Chapter 17, Solution 1.

(a) This is **periodic** with $\omega = \pi$ which leads to $T = 2\pi/\omega =$ **2**.

(b) y(t) is **not periodic** although sin t and 4 cos 2πt are independently periodic.

(c) Since sin A cos B = 0.5[sin(A + B) + sin(A – B)],
g(t) = sin 3t cos 4t = 0.5[sin 7t + sin(–t)] = –0.5 sin t + 0.5 sin7t
which is harmonic or **periodic** with the fundamental frequency

$$\omega = 1 \text{ or } T = 2\pi/\omega = \mathbf{2\pi}.$$

(d) $h(t) = \cos^2 t = 0.5(1 + \cos 2t)$. Since the sum of a periodic function and a constant is also **periodic**, h(t) is periodic. $\omega = 2$ or $T = 2\pi/\omega = \boldsymbol{\pi}$.

(e) The frequency ratio 0.6|0.4 = 1.5 makes z(t) **periodic**.

$$\omega = 0.2\pi \text{ or } T = 2\pi/\omega = \mathbf{10}.$$

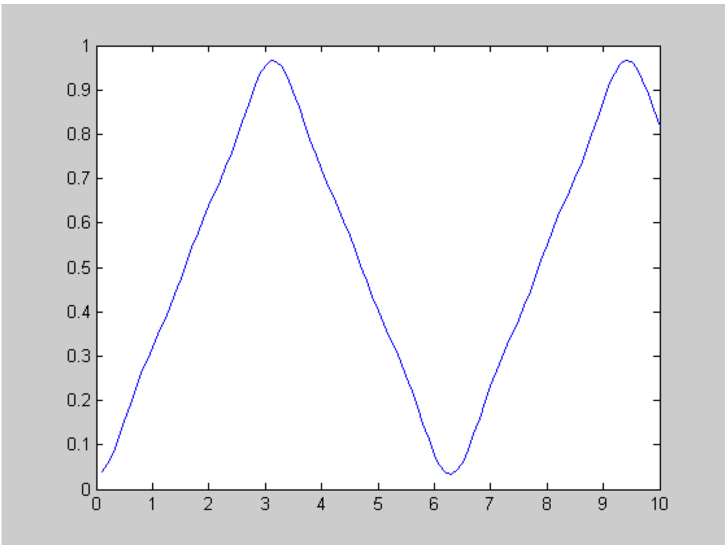
(f) p(t) = 10 is **not periodic**.

(g) g(t) is **not periodic**.

Chapter 17, Solution 2.

The function f(t) has a DC offset and is even. We use the following MATLAB code to plot f(t). The plot is shown below. If more terms are taken, the curve is clearly indicating a triangular wave shape which is easily represented with just the DC component and three, cosinusoidal terms of the expansion.

```
for n=1:100
   tn(n)=n/10;
   t=n/10;
   y1=cos(t);
   y2=(1/9)*cos(3*t);
   y3=(1/25)*cos(5*t);
   factor=4/(pi*pi);
   y(n)=0.5- factor*(y1+y2+y3);
end
plot(tn,y)
```



Chapter 17, Solution 3.

$T = 4, \ \omega_o = 2\pi/T = \pi/2$

$$g(t) = \begin{cases} 5, & 0 < t < 1 \\ 10, & 1 < t < 2 \\ 0, & 2 < t < 4 \end{cases}$$

$$a_o = (1/T)\int_0^T g(t)dt = 0.25\left[\int_0^1 5dt + \int_1^2 10dt\right] = \mathbf{3.75}$$

$$a_n = (2/T)\int_0^T g(t)\cos(n\omega_o t)dt = (2/4)\left[\int_0^1 5\cos(\frac{n\pi}{2}t)dt + \int_1^2 10\cos(\frac{n\pi}{2}t)dt\right]$$

$$= 0.5\left[5\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_0^1 + 10\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_1^2\right] = (-1/(n\pi))5\sin(n\pi/2)$$

$$a_n = \begin{cases} \mathbf{(5/(n\pi))(-1)^{(n+1)/2},} & \mathbf{n = odd} \\ \mathbf{0,} & \mathbf{n = even} \end{cases}$$

$$b_n = (2/T)\int_0^T g(t)\sin(n\omega_o t)dt = (2/4)\left[\int_0^1 5\sin(\frac{n\pi}{2}t)dt + \int_1^2 10\sin(\frac{n\pi}{2}t)dt\right]$$

$$= 0.5\left[\frac{-2x5}{n\pi}\cos\frac{n\pi}{2}t\Big|_0^1 - \frac{2x10}{n\pi}\cos\frac{n\pi}{2}t\Big|_1^2\right] = \mathbf{(5/(n\pi))[3 - 2\cos n\pi + \cos(n\pi/2)]}$$

n	$\mathbf{a_n}$	$\mathbf{b_n}$	$\mathbf{A_n}$	**phase**
1	**–1.59**	**7.95**	**8.11**	**–101.31**
2	**0**	**0**	**0**	**0**
3	**0.53**	**2.65**	**2.70**	**–78.69**
4	**0**	**0.80**	**0.80**	**–90**
5	**–0.32**	**1.59**	**1.62**	**–101.31**
6	**0**	**0**	**0**	**0**
7	**0.23**	**1.15**	**1.17**	**–78.69**
8	**0**	**0.40**	**0.40**	**–90**

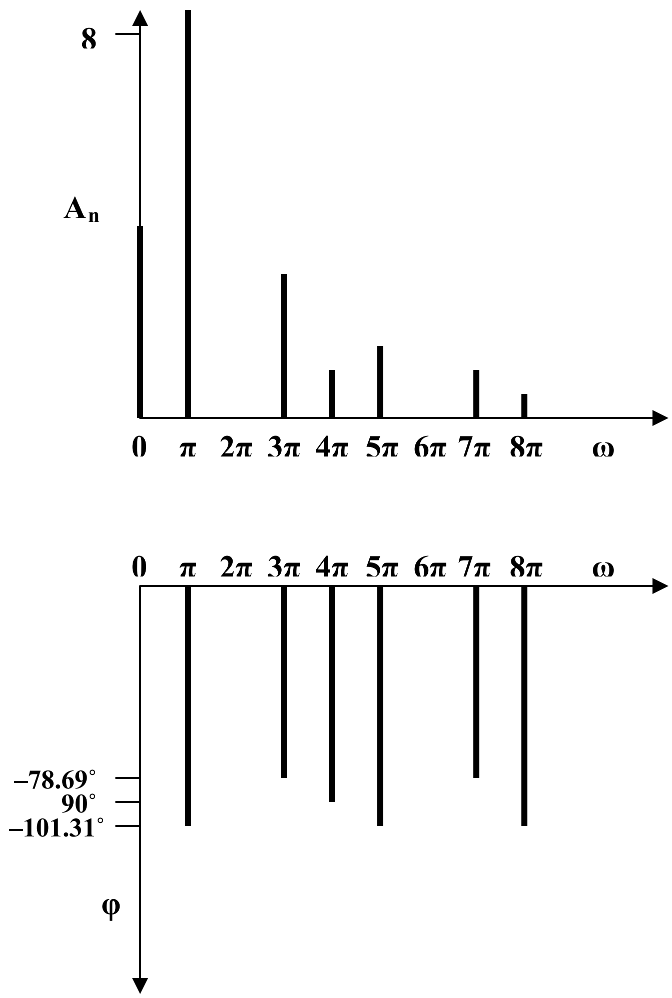


Figure D. 35
For Prob. 17.3.

Chapter 17, Solution 4.

$f(t) = 10 - 5t, \;\; 0 < t < 2, \;\; T = 2, \;\; \omega_o = 2\pi/T = \pi$

$$a_o = (1/T)\int_0^T f(t)dt = (1/2)\int_0^2 (10 - 5t)dt = 0.5[10t - (5t^2/2)]\Big|_0^2 = \mathbf{5}$$

$$a_n = (2/T)\int_0^T f(t)\cos(n\omega_o t)dt = (2/2)\int_0^2 (10 - 5t)\cos(n\pi t)dt$$

$$= \int_0^2 (10)\cos(n\pi t)dt - \int_0^2 (5t)\cos(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\cos n\pi t\Big|_0^2 + \frac{5t}{n\pi}\sin n\pi t\Big|_0^2 = [-5/(n^2\pi^2)](\cos 2n\pi - 1) = \mathbf{0}$$

$$b_n = (2/2)\int_0^2 (10 - 5t)\sin(n\pi t)dt$$

$$= \int_0^2 (10)\sin(n\pi t)dt - \int_0^2 (5t)\sin(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\sin n\pi t\Big|_0^2 + \frac{5t}{n\pi}\cos n\pi t\Big|_0^2 = 0 + [10/(n\pi)](\cos 2n\pi) = \mathbf{10/(n\pi)}$$

Hence

$$f(t) = \mathbf{5 + \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)}.$$

Chapter 17, Solution 5.

$$\mathrm{T} = 2\pi, \qquad \omega = 2\pi / \mathrm{T} = 1$$

$$a_o = \frac{1}{T}\int_0^T z(t)dt = \frac{1}{2\pi}[2x\pi - 4x\pi] = -1$$

$$a_n = \frac{2}{T}\int_0^T z(t)\cos(n\omega_o)dt = \frac{1}{\pi}\int_0^\pi 2\cos(nt)dt - \frac{1}{\pi}\int_\pi^{2\pi} 4\cos(nt)dt = \frac{2}{n\pi}\sin(nt)\Big|_0^\pi - \frac{4}{n\pi}\sin(nt)\Big|_\pi^{2\pi} = 0$$

$$b_n = \frac{2}{T}\int_0^T z(t)\sin(n\omega_o)dt = \frac{1}{\pi}\int_0^\pi 2\sin(nt)dt - \frac{1}{\pi}\int_\pi^{2\pi} 4\sin(nt)dt = -\frac{2}{n\pi}\cos(nt)\Big|_0^\pi + \frac{4}{n\pi}\cos(nt)\Big|_\pi^{2\pi}$$

$$= \begin{cases} \dfrac{12}{n\pi}, n = odd \\ 0, \; n = even \end{cases}$$

Thus,

$$\underline{\mathbf{z(t)} = -\mathbf{1} + \sum_{\substack{\mathbf{n=1} \\ \mathbf{n=odd}}}^{\infty} \frac{\mathbf{12}}{\mathbf{n\pi}}\mathbf{\sin(nt)}}$$

Chapter 17, Solution 6.

T=2π, ω_o=2π/T = 1

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} 5dt + \int_{\pi}^{2\pi} 10dt\right] = \frac{1}{2\pi}(5\pi + 10\pi) = 7.5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{2\pi}\left[\int_0^{\pi} 5\cos nt dt + \int_{\pi}^{2\pi} 10\cos nt dt\right] = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{2\pi}\left[\int_0^{\pi} 5\sin nt dt + \int_{\pi}^{2\pi} 10 s inn t dt\right] = \frac{1}{\pi}\left[-\frac{1}{n}\cos nt\Big|_0^{\pi} - \frac{1}{n}\cos nt\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{5}{n\pi}\left[\cos \pi n - 1\right] = \begin{cases} -\dfrac{10}{n\pi}, & n = odd \\ 0, & n = even \end{cases}$$

Thus,

$$\underline{f(t) = 7.5 - \sum_{n=odd}^{\infty} \frac{10}{n\pi}\sin nt}$$

Chapter 17, Solution 7.

$T = 3, \qquad \omega_o = 2\pi / T = 2\pi / 3$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{3}\left[\int_0^2 2dt + \int_2^3 (-1)dt\right] = \frac{1}{3}(4-1) = 1$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2n\pi t}{3}dt = \frac{2}{3}\left[\int_0^2 2\cos\frac{2n\pi t}{3}dt + \int_2^3 (-1)\cos\frac{2n\pi t}{3}dt\right]$$

$$= \frac{2}{3}\left[2\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\bigg|_0^2 - 1\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\bigg|_2^3\right] = \frac{3}{n\pi}\sin\frac{4n\pi}{3}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2n\pi t}{3}dt = \frac{2}{3}\left[\int_0^2 2\sin\frac{2n\pi t}{3}dt + \int_2^3 (-1)sin\frac{2n\pi t}{3}dt\right]$$

$$= \frac{2}{3}\left[-2x\frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\bigg|_0^2 + \frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\bigg|_2^3\right] = \frac{3}{n\pi}(1 - 2\cos\frac{4n\pi}{3})$$

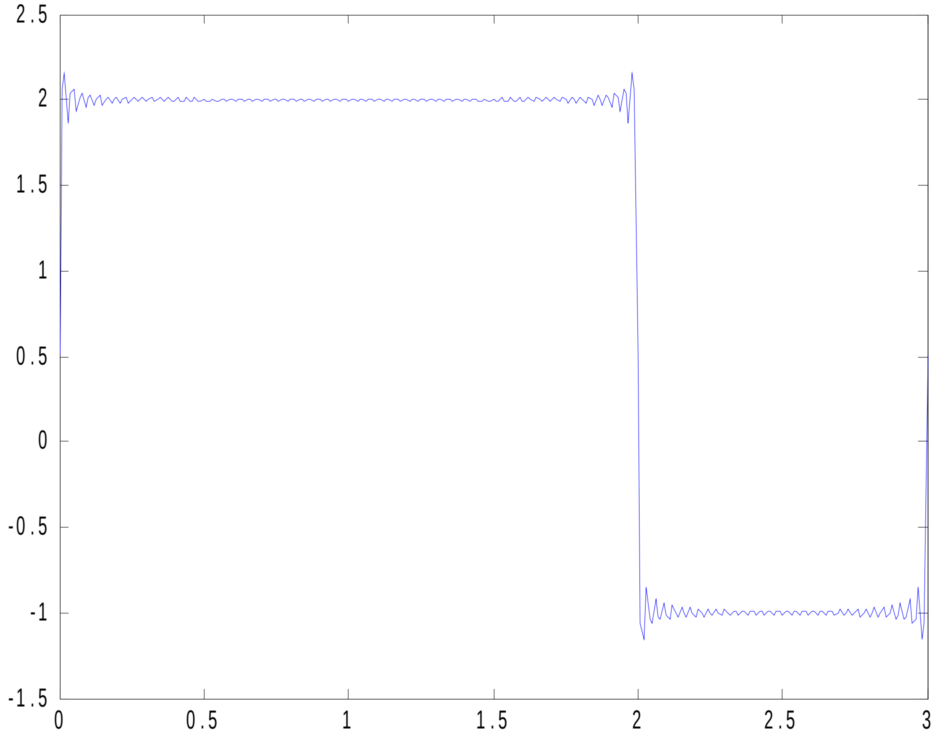
$$= \frac{1}{n\pi}\left(2 - 3\cos\frac{4n\pi}{3} + 1\right) = \frac{3}{n\pi}\left(1 - \cos\frac{4n\pi}{3}\right)$$

Hence,

$$f(t) = \underline{\mathbf{1} + \sum_{n=0}^{\infty}\left[\frac{3}{n\pi}\sin\frac{4n\pi}{3}\cos\frac{2n\pi t}{3} + \frac{3}{n\pi}\left(1 - \cos\frac{4n\pi}{3}\right)\sin\frac{2n\pi t}{3}\right]}$$

We can now use MATLAB to check our answer,

```
>> t=0:.01:3;
>> f=1*ones(size(t));
>> for n=1:1:99,
f=f+(3/(n*pi))*sin(4*n*pi/3)*cos(2*n*pi*t/3)+(3/(n*pi))*(1-
cos(4*n*pi/3))*sin(2*n*pi*t/3);
end
>> plot(t,f)
```



Clearly we have nearly the same figure we started with!!

Chapter 17, Solution 8.

Using Fig. 17.51, design a problem to help other students to better understand how to determine the exponential Fourier Series from a periodic wave shape.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain the exponential Fourier series of the function in Fig. 17.51.

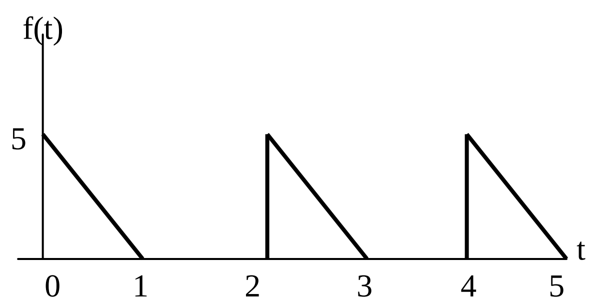


Figure 17.51 For Prob. 17.8.

Solution

$T = 2, \qquad \omega_o = 2\pi / T = \pi$

$$f(t) = \begin{cases} 5(1-t), & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{1}{2}\int_0^1 5(1-t)e^{-jn\pi t}dt$$

$$= \frac{5}{2}\int_0^1 e^{-jn\pi t}dt - \frac{5}{2}\int_0^1 te^{-jn\pi t}dt = \frac{5}{2}\frac{e^{-jn\pi t}}{-jn\pi}\Bigg|_0^1 - \frac{5}{2}\frac{e^{-jn\pi t}}{(-jn\pi)^2}(-jn\pi t - 1)\Bigg|_0^1$$

$$= \frac{5}{2}\frac{\left[e^{-jn\pi} - 1\right]}{-jn\pi} - \frac{5}{2}\frac{e^{-jn\pi}}{-n^2\pi^2}(-jn\pi - 1) + \frac{5}{2}\frac{(-1)}{-n^2\pi^2}$$

But $e^{-jn\pi} = \cos \pi n - j\sin n\pi = \cos n\pi + 0 = (-1)^n$

$$c_n = \frac{2.5[1-(-1)^n]}{jn\pi} - \frac{2.5(-1)^n[1 + jn\pi]}{n^2\pi^2} + \frac{2.5}{n^2\pi^2}$$

Chapter 17, Solution 9.

f(t) is an even function, b_n=0.

$$T = 8, \qquad \omega = 2\pi / T = \pi / 4$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{2}{8}\left[\int_0^2 10\cos\pi t/4dt + 0\right] = \frac{10}{4}(\frac{4}{\pi})\sin\pi t/4 \Big|_0^2 = \frac{10}{\pi} = 3.183$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{40}{8}[\int_0^2 10\cos\pi t/4\cos n\pi t/4dt + 0] = 5\int_0^2 [\cos\pi t(n+1)/4 + \cos\pi t(n-1)/4]dt$$

For n = 1,

$$a_1 = 5\int_0^2 [\cos\pi t/2 + 1]dt = 5\left[\frac{2}{\pi}\sin\pi t/2dt + t\right]_0^2 = 10$$

For n>1,

$$a_n = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)t}{4} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)t}{4}\Bigg|_0^2 = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)}{2} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)}{2}$$

$$a_2 = \frac{20}{3\pi}\sin 1.5\pi + \frac{20}{\pi}\sin\pi/2 = 2{,}122066\sin(270°) + 6.3662\sin(90°)$$

$$= -2.122066 + 6.3662 = 4.244, \qquad a_3 = \frac{20}{4\pi}\sin 2\pi + \frac{10}{\pi}\sin\pi = 0$$

Thus,

$$\underline{\mathbf{a_0 = 3.183, \quad a_1 = 10, \quad a_2 = 4.244, \quad a_3 = 0, \quad b_1 = 0 = b_2 = b_3}}$$

Chapter 17, Solution 10.

$T = 2\pi, \qquad \omega_o = 2\pi / T = 1$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{V_o}{2\pi}\int_0^\pi (1)e^{-jnt}dt = \frac{V_o}{2\pi}\frac{e^{=jnt}}{-jn}\Bigg|_0^\pi$$

$$= \frac{V_o}{2n\pi}\left[je^{-jn\pi} - j\right] = \frac{jV_o}{2n\pi}(\cos n\pi - 1)$$

$$f(t) = \underline{\sum_{n=-\infty}^{\infty} \frac{jV_o}{2n\pi}(\cos n\pi - 1)e^{jnt}}$$

Chapter 17, Solution 11.

$$T = 4, \qquad \omega_o = 2\pi / T = \pi / 2$$

$$c_n = \frac{1}{T}\int_0^T y(t)e^{-jn\omega_o t}dt = \frac{1}{4}\left[\int_{-1}^{0} 10(t+1)e^{-jn\pi/2}dt + \int_0^1 (10)e^{-jn\pi/2}dt\right]$$

$$c_n = \frac{10}{4}\left[\frac{e^{-jn\pi t/2}}{-n^2\pi^2/4}(-jn\pi t/2 - 1) - \frac{2}{jn\pi}e^{-jn\pi t/2}\Big|_{-1}^{0} - \frac{2}{jn\pi}e^{-jn\pi t/2}\Big|_{0}^{1}\right]$$

$$= \frac{10}{4}\left[\frac{4}{n^2\pi^2} - \frac{2}{jn\pi} + \frac{4}{n^2\pi^2}e^{jn\pi/2}(jn\pi/2 - 1) + \frac{2}{jn\pi}e^{jn\pi/2} - \frac{2}{jn\pi}e^{-jn\pi/2} + \frac{2}{jn\pi}\right]$$

But

$$e^{jn\pi/2} = \cos(n\pi/2) + j\sin(n\pi/2) = j\sin(n\pi/2),$$

$$e^{-jn\pi/2} = \cos(n\pi/2) - j\sin(n\pi/2) = -j\sin(n\pi/2)$$

$$c_n = \frac{10}{n^2\pi^2}\left[1 + j(jn\pi/2 - 1)\sin(n\pi/2) + n\pi\sin(n\pi/2)\right]$$

$$\mathbf{y(t)} = \underline{\sum_{n=-\infty}^{\infty} \frac{10}{n^2\pi^2}\left[1 + j([jn\pi/2] - 1)\sin(n\pi/2) + n\pi\sin(n\pi/2)\right] e^{jn\pi t/2}}$$

Chapter 17, Solution 12.

A voltage source has a periodic waveform defined over its period as
$v(t) = 10t(2\pi - t)$ V, for all $0 < t < 2\pi$

Find the Fourier series for this voltage.

$v(t) = 10(2\pi t - t^2)$, $0 < t < 2\pi$, $T = 2\pi$, $\omega_o = 2\pi/T = 1$

$a_o =$

$$(1/T)\int_0^T f(t)dt = \frac{1}{2\pi}\int_0^{2\pi} 10(2\pi t - t^2)dt$$

$$= \frac{10}{2\pi}(\pi t^2 - t^3/3)\Big|_0^{2\pi} = \frac{40\pi^3}{2\pi}(1 - 2/3) = \frac{20\pi^2}{3}$$

$$a_n = \frac{2}{T}\int_0^T 10(2\pi t - t^2)\cos(nt)dt = \frac{10}{\pi}\left[\frac{2\pi}{n^2}\cos(nt) + \frac{2\pi t}{n}\sin(nt)\right]\Bigg|_0^{2\pi}$$

$$-\frac{10}{\pi n^3}\left[2nt\cos(nt) - 2\sin(nt) + n^2 t^2 \sin(nt)\right]_0^{2\pi}$$

$$= \frac{20}{n^2}(1-1) - \frac{10}{\pi n^3}4n\pi\cos(2\pi n) = \frac{-40}{n^2}$$

$$b_n = \frac{20}{T}\int_0^T (2nt - t^2)\sin(nt)dt = \frac{10}{\pi}\int_0^T (2nt - t^2)\sin(nt)dt$$

$$= \frac{2n}{\pi}\frac{10}{n^2}(\sin(nt) - nt\cos(nt))\Big|_0^{\pi} - \frac{10}{\pi n^3}(2nt\sin(nt) + 2\cos(nt) - 1n^2t^2\cos(nt))\Big|_0^{2\pi}$$

$$= \frac{-40\pi}{n} + \frac{40\pi}{n} = 0$$

Hence, $$\mathbf{f(t) = \frac{20\pi^2}{3} - \sum_{n=1}^{\infty}\frac{40}{n^2}\cos(nt)}$$

Chapter 17, Solution 13.

Design a problem to help other students to better understand obtaining the Fourier series from a periodic function.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A periodic function is defined over its period as

$$h(t) = \begin{cases} 10\sin t, & 0 < t < \pi \\ 20\sin(t-\pi), & \pi < t < 2\pi \end{cases}$$

Find the Fourier series of h(t).

Solution

$T = 2\pi,\ \omega_o = 1$

$$a_o = (1/T)\int_0^T h(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} 10\sin t\,dt + \int_{\pi}^{2\pi} 20\sin(t-\pi)\,dt\right]$$

$$= \frac{1}{2\pi}\left[-10\cos t\Big|_0^{\pi} - 20\cos(t-\pi)\Big|_{\pi}^{2\pi}\right] = \frac{30}{\pi}$$

$$a_n = (2/T)\int_0^T h(t)\cos(n\omega_o t)dt$$

$$= [2/(2\pi)]\left[\int_0^{\pi} 10\sin t\cos(nt)dt + \int_{\pi}^{2\pi} 20\sin(t-\pi)\cos(nt)dt\right]$$

Since $\sin A\cos B = 0.5[\sin(A+B) + \sin(A-B)]$

$\sin t\cos nt = 0.5[\sin((n+1)t) + \sin((1-n))t]$

$\sin(t-\pi) = \sin t\cos\pi - \cos t\sin\pi = -\sin t$

$\sin(t-\pi)\cos(nt) = -\sin(t)\cos(nt)$

$$a_n = \frac{1}{2\pi}\left[10\int_0^{\pi}[\sin([1+n]t) + \sin([1-n]t)]dt - 20\int_{\pi}^{2\pi}[\sin([1+n]t) + \sin([1-n]t)]dt\right]$$

$$= \frac{5}{\pi}\left[\left(-\frac{\cos([1+n]t)}{1+n} - \frac{\cos([1-n]t)}{1-n}\right)\Bigg|_0^{\pi} + \left(\frac{2\cos([1+n]t)}{1+n} + \frac{2\cos([1-n]t)}{1-n}\right)\Bigg|_{\pi}^{2\pi}\right]$$

$$a_n = \frac{5}{\pi}\left[\frac{3}{1+n} + \frac{3}{1-n} - \frac{3\cos([1+n]\pi)}{1+n} - \frac{3\cos([1-n]\pi)}{1-n}\right]$$

But, $[1/(1+n)] + [1/(1-n)] = 2/(1-n^2)$

$$\cos([n-1]\pi) = \cos([n+1]\pi) = \cos\pi\cos n\pi - \sin\pi\sin n\pi = -\cos n\pi$$

$$a_n = (5/\pi)[(6/(1-n^2)) + (6\cos(n\pi)/(1-n^2))]$$

$$= [30/(\pi(1-n^2))](1+\cos n\pi) = \mathbf{[-60/(\pi(n^2-1))], n = even}$$
$$\mathbf{= 0, \quad n = odd}$$

$$b_n = (2/T)\int_0^T h(t)\sin n\omega_o t\, dt$$

$$= [2/(2\pi)][\int_0^{\pi} 10\sin t\sin nt\, dt + \int_{\pi}^{2\pi} 20(-\sin t)\sin nt\, dt$$

This is an interesting function which will have a value for b_1 but not for any of the other b_n terms (they will be zero).

$$b_1 = [2/(2\pi)][\left(\int_0^{\pi} 10\sin t\sin t\, dt = 10\int_0^{\pi}\frac{1-\cos(2t)}{2}dt = 5\pi\right.$$
$$\left.+\int_{\pi}^{2\pi} 20(-\sin t)\sin t\, dt = -20\int_{\pi}^{2\pi}(\sin t)^2 dt = -10\pi\right) = -5$$

Now we can calculate the rest of the b_n for values of n = 2 and greater than 2. We note that,

$$\sin A\sin B = 0.5[\cos(A-B) - \cos(A+B)]$$

$$\sin t\sin nt = 0.5[\cos([1-n]t) - \cos([1+n]t)]$$

$$b_n = (5/\pi)\{[(\sin([1-n]t)/(1-n)) - (\sin([1+n]t)/(1+n)]\Big|_0^{\pi}$$

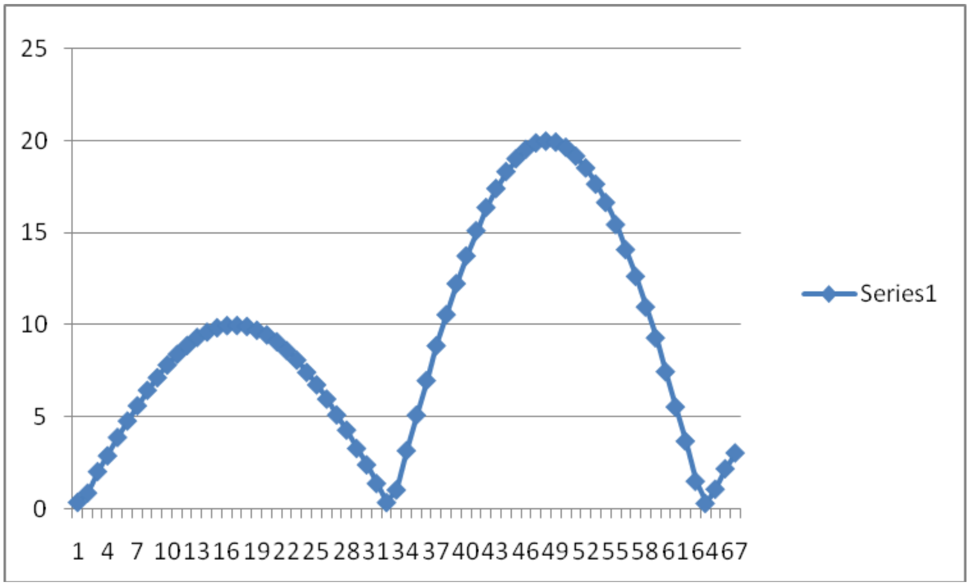
$$+ [(2\sin([1-n]t)/(1-n)) - (2\sin([1+n]t)/(1+n)]\Big|_{\pi}^{2\pi}\}$$

$$= \frac{5}{\pi}\left[-\frac{\sin([1-n]\pi)}{1-n} + \frac{\sin([1+n]\pi)}{1+n}\right] = 0$$

{Note, that if we substitute 1 for n, the first term is undefined!}

Thus, $$h(t) = \frac{30}{\pi} - 5\sin(t) - \frac{60}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2kt)}{(4k^2-1)}$$

This does make a very good approximation!



Chapter 17, Solution 14.

Since cos(A + B) = cos A cos B – sin A sin B.

$$f(t) = 5 + \sum_{n=1}^{\infty} \left(\frac{25}{n^3 + 1} \cos(n\pi / 4) \cos(2nt) - \frac{25}{n^3 + 1} \sin(n\pi / 4) \sin(2nt) \right)$$

Chapter 17, Solution 15.

(a) $$\text{Dcos } \omega t + \text{Esin } \omega t = \text{A cos}(\omega t - \theta)$$

where $$A = \sqrt{D^2 + E^2}, \quad \theta = \tan^{-1}(E/D)$$

$$A = \sqrt{\frac{16}{(n^2 + 1)^2} + \frac{1}{n^6}}, \quad \theta = \tan^{-1}((n^2+1)/(4n^3))$$

$$f(t) = \mathbf{10} + \sum_{n=1}^{\infty} \sqrt{\frac{16}{(n^2 + 1)^2} + \frac{1}{n^6}} \cos\left(10nt - \tan^{-1} \frac{n^2 + 1}{4n^3}\right)$$

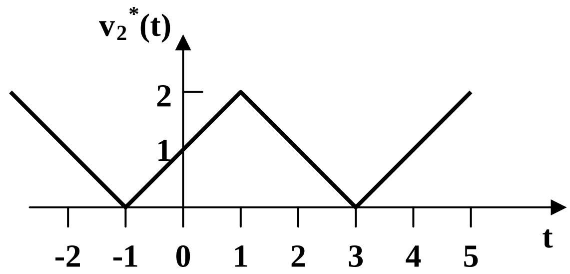
(b) $$\text{Dcos } \omega t + \text{Esin } \omega t = \text{A sin}(\omega t + \theta)$$

where $$A = \sqrt{D^2 + E^2}, \quad \theta = \tan^{-1}(D/E)$$

$$f(t) = \mathbf{10} + \sum_{n=1}^{\infty} \sqrt{\frac{16}{(n^2 + 1)^2} + \frac{1}{n^6}} \sin\left(10nt + \tan^{-1} \frac{4n^3}{n^2 + 1}\right)$$

Chapter 17, Solution 16.

If $v_2(t)$ is shifted by 1 along the vertical axis, we obtain $v_2^*(t)$ shown below, i.e. $v_2^*(t) = v_2(t) + 1$.



Comparing $v_2^*(t)$ with $v_1(t)$ shows that

$$v_2^*(t) = 2v_1((t + t_o)/2)$$

where $(t + t_o)/2 = 0$ at $t = -1$ or $t_o = 1$

Hence $$v_2^*(t) = 2v_1((t + 1)/2)$$

But $$v_2^*(t) = v_2(t) + 1$$

$$v_2(t) + 1 = 2v_1((t+1)/2)$$

$$v_2(t) = -1 + 2v_1((t+1)/2)$$

$$= -1 + 1 - \frac{8}{\pi^2}\left[\cos \pi\left(\frac{t+1}{2}\right) + \frac{1}{9}\cos 3\pi\left(\frac{t+1}{2}\right) + \frac{1}{25}\cos 5\pi\left(\frac{t+1}{2}\right) + \cdots\right]$$

$$v_2(t) = -\frac{8}{\pi^2}\left[\cos\left(\frac{\pi t}{2} + \frac{\pi}{2}\right) + \frac{1}{9}\cos\left(\frac{3\pi t}{2} + \frac{3\pi}{2}\right) + \frac{1}{25}\cos\left(\frac{5\pi t}{2} + \frac{5\pi}{2}\right) + \cdots\right]$$

$$v_2(t) = -\frac{\mathbf{8}}{\boldsymbol{\pi}^2}\left[\mathbf{sin}\left(\frac{\boldsymbol{\pi}\mathbf{t}}{\mathbf{2}}\right) + \frac{\mathbf{1}}{\mathbf{9}}\mathbf{sin}\left(\frac{\mathbf{3}\boldsymbol{\pi}\mathbf{t}}{\mathbf{2}}\right) + \frac{\mathbf{1}}{\mathbf{25}}\mathbf{sin}\left(\frac{\mathbf{5}\boldsymbol{\pi}\mathbf{t}}{\mathbf{2}}\right) + \cdots\right]$$

Chapter 17, Solution 17.

We replace t by –t in each case and see if the function remains unchanged.

(a) $1 - t$, **neither odd nor even.**

(b) $t^2 - 1$, **even**

(c) $\cos n\pi(-t) \sin n\pi(-t) = -\cos n\pi t \sin n\pi t$, **odd**

(d) $\sin^2 n(-t) = (-\sin \pi t)^2 = \sin^2 \pi t$, **even**

(e) e^t, **neither odd nor even**.

Chapter 17, Solution 18.

(a) $T = 2$ leads to $\omega_o = 2\pi/T = \pi$

$f_1(-t) = -f_1(t)$, showing that $f_1(t)$ is **odd and half-wave symmetric**.

(b) $T = 3$ leads to $\omega_o = 2\pi/3$

$f_2(t) = f_2(-t)$, showing that $f_2(t)$ is **even**.

(c) $T = 4$ leads to $\omega_o = \pi/2$

$f_3(t)$ is **even and half-wave symmetric**.

Chapter 17, Solution 19.

$T = 4, \qquad \omega_o = 2\pi / T = \pi / 2$

$$f(t) = \begin{cases} 10t, & 0 < t < 1 \\ 10(2-t), & 1 < t < 2 \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\int_0^1 10t\,dt + \frac{1}{4}\int_1^2 10(2-t)dt = \frac{1}{4}5t^2\Big|_0^1 + \frac{10}{4}(2t - \frac{t^2}{2})\Big|_1^2 = 2.5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t\,dt = \frac{2}{4}\int_0^1 10t\cos n\omega_o t\,dt + \frac{2}{4}\int_1^2 10(2-t)\cos n\omega_o t\,dt$$

$$= \frac{20}{n\omega_o}\cos n\omega_o t + \frac{t}{n\omega_o}\sin n\omega_o t\Big|_0^1 + \frac{10}{n\omega_o}\sin n\omega_o t\Big|_1^2 + \frac{5}{n^2\omega_o^2}\cos n\omega_o t + \frac{5t}{n\omega_o}\sin n\omega_o t\Big|_1^2$$

$$= \frac{20}{n\omega_o}(\cos n\pi/2 - 1) + \frac{1}{n\omega_o}\sin n\pi/2 + \frac{10}{n\omega_o}(\sin n\pi - \sin n\pi/2) + \frac{5}{n^2\pi^2/4}\cos n\pi$$

$$- \frac{5}{n^2\pi^2/4}\cos n\pi/2 + \frac{10}{n\omega_o}\sin n\pi - \frac{5}{n\pi/2}\sin n\pi/2$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t\,dt = \frac{2}{4}\int_0^1 10t\sin n\omega_o t\,dt + \frac{2}{4}\int_1^2 10(2-t)\sin n\omega_o t\,dt$$

$$= \frac{5}{n\omega_o}\sin n\omega_o t\Big|_0^1 - \frac{10}{n\omega_o}\cos n\omega_o t\Big|_0^1 - \frac{5}{n^2\omega_o^2}\sin n\omega_o t\Big|_1^2 + \frac{t}{n\omega_o}\cos n\omega_o t\Big|_1^2$$

$$= \frac{5}{n^2\omega_o^2}\sin n\pi/2 - \frac{10}{n\omega_o}(\cos\pi n - \cos n\pi/2) - \frac{5}{n^2\omega_o^2}(\sin\pi n - \sin n\pi/2)$$

$$- \frac{2}{n\omega_o}\cos n\pi - \frac{\cos\pi n/2}{n\omega_o}$$

Chapter 17, Solution 20.

This is an even function.

$$b_n = 0, \ T = 6, \ \omega = 2\pi/6 = \pi/3$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{6}\left[\int_1^2 (4t - 4)dt \int_2^3 4\, dt\right]$$

$$= \frac{1}{3}\left[(2t^2 - 4t)\Big|_1^2 + 4(3-2)\right] = 2$$

$$a_n = \frac{4}{T}\int_0^{T/4} f(t)\cos(n\pi t/3)dt$$

$$= (4/6)\left[\int_1^2 (4t - 4)\cos(n\pi t/3)dt + \int_2^3 4\cos(n\pi t/3)dt\right]$$

$$= \frac{16}{6}\left[\frac{9}{n^2\pi^2}\cos\left(\frac{n\pi t}{3}\right) + \frac{3t}{n\pi}\sin\left(\frac{n\pi t}{3}\right) - \frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_1^2 + \frac{16}{6}\left[\frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_2^3$$

$$= [24/(n^2\pi^2)][\cos(2n\pi/3) - \cos(n\pi/3)]$$

Thus $$f(t) = \mathbf{2} + \frac{\mathbf{24}}{\pi^2}\sum_{n=1}^{\infty}\frac{\mathbf{1}}{\mathbf{n}^2}\left[\cos\left(\frac{2\pi n}{3}\right) - \cos\left(\frac{\pi n}{3}\right)\right]\cos\left(\frac{n\pi t}{3}\right)$$

At $t = 2$,

$$f(2) = 2 + (24/\pi^2)[(\cos(2\pi/3) - \cos(\pi/3))\cos(2\pi/3)$$

$$+ (1/4)(\cos(4\pi/3) - \cos(2\pi/3))\cos(4\pi/3)$$

$$+ (1/9)(\cos(2\pi) - \cos(\pi))\cos(2\pi) + \text{-----}]$$

$$= 2 + 2.432(0.5 + 0 + 0.2222 + \text{-----})$$

$f(2) = \mathbf{3.756}$

Chapter 17, Solution 21.

This is an even function.

$b_n = 0, \; T = 4, \; \omega_o = 2\pi/T = \pi/2.$

$$f(t) = \begin{cases} 2 - 2t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$a_o = \frac{2}{4}\int_0^1 2(1-t)dt = \left[t - \frac{t^2}{2}\right]_0^1 = 0.5$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{4}\int_0^1 2(1-t)\cos\left(\frac{n\pi t}{2}\right)dt$$

$$= [8/(\pi^2 n^2)][1 - \cos(n\pi/2)]$$

$$f(t) = \mathbf{\frac{1}{2} + \sum_{n=1}^{\infty} \frac{8}{n^2\pi^2}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]\cos\left(\frac{n\pi t}{2}\right)}$$

Chapter 17, Solution 22.

Calculate the Fourier coefficients for the function in Fig. 16.54.

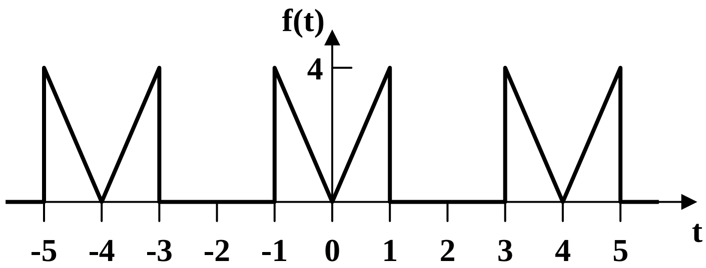


Figure 16.54 For Prob. 16.15

This is an even function, therefore $b_n = 0$. In addition, T=4 and $\omega_o = \pi/2$.

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{4}\int_0^1 4t dt = t^2\Big|_0^1 = \underline{\mathbf{1}}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(\omega_o nt)dt = \frac{4}{4}\int_0^1 4t\cos(n\pi t/2)dt$$

$$= 4\left[\frac{4}{n^2\pi^2}\cos(n\pi t/2) + \frac{2t}{n\pi}\sin(n\pi t/2)\right]_0^1$$

$$a_n = \mathbf{\frac{16}{n^2\pi^2}(\cos(n\pi/2) - 1) + \frac{8}{n\pi}\sin(n\pi/2)}$$

Chapter 17, Solution 23.

Using Fig. 17.61, design a problem to help other students to better understand finding the Fourier series of a periodic wave shape.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Fourier series of the function shown in Fig. 17.61.

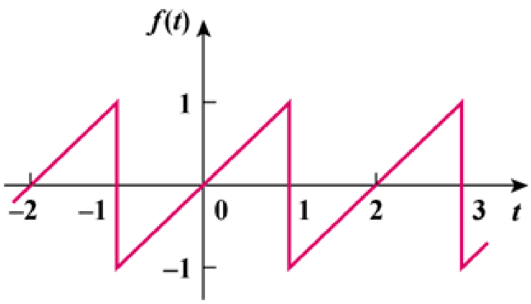


Figure 17.61

Solution

f(t) is an odd function.

$f(t) = t, \quad -1 < t < 1$

$a_o = 0 = a_n, \quad T = 2, \quad \omega_o = 2\pi/T = \pi$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{2}\int_0^1 t\sin(n\pi t)dt$$

$$= \frac{2}{n^2\pi^2}\left[\sin(n\pi t) - n\pi t\cos(n\pi t)\right]_0^1$$

$$= -[2/(n\pi)]\cos(n\pi) = 2(-1)^{n+1}/(n\pi)$$

$$f(t) = \mathbf{\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(n\pi t)}$$

Chapter 17, Solution 24.

(a) This is an odd function.

$$a_o = 0 = a_n, \quad T = 2\pi, \quad \omega_o = 2\pi/T = 1$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(\omega_o nt)dt$$

$$f(t) = 1 + t/\pi, \qquad 0 < t < \pi$$

$$b_n = \frac{4}{2\pi}\int_0^{\pi}(1 + t/\pi)\sin(nt)dt$$

$$= \frac{2}{\pi}\left[-\frac{1}{n}\cos(nt) + \frac{1}{n^2\pi}\sin(nt) - \frac{t}{n\pi}\cos(nt)\right]_0^{\pi}$$

$$= [2/(n\pi)][1 - 2\cos(n\pi)] = [2/(n\pi)][1 + 2(-1)^{n+1}]$$

a_2 = <u>**0**</u>, $b_2 = [2/(2\pi)][1 + 2(-1)] = -1/\pi =$ **–0.3183**

(b) $\omega n = n\omega_o = 10$ or $n = 10$

$$a_{10} = 0, \quad b_{10} = [2/(10\pi)][1 - \cos(10\pi)] = -1/(5\pi)$$

Thus the magnitude is $A_{10} = \sqrt{a_{10}^2 + b_{10}^2} = 1/(5\pi) =$ **0.06366**

and the phase is $\phi_{10} = \tan^{-1}(b_n/a_n) =$ **–90°**

(c) $$f(t) = \sum_{n=1}^{\infty}\frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(nt)\,\pi$$

$$f(\pi/2) = \sum_{n=1}^{\infty}\frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(n\pi/2)\,\pi$$

For n = 1, $f_1 = (2/\pi)(1 + 2) = 6/\pi$

For n = 2, $f_2 = 0$

For n = 3, $f_3 = [2/(3\pi)][1 - 2\cos(3\pi)]\sin(3\pi/2) = -6/(3\pi)$

For n = 4, $f_4 = 0$

For $n = 5$, $f_5 = 6/(5\pi)$, ----

Thus, $f(\pi/2) = 6/\pi - 6/(3\pi) + 6/(5\pi) - 6/(7\pi)$ ---------

$= (6/\pi)[1 - 1/3 + 1/5 - 1/7 + \text{--------}]$

$f(\pi/2) \cong \mathbf{1.3824}$

which is within 8% of the exact value of 1.5.

(d) From part (c)

$f(\pi/2) = 1.5 = (6/\pi)[1 - 1/3 + 1/5 - 1/7 + \text{- - -}]$

$(3/2)(\pi/6) = [1 - 1/3 + 1/5 - 1/7 + \text{- - -}]$

or $\pi/4 = \mathbf{1 - 1/3 + 1/5 - 1/7 + \text{- - -}}$

Chapter 17, Solution 25.

This is a half-wave (odd) function since $f(t–T/2) = –f(t)$.

$a_o = 0$, $a_n = b_n = 0$ for n = even, $T = 3$, $\omega_o = 2\pi/3$.

For n = odd,

$$a_n = \frac{4}{3}\int_0^{1.5} f(t)\cos n\omega_0 t\,dt = \frac{4}{3}\int_0^{1} t\cos n\omega_0 t\,dt$$

$$= \frac{4}{3}\left[\frac{9}{4\pi^2 n^2}\cos\left(\frac{2\pi nt}{3}\right) + \frac{3t}{2\pi n}\sin\left(\frac{2\pi nt}{3}\right)\right]_0^1$$

$$= \left[\frac{3}{\pi^2 n^2}\left(\cos\left(\frac{2\pi n}{3}\right) - 1\right) + \frac{2}{\pi n}\sin\left(\frac{2\pi n}{3}\right)\right]$$

$$b_n = \frac{4}{3}\int_0^{1.5} f(t)\sin(n\omega_o t)dt = \frac{4}{3}\int_0^{1} t\sin(2\pi nt/3)dt$$

$$= \frac{4}{3}\left[\frac{9}{4\pi^2 n^2}\sin\left(\frac{2\pi nt}{3}\right) - \frac{3t}{2n\pi}\cos\left(\frac{2\pi nt}{3}\right)\right]_0^1$$

$$= \left[\frac{3}{\pi^2 n^2}\sin\left(\frac{2\pi n}{3}\right) - \frac{2}{\pi n}\cos\left(\frac{2\pi n}{3}\right)\right]$$

$$f(t) = \sum_{\substack{n=1\\ n=odd}}^{\infty} \left\{ \begin{array}{l} \left[\frac{3}{\pi^2 n^2}\left(\cos\left(\frac{2\pi n}{3}\right) - 1\right) + \frac{2}{\pi n}\sin\left(\frac{2\pi n}{3}\right)\right]\cos\left(\frac{2\pi nt}{3}\right) \\ + \left[\frac{3}{\pi^2 n^2}\sin\left(\frac{2\pi n}{3}\right) - \frac{2}{n\pi}\cos\left(\frac{2\pi n}{3}\right)\right]\sin\left(\frac{2\pi nt}{3}\right) \end{array} \right\}$$

Chapter 17, Solution 26.

$$T = 4, \ \omega_o = 2\pi/T = \pi/2$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\left[\int_0^1 1\,dt + \int_1^3 2\,dt + \int_3^4 1\,dt\right] = 1$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_o t)dt$$

$$a_n = \frac{2}{4}\left[\int_1^2 1\cos(n\pi t/2)dt + \int_2^3 2\cos(n\pi t/2)dt + \int_3^4 1\cos(n\pi t/2)dt\right]$$

$$= 2\left[\frac{2}{n\pi}\sin\frac{n\pi t}{2}\bigg|_1^2 + \frac{4}{n\pi}\sin\frac{n\pi t}{2}\bigg|_2^3 + \frac{2}{n\pi}\sin\frac{n\pi t}{2}\bigg|_3^4\right]$$

$$= \frac{4}{n\pi}\left[\sin\frac{3n\pi}{2} - \sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_o t)dt$$

$$= \frac{2}{4}\left[\int_1^2 1\sin\frac{n\pi t}{2}dt + \int_2^3 2\sin\frac{n\pi t}{2}dt + \int_3^4 1\sin\frac{n\pi t}{2}dt\right]$$

$$= 2\left[-\frac{2}{n\pi}\cos\frac{n\pi t}{2}\bigg|_1^2 - \frac{4}{n\pi}\cos\frac{n\pi t}{2}\bigg|_2^3 - \frac{2}{n\pi}\cos\frac{n\pi t}{2}\bigg|_3^4\right]$$

$$= \frac{4}{n\pi}[\cos(n\pi) - 1]$$

Hence

$f(t) =$

$$\mathbf{1 + \sum_{n=1}^{\infty}\frac{4}{n\pi}\left[(\sin(3n\pi/2) - \sin(n\pi/2))\cos(n\pi t/2) + (\cos(n\pi) - 1)\sin(n\pi t/2)\right]}$$

Chapter 17, Solution 27.

(a) **odd** symmetry.

(b) $a_o = 0 = a_n,\ T = 4,\ \omega_o = 2\pi/T = \pi/2$

$$f(t) = t, \quad 0 < t < 1$$

$$= 0, \quad 1 < t < 2$$

$$b_n = \frac{4}{4}\int_0^1 t \sin\frac{n\pi t}{2}\,dt = \left[\frac{4}{n^2\pi^2}\sin\frac{n\pi t}{2} - \frac{2t}{n\pi}\cos\frac{n\pi t}{2}\right]_0^1$$

$$= \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{2}{n\pi}\cos\frac{n\pi}{2} - 0$$

$$= 4(-1)^{(n-1)/2}/(n^2\pi^2), \quad n = \text{odd}$$

$$-2(-1)^{n/2}/(n\pi), \quad n = \text{even}$$

$$a_3 = \mathbf{0},\ b_3 = 4(-1)/(9\pi^2) = \mathbf{-0.04503}$$

(c) $b_1 = 4/\pi^2,\ b_2 = 1/\pi,\ b_3 = -4/(9\pi^2),\ b_4 = -1/(2\pi),\ b_5 = 4/(25\pi^2)$

$$F_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum\left(a_n^2 + b_n^2\right)}$$

$$F_{rms}^2 = 0.5\Sigma b_n^2 = [1/(2\pi^2)][(16/\pi^2) + 1 + (16/(81\pi^2)) + (1/4) + (16/(625\pi^2))]$$

$$= (1/19.729)(2.6211 + 0.27 + 0.00259)$$

$$F_{rms} = \sqrt{0.14667} = \mathbf{0.383}$$

Compare this with the exact value of $F_{rms} = \sqrt{\frac{2}{T}\int_0^1 t^2 dt} = \sqrt{1/6} = 0.4082$ or

(0.383/0.4082)x100 = 93.83%, close.

Chapter 17, Solution 28.

This is half-wave symmetric since $f(t - T/2) = -f(t)$.

$$a_o = 0, \ T = 2, \ \omega_o = 2\pi/2 = \pi$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{2}\int_0^1 (2-2t)\cos(n\pi t)dt$$

$$= 4\left[\frac{1}{n\pi}\sin(n\pi t) - \frac{1}{n^2\pi^2}\cos(n\pi t) - \frac{t}{n\pi}\sin(n\pi t)\right]_0^1$$

$$= [4/(n^2\pi^2)][1 - \cos(n\pi)] = \begin{cases} 8/(n^2\pi^2), & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$$

$$b_n = 4\int_0^1 (1-t)\sin(n\pi t)dt$$

$$= 4\left[-\frac{1}{n\pi}\cos(n\pi t) - \frac{1}{n^2\pi^2}\sin(n\pi t) + \frac{t}{n\pi}\cos(n\pi t)\right]_0^1$$

$$= 4/(n\pi), \ n = \text{odd}$$

$$f(t) = \boldsymbol{\sum_{k=1}^{\infty}\left(\frac{8}{n^2\pi^2}\cos(n\pi t) + \frac{4}{n\pi}\sin(n\pi t)\right), n = 2k - 1}$$

Chapter 17, Solution 29.

This function is half-wave symmetric.

$$T = 2\pi, \ \omega_o = 2\pi/T = 1, \ f(t) = -t, \ 0 < t < \pi$$

For odd n,

$$a_n = \frac{2}{T}\int_0^{\pi}(-t)\cos(nt)dt = -\frac{2}{n^2\pi}\left[\cos(nt) + nt\sin(nt)\right]_0^{\pi} = 4/(n^2\pi)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi}(-t)\sin(nt)dt = -\frac{2}{n^2\pi}\left[\sin(nt) - nt\cos(nt)\right]_0^{\pi} = -2/n$$

Thus,

$$f(t) = \mathbf{2\sum_{k=1}^{\infty}\left[\frac{2}{n^2\pi}\cos(nt) - \frac{1}{n}\sin(nt)\right]}, \qquad \mathbf{n = 2k - 1}$$

Chapter 17, Solution 30.

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_o t}dt = \frac{1}{T}\left[\int_{-T/2}^{T/2} f(t)\cos n\omega_o t dt - j\int_{-T/2}^{T/2} f(t)\sin n\omega_o t dt\right] \qquad (1)$$

(a) The second term on the right hand side vanishes if f(t) is even. Hence

$$\mathbf{c_n = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_o t dt}$$

(b) The first term on the right hand side of (1) vanishes if f(t) is odd. Hence,

$$\mathbf{c_n = -\frac{j2}{T}\int_0^{T/2} f(t)\sin n\omega_o t dt}$$

Chapter 17, Solution 31.

If $\mathbf{h(t) = f(\alpha t)}$, $\quad \mathbf{T' = T/\alpha} \quad \longrightarrow \quad \omega_o' = \frac{2\pi}{T'} = \frac{2\pi}{T/\alpha} = \underline{\alpha\omega_o}$

$$\mathbf{a_n' = \frac{2}{T'}\int_0^{T'} h(t)\cos n\omega_o' t dt = \frac{2}{T'}\int_0^{T'} f(\alpha t)\cos n\omega_o' t dt}$$

Let $\alpha \mathbf{t} = \lambda,,$ $\quad \mathbf{dt = d\lambda/\alpha,} \quad \alpha\mathbf{T' = T}$

$$\mathbf{a_n' = \frac{2\alpha}{T}\int_0^{T} f(\lambda)\cos n\omega_o \lambda d\lambda/\alpha = a_n}$$

Similarly, $\quad \underline{\mathbf{b_n' = b_n}}$

Chapter 17, Solution 32.

When $i_s = 1$ (DC component)

$$i = 1/(1 + 2) = 1/3$$

For $n \geq 1$, $\omega_n = 3n$, $I_s = 1/n^2\angle 0°$

$$I = [1/(1 + 2 + j\omega_n^2)]I_s = I_s/(3 + j6n)$$

$$= \frac{\frac{1}{n^2}\angle 0°}{3\sqrt{1+4n^2}\angle\tan^{-1}(6n/3)} = \frac{1}{3n^2\sqrt{1+4n^2}}\angle -\tan(2n)$$

Thus,

$$i(t) = \mathbf{\frac{1}{3} + \sum_{n=1}^{\infty} \frac{1}{3n^2\sqrt{1+4n^2}} \cos(3n - \tan^{-1}(2n))}$$

Chapter 17, Solution 33.

For the DC case, the inductor acts like a short, $V_o = 0$.

For the AC case, we obtain the following:

$$\frac{V_o - V_s}{10} + \frac{V_o}{j2n\pi} + \frac{jn\pi V_o}{4} = 0$$

$$\left(1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)\right)V_o = V_s$$

$$V_o = \frac{V_s}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)}$$

$$A_n\angle\Theta_n = \frac{4}{n\pi}\frac{1}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)} = \frac{4}{n\pi + j(2.5n^2\pi^2 - 5)}$$

$$\mathbf{A_n = \frac{4}{\sqrt{n^2\pi^2 + (2.5n^2\pi^2 - 5)^2}}; \;\; \Theta_n = -\tan^{-1}\left(\frac{2.5n^2\pi^2 - 5}{n\pi}\right)}$$

$$\mathbf{v_o(t) = \underline{\sum_{n=1}^{\infty} A_n \sin(n\pi t + \Theta_n)}\,V}$$

Chapter 17, Solution 34.

Using Fig. 17.70, design a problem to help other students to better understand circuit responses to a Fourier series.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain $v_o(t)$ in the network of Fig. 17.70 if

$$v(t) = \sum_{n=1}^{\infty} \frac{10}{n^2} \cos\left(nt + \frac{n\pi}{4} \right) V$$

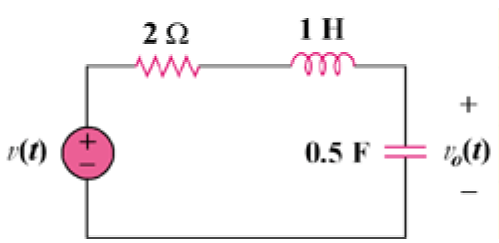
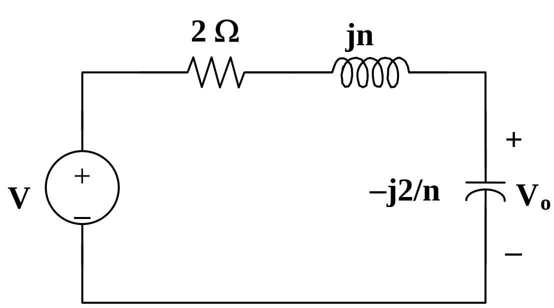


Figure 17.70

Solution

For any n, V = $[10/n^2]\angle(n\pi/4)$, ω = n.

1 H becomes $j\omega_n L$ = jn and 0.5 F becomes $1/(j\omega_n C)$ = −j2/n



V_o = {−j(2/n)/[2 + jn − j(2/n)]}V = $\{-j2/[2n + j(n^2 - 2)]\}[(10/n^2)\angle(n\pi/4)]$

$$= \frac{20\angle((n\pi/4)-\pi/2)}{n^2\sqrt{4n^2+(n^2-2)^2}\angle\tan^{-1}((n^2-2)/2n)}$$

$$= \frac{20}{n^2\sqrt{n^4+4}}\angle[(n\pi/4)-(\pi/2)-\tan^{-1}((n^2-2)/2n)]$$

$$v_o(t) = \sum_{\mathbf{n=1}}^{\infty}\frac{\mathbf{20}}{\mathbf{n^2\sqrt{n^4+4}}}\mathbf{cos}\left(\mathbf{nt}+\frac{\mathbf{n\pi}}{\mathbf{4}}-\frac{\pi}{\mathbf{2}}-\mathbf{tan}^{-1}\frac{\mathbf{n^2-2}}{\mathbf{2n}}\right)$$

Chapter 17, Solution 35.

If v_s in the circuit of Fig. 17.72 is the same as function $f_2(t)$ in Fig. 17.57(b), determine the dc component and the first three nonzero harmonics of $v_o(t)$.

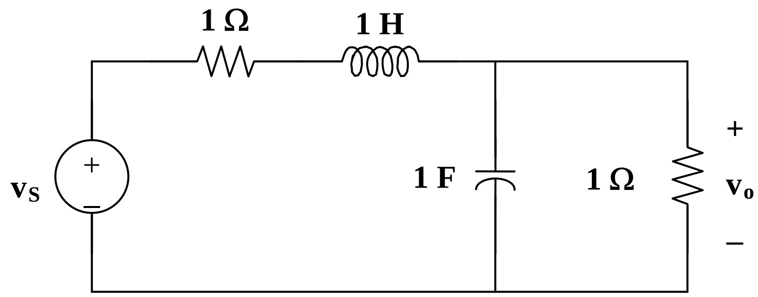


Figure 16.64 For Prob. 16.25

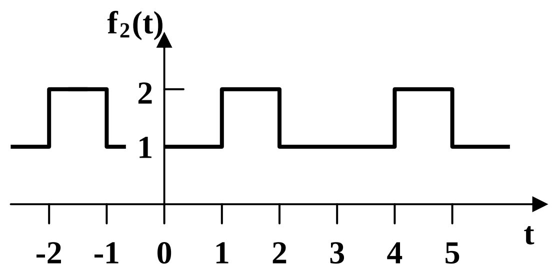


Figure 16.50(b) For Prob. 16.25

The signal is even, hence, $b_n = 0$. In addition, $T = 3$, $\omega_o = 2\pi/3$.

$$v_s(t) \quad = \; 1 \;\text{ for all } 0 < t < 1$$
$$= \; 2 \;\text{ for all } 1 < t < 1.5$$

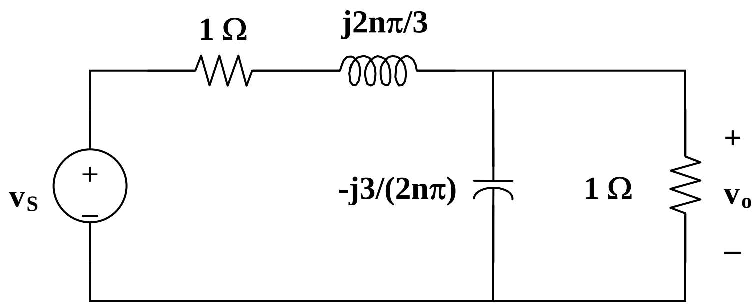
$$a_o = \frac{2}{3}\left[\int_0^1 1dt + \int_1^{1.5} 2dt\right] = \frac{4}{3}$$

$$a_n = \frac{4}{3}\left[\int_0^1 \cos(2n\pi t/3)dt + \int_1^{1.5} 2\cos(2n\pi t/3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin(2n\pi t/3)\Big|_0^1 + \frac{6}{2n\pi}\sin(2n\pi t/3)\Big|_1^{1.5}\right] = -\frac{2}{n\pi}\sin(2n\pi/3)$$

$$v_s(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\pi/3)\cos(2n\pi t/3)$$

Now consider this circuit,



Let Z = [-j3/(2nπ)](1)/(1 – j3/(2nπ)) = -j3/(2nπ - j3)

Therefore, $v_o = Zv_s/(Z + 1 + j2n\pi/3)$. Simplifying, we get

$$v_o = \frac{-j9v_s}{12n\pi + j(4n^2\pi^2 - 18)}$$

For the dc case, n = 0 and v_s = ¾ V and $v_o = v_s/2$ = 3/8 V.

We can now solve for $v_o(t)$

$$v_o(t) = \left[\frac{3}{8} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2n\pi t}{3} + \Theta_n\right)\right] \text{volts}$$

where $$A_n = \frac{\frac{6}{n\pi}\sin(2n\pi/3)}{\sqrt{16n^2\pi^2 + \left(\frac{4n^2\pi^2}{3} - 6\right)^2}} \text{ and } \Theta_n = 90^\circ - \tan^{-1}\left(\frac{n\pi}{3} - \frac{3}{2n\pi}\right)$$

where we can further simplify A_n to this, $$A_n = \frac{9\sin(2n\pi/3)}{n\pi\sqrt{4n^4\pi^4 + 81}}$$

Chapter 17, Solution 36.

We first find the Fourier series expansion of v_s. $T = 1, \quad \omega_o = 2\pi / T = 2\pi$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\int_0^1 10(1-t)tdt = 10(t - \frac{t^2}{2})\bigg|_1^0 = 5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = 2\int_0^1 10(1-t)\cos 2n\pi t dt$$

$$= 20\left[\frac{1}{2\pi n}\sin 2n\pi t - \frac{1}{4n^2\pi^2}\cos 2n\pi t - \frac{t}{2n\pi}\sin 2n\pi t\right]\bigg|_o^1 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{2}\int_0^1 10(1-t)t\sin n\omega_o t dt$$

$$= 20\left[-\frac{1}{2n\pi}\cos 2n\pi t - \frac{1}{4n^2\pi^2}\sin 2n\pi t + \frac{1}{2n\pi}\cos 2n\pi t\right]\bigg|_0^1 = \frac{10}{n\pi}$$

$$v_s(t) = 5 + \sum_{n=1}^{\infty}\frac{10}{n\pi}\sin 2n\pi t$$

$$1H \longrightarrow j\omega_n L = j\omega_n$$

$$10mF \longrightarrow \frac{1}{j\omega_n C} = \frac{1}{j\omega_n 0.01} = \frac{-j100}{\omega_n}$$

$$I_o = \frac{V_s}{5 + j\omega_n - \frac{j100}{\omega_n}}$$

For dc component, $\omega_0 = 0$ which leads to $I_0 = 0$.

For the nth harmonic,

$$\mathbf{I_n} = \frac{\frac{10}{n\pi}\angle 0^\circ}{5 + j2n\pi - \frac{j100}{2n\pi}} = \frac{10}{5n\pi + j(2n^2\pi^2 - 50)} = \mathbf{A_n}\angle\phi_n$$

where

$$A_n = \frac{10}{\sqrt{25n^2\pi^2 + (2n^2\pi^2 - 50)^2}}, \qquad \phi_n = -\tan^{-1}\frac{2n^2\pi^2 - 50}{5n\pi}$$

$$i_o(t) = \underline{\sum_{n=1}^{\infty} A_n \sin(2n\pi t + \phi_n)}$$

Chapter 17, Solution 37.

We first need to express i_s in Fourier series. $T = 2, \quad \omega_o = 2\pi / T = \pi$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 3dt + \int_1^2 1dt\right] = \frac{1}{2}(3+1) = 2$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{2}\left[\int_0^1 3\cos n\pi t dt + \int_1^2 \cos n\pi t dt\right] = \frac{3}{n\pi}\sin n\pi t\Big|_0^1 + \frac{1}{n\pi}\sin n\pi t\Big|_1^2 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{2}\left[\int_0^1 3\sin n\pi t dt + \int_1^2 \sin n\pi t dt\right] = \frac{-3}{n\pi}\cos n\pi t\Big|_0^1 + \frac{-1}{n\pi}\cos n\pi t\Big|_1^2 = \frac{2}{n\pi}(1-\cos n\pi$$

$$i_s(t) = 2 + \sum_{n=1}^{\infty}\frac{2}{n\pi}(1-\cos n\pi)\sin n\pi t$$

By current division,

$$I_o = \frac{1}{1+2+j\omega_n L} I_s = \frac{I_s}{3+j3\omega_n}$$

$$V_o = j\omega_n L I_o = \frac{j\omega_n 3 I_s}{3+j3\omega_n} = \frac{j\omega_n I_s}{1+j\omega_n}$$

For dc component (n=0), $V_o = 0$.
For the nth harmonic,

$$V_o = \frac{jn\pi}{1+jn\pi}\frac{2}{n\pi}(1-\cos n\pi)\angle -90^\circ = \frac{2(1-\cos n\pi)}{\sqrt{1+n^2\pi^2}}\angle(90^\circ - \tan^{-1} n\pi - 90^\circ)$$

$$\underline{v_o(t) = \sum_{n=1}^{\infty}\frac{2(1-\cos \pi n)}{\sqrt{1+n^2\pi^2}}\cos(n\pi t - \tan^{-1} n\pi)}$$

Chapter 17, Solution 38.

$$v_s(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t, \qquad n = 2k+1$$

$$V_o = \frac{j\omega_n}{1+j\omega_n}V_s, \qquad \omega_n = n\pi$$

For dc, $\omega_n = 0, \quad V_s = 0.5, \quad V_o = 0$

For nth harmonic, $V_s = \frac{2}{n\pi}\angle -90^o$

$$V_o = \frac{n\pi\angle 90^o}{\sqrt{1+n^2\pi^2}\angle \tan^{-1} n\pi} \bullet \frac{2}{n\pi}\angle 90^o = \frac{2\angle -\tan^{-1} n\pi}{\sqrt{1+n^2\pi^2}}$$

$$\mathbf{v_o(t) = \underline{\sum_{k=1}^{\infty}\frac{2}{\sqrt{1+n^2\pi^2}}\cos(n\pi t - \tan^{-1} n\pi), \qquad n = 2k-1}}$$

Chapter 17, Solution 39.

Comparing $v_s(t)$ with f(t) in Figure 15.1, v_s is shifted by 2.5 and the magnitude is 5 times that of f(t).
Hence

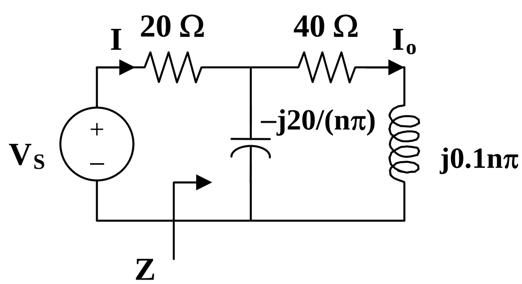
$$v_s(t) = 5 + \frac{10}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin(n\pi t), \qquad n = 2k - 1$$

$$T = 2, \ \omega_o = 2\pi//T = \pi, \ \omega_n = n\omega_o = n\pi$$

For the DC component, $\quad i_o = 5/(20 + 40) = 1/12$

For the kth harmonic, $\quad V_s = (10/(n\pi))\angle 0°$

100 mH becomes $j\omega_n L = jn\pi x0.1 = j0.1n\pi$
50 mF becomes $1/(j\omega_n C) = -j20/(n\pi)$



Let $Z = -j20/(n\pi)||(40 + j0.1n\pi) = \dfrac{-\frac{j20}{n\pi}(40 + j0.1n\pi)}{-\frac{j20}{n\pi} + 40 + j0.1n\pi}$

$$= \frac{-j20(40 + j0.1n\pi}{-j20 + 40n\pi + j0.1n^2\pi^2} = \frac{2n\pi - j800}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$Z_{in} = 20 + Z = \frac{802n\pi + j(2n^2\pi^2 - 1200)}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$I = \frac{V_s}{Z_{in}} = \frac{400n\pi + j(n^2\pi^2 - 200)}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$I_o = \frac{-\frac{j20}{n\pi}I}{-\frac{j20}{n\pi} + (40 + j0.1n\pi)} = \frac{-j20I}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$= \frac{-j200}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$= \frac{200\angle -90° - \tan^{-1}\{(2n^2\pi^2 - 1200)/(802n\pi)\}}{n\pi\sqrt{(802)^2 + (2n^2\pi^2 - 1200)^2}}$$

Thus

$$i_o(t) = \frac{1}{20} + \frac{200}{\pi}\sum_{k=1}^{\infty} I_n \sin(n\pi t - \theta_n), \qquad n = 2k - 1$$

where $$\theta_n = 90° + \tan^{-1}\frac{2n^2\pi^2 - 1200}{802n\pi}$$

$$I_n = \frac{1}{n\sqrt{(804n\pi)^2 + (2n^2\pi^2 - 1200)}}$$

Chapter 17, Solution 40.

$T = 2,\ \omega_o = 2\pi/T = \pi$

$$a_o = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{2}\int_0^1 (2 - 2t)dt = \left[t - \frac{t^2}{2}\right]\Big|_0^1 = 1/2$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\pi t)dt = \int_0^1 2(1 - t)\cos(n\pi t)dt$$

$$= 2\left[\frac{1}{n\pi}\sin(n\pi t) - \frac{1}{n^2\pi^2}\cos(n\pi t) - \frac{t}{n\pi}\sin(n\pi t)\right]\Big|_0^1$$

$$= \frac{2}{n^2\pi^2}(1 - \cos n\pi) = \begin{cases} 0, & n = \text{even} \\ \dfrac{4}{n^2\pi^2}, & n = \text{odd} \end{cases} = \frac{4}{\pi^2(2n-1)^2}$$

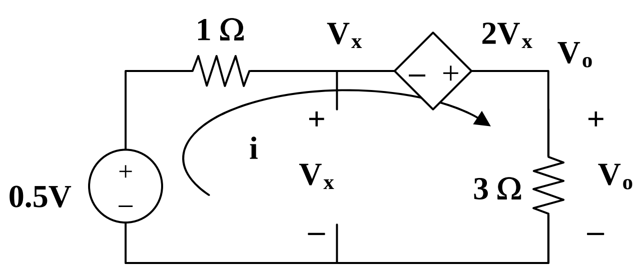
$$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\pi t)dt = 2\int_0^1 (1 - t)\sin(n\pi t)dt$$

$$= 2\left[-\frac{1}{n\pi}\cos(n\pi t) - \frac{1}{n^2\pi^2}\sin(n\pi t) + \frac{t}{n\pi}\cos(n\pi t)\right]\Big|_0^1 = \frac{2}{n\pi}$$

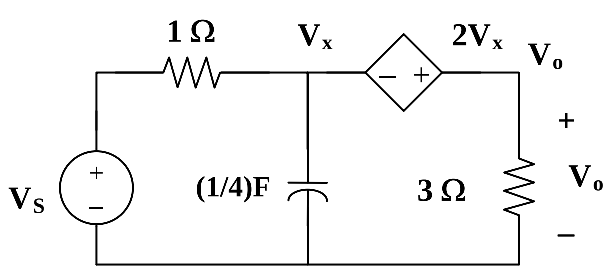
$$v_s(t) = \frac{1}{2} + \sum A_n\cos(n\pi t - \varphi_n)$$

$$\text{where}\quad \phi_n = \tan^{-1}\frac{\pi(2n-1)^2}{2n},\quad A_n = \sqrt{\frac{4}{n^2\pi^2} + \frac{16}{\pi^4(2n-1)^4}}$$

For the DC component, $v_s = 1/2$. As shown in Figure (a), the capacitor acts like an open circuit.



(a)



(b)

Applying KVL to the circuit in Figure (a) gives

$$-0.5 - 2V_x + 4i = 0 \qquad (1)$$

But $\quad -0.5 + i + V_x = 0$ or $-1 + 2V_x + 2i = 0 \qquad (2)$

Adding (1) and (2), $\quad -1.5 + 6i = 0$ or $i = 0.25$

$$V_o = 3i = 0.75$$

For the nth harmonic, we consider the circuit in Figure (b).

$$\omega_n = n\pi,\ V_s = A_n\angle-\phi,\ 1/(j\omega_n C) = -j4/(n\pi)$$

At the supernode,

$$(V_s - V_x)/1 = -[n\pi/(j4)]V_x + V_o/3$$

$$V_s = [1 + jn\pi/4]V_x + V_o/3 \qquad (3)$$

But $\quad -V_x - 2V_x + V_o = 0$ or $V_o = 3V_x$

Substituting this into (3),

$$V_s = [1 + jn\pi/4]V_x + V_x = [2 + jn\pi/4]V_x$$

$$= (1/3)[2 + jn\pi/4]V_o = (1/12)[8 + jn\pi]V_o$$

$$V_o = 12V_s/(8 + jn\pi) = \frac{12A_n\angle-\phi}{\sqrt{64+n^2\pi^2}\angle\tan^{-1}(n\pi/8)}$$

$$V_o = \frac{12}{\sqrt{64+n^2\pi^2}}\sqrt{\frac{4}{n^2\pi^2}+\frac{16}{\pi^4(2n-1)^4}}\angle[\tan^{-1}(n\pi/8)-\tan^{-1}(\pi(2n-1)/(2n))]$$

Thus

$$v_o(t) = \mathbf{\frac{3}{4} + \sum_{n=1}^{\infty} V_n \cos(n\pi t + \theta_n)\,volts}$$

where

$$V_n = \mathbf{\frac{12}{\sqrt{64+n^2\pi^2}}\sqrt{\frac{4}{n^2\pi^2}+\frac{16}{\pi^4(2n-1)^4}}} \text{ and}$$

$$\theta_n = \mathbf{\tan^{-1}(n\pi/8) - \tan^{-1}(\pi(2n-1)/(2n))}$$

Chapter 17, Solution 41.

For the full wave rectifier,

$$T = \pi, \ \omega_o = 2\pi/T = 2, \ \omega_n = n\omega_o = 2n$$

Hence

$$v_{in}(t) = \left[\frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos(2nt)\right] volts$$

For the DC component,

$$V_{in} = 2/\pi$$

The inductor acts like a short-circuit, while the capacitor acts like an open circuit.

$$V_o = V_{in} = 2/\pi$$

For the nth harmonic,

$$V_{in} = [-4/(\pi(4n^2 - 1))]\angle 0°$$
2 H becomes $j\omega_n L = j4n$
0.1 F becomes $1/(j\omega_n C) = -j5/n$
$$Z = 10 \| (-j5/n) = -j10/(2n - j)$$
$$V_o = [Z/(Z + j4n)]V_{in} = -j10V_{in}/(4 + j(8n - 10))$$
$$= -\frac{j10}{4 + j(8n-10)}\left(-\frac{4\angle 0°}{\pi(4n^2-1)}\right)$$
$$= \frac{40\angle\{90° - \tan^{-1}(2n-2.5)\}}{\pi(4n^2-1)\sqrt{16+(8n-10)^2}}$$

Hence

$$v_o(t) = \left[\frac{\mathbf{2}}{\boldsymbol{\pi}} + \sum_{\mathbf{n=1}}^{\infty}\mathbf{A_n}\,\mathbf{cos(2nt + \boldsymbol{\theta}_n)}\right]\textbf{volts}$$

where

$$A_n = \frac{\mathbf{20}}{\boldsymbol{\pi}\mathbf{(4n^2 - 1)}\sqrt{\mathbf{16n^2 - 40n + 29}}}$$

$$\theta_n = \mathbf{90° - tan^{-1}(2n - 2.5)}$$

Chapter 17, Solution 42.

$$v_s = 5 + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t,\ \ n = 2k - 1$$

$$\frac{V_s - 0}{R} = j\omega_n C(0 - V_o) \quad \longrightarrow \quad V_o = \frac{j}{\omega_n RC}V_s,\ \ \omega_n = n\omega_o = n\pi$$

For n = 0 (dc component), V_o=0.

For the nth harmonic,

$$V_o = \frac{1\angle 90^o}{n\pi RC}\frac{20}{n\pi}\angle -90^o = \frac{20}{n^2\pi^2 x10^4 x40x10^{-9}} = \frac{10^5}{2n^2\pi^2}$$

Hence,

$$\mathbf{v_o(t) = \underline{\frac{10^5}{2\pi^2}\sum_{k=1}^{\infty}\frac{1}{n^2}\cos n\pi t,\ \ n = 2k - 1}}$$

Alternatively, we notice that this is an integrator so that

$$\mathbf{v_o(t) = -\frac{1}{RC}\int v_s dt = \underline{\frac{10^5}{2\pi^2}\sum_{k=1}^{\infty}\frac{1}{n^2}\cos n\pi t,\ \ n = 2k - 1}}$$

Chapter 17, Solution 43.

(a) $$V_{rms} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)} = \sqrt{30^2 + \frac{1}{2}(20^2 + 10^2)} = \mathbf{33.91\ V}$$

(b) $$I_{rms} = \sqrt{6^2 + \frac{1}{2}(4^2 + 2^2)} = \mathbf{6.782\ A}$$

(c) $$P = V_{dc}I_{dc} + \frac{1}{2}\sum V_n I_n \cos(\Theta_n - \Phi_n)$$

$$= 30x6 + 0.5[20x4\cos(45^o\text{-}10^o) – 10x2\cos(\text{-}45^o+60^o)]$$

$$= 180 + 32.76 – 9.659 = \mathbf{203.1\ W}$$

Chapter 17, Solution 44.

Design a problem to help other students to better understand how to find the rms voltage across and the rms current through an electrical element given a Fourier series for both the current and the voltage. In addition, have them calculate the average power delivered to the element and the power spectrum.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The voltage and current through an element are respectively

$v(t)$ = [30 cos (t + 35°) + 10cos($2t$ – 55°) + 4 cos($3t$ - 10°)] V

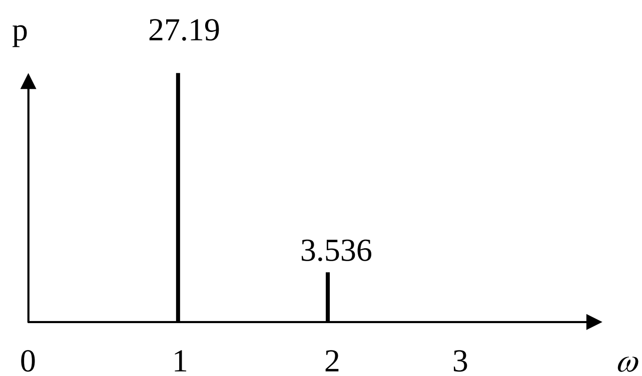
$i(t)$ = [2 cos (t–10°) + cos($2t$ – 10°)] A

(a) Find the average power delivered to the element
(b) Plot the power spectrum.

Solution

(a) $$\text{p} = \text{vi} = \frac{1}{2}\left[60\cos(-25^\circ) + 10\cos 45^\circ + 0\right] = 27.19 + 3.536 = \mathbf{30.73\ W}.$$

(b) The power spectrum is shown below.



Chapter 17, Solution 45.

$\omega_n = 1000n$

$j\omega_n L = j1000n\text{x}2\text{x}10^{-3} = j2n$

$1/(j\omega_n C) = -j/(1000n\text{x}40\text{x}10^{-6}) = -j25/n$

$Z = R + j\omega_n L + 1/(j\omega_n C) = 10 + j2n - j25/n$

$I = V/Z$

For $n = 1$, $V_1 = 100$, $Z = 10 + j2 - j25 = 10 - j23$

$I_1 = 100/(10 - j23) = 3.987\angle 73.89°$

For $n = 2$, $V_2 = 50$, $Z = 10 + j4 - j12.5 = 10 - j8.5$

$I_2 = 50/(10 - j8.5) = 3.81\angle 40.36°$

For $n = 3$, $V_3 = 25$, $Z = 10 + j6 - j25/3 = 10 - j2.333$

$I_3 = 25/(10 - j2.333) = 2.435\angle 13.13°$

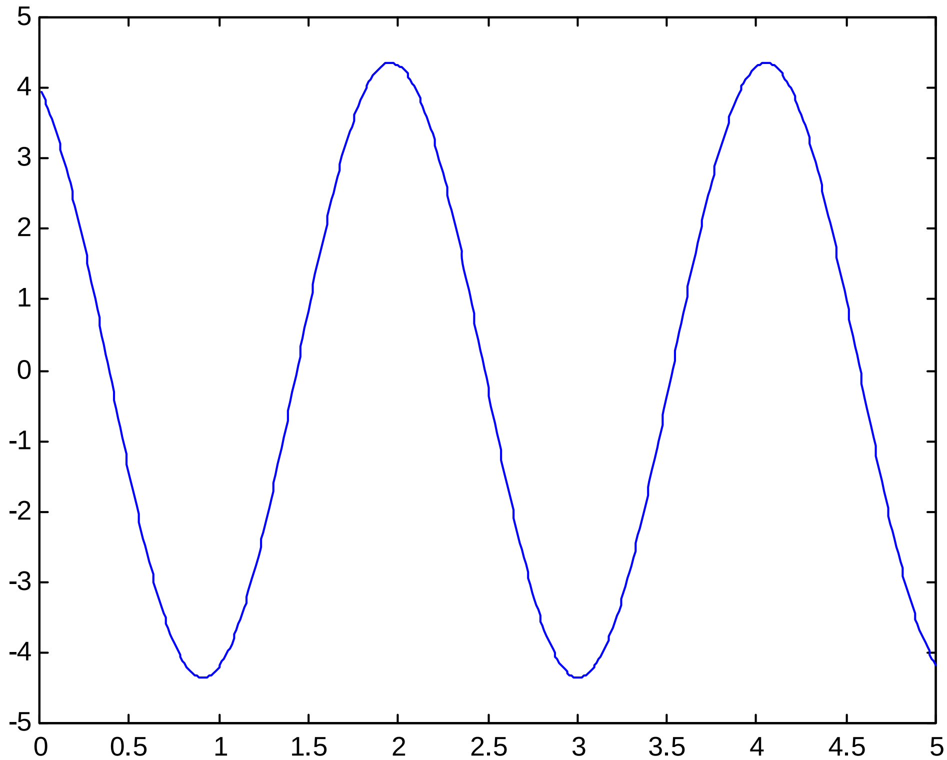
$I_{rms} = \sqrt{0.5(3.987^2 + 3.81^2 + 2.435^2)} = \mathbf{4.263\ A}$

$p = R(I_{rms})^2 = \mathbf{181.7W}$

Chapter 17, Solution 46.

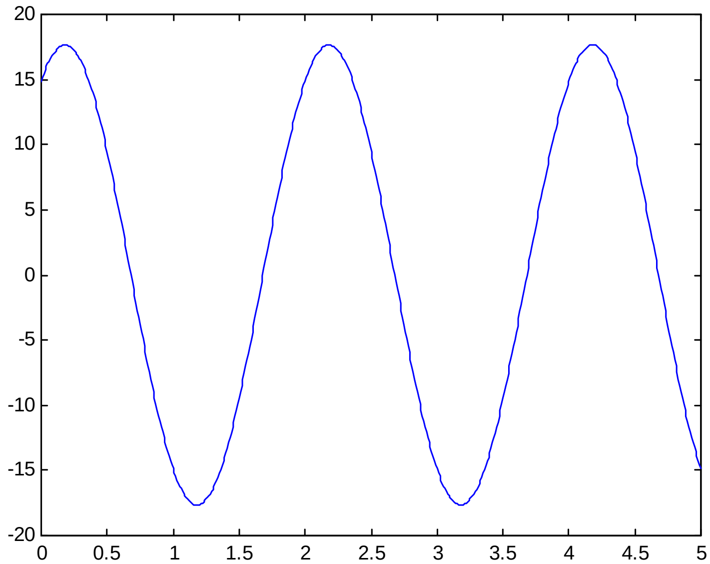
(a)The MATLAB commands are:

```
t=0:0.01:5;
y=5*cos(3*t) – 2*cos(3*t-pi/3);
plot(t,y)
```



(b) The MATLAB commands are:

```
t=0:0.01:5;
» x=8*sin(pi*t+pi/4)+10*cos(pi*t-pi/8);
» plot(t,x)
» plot(t,x)
```



20
15
10
5
0
-5
-10
-15
-20
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Chapter 17, Solution 47.

$T = 2, \qquad \omega_o = 2\pi / T = \pi$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 4dt + \int_1^2 (-2)dt\right] = \frac{1}{2}(4-2) = 1$$

$$P = Ri_{rms}^2 = \frac{R}{T}\int_0^T f^2(t)dt = \frac{R}{2}\left[\int_0^1 4^2\,dt + \int_1^2 (-2)^2\,dt\right] = 10R$$

The average power dissipation caused by the dc component is

$$P_0 = Ra_o^2 = R = \underline{10\%} \text{ of P}$$

Chapter 17, Solution 48.

(a) For the DC component, i(t) = 20 mA. The capacitor acts like an open circuit so that

$$v = Ri(t) = 2x10^3x20x10^{-3} = 40$$

For the AC component,

$$\omega_n = 10n, \quad n = 1,2$$

$$1/(j\omega_n C) = -j/(10nx100x10^{-6}) = (-j/n)\ k\Omega$$

$$Z = 2||(-j/n) = 2(-j/n)/(2 - j/n) = -j2/(2n - j)$$

$$V = ZI = [-j2/(2n - j)]I$$

For n = 1, $V_1 = [-j2/(2 - j)]16\angle 45° = 14.311\angle{-18.43°}$ mV

For n = 2, $V_2 = [-j2/(4 - j)]12\angle{-60°} = 5.821\angle{-135.96°}$ mV

v(t) = **40 + 0.014311cos(10t – 18.43°) + 0.005821cos(20t – 135.96°) V**

(b) $$p = V_{DC}I_{DC} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

$$= 20x40 + 0.5x10x0.014311\cos(45° + 18.43°)$$
$$+0.5x12x0.005821\cos(-60° + 135.96°)$$

= **800.1 mW**

Chapter 17, Solution 49.

(a) $$Z^2{}_{rms} = \frac{1}{T}\int_0^T z^2(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} 4dt + \int_{\pi}^{2\pi} 16dt\right] = \frac{1}{2\pi}(20\pi) = 10$$

Z_{rms} = **3.162**

(b)

$$Z^2{}_{rms} = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2) = 1 + \frac{1}{2}\sum_{\substack{n=1 \\ n=odd}}^{\infty}\frac{144}{n^2\pi^2} = 1 + \frac{72}{\pi^2}\left(1 + 0 + \frac{1}{9} + 0 + \frac{1}{25} + ...\right) = 9.396$$

Z_{rms} = **3.065**

(c) $$\%\text{error} = \left(1 - \frac{3.065}{3.162}\right)x100 = \mathbf{3.068\%}$$

Chapter 17, Solution 50.

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \qquad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_{-}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$u = t$ and $du = dt$

$dv = e^{-jn\pi t}dt$ which leads to $v = -[1/(2jn\pi)]e^{-jn\pi t}$

$$c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Big|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1} e^{-jn\pi t}dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi t}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\Big|_{-1}^{1}$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty} \mathbf{c_n e^{jn\omega_o t}} = \sum_{n=-\infty}^{\infty} \mathbf{(-1)^n \frac{j}{n\pi} e^{jn\pi t}}$$

Chapter 17, Solution 51.

Design a problem to help other students to better understand how to find the exponential Fourier series of a given periodic function.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Given the periodic function

$$f(t) = t^2, \qquad 0 < t < T$$

obtain the exponential Fourier series for the special case $T = 2$.

Solution

$$T = 2, \qquad \omega_o = 2\pi / T = \pi$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{1}{2}\int_0^2 t^2 e^{-jn\pi t}dt = \frac{1}{2}\frac{e^{-jn\pi t}}{(-jn\pi)^3}\left(-n^2\pi^2 t^2 + 2jn\pi t + 2\right)\Big|_0^2$$

$$c_n = \frac{1}{j2n^3\pi^3}(-4n^2\pi^2 + j4n\pi) = \frac{2}{n^2\pi^2}(1 + jn\pi)$$

$$\underline{\mathbf{f(t) = \sum_{n=-\infty}^{\infty} \frac{2}{n^2\pi^2}(1 + jn\pi)e^{jn\pi t}}}$$

Chapter 17, Solution 52.

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \qquad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_{-}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$

$$dv = e^{-jn\pi t}dt \text{ which leads to } v = -[1/(2jn\pi)]e^{-jn\pi t}$$

$$c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Big|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1} e^{-jn\pi t}dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi t}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\Big|_{-1}^{1}$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t} = \boldsymbol{\sum_{n=-\infty}^{\infty} (-1)^n \frac{j}{n\pi} e^{jn\pi t}}$$

Chapter 17, Solution 53.

$\omega_o = 2\pi/T = 2\pi$

$c_n = \int_0^T e^{-t} e^{-jn\omega_o t} dt = \int_0^1 e^{-(1+jn\omega_o)t} dt$

$= \frac{-1}{1 + j2\pi n} e^{-(1+j2n\pi)t} \Big|_0^1 = \frac{-1}{1 + j2n\pi} \left[e^{-(1+j2n\pi)} - 1\right]$

$= [1/(j2n\pi)][1 - e^{-1}(\cos(2\pi n) - j\sin(2n\pi))]$

$= (1 - e^{-1})/(1 + j2n\pi) = 0.6321/(1 + j2n\pi$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{0.6321 e^{j2n\pi t}}{1 + j2n\pi}$$

Chapter 17, Solution 54.

$$T = 4, \ \omega_o = 2\pi/T = \pi/2$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt$$

$$= \frac{1}{4}\left[\int_0^1 2e^{-jn\pi t/2}dt + \int_1^2 1e^{-jn\pi t/2}dt - \int_2^4 1e^{-jn\pi t/2}dt\right]$$

$$= \frac{j}{2n\pi}\left[2e^{-jn\pi/2} - 2 + e^{-jn\pi} - e^{-jn\pi/2} - e^{-j2n\pi} + e^{-jn\pi}\right]$$

$$= \boldsymbol{\frac{j}{2n\pi}\left[3e^{-jn\pi/2} - 3 + 2e^{-jn\pi}\right]}$$

$$f(t) = \boldsymbol{\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}}$$

Chapter 17, Solution 55.

$T = 2\pi, \ \omega_o = 2\pi/T = 1$

$$c_n = \frac{1}{T}\int_0^T i(t)e^{-jn\omega_o t}dt$$

But $$i(t) = \begin{cases} \sin(t), & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

$$c_n = \frac{1}{2\pi}\int_0^\pi \sin(t)e^{-jnt}dt = \frac{1}{2\pi}\int_0^\pi \frac{1}{2j}(e^{jt} - e^{-jt})e^{-jnt}dt$$

$$= \frac{1}{4\pi j}\left[\frac{e^{jt(1-n)}}{j(1-n)} + \frac{e^{-jt(1+n)}}{j(1+n)}\right]\Bigg|_0^\pi$$

$$= -\frac{1}{4\pi}\left[\frac{e^{j\pi(1-n)} - 1}{1-n} + \frac{e^{-j\pi(n+1)} - 1}{1+n}\right]$$

$$= \frac{1}{4\pi(n^2 - 1)}\left[e^{j\pi(1-n)} - 1 + ne^{j\pi(1-n)} - n + e^{-j\pi(1+n)} - 1 - ne^{-j\pi(1+n)} + n\right]$$

But $e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 = e^{-j\pi}$

$$c_n = \frac{1}{4\pi(n^2 - 1)}\left[- e^{-jn\pi} - e^{-jn\pi} - ne^{-jn\pi} + ne^{-jn\pi} - 2\right] = \frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)}$$

Thus

$$i(t) = \sum_{n=-\infty}^{\infty} \frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)}e^{jnt}$$

Chapter 17, Solution 56.

$c_o = a_o = 10, \ \omega_o = \pi$

$c_o = (a_n - jb_n)/2 = (1 - jn)/[2(n^2 + 1)]$

$$\mathbf{f(t) = 10 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{(1 - jn)}{2(n^2 + 1)} e^{jn\pi t}}$$

Chapter 17, Solution 57.

$a_o = (6/-2) = -3 = c_o$

$c_n = 0.5(a_n - jb_n) = a_n/2 = 3/(n^3 - 2)$

$$f(t) = -3 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{3}{n^3 - 2} e^{j50nt}$$

Chapter 17, Solution 58.

$c_n = (a_n - jb_n)/2$, $(-1)^n = \cos(n\pi)$, $\omega_o = 2\pi/T = 1$

$c_n = [(\cos(n\pi) - 1)/(2\pi n^2)] - j\cos(n\pi)/(2n)$

Thus

$$f(t) = \frac{\pi}{4} + \sum\left(\frac{\cos(n\pi) - 1}{2\pi n^2} - j\frac{\cos(n\pi)}{2n}\right)e^{jnt}$$

Chapter 17, Solution 59.

For f(t), $T = 2\pi$, $\omega_o = 2\pi/T = 1$.

a_o = DC component = $(1x\pi + 0)/2\pi = 0.5$

For h(t), $T = 2$, $\omega_o = 2\pi/T = \pi$.

$a_o = (2x1 - 2x1)/2 = 0$

Thus by replacing $\omega_o = 1$ with $\omega_o = \pi$ and multiplying the magnitude by four, we obtain

$$h(t) = -\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j4e^{-j(2n+1)\pi t}}{(2n+1)\pi}$$

Chapter 17, Solution 60.

From Problem 17.24,

$a_o = 0 = a_n,\;\; b_n = [2/(n\pi)][1 - 2\cos(n\pi)],\;\; c_o = \mathbf{0}$

$c_n = (a_n - jb_n)/2 = \mathbf{[j/(n\pi)][2\cos(n\pi) - 1],\;\; n \neq 0}.$

Chapter 17, Solution 61.

(a) $\omega_o = 1.$

$$f(t) = a_o + \sum A_n \cos(n\omega_o t - \phi_n)$$

$$= 6 + 4\cos(t + 50°) + 2\cos(2t + 35°) + \cos(3t + 25°) + 0.5\cos(4t + 20°)$$

$$= 6 + 4\cos(t)\cos(50°) - 4\sin(t)\sin(50°) + 2\cos(2t)\cos(35°) - 2\sin(2t)\sin(35°) + \cos(3t)\cos(25°) - \sin(3t)\sin(25°) + 0.5\cos(4t)\cos(20°) - 0.5\sin(4t)\sin(20°)$$

$$= \mathbf{6 + 2.571\cos(t) - 3.73\sin(t) + 1.635\cos(2t) - 1.147\sin(2t) + 0.906\cos(3t) - 0.423\sin(3t) + 0.47\cos(4t) - 0.171\sin(4t)}$$

(b) $$f_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2}$$

$$f_{rms}^2 = 6^2 + 0.5[4^2 + 2^2 + 1^2 + (0.5)^2] = 46.625$$

$$f_{rms} = \mathbf{6.828}$$

Chapter 17, Solution 62.

(a)

$$f(t) = 12 + 10\cos(2\omega_o t + 90^o) + 8\cos(4\omega_o t - 90^o) + 5\cos(6\omega_o t + 90^o) + 3\cos(8\omega_o t - 90^o)$$

(b) f(t) is an **even** function of t.

Chapter 17, Solution 63.

This is an even function.

$T = 3,\ \omega_o = 2\pi/3,\ b_n = 0.$

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 2, & 1 < t < 1.5 \end{cases}$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{3}\left[\int_0^1 1dt + \int_1^{1.5} 2\,dt\right] = (2/3)[1 + 1] = 4/3$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{3}\left[\int_0^1 1\cos(2n\pi t/3)dt + \int_1^{1.5} 2\cos(2n\pi t/3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\Bigg|_0^1 + \frac{6}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\Bigg|_1^{1.5}\right]$$

$$= [-2/(n\pi)]\sin(2n\pi/3)$$

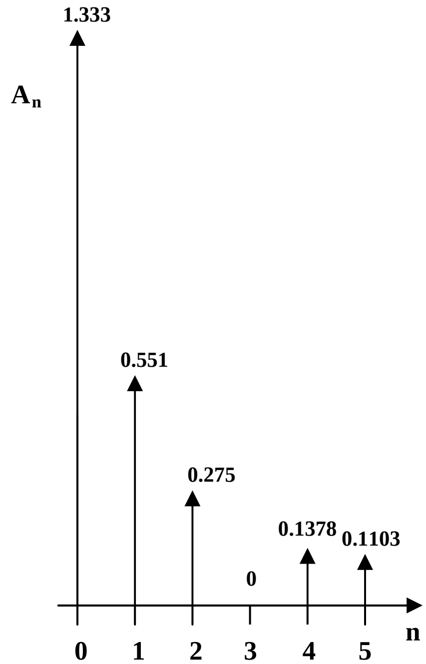
$$f_2(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{3n\pi}{3}\right)\cos\left(\frac{2n\pi t}{3}\right)$$

$a_o = 4/3 = 1.3333,\ \omega_o = 2\pi/3,\ a_n = -[2/(n\pi)]\sin(2n\pi t/3)$

$$A_n = \sqrt{a_n^2 + b_n^2} = \left|\frac{2}{n\pi}\sin\left(\frac{2n\pi}{3}\right)\right|$$

$A_1 = 0.5513,\ A_2 = 0.2757,\ A_3 = 0,\ A_4 = 0.1375,\ A_5 = 0.1103$

The amplitude spectra are shown below.



Chapter 17, Solution 64.

Design a problem to help other students to better understand the amplitude and phase spectra of a given Fourier series.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

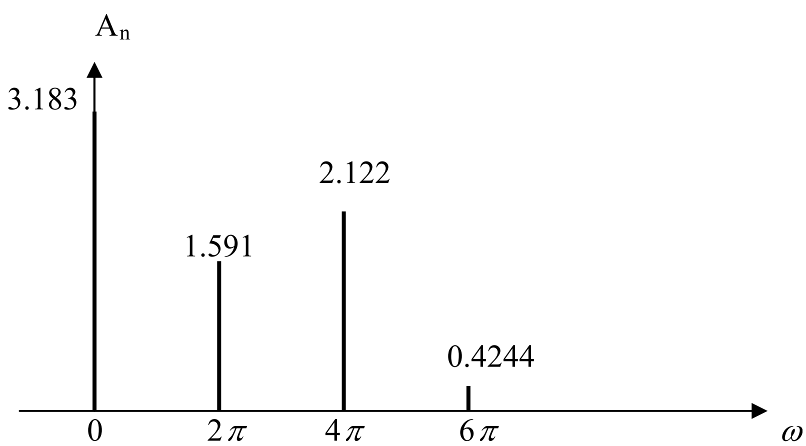
Given that

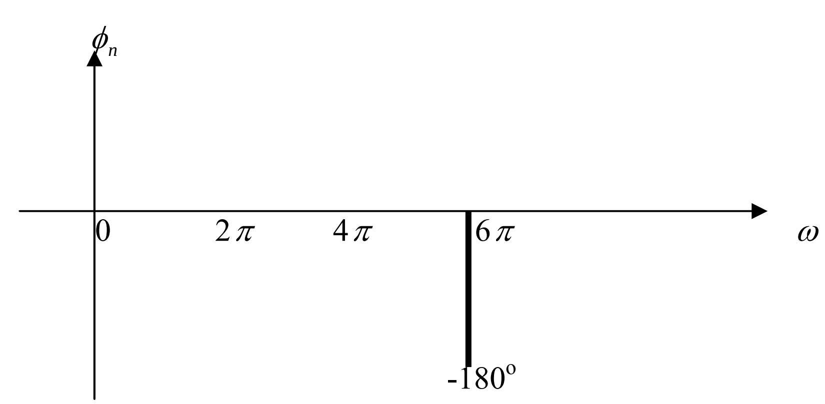
$$v(t) = (10/\pi)[1 + (1/2)\cos(2\pi t) + (2/3)\cos(4\pi t) - (2/15)\cos(6\pi t)] \text{ V}$$

draw the amplitude and phase spectra for $v(t)$.

Solution

The amplitude and phase spectra are shown below.





Chapter 17, Solution 65.

$a_n = 20/(n^2\pi^2)$, $b_n = -3/(n\pi)$, $\omega_n = 2n$

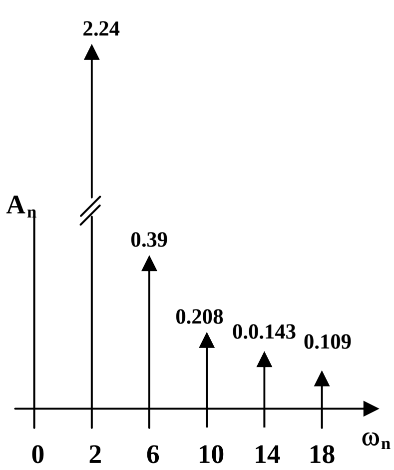
$$A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{\frac{400}{n^4\pi^4} + \frac{9}{n^2\pi^2}}$$

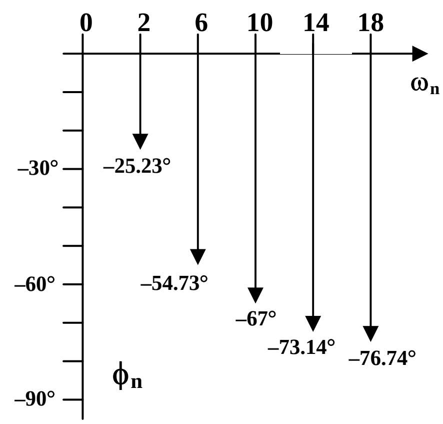
$$= \frac{3}{n\pi}\sqrt{1 + \frac{44.44}{n^2\pi^2}}, \quad n = 1,\ 3,\ 5,\ 7,\ 9,\ \text{etc.}$$

n	A_n
1	2.24
3	0.39
5	0.208
7	0.143
9	0.109

$$\phi_n = \tan^{-1}(b_n/a_n) = \tan^{-1}\{[-3/(n\pi)][n^2\pi^2/20]\} = \tan^{-1}(-nx0.4712)$$

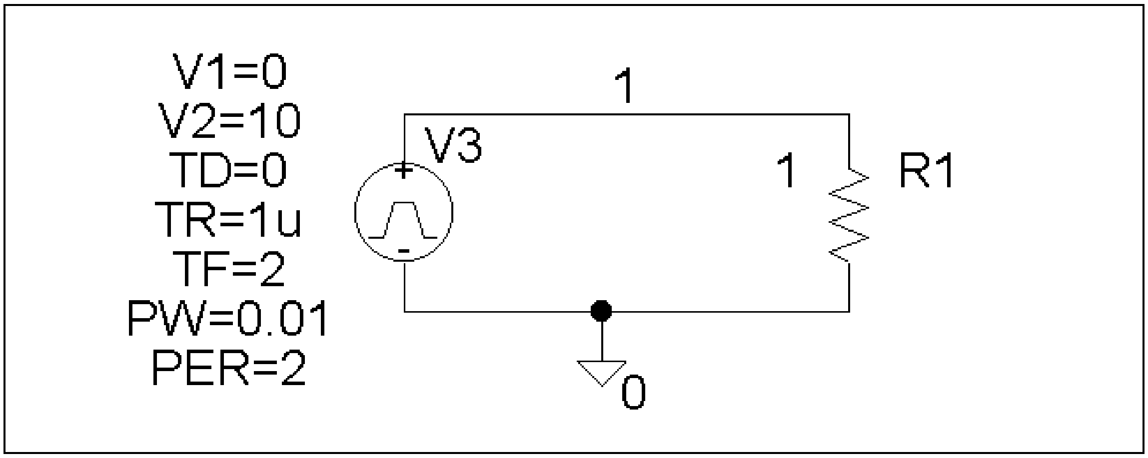
n	ϕ_n
1	–25.23°
3	–54.73°
5	–67°
7	–73.14°
9	–76.74°
∞	–90°





Chapter 17, Solution 66.

The schematic is shown below. The waveform is inputted using the attributes of VPULSE. In the Transient dialog box, we enter Print Step = 0.05, Final Time = 12, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, the output plot is shown below. The output file includes the following Fourier components.



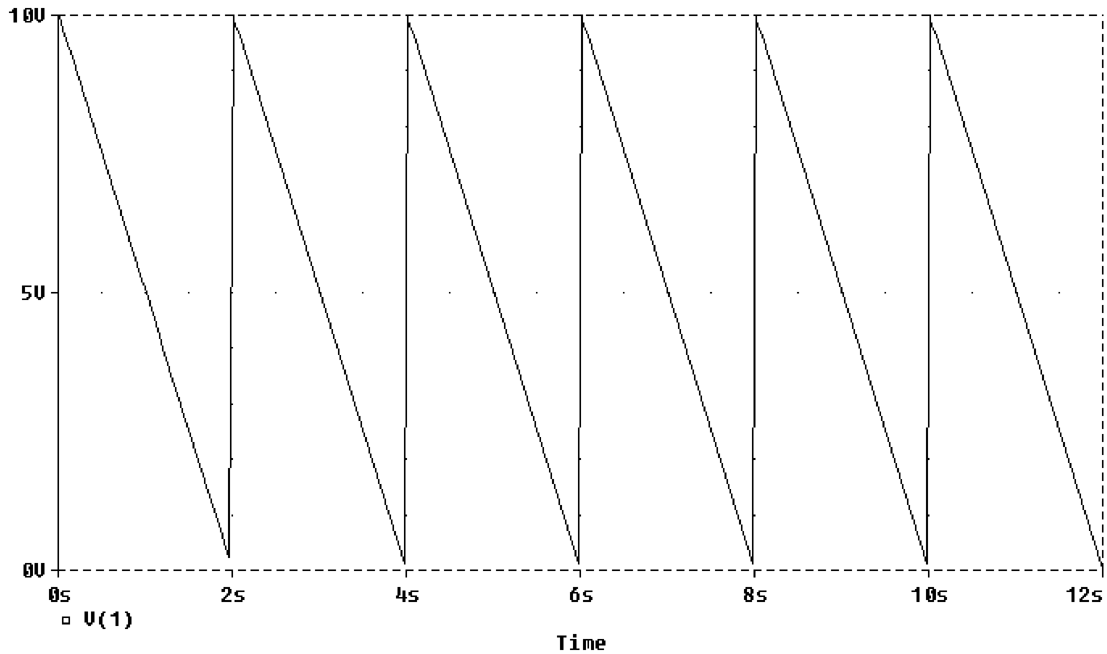
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.099510E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	3.184E+00	1.000E+00	1.782E+00	0.000E+00
2	1.000E+00	1.593E+00	5.002E-01	3.564E+00	1.782E+00
3	1.500E+00	1.063E+00	3.338E-01	5.347E+00	3.564E+00
4	2.000E+00	7.978E-01	2.506E-01	7.129E+00	5.347E+00
5	2.500E+00	6.392E-01	2.008E-01	8.911E+00	7.129E+00
6	3.000E+00	5.336E-01	1.676E-01	1.069E+01	8.911E+00
7	3.500E+00	4.583E-01	1.440E-01	1.248E+01	1.069E+01
8	4.000E+00	4.020E-01	1.263E-01	1.426E+01	1.248E+01
9	4.500E+00	3.583E-01	1.126E-01	1.604E+01	1.426E+01

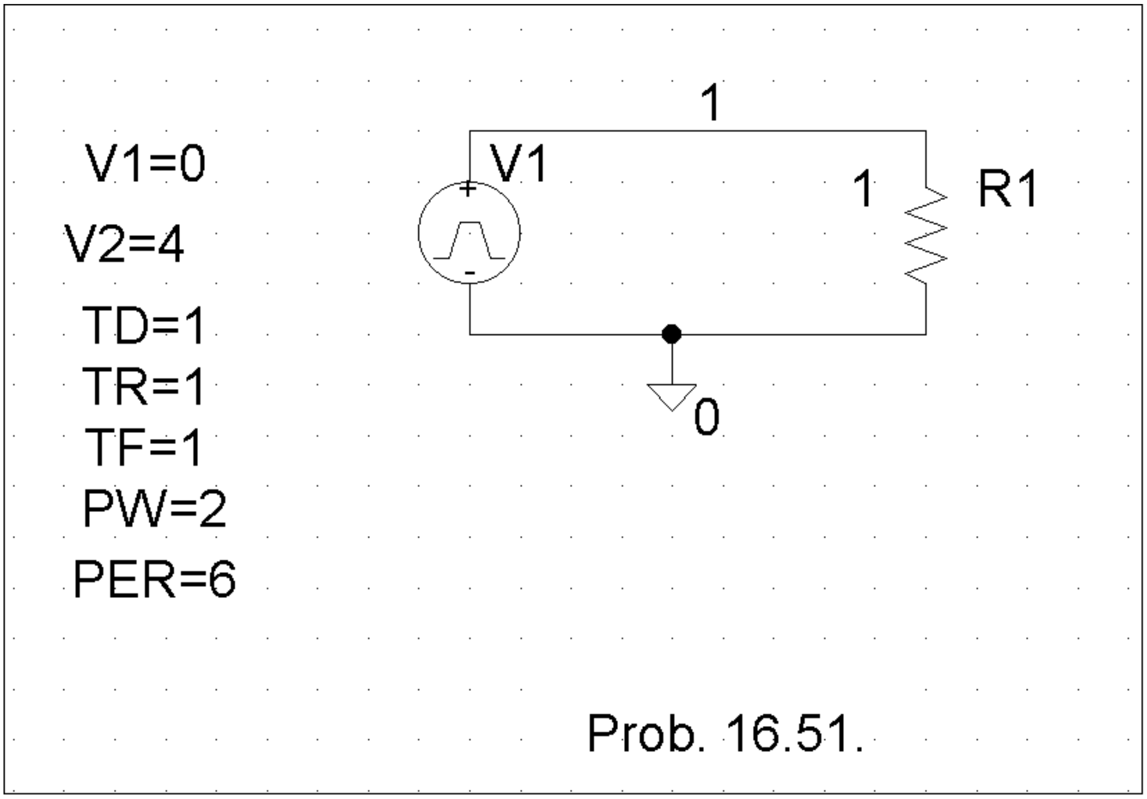
TOTAL HARMONIC DISTORTION = 7.363360E+01 PERCENT

From Prob. 17.4, we know the phase angle should be zero. Why do we have a phase angle equal to n(1.782)? The answer is actually quite straight forward. ***The angle comes from the approximation of the leading edge of the pulse. The graph shows an instantaneous rise whereas PSpice needs a finite rise time, thus artificially creating a phase shift.***


10V
5V
0V
0s
2s
4s
6s
8s
10s
12s
V(1)
Time

Chapter 17, Solution 67.

The Schematic is shown below. In the Transient dialog box, we type "Print step = 0.01s, Final time = 36s, Center frequency = 0.1667, Output vars = v(1)," and click Enable Fourier. After simulation, the output file includes the following Fourier components,



FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

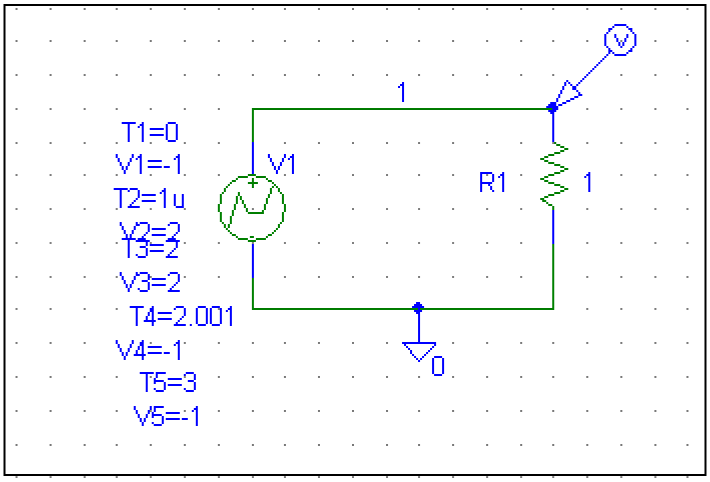
DC COMPONENT = 2.000396E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.667E-01	2.432E+00	1.000E+00	-8.996E+01	0.000E+00
2	3.334E-01	6.576E-04	2.705E-04	-8.932E+01	6.467E-01
3	5.001E-01	5.403E-01	2.222E-01	9.011E+01	1.801E+02
4	6.668E-01	3.343E-04	1.375E-04	9.134E+01	1.813E+02
5	8.335E-01	9.716E-02	3.996E-02	-8.982E+01	1.433E-01
6	1.000E+00	7.481E-06	3.076E-06	-9.000E+01	-3.581E-02
7	1.167E+00	4.968E-02	2.043E-02	-8.975E+01	2.173E-01
8	1.334E+00	1.613E-04	6.634E-05	-8.722E+01	2.748E+00
9	1.500E+00	6.002E-02	2.468E-02	9.032E+01	1.803E+02

TOTAL HARMONIC DISTORTION = 2.280065E+01 PERCENT

Chapter 17, Solution 68.

Since T=3, f =1/3 = 0.333 Hz. We use the schematic below.



We use VPWL to enter in the signal as shown. In the transient dialog box, we enable Fourier, select 15 for Final Time, 0.01s for Print Step, and 10ms for the Step Ceiling. When the file is saved and run, we obtain the Fourier coefficients as part of the output file as shown below.

Why is this problem wrong? Clearly the source is not periodic. The DC value must be +1!!!!!!!!!!

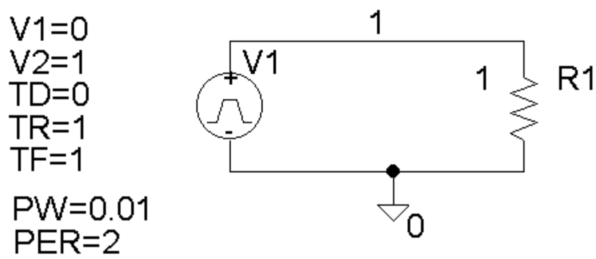
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = -1.000000E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	3.330E-01	1.615E-16	1.000E+00	1.762E+02	0.000E+00
2	6.660E-01	5.133E-17	3.179E-01	2.999E+01	-3.224E+02
3	9.990E-01	6.243E-16	3.867E+00	6.687E+01	-4.617E+02
4	1.332E+00	1.869E-16	1.158E+00	7.806E+01	-6.267E+02
5	1.665E+00	6.806E-17	4.215E-01	1.404E+02	-7.406E+02
6	1.998E+00	1.949E-16	1.207E+00	-1.222E+02	-1.179E+03
7	2.331E+00	1.465E-16	9.070E-01	-4.333E+01	-1.277E+03
8	2.664E+00	3.015E-16	1.867E+00	-1.749E+02	-1.584E+03
9	2.997E+00	1.329E-16	8.233E-01	-9.565E+01	-1.681E+03

Chapter 17, Solution 69.

The schematic is shown below. In the Transient dialog box, set Print Step = 0.05 s, Final Time = 120, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, we obtain V(1) as shown below. We also obtain an output file which includes the following Fourier components.

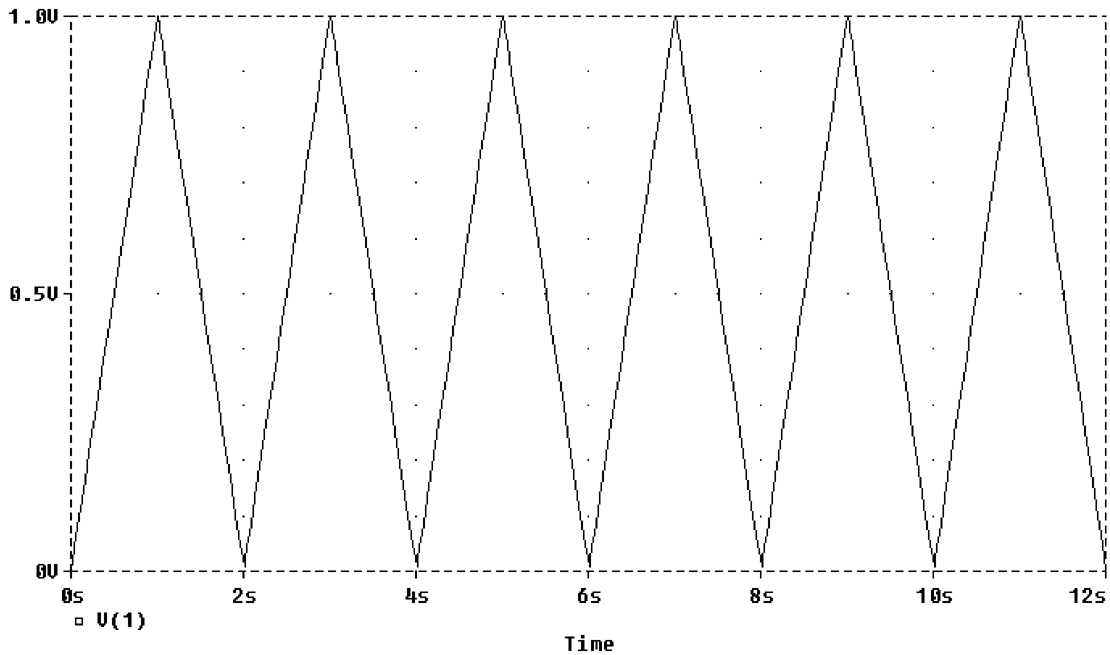


FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.048510E-01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	4.056E-01	1.000E+00	-9.090E+01	0.000E+00
2	1.000E+00	2.977E-04	7.341E-04	-8.707E+01	3.833E+00
3	1.500E+00	4.531E-02	1.117E-01	-9.266E+01	-1.761E+00
4	2.000E+00	2.969E-04	7.320E-04	-8.414E+01	6.757E+00
5	2.500E+00	1.648E-02	4.064E-02	-9.432E+01	-3.417E+00
6	3.000E+00	2.955E-04	7.285E-04	-8.124E+01	9.659E+00
7	3.500E+00	8.535E-03	2.104E-02	-9.581E+01	-4.911E+00
8	4.000E+00	2.935E-04	7.238E-04	-7.836E+01	1.254E+01
9	4.500E+00	5.258E-03	1.296E-02	-9.710E+01	-6.197E+00

TOTAL HARMONIC DISTORTION = 1.214285E+01 PERCENT


1.0V
0.5V
0V
0s
2s
4s
6s
8s
10s
12s
V(1)
Time

Chapter 17, Solution 70.

Design a problem to help other students to better understand how to use *PSpice* to solve circuit problems with periodic inputs.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Rework Prob. 17.40 using *PSpice*.

Chapter 17, Problem 40.

The signal in Fig. 17.77(a) is applied to the circuit in Fig. 17.77(b). Find $v_o(t)$.

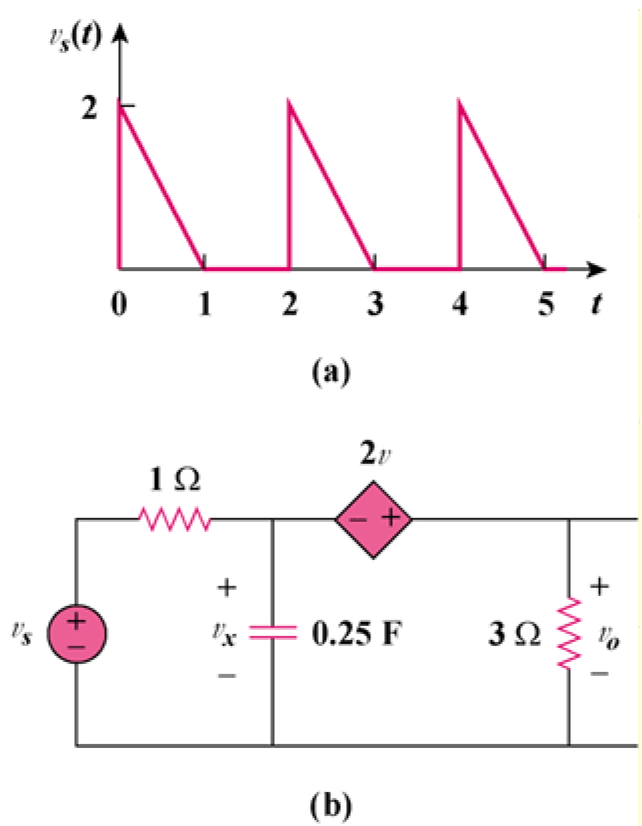
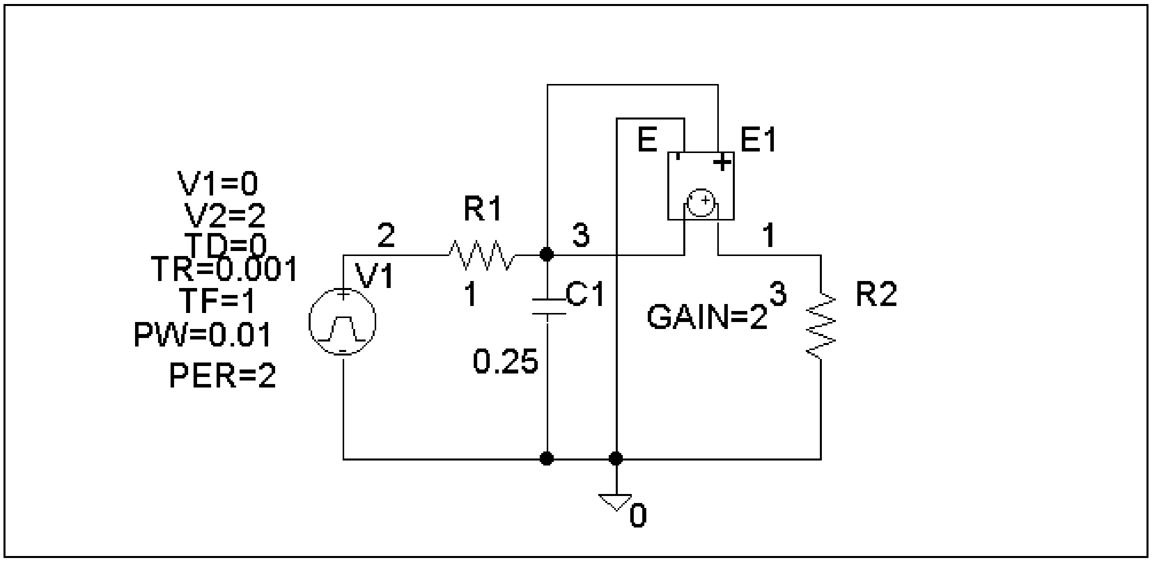


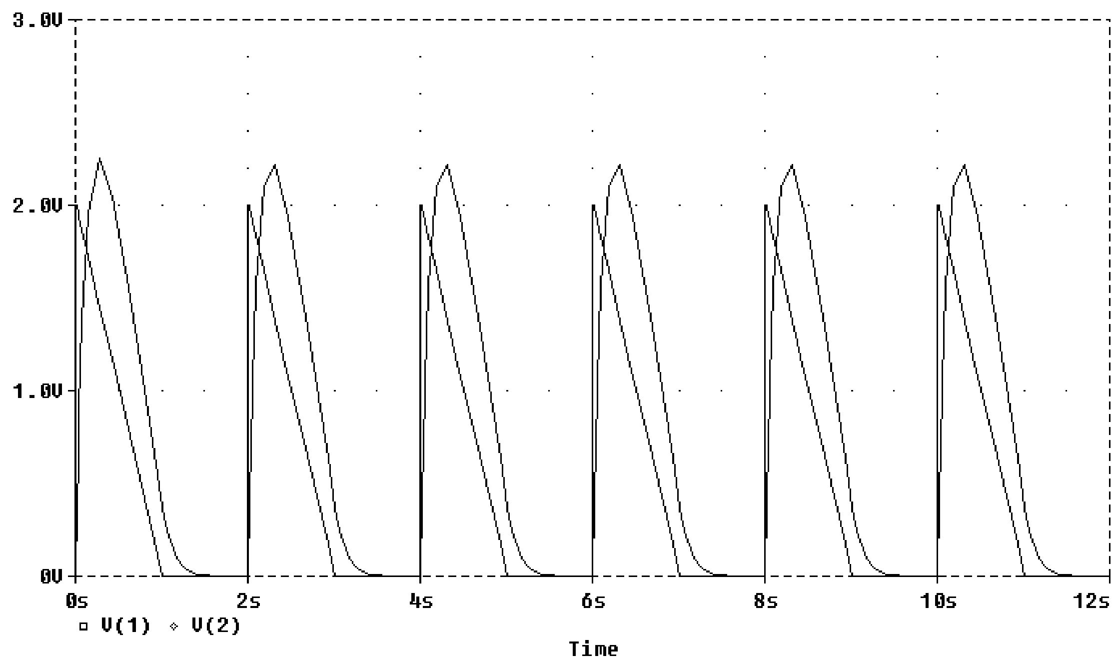
Figure 17.77

Solution

The schematic is shown below. In the Transient dialog box, we set Print Step = 0.02 s, Final Step = 12 s, Center Frequency = 0.5, Output Vars = V(1) and V(2), and click enable Fourier.

After simulation, we compare the output and output waveforms as shown. The output includes the following Fourier components.





FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

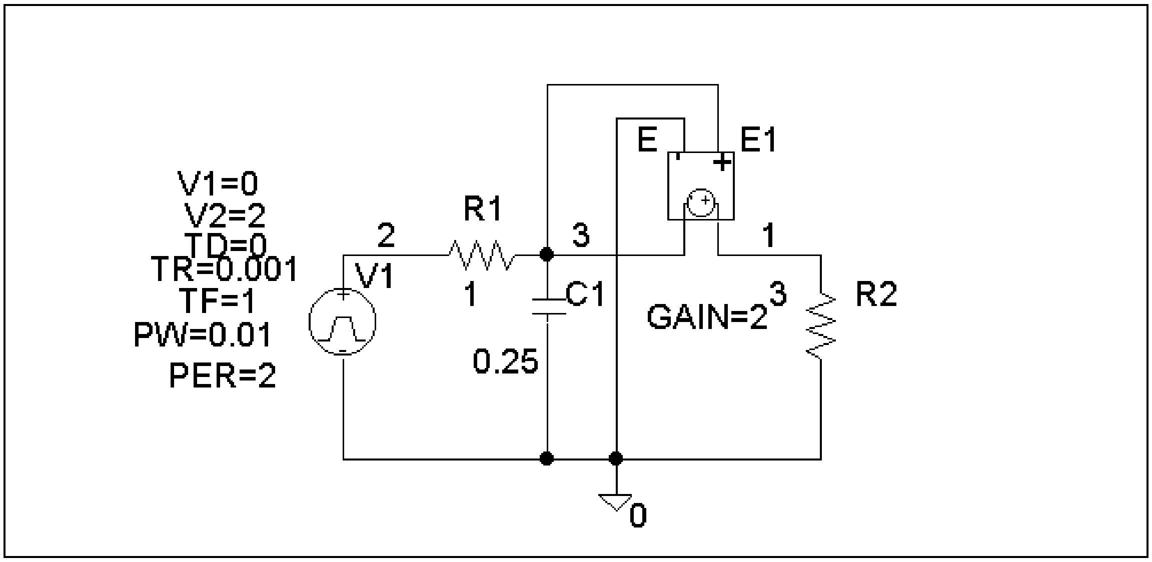
DC COMPONENT = 7.658051E-01

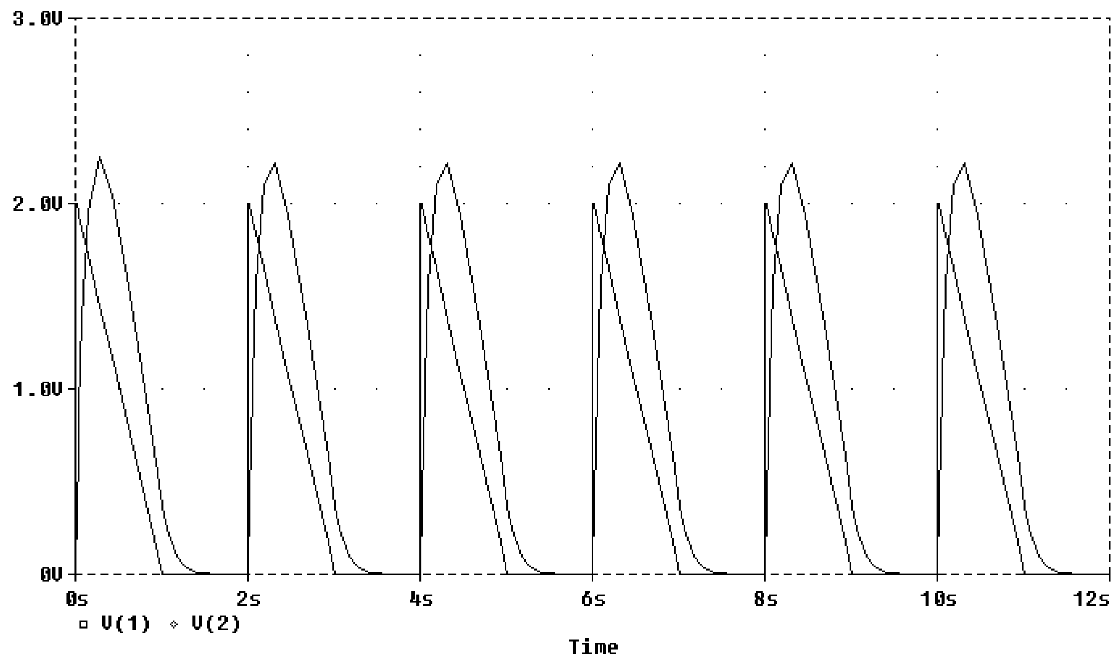
HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	1.070E+00	1.000E+00	1.004E+01	0.000E+00
2	1.000E+00	3.758E-01	3.512E-01	-3.924E+01	-4.928E+01
3	1.500E+00	2.111E-01	1.973E-01	-3.985E+01	-4.990E+01
4	2.000E+00	1.247E-01	1.166E-01	-5.870E+01	-6.874E+01
5	2.500E+00	8.538E-02	7.980E-02	-5.680E+01	-6.685E+01
6	3.000E+00	6.139E-02	5.738E-02	-6.563E+01	-7.567E+01
7	3.500E+00	4.743E-02	4.433E-02	-6.520E+01	-7.524E+01
8	4.000E+00	3.711E-02	3.469E-02	-7.222E+01	-8.226E+01
9	4.500E+00	2.997E-02	2.802E-02	-7.088E+01	-8.092E+01

TOTAL HARMONIC DISTORTION = 4.352895E+01 PERCENT

Chapter 17, Solution 71.

The schematic is shown below. In the Transient dialog box, we set Print Step = 0.02 s, Final Step = 12 s, Center Frequency = 0.5, Output Vars = V(1) and V(2), and click enable Fourier. After simulation, we compare the output and output waveforms as shown. The output includes the following Fourier components.





FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 7.658051E-01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	1.070E+00	1.000E+00	1.004E+01	0.000E+00
2	1.000E+00	3.758E-01	3.512E-01	-3.924E+01	-4.928E+01
3	1.500E+00	2.111E-01	1.973E-01	-3.985E+01	-4.990E+01
4	2.000E+00	1.247E-01	1.166E-01	-5.870E+01	-6.874E+01
5	2.500E+00	8.538E-02	7.980E-02	-5.680E+01	-6.685E+01
6	3.000E+00	6.139E-02	5.738E-02	-6.563E+01	-7.567E+01
7	3.500E+00	4.743E-02	4.433E-02	-6.520E+01	-7.524E+01
8	4.000E+00	3.711E-02	3.469E-02	-7.222E+01	-8.226E+01
9	4.500E+00	2.997E-02	2.802E-02	-7.088E+01	-8.092E+01

TOTAL HARMONIC DISTORTION = 4.352895E+01 PERCENT

Chapter 17, Solution 72.

$T = 5, \ \omega_o = 2\pi/T = 2\pi/5$

f(t) is an odd function. $a_o = 0 = a_n$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{5}\int_0^{10} 10\sin(0.4n\pi t)dt$$

$$= -\frac{8x5}{2n\pi}\cos(0.4\pi nt)\Big|_0^1 = \frac{20}{n\pi}[1 - \cos(0.4n\pi)]$$

$$f(t) = \boldsymbol{\frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}[1 - \cos(0.4n\pi)]\sin(0.4n\pi t)}$$

Chapter 17, Solution 73.

$$p = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum \frac{V_n^2}{R}$$

$$= 0 + 0.5[(2^2 + 1^2 + 1^2)/10] = \mathbf{300\ mW}$$

Chapter 17, Solution 74.

(a) $$A_n = \sqrt{a_n^2 + b_n^2}, \qquad \phi = \tan^{-1}(b_n/a_n)$$

$$A_1 = \sqrt{6^2 + 8^2} = 10, \qquad \phi_1 = \tan^{-1}(6/8) = 36.87°$$

$$A_2 = \sqrt{3^2 + 4^2} = 5, \qquad \phi_2 = \tan^{-1}(3/4) = 36.87°$$

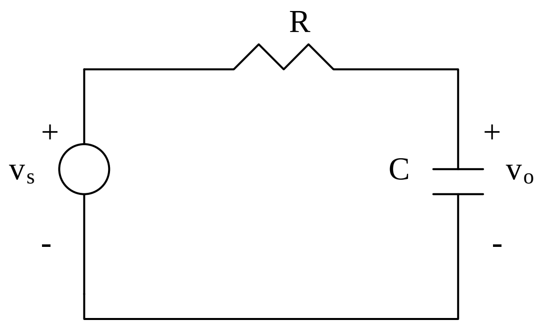
$$i(t) = \mathbf{\{4 + 10\cos(100\pi t - 36.87°) - 5\cos(200\pi t - 36.87°)\}\ A}$$

(b) $$p = I_{DC}^2 R + 0.5\sum I_n^2 R$$

$$= 2[4^2 + 0.5(10^2 + 5^2)] = \mathbf{157\ W}$$

Chapter 17, Solution 75.

The lowpass filter is shown below.



$$v_s = \frac{A\tau}{T} + \frac{2A}{T}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi\tau}{T}\cos n\omega_o t$$

$$V_o = \frac{\frac{1}{j\omega_n C}}{R + \frac{1}{j\omega_n C}} V_s = \frac{1}{1 + j\omega_n RC} V_s, \qquad \omega_n = n\omega_o = 2n\pi / T$$

For n=0, (dc component), $V_o = V_s = \dfrac{A\tau}{T}$ (1)

For the nth harmonic,

$$V_o = \frac{1}{\sqrt{1 + \omega^2{}_n R^2 C^2} \angle \tan^{-1} \omega_n RC} \bullet \frac{2A}{nT}\sin\frac{n\pi\tau}{T}\angle -90^o$$

When n=1, $| V_o |= \dfrac{2A}{T}\sin\dfrac{n\pi\tau}{T} \bullet \dfrac{1}{\sqrt{1 + \dfrac{4\pi^2}{T} R^2 C^2}}$ (2)

From (1) and (2),

$$\frac{A\tau}{T} = 50x\frac{2A}{T}\sin\frac{\pi}{10}\frac{1}{\sqrt{1 + \frac{4\pi^2}{T} R^2 C^2}} \quad \longrightarrow \quad \sqrt{1 + \frac{4\pi^2}{T} R^2 C^2} = \frac{30.9}{\tau} = 3.09x10^4$$

$$1+\frac{4\pi^2}{T}R^2C^2=10^{10} \quad \longrightarrow \quad C=\frac{T}{2\pi R}10^5=\frac{10^{-2}x3.09x10^4}{4\pi x10^3}=\underline{24.59\,mF}$$

Chapter 17, Solution 76.

$v_s(t)$ is the same as f(t) in Figure 16.1 except that the magnitude is multiplied by 10. Hence

$$v_o(t) = 5 + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin(n\pi t), \quad n = 2k - 1$$

$$T = 2, \ \omega_o = 2\pi/T = 2\pi, \ \omega_n = n\omega_o = 2n\pi$$

$$j\omega_n L = j2n\pi; \ Z = R\|10 = 10R/(10 + R)$$

$$V_o = ZV_s/(Z + j2n\pi) = [10R/(10R + j2n\pi(10 + R))]V_s$$

$$V_o = \frac{10R\angle -\tan^{-1}\{(n\pi/5R)(10+R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10+R)^2}}V_s$$

$$V_s = [20/(n\pi)]\angle 0°$$

The source current I_s is

$$I_s = \frac{V_s}{Z + j2n\pi} = \frac{V_s}{\dfrac{10R}{10+R} + j2n\pi} = \frac{(10+R)\dfrac{20}{n\pi}}{10R + j2n\pi(10+R)}$$

$$= \frac{(10+R)\dfrac{20}{n\pi}\angle -\tan^{-1}\{(n\pi/3)(10+R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10+R)^2}}$$

$$p_s = V_{DC}I_{DC} + \frac{1}{2}\sum V_{sn}I_{sn}\cos(\theta_n - \phi_n)$$

For the DC case, L acts like a short-circuit.

$$I_s = \frac{5}{\dfrac{10R}{10+R}} = \frac{5(10+R)}{10R}, \ V_s = 5 = V_o$$

$$p_s = \frac{25(10+R)}{10R} + \frac{1}{2}\left[\left(\frac{20}{\pi}\right)^2 \frac{(10+R)\cos\left(\tan^{-1}\left(\frac{\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 4\pi^2(10+R)^2}}\right.$$

$$\left. + \left(\frac{10}{\pi}\right)^2 \frac{(10+R)^2\cos\left(\tan^{-1}\left(\frac{2\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 16\pi^2(10+R)^2}} + \cdots\right]$$

$$p_s = \frac{V_{DC}}{R} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{V_{on}}{R}$$

$$= \frac{25}{R} + \frac{1}{2}\left[\frac{100R}{100R^2 + 4\pi^2(10+R)^2} + \frac{100R}{100R^2 + 10\pi^2(10+R)^2} + \cdots\right]$$

We want $p_o = (70/100)p_s = 0.7p_s$. Due to the complexity of the terms, we consider only the DC component as an approximation. In fact the DC component has the latgest share of the power for both input and output signals.

$$\frac{25}{R} = \frac{7}{10} \text{ x } \frac{25(10+R)}{10R}$$

100 = 70 + 7R which leads to R = 30/7 = **4.286 Ω**

Chapter 17, Solution 77.

(a) For the first two AC terms, the frequency ratio is 6/4 = 1.5 so that the highest common factor is 2. Hence $\omega_o = 2$.

$$T = 2\pi/\omega_o = 2\pi/2 = \boldsymbol{\pi}$$

(b) The average value is the DC component = **–2**

(c) $$V_{rms} = \sqrt{a_o + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)}$$

$$V_{rms}^2 = (-2)^2 + \frac{1}{2}(10^2 + 8^2 + 6^2 + 3^2 + 1^2) = 121.5$$

$$V_{rms} = \mathbf{11.02\ V}$$

Chapter 17, Solution 78.

(a) $$p = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum \frac{V_n^2}{R} = \frac{V_{DC}^2}{R} + \sum \frac{V_{n,rms}^2}{R}$$

$$= 0 + (40^2/5) + (20^2/5) + (10^2/5) = \mathbf{420\ W}$$

(b) 5% increase = (5/100)420 = 21

$$p_{DC} = 21\text{ W} = \frac{V_{DC}^2}{R} \quad \text{which leads to} \quad V_{DC}^2 = 21R = 105$$

V_{DC} = **10.25 V**

Chapter 17, Solution 79.

From Table 17.3, it is evident that $a_n = 0$,

$$b_n = 4A/[\pi(2n - 1)], \quad A = 10.$$

A Fortran program to calculate b_n is shown below. The result is also shown.

```
C       FOR PROBLEM 17.79
              DIMENSION B(20)

              A  =  10
              PIE  =  3.142
              C  =  4.*A/PIE
              DO 10 N = 1, 10
              B(N)  =  C/(2.*FLOAT(N) – 1.)
              PRINT  *, N, B(N)
10            CONTINUE
              STOP
              END
```

n	b_n
1	12.731
2	4.243
3	2.546
4	1.8187
5	1.414
6	1.1573
7	0.9793
8	0.8487
9	0.7498
10	0.6700

Chapter 17, Solution 80.

From Problem 17.55,

$$c_n = [1 + e^{-jn\pi}]/[2\pi(1 - n^2)]$$

This is calculated using the Fortran program shown below. The results are also shown.

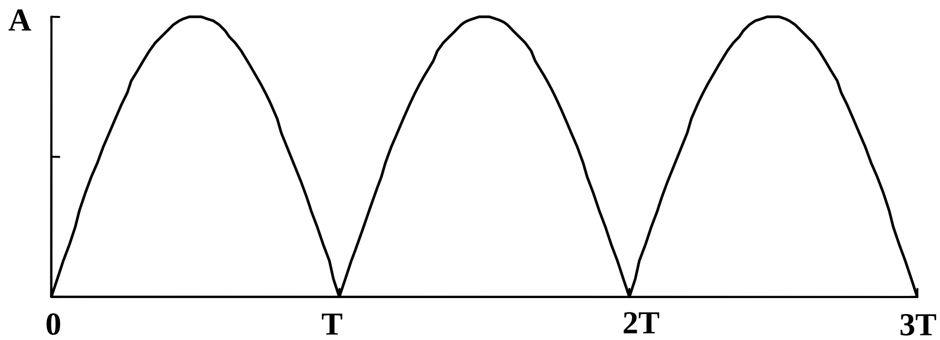
```
C       FOR PROBLEM 17.80
              COMPLEX X, C(0:20)

              PIE  =  3.1415927
              A  =  2.0*PIE
              DO  10  N = 0, 10
              IF(N.EQ.1) GO TO 10
              X  =  CMPLX(0, PIE*FLOAT(N))
              C(N)  =  (1.0 + CEXP(–X))/(A*(1 – FLOAT(N*N)))
              PRINT *, N, C(N)
10            CONTINUE
              STOP
              END
```

n	c_n
0	0.3188 + j0
1	0
2	–0.1061 + j0
3	0
4	$-0.2121\text{x}10^{-1} + j0$
5	0
6	$-0.9095\text{x}10^{-2} + j0$
7	0
8	$-0.5052\text{x}10^{-2} + j0$
9	0
10	$-0.3215\text{x}10^{-2} + j0$

Chapter 17, Solution 81.

(a)



$$f(t) = \frac{2A}{\pi} - \frac{4A}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos(n\omega_o t)$$

The total average power is $p_{avg} = F_{rms}{}^2R = F_{rms}{}^2$ since R = 1 ohm.

$$P_{avg} = F_{rms}{}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \mathbf{0.5A^2}$$

(b) From the Fourier series above

$$|c_o| = 2A/\pi,\ |c_n| = |a_n|/2 = 2A/[\pi(4n^2 - 1)]$$

n	ω_o	$\lvert c_n\rvert$	$\lvert c_o\rvert^2$ or $2\lvert c_n\rvert^2$	% power
0	0	$2A/\pi$	$4A^2/(\pi^2)$	81.1%
1	$2\omega_o$	$2A/(3\pi)$	$8A^2/(9\pi^2)$	18.01%
2	$4\omega_o$	$2A/(15\pi)$	$8A^2/(225\pi^2)$	0.72%
3	$6\omega_o$	$2A/(35\pi)$	$8A^2/(1225\pi^2)$	0.13%
4	$8\omega_o$	$2A/(63\pi)$	$8A^2/(3969\pi^2)$	0.04%

(c) **81.1%**

(d) **0.72%**

Chapter 17, Solution 82.

$$P = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{V_n^2}{R}$$

Assuming V is an amplitude-phase form of Fourier series. But

$$|A_n| = 2|C_n|, \; c_o = a_o$$

$$|A_n|^2 = 4|C_n|^2$$

Hence,

$$P = \frac{c_o^2}{R} + 2\sum_{n=1}^{\infty}\frac{c_n^2}{R}$$

Alternatively,

$$P = \frac{V_{rms}^2}{R}$$

where

$$V_{rms}^2 = a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty}A_n^2 = c_o^2 + 2\sum_{n=1}^{\infty}c_n^2 = \sum_{n=-\infty}^{\infty}c_n^2$$

$$= 10^2 + 2(8.5^2 + 4.2^2 + 2.1^2 + 0.5^2 + 0.2^2)$$

$$= 100 + 2x94.57 = 289.14$$

$P = 289.14/4 =$ **72.3 W**