

Chapter 3, Solution 1

Using Fig. 3.50, design a problem to help other students to better understand nodal analysis.

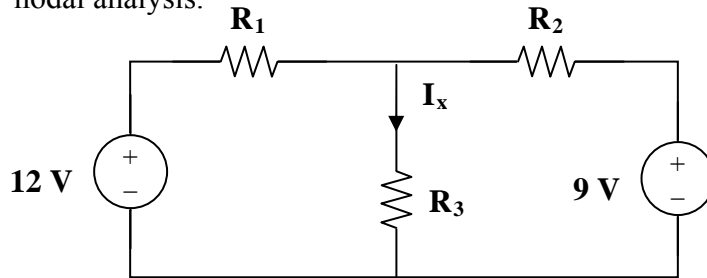


Figure 3.50
For Prob. 3.1 and Prob. 3.39.

Solution

Given $R_1 = 4\text{ k}\Omega$, $R_2 = 2\text{ k}\Omega$, and $R_3 = 2\text{ k}\Omega$, determine the value of I_x using nodal analysis.

Let the node voltage in the top middle of the circuit be designated as V_x .

$$[(V_x - 12)/4k] + [(V_x - 0)/2k] + [(V_x - 9)/2k] = 0 \text{ or (multiply this by 4 k)}$$

$$(1 + 2 + 2)V_x = 12 + 18 = 30 \text{ or } V_x = 30/5 = 6 \text{ volts and}$$

$$I_x = 6/(2k) = \mathbf{3\text{ mA}}.$$

Chapter 3, Solution 2

At node 1,

$$\frac{-v_1}{10} - \frac{v_1}{5} = 6 + \frac{v_1 - v_2}{2} \longrightarrow 60 = -8v_1 + 5v_2 \quad (1)$$

At node 2,

$$\frac{v_2}{4} = 3 + 6 + \frac{v_1 - v_2}{2} \longrightarrow 36 = -2v_1 + 3v_2 \quad (2)$$

Solving (1) and (2),

$$v_1 = \mathbf{0 \text{ V}}, v_2 = \mathbf{12 \text{ V}}$$

Chapter 3, Solution 3

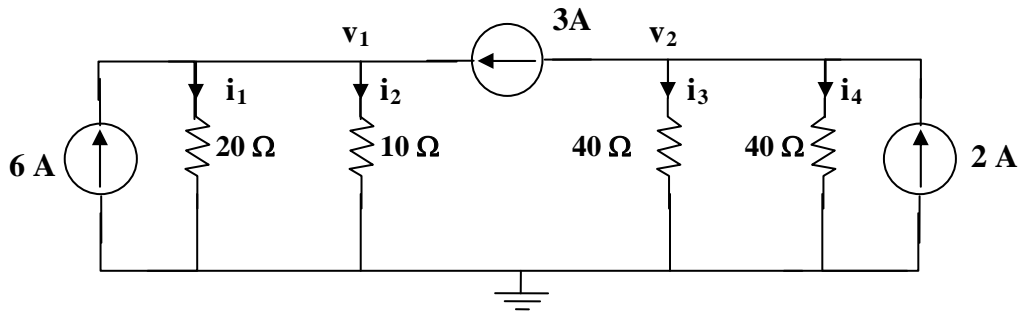
Applying KCL to the upper node,

$$-8 + \frac{v_0}{10} + \frac{v_0}{20} + \frac{v_0}{30} + 20 + \frac{v_0}{60} = 0 \text{ or } v_0 = \mathbf{-60 \text{ V}}$$

$$i_1 = \frac{v_0}{10} = \mathbf{-6 \text{ A}}, i_2 = \frac{v_0}{20} = \mathbf{-3 \text{ A}},$$

$$i_3 = \frac{v_0}{30} = \mathbf{-2 \text{ A}}, i_4 = \frac{v_0}{60} = \mathbf{1 \text{ A}}.$$

Chapter 3, Solution 4



At node 1,

$$-6 - 3 + v_1/(20) + v_1/(10) = 0 \text{ or } v_1 = 9(200/30) = 60 \text{ V}$$

At node 2,

$$3 - 2 + v_2/(10) + v_2/(5) = 0 \text{ or } v_2 = -1(1600/80) = -20 \text{ V}$$

$$i_1 = v_1/(20) = \mathbf{3 \text{ A}}, i_2 = v_1/(10) = \mathbf{6 \text{ A}},$$
$$i_3 = v_2/(40) = \mathbf{-500 \text{ mA}}, i_4 = v_2/(40) = \mathbf{-500 \text{ mA}}.$$

Chapter 3, Solution 5

Apply KCL to the top node.

$$\frac{30 - v_0}{2k} + \frac{20 - v_0}{5k} = \frac{v_0}{4k} \longrightarrow v_0 = \mathbf{20\text{ V}}$$

Chapter 3, Solution 6.

Solve for V_1 using nodal analysis.

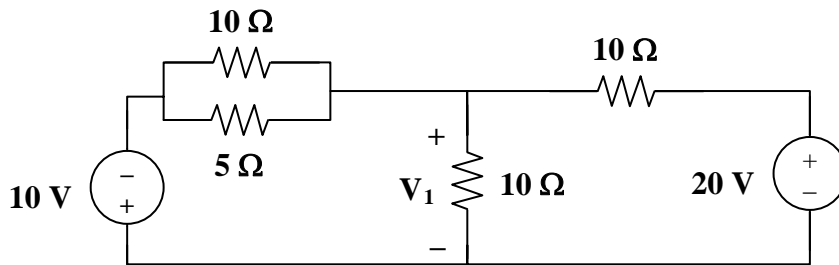


Figure 3.55
For Prob. 3.6.

Step 1. The first thing to do is to select a reference node and to identify all the unknown nodes. We select the bottom of the circuit as the reference node. The only unknown node is the one connecting all the resistors together and we will call that node V_1 . The other two nodes are at the top of each source. Relative to the reference, the one at the top of the 10-volt source is -10 V. At the top of the 20-volt source is $+20$ V.

Step 2. Setup the nodal equation (there is only one since there is only one unknown).

$$\frac{(V_1 - (-10))}{5} + \frac{(V_1 - (-10))}{10} + \frac{(V_1 - 0)}{10} + \frac{(V_1 - 20)}{10} = 0$$

Step 3. Simplify and solve.

$$\begin{aligned} \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}\right)V_1 &= -\frac{10}{5} - \frac{10}{10} + \frac{20}{10} \\ (0.2 + 0.1 + 0.1 + 0.1)V_1 &= 0.5V_1 = -2 - 1 + 2 = -1 \end{aligned}$$

or

$$V_1 = -2 \text{ V.}$$

The answer can be checked by calculating all the currents and see if they add up to zero. The top two currents on the left flow right to left and are 0.8 A and 1.6 A respectively. The current flowing up through the 10-ohm resistor is 0.2 A. The current flowing right to left through the 10-ohm resistor is 2.2 A. Summing all the currents flowing out of the node, V_1 , we get, $+0.8 + 1.6 - 0.2 - 2.2 = 0$. The answer checks.

Chapter 3, Solution 7

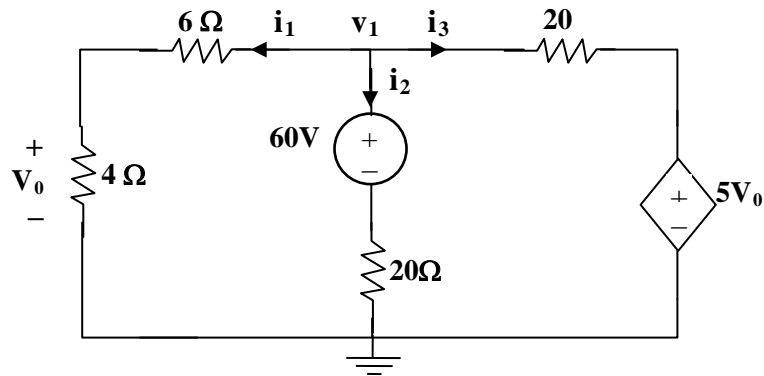
$$-2 + \frac{V_x - 0}{10} + \frac{V_x - 0}{20} + 0.2V_x = 0$$

$$0.35V_x = 2 \text{ or } V_x = \mathbf{5.714 \text{ V}}.$$

Substituting into the original equation for a check we get,

$$0.5714 + 0.2857 + 1.1428 = 1.9999 \text{ checks!}$$

Chapter 3, Solution 8



$$i_1 + i_2 + i_3 = 0 \longrightarrow \frac{v_1}{10} + \frac{(v_1 - 60) - 0}{20} + \frac{v_1 - 5v_o}{20} = 0$$

But $v_o = \frac{2}{5}v_1$ so that $2v_1 + v_1 - 60 + v_1 - 2v_1 = 0$

or $v_1 = 60/2 = 30$ V, therefore $v_o = 2v_1/5 = \mathbf{12}$ V.

Chapter 3, Solution 9

Let V_1 be the unknown node voltage to the right of the $250\text{-}\Omega$ resistor. Let the ground reference be placed at the bottom of the $50\text{-}\Omega$ resistor. This leads to the following nodal equation:

$$\frac{V_1 - 24}{250} + \frac{V_1 - 0}{50} + \frac{V_1 - 60I_b - 0}{150} = 0$$

simplifying we get

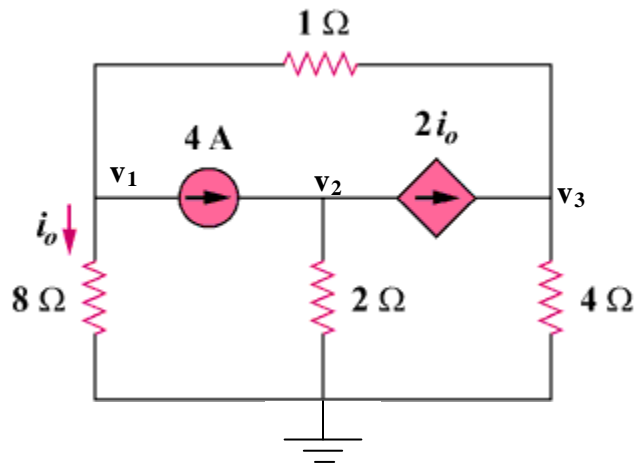
$$3V_1 - 72 + 15V_1 + 5V_1 - 300I_b = 0$$

But $I_b = \frac{24 - V_1}{250}$. Substituting this into the nodal equation leads to

$$24.2V_1 - 100.8 = 0 \quad \text{or} \quad V_1 = 4.165 \text{ V.}$$

Thus, $I_b = (24 - 4.165)/250 = \mathbf{79.34 \text{ mA}}$.

Chapter 3, Solution 10



At node 1. $[(v_1-0)/8] + [(v_1-v_3)/1] + 4 = 0$

At node 2. $-4 + [(v_2-0)/2] + 2i_o = 0$

At node 3. $-2i_o + [(v_3-0)/4] + [(v_3-v_1)/1] = 0$

Finally, we need a constraint equation, $i_o = v_1/8$

This produces,

$$1.125v_1 - v_3 = 4 \quad (1)$$

$$0.25v_1 + 0.5v_2 = 4 \quad (2)$$

$$-1.25v_1 + 1.25v_3 = 0 \text{ or } v_1 = v_3 \quad (3)$$

Substituting (3) into (1) we get $(1.125-1)v_1 = 4$ or $v_1 = 4/0.125 = 32$ volts. This leads to,

$$i_o = 32/8 = \mathbf{4 \text{ amps.}}$$

Chapter 3, Solution 11

Find V_o and the power absorbed by all the resistors in the circuit of Fig. 3.60.

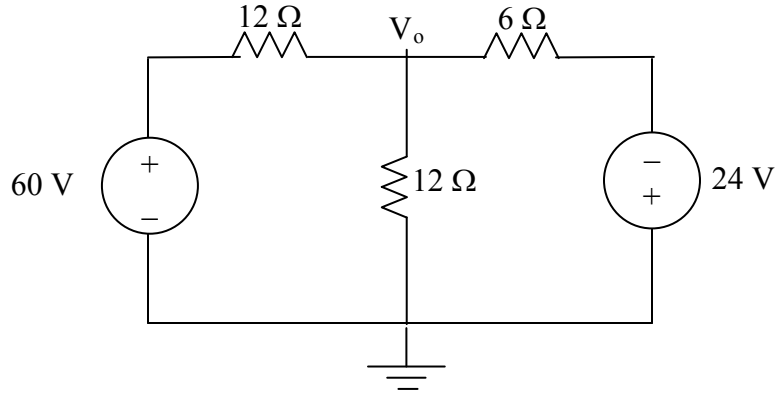


Figure 3.60
For Prob. 3.11.

Solution

At the top node, KCL produces $\frac{V_o - 60}{12} + \frac{V_o - 0}{12} + \frac{V_o - (-24)}{6} = 0$

$$(1/3)V_o = 1 \text{ or } V_o = \mathbf{3 \text{ V.}}$$

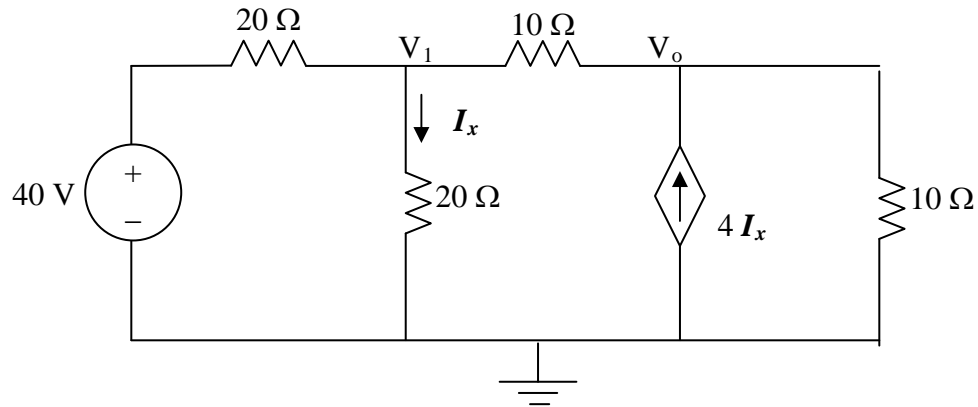
$P_{12\Omega} = (3-60)^2/12 = \mathbf{293.9 \text{ W}}$ (this is for the 12Ω resistor in series with the 60 V source)

$P_{12\Omega} = (V_o)^2/12 = 9/12 = \mathbf{750 \text{ mW}}$ (this is for the 12Ω resistor connecting V_o to ground)

$$P_{6\Omega} = (3-(-24))^2/6 = \mathbf{121.5 \text{ W.}}$$

Chapter 3, Solution 12

There are two unknown nodes, as shown in the circuit below.



At node 1,

$$\begin{aligned}\frac{V_1 - 40}{20} + \frac{V_1 - 0}{20} + \frac{V_1 - V_o}{10} &= 0 \text{ or} \\ (0.05 + 0.05 + 0.1)V_1 - 0.1V_o &= 0.2V_1 - 0.1V_o = 2\end{aligned}\quad (1)$$

At node o,

$$\begin{aligned}\frac{V_o - V_1}{10} - 4I_x + \frac{V_o - 0}{10} &= 0 \text{ and } I_x = V_1/20 \\ -0.1V_1 - 0.2V_1 + 0.2V_o &= -0.3V_1 + 0.2V_o = 0 \text{ or}\end{aligned}\quad (2)$$

$$V_1 = (2/3)V_o \quad (3)$$

Substituting (3) into (1),

$$0.2(2/3)V_o - 0.1V_o = 0.03333V_o = 2 \text{ or}$$

$$V_o = \mathbf{60 \text{ V.}}$$

Chapter 3, Solution 13

Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

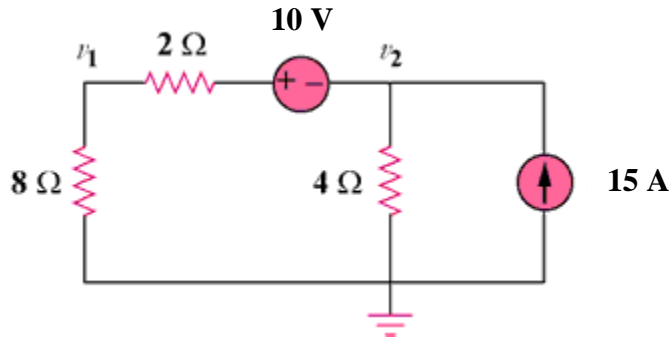


Figure 3.62
For Prob. 3.13.

Solution

At node number 2, $[(v_2 + 10) - 0]/10 + [(v_2 - 0)/4] - 15 = 0$ or
 $(0.1 + 0.25)v_2 = -1 + 15 = 14$ or

$$v_2 = \mathbf{40 \text{ volts.}}$$

Next, $I = [(v_2 + 10) - 0]/10 = (40 + 10)/10 = 5 \text{ amps}$ and

$$v_1 = 8 \times 5 = \mathbf{40 \text{ volts.}}$$

Chapter 3, Solution 14

Using nodal analysis, find v_o in the circuit of Fig. 3.63.

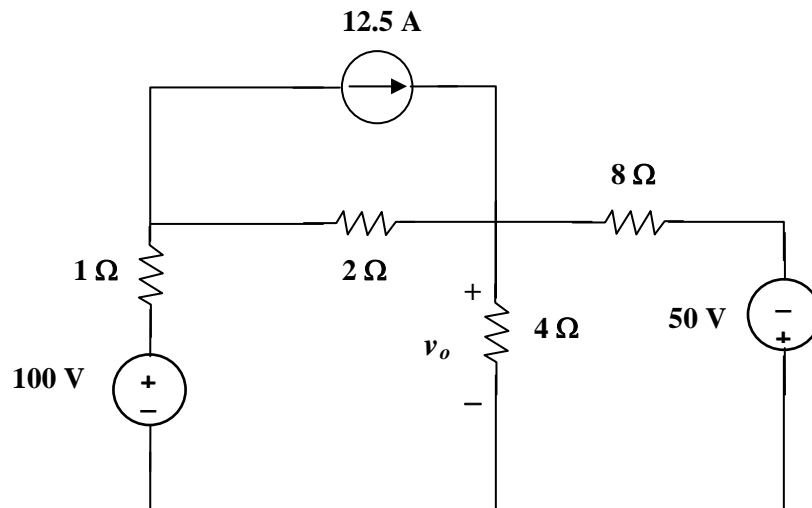
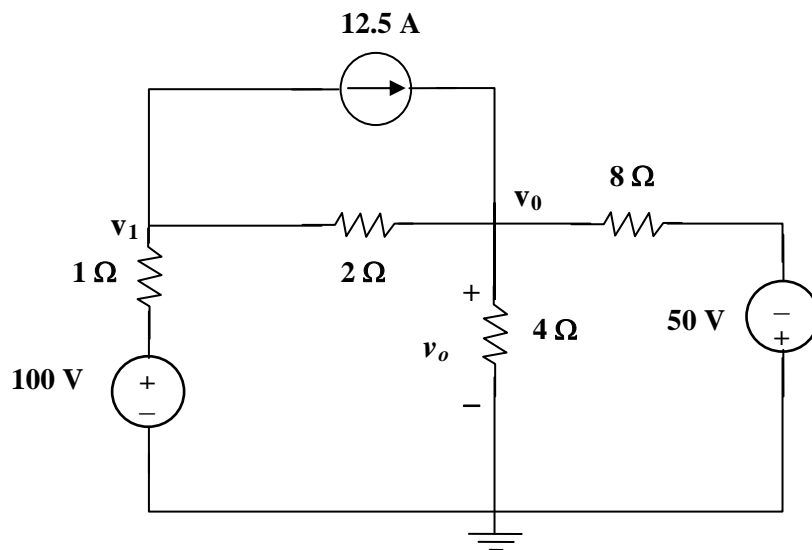


Figure 3.63
For Prob. 3.14.

Solution



At node 1,

$$[(v_1 - 100)/1] + [(v_1 - v_o)/2] + 12.5 = 0 \text{ or } 3v_1 - v_o = 200 - 25 = 175 \quad (1)$$

At node o,

$$[(v_o - v_1)/2] - 12.5 + [(v_o - 0)/4] + [(v_o + 50)/8] = 0 \text{ or } -4v_1 + 7v_o = 50 \quad (2)$$

Adding 4x(1) to 3x(2) yields,

$$4(1) + 3(2) = -4v_o + 21v_o = 700 + 150 \text{ or } 17v_o = 850 \text{ or}$$

$$v_o = \mathbf{50 \text{ V}}.$$

Checking, we get $v_1 = (175 + v_o)/3 = 75 \text{ V}$.

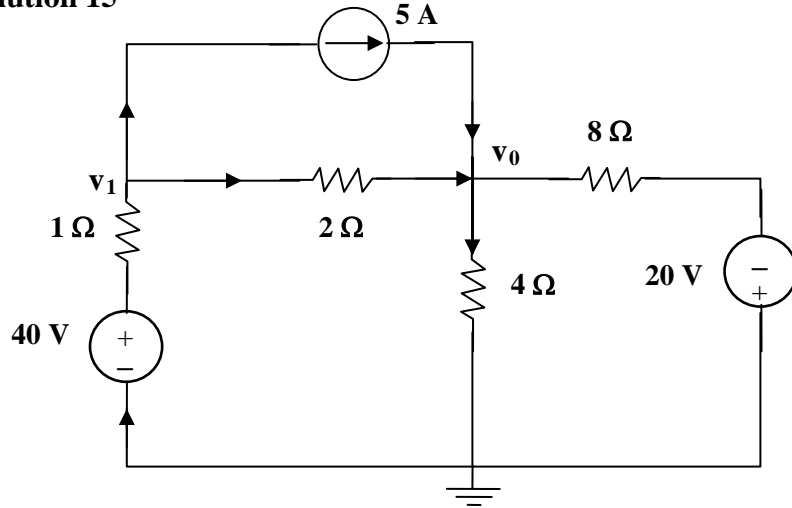
At node 1,

$$[(75-100)/1] + [(75-50)/2] + 12.5 = -25 + 12.5 + 12.5 = 0!$$

At node o,

$$[(50-75)/2] + [(50-0)/4] + [(50+50)/8] - 12.5 = -12.5 + 12.5 + 12.5 - 12.5 = 0!$$

Chapter 3, Solution 15



Nodes 1 and 2 form a supernode so that $v_1 = v_2 + 10$ (1)

At the supernode, $2 + 6v_1 + 5v_2 = 3(v_3 - v_2) \longrightarrow 2 + 6v_1 + 8v_2 = 3v_3$ (2)

At node 3, $2 + 4 = 3(v_3 - v_2) \longrightarrow v_3 = v_2 + 2$ (3)

Substituting (1) and (3) into (2),

$$2 + 6v_2 + 60 + 8v_2 = 3v_2 + 6 \longrightarrow v_2 = \frac{-56}{11}$$

$$v_1 = v_2 + 10 = \frac{54}{11}$$

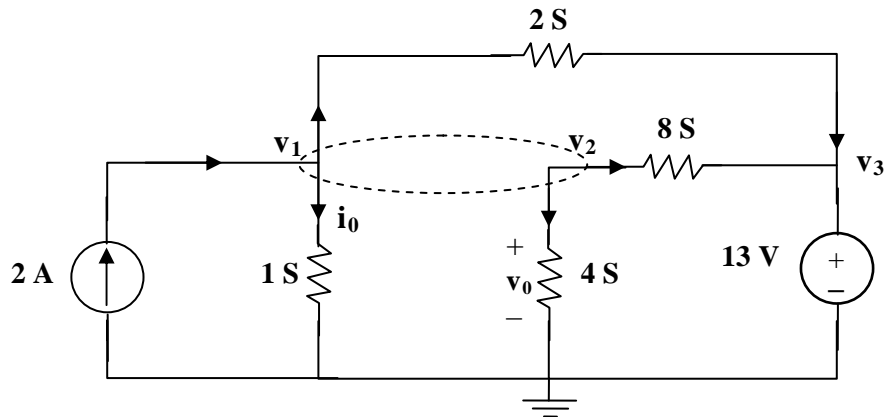
$$i_0 = 6v_1 = \mathbf{29.45 \text{ A}}$$

$$P_{65} = \frac{v_1^2}{R} = v_1^2 G = \left(\frac{54}{11}\right)^2 6 = \mathbf{144.6 \text{ W}}$$

$$P_{55} = v_2^2 G = \left(\frac{-56}{11}\right)^2 5 = \mathbf{129.6 \text{ W}}$$

$$P_{35} = (v_L - v_3)^2 G = (2)^2 3 = \mathbf{12 \text{ W}}$$

Chapter 3, Solution 16



At the supernode,

$$2 = v_1 + 2(v_1 - v_3) + 8(v_2 - v_3) + 4v_2, \text{ which leads to } 2 = 3v_1 + 12v_2 - 10v_3 \quad (1)$$

But

$$v_1 = v_2 + 2v_0 \text{ and } v_0 = v_2.$$

Hence

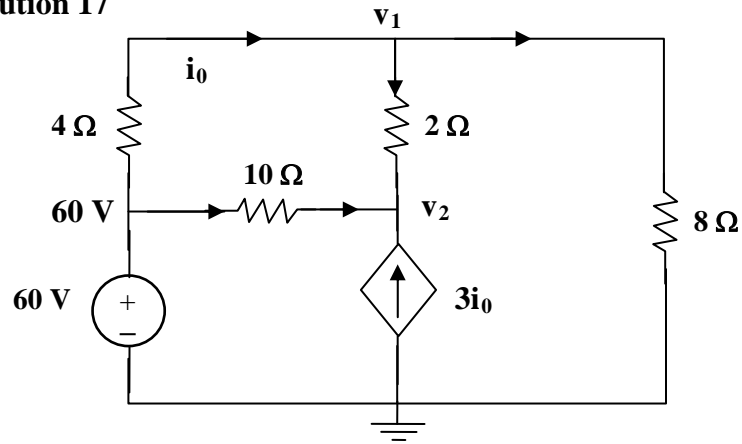
$$v_1 = 3v_2 \quad (2)$$

$$v_3 = 13V \quad (3)$$

Substituting (2) and (3) with (1) gives,

$$v_1 = 18.858 \text{ V}, v_2 = 6.286 \text{ V}, v_3 = 13 \text{ V}$$

Chapter 3, Solution 17



At node 1, $\frac{60 - v_1}{4} = \frac{v_1}{8} + \frac{v_1 - v_2}{2}$ $120 = 7v_1 - 4v_2$ (1)

At node 2, $3i_0 + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0$

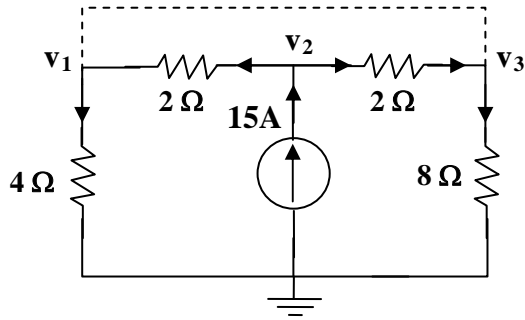
But $i_0 = \frac{60 - v_1}{4}$.

Hence

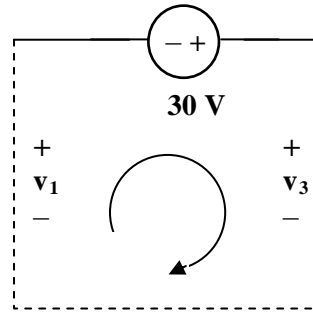
$$\frac{3(60 - v_1)}{4} + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0 \longrightarrow 1020 = 5v_1 + 12v_2 \quad (2)$$

Solving (1) and (2) gives $v_1 = 53.08$ V. Hence $i_0 = \frac{60 - v_1}{4} = \mathbf{1.73 \text{ A}}$

Chapter 3, Solution 18



(a)



(b)

At node 2, in Fig. (a), $\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{2} - 15 = 0$ or $-0.5v_1 + v_2 - 0.5v_3 = 15$ (1)

At the supernode, $\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{2} - \frac{v_1}{4} - \frac{v_3}{8} = 0$ and $(v_1/4) - 15 + (v_3/8) = 0$ or $2v_1 + v_3 = 120$ (2)

From Fig. (b), $-v_1 - 30 + v_3 = 0$ or $v_3 = v_1 + 30$ (3)

Solving (1) to (3), we obtain,

$$v_1 = \mathbf{30\text{ V}}, v_2 = \mathbf{60\text{ V}} = v_3$$

Chapter 3, Solution 19

At node 1,

$$5 = 3 + \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{8} + \frac{V_1}{4} \longrightarrow 16 = 7V_1 - V_2 - 4V_3 \quad (1)$$

At node 2,

$$\frac{V_1 - V_2}{8} = \frac{V_2}{2} + \frac{V_2 - V_3}{4} \longrightarrow 0 = -V_1 + 7V_2 - 2V_3 \quad (2)$$

At node 3,

$$3 + \frac{12 - V_3}{8} + \frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{4} = 0 \longrightarrow -36 = 4V_1 + 2V_2 - 7V_3 \quad (3)$$

From (1) to (3),

$$\begin{pmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ 4 & 2 & -7 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \\ -36 \end{pmatrix} \longrightarrow AV = B$$

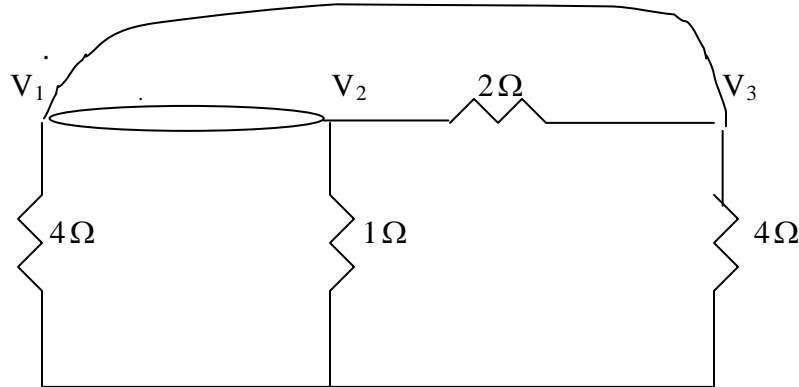
Using MATLAB,

$$V = A^{-1}B = \begin{bmatrix} 10 \\ 4.933 \\ 12.267 \end{bmatrix} \longrightarrow \underline{V_1 = 10 \text{ V}, V_2 = 4.933 \text{ V}, V_3 = 12.267 \text{ V}}$$

Chapter 3, Solution 20

Nodes 1 and 2 form a supernode; so do nodes 1 and 3. Hence

$$\frac{V_1}{4} + \frac{V_2}{1} + \frac{V_3}{4} = 0 \quad \longrightarrow \quad V_1 + 4V_2 + V_3 = 0 \quad (1)$$



Between nodes 1 and 3,

$$-V_1 + 12 + V_3 = 0 \quad \longrightarrow \quad V_3 = V_1 - 12 \quad (2)$$

Similarly, between nodes 1 and 2,

$$V_1 = V_2 + 2i \quad (3)$$

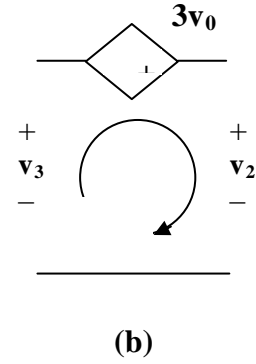
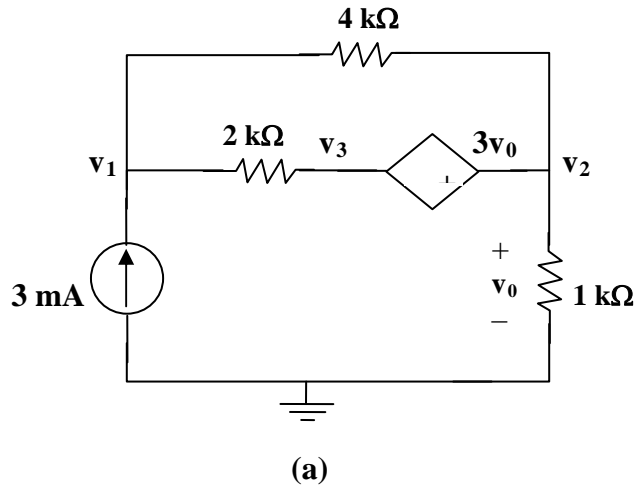
But $i = V_3 / 4$. Combining this with (2) and (3) gives

$$V_2 = 6 + V_1 / 2 \quad (4)$$

Solving (1), (2), and (4) leads to

$$\underline{V_1 = -3\text{V}, \quad V_2 = 4.5\text{V}, \quad V_3 = -15\text{V}}$$

Chapter 3, Solution 21



Let v_3 be the voltage between the $2\text{ k}\Omega$ resistor and the voltage-controlled voltage source. At node 1,

$$3 \times 10^{-3} = \frac{v_1 - v_2}{4000} + \frac{v_1 - v_3}{2000} \longrightarrow 12 = 3v_1 - v_2 - 2v_3 \quad (1)$$

At node 2,

$$\frac{v_1 - v_2}{4} + \frac{v_1 - v_3}{2} = \frac{v_2}{1} \longrightarrow 3v_1 - 5v_2 - 2v_3 = 0 \quad (2)$$

Note that $v_0 = v_2$. We now apply KVL in Fig. (b)

$$-v_3 - 3v_2 + v_2 = 0 \longrightarrow v_3 = -2v_2 \quad (3)$$

From (1) to (3),

$$v_1 = 1 \text{ V}, \quad v_2 = 3 \text{ V}$$

Chapter 3, Solution 22

$$\text{At node 1, } \frac{12 - v_0}{2} = \frac{v_1}{4} + 3 + \frac{v_1 - v_0}{8} \longrightarrow 24 = 7v_1 - v_2 \quad (1)$$

$$\text{At node 2, } 3 + \frac{v_1 - v_2}{8} = \frac{v_2 + 5v_2}{1}$$

$$\text{But, } v_1 = 12 - v_1$$

$$\text{Hence, } 24 + v_1 - v_2 = 8(v_2 + 60 + 5v_1) = 4 \text{ V}$$

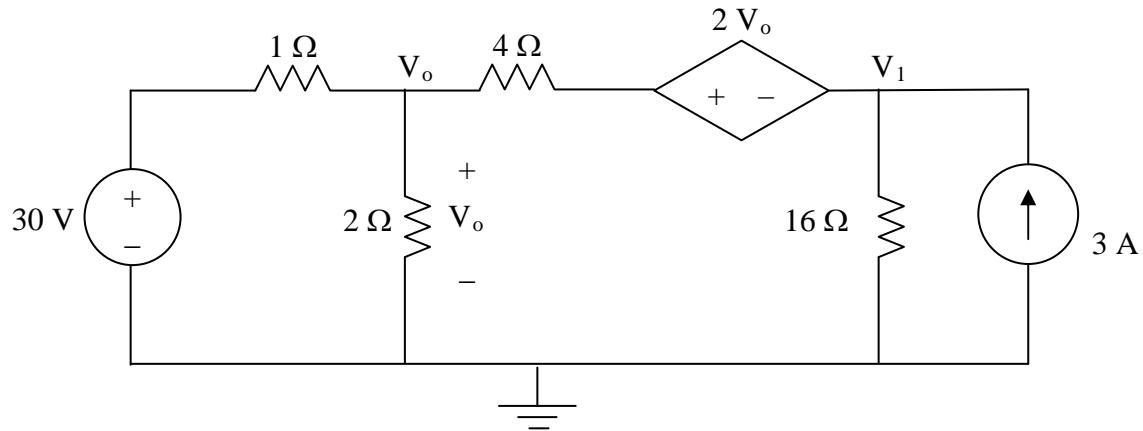
$$456 = 41v_1 - 9v_2 \quad (2)$$

Solving (1) and (2),

$$v_1 = -10.91 \text{ V, } v_2 = -100.36 \text{ V}$$

Chapter 3, Solution 23

We apply nodal analysis to the circuit shown below.



At node o,

$$\frac{V_o - 30}{1} + \frac{V_o - 0}{2} + \frac{V_o - (2V_o + V_1)}{4} = 0 \rightarrow 1.25V_o - 0.25V_1 = 30 \quad (1)$$

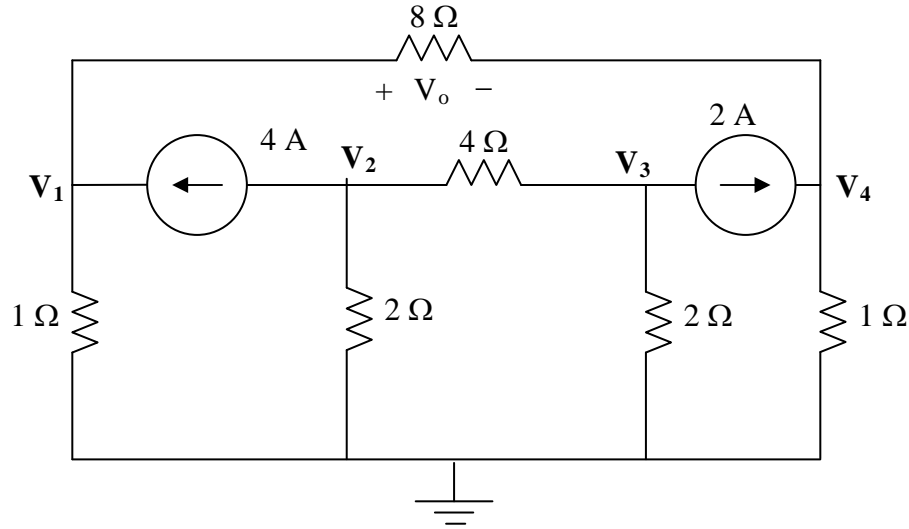
At node 1,

$$\frac{(2V_o + V_1) - V_o}{4} + \frac{V_1 - 0}{16} - 3 = 0 \rightarrow 5V_1 + 4V_o = 48 \quad (2)$$

From (1), $V_1 = 5V_o - 120$. Substituting this into (2) yields
 $29V_o = 648$ or $V_o = \mathbf{22.34\text{ V}}$.

Chapter 3, Solution 24

Consider the circuit below.



$$\frac{V_1 - 0}{1} - 4 + \frac{V_1 - V_4}{8} = 0 \rightarrow 1.125V_1 - 0.125V_4 = 4 \quad (1)$$

$$+4 + \frac{V_2 - 0}{2} + \frac{V_2 - V_3}{4} = 0 \rightarrow 0.75V_2 - 0.25V_3 = -4 \quad (2)$$

$$\frac{V_3 - V_2}{4} + \frac{V_3 - 0}{2} + 2 = 0 \rightarrow -0.25V_2 + 0.75V_3 = -2 \quad (3)$$

$$-2 + \frac{V_4 - V_1}{8} + \frac{V_4 - 0}{1} = 0 \rightarrow -0.125V_1 + 1.125V_4 = 2 \quad (4)$$

$$\begin{bmatrix} 1.125 & 0 & 0 & -0.125 \\ 0 & 0.75 & -0.25 & 0 \\ 0 & -0.25 & 0.75 & 0 \\ -0.125 & 0 & 0 & 1.125 \end{bmatrix} \mathbf{V} = \begin{bmatrix} 4 \\ -4 \\ -2 \\ 2 \end{bmatrix}$$

Now we can use MATLAB to solve for the unknown node voltages.

```
>> Y=[1.125,0,0,-0.125;0,0.75,-0.25,0;0,-0.25,0.75,0;-0.125,0,0,1.125]
```

```
Y =
```

```

1.1250    0    0 -0.1250
    0 0.7500 -0.2500    0
    0 -0.2500 0.7500    0
-0.1250    0    0 1.1250

```

```
>> I=[4,-4,-2,2]'
```

```
I =
```

```

4
-4
-2
2

```

```
>> V=inv(Y)*I
```

```
V =
```

```

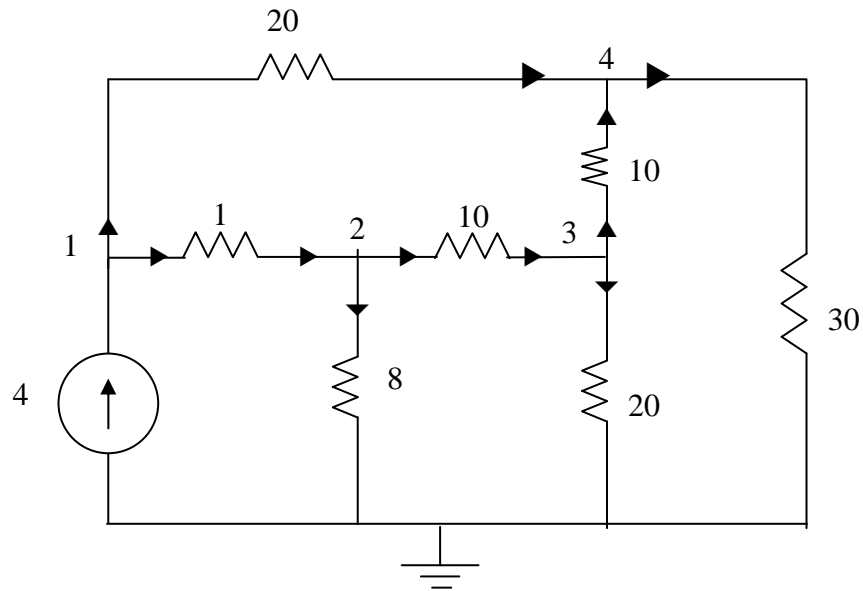
3.8000
-7.0000
-5.0000
2.2000

```

$$V_o = V_1 - V_4 = 3.8 - 2.2 = \mathbf{1.6 \text{ V.}}$$

Chapter 3, Solution 25

Consider the circuit shown below.



At node 1,

$$4 = \frac{V_1 - V_2}{1} + \frac{V_1 - V_4}{20} \longrightarrow 80 = 21V_1 - 20V_2 - V_4 \quad (1)$$

At node 2,

$$\frac{V_1 - V_2}{1} = \frac{V_2}{8} + \frac{V_2 - V_3}{10} \longrightarrow 0 = -80V_1 + 98V_2 - 8V_3 \quad (2)$$

At node 3,

$$\frac{V_2 - V_3}{10} = \frac{V_3}{20} + \frac{V_3 - V_4}{10} \longrightarrow 0 = -2V_2 + 5V_3 - 2V_4 \quad (3)$$

At node 4,

$$\frac{V_1 - V_4}{20} + \frac{V_3 - V_4}{10} = \frac{V_4}{30} \longrightarrow 0 = 3V_1 + 6V_3 - 11V_4 \quad (4)$$

Putting (1) to (4) in matrix form gives:

$$\begin{bmatrix} 80 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 21 & -20 & 0 & -1 \\ -80 & 98 & -8 & 0 \\ 0 & -2 & 5 & -2 \\ 3 & 0 & 6 & -11 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$\mathbf{B} = \mathbf{A} \mathbf{V} \longrightarrow \mathbf{V} = \mathbf{A}^{-1} \mathbf{B}$$

Using MATLAB leads to

$$V_1 = \mathbf{25.52\ V}, \quad V_2 = \mathbf{22.05\ V}, \quad V_3 = \mathbf{14.842\ V}, \quad V_4 = \mathbf{15.055\ V}$$

Chapter 3, Solution 26

At node 1,

$$\frac{15 - V_1}{20} = 3 + \frac{V_1 - V_3}{10} + \frac{V_1 - V_2}{5} \longrightarrow -45 = 7V_1 - 4V_2 - 2V_3 \quad (1)$$

At node 2,

$$\frac{V_1 - V_2}{5} + \frac{4I_o - V_2}{5} = \frac{V_2 - V_3}{5} \quad (2)$$

But $I_o = \frac{V_1 - V_3}{10}$. Hence, (2) becomes

$$0 = 7V_1 - 15V_2 + 3V_3 \quad (3)$$

At node 3,

$$3 + \frac{V_1 - V_3}{10} + \frac{-10 - V_3}{15} + \frac{V_2 - V_3}{5} = 0 \longrightarrow 70 = -3V_1 - 6V_2 + 11V_3 \quad (4)$$

Putting (1), (3), and (4) in matrix form produces

$$\begin{pmatrix} 7 & -4 & -2 \\ 7 & -15 & 3 \\ -3 & -6 & 11 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} -45 \\ 0 \\ 70 \end{pmatrix} \longrightarrow \mathbf{AV} = \mathbf{B}$$

Using MATLAB leads to

$$\mathbf{V} = \mathbf{A}^{-1}\mathbf{B} = \begin{pmatrix} -7.19 \\ -2.78 \\ 2.89 \end{pmatrix}$$

Thus,

$$V_1 = -7.19\text{V}; V_2 = -2.78\text{V}; V_3 = 2.89\text{V}.$$

Chapter 3, Solution 27

At node 1,

$$2 = 2v_1 + v_1 - v_2 + (v_1 - v_3)4 + 3i_0, \quad i_0 = 4v_2. \text{ Hence,}$$

$$2 = 7v_1 + 11v_2 - 4v_3 \quad (1)$$

At node 2,

$$v_1 - v_2 = 4v_2 + v_2 - v_3 \longrightarrow 0 = -v_1 + 6v_2 - v_3 \quad (2)$$

At node 3,

$$2v_3 = 4 + v_2 - v_3 + 12v_2 + 4(v_1 - v_3)$$

$$\text{or} \quad -4 = 4v_1 + 13v_2 - 7v_3 \quad (3)$$

In matrix form,

$$\begin{bmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{vmatrix} = 176, \quad \Delta_1 = \begin{vmatrix} 2 & 11 & -4 \\ 0 & -6 & 1 \\ -4 & 13 & -7 \end{vmatrix} = 110$$

$$\Delta_2 = \begin{vmatrix} 7 & 2 & -4 \\ 1 & 0 & 1 \\ 4 & -4 & -7 \end{vmatrix} = 66, \quad \Delta_3 = \begin{vmatrix} 7 & 11 & 2 \\ 1 & -6 & 0 \\ 4 & 13 & -4 \end{vmatrix} = 286$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{110}{176} = 0.625\text{V}, \quad v_2 = \frac{\Delta_2}{\Delta} = \frac{66}{176} = 0.375\text{V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{286}{176} = 1.625\text{V}.$$

$$v_1 = \mathbf{625\text{ mV}}, \quad v_2 = \mathbf{375\text{ mV}}, \quad v_3 = \mathbf{1.625\text{ V}}.$$

Chapter 3, Solution 28

At node c,

$$\frac{V_d - V_c}{10} = \frac{V_c - V_b}{4} + \frac{V_c}{5} \longrightarrow 0 = -5V_b + 11V_c - 2V_d \quad (1)$$

At node b,

$$\frac{V_a + 90 - V_b}{8} + \frac{V_c - V_b}{4} = \frac{V_b}{8} \longrightarrow -90 = V_a - 4V_b + 2V_c \quad (2)$$

At node a,

$$\frac{V_a - 60 - V_d}{4} + \frac{V_a}{16} + \frac{V_a + 90 - V_b}{8} = 0 \longrightarrow 60 = 7V_a - 2V_b - 4V_d \quad (3)$$

At node d,

$$\frac{V_a - 60 - V_d}{4} = \frac{V_d}{20} + \frac{V_d - V_c}{10} \longrightarrow 300 = 5V_a + 2V_c - 8V_d \quad (4)$$

In matrix form, (1) to (4) become

$$\begin{pmatrix} 0 & -5 & 11 & -2 \\ 1 & -4 & 2 & 0 \\ 7 & -2 & 0 & -4 \\ 5 & 0 & 2 & -8 \end{pmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \\ V_d \end{pmatrix} = \begin{pmatrix} 0 \\ -90 \\ 60 \\ 300 \end{pmatrix} \longrightarrow AV = B$$

We use MATLAB to invert A and obtain

$$V = A^{-1}B = \begin{pmatrix} -10.56 \\ 20.56 \\ 1.389 \\ -43.75 \end{pmatrix}$$

Thus,

$$V_a = -10.56 \text{ V}; V_b = 20.56 \text{ V}; V_c = 1.389 \text{ V}; V_d = -43.75 \text{ V}.$$

Chapter 3, Solution 29

At node 1,

$$5 + V_1 - V_4 + 2V_1 + V_1 - V_2 = 0 \longrightarrow -5 = 4V_1 - V_2 - V_4 \quad (1)$$

At node 2,

$$V_1 - V_2 = 2V_2 + 4(V_2 - V_3) = 0 \longrightarrow 0 = -V_1 + 7V_2 - 4V_3 \quad (2)$$

At node 3,

$$6 + 4(V_2 - V_3) = V_3 - V_4 \longrightarrow 6 = -4V_2 + 5V_3 - V_4 \quad (3)$$

At node 4,

$$2 + V_3 - V_4 + V_1 - V_4 = 3V_4 \longrightarrow 2 = -V_1 - V_3 + 5V_4 \quad (4)$$

In matrix form, (1) to (4) become

$$\begin{pmatrix} 4 & -1 & 0 & -1 \\ -1 & 7 & -4 & 0 \\ 0 & -4 & 5 & -1 \\ -1 & 0 & -1 & 5 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 6 \\ 2 \end{pmatrix} \longrightarrow AV = B$$

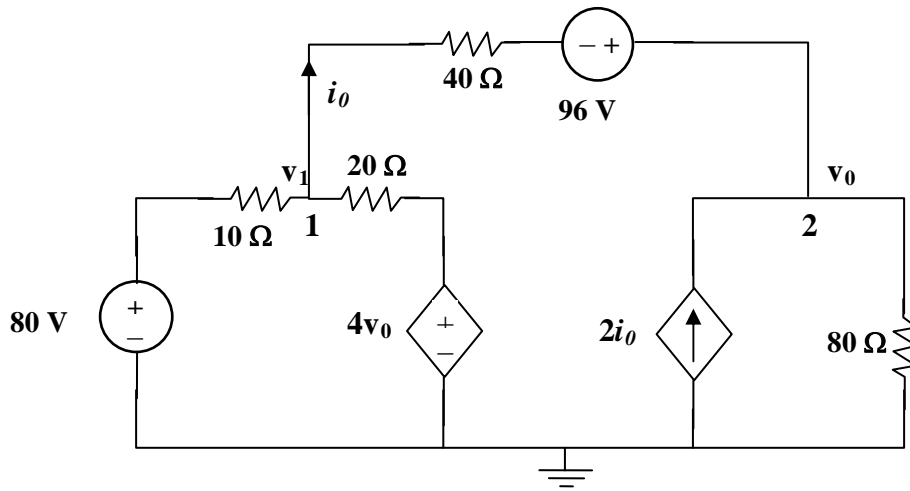
Using MATLAB,

$$V = A^{-1}B = \begin{pmatrix} -0.7708 \\ 1.209 \\ 2.309 \\ 0.7076 \end{pmatrix}$$

i.e.

$$\underline{V_1 = -0.7708 \text{ V}, V_2 = 1.209 \text{ V}, V_3 = 2.309 \text{ V}, V_4 = 0.7076 \text{ V}}$$

Chapter 3, Solution 30



At node 1,

$$\begin{aligned} [(v_1 - 80)/10] + [(v_1 - 4v_o)/20] + [(v_1 - (v_o - 96))/40] &= 0 \text{ or} \\ (0.1 + 0.05 + 0.025)v_1 - (0.2 + 0.025)v_o &= \\ 0.175v_1 - 0.225v_o &= 8 - 2.4 = 5.6 \end{aligned} \quad (1)$$

At node 2,

$$\begin{aligned} -2i_o + [(v_o - 96) - v_1]/40 + [(v_o - 0)/80] &= 0 \text{ and } i_o = [(v_1 - (v_o - 96))/40] \\ -2[(v_1 - (v_o - 96))/40] + [(v_o - 96) - v_1]/40 + [(v_o - 0)/80] &= 0 \\ -3[(v_1 - (v_o - 96))/40] + [(v_o - 0)/80] &= 0 \text{ or} \\ -0.075v_1 + (0.075 + 0.0125)v_o &= 7.2 = \\ -0.075v_1 + 0.0875v_o &= 7.2 \end{aligned} \quad (2)$$

Using (1) and (2) we get,

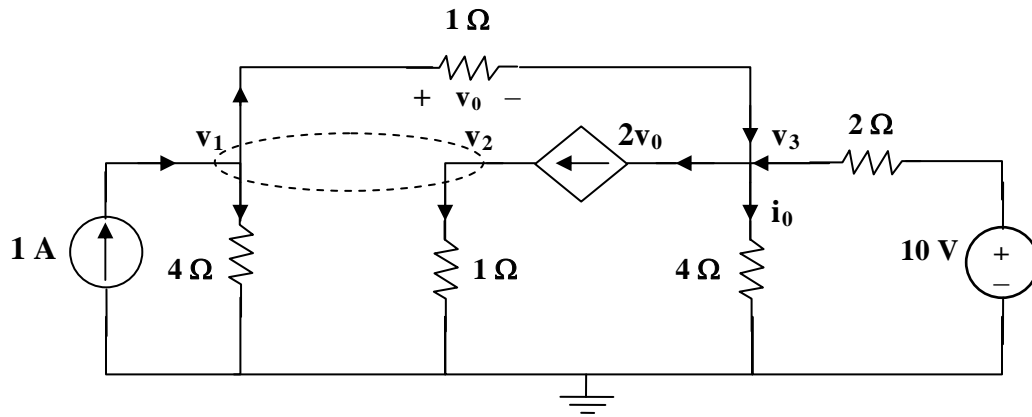
$$\begin{aligned} \begin{bmatrix} 0.175 & -0.225 \\ -0.075 & 0.0875 \end{bmatrix} \begin{bmatrix} v_1 \\ v_o \end{bmatrix} &= \begin{bmatrix} 5.6 \\ 7.2 \end{bmatrix} \text{ or} \\ \begin{bmatrix} v_1 \\ v_o \end{bmatrix} &= \frac{\begin{bmatrix} 0.0875 & 0.225 \\ 0.075 & 0.175 \end{bmatrix}}{0.0153125 - 0.016875} \begin{bmatrix} 5.6 \\ 7.2 \end{bmatrix} = \frac{\begin{bmatrix} 0.0875 & 0.225 \\ 0.075 & 0.175 \end{bmatrix}}{-0.0015625} \begin{bmatrix} 5.6 \\ 7.2 \end{bmatrix} \end{aligned}$$

$$v_1 = -313.6 - 1036.8 = -1350.4$$

$$v_o = -268.8 - 806.4 = -1.0752 \text{ kV}$$

$$\text{and } i_o = [(v_1 - (v_o - 96))/40] = [(-1350.4 - (-1075.2 - 96))/40] = -4.48 \text{ amps.}$$

Chapter 3, Solution 31



At the supernode,

$$1 + 2v_0 = \frac{v_1}{4} + \frac{v_2}{1} + \frac{v_1 - v_3}{1} \quad (1)$$

But $v_0 = v_1 - v_3$. Hence (1) becomes,

$$4 = -3v_1 + 4v_2 + 4v_3 \quad (2)$$

At node 3,

$$2v_0 + \frac{v_3}{4} = v_1 - v_3 + \frac{10 - v_3}{2}$$

or

$$20 = 4v_1 + 0v_2 - v_3 \quad (3)$$

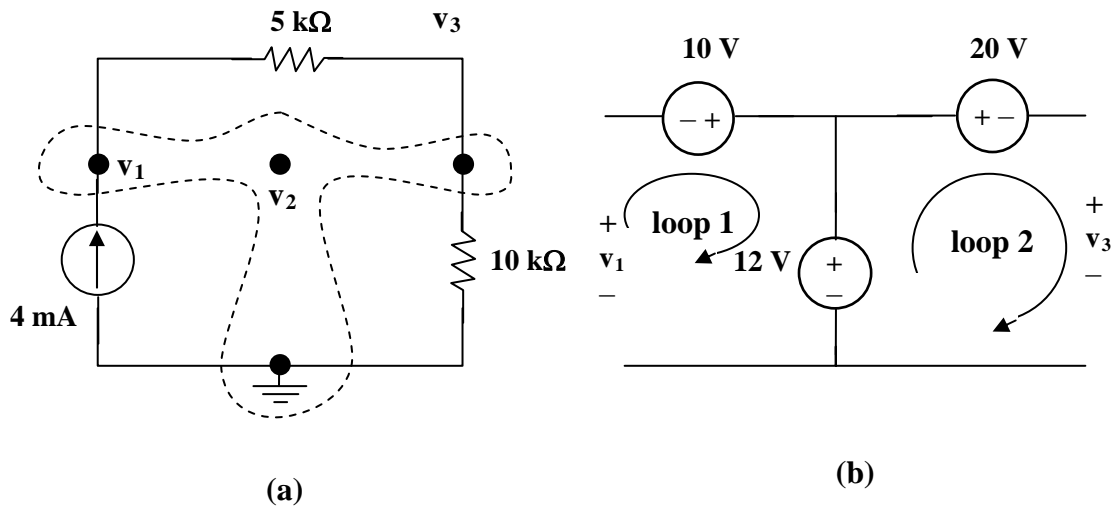
At the supernode, $v_2 = v_1 + 4i_0$. But $i_0 = \frac{v_3}{4}$. Hence,

$$v_2 = v_1 + v_3 \quad (4)$$

Solving (2) to (4) leads to,

$$v_1 = 4.97\text{V}, \quad v_2 = 4.85\text{V}, \quad v_3 = -0.12\text{V}.$$

Chapter 3, Solution 32



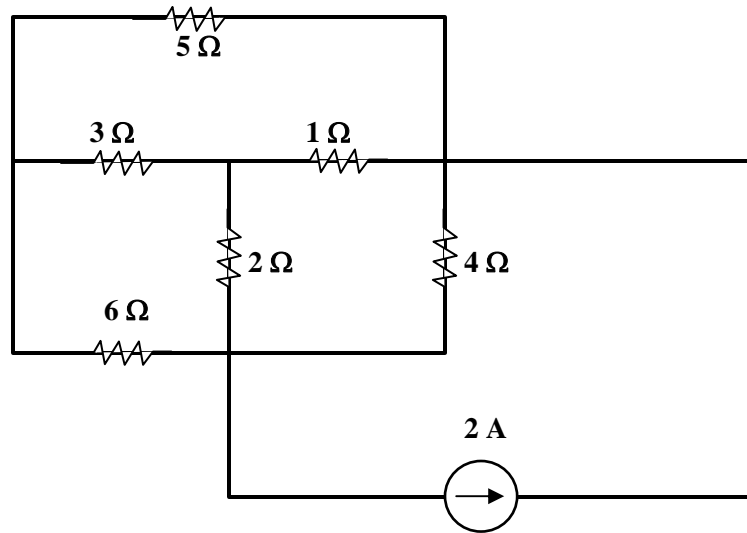
We have a supernode as shown in figure (a). It is evident that $v_2 = 12 \text{ V}$. Applying KVL to loops 1 and 2 in figure (b), we obtain,

$$-v_1 - 10 + 12 = 0 \text{ or } v_1 = 2 \text{ and } -12 + 20 + v_3 = 0 \text{ or } v_3 = -8 \text{ V}$$

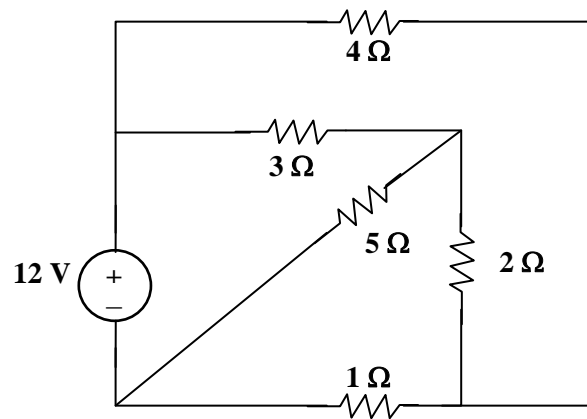
Thus, $v_1 = 2 \text{ V}$, $v_2 = 12 \text{ V}$, $v_3 = -8 \text{ V}$.

Chapter 3, Solution 33

(a) This is a **planar** circuit. It can be redrawn as shown below.

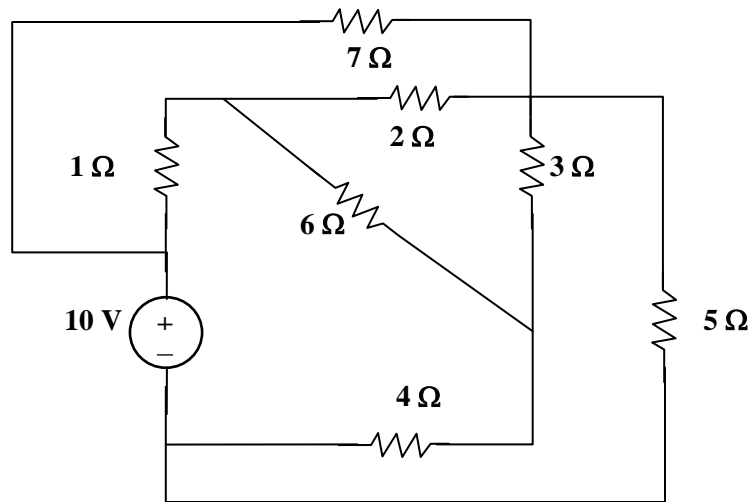


(b) This is a **planar** circuit. It can be redrawn as shown below.



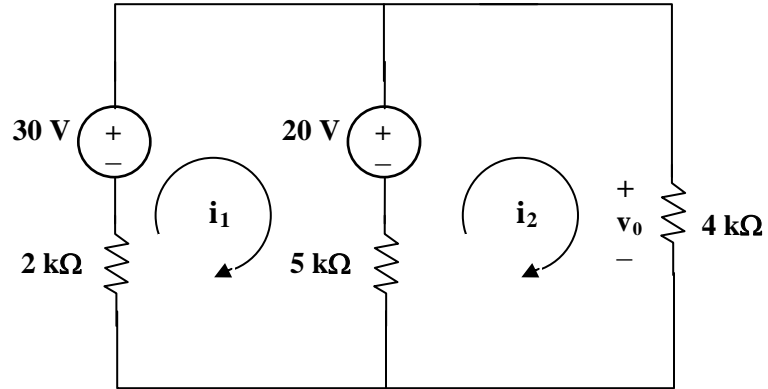
Chapter 3, Solution 34

- (a) This is a **planar** circuit because it can be redrawn as shown below,



- (b) This is a **non-planar** circuit.

Chapter 3, Solution 35



Assume that i_1 and i_2 are in mA. We apply mesh analysis. For mesh 1,

$$-30 + 20 + 7i_1 - 5i_2 = 0 \text{ or } 7i_1 - 5i_2 = 10 \quad (1)$$

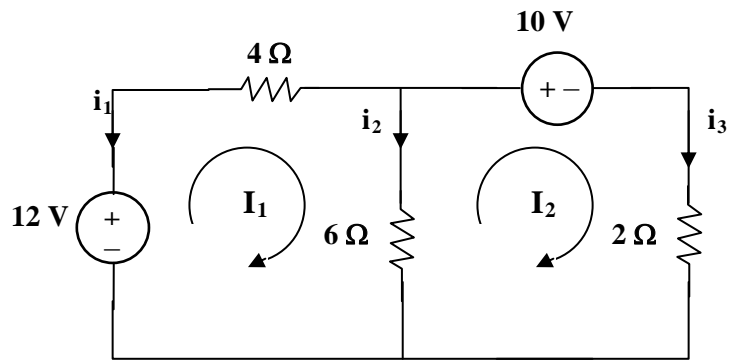
For mesh 2,

$$-20 + 9i_2 - 5i_1 = 0 \text{ or } -5i_1 + 9i_2 = 20 \quad (2)$$

Solving (1) and (2), we obtain, $i_2 = 5$.

$$v_0 = 4i_2 = \mathbf{20 \text{ volts.}}$$

Chapter 3, Solution 36



Applying mesh analysis gives,

$$10I_1 - 6I_2 = 12 \quad \text{and} \quad -6I_1 + 8I_2 = -10$$

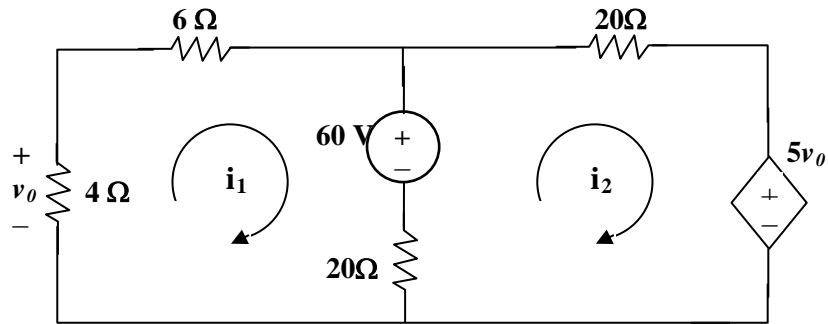
or

$$\begin{bmatrix} 6 \\ -5 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}}{11} \begin{bmatrix} 6 \\ -5 \end{bmatrix}$$

$$I_1 = (24-15)/11 = 0.8182 \quad \text{and} \quad I_2 = (18-25)/11 = -0.6364$$

$$i_1 = -I_1 = \mathbf{-818.2 \text{ mA}}; \quad i_2 = I_1 - I_2 = 0.8182 + 0.6364 = \mathbf{1.4546 \text{ A}}; \quad \text{and} \\ i_3 = I_2 = \mathbf{-636.4 \text{ mA}}.$$

Chapter 3, Solution 37



Applying mesh analysis to loops 1 and 2, we get,

$$30i_1 - 20i_2 + 60 = 0 \text{ which leads to } i_2 = 1.5i_1 + 3 \quad (1)$$

$$-20i_1 + 40i_2 - 60 + 5v_0 = 0 \quad (2)$$

$$\text{But, } v_0 = -4i_1 \quad (3)$$

Using (1), (2), and (3) we get $-20i_1 + 60i_1 + 120 - 60 - 20i_1 = 0$ or

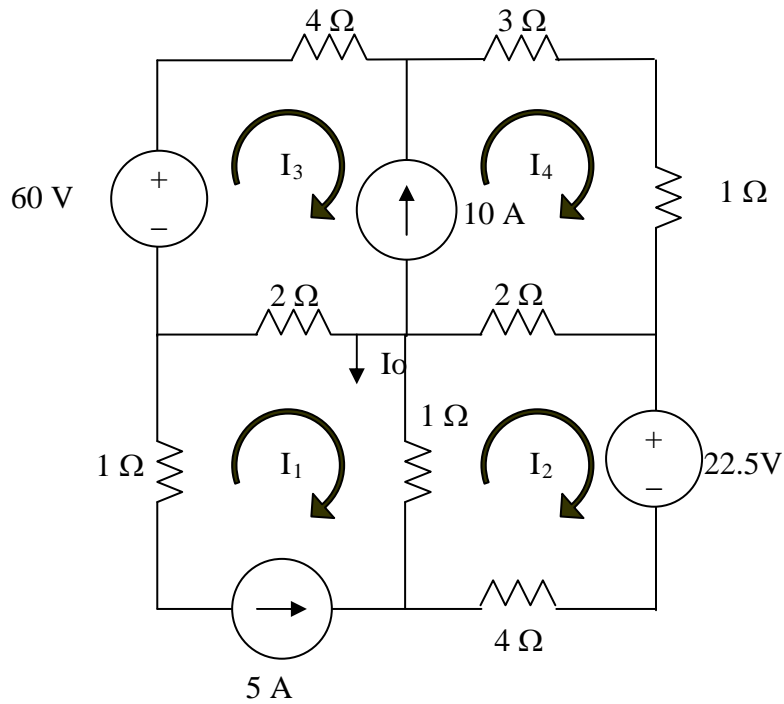
$$20i_1 = -60 \text{ or } i_1 = -3 \text{ amps and } i_2 = 7.5 \text{ amps.}$$

Therefore, we get,

$$v_0 = -4i_1 = \mathbf{12 \text{ volts.}}$$

Chapter 3, Solution 38

Consider the circuit below with the mesh currents.



$$I_1 = -5 \text{ A} \quad (1)$$

$$\begin{aligned} 1(I_2 - I_1) + 2(I_2 - I_4) + 22.5 + 4I_2 &= 0 \\ 7I_2 - I_4 &= -27.5 \end{aligned} \quad (2)$$

$$\begin{aligned} -60 + 4I_3 + 3I_4 + 1I_4 + 2(I_4 - I_2) + 2(I_3 - I_1) &= 0 \text{ (super mesh)} \\ -2I_2 + 6I_3 + 6I_4 &= +60 - 10 = 50 \end{aligned} \quad (3)$$

But, we need one more equation, so we use the constraint equation $-I_3 + I_4 = 10$. This now gives us three equations with three unknowns.

$$\begin{bmatrix} 7 & 0 & -1 \\ -2 & 6 & 6 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -27.5 \\ 50 \\ 10 \end{bmatrix}$$

We can now use MATLAB to solve the problem.

$$>> Z=[7,0,-1;-2,6,6;0,-1,0]$$

Z =

```
    7    0   -1  
   -2    6    6  
    0   -1    0  
>> V=[-27.5,50,10]'
```

V =

```
  -27.5  
    50  
    10  
>> I=inv(Z)*V
```

I =

```
  -1.3750  
 -10.0000  
  17.8750
```

$$I_o = I_1 - I_2 = -5 - 1.375 = \mathbf{-6.375 \text{ A.}}$$

Check using the super mesh (equation (3)):

$$-2I_2 + 6 I_3 + 6I_4 = 2.75 - 60 + 107.25 = 50!$$

Chapter 3, Solution 39

Using Fig. 3.50 from Prob. 3.1, design a problem to help other students to better understand mesh analysis.

Solution

Given $R_1 = 4\text{ k}\Omega$, $R_2 = 2\text{ k}\Omega$, and $R_3 = 2\text{ k}\Omega$, determine the value of I_x using mesh analysis.

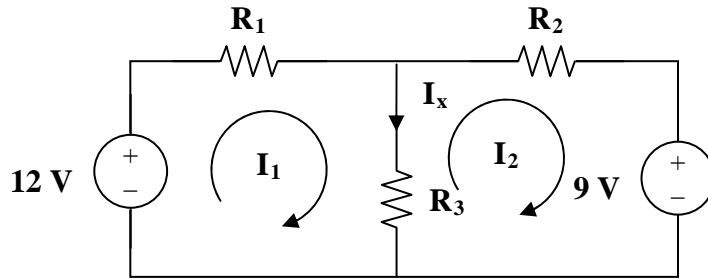


Figure 3.50
For Prob. 3.1 and 3.39.

For loop 1 we get $-12 + 4kI_1 + 2k(I_1 - I_2) = 0$ or $6I_1 - 2I_2 = 0.012$ and at

loop 2 we get $2k(I_2 - I_1) + 2kI_2 + 9 = 0$ or $-2I_1 + 4I_2 = -0.009$.

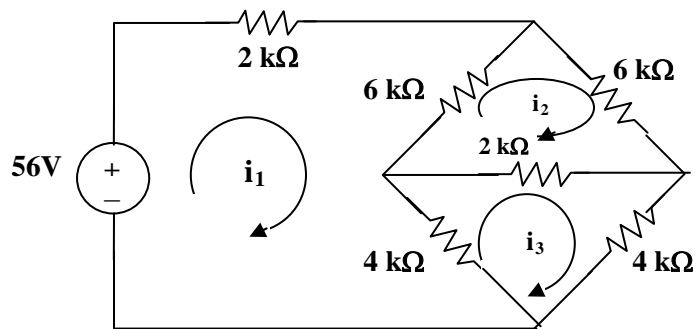
Now $6I_1 - 2I_2 = 0.012 + 3[-2I_1 + 4I_2 = -0.009]$ leads to,

$10I_2 = 0.012 - 0.027 = -0.015$ or $I_2 = -1.5\text{ mA}$ and $I_1 = (-0.003 + 0.012)/6 = 1.5\text{ mA}$.

Thus,

$$I_x = I_1 - I_2 = (1.5 + 1.5)\text{ mA} = \mathbf{3\text{ mA}}.$$

Chapter 3, Solution 40



Assume all currents are in mA and apply mesh analysis for mesh 1.

$$-56 + 12i_1 - 6i_2 - 4i_3 = 0 \text{ or } 6i_1 - 3i_2 - 2i_3 = 28 \quad (1)$$

for mesh 2,

$$-6i_1 + 14i_2 - 2i_3 = 0 \text{ or } -3i_1 + 7i_2 - i_3 = 0 \quad (2)$$

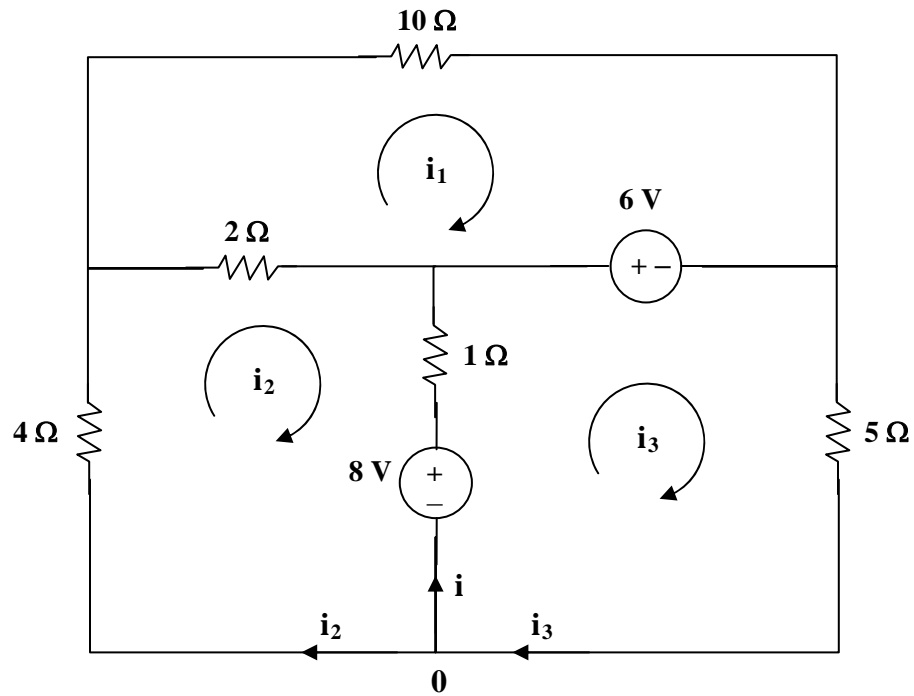
for mesh 3,

$$-4i_1 - 2i_2 + 10i_3 = 0 \text{ or } -2i_1 - i_2 + 5i_3 = 0 \quad (3)$$

Solving (1), (2), and (3) using MATLAB, we obtain,

$$i_o = i_1 = \mathbf{8 \text{ mA}}.$$

Chapter 3, Solution 41



For loop 1,

$$6 = 12i_1 - 2i_2 \quad \longrightarrow \quad 3 = 6i_1 - i_2 \quad (1)$$

For loop 2,

$$-8 = -2i_1 + 7i_2 - i_3 \quad (2)$$

For loop 3,

$$-8 + 6 + 6i_3 - i_2 = 0 \quad \longrightarrow \quad 2 = -i_2 + 6i_3 \quad (3)$$

We put (1), (2), and (3) in matrix form,

$$\begin{bmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{vmatrix} = -234, \quad \Delta_2 = \begin{vmatrix} 6 & 3 & 0 \\ 2 & 8 & 1 \\ 0 & 2 & 6 \end{vmatrix} = 240$$

$$\Delta_3 = \begin{vmatrix} 6 & -1 & 3 \\ 2 & -7 & 8 \\ 0 & -1 & 2 \end{vmatrix} = -38$$

At node 0, $i + i_2 = i_3$ or $i = i_3 - i_2 = \frac{\Delta_3 - \Delta_2}{\Delta} = \frac{-38 - 240}{-234} = \mathbf{1.188 \text{ A}}$

Chapter 3, Solution 42

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the mesh currents in the circuit of Fig. 3.88.

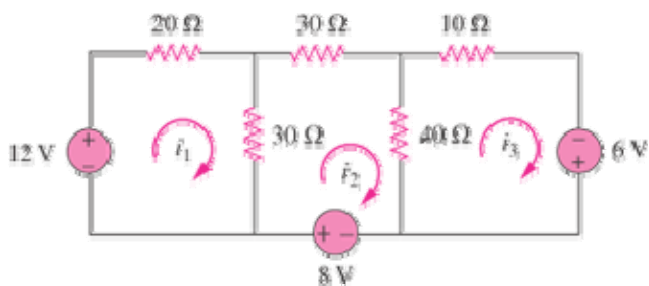


Figure 3.88

Solution

For mesh 1,

$$-12 + 50I_1 - 30I_2 = 0 \quad \longrightarrow \quad 12 = 50I_1 - 30I_2 \quad (1)$$

For mesh 2,

$$-8 + 100I_2 - 30I_1 - 40I_3 = 0 \quad \longrightarrow \quad 8 = -30I_1 + 100I_2 - 40I_3 \quad (2)$$

For mesh 3,

$$-6 + 50I_3 - 40I_2 = 0 \quad \longrightarrow \quad 6 = -40I_2 + 50I_3 \quad (3)$$

Putting eqs. (1) to (3) in matrix form, we get

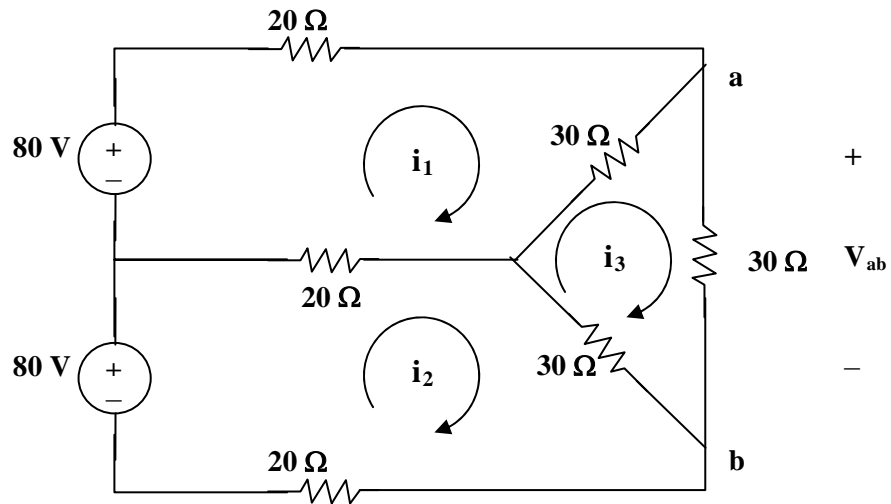
$$\begin{pmatrix} 50 & -30 & 0 \\ -30 & 100 & -40 \\ 0 & -40 & 50 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \\ 6 \end{pmatrix} \quad \longrightarrow \quad \mathbf{AI} = \mathbf{B}$$

Using Matlab,

$$\mathbf{I} = \mathbf{A}^{-1}\mathbf{B} = \begin{pmatrix} 0.48 \\ 0.40 \\ 0.44 \end{pmatrix}$$

$$\text{i.e. } I_1 = 480 \text{ mA}, I_2 = 400 \text{ mA}, I_3 = 440 \text{ mA}$$

Chapter 3, Solution 43



For loop 1,

$$80 = 70i_1 - 20i_2 - 30i_3 \quad \longrightarrow \quad 8 = 7i_1 - 2i_2 - 3i_3 \quad (1)$$

For loop 2,

$$80 = 70i_2 - 20i_1 - 30i_3 \quad \longrightarrow \quad 8 = -2i_1 + 7i_2 - 3i_3 \quad (2)$$

For loop 3,

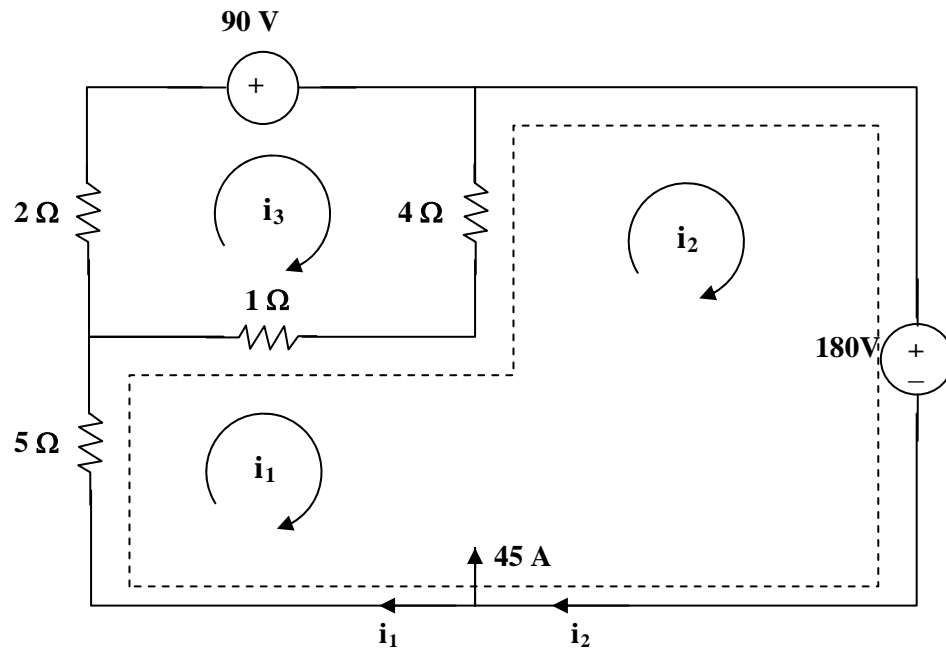
$$0 = -30i_1 - 30i_2 + 90i_3 \quad \longrightarrow \quad 0 = i_1 + i_2 - 3i_3 \quad (3)$$

Solving (1) to (3), we obtain $i_3 = 16/9$

$$I_o = i_3 = 16/9 = \mathbf{1.7778 \text{ A}}$$

$$V_{ab} = 30i_3 = \mathbf{53.33 \text{ V.}}$$

Chapter 3, Solution 44



Loop 1 and 2 form a supermesh. For the supermesh,

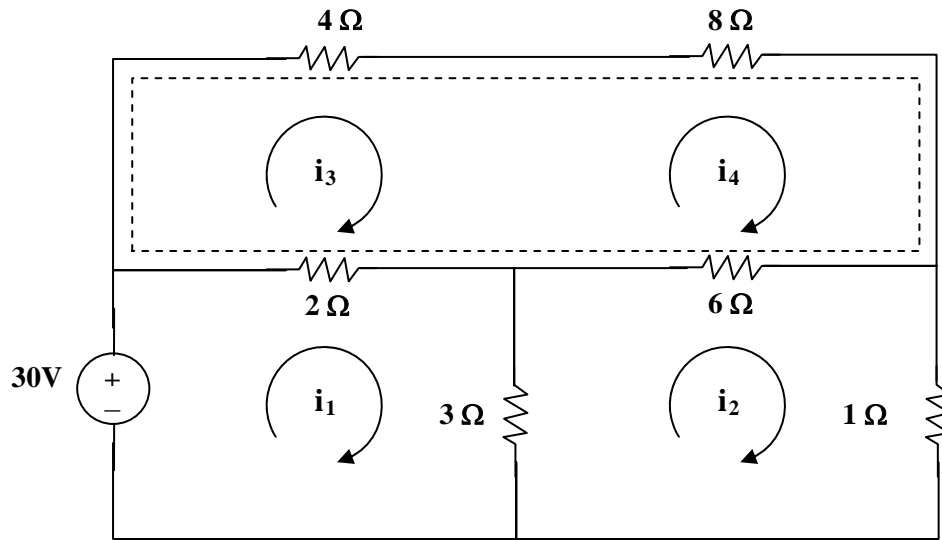
$$6i_1 + 4i_2 - 5i_3 + 180 = 0 \quad (1)$$

For loop 3,
$$-i_1 - 4i_2 + 7i_3 + 90 = 0 \quad (2)$$

Also,
$$i_2 = 45 + i_1 \quad (3)$$

Solving (1) to (3), $i_1 = -46$, $i_3 = -20$; $i_o = i_1 - i_3 = -26 \text{ A}$

Chapter 3, Solution 45



For loop 1, $30 = 5i_1 - 3i_2 - 2i_3$ (1)

For loop 2, $10i_2 - 3i_1 - 6i_4 = 0$ (2)

For the supermesh, $6i_3 + 14i_4 - 2i_1 - 6i_2 = 0$ (3)

But $i_4 - i_3 = 4$ which leads to $i_4 = i_3 + 4$ (4)

Solving (1) to (4) by elimination gives $i = i_1 = \mathbf{8.561\text{ A}}$.

Chapter 3, Solution 46

For loop 1,

$$-12 + 3i_1 + 8(i_1 - i_2) = -12 + 11i_1 - 8i_2 = 0 \quad \longrightarrow \quad 11i_1 - 8i_2 = 12 \quad (1)$$

For loop 2,

$$8(i_2 - i_1) + 6i_2 + 2v_o = -8i_1 + 14i_2 + 2v_o = 0$$

But $v_o = 3i_1$,

$$-8i_1 + 14i_2 + 6i_1 = 0 \quad \longrightarrow \quad i_1 = 7i_2 \quad (2)$$

Substituting (2) into (1),

$$77i_2 - 8i_2 = 12 \quad \longrightarrow \quad \underline{i_2 = 0.1739 \text{ A}} \text{ and } \underline{i_1 = 7i_2 = 1.217 \text{ A}}$$

First, transform the current sources as shown below.

$$I_1 = \frac{20 - V}{4} \longrightarrow V_1 = 20 - 4I_1 = \mathbf{10 \text{ V}}$$

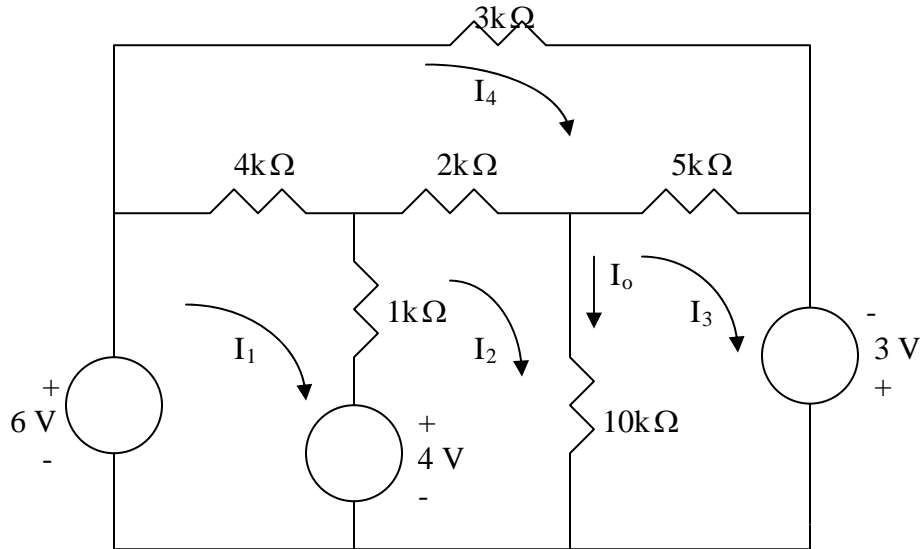
$$V_2 = 2(I_1 - I_2) = \mathbf{4.933 \text{ V}}$$

Also,

$$I_2 = \frac{V_3 - 12}{8} \longrightarrow V_3 = 12 + 8I_2 = \mathbf{12.267 \text{ V}}.$$

Chapter 3, Solution 48

We apply mesh analysis and let the mesh currents be in mA.



For mesh 1,

$$-6 + 8 + 5I_1 - I_2 - 4I_4 = 0 \quad \longrightarrow \quad 2 = 5I_1 - I_2 - 4I_4 \quad (1)$$

For mesh 2,

$$-4 + 13I_2 - I_1 - 10I_3 - 2I_4 = 0 \quad \longrightarrow \quad 4 = -I_1 + 13I_2 - 10I_3 - 2I_4 \quad (2)$$

For mesh 3,

$$-3 + 15I_3 - 10I_2 - 5I_4 = 0 \quad \longrightarrow \quad 3 = -10I_2 + 15I_3 - 5I_4 \quad (3)$$

For mesh 4,

$$-4I_1 - 2I_2 - 5I_3 + 14I_4 = 0 \quad (4)$$

Putting (1) to (4) in matrix form gives

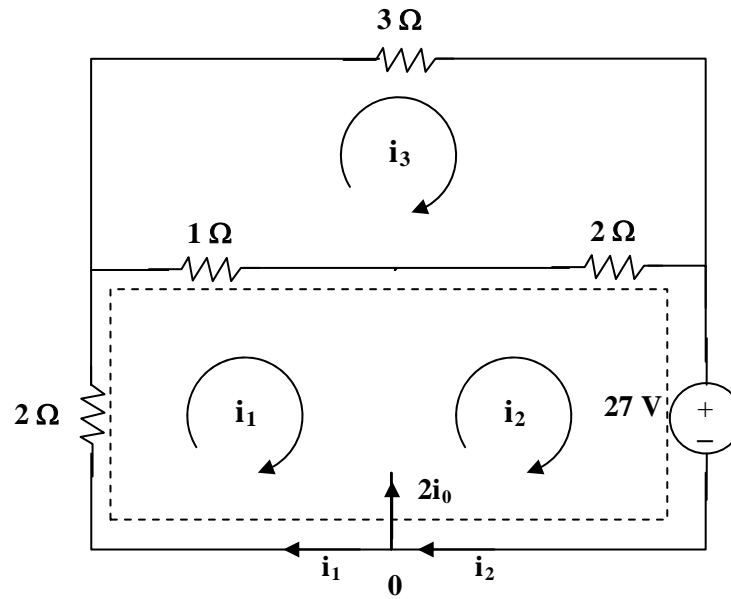
$$\begin{pmatrix} 5 & -1 & 0 & -4 \\ -1 & 13 & -10 & -2 \\ 0 & -10 & 15 & -5 \\ -4 & -2 & -5 & 14 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 0 \end{pmatrix} \quad \longrightarrow \quad \mathbf{AI} = \mathbf{B}$$

Using MATLAB,

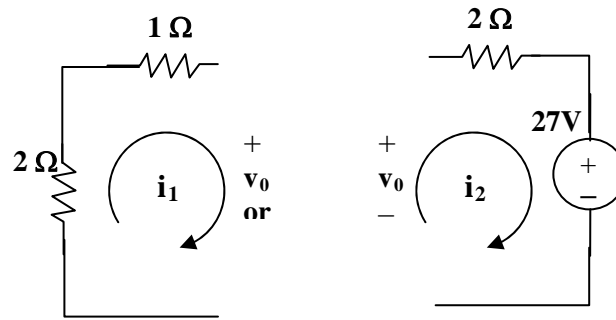
$$\mathbf{I} = \mathbf{A}^{-1}\mathbf{B} = \begin{pmatrix} 3.608 \\ 4.044 \\ 3.896 \\ 3 \end{pmatrix} \times 0.148$$

The current through the $10\text{k}\Omega$ resistor is $I_o = I_2 - I_3 = \mathbf{148\text{ mA}}$.

Chapter 3, Solution 49



(a)



(b)

For the supermesh in figure (a),

$$3i_1 + 2i_2 - 3i_3 + 27 = 0 \quad (1)$$

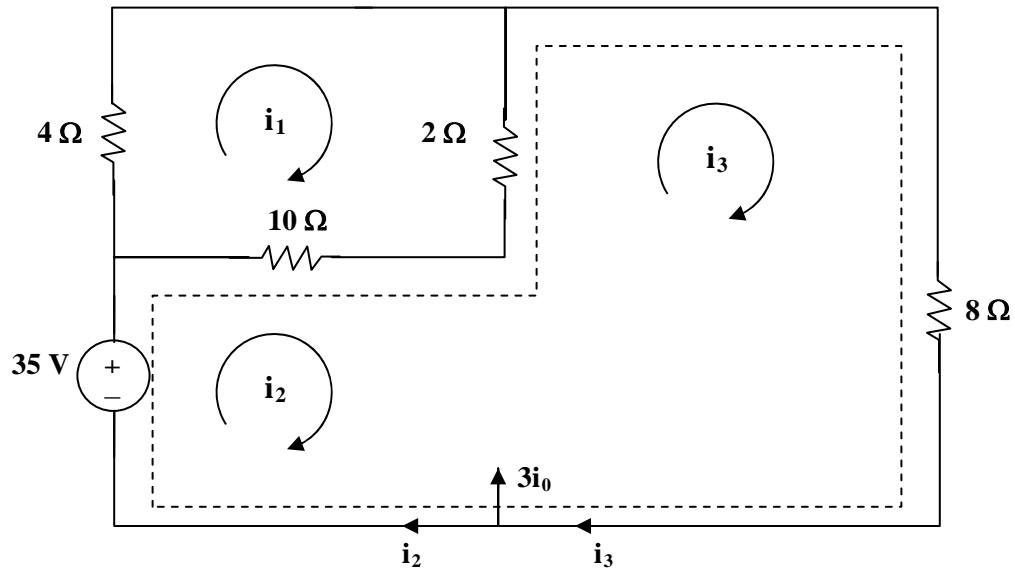
$$\text{At node 0, } i_2 - i_1 = 2i_0 \text{ and } i_0 = -i_1 \text{ which leads to } i_2 = -i_1 \quad (2)$$

$$\text{For loop 3, } -i_1 - 2i_2 + 6i_3 = 0 \text{ which leads to } 6i_3 = -i_1 \quad (3)$$

Solving (1) to (3), $i_1 = (-54/3)\text{A}$, $i_2 = (54/3)\text{A}$, $i_3 = (27/9)\text{A}$

$$i_0 = -i_1 = \mathbf{18\text{ A}}, \text{ from fig. (b), } v_0 = i_3 - 3i_1 = (27/9) + 54 = \mathbf{57\text{ V}}.$$

Chapter 3, Solution 50



For loop 1, $16i_1 - 10i_2 - 2i_3 = 0$ which leads to $8i_1 - 5i_2 - i_3 = 0$ (1)

For the supermesh, $-35 + 10i_2 - 10i_1 + 10i_3 - 2i_1 = 0$

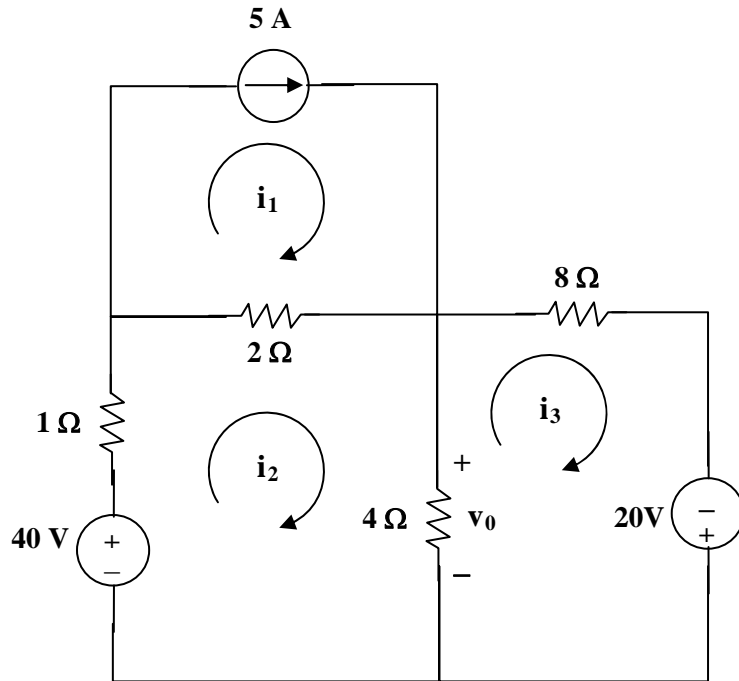
or $-6i_1 + 5i_2 + 5i_3 = 17.5$ (2)

Also, $3i_0 = i_3 - i_2$ and $i_0 = i_1$ which leads to $3i_1 = i_3 - i_2$ (3)

Solving (1), (2), and (3), we obtain $i_1 = 1.0098$ and

$$i_0 = i_1 = \mathbf{1.0098\text{ A}}$$

Chapter 3, Solution 51



For loop 1, $i_1 = 5\text{ A}$ (1)

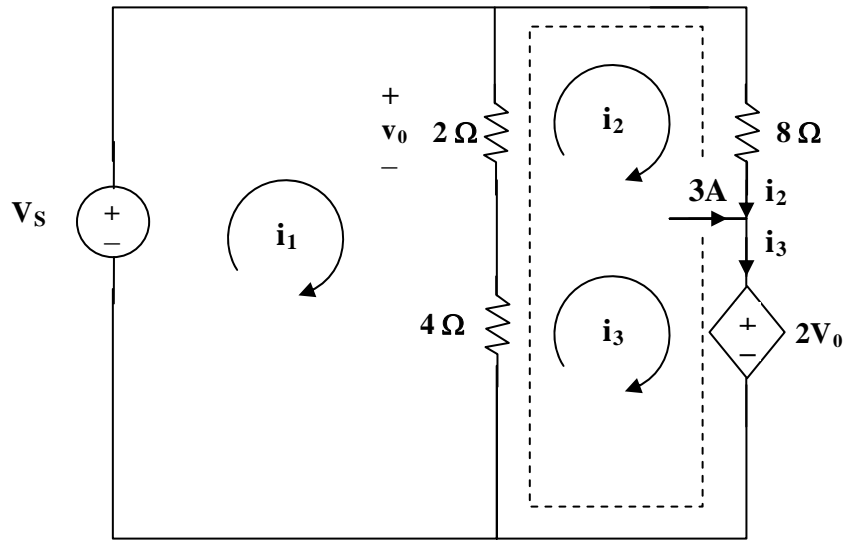
For loop 2, $-40 + 7i_2 - 2i_1 - 4i_3 = 0$ which leads to $50 = 7i_2 - 4i_3$ (2)

For loop 3, $-20 + 12i_3 - 4i_2 = 0$ which leads to $5 = -i_2 + 3i_3$ (3)

Solving with (2) and (3), $i_2 = 10\text{ A}$, $i_3 = 5\text{ A}$

And, $v_0 = 4(i_2 - i_3) = 4(10 - 5) = \mathbf{20\text{ V}}$.

Chapter 3, Solution 52



For mesh 1,

$$2(i_1 - i_2) + 4(i_1 - i_3) - 12 = 0 \text{ which leads to } 3i_1 - i_2 - 2i_3 = 6 \quad (1)$$

For the supermesh, $2(i_2 - i_1) + 8i_2 + 2v_0 + 4(i_3 - i_1) = 0$

But $v_0 = 2(i_1 - i_2)$ which leads to $-i_1 + 3i_2 + 2i_3 = 0$
(2)

For the independent current source, $i_3 = 3 + i_2$ (3)

Solving (1), (2), and (3), we obtain,

$$i_1 = \mathbf{3.5 \text{ A}}, \quad i_2 = \mathbf{-0.5 \text{ A}}, \quad i_3 = \mathbf{2.5 \text{ A}}.$$

Chapter 3, Solution 53

Applying mesh analysis leads to;

$$-12 + 4kI_1 - 3kI_2 - 1kI_3 = 0 \quad (1)$$

$$\begin{aligned} -3kI_1 + 7kI_2 - 4kI_4 &= 0 \\ -3kI_1 + 7kI_2 &= -12 \end{aligned} \quad (2)$$

$$\begin{aligned} -1kI_1 + 15kI_3 - 8kI_4 - 6kI_5 &= 0 \\ -1kI_1 + 15kI_3 - 6k &= -24 \end{aligned} \quad (3)$$

$$I_4 = -3\text{mA} \quad (4)$$

$$\begin{aligned} -6kI_3 - 8kI_4 + 16kI_5 &= 0 \\ -6kI_3 + 16kI_5 &= -24 \end{aligned} \quad (5)$$

Putting these in matrix form (having substituted $I_4 = 3\text{mA}$ in the above),

$$\begin{bmatrix} 4 & -3 & -1 & 0 \\ -3 & 7 & 0 & 0 \\ -1 & 0 & 15 & -6 \\ 0 & 0 & -6 & 16 \end{bmatrix} k \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_5 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ -24 \\ -24 \end{bmatrix}$$

$$ZI = V$$

Using MATLAB,

$$>> Z = [4,-3,-1,0;-3,7,0,0;-1,0,15,-6;0,0,-6,16]$$

$$Z =$$

$$\begin{bmatrix} 4 & -3 & -1 & 0 \\ -3 & 7 & 0 & 0 \\ -1 & 0 & 15 & -6 \\ 0 & 0 & -6 & 16 \end{bmatrix}$$

$$>> V = [12,-12,-24,-24]'$$

$$V =$$

$$\begin{bmatrix} 12 \\ -12 \\ -24 \\ -24 \end{bmatrix}$$

We obtain,

$$>> I = \text{inv}(Z)*V$$

I =

1.6196 mA

-1.0202 mA

-2.461 mA

3 mA

-2.423 mA

Chapter 3, Solution 54

Let the mesh currents be in mA. For mesh 1,

$$-12 + 10 + 2I_1 - I_2 = 0 \quad \longrightarrow \quad 2 = 2I_1 - I_2 \quad (1)$$

For mesh 2,

$$-10 + 3I_2 - I_1 - I_3 = 0 \quad \longrightarrow \quad 10 = -I_1 + 3I_2 - I_3 \quad (2)$$

For mesh 3,

$$-12 + 2I_3 - I_2 = 0 \quad \longrightarrow \quad 12 = -I_2 + 2I_3 \quad (3)$$

Putting (1) to (3) in matrix form leads to

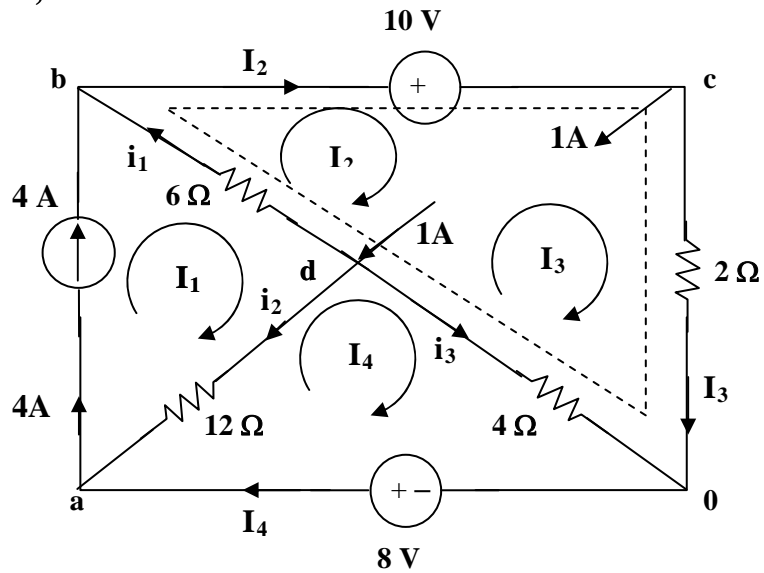
$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \\ 12 \end{pmatrix} \quad \longrightarrow \quad AI = B$$

Using MATLAB,

$$I = A^{-1}B = \begin{bmatrix} 5.25 \\ 8.5 \\ 10.25 \end{bmatrix} \quad \longrightarrow \quad \underline{I_1 = 5.25 \text{ mA}, I_2 = 8.5 \text{ mA}, I_3 = 10.25 \text{ mA}}$$

$$I_1 = \mathbf{5.25 \text{ mA}}, I_2 = \mathbf{8.5 \text{ mA}}, \text{ and } I_3 = \mathbf{10.25 \text{ mA}}.$$

Chapter 3, Solution 55



It is evident that $I_1 = 4$ (1)

For mesh 4, $12(I_4 - I_1) + 4(I_4 - I_3) - 8 = 0$ (2)

For the supermesh $6(I_2 - I_1) + 10 + 2I_3 + 4(I_3 - I_4) = 0$
or $-3I_1 + 3I_2 + 3I_3 - 2I_4 = -5$ (3)

At node c, $I_2 = I_3 + 1$ (4)

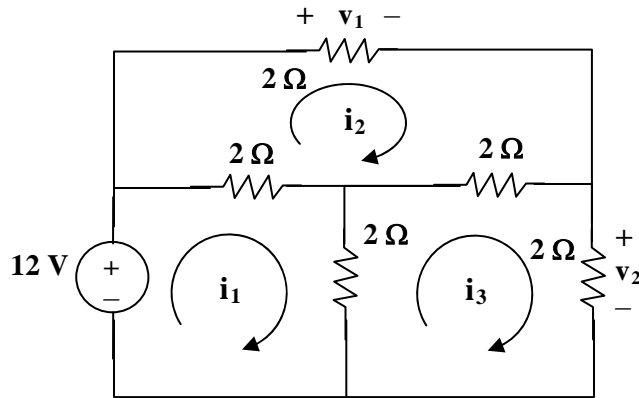
Solving (1), (2), (3), and (4) yields, $I_1 = 4\text{A}$, $I_2 = 3\text{A}$, $I_3 = 2\text{A}$, and $I_4 = 4\text{A}$

At node b, $i_1 = I_2 - I_1 = -1\text{A}$

At node a, $i_2 = 4 - I_4 = 0\text{A}$

At node 0, $i_3 = I_4 - I_3 = 2\text{A}$

Chapter 3, Solution 56



For loop 1, $12 = 4i_1 - 2i_2 - 2i_3$ which leads to $6 = 2i_1 - i_2 - i_3$ (1)

For loop 2, $0 = 6i_2 - 2i_1 - 2i_3$ which leads to $0 = -i_1 + 3i_2 - i_3$ (2)

For loop 3, $0 = 6i_3 - 2i_1 - 2i_2$ which leads to $0 = -i_1 - i_2 + 3i_3$ (3)

In matrix form (1), (2), and (3) become,

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} = 8, \quad \Delta_2 = \begin{vmatrix} 2 & 6 & -1 \\ -1 & 3 & -1 \\ -1 & 0 & 3 \end{vmatrix} = 24$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 6 \\ -1 & 3 & 0 \\ -1 & -1 & 0 \end{vmatrix} = 24, \text{ therefore } i_2 = i_3 = 24/8 = 3\text{A},$$

$$v_1 = 2i_2 = \mathbf{6 \text{ volts}}, \quad v_2 = 2i_3 = \mathbf{6 \text{ volts}}$$

Chapter 3, Solution 57

Assume R is in kilo-ohms.

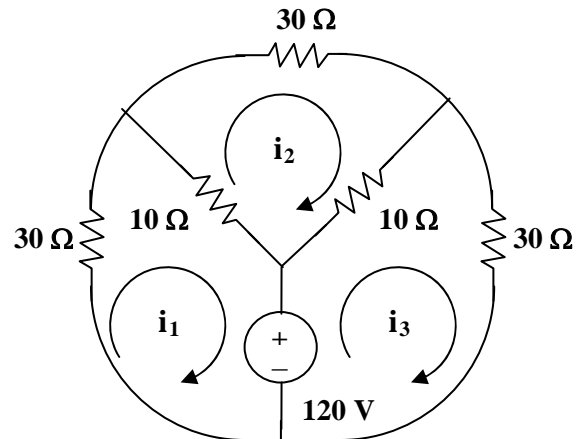
$$V_2 = 4\text{k}\Omega \times 15\text{mA} = \underline{60\text{V}}, \quad V_1 = 90 - V_2 = 90 - 60 = \underline{30\text{V}}$$

Current through R is

$$i_R = \frac{3}{3+R} i_o, \quad V_1 = i_R R \quad \longrightarrow \quad 30 = \frac{3}{3+R} (15)R$$

This leads to $R = 90/15 = \mathbf{6\text{ k}\Omega}$.

Chapter 3, Solution 58



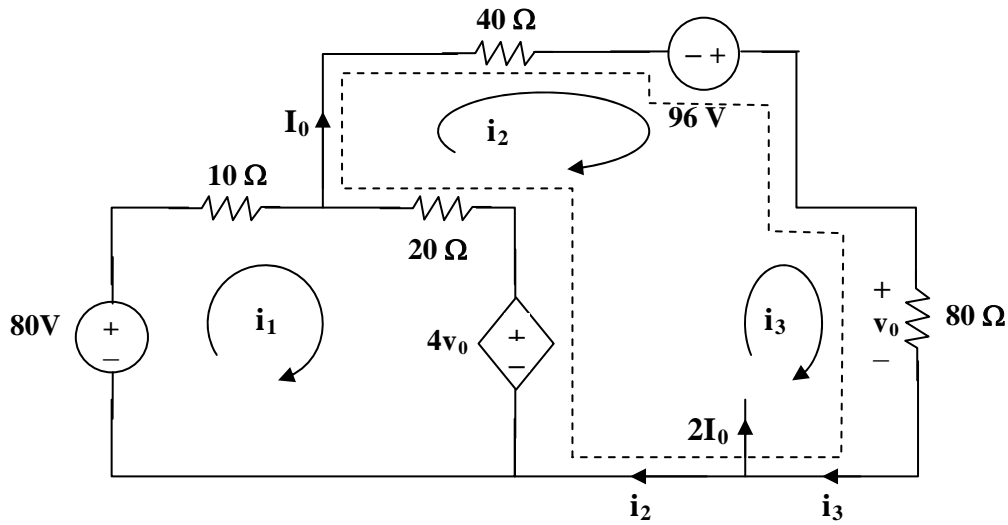
For loop 1, $120 + 40i_1 - 10i_2 = 0$, which leads to $-12 = 4i_1 - i_2$ (1)

For loop 2, $50i_2 - 10i_1 - 10i_3 = 0$, which leads to $-i_1 + 5i_2 - i_3 = 0$ (2)

For loop 3, $-120 - 10i_2 + 40i_3 = 0$, which leads to $12 = -i_2 + 4i_3$ (3)

Solving (1), (2), and (3), we get, $i_1 = \mathbf{-3A}$, $i_2 = \mathbf{0}$, and $i_3 = \mathbf{3A}$

Chapter 3, Solution 59



For loop 1, $-80 + 30i_1 - 20i_2 + 4v_0 = 0$, where $v_0 = 80i_3$
 or $4 = 1.5i_1 - i_2 + 16i_3$ (1)

For the supermesh, $60i_2 - 20i_1 - 96 + 80i_3 - 4v_0 = 0$, where $v_0 = 80i_3$
 or $4.8 = -i_1 + 3i_2 - 12i_3$ (2)

Also, $2I_0 = i_3 - i_2$ and $I_0 = i_2$, hence, $3i_2 = i_3$ (3)

From (1), (2), and (3),

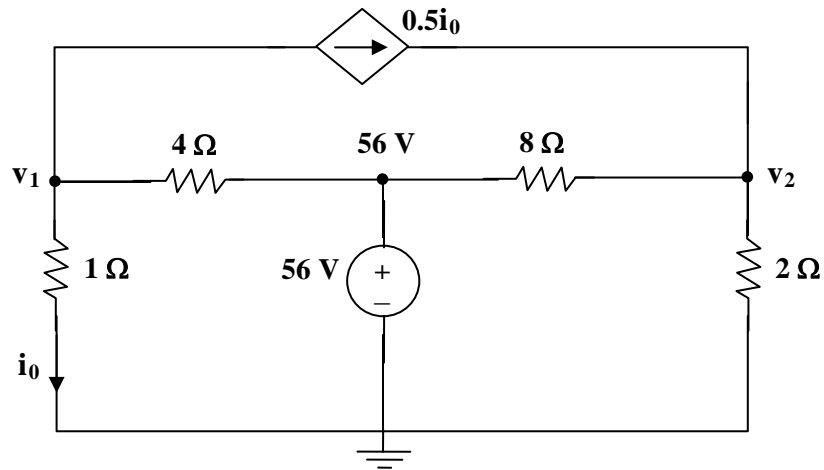
$$\begin{bmatrix} 3 & -2 & 32 \\ -1 & 3 & -12 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 4.8 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 & 32 \\ -1 & 3 & -12 \\ 0 & 3 & -1 \end{vmatrix} = 5, \quad \Delta_2 = \begin{vmatrix} 3 & 8 & 32 \\ -1 & 4.8 & -12 \\ 0 & 0 & -1 \end{vmatrix} = -22.4, \quad \Delta_3 = \begin{vmatrix} 3 & -2 & 8 \\ -1 & 3 & 4.8 \\ 0 & 3 & 0 \end{vmatrix} = -67.2$$

$$I_0 = i_2 = \Delta_2 / \Delta = -22.4 / 5 = -4.48 \text{ A}$$

$$v_0 = 80i_3 = (-84/5)80 = -1.0752 \text{ kvolts}$$

Chapter 3, Solution 60



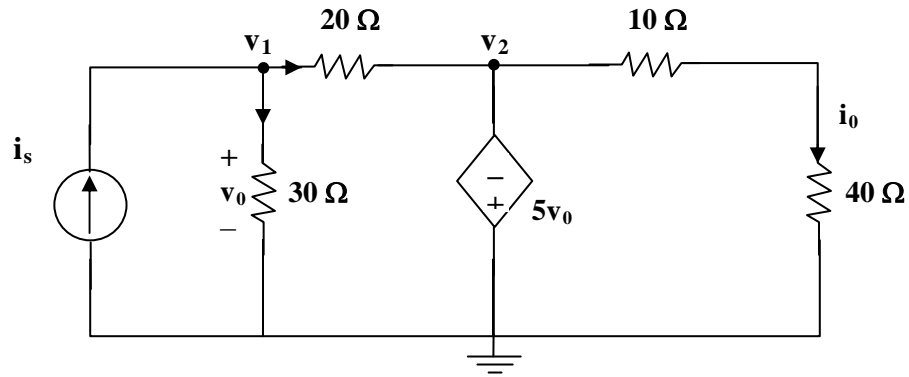
At node 1, $[(v_1 - 0)/1] + [(v_1 - 56)/4] + 0.5[(v_1 - 0)/1] = 0$ or $1.75v_1 = 14$ or $v_1 = 8$ V

At node 2, $[(v_2 - 56)/8] - 0.5[8/1] + [(v_2 - 0)/2] = 0$ or $0.625v_2 = 11$ or $v_2 = 17.6$ V

$$P_{1\Omega} = (v_1)^2/1 = \mathbf{64 \text{ watts}}, P_{2\Omega} = (v_2)^2/2 = \mathbf{154.88 \text{ watts}},$$

$$P_{4\Omega} = (56 - v_1)^2/4 = \mathbf{576 \text{ watts}}, P_{8\Omega} = (56 - v_2)^2/8 = \mathbf{1.84.32 \text{ watts}}.$$

Chapter 3, Solution 61



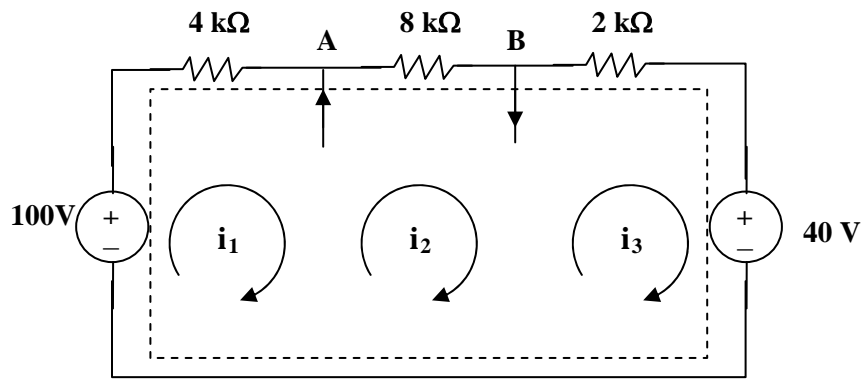
At node 1, $i_s = (v_1/30) + ((v_1 - v_2)/20)$ which leads to $60i_s = 5v_1 - 3v_2$ (1)

But $v_2 = -5v_0$ and $v_0 = v_1$ which leads to $v_2 = -5v_1$

Hence, $60i_s = 5v_1 + 15v_1 = 20v_1$ which leads to $v_1 = 3i_s$, $v_2 = -15i_s$

$i_0 = v_2/50 = -15i_s/50$ which leads to $i_0/i_s = -15/50 = \mathbf{-0.3}$

Chapter 3, Solution 62



We have a supermesh. Let all R be in $k\Omega$, i in mA , and v in volts.

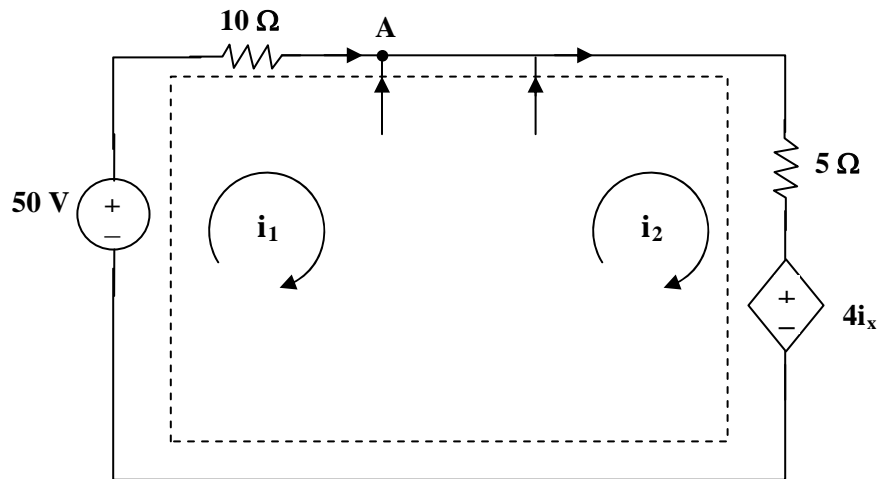
For the supermesh, $-100 + 4i_1 + 8i_2 + 2i_3 + 40 = 0$ or $30 = 2i_1 + 4i_2 + i_3$ (1)

At node A, $i_1 + 4 = i_2$ (2)

At node B, $i_2 = 2i_1 + i_3$ (3)

Solving (1), (2), and (3), we get $i_1 = \mathbf{2\text{ mA}}$, $i_2 = \mathbf{6\text{ mA}}$, and $i_3 = \mathbf{2\text{ mA}}$.

Chapter 3, Solution 63



For the supermesh, $-50 + 10i_1 + 5i_2 + 4i_x = 0$, but $i_x = i_1$. Hence,

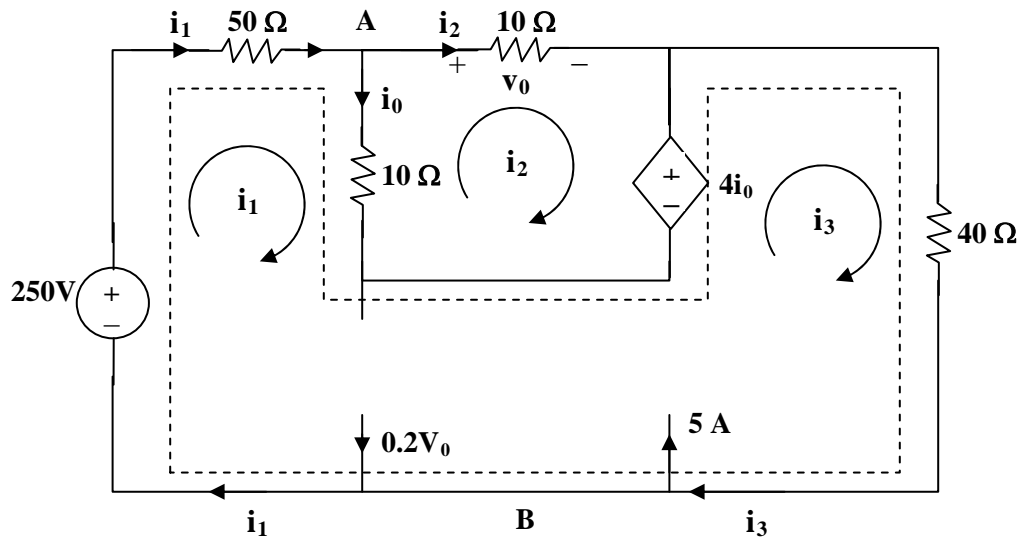
$$50 = 14i_1 + 5i_2 \quad (1)$$

At node A, $i_1 + 3 + (v_x/4) = i_2$, but $v_x = 2(i_1 - i_2)$, hence, $i_1 + 2 = i_2$ (2)

Solving (1) and (2) gives $i_1 = 2.105 \text{ A}$ and $i_2 = 4.105 \text{ A}$

$$v_x = 2(i_1 - i_2) = \mathbf{-4 \text{ volts}} \text{ and } i_x = i_2 - 2 = \mathbf{2.105 \text{ amp}}$$

Chapter 3, Solution 64



For mesh 2, $20i_2 - 10i_1 + 4i_0 = 0$ (1)

But at node A, $i_0 = i_1 - i_2$ so that (1) becomes $i_1 = (16/6)i_2$ (2)

For the supermesh, $-250 + 50i_1 + 10(i_1 - i_2) - 4i_0 + 40i_3 = 0$
or $28i_1 - 3i_2 + 20i_3 = 125$ (3)

At node B, $i_3 + 0.2v_0 = 2 + i_1$ (4)

But, $v_0 = 10i_2$ so that (4) becomes $i_3 = 5 + (2/3)i_2$ (5)

Solving (1) to (5), $i_2 = 0.2941$ A,

$$v_0 = 10i_2 = \mathbf{2.941 \text{ volts}}, i_0 = i_1 - i_2 = (5/3)i_2 = \mathbf{490.2 \text{ mA}}.$$

Chapter 3, Solution 65

For mesh 1,

$$\begin{aligned} -12 + 12I_1 - 6I_2 - I_4 &= 0 \text{ or} \\ 12 &= 12I_1 - 6I_2 - I_4 \end{aligned} \quad (1)$$

For mesh 2,

$$-6I_1 + 16I_2 - 8I_3 - I_4 - I_5 = 0 \quad (2)$$

For mesh 3,

$$\begin{aligned} -8I_2 + 15I_3 - I_5 - 9 &= 0 \text{ or} \\ 9 &= -8I_2 + 15I_3 - I_5 \end{aligned} \quad (3)$$

For mesh 4,

$$\begin{aligned} -I_1 - I_2 + 7I_4 - 2I_5 - 6 &= 0 \text{ or} \\ 6 &= -I_1 - I_2 + 7I_4 - 2I_5 \end{aligned} \quad (4)$$

For mesh 5,

$$\begin{aligned} -I_2 - I_3 - 2I_4 + 8I_5 - 10 &= 0 \text{ or} \\ 10 &= -I_2 - I_3 - 2I_4 + 8I_5 \end{aligned} \quad (5)$$

Casting (1) to (5) in matrix form gives

$$\begin{pmatrix} 12 & -6 & 0 & 1 & 0 \\ -6 & 16 & -8 & -1 & -1 \\ 0 & -8 & 15 & 0 & -1 \\ -1 & -1 & 0 & 7 & -2 \\ 0 & -1 & -1 & -2 & 8 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 9 \\ 6 \\ 10 \end{pmatrix} \longrightarrow \mathbf{AI} = \mathbf{B}$$

Using MATLAB we input:

Z=[12,-6,0,-1,0;-6,16,-8,-1,-1;0,-8,15,0,-1;-1,-1,0,7,-2;0,-1,-1,-2,8]

and V=[12;0;9;6;10]

This leads to

>> Z=[12,-6,0,-1,0;-6,16,-8,-1,-1;0,-8,15,0,-1;-1,-1,0,7,-2;0,-1,-1,-2,8]

Z =

```

12  -6   0  -1   0
-6  16  -8  -1  -1
 0  -8  15   0  -1
-1  -1   0   7  -2
 0  -1  -1  -2   8

```

>> V=[12;0;9;6;10]

V =

```

12

```

0
9
6
10

```
>> I=inv(Z)*V
```

I =

2.1701
1.9912
1.8119
2.0942
2.2489

Thus,

I = [2.17, 1.9912, 1.8119, 2.094, 2.249] A.

Chapter 3, Solution 66

The mesh equations are obtained as follows.

$$-12 + 24 + 30I_1 - 4I_2 - 6I_3 - 2I_4 = 0$$

or

$$\begin{aligned} 30I_1 - 4I_2 - 6I_3 - 2I_4 &= -12 \\ -24 + 40 - 4I_1 + 30I_2 - 2I_4 - 6I_5 &= 0 \end{aligned} \quad (1)$$

or

$$-4I_1 + 30I_2 - 2I_4 - 6I_5 = -16 \quad (2)$$

$$-6I_1 + 18I_3 - 4I_4 = 30 \quad (3)$$

$$-2I_1 - 2I_2 - 4I_3 + 12I_4 - 4I_5 = 0 \quad (4)$$

$$-6I_2 - 4I_4 + 18I_5 = -32 \quad (5)$$

Putting (1) to (5) in matrix form

$$\begin{bmatrix} 30 & -4 & -6 & -2 & 0 \\ -4 & 30 & 0 & -2 & -6 \\ -6 & 0 & 18 & -4 & 0 \\ -2 & -2 & -4 & 12 & -4 \\ 0 & -6 & 0 & -4 & 18 \end{bmatrix} \mathbf{I} = \begin{bmatrix} -12 \\ -16 \\ 30 \\ 0 \\ -32 \end{bmatrix}$$

$$\mathbf{Z}\mathbf{I} = \mathbf{V}$$

Using MATLAB,

```
>> Z = [30,-4,-6,-2,0;  
-4,30,0,-2,-6;  
-6,0,18,-4,0;  
-2,-2,-4,12,-4;  
0,-6,0,-4,18]
```

Z =

```
30  -4  -6  -2   0  
-4  30   0  -2  -6  
-6   0  18  -4   0  
-2  -2  -4  12  -4
```

0 -6 0 -4 18

>> V = [-12,-16,30,0,-32]'

V =

-12

-16

30

0

-32

>> I = inv(Z)*V

I =

-0.2779 A

-1.0488 A

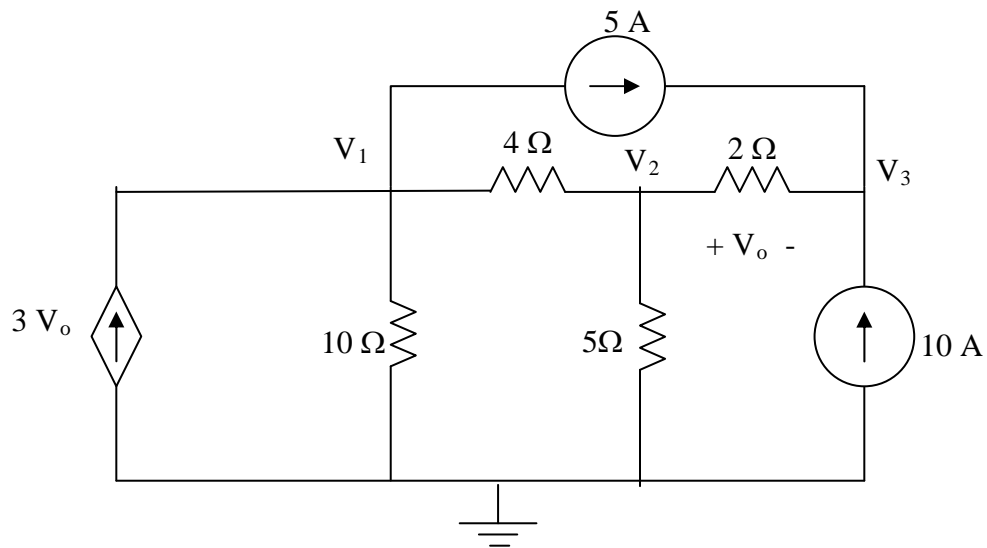
1.4682 A

-0.4761 A

-2.2332 A

Chapter 3, Solution 67

Consider the circuit below.



$$\begin{bmatrix} 0.35 & -0.25 & 0 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} \mathbf{V} = \begin{bmatrix} -5 + 3V_o \\ 0 \\ 15 \end{bmatrix}$$

Since we actually have four unknowns and only three equations, we need a constraint equation.

$$V_o = V_2 - V_3$$

Substituting this back into the matrix equation, the first equation becomes,

$$0.35V_1 - 3.25V_2 + 3V_3 = -5$$

This now results in the following matrix equation,

$$\begin{bmatrix} 0.35 & -3.25 & 3 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} \mathbf{V} = \begin{bmatrix} -5 \\ 0 \\ 15 \end{bmatrix}$$

Now we can use MATLAB to solve for V.

```
>> Y=[0.35,-3.25,3;-0.25,0.95,-0.5;0,-0.5,0.5]
```

```
Y =
```

```
    0.3500   -3.2500    3.0000
   -0.2500    0.9500   -0.5000
    0   -0.5000    0.5000
```

```
>> I=[-5,0,15]'
```

```
I =
```

```
    -5
     0
    15
```

```
>> V=inv(Y)*I
```

```
V =
```

```
  -410.5262
  -194.7368
  -164.7368
```

$$V_o = V_2 - V_3 = -77.89 + 65.89 = \mathbf{-30 \text{ V.}}$$

Let us now do a quick check at node 1.

$$\begin{aligned} & -3(-30) + 0.1(-410.5) + 0.25(-410.5 + 194.74) + 5 = \\ & 90 - 41.05 - 102.62 + 48.68 + 5 = 0.01; \text{ essentially zero considering the} \\ & \text{accuracy we are using. The answer checks.} \end{aligned}$$

Chapter 3, Solution 68

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the voltage V_o in the circuit of Fig. 3.112.

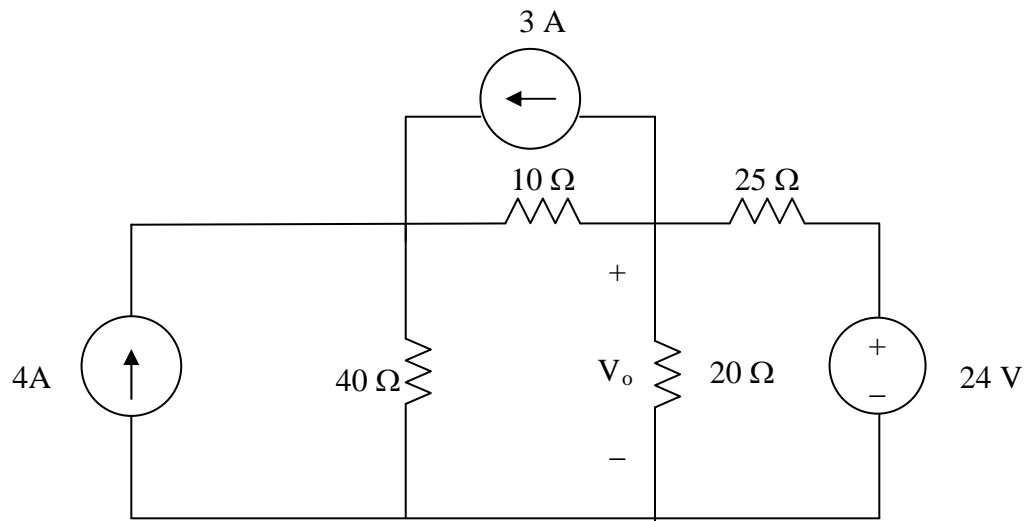
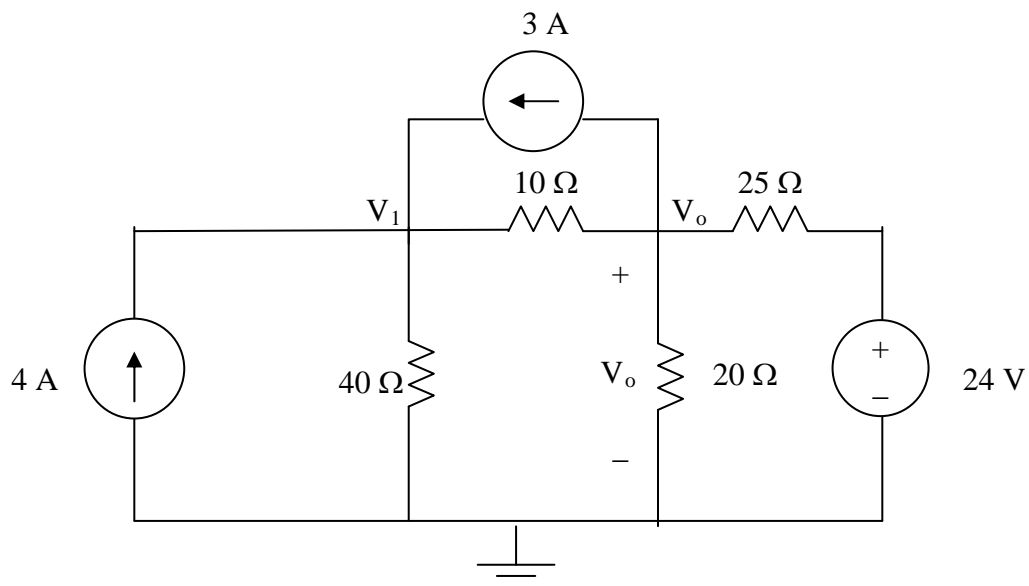


Figure 3.112
For Prob. 3.68.

Solution

Consider the circuit below. There are two non-reference nodes.



$$\begin{bmatrix} 0.125 & -0.1 \\ -0.1 & 0.19 \end{bmatrix} \mathbf{V} = \begin{bmatrix} +4+3 \\ -3+24/25 \end{bmatrix} = \begin{bmatrix} 7 \\ -2.04 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Y=[0.125,-0.1;-0.1,0.19]
```

Y =

```
    0.1250   -0.1000
   -0.1000    0.1900
```

```
>> I=[7,-2.04]'
```

I =

```
    7.0000
   -2.0400
```

```
>> V=inv(Y)*I
```

V =

```
    81.8909
    32.3636
```

Thus, $V_o = \mathbf{32.36\ V}$.

We can perform a simple check at node V_o ,

$$3 + 0.1(32.36-81.89) + 0.05(32.36) + 0.04(32.36-24) = 3 - 4.953 + 1.618 + 0.3344 = -0.0004; \text{ answer checks!}$$

Chapter 3, Solution 69

Assume that all conductances are in mS, all currents are in mA, and all voltages are in volts.

$$\begin{aligned}G_{11} &= (1/2) + (1/4) + (1/1) = 1.75, \quad G_{22} = (1/4) + (1/4) + (1/2) = 1, \\G_{33} &= (1/1) + (1/4) = 1.25, \quad G_{12} = -1/4 = -0.25, \quad G_{13} = -1/1 = -1, \\G_{21} &= -0.25, \quad G_{23} = -1/4 = -0.25, \quad G_{31} = -1, \quad G_{32} = -0.25\end{aligned}$$

$$i_1 = 20, \quad i_2 = 5, \quad \text{and} \quad i_3 = 10 - 5 = 5$$

The node-voltage equations are:

$$\begin{bmatrix} 1.75 & -0.25 & -1 \\ -0.25 & 1 & -0.25 \\ -1 & -0.25 & 1.25 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 5 \\ 5 \end{bmatrix}$$

Chapter 3, Solution 70

$$\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} \mathbf{V} = \begin{bmatrix} 4\mathbf{I}_x + 20 \\ -4\mathbf{I}_x - 7 \end{bmatrix}$$

With two equations and three unknowns, we need a constraint equation,

$\mathbf{I}_x = 2\mathbf{V}_1$, thus the matrix equation becomes,

$$\begin{bmatrix} -5 & 0 \\ 8 & 5 \end{bmatrix} \mathbf{V} = \begin{bmatrix} 20 \\ -7 \end{bmatrix}$$

This results in $\mathbf{V}_1 = 20/(-5) = \mathbf{-4\ V}$ and
 $\mathbf{V}_2 = [-8(-4) - 7]/5 = [32 - 7]/5 = \mathbf{5\ V}$.

Chapter 3, Solution 71

$$\begin{bmatrix} 9 & -4 & -5 \\ -4 & 7 & -1 \\ -5 & -1 & 9 \end{bmatrix} \mathbf{I} = \begin{bmatrix} 30 \\ -15 \\ 0 \end{bmatrix}$$

We can now use MATLAB solve for our currents.

```
>> R=[9,-4,-5;-4,7,-1;-5,-1,9]
```

```
R =
```

```
    9    -4    -5  
   -4     7    -1  
   -5    -1     9
```

```
>> V=[30,-15,0]'
```

```
V =
```

```
    30  
   -15  
     0
```

```
>> I=inv(R)*V
```

```
I =
```

```
6.255 A  
1.9599 A  
3.694 A
```

Chapter 3, Solution 72

$R_{11} = 5 + 2 = 7$, $R_{22} = 2 + 4 = 6$, $R_{33} = 1 + 4 = 5$, $R_{44} = 1 + 4 = 5$,
 $R_{12} = -2$, $R_{13} = 0 = R_{14}$, $R_{21} = -2$, $R_{23} = -4$, $R_{24} = 0$, $R_{31} = 0$,
 $R_{32} = -4$, $R_{34} = -1$, $R_{41} = 0 = R_{42}$, $R_{43} = -1$, we note that $R_{ij} = R_{ji}$ for
all i not equal to j .

$$v_1 = 8, \quad v_2 = 4, \quad v_3 = -10, \quad \text{and} \quad v_4 = -4$$

Hence the mesh-current equations are:

$$\begin{bmatrix} 7 & -2 & 0 & 0 \\ -2 & 6 & -4 & 0 \\ 0 & -4 & 5 & -1 \\ 0 & 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -10 \\ -4 \end{bmatrix}$$

Chapter 3, Solution 73

$$R_{11} = 2 + 3 + 4 = 9, \quad R_{22} = 3 + 5 = 8, \quad R_{33} = 1 + 1 + 4 = 6, \quad R_{44} = 1 + 1 = 2, \\ R_{12} = -3, \quad R_{13} = -4, \quad R_{14} = 0, \quad R_{23} = 0, \quad R_{24} = 0, \quad R_{34} = -1$$

$$v_1 = 6, \quad v_2 = 4, \quad v_3 = 2, \quad \text{and} \quad v_4 = -3$$

Hence,

$$\begin{bmatrix} 9 & -3 & -4 & 0 \\ -3 & 8 & 0 & 0 \\ -4 & 0 & 6 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \\ -3 \end{bmatrix}$$

Chapter 3, Solution 74

$$\begin{aligned} R_{11} &= R_1 + R_4 + R_6, \quad R_{22} = R_2 + R_4 + R_5, \quad R_{33} = R_6 + R_7 + R_8, \\ R_{44} &= R_3 + R_5 + R_8, \quad R_{12} = -R_4, \quad R_{13} = -R_6, \quad R_{14} = 0, \quad R_{23} = 0, \\ R_{24} &= -R_5, \quad R_{34} = -R_8, \quad \text{again, we note that } R_{ij} = R_{ji} \text{ for all } i \text{ not equal to } j. \end{aligned}$$

$$\text{The input voltage vector is } = \begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4 \end{bmatrix}$$

$$\begin{bmatrix} R_1 + R_4 + R_6 & -R_4 & -R_6 & 0 \\ -R_4 & R_2 + R_4 + R_5 & 0 & -R_5 \\ -R_6 & 0 & R_6 + R_7 + R_8 & -R_8 \\ 0 & -R_5 & -R_8 & R_3 + R_5 + R_8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4 \end{bmatrix}$$

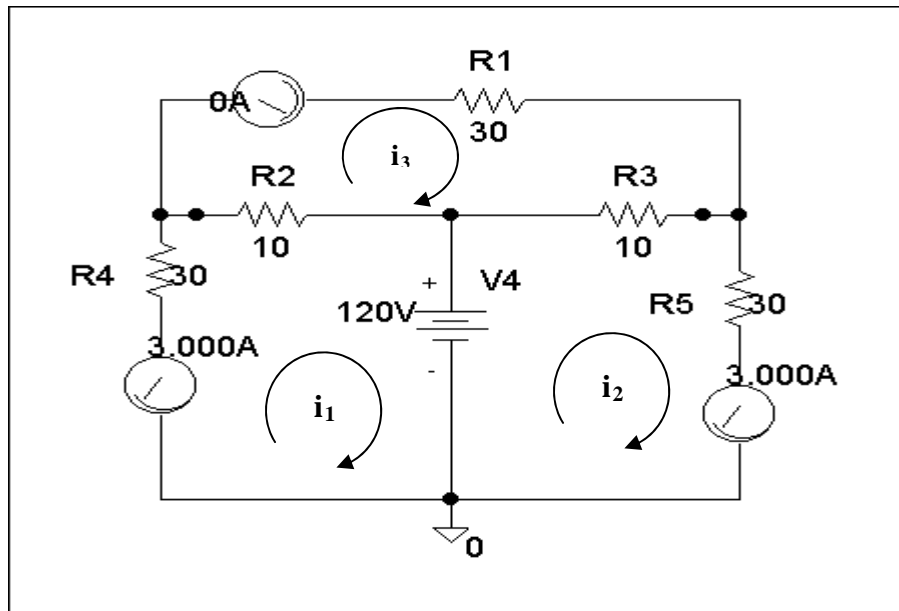
Chapter 3, Solution 75

* Schematics Netlist *

```

R_R4      $N_0002 $N_0001 30
R_R2      $N_0001 $N_0003 10
R_R1      $N_0005 $N_0004 30
R_R3      $N_0003 $N_0004 10
R_R5      $N_0006 $N_0004 30
V_V4      $N_0003 0 120V
v_V3      $N_0005 $N_0001 0
v_V2      0 $N_0006 0
v_V1      0 $N_0002 0

```



Clearly, $i_1 = -3$ amps, $i_2 = 0$ amps, and $i_3 = 3$ amps, which agrees with the answers in Problem 3.44.

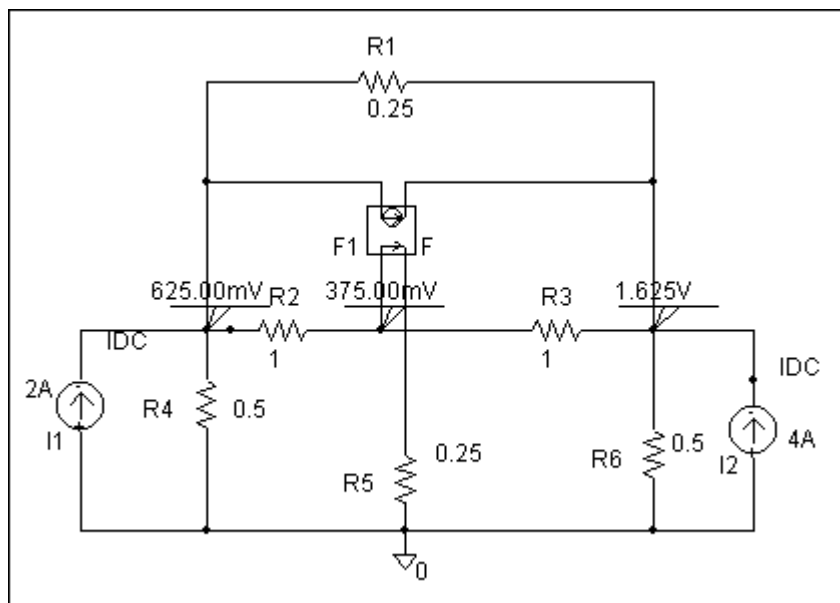
Chapter 3, Solution 76

* Schematics Netlist *

```

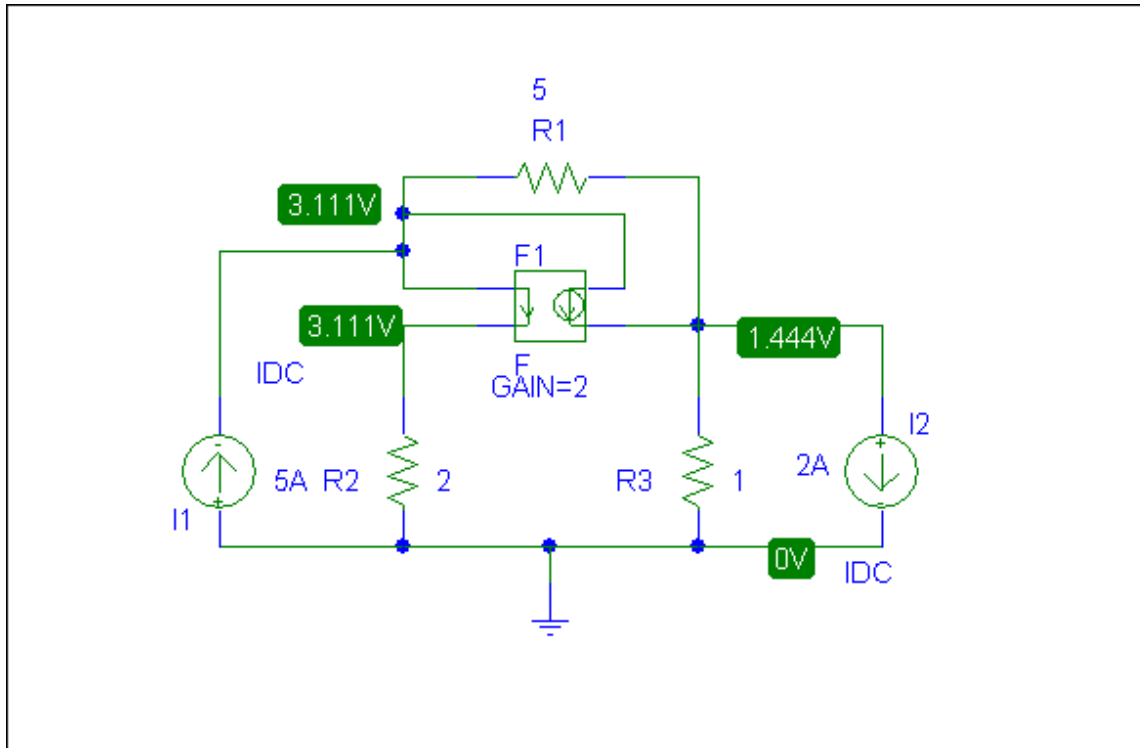
I_I2      0 $N_0001 DC 4A
R_R1      $N_0002 $N_0001 0.25
R_R3      $N_0003 $N_0001 1
R_R2      $N_0002 $N_0003 1
F_F1      $N_0002 $N_0001 VF_F1 3
VF_F1     $N_0003 $N_0004 0V
R_R4      0 $N_0002 0.5
R_R6      0 $N_0001 0.5
I_I1      0 $N_0002 DC 2A
R_R5      0 $N_0004 0.25

```



Clearly, $v_1 = 625 \text{ mVolts}$, $v_2 = 375 \text{ mVolts}$, and $v_3 = 1.625 \text{ volts}$, which agrees with the solution obtained in Problem 3.27.

Chapter 3, Solution 77



As a check we can write the nodal equations,

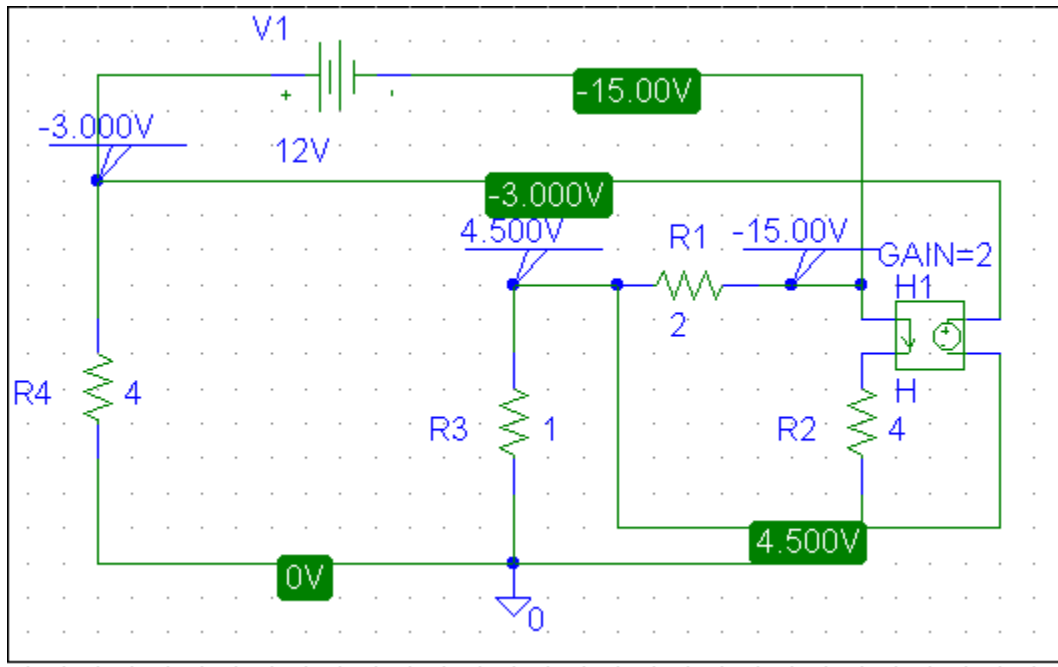
$$\begin{bmatrix} 1.7 & -0.2 \\ -1.2 & 1.2 \end{bmatrix} \mathbf{V} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

Solving this leads to $V_1 = 3.111 \text{ V}$ and $V_2 = 1.4444 \text{ V}$. The answer checks!

Chapter 3, Solution 78

The schematic is shown below. When the circuit is saved and simulated the node voltages are displayed on the pseudocomponents as shown. Thus,

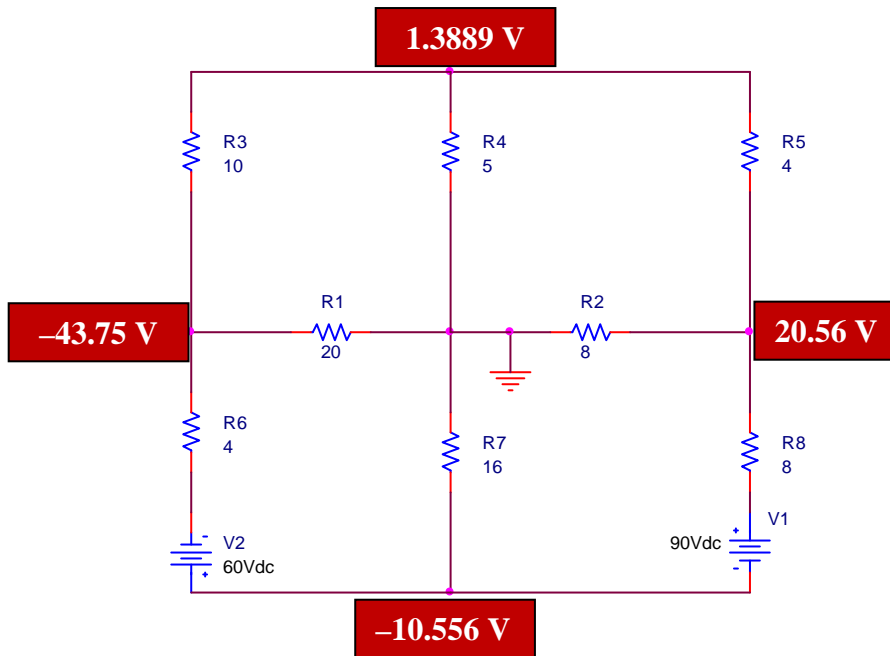
$$V_1 = -3\text{V}, \quad V_2 = 4.5\text{V}, \quad V_3 = -15\text{V},$$



Chapter 3, Solution 79

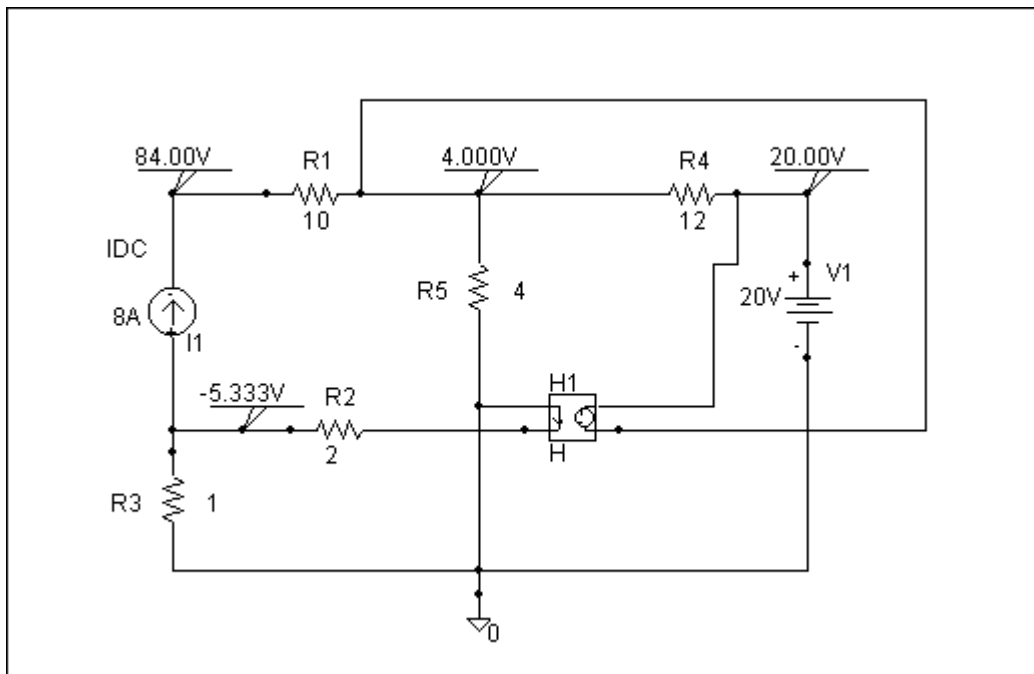
The schematic is shown below. When the circuit is saved and simulated, we obtain the node voltages as displayed. Thus,

$$V_a = -10.556 \text{ volts}; V_b = 20.56 \text{ volts}; V_c = 1.3889 \text{ volts}; \text{ and } V_d = -43.75 \text{ volts.}$$



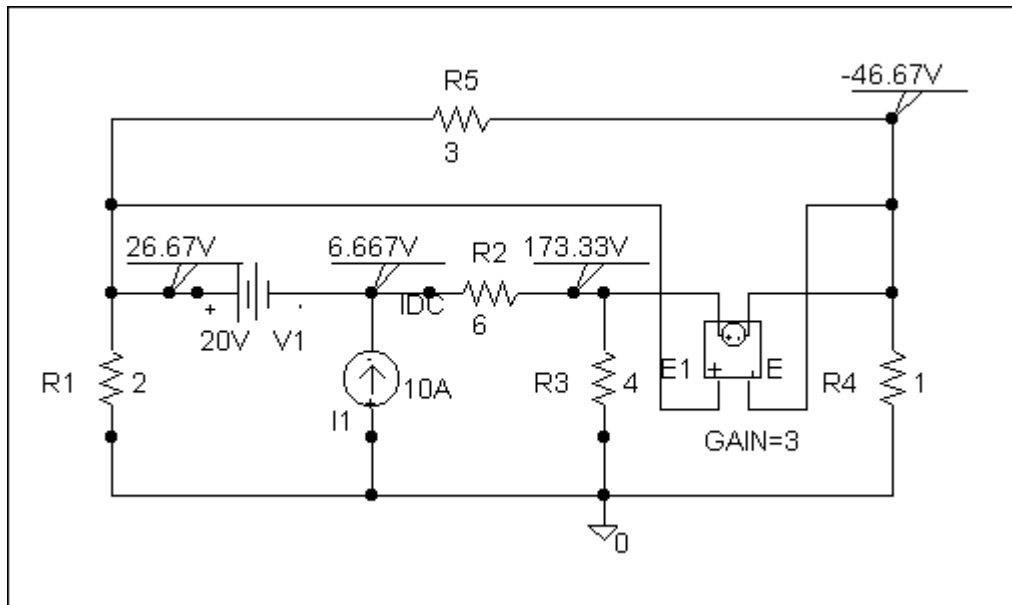
```
* Schematics Netlist *
```

H_H1	\$N_0002	\$N_0003	VH_H1	6
VH_H1	0	\$N_0001	0V	
I_I1	\$N_0004	\$N_0005	DC	8A
V_V1	\$N_0002	0	20V	
R_R4	0	\$N_0003	4	
R_R1	\$N_0005	\$N_0003	10	
R_R2	\$N_0003	\$N_0002	12	
R_R5	0	\$N_0004	1	
R_R3	\$N_0004	\$N_0001	2	



Clearly, $v_1 = 84$ volts, $v_2 = 4$ volts, $v_3 = 20$ volts, and $v_4 = -5.333$ volts

Chapter 3, Solution 81



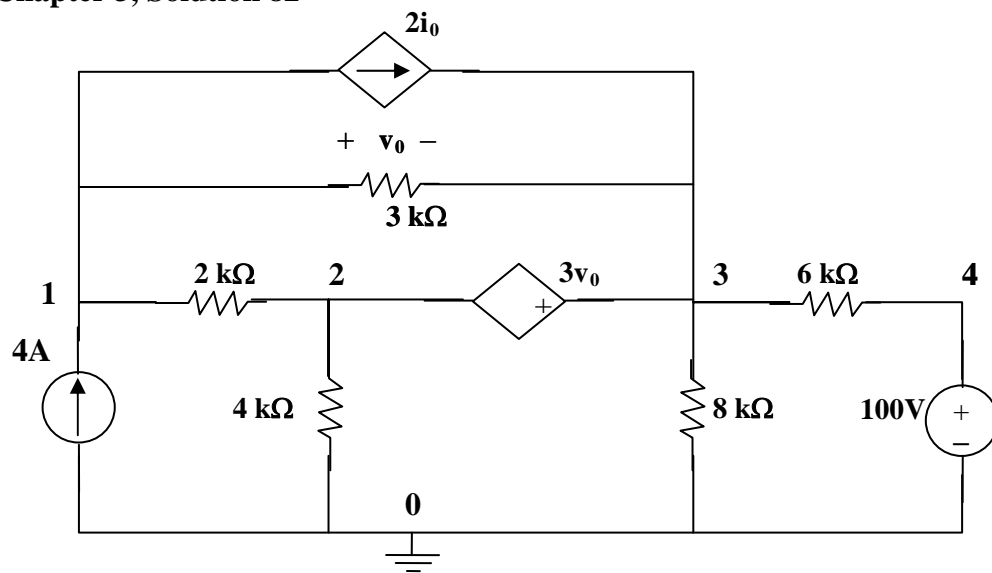
Clearly, $v_1 = 26.67$ volts, $v_2 = 6.667$ volts, $v_3 = 173.33$ volts, and $v_4 = -46.67$ volts which agrees with the results of Example 3.4.

This is the netlist for this circuit.

* Schematics Netlist *

```
R_R1      0 $N_0001  2
R_R2      $N_0003 $N_0002  6
R_R3      0 $N_0002  4
R_R4      0 $N_0004  1
R_R5      $N_0001 $N_0004  3
I_I1      0 $N_0003 DC 10A
V_V1      $N_0001 $N_0003 20V
E_E1      $N_0002 $N_0004 $N_0001 $N_0004 3
```

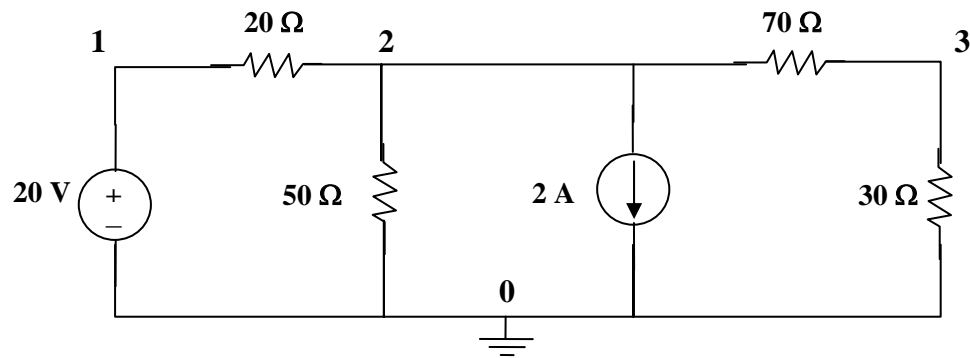
Chapter 3, Solution 82



This network corresponds to the Netlist.

Chapter 3, Solution 83

The circuit is shown below.



When the circuit is saved and simulated, we obtain $v_2 = -12.5$ volts

Chapter 3, Solution 84

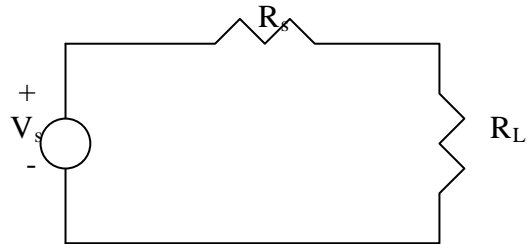
$$\text{From the output loop, } v_0 = 50i_0 \times 20 \times 10^3 = 10^6 i_0 \quad (1)$$

$$\text{From the input loop, } 15 \times 10^{-3} + 4000i_0 - v_0/100 = 0 \quad (2)$$

From (1) and (2) we get, $i_0 = \mathbf{2.5 \mu A}$ and $v_0 = \mathbf{2.5 \text{ volt}}$.

Chapter 3, Solution 85

The amplifier acts as a source.



For maximum power transfer,

$$R_L = R_s = \underline{9\Omega}$$

Chapter 3, Solution 86

Let v_1 be the potential across the 2 k-ohm resistor with plus being on top. Then,

Since $i = [(0.047 - v_1)/1k]$

$$[(v_1 - 0.047)/1k] - 400[(0.047 - v_1)/1k] + [(v_1 - 0)/2k] = 0 \text{ or}$$

$$401[(v_1 - 0.047)] + 0.5v_1 = 0 \text{ or } 401.5v_1 = 401 \times 0.047 \text{ or}$$

$$v_1 = 0.04694 \text{ volts and } i = (0.047 - 0.04694)/1k = 60 \text{ nA}$$

Thus,

$$v_0 = -5000 \times 400 \times 60 \times 10^{-9} = \mathbf{-120 \text{ mV}}.$$

Chapter 3, Solution 87

$$v_1 = 500(v_s)/(500 + 2000) = v_s/5$$

$$v_0 = -400(60v_1)/(400 + 2000) = -40v_1 = -40(v_s/5) = -8v_s,$$

Therefore, $v_0/v_s = -8$

Chapter 3, Solution 88

Let v_1 be the potential at the top end of the 100-ohm resistor.

$$(v_s - v_1)/200 = v_1/100 + (v_1 - 10^{-3}v_0)/2000 \quad (1)$$

For the right loop, $v_0 = -40i_0(10,000) = -40(v_1 - 10^{-3})10,000/2000$,

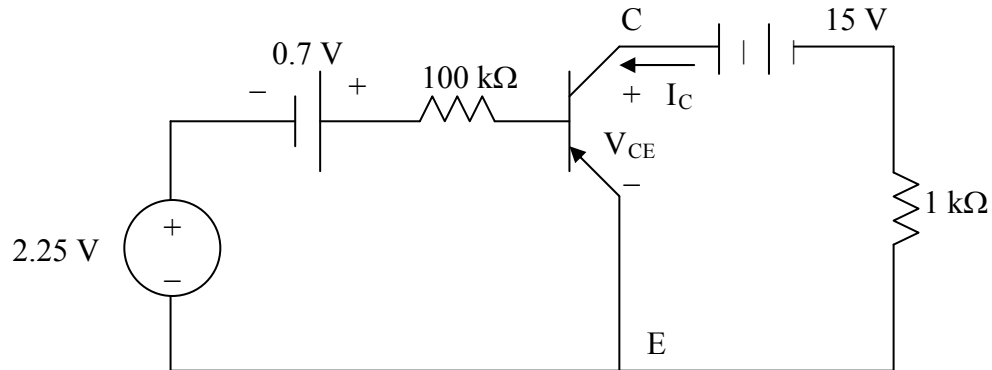
$$\text{or, } v_0 = -200v_1 + 0.2v_0 = -4 \times 10^{-3}v_0 \quad (2)$$

Substituting (2) into (1) gives, $(v_s + 0.004v_1)/2 = -0.004v_0 + (-0.004v_1 - 0.001v_0)/20$

This leads to $0.125v_0 = 10v_s$ or $(v_0/v_s) = 10/0.125 = \mathbf{-80}$

Chapter 3, Solution 89

Consider the circuit below.



For the left loop, applying KVL gives

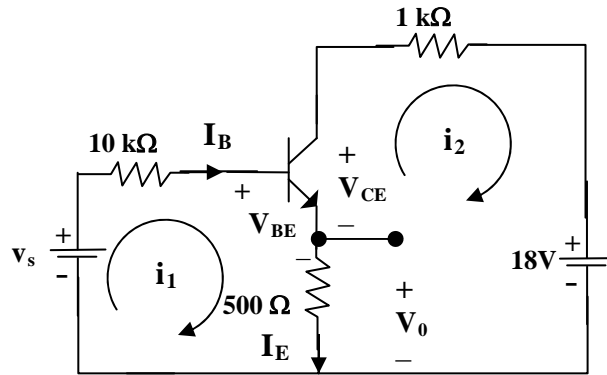
$$-2.25 - 0.7 + 10^5 I_B + V_{BE} = 0 \text{ but } V_{BE} = 0.7 \text{ V means } 10^5 I_B = 2.25 \text{ or}$$

$$I_B = \mathbf{22.5 \mu A}.$$

For the right loop, $-V_{CE} + 15 - I_C \times 10^3 = 0$. Additionally, $I_C = \beta I_B = 100 \times 22.5 \times 10^{-6} = 2.25 \text{ mA}$. Therefore,

$$V_{CE} = 15 - 2.25 \times 10^{-3} \times 10^3 = \mathbf{12.75 \text{ V}}.$$

Chapter 3, Solution 90



For loop 1, $-v_s + 10\text{k}(I_B) + V_{BE} + I_E (500) = 0 = -v_s + 0.7 + 10,000I_B + 500(1 + \beta)I_B$

which leads to $v_s + 0.7 = 10,000I_B + 500(151)I_B = 85,500I_B$

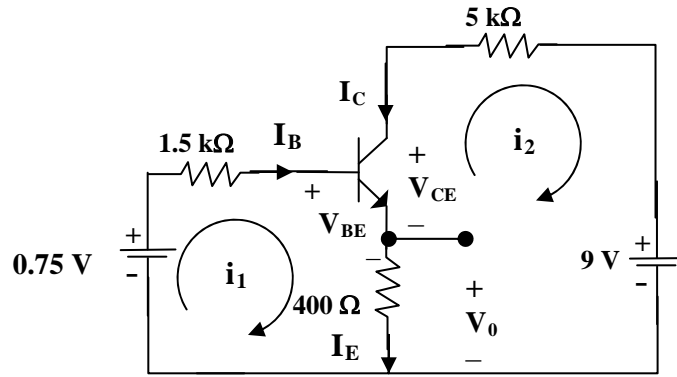
But, $v_0 = 500I_E = 500 \times 151I_B = 4$ which leads to $I_B = 5.298 \times 10^{-5}$

Therefore, $v_s = 0.7 + 85,500I_B = \mathbf{5.23\text{ volts}}$

Chapter 3, Solution 91

We first determine the Thevenin equivalent for the input circuit.

$$R_{Th} = 6 \parallel 2 = 6 \times 2 / 8 = 1.5 \text{ k}\Omega \text{ and } V_{Th} = 2(3)/(2+6) = 0.75 \text{ volts}$$



For loop 1, $-0.75 + 1.5kI_B + V_{BE} + 400I_E = 0 = -0.75 + 0.7 + 1500I_B + 400(1 + \beta)I_B$

$$I_B = 0.05/81,900 = \mathbf{0.61 \mu A}$$

$$v_0 = 400I_E = 400(1 + \beta)I_B = \mathbf{49 \text{ mV}}$$

For loop 2, $-400I_E - V_{CE} - 5kI_C + 9 = 0$, but, $I_C = \beta I_B$ and $I_E = (1 + \beta)I_B$

$$V_{CE} = 9 - 5k\beta I_B - 400(1 + \beta)I_B = 9 - 0.659 = \mathbf{8.641 \text{ volts}}$$

Chapter 3, Solution 92

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find I_B and V_C for the circuit in Fig. 3.128. Let $\beta = 100$, $V_{BE} = 0.7\text{V}$.

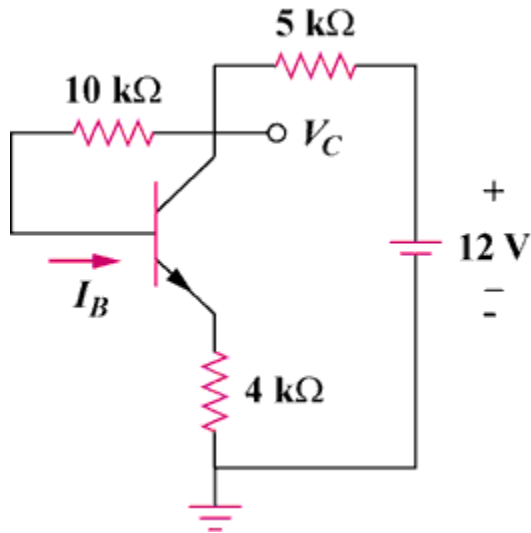
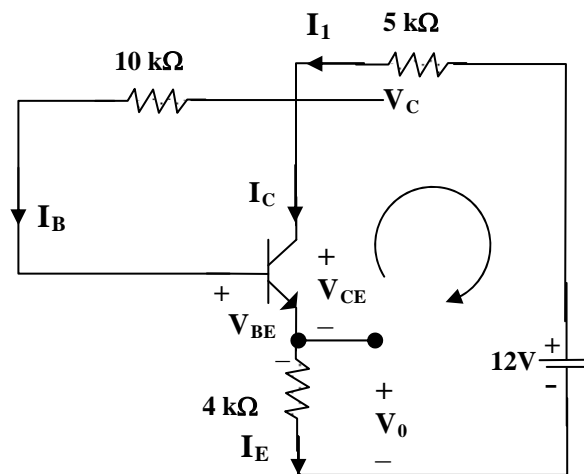


Figure 3.128

Solution



$$I_1 = I_B + I_C = (1 + \beta)I_B \quad \text{and} \quad I_E = I_B + I_C = I_1$$

Applying KVL around the outer loop,

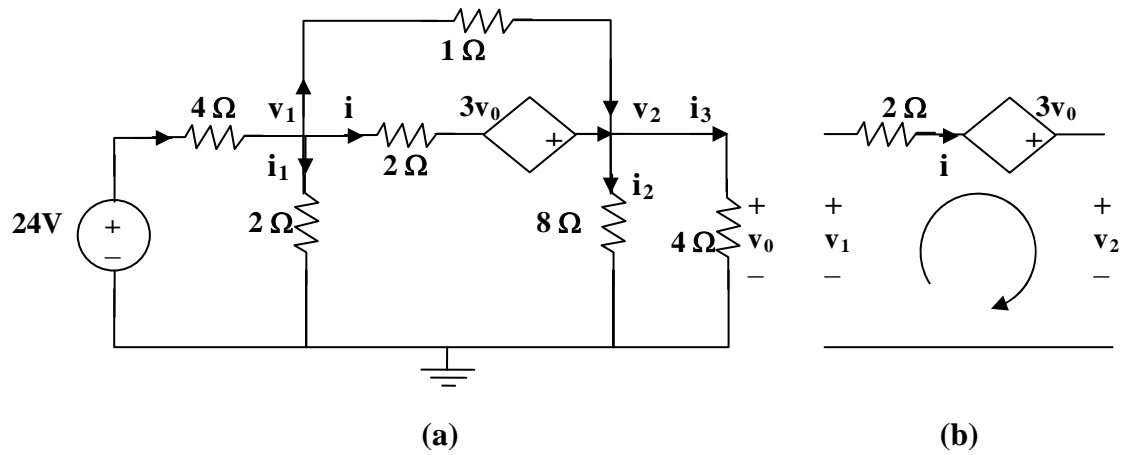
$$4kI_E + V_{BE} + 10kI_B + 5kI_1 = 12$$

$$12 - 0.7 = 5k(1 + \beta)I_B + 10kI_B + 4k(1 + \beta)I_B = 919kI_B$$

$$I_B = 11.3/919k = 12.296 \mu A$$

Also, $12 = 5kI_1 + V_C$ which leads to $V_C = 12 - 5k(101)I_B = \mathbf{5.791 \text{ volts}}$

Chapter 3, Solution 93



From (b), $-v_1 + 2i - 3v_0 + v_2 = 0$ which leads to $i = (v_1 + 3v_0 - v_2)/2$

At node 1 in (a), $((24 - v_1)/4) = (v_1/2) + ((v_1 + 3v_0 - v_2)/2) + ((v_1 - v_2)/1)$, where $v_0 = v_2$

or $24 = 9v_1$ which leads to $v_1 = \mathbf{2.667 \text{ volts}}$

At node 2, $((v_1 - v_2)/1) + ((v_1 + 3v_0 - v_2)/2) = (v_2/8) + v_2/4$, $v_0 = v_2$

$v_2 = 4v_1 = \mathbf{10.66 \text{ volts}}$

Now we can solve for the currents, $i_1 = v_1/2 = \mathbf{1.333 \text{ A}}$, $i_2 = \mathbf{1.333 \text{ A}}$, and

$i_3 = \mathbf{2.6667 \text{ A}}$.