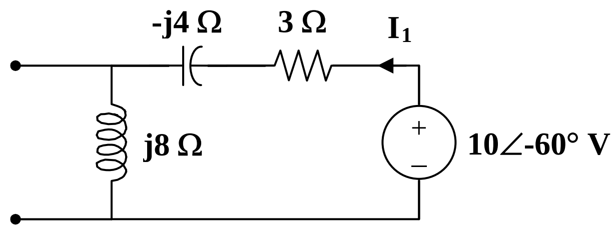
$$I_2 = \frac{-Z_1}{Z_1 + Z_2}(0.5 < 45^o) = -0.5275 - j0.3077$$

Hence, $\mathbf{I_o} = \mathbf{I_1} + \mathbf{I_2} =$ 0.0942+j0.0509 = $\mathbf{109\angle 30^\circ}$ **mA**.

Chapter 10, Solution 43. Let $\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2$, where $\mathbf{I}_1$ is due to the voltage source and $\mathbf{I}_2$ is due to the current source.

$$\omega = 2$$
$$5\cos(2t + 10°) \longrightarrow 5\angle 10°$$
$$10\cos(2t - 60°) \longrightarrow 10\angle -60°$$
$$4 \text{ H} \longrightarrow j\omega L = j8$$
$$\frac{1}{8}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/8)} = -j4$$

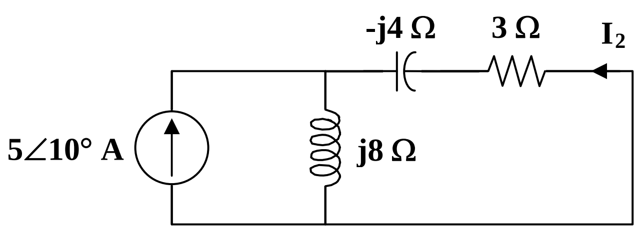
For $\mathbf{I}_1$, consider the circuit in Fig. (a).



(a)

$$\mathbf{I}_1 = \frac{10\angle -60°}{3 + j8 - j4} = \frac{10\angle -60°}{3 + j4}$$

For $\mathbf{I}_2$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = \frac{-j8}{3 + j8 - j4}(5\angle 10°) = \frac{-j40\angle 10°}{3 + j4}$$

$$\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2 = \frac{1}{3 + j4}(10\angle -60° - j40\angle 10°)$$
$$\mathbf{I}_x = \frac{49.51\angle -76.04°}{5\angle 53.13°} = 9.902\angle -129.17°$$

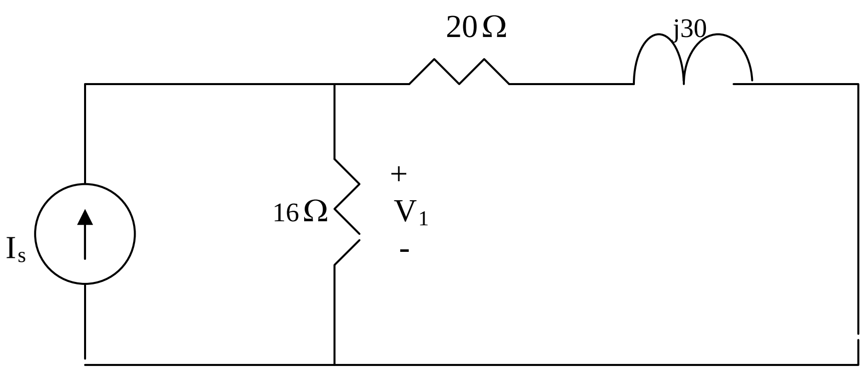
Therefore, $i_x =$ **9.902 cos(2t – 129.17°) A**

Chapter 10, Solution 44.

Let $v_x = v_1 + v_2$, where v_1 and v_2 are due to the current source and voltage source respectively.

For v_1, $\omega = 6$, $5\,H \longrightarrow j\omega L = j30$

The frequency-domain circuit is shown below.

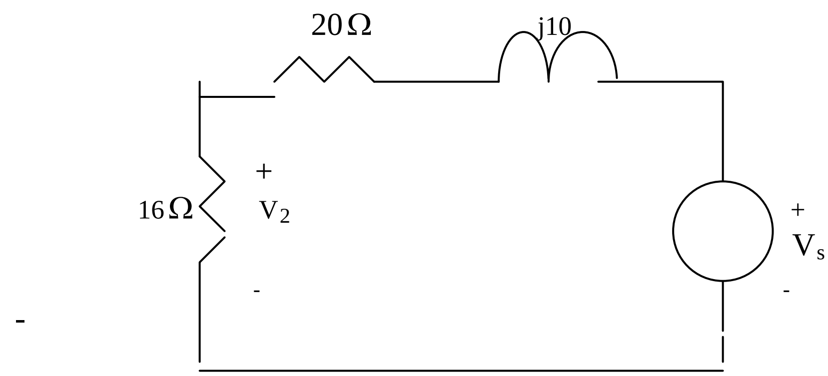


Let $Z = 16//(20 + j30) = \dfrac{16(20 + j30)}{36 + j30} = 11.8 + j3.497 = 12.31\angle 16.5^o$

$V_1 = I_s Z = (12\angle 10^o)(12.31\angle 16.5^o) = 147.7\angle 26.5^o \longrightarrow v_1 = 147.7\cos(6t + 26.5^o)$ V

For v_2, $\omega = 2$, $5\,H \longrightarrow j\omega L = j10$

The frequency-domain circuit is shown below.



Using voltage division,

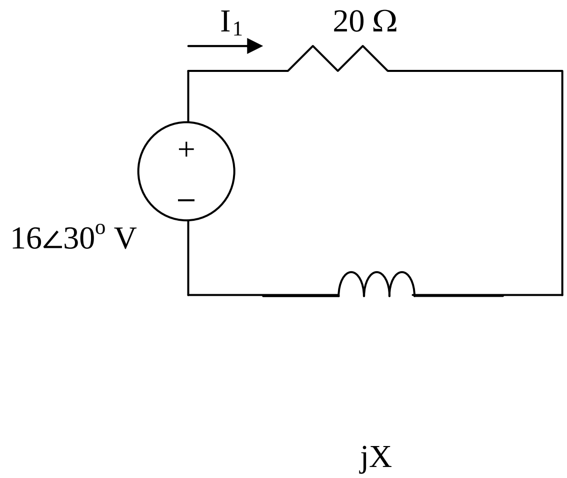
$$V_2 = \frac{16}{16+20+j10} V_s = \frac{16(50\angle 0^o)}{36+j10} = 21.41\angle -15.52^o \quad \longrightarrow \quad v_2 = 21.41\sin(2t - 15.52^o)\ V$$

Thus,

$$v_x = \mathbf{[147.7cos(6t+26.5°)+21.41sin(2t–15.52°)]\ V}$$

Chapter 10, Solution 45.

Let $i = i_1 + i_2$, where i_1 and i_2 are due to $16\cos(10t + 30^o)$ and $6\sin 4t$ sources respectively. To find i_1, consider the circuit below.



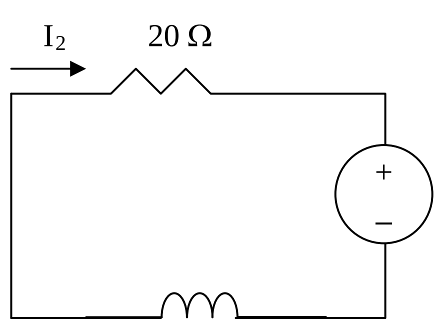
$$X = \omega L = 10x300x10^{-3} = 3$$

Type equation here.

$$\mathbf{I}_1 = \frac{16\angle 30^\circ}{20 + j3} = \frac{16\angle 30^\circ}{20.22\angle 8.53^\circ} = 0.7913\angle 21.47^\circ$$

$i_1(t) = 791.1\cos(10t + 21.47^\circ)$ mA.

To find $i_2(t)$, consider the circuit below,



$6\angle 0^o$ V

jX

$X = \omega L = 4x300x10^{-3} = 1.2$

$$I_2 = -\frac{6\angle 0^\circ}{20 + j1.2} = \frac{6\angle 180^\circ}{20.036\angle 3.43^\circ}$$ = $0.2995\angle 176.57°$ or

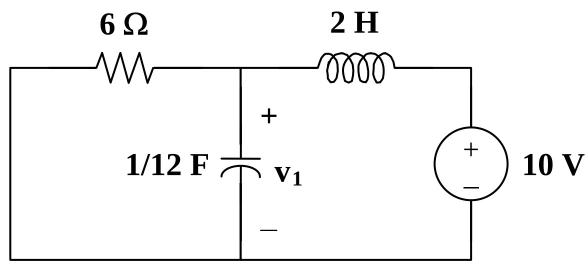
$i_2(t)$ = 299.5sin(4t+176.57°) mA.

Thus,

$i(t) = i_1(t) + i_2(t)$ = **[791.1cos(10t+21.47°) + 299.5sin(4t+176.57°)] mA**.

Chapter 10, Solution 46.

Let $v_o = v_1 + v_2 + v_3$, where v_1, v_2, and v_3 are respectively due to the 10-V dc source, the ac current source, and the ac voltage source. For v_1 consider the circuit in Fig. (a).



(a)

The capacitor is open to dc, while the inductor is a short circuit. Hence,

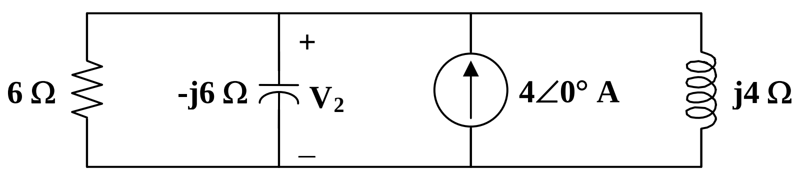
$$v_1 = 10 \text{ V}$$

For v_2, consider the circuit in Fig. (b).

$$\omega = 2$$

$$2 \text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12} \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/12)} = \text{-}j6$$



(b)

Applying nodal analysis,

$$4 = \frac{\mathbf{V}_2}{6} + \frac{\mathbf{V}_2}{\text{-}j6} + \frac{\mathbf{V}_2}{j4} = \left(\frac{1}{6} + \frac{j}{6} - \frac{j}{4}\right)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{24}{1 - j0.5} = 21.45\angle 26.56°$$

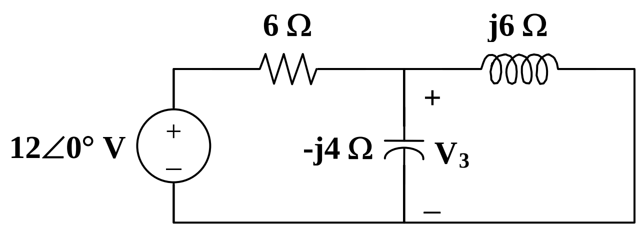
Hence, $$v_2 = 21.45\sin(2t + 26.56°) \text{ V}$$

For v_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\text{ H} \longrightarrow j\omega L = j6$$

$$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/12)} = -j4$$



(c)

At the non-reference node,

$$\frac{12 - \mathbf{V}_3}{6} = \frac{\mathbf{V}_3}{-j4} + \frac{\mathbf{V}_3}{j6}$$

$$\mathbf{V}_3 = \frac{12}{1 + j0.5} = 10.73\angle -26.56°$$

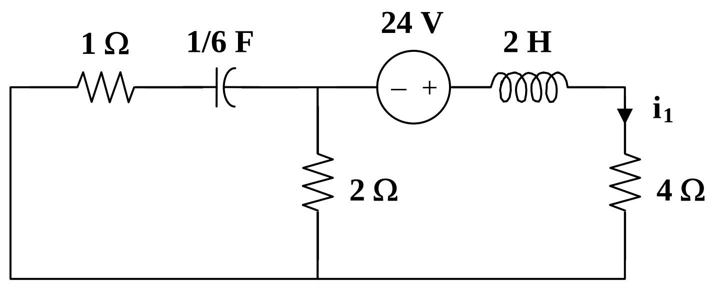
Hence, $v_3 = 10.73\cos(3t - 26.56°)\text{ V}$

Therefore,

$$v_o = \mathbf{[10+21.45\sin(2t+26.56°)+10.73\cos(3t–26.56°)]\ V}$$

Chapter 10, Solution 47.

Let $i_o = i_1 + i_2 + i_3$, where i_1, i_2, and i_3 are respectively due to the 24-V dc source, the ac voltage source, and the ac current source. For i_1, consider the circuit in Fig. (a).



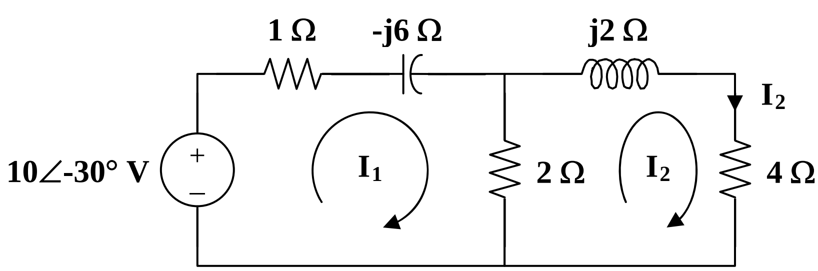
(a)

Since the capacitor is an open circuit to dc,

$$i_1 = \frac{24}{4+2} = 4 \text{ A}$$

For i_2, consider the circuit in Fig. (b).

$$\omega = 1$$
$$2 \text{ H} \longrightarrow j\omega L = j2$$
$$\frac{1}{6} \text{ F} \longrightarrow \frac{1}{j\omega C} = -j6$$



(b)

For mesh 1,

$$-10\angle -30° + (3 - j6)\mathbf{I}_1 - 2\mathbf{I}_2 = 0$$
$$10\angle -30° = 3(1 - 2j)\mathbf{I}_1 - 2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$0 = -2\mathbf{I}_1 + (6 + j2)\mathbf{I}_2$$
$$\mathbf{I}_1 = (3 + j)\mathbf{I}_2 \qquad (2)$$

Substituting (2) into (1)

$$10\angle -30° = 13 - j15\,\mathbf{I}_2$$

$$\mathbf{I}_2 = 0.504\angle 19.1°$$

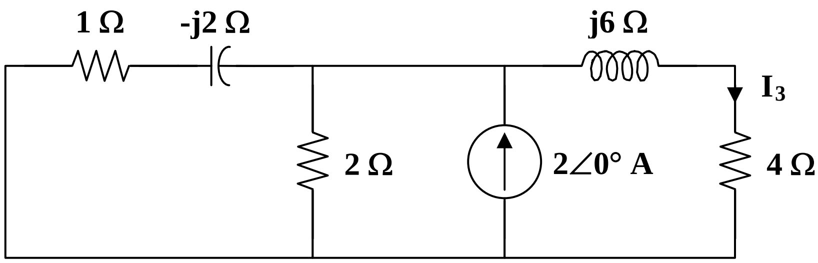
Hence, $$i_2 = 0.504\sin(t + 19.1°)\text{ A}$$

For i_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\text{ H} \longrightarrow j\omega L = j6$$

$$\frac{1}{6}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = -j2$$



(c)

$$2 \parallel (1 - j2) = \frac{2(1 - j2)}{3 - j2}$$

Using current division,

$$\mathbf{I}_3 = \frac{\dfrac{2(1 - j2)}{3 - j2}\cdot(2\angle 0°)}{4 + j6 + \dfrac{2(1 - j2)}{3 - j2}} = \frac{2(1 - j2)}{13 + j3}$$

$$\mathbf{I}_3 = 0.3352\angle -76.43°$$

Hence $$i_3 = 0.3352\cos(3t - 76.43°)\text{ A}$$

Therefore, $i_o =$ **[4 + 0.504 sin(t + 19.1°) + 0.3352 cos(3t – 76.43°)] A**

Chapter 10, Solution 48.

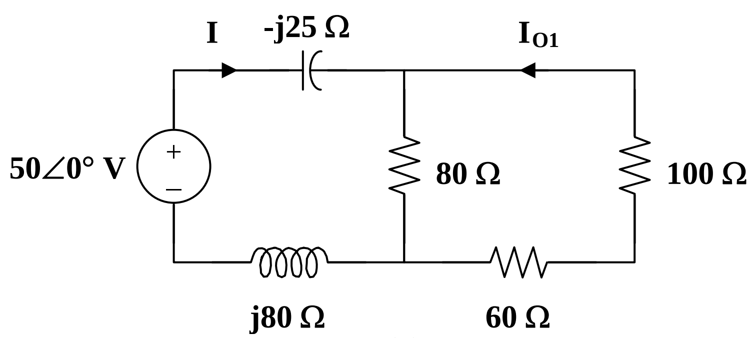
Let $i_O = i_{O1} + i_{O2} + i_{O3}$, where i_{O1} is due to the ac voltage source, i_{O2} is due to the dc voltage source, and i_{O3} is due to the ac current source. For i_{O1}, consider the circuit in Fig. (a).

$$\omega = 2000$$

$$50\cos(2000t) \longrightarrow 50\angle 0°$$

$$40 \text{ mH} \longrightarrow j\omega L = j(2000)(40\times 10^{-3}) = j80$$

$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2000)(20\times 10^{-6})} = -j25$$



(a)

$$80 \,||\, (60+100) = 160/3$$

$$\mathbf{I} = \frac{50}{160/3 + j80 - j25} = \frac{30}{32 + j33}$$

Using current division,

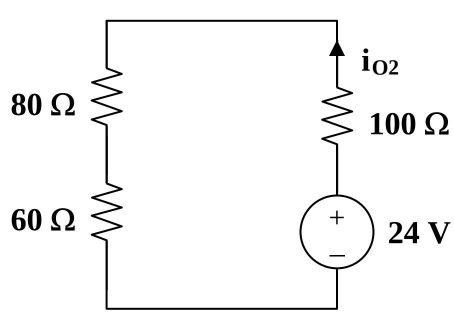
$$\mathbf{I}_{O1} = \frac{-80\,\mathbf{I}}{80+160} = \frac{-1}{3}\mathbf{I} = \frac{10\angle 180°}{46\angle 45.9°}$$

$$\mathbf{I}_{O1} = 0.217\angle 134.1°$$

Hence,

$$i_{O1} = 0.217\cos(2000t + 134.1°) \text{ A}$$

For i_{O2}, consider the circuit in Fig. (b).



(b)

$$i_{O2} = \frac{24}{80+60+100} = 0.1\text{ A}$$

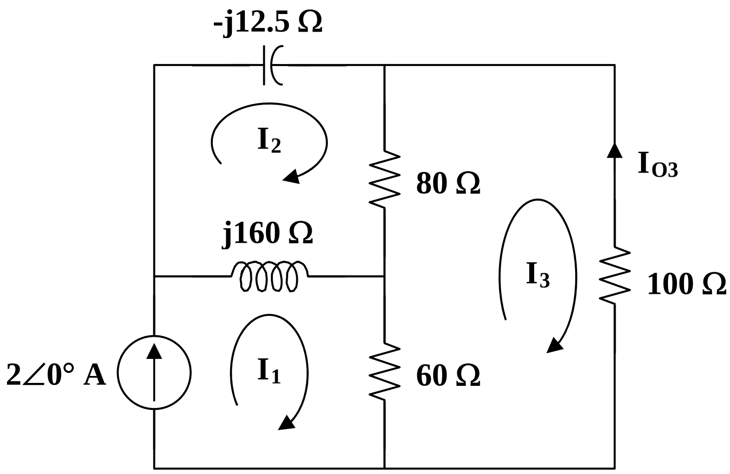
For i_{O3}, consider the circuit in Fig. (c).

$$\omega = 4000$$

$$2\cos(4000t) \longrightarrow 2\angle 0°$$

$$40\text{ mH} \longrightarrow j\omega L = j(4000)(40\times 10^{-3}) = j160$$

$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4000)(20\times 10^{-6})} = -j12.5$$



(c)

`

For mesh 1,

$$\mathbf{I}_1 = 2 \quad (1)$$

For mesh 2,

$$(80 + j160 - j12.5)\mathbf{I}_2 - j160\mathbf{I}_1 - 80\mathbf{I}_3 = 0$$

Simplifying and substituting (1) into this equation yields

$$(8 + j14.75)\mathbf{I}_2 - 8\mathbf{I}_3 = j32 \quad (2)$$

For mesh 3,

$$240\mathbf{I}_3 - 60\mathbf{I}_1 - 80\mathbf{I}_2 = 0$$

Simplifying and substituting (1) into this equation yields

$$\mathbf{I}_2 = 3\mathbf{I}_3 - 1.5 \quad (3)$$

Substituting (3) into (2) yields

$$(16 + j44.25)\mathbf{I}_3 = 12 + j54.125$$

$$\mathbf{I}_3 = \frac{12 + j54.125}{16 + j44.25} = 1.1782\angle 7.38°$$

$$\mathbf{I}_{O3} = -\mathbf{I}_3 = -1.1782\angle 7.38°$$

Hence, $i_{O3} = -1.1782\sin(4000t + 7.38°)$ A

Therefore,

$i_O =$ **{0.1 + 0.217 cos(2000t + 134.1°) – 1.1782 sin(4000t + 7.38°)} A**

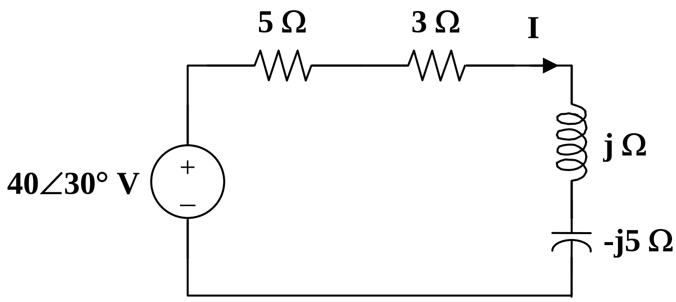
Chapter 10, Solution 49.

$$8\sin(200t + 30°) \longrightarrow 8\angle 30°, \quad \omega = 200$$

$$5 \text{ mH} \longrightarrow j\omega L = j(200)(5\times10^{-3}) = j$$

$$1 \text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(1\times10^{-3})} = -j5$$

After transforming the current source, the circuit becomes that shown in the figure below.



$$\mathbf{I} = \frac{40\angle 30°}{5 + 3 + j - j5} = \frac{40\angle 30°}{8 - j4} = 4.472\angle 56.56°$$

$$i = \mathbf{[4.472\sin(200t+56.56°)]\ A}$$

Chapter 10, Solution 50.

Using Fig. 10.95, design a problem to help other students to better understand source transformation.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use source transformation to find v_o in the circuit in Fig. 10.95.

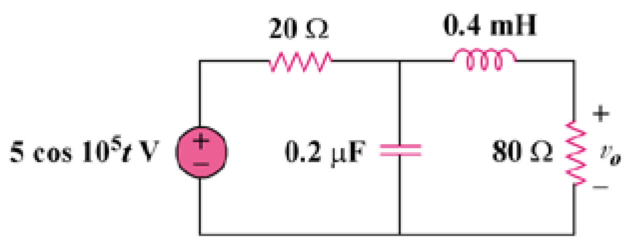
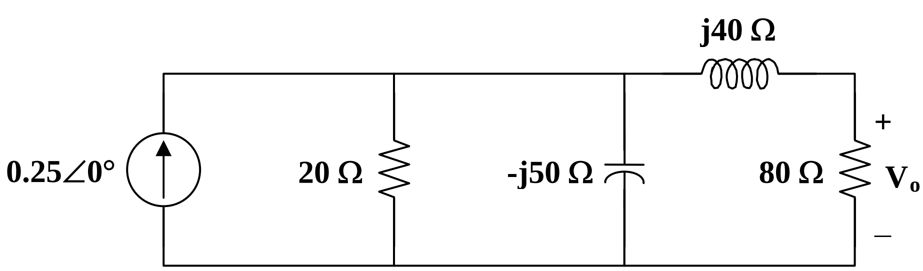


Figure 10.95

Solution

$$5\cos(10^5 t) \longrightarrow 5\angle 0°, \quad \omega = 10^5$$
$$0.4 \text{ mH} \longrightarrow j\omega L = j(10^5)(0.4\times 10^{-3}) = j40$$
$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^5)(0.2\times 10^{-6})} = -j50$$

After transforming the voltage source, we get the circuit in Fig. (a).

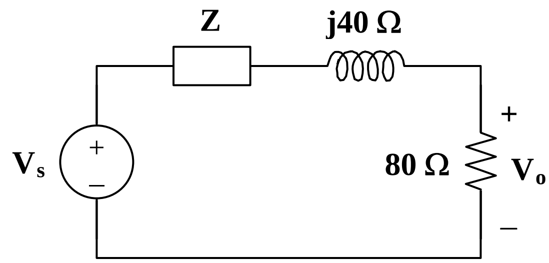


(a)

Let $\quad \mathbf{Z} = 20 \,||\, -j50 = \dfrac{-j100}{2 - j5}$

and $\quad \mathbf{V}_s = (0.25\angle 0°)\,\mathbf{Z} = \dfrac{-j25}{2 - j5}$

With these, the current source is transformed to obtain the circuit in Fig.(b).



(b)

By voltage division,

$$\mathbf{V_o} = \frac{80}{\mathbf{Z} + 80 + j40}\mathbf{V_s} = \frac{80}{\frac{-j100}{2-j5} + 80 + j40} \cdot \frac{-j25}{2-j5}$$

$$\mathbf{V_o} = \frac{8(-j25)}{36 - j42} = 3.615\angle -40.6°$$

Therefore, $v_o =$ **$3.615 \cos(10^5 t - 40.6°)$ V**

Chapter 10, Solution 51.

There are many ways to create this problem, here is one possible solution. Let V_1 =

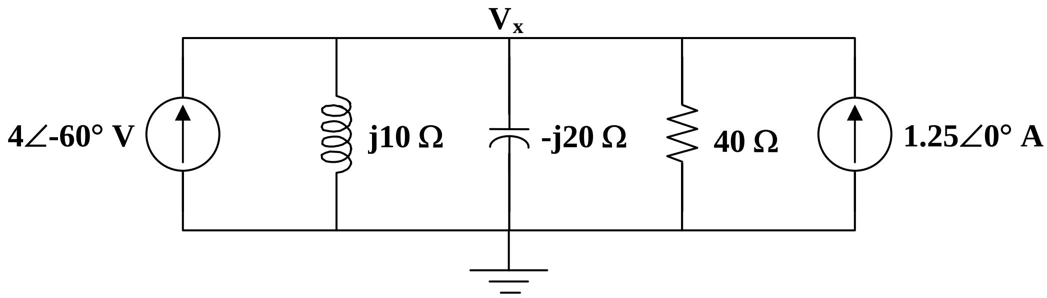
$40\angle30°$ V, X_L = 10 Ω, X_C = 20 Ω, $R_1 = R_2$ = 80 Ω, and V_2 = 50 V.

If we let the voltage across the capacitor be equal to V_x, then

$\mathbf{I_o} = [\mathbf{V_x}/(-j20)] + [(\mathbf{V_x}-50)/80] = (0.0125+j0.05)\mathbf{V_x} - 0.625 = (0.051539\angle75.96°)\mathbf{V_x} -$

0.625.

The following circuit is obtained by transforming the voltage sources.



$\mathbf{V_x} = (4\angle\text{-}60°+1.25)/(-j0.1+j0.05+0.025) = (2-j3.4641+1.25)/(0.025-j0.05)$

$= (3.25-j3.4641)/(0.025-j0.05) = (4.75\angle-46.826°)/(0.055902\angle-63.435°)$

$= 84.97\angle16.609°$ V.

Therefore,

$\mathbf{I_o} = (0.051539\angle 75.96°)(84.97\angle 16.609°) – 0.625 = 4.3793\angle 92.569° – 0.625$

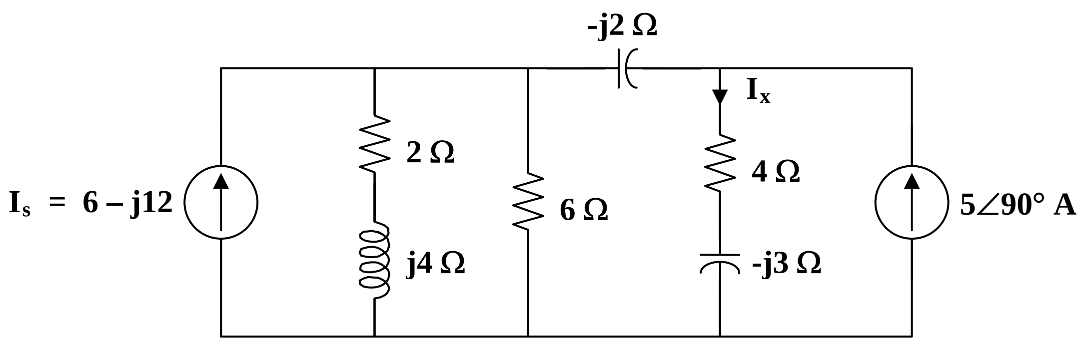
$= –0.196291+j4.3749 – 0.625 = –0.821291+j4.3749 =$ $\mathbf{4.451\angle 100.63°\ A}$.

Chapter 10, Solution 52.

We transform the voltage source to a current source.

$$\mathbf{I}_s = \frac{60\angle 0°}{2 + j4} = 6 - j12$$

The new circuit is shown in Fig. (a).

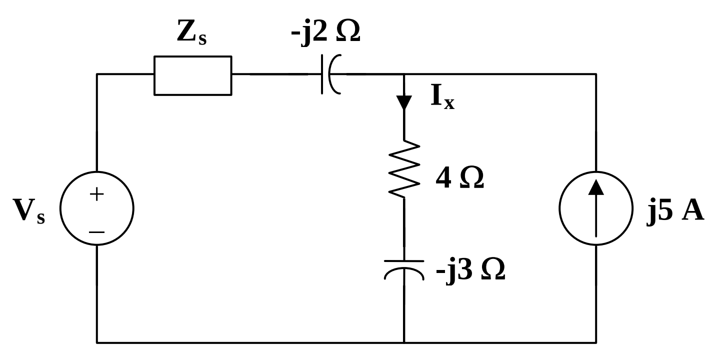


(a)

Let $\qquad \mathbf{Z}_s = 6 \,||\, (2 + j4) = \dfrac{6(2 + j4)}{8 + j4} = 2.4 + j1.8$

$$\mathbf{V}_s = \mathbf{I}_s\, \mathbf{Z}_s = (6 - j12)(2.4 + j1.8) = 36 - j18 = 18(2 - j)$$

With these, we transform the current source on the left hand side of the circuit to a voltage source. We obtain the circuit in Fig. (b).

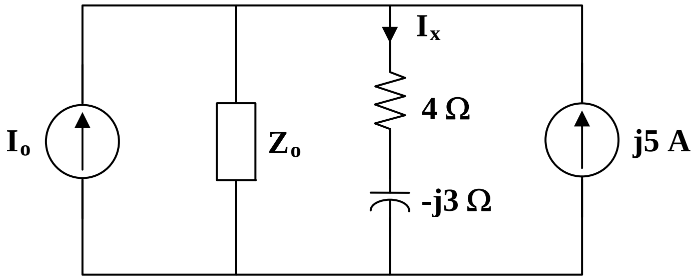


(b)

Let $\qquad \mathbf{Z}_o = \mathbf{Z}_s - j2 = 2.4 - j0.2 = 0.2(12 - j)$

$$\mathbf{I}_o = \frac{\mathbf{V}_s}{\mathbf{Z}_o} = \frac{18(2 - j)}{0.2(12 - j)} = 15.517 - j6.207$$

With these, we transform the voltage source in Fig. (b) to a current source. We obtain the circuit in Fig. (c).



(c)

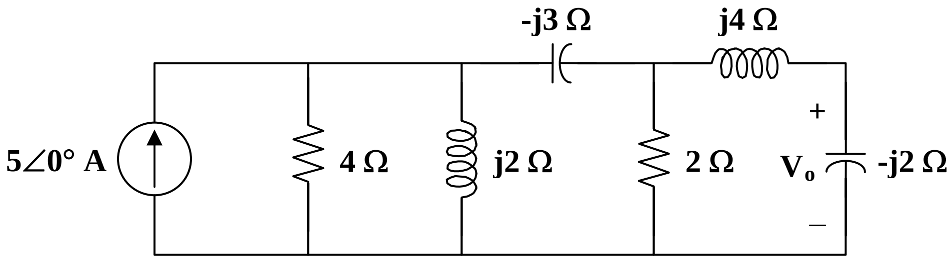
Using current division,

$$\mathbf{I}_x = \frac{\mathbf{Z}_o}{\mathbf{Z}_o + 4 - j3}(\mathbf{I}_o + j5) = \frac{2.4 - j0.2}{6.4 - j3.2}(15.517 - j1.207)$$

$$\mathbf{I}_x = 5 + j1.5625 = \mathbf{5.238\angle 17.35° \; A}$$

Chapter 10, Solution 53.

We transform the voltage source to a current source to obtain the circuit in Fig. (a).

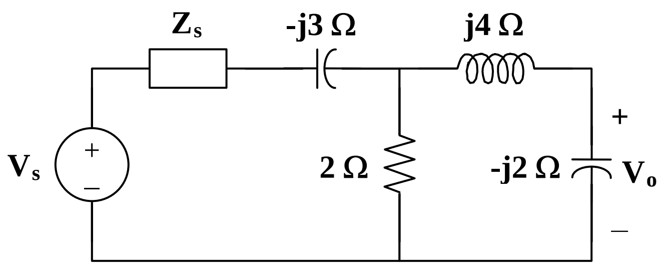


(a)

Let $$\mathbf{Z}_s = 4 \parallel j2 = \frac{j8}{4 + j2} = 0.8 + j1.6$$

$$\mathbf{V}_s = (5\angle 0°)\,\mathbf{Z}_s = (5)(0.8 + j1.6) = 4 + j8$$

With these, the current source is transformed so that the circuit becomes that shown in Fig. (b).

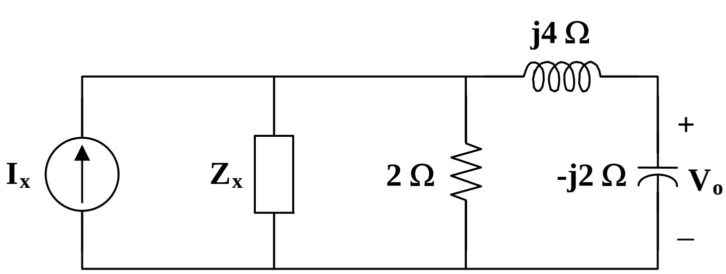


(b)

Let $$\mathbf{Z}_x = \mathbf{Z}_s - j3 = 0.8 - j1.4$$

$$\mathbf{I}_x = \frac{\mathbf{V}_s}{\mathbf{Z}_s} = \frac{4 + j8}{0.8 - j1.4} = -3.0769 + j4.6154$$

With these, we transform the voltage source in Fig. (b) to obtain the circuit in Fig. (c).

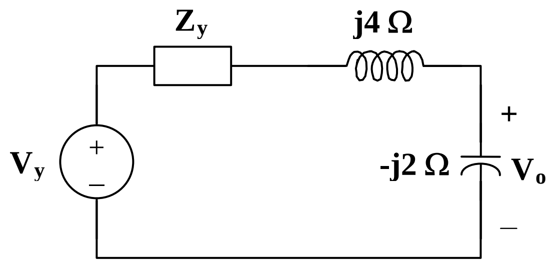


(c)

Let $$\mathbf{Z}_y = 2 \parallel \mathbf{Z}_x = \frac{1.6 - j2.8}{2.8 - j1.4} = 0.8571 - j0.5714$$

$$\mathbf{V}_y = \mathbf{I}_x \mathbf{Z}_y = (-3.0769 + j4.6154) \cdot (0.8571 - j0.5714) = j5.7143$$

With these, we transform the current source to obtain the circuit in Fig. (d).



(d)

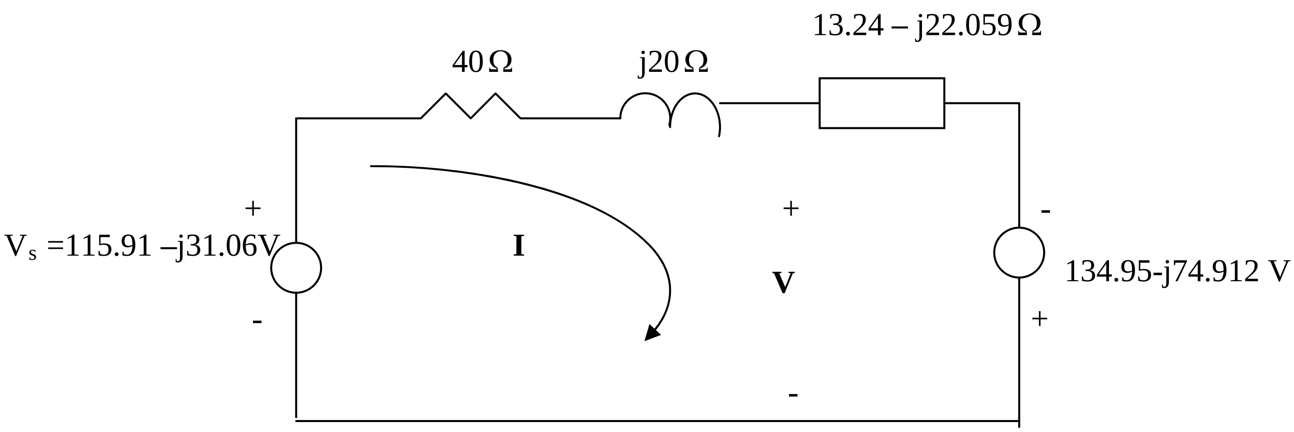
Using current division,

$$\mathbf{V}_o = \frac{-j2}{\mathbf{Z}_y + j4 - j2}\mathbf{V}_y = \frac{-j2(j5.7143)}{0.8571 - j0.5714 + j4 - j2} = \mathbf{(3.529 - j5.883)\ V}$$

Chapter 10, Solution 54.

$$50\,//(-j30) = \frac{50x(-j30)}{50-j30} = 13.24 - j22.059$$

We convert the current source to voltage source and obtain the circuit below.



Applying KVL gives

-115.91 + j31.058 + (53.24-j2.059)I -134.95 + j74.912 = 0

or $\quad I = \dfrac{-250.86 + j105.97}{53.24 - j2.059} = -4.7817 + j1.8055$

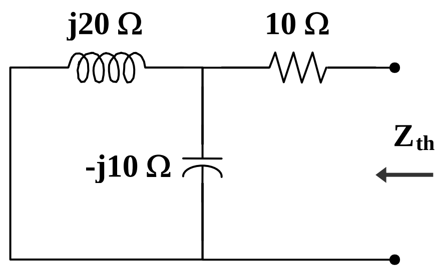
But $\; -V_s + (40 + j20)I + V = 0 \quad \longrightarrow \quad V = V_s - (40 + j20)I$

$$\boldsymbol{V = 115.91 - j31.05 - (40 + j20)(-4.7817 + j1.8055) = \underline{124.06\angle -154^o\ \mathrm{V}}}$$

which agrees with the result in Prob. 10.7.

Chapter 10, Solution 55.

(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).

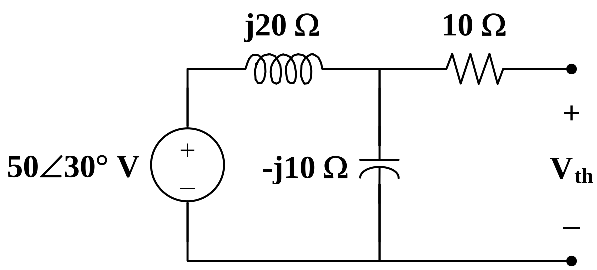


(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 10 + j20 \,||\, (-j10) = 10 + \frac{(j20)(-j10)}{j20 - j10}$$

$$= 10 - j20 = \mathbf{22.36\angle{-63.43}°\ \Omega}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).

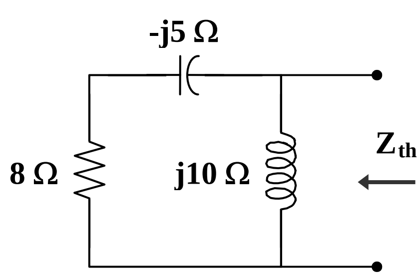


(b)

$$\mathbf{V}_{th} = \frac{-j10}{j20 - j10}(50\angle 30°) = \mathbf{-50\angle 30°\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{-50\angle 30°}{22.36\angle{-63.43}°} = \mathbf{2.236\angle 273.4°\ A}$$

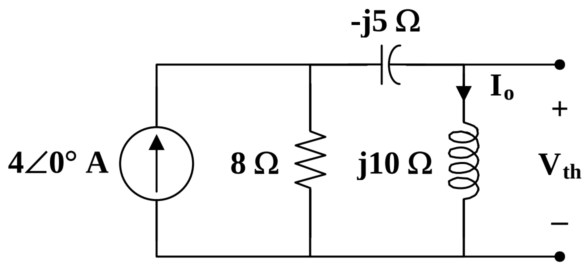
(b) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (c).



(c)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = j10 \,||\, (8 - j5) = \frac{(j10)(8 - j5)}{j10 + 8 - j5} = \mathbf{10\angle 26^\circ\ \Omega}$$

To obtain $\mathbf{V}_{th}$, consider the circuit in Fig. (d).



(d)

By current division,

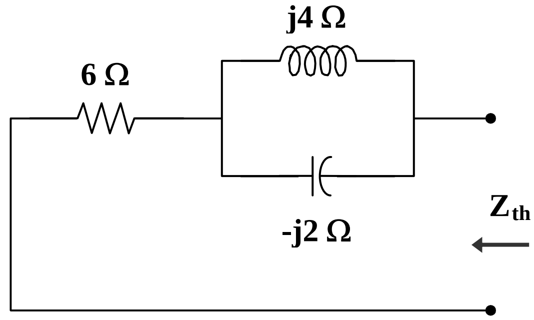
$$\mathbf{I}_o = \frac{8}{8 + j10 - j5}(4\angle 0^\circ) = \frac{32}{8 + j5}$$

$$\mathbf{V}_{th} = j10\,\mathbf{I}_o = \frac{j320}{8 + j5} = \mathbf{33.92\angle 58^\circ\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{33.92\angle 58^\circ}{10\angle 26^\circ} = \mathbf{3.392\angle 32^\circ\ A}$$

Chapter 10, Solution 56.

(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

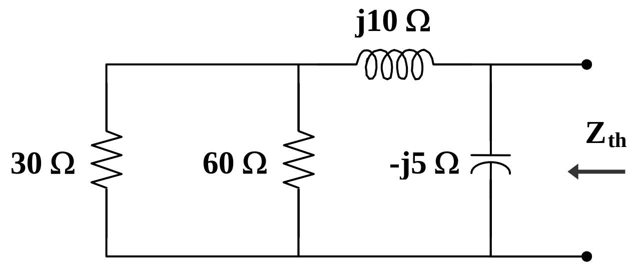
$$\mathbf{Z}_N = \mathbf{Z}_{th} = 6 + j4 \,||\, (\text{-}j2) = 6 + \frac{(j4)(\text{-}j2)}{j4 - j2} = 6 - j4$$

$$= \mathbf{7.211\angle\text{-}33.69^\circ\ \Omega}$$

By placing short circuit at terminals a-b, we obtain,
$\mathbf{I}_N = \mathbf{2\angle 0^\circ\ A}$

$$\mathbf{V}_{th} = \mathbf{Z}_{th}\,\mathbf{I}_{th} = (7.211\angle\text{-}33.69^\circ)(2\angle 0^\circ) = \mathbf{14.422\angle\text{-}33.69^\circ\ V}$$

(b) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (b).



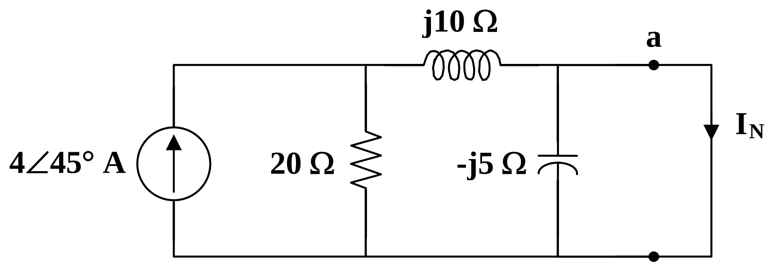
(b)

$$30 \,||\, 60 = 20$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \text{-}j5 \,||\, (20 + j10) = \frac{(\text{-}j5)(20 + j10)}{20 + j5}$$

$$= \mathbf{5.423\angle\text{-}77.47^\circ\ \Omega}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, we transform the voltage source and combine the 30 Ω and 60 Ω resistors. The result is shown in Fig. (c).



(c)

$$\mathbf{I}_N = \frac{20}{20 + j10}(4\angle 45°) = \frac{2}{5}(2 - j)(4\angle 45°)$$
$$= \mathbf{3.578\angle 18.43°\ A}$$

$$\mathbf{V}_{th} = \mathbf{Z}_{th}\,\mathbf{I}_N = (5.423\angle -77.47°)(3.578\angle 18.43°)$$
$$= \mathbf{19.4\angle -59°\ V}$$

Chapter 10, Solution 57.

Using Fig. 10.100, design a problem to help other students to better understand Thevenin and Norton equivalent circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Thevenin and Norton equivalent circuits for the circuit shown in Fig. 10.100.

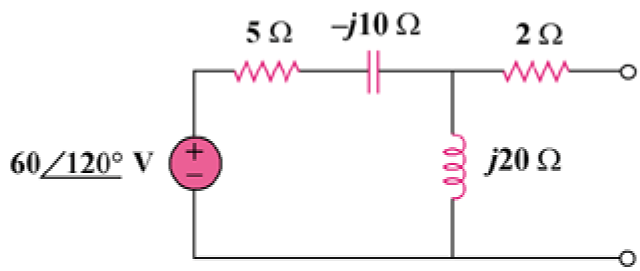
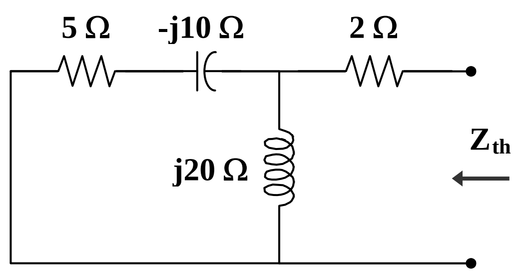


Figure 10.100

Solution

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).

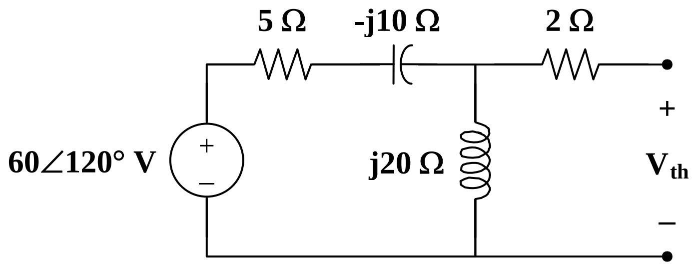


(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 2 + j20 \,||\, (5 - j10) = 2 + \frac{(j20)(5 - j10)}{5 + j10}$$

$$= 18 - j12 = \mathbf{21.633\angle\text{-}33.7^\circ \ \Omega}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



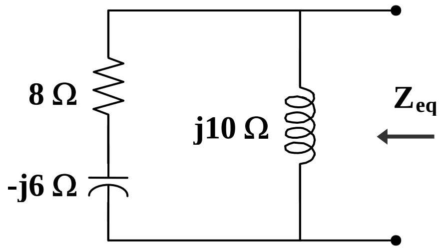
(b)

$$\mathbf{V}_{th} = \frac{j20}{5 - j10 + j20}(60\angle 120°) = \frac{j4}{1 + j2}(60\angle 120°)$$
$$= \mathbf{107.3\angle 146.56°\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{107.3\angle 146.56°}{21.633\angle -33.7°} = \mathbf{4.961\angle\text{-}179.7°\ A}$$

Chapter 10, Solution 58.

Consider the circuit in Fig. (a) to find $\mathbf{Z}_{eq}$.

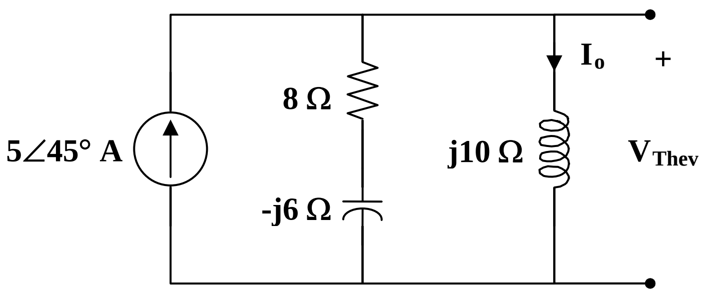


(a)

$$\mathbf{Z}_{eq} = j10 \,\|\, (8 - j6) = \frac{(j10)(8 - j6)}{8 + j4} = 5(2 + j)$$

$$= \mathbf{11.18\angle 26.56^\circ\ \Omega}$$

Consider the circuit in Fig. (b) to find V_{Thev}.



(b)

$$\mathbf{I}_o = \frac{8 - j6}{8 - j6 + j10}(5\angle 45^\circ) = \frac{4 - j3}{4 + j2}(5\angle 45^\circ)$$

$$\mathbf{V}_{Thev} = j10\,\mathbf{I}_o = \frac{(j10)(4 - j3)(5\angle 45^\circ)}{(2)(2 + j)}$$

$$= \mathbf{55.9\angle 71.56^\circ\ V}$$

Chapter 10, Solution 59.

Calculate the output impedance of the circuit shown in Fig. 10.102.

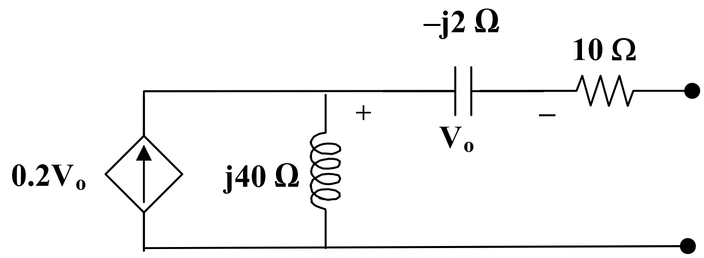
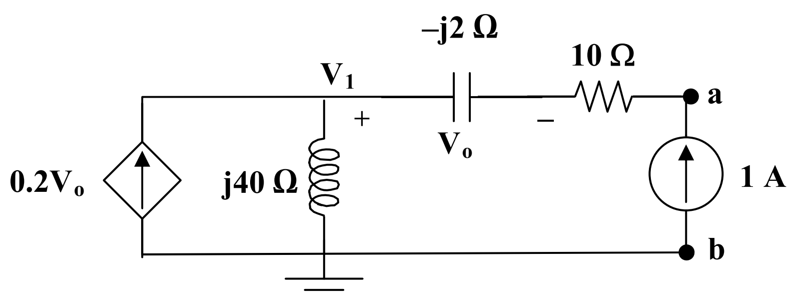


Figure 10.102
For Prob. 10.59.

Solution

Since there are no independent sources, we need to inject a current, best value is to make it 1 amp, into the terminals on the right and then to determine the voltage at the terminals.



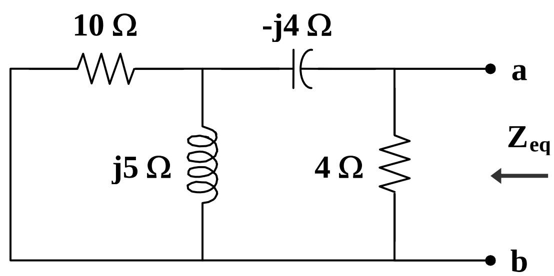
Clearly $\mathbf{V_o} = -(-j2) = j2$ and $\mathbf{V_1} = (0.2V_o + 1)j40 = (1+j0.4)j40 = -16+j40$ V.

Next, $\mathbf{V_{ab}} = 10 - j2 - 16 + j40 = -6+j38 = 38.47\angle 98.97°$ V or

$$\mathbf{Z_{eq} = (-6+j38)\ \Omega}.$$

Chapter 10, Solution 60.

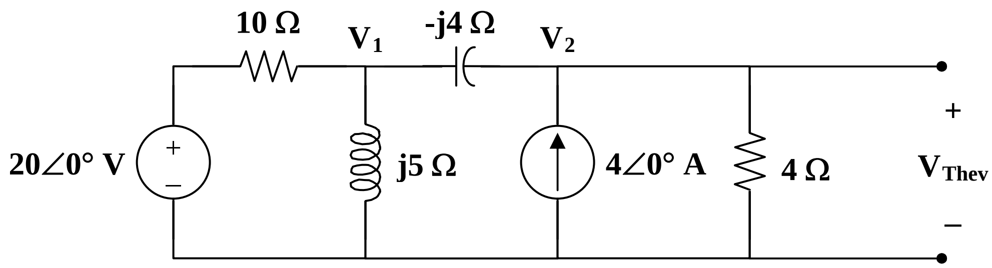
(a) To find $\mathbf{Z}_{eq}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_{eq} = 4 \,\|\, (\text{-}j4 + 10 \,\|\, j5) = 4 \,\|\, (\text{-}j4 + 2 + j4)$$
$$\mathbf{Z}_{eq} = 4 \,\|\, 2$$
$$= \mathbf{1.333\ \Omega}$$

To find $\mathbf{V}_{Thev}$, consider the circuit in Fig. (b).



(b)

At node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{\text{j}5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\text{- j}4}$$

$$(1 + \text{j}0.5)\,\mathbf{V}_1 - \text{j}2.5\,\mathbf{V}_2 = 20 \qquad (1)$$

At node 2,

$$4 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\text{- j}4} = \frac{\mathbf{V}_2}{4}$$

$$\mathbf{V}_1 = (1 - \text{j})\,\mathbf{V}_2 + \text{j}16 \qquad (2)$$

Substituting (2) into (1) leads to

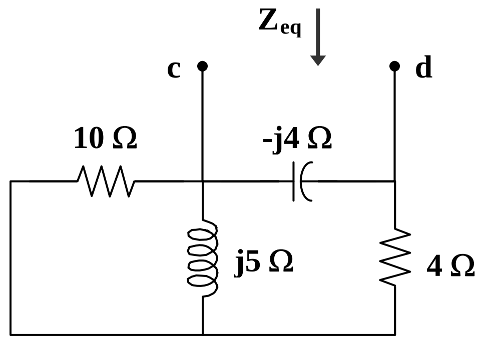
$$28 - \text{j}16 = (1.5 - \text{j}3)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{28 - \text{j}16}{1.5 - \text{j}3} = 8 + \text{j}5.333$$

Therefore,

$$\mathbf{V}_{Thev} = \mathbf{V}_2 = \underline{\mathbf{9.615\angle 33.69° \ V}}$$

(b) To find $\mathbf{Z}_{eq}$, consider the circuit in Fig. (c).



(c)

$$\mathbf{Z}_{eq} = -j4 \,||\, (4 + 10 \,||\, j5) = -j4 \,||\left(4 + \frac{j10}{2+j}\right)$$

$$\mathbf{Z}_{eq} = -j4 \,||\, (6 + j4) = \frac{-j4}{6}(6 + j4) = \mathbf{(2.667 - j4)\ \Omega}$$

To find $\mathbf{V}_{Thev}$,we will make use of the result in part (a).

$\mathbf{V}_2 = 8 + \mathrm{j}5.333 = (8/3)(3 + \mathrm{j}2)$

$\mathbf{V}_1 = (1 - \mathrm{j})\mathbf{V}_2 + \mathrm{j}16 = \mathrm{j}16 + (8/3)(5 - \mathrm{j})$

$$\mathbf{V}_{Thev} = \mathbf{V}_1 - \mathbf{V}_2 = 16/3 + j8 = \mathbf{9.614\angle 56.31° \ V}$$

Chapter 10, Solution 61.

Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 10.104.

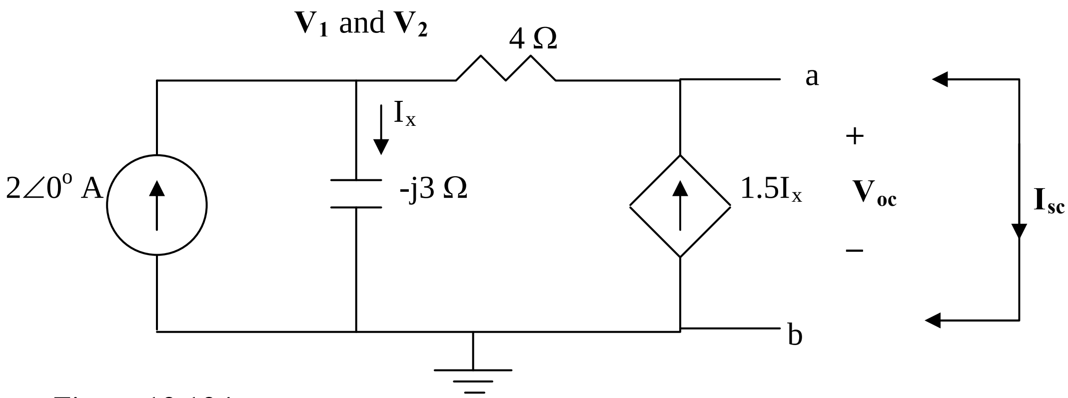


Figure 10.104
For Prob. 10.61.

Solution

Step 1. First we solve for the open circuit voltage using the above circuit and writing two node equations. Then we solve for the short circuit current which only need one node equation. For being able to solve for $\mathbf{V}_{oc}$, we need to solve these three equations,

$$-2 + [(\mathbf{V}_1-0)/(-j3)] + [(\mathbf{V}_1-\mathbf{V}_{oc})/4] = 0 \text{ and}$$

$$[(\mathbf{V}_{oc}-\mathbf{V}_1)/4] - 1.5\mathbf{I}_x = 0 \text{ where } \mathbf{I}_x = [(\mathbf{V}_1-0)/(-j3)].$$

To solve for $\mathbf{I}_{sc}$, all we need to do is to solve these three equations,

$$-2 + [(\mathbf{V}_2-0)/(-j3)] + [(\mathbf{V}_2-0)/4] = 0,\ \mathbf{I}_{sc} = [\mathbf{V}_2/4] + 1.5\mathbf{I}_x, \text{ and}$$

$$\mathbf{I}_x = [\mathbf{V}_2/-j3].$$

Finally, $\mathbf{V}_{Thev} = \mathbf{V}_{oc}$ and $\mathbf{Z}_{eq} = \mathbf{V}_{oc}/\mathbf{I}_{sc}$.

Step 2. Now all we need to do is to solve for the unknowns. For $\mathbf{V}_{oc}$,

$$\mathbf{I}_x = j0.33333\mathbf{V}_1 \text{ and } (0.25+(1.5)(j0.33333))\mathbf{V}_1 = 0.25\mathbf{V}_{oc} \text{ or}$$

$$(0.25+j0.5)\mathbf{V}_1 = (0.55902\angle 63.43°)\mathbf{V}_1 = 0.25\mathbf{V}_{oc} \text{ or}$$

$\mathbf{V_1} = (0.44721\angle{-63.43°})\mathbf{V_{oc}}$ which leads to,

$(0.25+j0.33333)\mathbf{V_1} - 0.25\mathbf{V_{oc}} = 2$

$= (0.41666\angle{+53.13°})(0.44721\angle{-63.43°})V_{oc} - 0.25\mathbf{V_{oc}}$

$= (0.186335\angle{-10.3°})\mathbf{V_{oc}} - 0.25\mathbf{V_{oc}} = (0.183333-0.25-j0.033333)\mathbf{V_{oc}}$

$= (-0.066667-j0.033333)\mathbf{V_{oc}} = (0.074536\angle{-153.435°})\mathbf{V_{oc}} = 2$ or

$$\mathbf{V_{oc}} = \mathbf{V_{Thev}} = \mathbf{26.83\angle 153.44° \ V = (-24+j12)\ V}.$$

Now for $\mathbf{I_{sc}}$,

$\mathbf{I_{sc}} = [\mathbf{V_2}/4] + 1.5\mathbf{I_x} = (0.25+(1.5)(j0.33333))\mathbf{V_2} = (0.25+j0.5)\mathbf{V_2}$.

$[(\mathbf{V_2}-0)/(-j3)] + [(\mathbf{V_2}-0)/4] = 2 = (0.25+j0.3333)\mathbf{V_2}$

$= (0.41667\angle 53.13°)\mathbf{V_2} = 2$ or $\mathbf{V_2} = 4.8\angle{-53.13°}$

$\mathbf{I_{sc}} = (0.25+j0.5)\mathbf{V_2} = (0.55901\angle 63.435°)(4.8\angle{-53.13°})$

$= 2.6832\angle 10.305°$ A

Finally,

$\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}} = 26.833\angle 153.435°/2.6832\angle 10.305°$

$= \mathbf{10\angle 143.13°\ \Omega}$ or $= \mathbf{(-8+j6)\ \Omega}$.

Chapter 10, Solution 62.

First, we transform the circuit to the frequency domain.

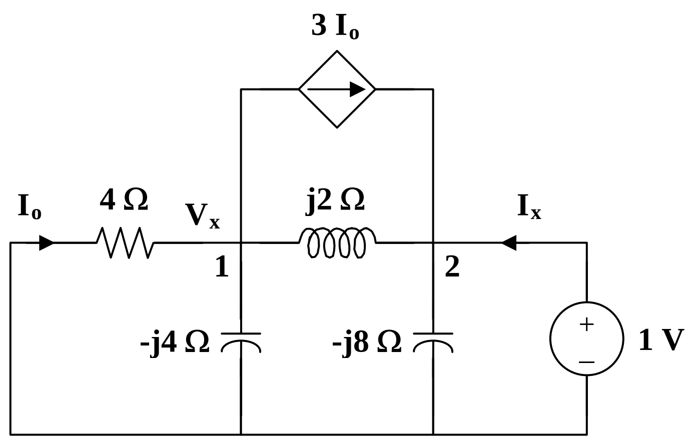
$$12\cos(t) \longrightarrow 12\angle 0°, \quad \omega = 1$$

$$2\text{ H} \longrightarrow j\omega L = j2$$

$$\frac{1}{4}\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j4$$

$$\frac{1}{8}\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j8$$

To find $\mathbf{Z}_{eq}$, consider the circuit in Fig. (a).



(a)

At node 1,

$$\frac{\mathbf{V}_x}{4} + \frac{\mathbf{V}_x}{\text{-}j4} + 3\mathbf{I}_o = \frac{1 - \mathbf{V}_x}{j2}, \qquad \text{where } \mathbf{I}_o = \frac{\text{-}\mathbf{V}_x}{4}$$

Thus,

$$\frac{\mathbf{V}_x}{\text{-}j4} - \frac{2\mathbf{V}_x}{4} = \frac{1 - \mathbf{V}_x}{j2}$$

$$\mathbf{V}_x = 0.4 + j0.8$$

At node 2,

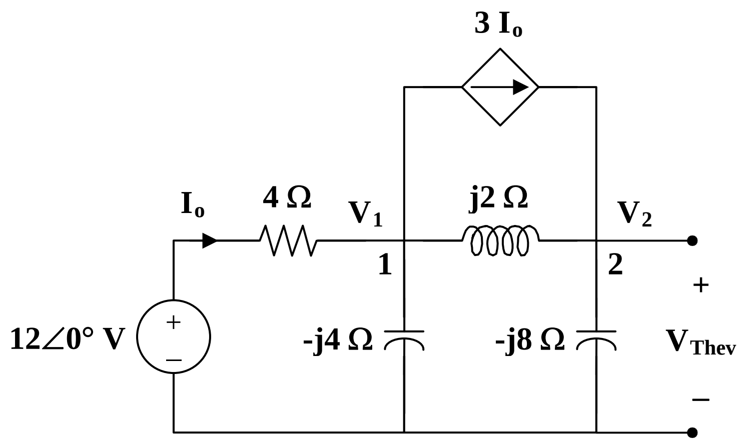
$$\mathbf{I}_x + 3\mathbf{I}_o = \frac{1}{\text{-}j8} + \frac{1 - \mathbf{V}_x}{j2}$$

$$\mathbf{I}_x = (0.75 + j0.5)\mathbf{V}_x - j\frac{3}{8}$$

$$\mathbf{I}_x = \text{-}0.1 + j0.425$$

$$\mathbf{Z}_{eq} = \frac{1}{\mathbf{I}_x} = \text{-}0.5246 - j2.229 = 2.29\angle\text{-}103.24°\ \Omega$$

To find $\mathbf{V}_{Thev}$, consider the circuit in Fig. (b).



(b)

At node 1,

$$\frac{12-\mathbf{V}_1}{4} = 3\mathbf{I}_o + \frac{\mathbf{V}_1}{-j4} + \frac{\mathbf{V}_1-\mathbf{V}_2}{j2}, \quad \text{where} \quad \mathbf{I}_o = \frac{12-\mathbf{V}_1}{4}$$

$$24 = (2+j)\mathbf{V}_1 - j2\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1-\mathbf{V}_2}{j2} + 3\mathbf{I}_o = \frac{\mathbf{V}_2}{-j8}$$

$$72 = (6+j4)\mathbf{V}_1 - j3\mathbf{V}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 24 \\ 72 \end{bmatrix} = \begin{bmatrix} 2+j & -j2 \\ 6+j4 & -j3 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Delta = -5+j6, \qquad \Delta_2 = -j24$$

$$\mathbf{V}_{th} = \mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 3.073\angle -219.8°$$

Thus,

$$\mathbf{V}_o = \frac{2}{2+\mathbf{Z}_{th}}\mathbf{V}_{th} = \frac{(2)(3.073\angle -219.8°)}{1.4754 - j2.229}$$

$$\mathbf{V}_o = \frac{6.146\angle -219.8°}{2.673\angle -56.5°} = 2.3\angle -163.3°$$

Therefore,

$$v_o = \mathbf{2.3cos(t–163.3°)\ V}$$

Chapter 10, Solution 63.

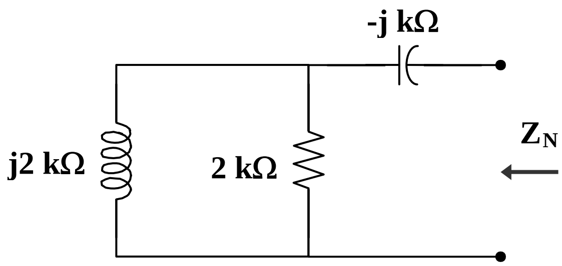
Transform the circuit to the frequency domain.

$$4\cos(200t + 30°) \longrightarrow 4\angle 30°, \quad \omega = 200$$

$$10\text{ H} \longrightarrow j\omega L = j(200)(10) = j2\text{ k}\Omega$$

$$5\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(5\times 10^{-6})} = -j\text{ k}\Omega$$

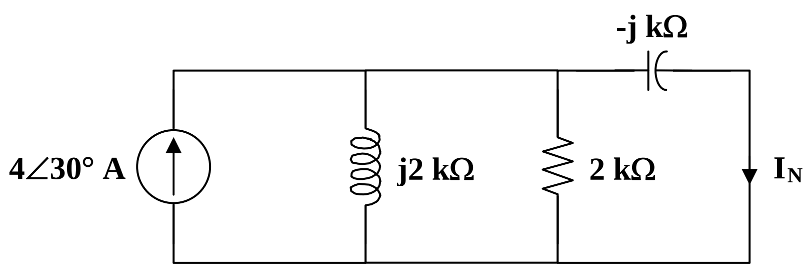
$\mathbf{Z}_N$ is found using the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = -j + 2 \,||\, j2 = -j + 1 + j = 1\text{ k}\Omega$$

We find $\mathbf{I}_N$ using the circuit in Fig. (b).



(b)

$$j2 \,||\, 2 = 1 + j$$

By the current division principle,

$$\mathbf{I}_N = \frac{1+j}{1+j-j}(4\angle 30°) = 5.657\angle 75°$$

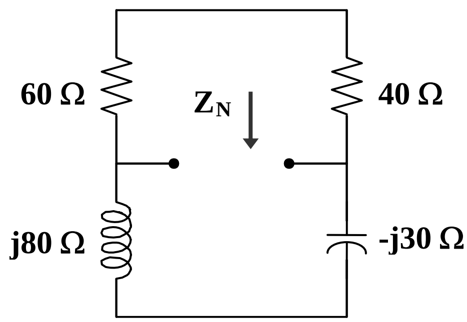
Therefore,

$i_N(t) =$ **5.657 cos(200t + 75°) A**

$\mathbf{Z}_N =$ **1 kΩ**

Chapter 10, Solution 64.

$\mathbf{Z}_N$ is obtained from the circuit in Fig. (a).

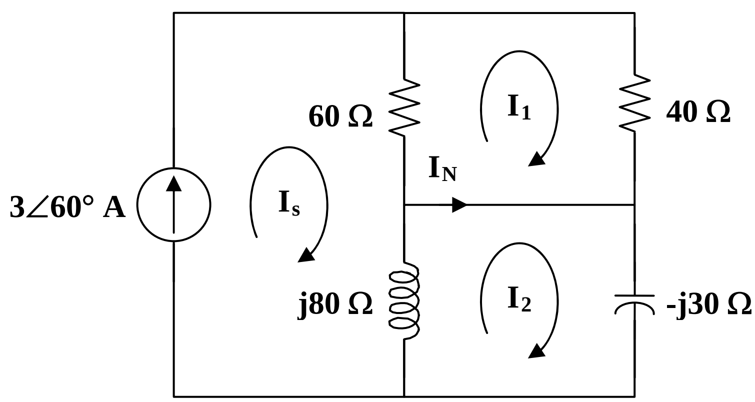


(a)

$$\mathbf{Z}_N = (60+40)\,||\,(j80-j30) = 100\,||\,j50 = \frac{(100)(j50)}{100+j50}$$

$$\mathbf{Z}_N = 20+j40 = \mathbf{44.72\angle 63.43°\ \Omega}$$

To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_s = 3\angle 60°$$

For mesh 1,

$$100\mathbf{I}_1 - 60\mathbf{I}_s = 0$$

$$\mathbf{I}_1 = 1.8\angle 60°$$

For mesh 2,

$$(j80-j30)\mathbf{I}_2 - j80\mathbf{I}_s = 0$$

$$\mathbf{I}_2 = 4.8\angle 60°$$

$I_N = I_2 - I_1 = \mathbf{3\angle 60°\ A}$

Chapter 10, Solution 65.

Using Fig. 10.108, design a problem to help other students to better understand Norton's theorem.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Compute i_o in Fig. 10.108 using Norton's theorem.

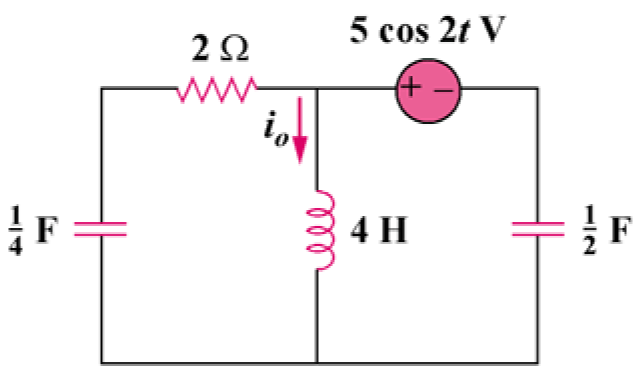
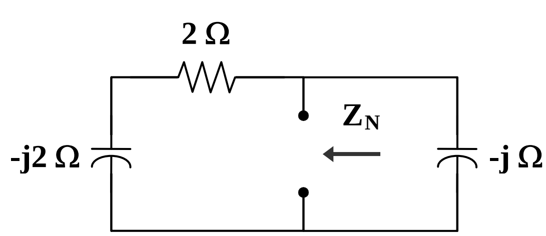


Figure 10.108

Solution

$$5\cos(2t) \longrightarrow 5\angle 0°, \quad \omega = 2$$
$$4\text{ H} \longrightarrow j\omega L = j(2)(4) = j8$$
$$\frac{1}{4}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = \text{-}j2$$
$$\frac{1}{2}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/2)} = \text{-}j$$

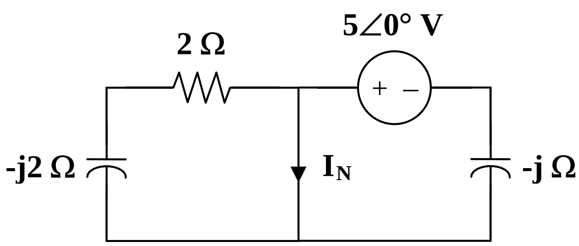
To find $\mathbf{Z}_N$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = -j \parallel (2 - j2) = \frac{-j(2 - j2)}{2 - j3} = \frac{1}{13}(2 - j10)$$

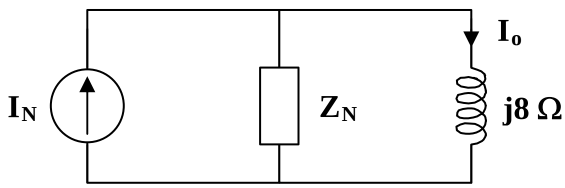
To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_N = \frac{5\angle 0°}{-j} = j5$$

The Norton equivalent of the circuit is shown in Fig. (c).



(c)

Using current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + j8}\mathbf{I}_N = \frac{(1/13)(2 - j10)(j5)}{(1/13)(2 - j10) + j8} = \frac{50 + j10}{2 + j94}$$

$$\mathbf{I}_o = 0.1176 - j0.5294 = 0542\angle -77.47°$$

Therefore, $i_o =$ **542 cos(2t – 77.47°) mA**

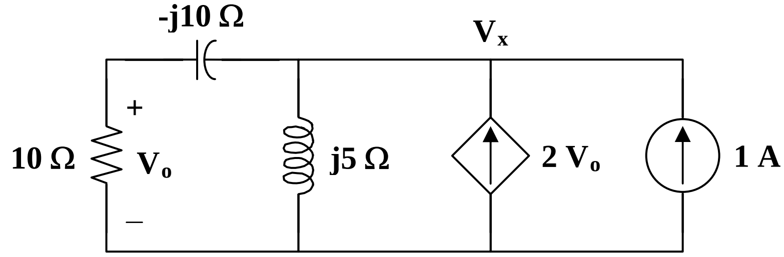
Chapter 10, Solution 66.

$$\omega = 10$$

$$0.5\text{ H} \longrightarrow j\omega L = j(10)(0.5) = j5$$

$$10\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(10\times 10^{-3})} = -j10$$

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



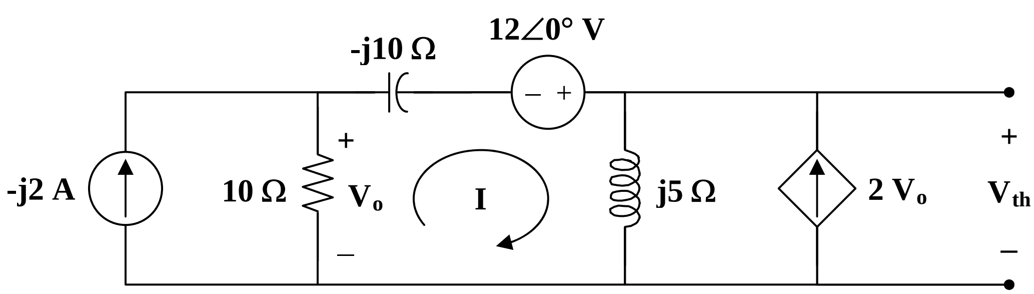
(a)

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_x}{j5} + \frac{\mathbf{V}_x}{10 - j10}, \qquad \text{where} \quad \mathbf{V}_o = \frac{10\mathbf{V}_x}{10 - j10}$$

$$1 + \frac{19\mathbf{V}_x}{10 - j10} = \frac{\mathbf{V}_x}{j5} \longrightarrow \mathbf{V}_x = \frac{-10 + j10}{21 + j2}$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \frac{\mathbf{V}_x}{1} = \frac{14.142\angle 135°}{21.095\angle 5.44°} = \mathbf{670\angle 129.56°\ m\Omega}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$(10 - j10 + j5)\mathbf{I} - (10)(-j2) + j5(2\mathbf{V}_o) - 12 = 0$$

where $\quad \mathbf{V}_o = (10)(-j2 - \mathbf{I})$

Thus,

$$(10 - j105)\mathbf{I} = -188 - j20$$

$$\mathbf{I} = \frac{188 + j20}{-10 + j105}$$

$$\mathbf{V}_{th} = j5(\mathbf{I} + 2\mathbf{V}_o) = j5(-19\mathbf{I} - j40) = -j95\mathbf{I} + 200$$

$$\mathbf{V}_{th} = \frac{-j95(188 + j20)}{-10 + j105} + 200 = \frac{(95\angle -90°)(189.06\angle 6.07°)}{105.48\angle 95.44} + 200$$

$$= 170.28\angle -179.37° + 200 = -170.27 - j1.8723 + 200 = 29.73 - j1.8723$$

$$\mathbf{V}_{th} = \mathbf{29.79\angle{-3.6°}\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{29.79\angle -3.6°}{0.67\angle 129.56°} = \mathbf{44.46\angle{-133.16°}\ A}$$

Chapter 10, Solution 67.

$$\boldsymbol{Z_N = Z_{Th} = 10//(13 - j5) + 12//(8 + j6) = \frac{10(13 - j5)}{23 - j5} + \frac{12(8 + j6)}{20 + j6} = \underline{11.243 + j1.079\Omega}}$$

$$V_a = \frac{10}{23 - j5}(60\angle 45^o) = 13.78 + j21.44, \qquad V_b = \frac{(8 + j6)}{20 + j6}(60\angle 45^o) = 12.069 + j26.08\Omega$$

$$\boldsymbol{V_{Th} = V_a - V_b = \underline{1.711 - j4.64 = 4.945\angle - 69.76^o}\ \text{V},}$$

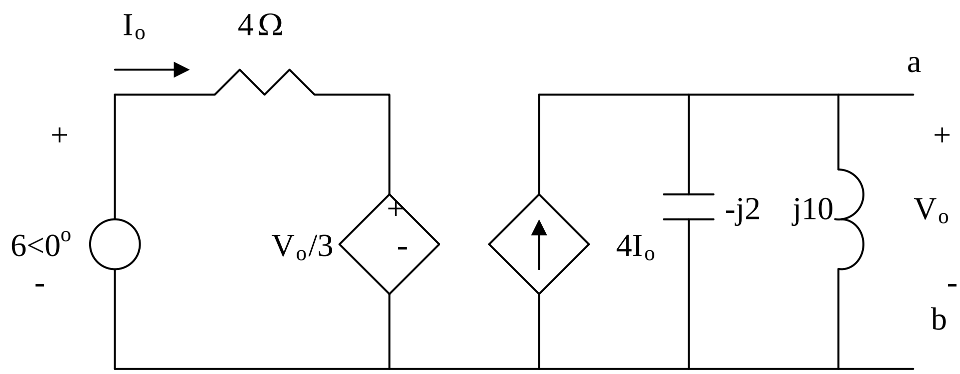
$$\boldsymbol{I_N = \frac{V_{Th}}{Z_{Th}} = \frac{4.945\angle - 69.76°}{11.295\angle 5.48°} = \underline{437.8\angle - 75.24^o}\ \text{mA}}$$

Chapter 10, Solution 68.

$$1\text{H} \longrightarrow j\omega L = j10x1 = j10$$

$$\frac{1}{20}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x\frac{1}{20}} = -j2$$

We obtain V_{Th} using the circuit below.



$$j10//(-j2) = \frac{j10(-j2)}{j10 - j2} = -j2.5$$

$$V_o = 4I_o x(-j2.5) = -j10I_o \qquad (1)$$

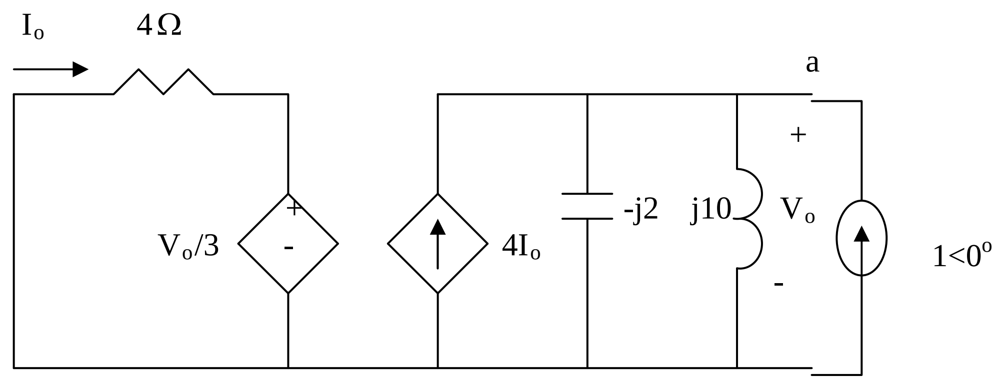
$$-6 + 4I_o + \frac{1}{3}V_o = 0 \qquad (2)$$

Combining (1) and (2) gives

$$I_o = \frac{6}{4 - j10/3}, \qquad V_{Th} = V_o = -j10I_o = \frac{-j60}{4 - j10/3} = 11.52\angle -50.19^o$$

$$\underline{\boldsymbol{v_{Th} = 11.52\sin(10t - 50.19^o)}}$$

To find $R_{Th,}$ we insert a 1-A source at terminals a-b, as shown below.



$$4I_o + \frac{1}{3}V_o = 0 \quad \longrightarrow \quad I_o = -\frac{V_o}{12}$$

$$1 + 4I_o = \frac{V_o}{-j2} + \frac{V_o}{j10}$$

Combining the two equations leads to

$$V_o = \frac{1}{0.333 + j0.4} = 1.2293 - j1.4766$$

$$\boldsymbol{Z_{Th} = \frac{V_o}{1} = \underline{1.2293 - 1.477\Omega}}$$

Chapter 10, Solution 69.

This is an inverting op amp so that

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-R}{1/j\omega C} = \mathbf{-j\omega RC}$$

When $\mathbf{V}_s = V_m$ and $\omega = 1/RC$,

$$\mathbf{V}_o = -j \cdot \frac{1}{RC} \cdot RC \cdot V_m = -jV_m = V_m \angle -90°$$

Therefore,

$$v_o(t) = V_m \sin(\omega t - 90°) = \mathbf{-\ V_m\ cos(\omega t)}$$

Chapter 10, Solution 70.

Using Fig. 10.113, design a problem to help other students to better understand op amps in AC circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The circuit in Fig. 10.113 is an integrator with a feedback resistor. Calculate $v_o(t)$ if v_s = 2 cos 4 × $10^4 t$ V.

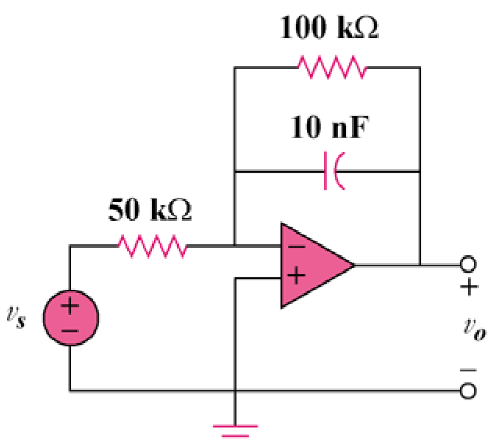


Figure 10.113

Solution

This may also be regarded as an inverting amplifier.

$$2\cos(4\times10^4 t) \longrightarrow 2\angle 0°, \quad \omega = 4\times10^4$$

$$10 \text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4\times10^4)(10\times10^{-9})} = \text{-j2.5 k}\Omega$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}$$

where $\mathbf{Z}_i = 50 \text{ k}\Omega$ and $\mathbf{Z}_f = 100\text{k} \parallel (\text{-j2.5k}) = \frac{\text{- j100}}{40 - \text{j}} \text{ k}\Omega$.

Thus, $$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-(\text{-j2})}{40 - \text{j}}$$

If $\mathbf{V}_s = 2\angle 0°$,

$$\mathbf{V}_o = \frac{j4}{40 - j} = \frac{4\angle 90°}{40.01\angle -1.43°} = 0.1\angle 91.43°$$

Therefore,

$$v_o(t) = \mathbf{100\ cos(4x10^4\ t + 91.43°)\ mV}$$

Chapter 10, Solution 71.

$$8\cos(2t+30^o) \longrightarrow 8\angle 30^o$$

$$0.5\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2x0.5x10^{-6}} = -j1M\Omega$$

At the inverting terminal,

$$\frac{V_o - 8\angle 30^o}{-j1000k} + \frac{V_o - 8\angle 30^o}{10k} = \frac{8\angle 30^o}{2k} \longrightarrow$$

$$V_o(1-j100) = 8\angle 30 + 800\angle -60° + 4000\angle -60°$$

$$V_o = \frac{6.928 + j4 + 2400 - j4157}{1-j100} = \frac{4800\angle -59.9°}{100\angle -89.43°} = 48\angle 29.53^o$$

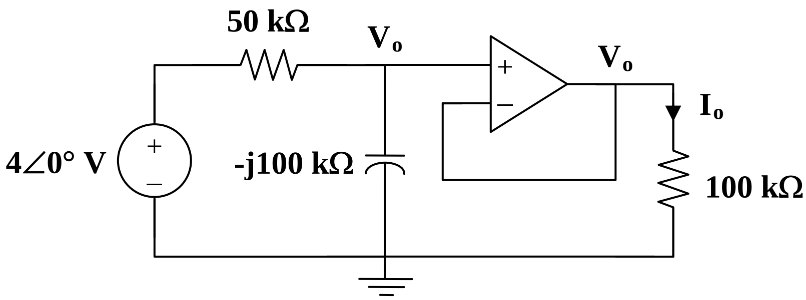
$v_o(t)$ = **$48\cos(2t+29.53^o)$ V**

Chapter 10, Solution 72.

$$4\cos(10^4 t) \longrightarrow 4\angle 0°, \quad \omega = 10^4$$

$$1\text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^4)(10^{-9})} = -j100\text{ k}\Omega$$

Consider the circuit as shown below.



At the noninverting node,

$$\frac{4 - \mathbf{V}_o}{50} = \frac{\mathbf{V}_o}{-j100} \longrightarrow \mathbf{V}_o = \frac{4}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{100\text{k}} = \frac{4}{(100)(1 + j0.5)}\text{ mA} = 35.78\angle -26.56°\ \mu\text{A}$$

Therefore,

$$i_o(t) = \mathbf{35.78\cos(10^4 t - 26.56°)\ \mu A}$$

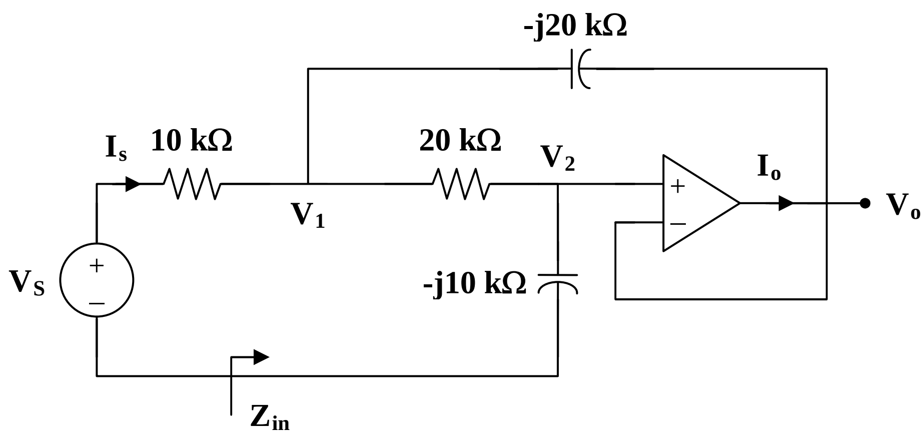
Chapter 10, Solution 73.

As a voltage follower, $\mathbf{V}_2 = \mathbf{V}_o$

$$C_1 = 10\text{ nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(5\times10^3)(10\times10^{-9})} = -j20\text{ k}\Omega$$

$$C_2 = 20\text{ nF} \longrightarrow \frac{1}{j\omega C_2} = \frac{1}{j(5\times10^3)(20\times10^{-9})} = -j10\text{ k}\Omega$$

Consider the circuit in the frequency domain as shown below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j20} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$2\mathbf{V}_s = (3+j)\mathbf{V}_1 - (1+j)\mathbf{V}_o \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{20} = \frac{\mathbf{V}_o - 0}{-j10}$$

$$\mathbf{V}_1 = (1+j2)\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1) gives

$$2\mathbf{V}_s = j6\mathbf{V}_o \quad \text{or} \quad \mathbf{V}_o = -j\frac{1}{3}\mathbf{V}_s$$

$$\mathbf{V}_1 = (1+j2)\mathbf{V}_o = \left(\frac{2}{3} - j\frac{1}{3}\right)\mathbf{V}_s$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s - \mathbf{V}_1}{10k} = \frac{(1/3)(1+j)}{10k}\mathbf{V}_s$$

$$\frac{\mathbf{I}_s}{\mathbf{V}_s} = \frac{1+j}{30k}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{30k}{1+j} = 15(1-j)\,k$$

$$\mathbf{Z}_{in} = \mathbf{21.21\angle{-45°}\ k\Omega}$$

Chapter 10, Solution 74.

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1}, \qquad \mathbf{Z}_f = R_2 + \frac{1}{j\omega C_2}$$

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = -\frac{R_2 + \dfrac{1}{j\omega C_2}}{R_1 + \dfrac{1}{j\omega C_1}} = -\left(\frac{C_1}{C_2}\right)\left(\frac{1 + j\omega R_2 C_2}{1 + j\omega R_1 C_1}\right)$$

At $\omega = 0$, $\qquad \mathbf{A}_v = -\dfrac{\mathbf{C_1}}{\mathbf{C_2}}$

As $\omega \to \infty$, $\qquad \mathbf{A}_v = -\dfrac{\mathbf{R_2}}{\mathbf{R_1}}$

At $\omega = \dfrac{1}{R_1 C_1}$, $\qquad \mathbf{A}_v = -\left(\dfrac{\mathbf{C_1}}{\mathbf{C_2}}\right)\left(\dfrac{\mathbf{1 + jR_2C_2/R_1C_1}}{\mathbf{1 + j}}\right)$

At $\omega = \dfrac{1}{R_2 C_2}$, $\qquad \mathbf{A}_v = -\left(\dfrac{\mathbf{C_1}}{\mathbf{C_2}}\right)\left(\dfrac{\mathbf{1 + j}}{\mathbf{1 + jR_1C_1/R_2C_2}}\right)$

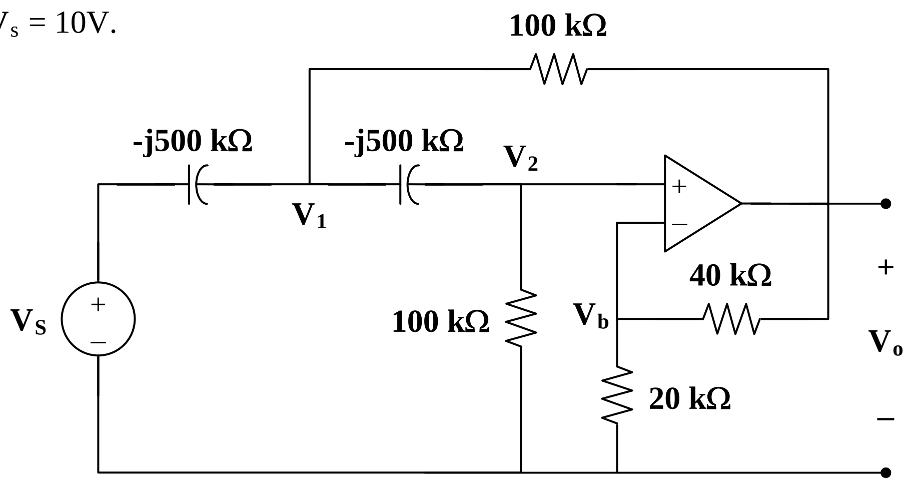
Chapter 10, Solution 75.

$$\omega = 2\times10^3$$

$$C_1 = C_2 = 1\,\text{nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(2\times10^3)(1\times10^{-9})} = \text{-j500 k}\Omega$$

Consider the circuit shown below.

Let $\mathbf{V}_s$ = 10V.



At node 1,

$[(\mathbf{V}_1–10)/(–j500k)] + [(\mathbf{V}_1–\mathbf{V}_o)/10^5] + [(\mathbf{V}_1–\mathbf{V}_2)/(–j500k)] = 0$

or $(1+j0.4)\mathbf{V}_1 – j0.2\mathbf{V}_2 – \mathbf{V}_o = j2$ (1)

At node 2,

$[(\mathbf{V}_2–\mathbf{V}_1)/(–j500k)] + [(\mathbf{V}_2–0)/100k] + 0 = 0$ or

$–j0.2\mathbf{V}_1 + (1+j0.2)\mathbf{V}_2 = 0$ or $\mathbf{V}_1 = [–(1+j0.2)/(–j0.2)]V_2$

$= (1–j5)\mathbf{V}_2$ (2)

At node b,

$$\mathbf{V_b} = \frac{R_3}{R_3+R_4}\mathbf{V}_o = \frac{\mathbf{V}_o}{3} = \mathbf{V_2} \quad (3)$$

From (2) and (3),

$\mathbf{V}_1 = (0.3333–j1.6667)\mathbf{V}_o$ (4)

Substituting (3) and (4) into (1),

$(1+j0.4)(0.3333–j1.6667)\mathbf{V}_o – j0.06667\mathbf{V}_o – \mathbf{V}_o = j2$

$(1+j0.4)(0.3333–j1.6667) = (1.077\angle21.8^\circ)(1.6997\angle–78.69^\circ)$

$= 1.8306\angle–56.89^\circ = 1–j1.5334$

$$(1–1+j(–1.5334–0.06667))\mathbf{V_o} = (–j1.6001)\mathbf{V_o} = 1.6001\angle–90^\circ$$

Therefore,

$$\mathbf{V}_o = 2\angle 90^\circ/(1.6001\angle–90^\circ) = 1.2499\angle 180^\circ$$

Since $\mathbf{V}_s = 10$,

$$\mathbf{V}_o/\mathbf{V}_s = \mathbf{0.12499\angle 180^\circ}.$$

Chapter 10, Solution 76.

Let the voltage between the -jkΩ capacitor and the 10kΩ resistor be $\mathbf{V}_1$.

$$\frac{2\angle 30^o - V_1}{-j4k} = \frac{V_1 - V_o}{10k} + \frac{V_1 - V_o}{20k} \quad \longrightarrow \qquad (1)$$

$$2\angle 30^o = (1 - j0.6)V_1 + j0.6V_o$$
$$= 1.7321+j1$$

Also,

$$\frac{V_1 - V_o}{10k} = \frac{V_o}{-j2k} \quad \longrightarrow \quad V_1 = (1 + j5)V_o \qquad (2)$$

Solving (2) into (1) yields

$$2\angle 30° = (1 - j0.6)(1 + j5)V_o + j0.6V_o = (1 + 3 - j0.6 + j5 + j6)V_o$$
$$= (4+j5)V_o$$
$$V_o = \frac{2\angle 30°}{6.403\angle 51.34°} = \underline{0.3124\angle -21.34^o \ \text{V}}$$

$$= \mathbf{312.4\angle{-21.34}^\circ\ mV}$$

$\mathbf{I}_o = (\mathbf{V}_1 - \mathbf{V}_o)/20k = \mathbf{V}_o/(-j4k) = (0.3124/4k)\angle(-21.43+90)^\circ$

$$= \mathbf{78.1\angle 68.57^\circ\ \mu A}$$

We can easily check this answer using MATLAM. Using equations (1) and (2) we can identify the following matrix equations:

YV = **I** where

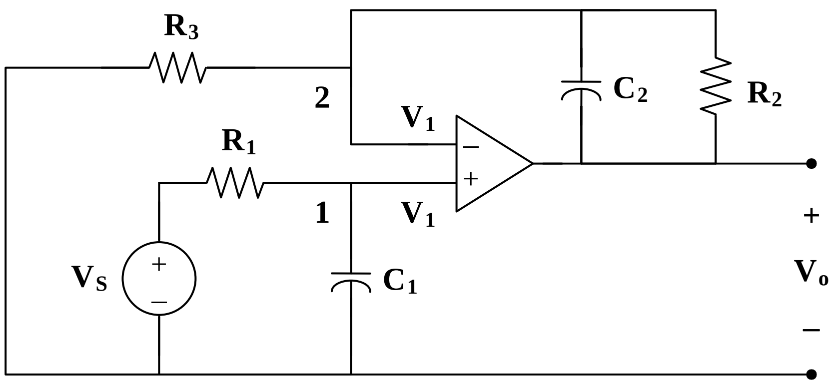
```
>> Y=[1-0.6i,0.6i;1,-1-0.5i]

Y =
   1.0000 - 0.6000i        0 + 0.6000i
   1.0000             -1.0000 - 5.0000i

>> I=[1.7321+1i;0]

I =
   1.7321 + 1.0000i
        0
```

```
>> V=inv(Y)*I

V =
   0.8593 + 1.3410i
   0.2909 - 0.1137i
```

$= V_o =$ **$312.3\angle{-21.35^\circ}$ mV**. The answer checks.

Chapter 10, Solution 77.

Consider the circuit below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{R_1} = j\omega C\,\mathbf{V}_1$$

$$\mathbf{V}_s = (1 + j\omega R_1 C_1)\,\mathbf{V}_1 \qquad (1)$$

At node 2,

$$\frac{0 - \mathbf{V}_1}{R_3} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{R_2} + j\omega C_2\,(\mathbf{V}_1 - \mathbf{V}_o)$$

$$\mathbf{V}_1 = (\mathbf{V}_o - \mathbf{V}_1)\left(\frac{R_3}{R_2} + j\omega C_2 R_3\right)$$

$$\mathbf{V}_o = \left(1 + \frac{1}{(R_3/R_2) + j\omega C_2 R_3}\right)\mathbf{V}_1 \qquad (2)$$

From (1) and (2),

$$\mathbf{V}_o = \frac{\mathbf{V}_s}{1 + j\omega R_1 C_1}\left(1 + \frac{R_2}{R_3 + j\omega C_2 R_2 R_3}\right)$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{R_2 + R_3 + j\omega C_2 R_2 R_3}}{\mathbf{(1 + j\omega R_1 C_1)(R_3 + j\omega C_2 R_2 R_3)}}$$

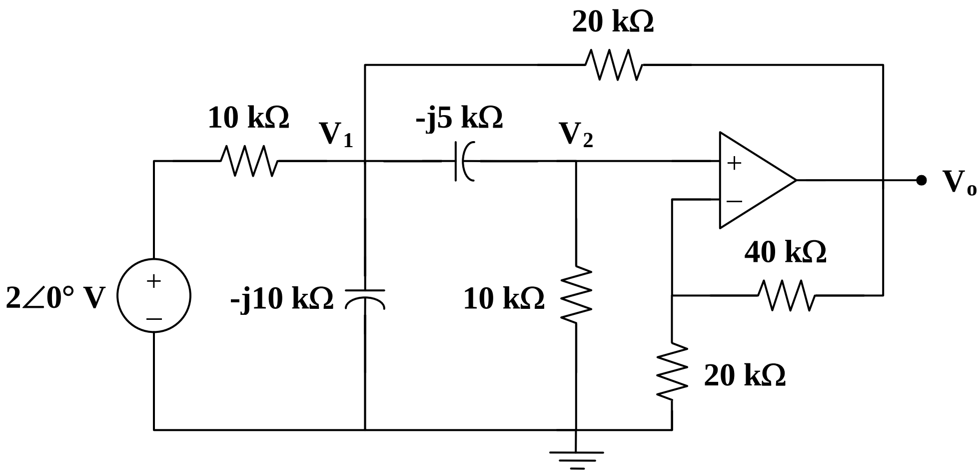
Chapter 10, Solution 78.

$$2\sin(400t) \longrightarrow 2\angle 0°, \quad \omega = 400$$

$$0.5\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.5\times 10^{-6})} = -j5\ k\Omega$$

$$0.25\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.25\times 10^{-6})} = -j10\ k\Omega$$

Consider the circuit as shown below.



At node 1,

$$\frac{2-\mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$4 = (3 + j6)\,\mathbf{V}_1 - j4\,\mathbf{V}_2 - \mathbf{V}_o \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} = \frac{\mathbf{V}_2}{10}$$

$$\mathbf{V}_1 = (1 - j0.5)\,\mathbf{V}_2 \qquad (2)$$

But

$$\mathbf{V}_2 = \frac{20}{20+40}\mathbf{V}_o = \frac{1}{3}\mathbf{V}_o \qquad (3)$$

From (2) and (3),

$$\mathbf{V}_1 = \frac{1}{3}\cdot(1 - j0.5)\,\mathbf{V}_o \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$4 = (3 + j6)\cdot\frac{1}{3}\cdot(1 - j0.5)\,\mathbf{V}_o - j\frac{4}{3}\mathbf{V}_o - \mathbf{V}_o = \left(1 + j\frac{1}{6}\right)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{24}{6+j} = 3.945\angle -9.46°$$

Therefore,

$$v_o(t) = \mathbf{3.945sin(400t–9.46°)\ V}$$

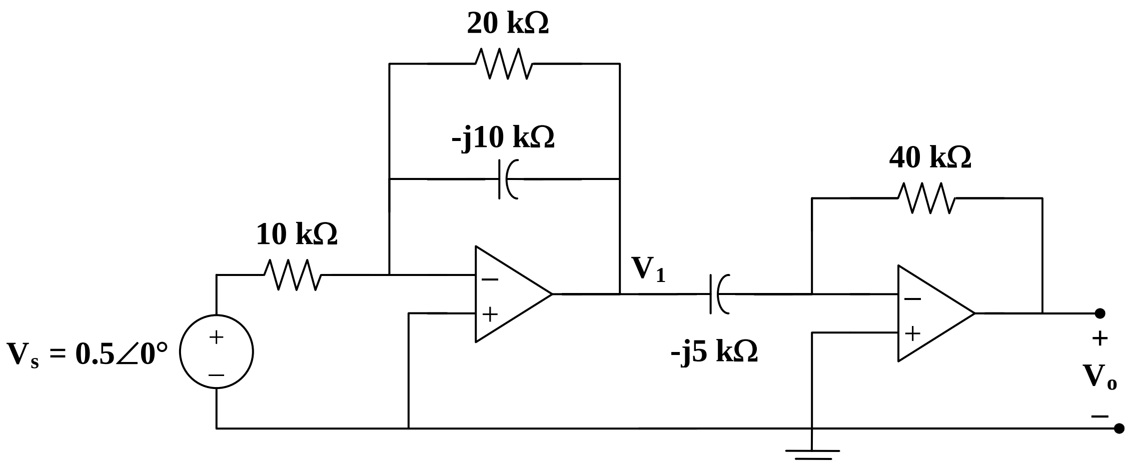
Chapter 10, Solution 79.

$$0.5\cos(1000t) \longrightarrow 0.5\angle 0°, \quad \omega = 1000$$

$$0.1\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.1\times 10^{-6})} = -j10\ k\Omega$$

$$0.2\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.2\times 10^{-6})} = -j5\ k\Omega$$

Consider the circuit shown below.



Since each stage is an inverter, we apply $\mathbf{V}_o = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}\mathbf{V}_i$ to each stage.

$$\mathbf{V}_o = \frac{-40}{-j5}\mathbf{V}_1 \qquad (1)$$

and

$$\mathbf{V}_1 = \frac{-20 \parallel (-j10)}{10}\mathbf{V}_s \qquad (2)$$

From (1) and (2),

$$\mathbf{V}_o = \left(\frac{-j8}{10}\right)\left(\frac{-(20)(-j10)}{20 - j10}\right) 0.5\angle 0°$$

$$\mathbf{V}_o = 1.6(2 + j) = 35.78\angle 26.56°$$

Therefore, $v_o(t) =$ **3.578cos(1000t + 26.56°) V**

Chapter 10, Solution 80.

$$4\cos(1000t - 60°) \longrightarrow 4\angle -60°, \quad \omega = 1000$$

$$0.1\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.1\times 10^{-6})} = -j10\ \text{k}\Omega$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.2\times 10^{-6})} = -j5\ \text{k}\Omega$$

The two stages are inverters so that

$$\mathbf{V}_o = \left(\frac{20}{-j10}\cdot(4\angle -60°) + \frac{20}{50}\mathbf{V}_o\right)\left(\frac{-j5}{10}\right)$$

$$= \frac{-j}{2}\cdot(j2)\cdot(4\angle -60°) + \frac{-j}{2}\cdot\frac{2}{5}\mathbf{V}_o$$

$$(1 + j/5)\,\mathbf{V}_o = 4\angle -60°$$

$$\mathbf{V}_o = \frac{4\angle -60°}{1 + j/5} = 3.922\angle -71.31°$$

Therefore, $v_o(t) =$ **3.922 cos(1000t – 71.31°) V**

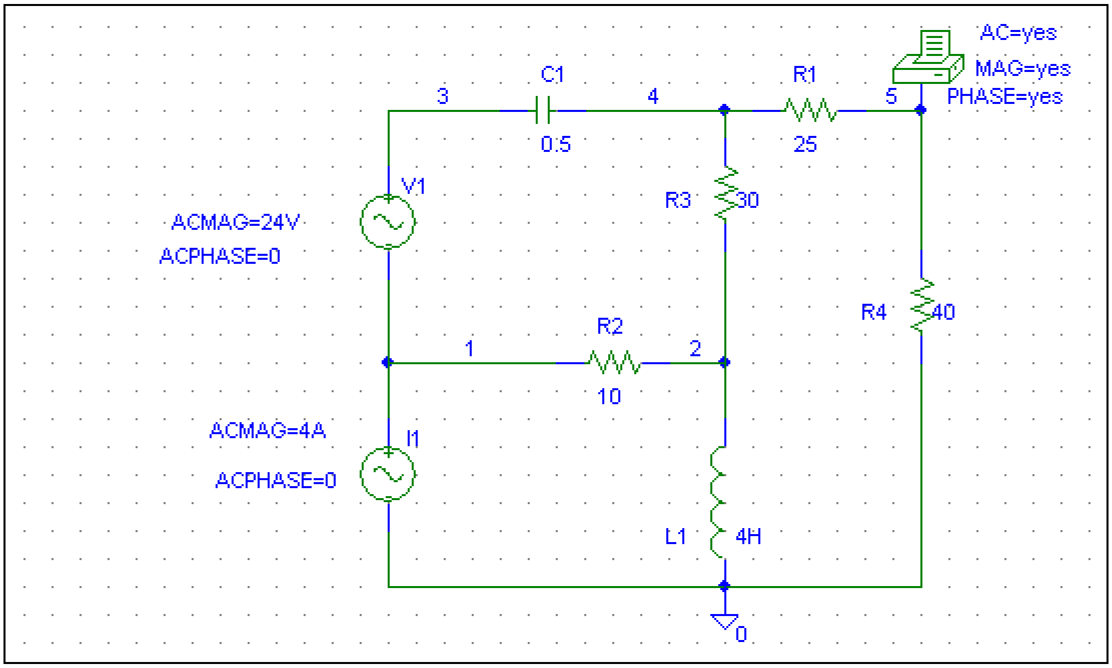
Chapter 10, Solution 81.

We need to get the capacitance and inductance corresponding to –j2 Ω and j4 Ω.

$$-j2 \quad \longrightarrow \quad C = \frac{1}{\omega X_c} = \frac{1}{1x2} = 0.5F$$

$$j4 \quad \longrightarrow \quad L = \frac{X_L}{\omega} = 4H$$

The schematic is shown below.



When the circuit is simulated, we obtain the following from the output file.

FREQ	VM(5)	VP(5)
1.592E-01	1.127E+01	-1.281E+02

From this, we obtain

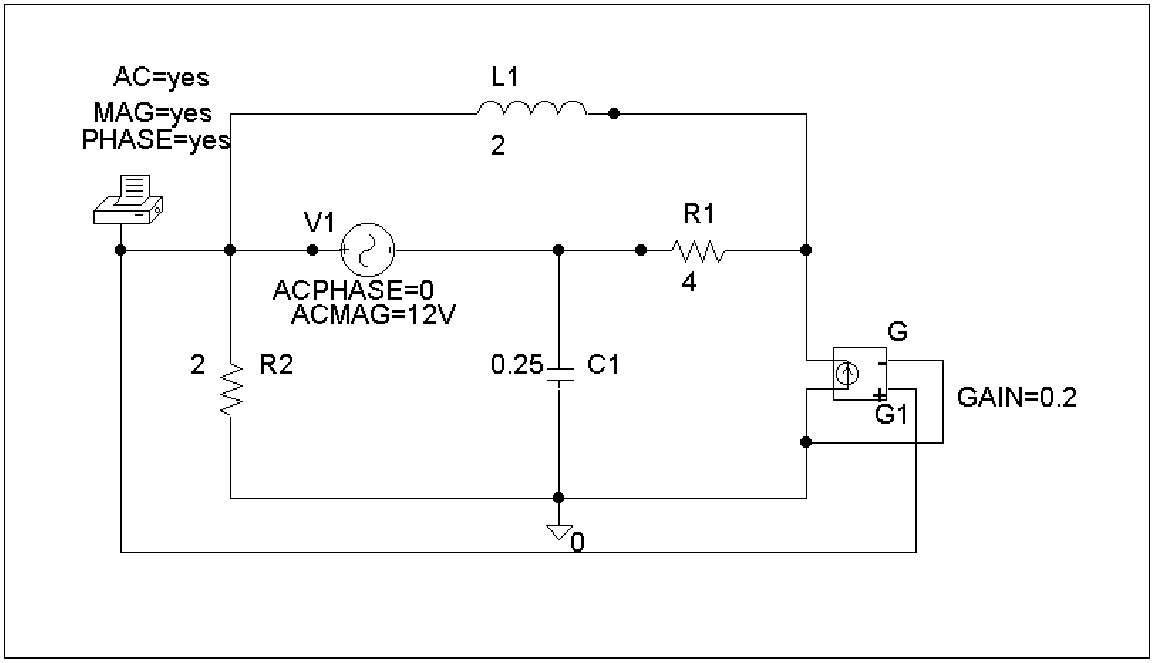
$$V_o = \mathbf{11.27\angle 128.1^o \ \ V}.$$

Chapter 10, Solution 82.

The schematic is shown below. We insert PRINT to print V_o in the output file. For AC Sweep, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we print out the output file which includes:

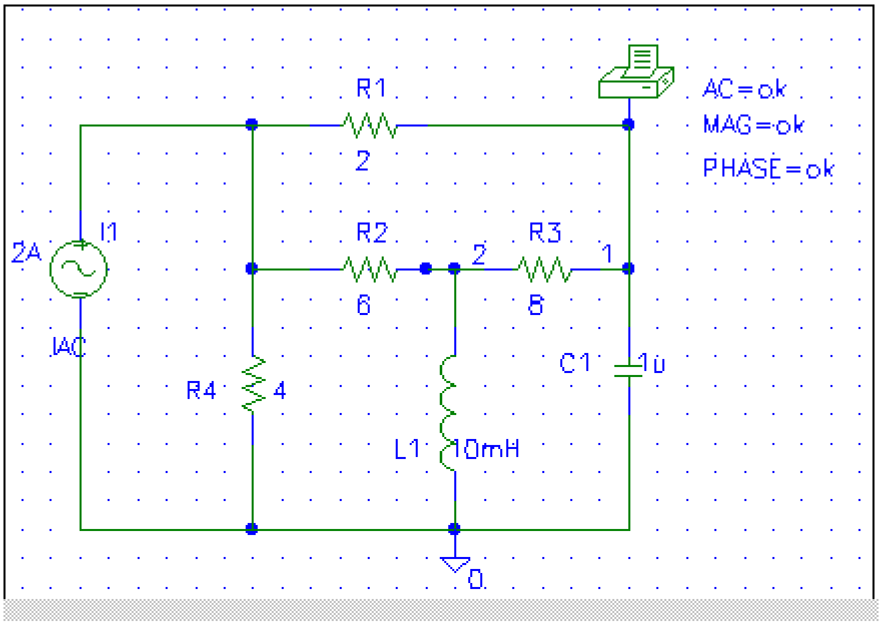
FREQ	VM($N_0001)	VP($N_0001)
1.592 E-01	7.684 E+00	5.019 E+01

which means that $V_o = \mathbf{7.684\angle 50.19^o\ V}$



Chapter 10, Solution 83.

The schematic is shown below. The frequency is $f = \omega / 2\pi = \dfrac{1000}{2\pi} = 159.15$



When the circuit is saved and simulated, we obtain from the output file

FREQ	VM(1)	VP(1)
1.592E+02	6.611E+00	-1.592E+02

Thus,

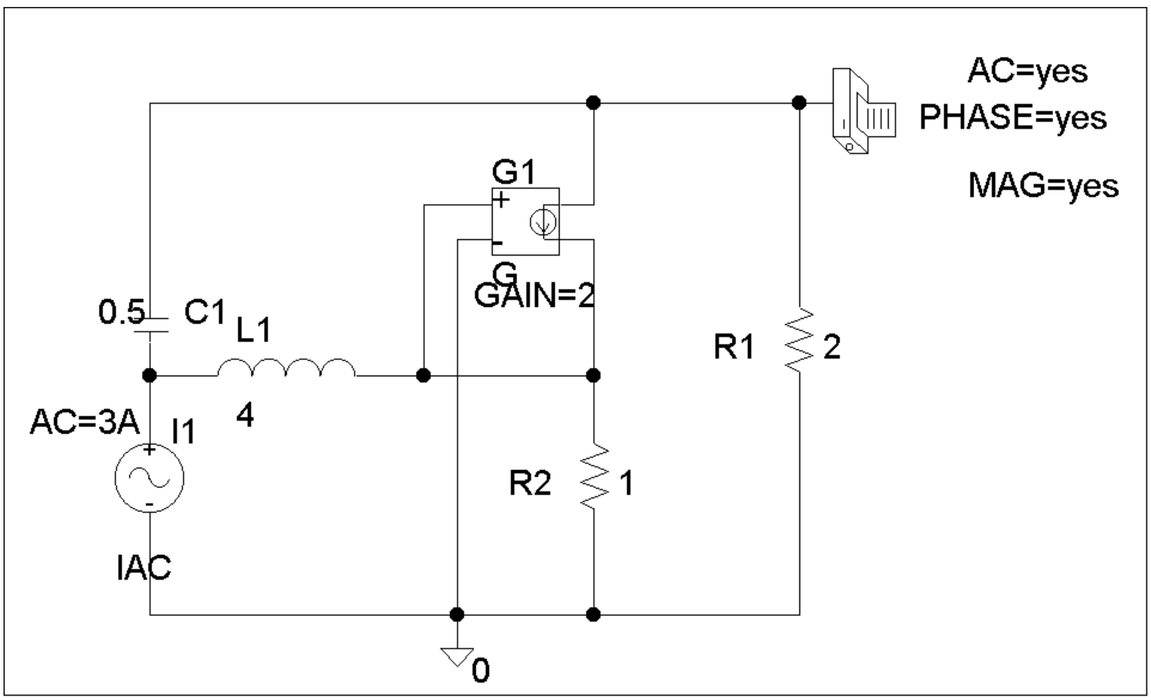
$$v_o = \mathbf{6.611\cos(1000t - 159.2^o)\ \ V}$$

Chapter 10, Solution 84.

The schematic is shown below. We set PRINT to print V_o in the output file. In AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain the output file which includes:

FREQ	VM($N_0003)	VP($N_0003)
1.592 E-01	1.664 E+00	–1.646 E+02

Namely, $V_o = \mathbf{1.664\angle{-146.4}^o\ V}$



Chapter 10, Solution 85.

Using Fig. 10.127, design a problem to help other students to better understand performing AC analysis with *PSpice*.

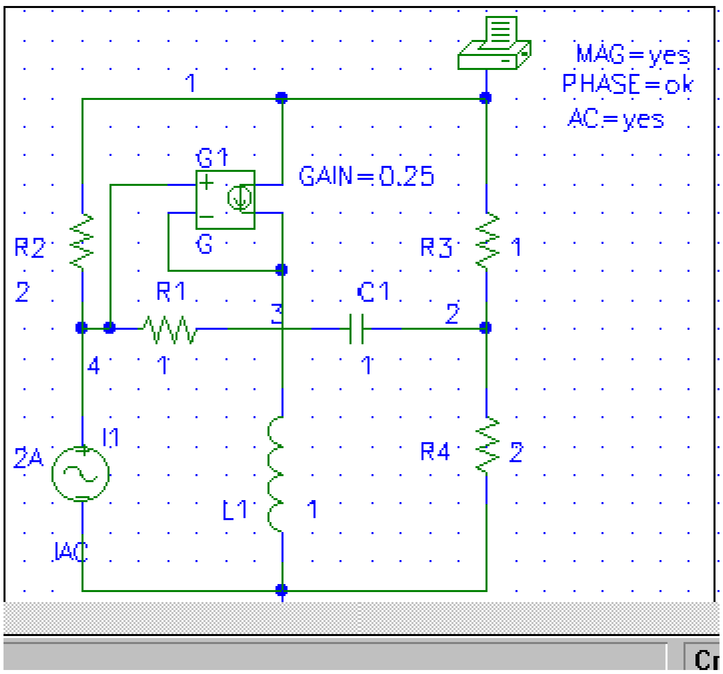
Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use *PSpice* to find V_o in the circuit of Fig. 10.127. Let $R_1 = 2\ \Omega$, $R_2 = 1\ \Omega$, $R_3 = 1\ \Omega$, $R_4 = 2\ \Omega$, $I_s = 2\angle 0^\circ$ A, $X_L = 1\ \Omega$, and $X_C = 1\ \Omega$.

Solution

The schematic is shown below. We let $\omega = 1$ rad/s so that L=1H and C=1F.



When the circuit is saved and simulated, we obtain from the output file

FREQ	VM(1)	VP(1)
1.591E-01	2.228E+00	-1.675E+02

From this, we conclude that

$$V_o = \mathbf{2.228\angle{-167.5^\circ}\ V}.$$

Chapter 10, Solution 86.

The schematic is shown below. We insert three pseudocomponent PRINTs at nodes 1, 2, and 3 to print V_1, V_2, and V_3, into the output file. Assume that w = 1, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After saving and simulating the circuit, we obtain the output file which includes:

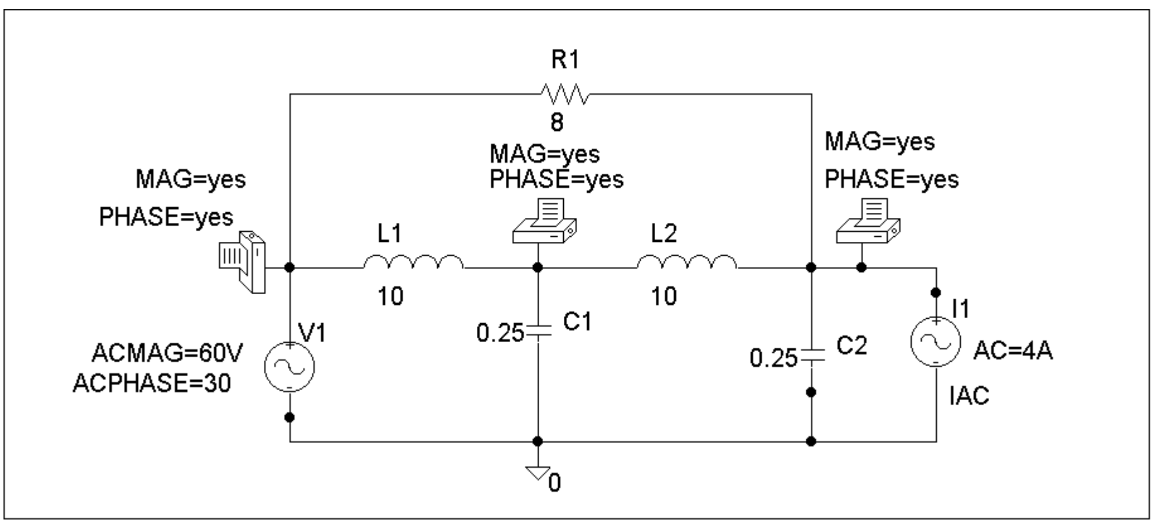
FREQ VM($N_0002) VP($N_0002)
1.592 E-01 6.000 E+01 3.000 E+01

FREQ VM($N_0003) VP($N_0003)
1.592 E-01 2.367 E+02 -8.483 E+01

FREQ VM($N_0001) VP($N_0001)
1.592 E-01 1.082 E+02 1.254 E+02

Therefore,

$$V_1 = \mathbf{60\angle 30^o\ V} \quad V_2 = \mathbf{236.7\angle -84.83^o\ V} \quad V_3 = \mathbf{108.2\angle 125.4^o\ V}$$



Chapter 10, Solution 87.

The schematic is shown below. We insert three PRINTs at nodes 1, 2, and 3. We set Total Pts = 1, Start Freq = 0.1592, End Freq = 0.1592 in the AC Sweep box. After simulation, the output file includes:

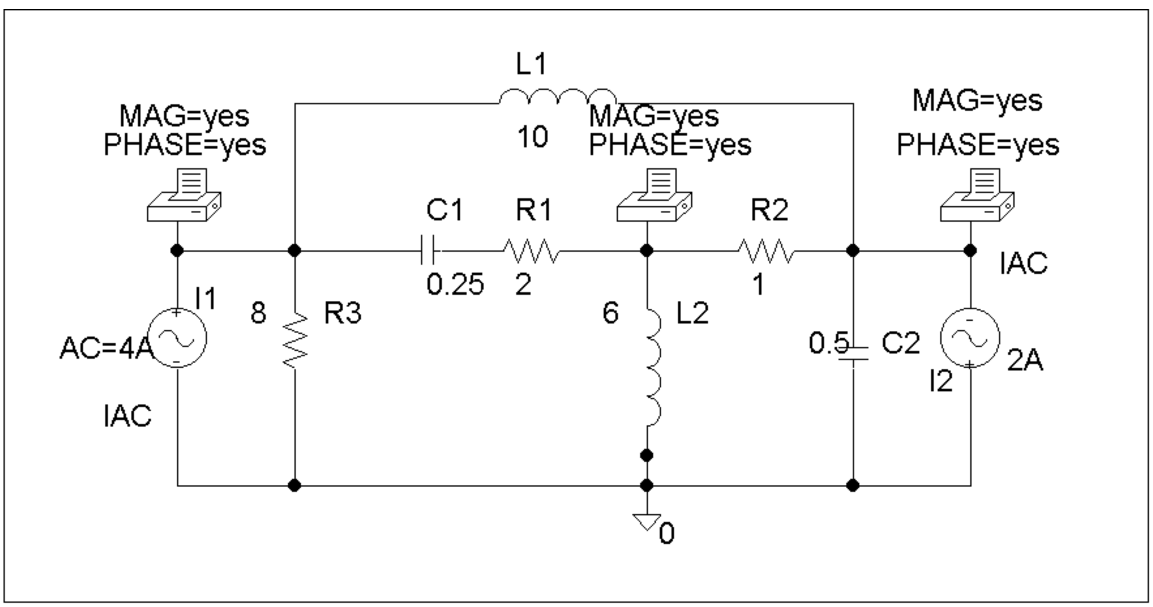
FREQ	VM($N_0004)	VP($N_0004)
1.592 E-01	1.591 E+01	1.696 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E-01	5.172 E+00	-1.386 E+02

FREQ	VM($N_0003)	VP($N_0003)
1.592 E-01	2.270 E+00	-1.524 E+02

Therefore,

$$V_1 = \mathbf{15.91\angle 169.6^o\ V} \quad V_2 = \mathbf{5.172\angle -138.6^o\ V} \quad V_3 = \mathbf{2.27\angle -152.4^o\ V}$$



Chapter 10, Solution 88.

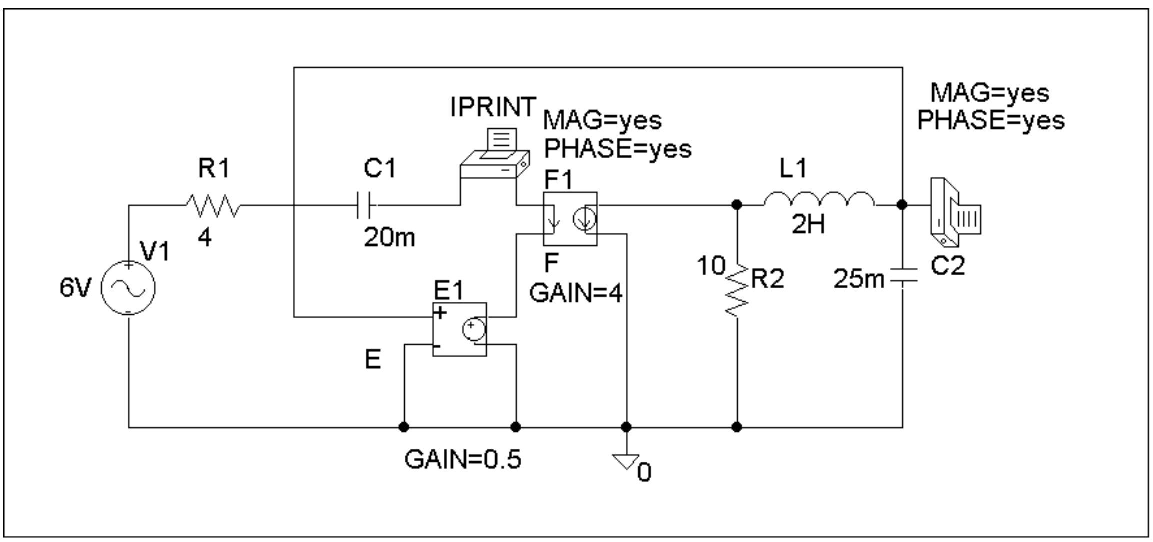
The schematic is shown below. We insert IPRINT and PRINT to print I_o and V_o in the output file. Since w = 4, f = w/2π = 0.6366, we set Total Pts = 1, Start Freq = 0.6366, and End Freq = 0.6366 in the AC Sweep box. After simulation, the output file includes:

FREQ	VM($N_0002)	VP($N_0002)
6.366 E-01	3.496 E+01	1.261 E+01

FREQ	IM(V_PRINT2)	IP (V_PRINT2)
6.366 E-01	8.912 E-01	-8.870 E+01

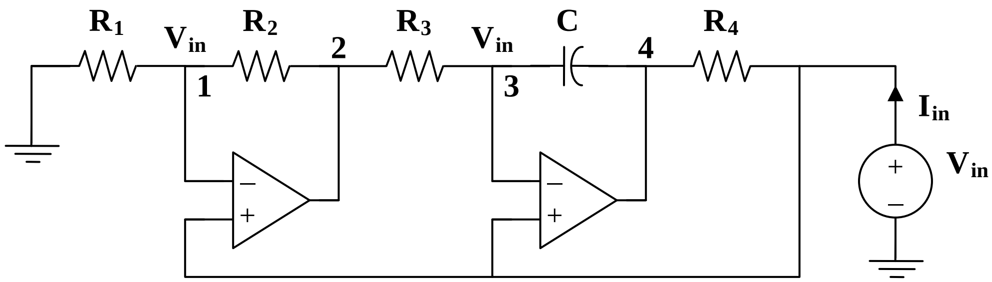
Therefore, $V_o = 34.96\angle 12.6^o$ V, $I_o = 0.8912\angle -88.7^o$ A

v_o = **$34.96 \cos(4t + 12.6^o)$V**, i_o = **$0.8912\cos(4t - 88.7^o)$A**



Chapter 10, Solution 89.

Consider the circuit below.



At node 1,

$$\frac{0-\mathbf{V}_{in}}{R_1} = \frac{\mathbf{V}_{in}-\mathbf{V}_2}{R_2}$$

$$-\mathbf{V}_{in} + \mathbf{V}_2 = \frac{R_2}{R_1}\mathbf{V}_{in} \qquad (1)$$

At node 3,

$$\frac{\mathbf{V}_2-\mathbf{V}_{in}}{R_3} = \frac{\mathbf{V}_{in}-\mathbf{V}_4}{1/j\omega C}$$

$$-\mathbf{V}_{in} + \mathbf{V}_4 = \frac{\mathbf{V}_{in}-\mathbf{V}_2}{j\omega C R_3} \qquad (2)$$

From (1) and (2),

$$-\mathbf{V}_{in} + \mathbf{V}_4 = \frac{-R_2}{j\omega C R_3 R_1}\mathbf{V}_{in}$$

Thus,

$$\mathbf{I}_{in} = \frac{\mathbf{V}_{in}-\mathbf{V}_4}{R_4} = \frac{R_2}{j\omega C R_3 R_1 R_4}\mathbf{V}_{in}$$

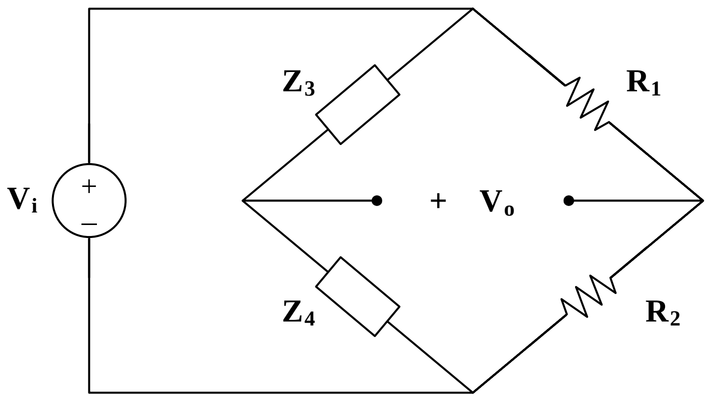
$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{\mathbf{I}_{in}} = \frac{j\omega C R_1 R_3 R_4}{R_2} = j\omega L_{eq}$$

where $\qquad \mathbf{L_{eq}} = \dfrac{\mathbf{R_1 R_3 R_4 C}}{\mathbf{R_2}}$

Chapter 10, Solution 90.

Let $$\mathbf{Z}_4 = R \parallel \frac{1}{j\omega C} = \frac{R}{1 + j\omega RC}$$
$$\mathbf{Z}_3 = R + \frac{1}{j\omega C} = \frac{1 + j\omega RC}{j\omega C}$$

Consider the circuit shown below.



$$\mathbf{V}_o = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4}\mathbf{V}_i - \frac{R_2}{R_1 + R_2}\mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{R}{1 + j\omega C}}{\dfrac{R}{1 + j\omega C} + \dfrac{1 + j\omega RC}{j\omega C}} - \frac{R_2}{R_1 + R_2}$$

$$= \frac{j\omega RC}{j\omega RC + (1 + j\omega RC)^2} - \frac{R_2}{R_1 + R_2}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega RC}{1 - \omega^2 R^2 C^2 + j3\omega RC} - \frac{R_2}{R_1 + R_2}$$

For $\mathbf{V}_o$ and $\mathbf{V}_i$ to be in phase, $\dfrac{\mathbf{V}_o}{\mathbf{V}_i}$ must be purely real. This happens when

$$1 - \omega^2 R^2 C^2 = 0$$
$$\omega = \frac{1}{RC} = 2\pi f$$

or $$f = \frac{1}{2\pi RC}$$

At this frequency,

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{3} - \frac{R_2}{R_1 + R_2}$$

Chapter 10, Solution 91.

(a) Let $\mathbf{V}_2 =$ voltage at the noninverting terminal of the op amp

$\mathbf{V}_o =$ output voltage of the op amp

$\mathbf{Z}_p = 10\ \text{k}\Omega = R_o$

$$\mathbf{Z}_s = R + j\omega L + \frac{1}{j\omega C}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{R_o}{R + R_o + j\omega L - \dfrac{j}{\omega C}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega C R_o}{\omega C (R + R_o) + j(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC - 1 = 0 \longrightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.4\times 10^{-3})(2\times 10^{-9})}}$$

$$f_o = \mathbf{180\ kHz}$$

(b) At oscillation,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o C R_o}{\omega_o C (R + R_o)} = \frac{R_o}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{80}{20} = 5$$

$$\frac{R_o}{R + R_o} = \frac{1}{5} \longrightarrow R = 4R_o = \mathbf{40\ k?}$$

Chapter 10, Solution 92.

Let $\mathbf{V}_2 =$ voltage at the noninverting terminal of the op amp

$\mathbf{V}_o =$ output voltage of the op amp

$\mathbf{Z}_s = R_o$

$$\mathbf{Z}_p = j\omega L \parallel \frac{1}{j\omega C} \parallel R = \frac{1}{\dfrac{1}{R} + j\omega C + \dfrac{1}{j\omega L}} = \frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{\dfrac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}{R_o + \dfrac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega RL}{\omega RL + \omega R_o L + jR_o R\,(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC = 1 \quad \longrightarrow \quad f_o = \frac{1}{2\pi\sqrt{LC}}$$

(a) At $\omega = \omega_o$,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o RL}{\omega_o RL + \omega_o R_o L} = \frac{R}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{R_f}{R_o} = 1 + \frac{1000k}{100k} = 11$$

Hence,

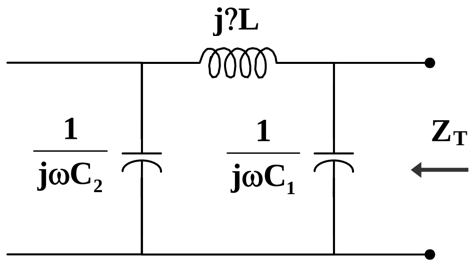
$$\frac{R}{R + R_o} = \frac{1}{11} \quad \longrightarrow \quad R_o = 10R = \mathbf{100\ k?}$$

(b) $$f_o = \frac{1}{2\pi\sqrt{(10\times 10^{-6})(2\times 10^{-9})}}$$

$$f_o = \mathbf{1.125\ MHz}$$

Chapter 10, Solution 93.

As shown below, the impedance of the feedback is



$$\mathbf{Z}_T = \frac{1}{j\omega C_1} \,||\, \left(j\omega L + \frac{1}{j\omega C_2} \right)$$

$$\mathbf{Z}_T = \frac{\frac{-j}{\omega C_1}\left(j\omega L + \frac{-j}{\omega C_2} \right)}{\frac{-j}{\omega C_1} + j\omega L + \frac{-j}{\omega C_2}} = \frac{\frac{1}{\omega} - \omega L C_2}{j(C_1 + C_2 - \omega^2 L C_1 C_2)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

$$C_1 + C_2 - \omega_o^2 L C_1 C_2 = 0$$

$$\omega_o^2 = \frac{C_1 + C_2}{L C_1 C_2} = \frac{1}{L C_T}$$

$$\mathbf{f_o = \frac{1}{2\pi\sqrt{LC_T}}}$$

Chapter 10, Solution 94.

If we select $C_1 = C_2 = 20$ nF

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1}{2} = 10 \text{ nF}$$

Since $f_o = \dfrac{1}{2\pi\sqrt{LC_T}}$,

$$L = \frac{1}{(2\pi f)^2 C_T} = \frac{1}{(4\pi^2)(2500 \times 10^6)(10 \times 10^{-9})} = 10.13 \text{ mH}$$

$$X_c = \frac{1}{\omega C_2} = \frac{1}{(2\pi)(50 \times 10^3)(20 \times 10^{-9})} = \text{ 159 ?}$$

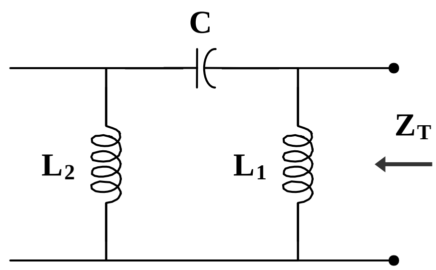
We may select $R_i = 20 \text{ k}\Omega$ and $R_f \geq R_i$, say $R_f = 20 \text{ k}\Omega$.

Thus,

$C_1 = C_2 =$ **20 nF**, $L =$ **10.13 mH** $R_f = R_i =$ **20 k?**

Chapter 10, Solution 95.

First, we find the feedback impedance.



$$\mathbf{Z}_T = j\omega L_1 \,||\left(j\omega L_2 + \frac{1}{j\omega C}\right)$$

$$\mathbf{Z}_T = \frac{j\omega L_1\left(j\omega L_2 - \dfrac{j}{\omega C}\right)}{j\omega L_1 + j\omega L_2 - \dfrac{j}{\omega C}} = \frac{\omega^2 L_1 C(1-\omega L_2)}{j(\omega^2 C(L_1 + L_2) - 1)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

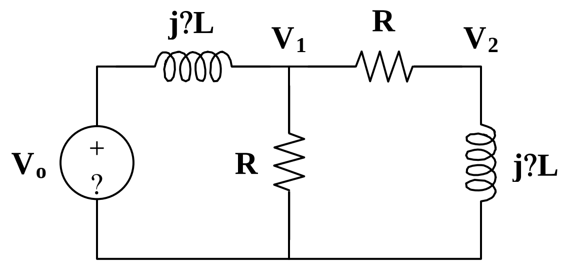
$$\omega_o^2 C(L_1 + L_2) - 1 = 0$$

$$\omega_o = 2\pi f_o = \frac{1}{C(L_1 + L_2)}$$

$$\boldsymbol{f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}}$$

Chapter 10, Solution 96.

(a) Consider the feedback portion of the circuit, as shown below.



$$\mathbf{V_2} = \frac{j\omega L}{R + j\omega L}\mathbf{V_1} \longrightarrow \mathbf{V}_1 = \frac{R + j\omega L}{j\omega L}\mathbf{V}_2 \qquad (1)$$

Applying KCL at node 1,

$$\frac{\mathbf{V}_o - \mathbf{V}_1}{j\omega L} = \frac{\mathbf{V}_1}{R} + \frac{\mathbf{V}_1}{R + j\omega L}$$

$$\mathbf{V}_o - \mathbf{V}_1 = j\omega L\,\mathbf{V}_1\left(\frac{1}{R} + \frac{1}{R + j\omega L}\right)$$

$$\mathbf{V}_o = \mathbf{V}_1\left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)}\right)$$

(2)

From (1) and (2),

$$\mathbf{V}_o = \left(\frac{R + j\omega L}{j\omega L}\right)\left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)}\right)\mathbf{V}_2$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{R^2 + j\omega RL + j2\omega RL - \omega^2 L^2}{j\omega RL}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + \dfrac{R^2 - \omega^2 L^2}{j\omega RL}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{1}}{\mathbf{3 + j(\omega L/R - R/\omega L)}}$$

(b) Since the ratio $\dfrac{\mathbf{V}_2}{\mathbf{V}_o}$ must be real,

$$\frac{\omega_o L}{R} - \frac{R}{\omega_o L} = 0$$

$$\omega_o L = \frac{R^2}{\omega_o L}$$

$$\omega_o = 2\pi f_o = \frac{R}{L}$$

$$\mathbf{f_o = \frac{R}{2\pi L}}$$

(c) When $\omega = \omega_o$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3}$$

This must be compensated for by $\mathbf{A}_v = 3$. But

$$\mathbf{A}_v = 1 + \frac{R_2}{R_1} = 3$$

$$\mathbf{R_2 = 2R_1}$$