Chapter 11, Solution 1.

$v(t) = 160\cos(50t)$

$i(t) = -33\sin(50t-30^\circ) = 33\cos(50t-30^\circ+180^\circ-90^\circ) = 33\cos(50t+60^\circ)$

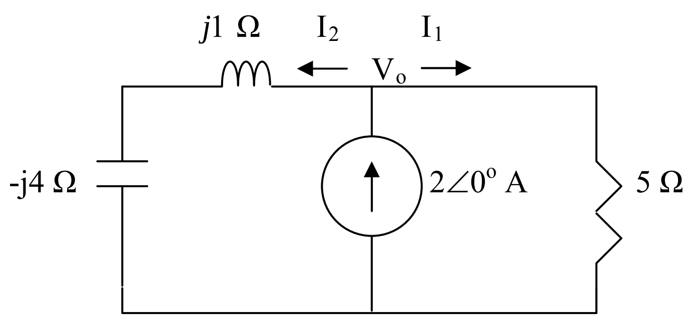
$p(t) = v(t)i(t) = 160x33\cos(50t)\cos(50t+60^\circ)$

$= 5280(1/2)[\cos(100t+60^\circ)+\cos(60^\circ)] =$ **$[1.320+2.640\cos(100t+60^\circ)]$ kW**.

$P = [V_m I_m/2]\cos(0-60^\circ) = 0.5x160x33x0.5 =$ **1.320 kW**.

Chapter 11, Solution 2.

Using current division,



$$I_1 = \frac{j1 - j4}{5 + j1 - j4}(2) = \frac{-j6}{5 - j3}$$

$$I_2 = \frac{5}{5 + j1 - j4}(2) = \frac{10}{5 - j3}$$

For the inductor and capacitor, the average power is zero. For the resistor,

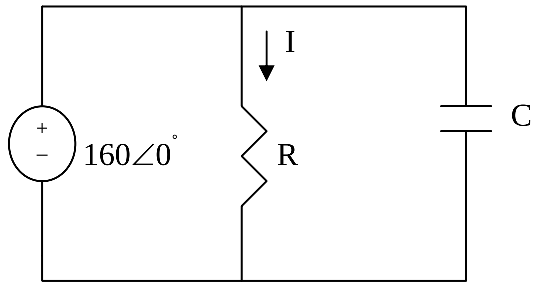
$$P = \frac{1}{2}|I_1|^2 R = \frac{1}{2}(1.029)^2(5) = 2.647 \text{ W}$$

$$V_o = 5I_1 = -2.6471 - j4.4118$$

$$S = \frac{1}{2}V_o I^* = \frac{1}{2}(-2.6471 - j4.4118)x2 = -2.6471 - j4.4118$$

Hence the average power supplied by the current source is **2.647 W**.

Chapter 11, Solution 3.



$$90\ \mu\text{F} \quad \longrightarrow \quad \frac{1}{\mathrm{j}\omega \mathrm{C}} = \frac{1}{j90x10^{-6}x2x10^{3}} = -j5.5556$$

I = 160/60 = 2.667A

The average power delivered to the load is the same as the average power absorbed by the resistor which is

$$P_{avg} = 0.5|I|^{2}60 = \mathbf{213.4\ W}.$$

Chapter 11, Solution 4.

Using Fig. 11.36, design a problem to help other students better understand instantaneous and average power.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the average power dissipated by the resistances in the circuit of Fig. 11.36. Additionally, verify the conservation of power. Note, we do not talk about rms values of voltages and currents until Section 11.4, all voltages and currents are peak values.

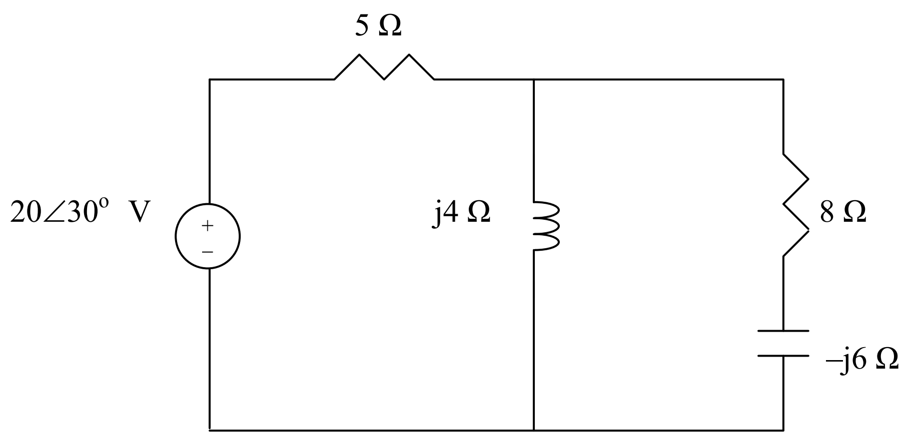
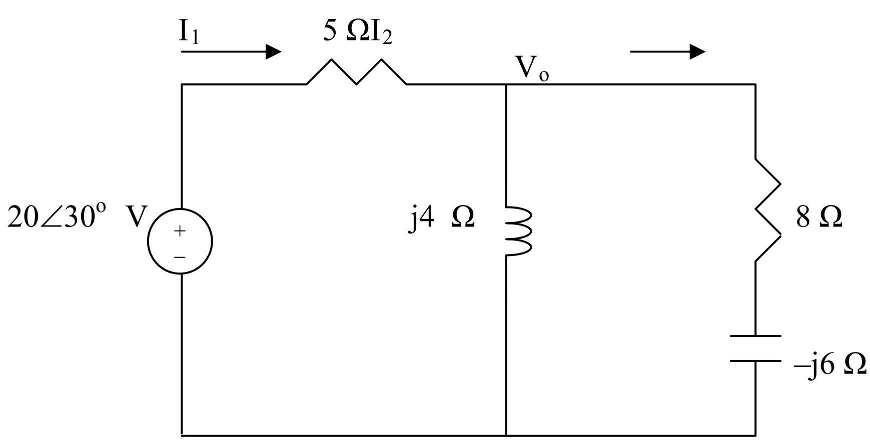


Figure 11.36 For Prob. 11.4.

Solution

We apply nodal analysis. At the main node,



$$\frac{20 < 30^o - V_o}{5} = \frac{V_o}{j4} + \frac{V_o}{8 - j6} \quad \longrightarrow \quad V_o = 5.152 + j10.639 = 11.821\angle 64.16^\circ$$

For the 5-Ω resistor,

$$I_1 = \frac{20 < 30^o - V_o}{5} = 2.438 < -3.0661^o \quad \text{A}$$

The average power dissipated by the resistor is

$$P_1 = \frac{1}{2} | I_1 |^2 R_1 = \frac{1}{2} x 2.438^2 x 5 = \underline{14.86 \text{ W}}$$

For the 8-Ω resistor,

$\mathbf{I_2} = V_o/(8–j6) = (11.812/10)\angle(64.16+36.87)^\circ = 1.1812\angle 101.03^\circ$ A

The average power dissipated by the resistor is

$P_2 = 0.5|I_2|^2 R_2 = 0.5(1.1812)^2 8 =$ **5.581 W**

The complex power supplied is

$\mathbf{S} = 0.5(V_s)(I_1)^* = 0.5(20\angle 30^\circ)(2.438\angle 3.07^\circ) = 24.38\angle 33.07^\circ$

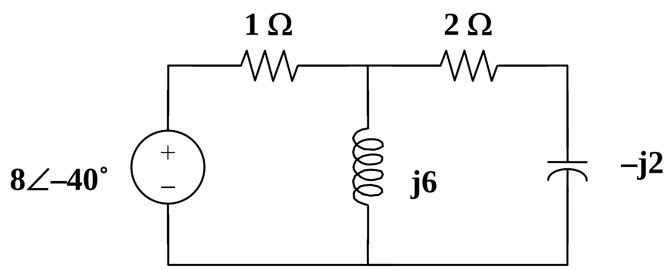
= **(20.43+13.303) VA**

Adding P_1 and P_2 gives the real part of **S**, showing the conservation of power.

P = 14.86+5.581 = **20.44 W** which checks nicely.

Chapter 11, Solution 5.

Converting the circuit into the frequency domain, we get:



$$I_{1\Omega} = \frac{8\angle -40°}{1+\dfrac{j6(2-j2)}{j6+2-j2}} = 1.6828\angle -25.38°$$

$$P_{1\Omega} = \frac{1.6828^2}{2}1 = \underline{1.4159\ \text{W}}$$

$P_{1\Omega}$ = **1.4159 W**

$P_{3H} = P_{0.25F}$ = **0 W**

$$|I_{2\Omega}| = \left|\frac{j6}{j6+2-j2}1.6828\angle -25.38°\right| = 2.258$$

$$P_{2\Omega} = \frac{2.258^2}{2}2 = \underline{5.097\ \text{W}}$$

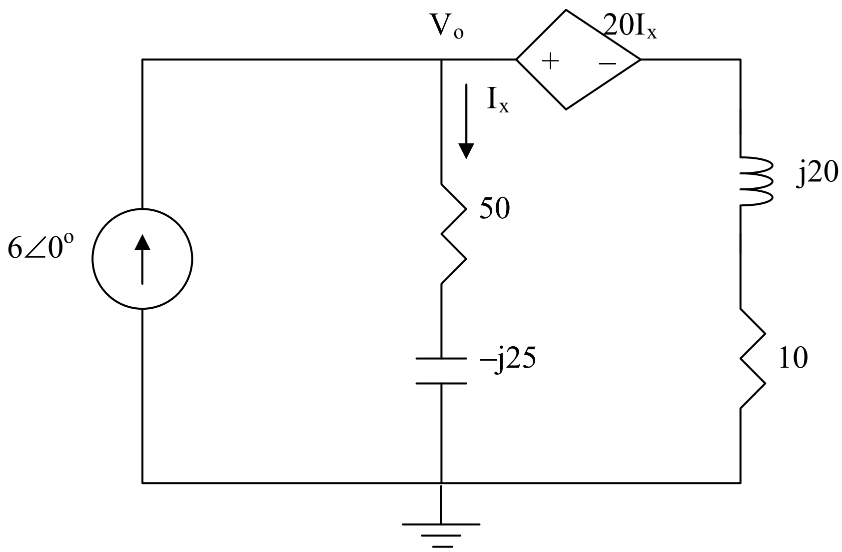
$P_{2\Omega}$ = **5.097 W**

Chapter 11, Solution 6.

$$20\text{ mH} \quad \longrightarrow \quad j\omega L = j10^3 x20x10^{-3} = j20$$

$$40\mu F \rightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 x40x10^{-6}} = -j25$$

We apply nodal analysis to the circuit below.



$$-6 + \frac{V_o - 20I_x}{10 + j20} + \frac{V_o - 0}{50 - j25} = 0$$

But $I_x = \frac{V_o}{50 - j25}$. Substituting this and solving for V_o leads

$$\left(\frac{1}{10 + j20} - \frac{20}{(10 + j20)}\frac{1}{(50 - j25)} + \frac{1}{50 - j25}\right)V_o = 6$$

$$\left(\frac{1}{22.36\angle 63.43^\circ} - \frac{20}{(22.36\angle 63.43^\circ)(55.9\angle -26.57^\circ)} + \frac{1}{55.9\angle -26.57^\circ}\right)V_o = 6$$

$$(0.02 - j0.04 - 0.012802 + j0.009598 + 0.016 + j0.008)V_o = 6$$

$(0.0232 - j0.0224)V_o = 6$ or $V_o = 6/(0.03225\angle{-43.99^\circ}) = 186.05\angle 43.99^\circ$ volts.

$|I_x| = 186.05/55.9 = 3.328$

We can now calculate the average power absorbed by the 50-Ω resistor.

$P_{avg} = [(3.328)^2/2]x50 =$ **276.8 W**.

Chapter 11, Solution 7.

Applying KVL to the left-hand side of the circuit,

$$8\angle 20° = 4\mathbf{I}_o + 0.1\mathbf{V}_o \qquad (1)$$

Applying KCL to the right side of the circuit,

$$8\mathbf{I}_o + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1}{10 - j5} = 0$$

But, $\mathbf{V}_o = \frac{10}{10 - j5}\mathbf{V}_1 \longrightarrow \mathbf{V}_1 = \frac{10 - j5}{10}\mathbf{V}_o$

Hence, $8\mathbf{I}_o + \frac{10 - j5}{j50}\mathbf{V}_o + \frac{\mathbf{V}_o}{10} = 0$

$$\mathbf{I}_o = j0.025\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1),

$$8\angle 20° = 0.1\mathbf{V}_o\,(1 + j)$$

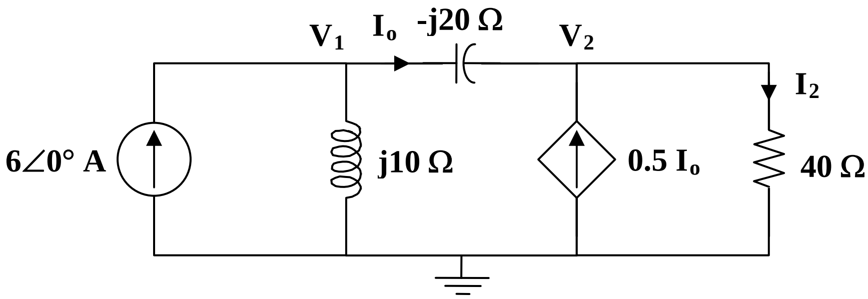
$$\mathbf{V}_o = \frac{80\angle 20°}{1 + j}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{10} = \frac{8}{\sqrt{2}}\angle -25°$$

$$P = \frac{1}{2}\left|\mathbf{I}_1\right|^2 R = \left(\frac{1}{2}\right)\left(\frac{64}{2}\right)(10) = \mathbf{160W}$$

Chapter 11, Solution 8.

We apply nodal analysis to the following circuit.



At node 1,

$$6 = \frac{\mathbf{V}_1}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} \quad \mathbf{V}_1 = j120 - \mathbf{V}_2 \qquad (1)$$

At node 2,

$$0.5\mathbf{I}_o + \mathbf{I}_o = \frac{\mathbf{V}_2}{40}$$

But, $$\mathbf{I}_o = \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$$

Hence, $$\frac{1.5(\mathbf{V}_1 - \mathbf{V}_2)}{-j20} = \frac{\mathbf{V}_2}{40}$$

$$3\mathbf{V}_1 = (3 - j)\mathbf{V}_2 \qquad (2)$$

Substituting (1) into (2),

$$j360 - 3\mathbf{V}_2 - 3\mathbf{V}_2 + j\mathbf{V}_2 = 0$$

$$\mathbf{V}_2 = \frac{j360}{6 - j} = \frac{360}{37}(-1 + j6)$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{40} = \frac{9}{37}(-1 + j6)$$

$$P = \frac{1}{2}\left|\mathbf{I}_2\right|^2 R = \frac{1}{2}\left(\frac{9}{\sqrt{37}}\right)^2 (40) = \mathbf{43.78\ W}$$

Chapter 11, Solution 9.

This is a non-inverting op amp circuit. At the output of the op amp,

$$V_o = \left(1 + \frac{Z_2}{Z_1}\right)V_s = \left(1 + \frac{(10 + j6)x10^3}{(2 + j4)x10^3}\right)(8.66 + j5) = 20.712 + j28.124$$

The current through the 20-kς resistor is

$$I_o = \frac{V_o}{20k - j12k} = 0.1411 + j1.491 \text{ mA} \quad \text{or } |I_o| = 1.4975 \text{ A}$$

$P = [|I_o|^2/2]R = [1.4875^2/2]10^{-6}x20x10^3$

= **22.42 mW**

Chapter 11, Solution 10.

No current flows through each of the resistors. Hence, for each resistor, $P = \mathbf{0\ W}$. It should be noted that the input voltage will appear at the output of each of the op amps.

Chapter 11, Solution 11.

$\omega = 377$, $\quad R = 10^4$, $\quad C = 200 \times 10^{-9}$

$\omega RC = (377)(10^4)(200 \times 10^{-9}) = 0.754$

$\tan^{-1}(\omega RC) = 37.02°$

$$Z_{ab} = \frac{10k}{\sqrt{1 + (0.754)^2}} \angle -37.02° = 7.985 \angle -37.02° \text{ k}\Omega$$

$i(t) = 33\sin(377t + 22°) = 33\cos(377t - 68°)\ mA$

$\mathbf{I} = 33\angle{-68°}$ mA

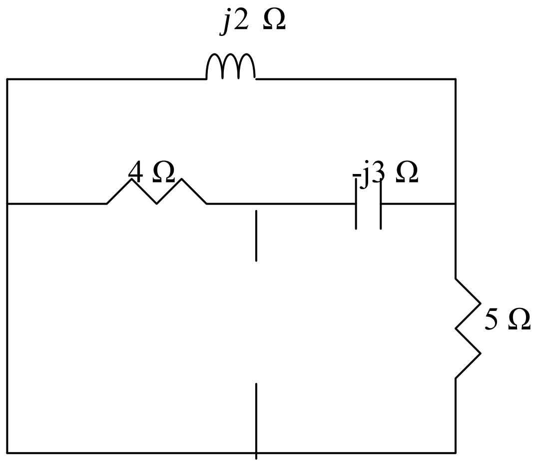
$$S = \frac{I^2 Z_{ab}}{2} = \frac{\left(33x10^{-3}\right)^2 (7.985\angle -37.02°) \times 10^3}{2}$$

$\mathbf{S} = 4.348\angle{-37.02°}$ VA

$$P = \left|S\right| \cos(37.02) = \mathbf{3.472\ W}$$

Chapter 11, Solution 12.

We find the Thevenin impedance using the circuit below.

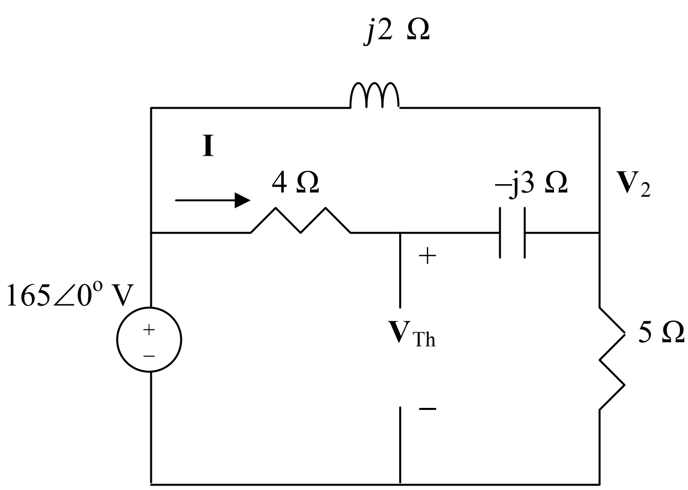


We note that the inductor is in parallel with the 5-Ω resistor and the combination is in series with the capacitor. That whole combination is in parallel with the 4-Ω resistor. Thus,

$$Z_{Thev} = \frac{4\left(-j3+\dfrac{5xj2}{5+j2}\right)}{4-j3+\dfrac{5xj2}{5+j2}} = \frac{4(0.6896-j1.2758)}{4.69-j1.2758} = \frac{4(1.4502\angle-61.61°)}{4.86\angle-15.22°}$$
$$= 1.1936\angle-46.39°$$

$Z_{Thev} = 0.8233 - j0.8642$ or $Z_L =$ **[823.3 + j864.2] mΩ**.

We obtain V_{Th} using the circuit below. We apply nodal analysis.



$$\frac{V_2 - 165}{4 - j3} + \frac{V_2 - 165}{j2} + \frac{V_2 - 0}{5} = 0$$

$$(0.16 + j0.12 - j0.5 + 0.2)V_2 = (0.16 + j0.12 - j0.5)165 \quad 4.125$$

$$(0.5235\angle -46.55°)V_2 = (0.4123\angle -67.17°)165$$

Thus, $V_2 = 129.94\angle{-20.62^\circ}\text{V} = 121.62 - j45.76$

$I = (165 - V_2)/(4 - j3) = (165 - 121.62 + j45.76)/(4 - j3)$
$= (63.06\angle 46.52^\circ)/(5\angle{-36.87^\circ}) = 12.613\angle 83.39^\circ = 1.4519 + j12.529$

$V_{Thev} = 165 - 4I = 165 - 5.808 - j50.12 = [159.19 - j50.12]$ V
$= 166.89\angle{-17.48^\circ}\text{V}$

We can check our value of V_{Thev} by letting $V_1 = V_{Thev}$. Now we can use nodal analysis to solve for V_1.

At node 1,

$$\frac{V_1 - 165}{4} + \frac{V_1 - V_2}{-j3} + \frac{V_2 - 0}{5} = 0 \rightarrow (0.25 + j0.3333)V_1 + (0.2 - j0.3333)V_2 = 41.25$$

At node 2,

$$\frac{V_2 - V_1}{-j3} + \frac{V_2 - 165}{j2} = 0 \rightarrow -j0.3333V_1 + (-j0.1667)V_2 = -j82.5$$

```
>> Y=[(0.25+0.3333i),-0.3333i;-0.3333i,(0.2-0.1667i)]

Y =

   0.2500 + 0.3333i        0 - 0.3333i
        0 - 0.3333i   0.2000 - 0.1667i

>> I=[41.25;–82.5i]

I =

  41.2500
       0 -20.0000i

>> V=inv(Y)*I

V =
```

159.2221 – 50.1018i
121.6421–45.7677i

Please note, these values check with the ones obtained above.

To calculate the maximum power to the load,

$|I_L| = (166.89/(2x0.8233)) = 101.34$ A

$P_{avg} = [(|I_L|_{rms})^2 0.8233]/2 =$ **4.228 mW**.

Chapter 11, Solution 13.

For maximum power transfer to the load, $\mathbf{Z_L}$ = **[120 – j60] Ω**.

$I_L = 165/(240) = 0.6875$ A

$$P_{avg} = [|I_L|^2 120]/2 = \mathbf{28.36\ W}.$$

Chapter 11, Solution 14.

Using Fig. 11.45, design a problem to help other students better understand maximum average power transfer.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

It is desired to transfer maximum power to the load Z in the circuit of Fig. 11.45. Find Z and the maximum power. Let $i_s = 5\cos 40t$ A.

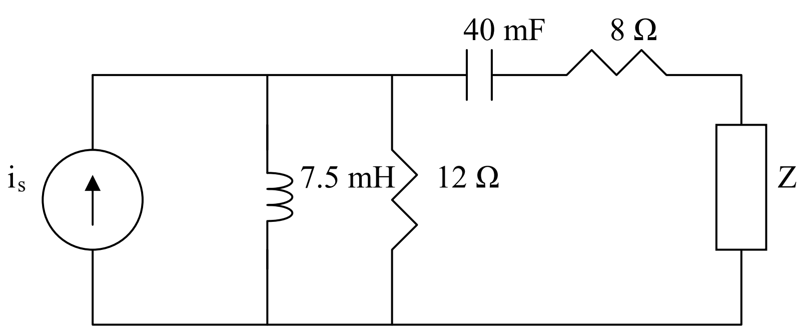


Figure 11.45 For Prob. 11.14.

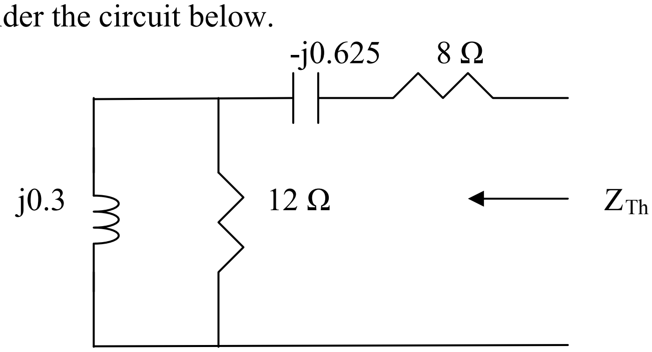
Solution

We find the Thevenin equivalent at the terminals of Z.

$$40\text{ mF} \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j40x40x10^{-3}} = j0.625$$

$$7.5\text{ mH} \quad \longrightarrow \quad j\omega L = j40x7.5x10^{-3} = j0.3$$

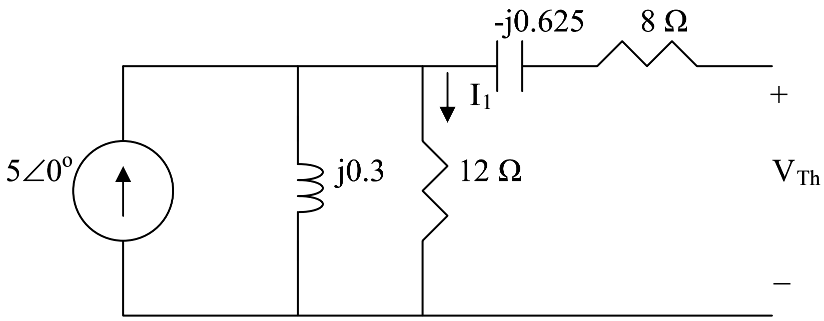
To find Z_{Th}, consider the circuit below.



$$Z_{Th} = 8 - j0.625 + 12 // j0.3 = 8 - j0.625 + \frac{12x0.3}{12+0.3} = 8.0075 - j0.3252$$

$Z_L = (Z_{Thev})^* =$ **[8.008 + j0.3252] Ω**.

To find V_{Th}, consider the circuit below.



By current division,

$I_1 = 5(j0.3)/(12+j0.3) = 1.5\angle 90^\circ/12.004\angle 1.43^\circ = 0.12496\angle 88.57^\circ$

$= 0.003118 + j0.12492A$

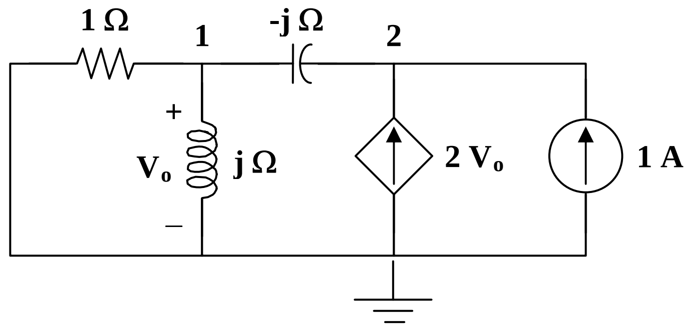
$V_{Thev\ rms} = 12I_1/\sqrt{2} = 1.0603\angle 88.57^\circ V$

$I_{Lrms} = 1.0603\angle 88.57^\circ/2(8.008) = 66.2\angle 88.57^\circ mA$

$P_{avg} = |I_{Lrms}|^2 8.008 =$ **35.09 mW**.

Chapter 11, Solution 15.

To find $\mathbf{Z}_{eq}$, insert a 1-A current source at the load terminals as shown in Fig. (a).



(a)

At node 1,

$$\frac{\mathbf{V}_o}{1} + \frac{\mathbf{V}_o}{j} = \frac{\mathbf{V}_2 - \mathbf{V}_o}{-j} \longrightarrow \mathbf{V}_o = j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_2 - \mathbf{V}_o}{-j} \longrightarrow 1 = j\mathbf{V}_2 - (2+j)\mathbf{V}_o \qquad (2)$$

Substituting (1) into (2),

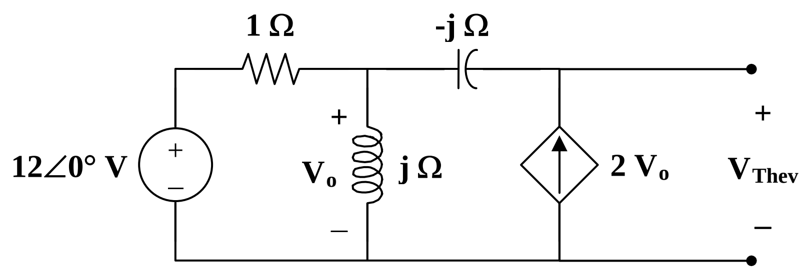
$$1 = j\mathbf{V}_2 - (2+j)(j)\mathbf{V}_2 = (1-j)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{1}{1-j}$$

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_2}{1} = \frac{1+j}{2} = 0.5 + j0.5$$

$$\mathbf{Z}_L = \mathbf{Z}_{eq}^{*} = \mathbf{[0.5 - \mathit{j}0.5]\,\Omega}$$

We now obtain $\mathbf{V}_{Thev}$ from Fig. (b).



(b)

$$-2\mathbf{V}_o + \frac{\mathbf{V}_o - 12}{1} + \frac{\mathbf{V}_o}{j} = 0$$

$$\mathbf{V}_o = \frac{-12}{1+j}$$

$$-\mathbf{V}_o - (-j \times 2\,\mathbf{V}_o) + \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Thev} = (1 - j2)\mathbf{V}_o = \frac{(-12)(1-j2)}{1+j}$$

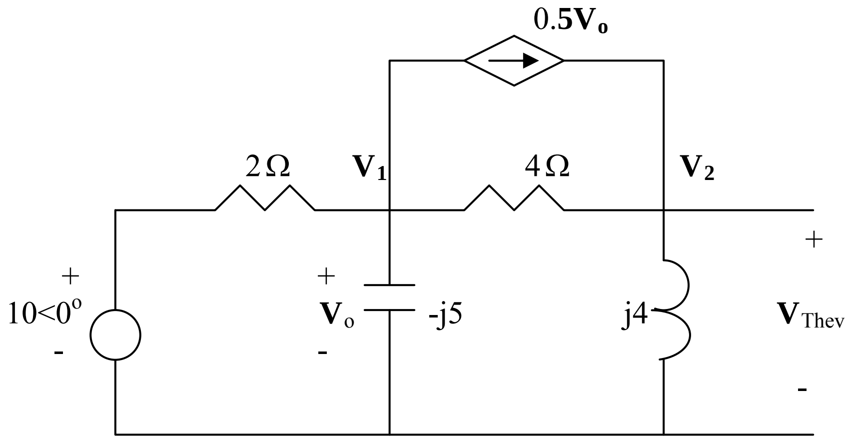
$$P_{\max} = \frac{\left|\left[\dfrac{V_{Thev}}{0.5 + j0.5 + 0.5 - j0.5}\right]\right|^2}{2} 0.5 = \frac{\left(\dfrac{12\sqrt{5}}{\sqrt{2}}\right)^2}{2(2x0.5)^2} 0.5$$

$$= \mathbf{90\ W}$$

Chapter 11, Solution 16.

$$\omega = 4, \quad 1\text{H} \longrightarrow j\omega L = j4, \quad 1/20\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x1/20} = -j5$$

We find the Thevenin equivalent at the terminals of Z_L. To find V_{Thev}, we use the circuit shown below.



At node 1,

$$\frac{10 - V_1}{2} = \frac{V_1}{-j5} + 0.5V_1 + \frac{V_1 - V_2}{4} \longrightarrow 5 = V_1(1.25 + j0.2) - 0.25V_2 \qquad (1)$$

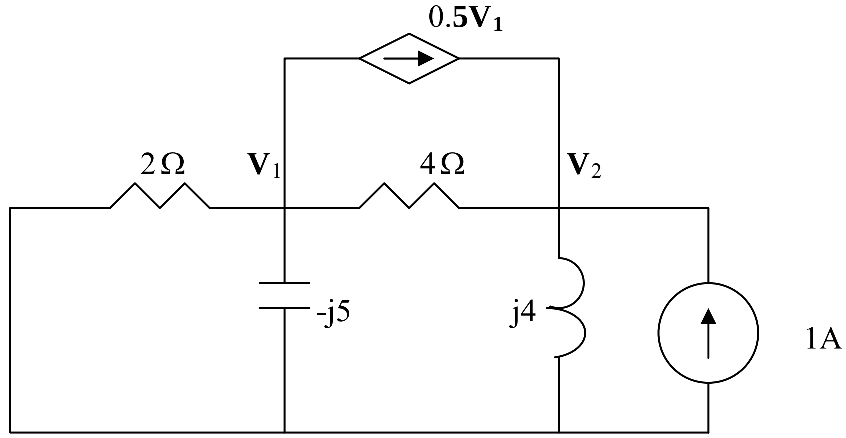
At node 2,

$$\frac{V_1 - V_2}{4} + 0.25V_1 = \frac{V_2}{j4} \longrightarrow 0 = 0.5V_1 + V_2(-0.25 + j0.25) \qquad (2)$$

Solving (1) and (2) leads to

$$V_{Thev} = V_2 = 6.1947 + j7.0796 = 9.4072\angle 48.81^o$$

To obtain R_{eq}, consider the circuit shown below. We replace Z_L by a 1-A current source.



At node 1,

$$\frac{V_1}{2}+\frac{V_1}{-j5}+0.25V_1+\frac{V_1-V_2}{4}=0 \longrightarrow 0=V_1(1+j0.2)-0.25V_2 \qquad (3)$$

At node 2,

$$1+\frac{V_1-V_2}{4}+0.25V_1=\frac{V_2}{j4} \longrightarrow -1=0.5V_1+V_2(-0.25+j0.25) \qquad (4)$$

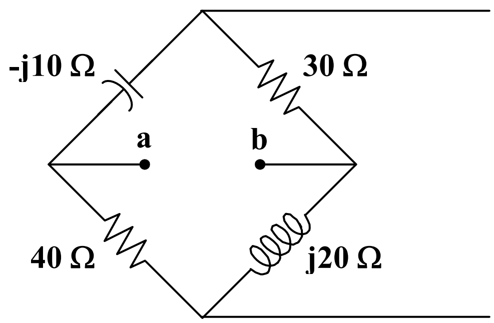
Solving (1) and (2) gives

$$Z_{eq}=\frac{V_2}{1}=1.9115+j3.3274=3.837\angle 60.12^o$$ and $\mathbf{Z_L} = 3.837\angle{-60.12°}\ \Omega$

$$P_{max}=\frac{|V_{Th}|^2}{2\left|Z_{eq}-Z_L\right|^2}1.9115=\frac{9.4072^2}{2x4x1.9115}=\mathbf{5.787\ W}$$

Chapter 11, Solution 17.

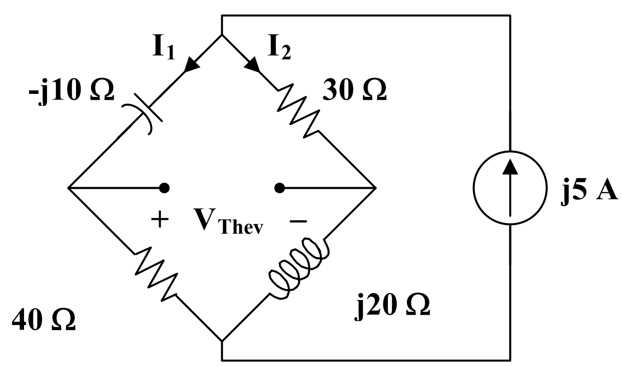
We find Z_{eq} at terminals a-b following Fig. (a).



(a)

$$\mathbf{Z}_{eq} = (-j10+30) \| (j20+40) = \frac{(30-j10)(40+j20)}{70+j10} = \mathbf{20\ \Omega} = \mathbf{Z_L}$$

We obtain $\mathbf{V}_{Thev}$ from Fig. (b).



(b)

Using current division,

$$\mathbf{I}_1 = \frac{30+\mathrm{j}20}{70+\mathrm{j}10}(\mathrm{j}5) = \text{-}1.1+\mathrm{j}2.3$$

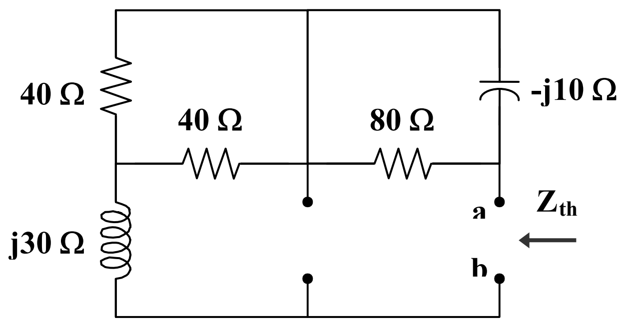
$$\mathbf{I}_2 = \frac{40-\mathrm{j}10}{70+\mathrm{j}10}(\mathrm{j}5) = 1.1+\mathrm{j}2.7$$

$$\mathbf{V}_{\mathrm{Th}} = 30\mathbf{I}_2 + \mathrm{j}10\mathbf{I}_1 = 10+\mathrm{j}70$$

$$P_{\max} = \frac{\left|\mathbf{V}_{Th}\right|^2}{2\left(Z_{eq}+Z_L\right)^2} Z_L = \frac{5000}{(2)(2x20)^2} 20 = \mathbf{31.25\ W}$$

Chapter 11, Solution 18.

We find $\mathbf{Z}_{\text{Th}}$ at terminals a-b as shown in the figure below.



$$\mathbf{Z}_{Th} = j30 + 40 \,||\, 40 + 80 \,||\, (\text{-j10}) = \text{j30} + 20 + \frac{(80)(\text{-j10})}{80 - \text{j10}}$$

$$\mathbf{Z}_{Th} = 21.23 + j20.154$$

$$\mathbf{Z}_{\text{L}} = \mathbf{Z}^*_{\text{Th}} = \mathbf{[21.23 – j20.15]\ \Omega}$$

Chapter 11, Solution 19.

At the load terminals,

$$\mathbf{Z}_{Th} = \text{-j2} + 6 \,||\, (3 + \text{j}) = \text{-j2} + \frac{(6)(3 + \text{j})}{9 + \text{j}}$$

$$\mathbf{Z}_{Th} = 2.049 - \text{j}1.561$$

$$R_L = \left| \mathbf{Z}_{Th} \right| = \mathbf{2.576\ \Omega}$$

To get $\mathbf{V}_{Th}$, let $\mathbf{Z} = 6 \,||\, (3 + \text{j}) = 2.049 + \text{j}0.439$.

By transforming the current sources, we obtain

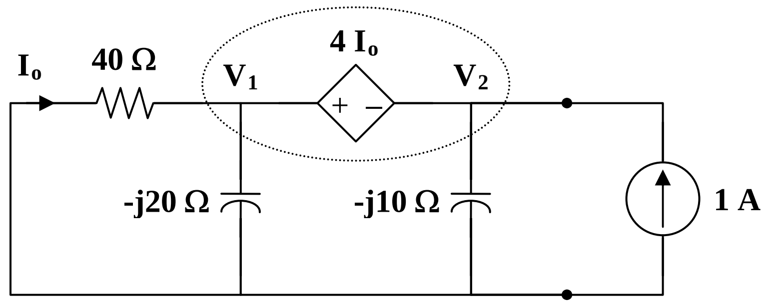
$$\mathbf{V}_{Th} = (33\angle 0°)\mathbf{Z} = 67.62 + j14.487 = 69.16\angle 12.09°$$

$$P_{max} = \left| \frac{69.16}{2.049 - j1.561 + 2.576} \right|^2 \frac{2.576}{2} = \mathbf{258.5\ W}.$$

Chapter 11, Solution 20.

Combine j20 Ω and -j10 Ω to get $j20 \parallel -j10 = -j20$.

To find $\mathbf{Z}_{Th}$, insert a 1-A current source at the terminals of R_L, as shown in Fig. (a).



(a)

At the supernode,

$$1 = \frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1}{-j20} + \frac{\mathbf{V}_2}{-j10}$$

$$40 = (1 + j2)\,\mathbf{V}_1 + j4\,\mathbf{V}_2 \qquad (1)$$

Also, $\mathbf{V}_1 = \mathbf{V}_2 + 4\mathbf{I}_o$, where $\mathbf{I}_o = \dfrac{-\mathbf{V}_1}{40}$

$$1.1\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{V}_1 = \frac{\mathbf{V}_2}{1.1} \qquad (2)$$

Substituting (2) into (1),

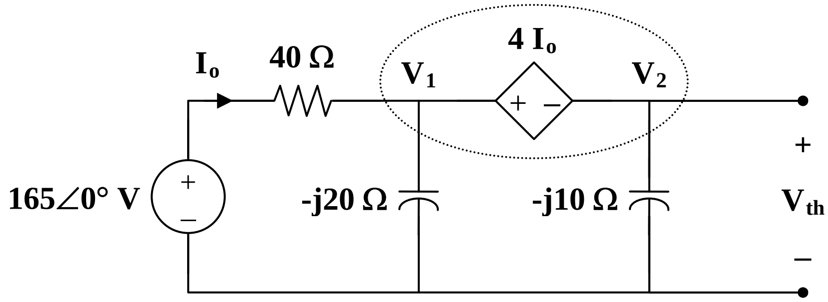
$$40 = (1 + j2)\left(\frac{\mathbf{V}_2}{1.1}\right) + j4\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{44}{1 + j6.4}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{1} = 1.05 - j6.71\,\Omega$$

$$R_L = \left|\mathbf{Z}_{Th}\right| = \mathbf{6.792\,\Omega}$$

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. (b).



(b)

At the supernode,

$$\frac{165-\mathbf{V}_1}{40} = \frac{\mathbf{V}_1}{-\text{j}20} + \frac{\mathbf{V}_2}{-\text{j}10}$$

$$165 = (1+j2)\mathbf{V}_1 + j4\mathbf{V}_2 \qquad (3)$$

Also, $\mathbf{V}_1 = \mathbf{V}_2 + 4\mathbf{I}_o$, where $\mathbf{I}_o = \dfrac{165-\mathbf{V}_1}{40}$

$$\mathbf{V}_1 = \frac{\mathbf{V}_2 + 16.5}{1.1} \qquad (4)$$

Substituting (4) into (3),

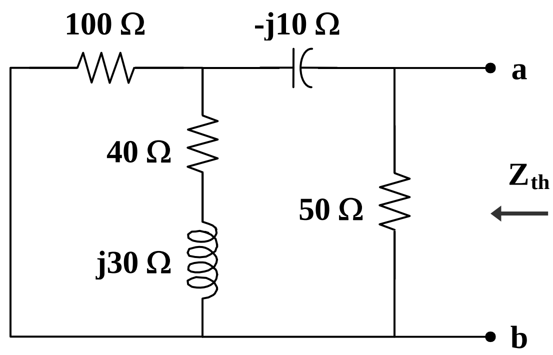
$$150\text{–j}30 = (0.9091 + j5.818)\mathbf{V}_2$$

$$\mathbf{V}_{Th} = \mathbf{V}_2 = \frac{150 - j30}{0.9091 + j5.818} = \frac{152.97\angle -11.31°}{5.889\angle 81.12°} = 25.98\angle -92.43°$$

$$\text{P}_{\text{max}} = \left|\frac{25.98}{1.05 - j6.71 + 6.792}\right|^2 \frac{6.792}{2} = \mathbf{21.51\ W}$$

Chapter 11, Solution 21.

We find $\mathbf{Z}_{Th}$ at terminals a-b, as shown in the figure below.



$$\mathbf{Z}_{Th} = 50 \,||\, [\, -j10 + 100 \,||\, (40 + j30)\,]$$

where $100 \,||\, (40 + j30) = \dfrac{(100)(40 + j30)}{140 + j30} = 31.707 + j14.634$

$$\mathbf{Z}_{Th} = 50 \,||\, (31.707 + j4.634) = \frac{(50)(31.707 + j4.634)}{81.707 + j4.634}$$

$$\mathbf{Z}_{Th} = 19.5 + j1.73$$

$$R_L = \left|\mathbf{Z}_{Th}\right| = \mathbf{19.58\ \Omega}$$

Chapter 11, Solution 22.

i(t) = [2–2cos(2t)] amps

$$I_{rms}^2 = \frac{1}{\pi}\left[\int_0^{\pi}[2-2\cos(2t)]^2 dt\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} 4dt + \int_0^{\pi}[-4\cos(2t)]dt + \int_0^{\pi} 4\cos^2(2t)dt\right]$$

$$= \frac{1}{\pi}\left[4\pi + 0 + 4\int_0^{\pi}\left[\frac{1+\cos(4t)}{2}\right]dt\right] = \frac{1}{\pi}\left[4\pi + 4\left(\frac{\pi}{2}\right)\right] = 6$$

$$I_{rms} = \sqrt{6} = 2.449 \text{ amps}$$

Chapter 11, Solution 23.

Using Fig. 11.54, design a problem to help other students to better understand how to find the rms value of a waveshape.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the rms value of the voltage shown in Fig. 11.54.

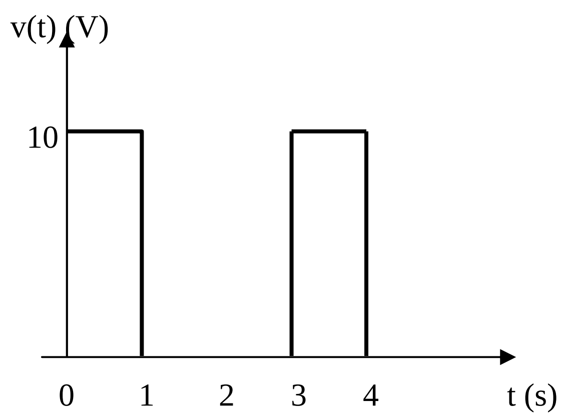


Figure 11.54 For Prob. 11.23.

Solution

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{3}\int_0^1 10^2 dt = \frac{100}{3}$$

V_{rms} = **5.7735 V**

Chapter 11, Solution 24.

$$T = 2, \qquad v(t) = \begin{cases} 5, & 0 < t < 1 \\ -5, & 1 < t < 2 \end{cases}$$

$$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 5^2 \, dt + \int_1^2 (-5)^2 \, dt\right] = \frac{25}{2}[1+1] = 25$$

$$V_{rms} = \mathbf{5\ V}$$

Chapter 11, Solution 25.

$$f_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{3}\left[\int_0^1 (-4)^2 dt + \int_1^2 0dt + \int_2^3 4^2 dt\right]$$

$$= \frac{1}{3}[16 + 0 + 16] = \frac{32}{3}$$

$$f_{rms} = \sqrt{\frac{32}{3}} = \underline{3.266}$$

f_{rms} = **3.266**

Chapter 11, Solution 26.

$$\mathrm{T} = 4, \qquad v(t) = \begin{cases} 5 & 0 < t < 2 \\ 20 & 2 < \mathrm{t} < 4 \end{cases}$$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 10^2\ dt + \int_2^4 (20)^2\ dt\right] = \frac{1}{4}[200 + 800] = 250$$

$$\mathrm{V_{rms}} = \mathbf{15.811\ V}.$$

Chapter 11, Solution 27.

$$T = 5, \qquad i(t) = t, \quad 0 < t < 5$$

$$I_{rms}^2 = \frac{1}{5}\int_0^5 t^2\,dt = \frac{1}{5}\cdot\frac{t^3}{3}\bigg|_0^5 = \frac{125}{15} = 8.333$$

$$I_{rms} = \mathbf{2.887\ A}$$

Chapter 11, Solution 28.

$$V_{rms}^2 = \frac{1}{5}\left[\int_0^2 (4t)^2\ dt + \int_2^5 0^2\ dt\right]$$

$$V_{rms}^2 = \frac{1}{5}\cdot\frac{16\,t^3}{3}\Big|_0^2 = \frac{16}{15}(8) = 8.533$$

$$V_{rms} = \mathbf{2.92\ V}$$

$$P = \frac{V_{rms}^2}{R} = \frac{8.533}{2} = \mathbf{4.267\ W}$$

Chapter 11, Solution 29.

$$T = 20, \qquad i(t) = \begin{cases} 60 - 6t & 5 < t < 15 \\ -120 + 6t & 15 < t < 25 \end{cases}$$

$$I_{eff}^2 = \frac{1}{20}\left[\int_5^{15} (60 - 6t)^2 \, dt + \int_{15}^{25} (-120 + 6t)^2 \, dt \right]$$

$$I_{eff}^2 = \frac{1}{5}\left[\int_5^{15} (900 - 180t + 9t^2) \, dt + \int_{15}^{25} (9t^2 - 360t + 3600) \, dt \right]$$

$$I_{eff}^2 = \frac{1}{5}\left[\left(900t - 90t^2 + 3t^3\right)\Big|_5^{15} + \left(3t^3 - 180t^2 + 3600t\right)\Big|_{15}^{25} \right]$$

$$I_{eff}^2 = \frac{1}{5}[750 + 750] = 300$$

$$I_{eff} = \mathbf{17.321\ A}$$

$P = I_{eff}^2 R = (17.321)^2 x12 =$ **3.6 kW**.

Chapter 11, Solution 30.

$$v(t) = \begin{cases} t & 0 < t < 2 \\ -1 & 2 < t < 4 \end{cases}$$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 t^2\ dt + \int_2^4 (-1)^2\ dt\right] = \frac{1}{4}\left[\frac{8}{3} + 2\right] = 1.1667$$

$$V_{rms} = \mathbf{1.08\ V}$$

Chapter 11, Solution 31.

$$V^2{}_{rms} = \frac{1}{2}\int_0^2 v(t)dt = \frac{1}{2}\left[\int_0^1 (2t)^2\,dt + \int_1^2 (-4)^2\,dt\right] = \frac{1}{2}\left[\frac{4}{3} + 16\right] = 8.6667$$

$$\boldsymbol{V_{rms}} = \underline{\mathbf{2.944\ V}}$$

Chapter 11, Solution 32.

$$I_{rms}^2 = \frac{1}{2}\left[\int_0^1 (10t^2)^2 \, dt + \int_1^2 0 \, dt\right]$$

$$I_{rms}^2 = 50\int_0^1 t^4 \, dt = 50 \cdot \frac{t^5}{5}\Big|_0^1 = 10$$

$$I_{rms} = \mathbf{3.162\ A}$$

Chapter 11, Solution 33.

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2(t)dt = \frac{1}{6}\left[\int_0^1 25t^2 dt + \int_1^3 25dt + \int_3^4 (-5t+20)^2 dt\right]$$

$$I_{rms}^2 = \frac{1}{6}\left[25\frac{t^3}{3}\Bigg|_0^1 + 25(3-1) + (25\frac{t^3}{3} - 100t^2 + 400t)\Bigg|_3^4\right] = 11.1056$$

I_{rms} = **3.332 A**

Chapter 11, Solution 34.

$$f_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{3}\left[\int_0^2 (3t)^2\, dt + \int_2^3 6^2\, dt\right]$$

$$= \frac{1}{3}\left[\frac{9t^3}{3}\Bigg|_0^2 + 36\right] = 20$$

$$f_{rms} = \sqrt{20} = 4.472$$

f_{rms} = **4.472**

Chapter 11, Solution 35.

$$V_{rms}^2 = \frac{1}{6}\left[\int_0^1 10^2\ dt + \int_1^2 20^2\ dt + \int_2^4 30^2\ dt + \int_4^5 20^2\ dt + \int_5^6 10^2\ dt\right]$$

$$V_{rms}^2 = \frac{1}{6}[100 + 400 + 1800 + 400 + 100] = 466.67$$

$$V_{rms} = \mathbf{21.6\ V}$$

Chapter 11, Solution 36.

(a) I_{rms} = $\underline{10\ \text{A}}$

(b) $V^2_{rms} = 4^2 + \left(\frac{3}{\sqrt{2}}\right)^2 \longrightarrow V_{rms} = \sqrt{16 + \frac{9}{2}} = \underline{4.528\ \text{V}}$ (checked)

(c) $I_{rms} = \sqrt{64 + \frac{36}{2}} = \underline{9.055\ \text{A}}$

(d) $V_{rms} = \sqrt{\frac{25}{2} + \frac{16}{2}} = \underline{4.528\ \text{V}}$

Chapter 11, Solution 37.

Design a problem to help other students to better understand how to determine the rms value of the sum of multiple currents.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Calculate the rms value of the sum of these three currents:

$i_1 = 8$, $i_2 = 4\sin(t + 10°)$, $i_3 = 6\cos(2t + 30°)$ A

Solution

$$i = i_1 + i_2 + i_3 = 8 + 4\sin(t + 10^o) + 6\cos(2t + 30^o)$$

$$\boldsymbol{I_{rms} = \sqrt{I^2{}_{1rms} + I^2{}_{2rms} + I^2{}_{3rms}} = \sqrt{64 + \frac{16}{2} + \frac{36}{2}} = \sqrt{90} = \underline{9.487\ \text{A}}}$$

Chapter 11, Solution 38.

$$S_1 = \frac{V^2}{Z_1^*} = \frac{220^2}{124} = 390.32$$

$$S_2 = \frac{V^2}{Z_2^*} = \frac{220^2}{20 + j25} = 944.4 - j1180.5$$

$$S_3 = \frac{V^2}{Z_3^*} = \frac{220^2}{90 - j80} = 300 + j267.03$$

$$S = S_1 + S_2 + S_3 = 1634.7 - j913.47 = 1872.6 < -29.196^o \text{ VA}$$

(a) P = Re(S) = **1634.7 W**

(b) Q = Im (S) = **913.47 VA (leading)**

(c) pf = cos (29.196^o) = **0.8732**

Chapter 11, Solution 39.

(a) $Z_L = 4.2 + j3.6 = 5.5317 \angle 40.6^o$

pf = cos 40.6 = **0.7592**

$$S = \frac{V_{rms}^2}{Z^*} = \frac{220^2}{5.5317\angle -40.6^o} = 6.643 + j5.694 \text{ kVA}$$

P = **6.643 kW**

Q = **5.695 kVAR**

(b) $$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \frac{6.643x10^3(\tan 40.6^o - \tan 0^o)}{2\pi x60x220^2} = \underline{312\ \mu\text{F}},$$

{It is important to note that this capacitor will see a peak voltage of 220$\sqrt{2}$ = 311.08V, this means that the specifications on the capacitor must be at least this or greater!}

Chapter 11, Solution 40.

Design a problem to help other students to better understand apparent power and power factor.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A load consisting of induction motors is drawing 80 kW from a 220-V, 60 Hz power line at a pf of 0.72 lagging. Find the capacitance of a capacitor required to raise the pf to 0.92.

Solution

$$pf1 = 0.72 = \cos\theta_1 \quad \longrightarrow \quad \theta_1 = 43.94^0$$

$$pf2 = 0.92 = \cos\theta_2 \quad \longrightarrow \quad \theta_2 = 23.07^0$$

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \frac{80x10^3(0.9637 - 0.4259)}{2\pi x 60 x (220)^2} = \underline{2.4 \text{ mF}},$$

{Again, we need to note that this capacitor will be exposed to a peak voltage of 311.08V and must be rated to at least this level, preferably higher!}

Chapter 11, Solution 41.

(a) $$-j2 \,\|\, (j5 - j2) = -j2 \,\|\, -j3 = \frac{(-j2)(-j3)}{j} = -j6$$

$$\mathbf{Z}_T = 4 - j6 = 7.211\angle -56.31°$$

pf $= \cos(-56.31°) =$ **0.5547 (leading)**

(b) $$j2 \,\|\, (4 + j) = \frac{(j2)(4 + j)}{4 + j3} = 0.64 + j1.52$$

$$\mathbf{Z} = 1 \,\|\, (0.64 + j1.52 - j) = \frac{0.64 + j0.44}{1.64 + j0.44} = 0.4793\angle 21.5°$$

pf $= \cos(21.5°) =$ **0.9304 (lagging)**

Chapter 11, Solution 42.

(a) S=120, $pf = 0.707 = \cos\theta \longrightarrow \theta = 45^o$

$$S = S\cos\theta + jS\sin\theta = \underline{84.84 + j84.84 \text{ VA}}$$

(b) $S = V_{rms} I_{rms} \longrightarrow I_{rms} = \dfrac{S}{V_{rms}} = \dfrac{120}{110} = \underline{1.091 \text{ A rms}}$

(c) $S = I_{rms}^2 Z \longrightarrow Z = \dfrac{S}{I_{rms}^2} = \underline{71.278 + j71.278\ \Omega}$

(d) If Z = R + jϖL, then R = **71.278 Ω**

$$\omega L = 2\pi fL = 71.278 \longrightarrow L = \frac{71.278}{2\pi x 60} = \underline{0.1891 \text{ H}} = \textbf{189.1 mH}.$$

Chapter 11, Solution 43.

Design a problem to help other students to better understand complex power.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The voltage applied to a 10-ohm resistor is

$$v(t) = 5 + 3\cos(t + 10°) + \cos(2t + 30°)\ \text{V}$$

(a) Calculate the rms value of the voltage.

(b) Determine the average power dissipated in the resistor.

Solution

(a) $V_{rms} = \sqrt{V^2_{1rms} + V^2_{2rms} + V^2_{3rms}} = \sqrt{25 + \frac{9}{2} + \frac{1}{2}} = \sqrt{30} = \underline{5.477\ \text{V}}$

(b) $P = \frac{V^2_{rms}}{R} = 30/10 = \underline{3\ \text{W}}$

Chapter 11, Solution 44.

$$40\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2000x40x10^{-6}} = -j12.5$$

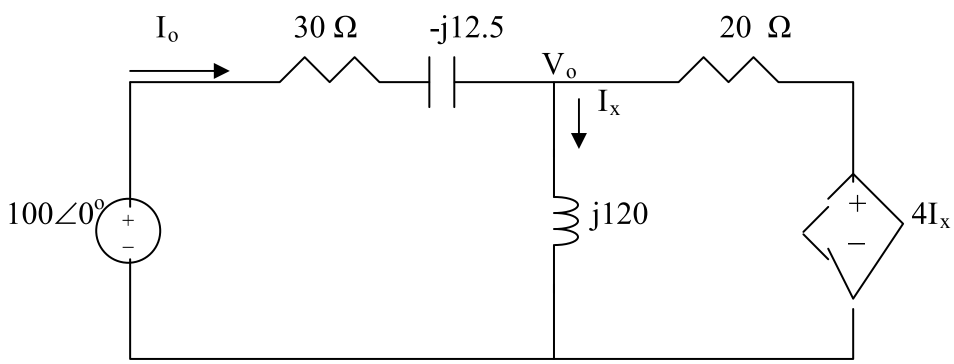
$$60mH \longrightarrow j\omega L = j2000x60x10^{-3} = j120$$

We apply nodal analysis to the circuit shown below.

$$\frac{100 - V_o}{30 - j12.5} + \frac{4I_x - V_o}{20} = \frac{V_o}{j120}$$

But $I_x = \dfrac{V_o}{j120}$. Solving for V_o leads to

$$V_o = 2.9563 + j1.126$$



$$I_o = \frac{100 - V_o}{30 - j12.5} = 2.7696 + j1.1165$$

$$S = \frac{1}{2}V_s I_o^* = \frac{1}{2}(100)(2.7696 - j.1165) = \underline{138.48 - j55.825 \text{ VA}}$$

S = (138.48 – j55.82) VA

Chapter 11, Solution 45.

(a) $V^2_{rms} = 20^2 + \dfrac{60^2}{2} = 2200 \quad \longrightarrow \quad V_{rms} = \underline{46.9\text{ V}}$

$$I_{rms} = \sqrt{1^2 + \frac{0.5^2}{2}} = \sqrt{1.125} = \underline{1.061A}$$

(b) p(t) = v(t)i(t) = 20 + 60cos100t – 10sin100t – 30(sin100t)(cos100t); clearly the average power = **20W**.

Chapter 11, Solution 46.

(a) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (220\angle 30°)(0.5\angle -60°) = 110\angle -30°$

$\mathbf{S} = \mathbf{[95.26 - \mathit{j}55]}\,\boldsymbol{VA}$

Apparent power = **110 VA**
Real power = **95.26 W**
Reactive power = **55 VAR**
pf is **leading** because current leads voltage

(b) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (250\angle -10°)(6.2\angle 25°) = 1550\angle 15°$

$\mathbf{S} = \mathbf{[497.2 + \mathit{j}401.2]}\,\boldsymbol{VA}$

Apparent power = **1550 VA**
Real power = **1497.2 W**
Reactive power = **401.2 VAR**
pf is **lagging** because current lags voltage

(c) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (120\angle 0°)(2.4\angle 15°) = 288\angle 15°$

$\mathbf{S} = \mathbf{[278.2 + \mathit{j}74.54]}\,\boldsymbol{VA}$

Apparent power = **288 VA**
Real power = **278.2 W**
Reactive power = **74.54 VAR**
pf is **lagging** because current lags voltage

(d) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (160\angle 45°)(8.5\angle -90°) = 1360\angle -45°$

S = [961.7 – j961.7] VA

Apparent power = **1360 VA**
Real power = **961.7 W**
Reactive power = **- 961.7 VAR**
pf is **leading** because current leads voltage

Chapter 11, Solution 47.

(a) $\mathbf{V} = 112\angle 10°$, $\mathbf{I} = 4\angle -50°$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\,\mathbf{I}^* = 224\angle 60° = \mathbf{[112 + \mathit{j}194]}\,\boldsymbol{VA}$$

Average power = **112 W**

Reactive power = **194 VAR**

(b) $\mathbf{V} = 160\angle 0°$, $\mathbf{I} = 4\angle 45°$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\,\mathbf{I}^* = 320\angle -45° = \mathbf{226.3 - j226.3}$$

Average power = **226.3 W**
Reactive power = **–226.3 VAR**

(c) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(80)^2}{50\angle -30°} = 128\angle 30° = \mathbf{110.85 + j64}$$

Average power = **110.85 W**
Reactive power = **64 VAR**

(d) $$\mathbf{S} = |\mathbf{I}|^2\,\mathbf{Z} = (100)(100\angle 45°) = \mathbf{[7.071 + \mathit{j}7.071]}\,\boldsymbol{kVA}$$

Average power = **7.071 kW**

Reactive power = **7.071 kVAR**

Chapter 11, Solution 48.

(a) $\mathbf{S} = \mathrm{P} - \mathrm{jQ} = \mathbf{[269 - \mathit{j}150]}\,\boldsymbol{VA}$

(b) $\mathrm{pf} = \cos\theta = 0.9 \longrightarrow \theta = 25.84°$

$$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{2000}{\sin(25.84°)} = 4588.31$$

$$P = S\cos\theta = 4129.48$$

$\mathbf{S} = \mathbf{[4.129 - \mathit{j}2]}\,\boldsymbol{kVA}$

(c) $Q = S\sin\theta \longrightarrow \sin\theta = \dfrac{Q}{S} = \dfrac{450}{600} = 0.75$

$\theta = 48.59$, $\quad \mathrm{pf} = 0.6614$

$$P = S\cos\theta = (600)(0.6614) = 396.86$$

$\mathbf{S} = \mathbf{[396.9 + \mathit{j}450]}\,\boldsymbol{VA}$

(d) $S = \dfrac{|\mathbf{V}|^2}{|\mathbf{Z}|} = \dfrac{(220)^2}{40} = 1210$

$P = S\cos\theta \longrightarrow \cos\theta = \dfrac{P}{S} = \dfrac{1000}{1210} = 0.8264$

$\theta = 34.26°$

$$Q = S\sin\theta = 681.25$$

$\mathbf{S} = \mathbf{[1 + \mathit{j}0.6812]}\,\boldsymbol{kVA}$

Chapter 11, Solution 49.

(a) $\mathbf{S} = 4 + j\dfrac{4}{0.86}\sin(\cos^{-1}(0.86))$ kVA

$\mathbf{S} = \mathbf{[4 + \textit{j}2.373]}\ \boldsymbol{kVA}$

(b) $\text{pf} = \dfrac{P}{S} = \dfrac{1.6}{2}0.8 = \cos\theta \longrightarrow \sin\theta = 0.6$

$\mathbf{S} = 1.6 - j2\sin\theta = \mathbf{[1.6 - \textit{j}1.2]}\ \boldsymbol{kVA}$

(c) $\mathbf{S} = \mathbf{V}_{rms}\,\mathbf{I}^*_{rms} = (208\angle 20°)(6.5\angle 50°)$ VA

$\mathbf{S} = 1.352\angle 70° = \mathbf{[0.4624 + \textit{j}1.2705]}\ \boldsymbol{kVA}$

(d) $\mathbf{S} = \dfrac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \dfrac{(120)^2}{40 - j60} = \dfrac{14400}{72.11\angle -56.31°}$

$\mathbf{S} = 199.7\angle 56.31° = \mathbf{[110.77 + \textit{j}166.16]}\ \boldsymbol{VA}$

Chapter 11, Solution 50.

(a) $\mathbf{S} = \mathrm{P} - \mathrm{jQ} = 1000 - \mathrm{j}\dfrac{1000}{0.8}\sin(\cos^{-1}(0.8))$

$\mathbf{S} = 1000 - \mathrm{j}750$

But, $\mathbf{S} = \dfrac{\left|\mathbf{V}_{\mathrm{rms}}\right|^2}{\mathbf{Z}^*}$

$\mathbf{Z}^* = \dfrac{\left|\mathbf{V}_{\mathrm{rms}}\right|^2}{\mathbf{S}} = \dfrac{(220)^2}{1000 - \mathrm{j}750} = 30.98 + \mathrm{j}23.23$

$\mathbf{Z} = \mathbf{[30.98 - \textit{j}23.23]\,\Omega}$

(b) $\mathbf{S} = \left|\mathbf{I}_{\mathrm{rms}}\right|^2 \mathbf{Z}$

$\mathbf{Z} = \dfrac{\mathbf{S}}{\left|\mathbf{I}_{\mathrm{rms}}\right|^2} = \dfrac{1500 + \mathrm{j}2000}{(12)^2} = \mathbf{[10.42 + \textit{j}13.89]\,\Omega}$

(c) $\mathbf{Z}^* = \dfrac{\left|\mathbf{V}_{\mathrm{rms}}\right|^2}{\mathbf{S}} = \dfrac{\left|\mathbf{V}\right|^2}{2\mathbf{S}} = \dfrac{(120)^2}{(2)(4500\angle 60°)} = 1.6\angle -60°$

$\mathbf{Z} = 1.6\angle 60° = \mathbf{[0.8 + \textit{j}1.386]\,\Omega}$

Chapter 11, Solution 51.

(a) $\mathbf{Z}_T = 2 + (10 - j5) \,||\, (8 + j6)$

$$\mathbf{Z}_T = 2 + \frac{(10 - j5)(8 + j6)}{18 + j} = 2 + \frac{110 + j20}{18 + j}$$

$$\mathbf{Z}_T = 8.152 + j0.768 = 8.188\angle 5.382°$$

pf $= \cos(5.382°) =$ **0.9956 (lagging)**

(b) $$\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(16)^2}{(8.188\angle -5.382°)}$$

$$\mathbf{S} = 31.26\angle 5.382°$$

$P = S\cos\theta =$ **31.12 W**

(c) $Q = S\sin\theta =$ **2.932 VAR**

(d) $S = |\mathbf{S}| =$ **31.26 VA**

(e) $\mathbf{S} = 31.26\angle 5.382° =$ **(31.12+j2.932) VA**

(a) 0.9956 (lagging, (b) 31.12 W, (c) 2.932 VAR, (d) 31.26 VA, (e) [31.12+j2.932] VA

Chapter 11, Solution 52.

$$S_A = 2000 + j\frac{2000}{0.8}0.6 = 2000 + j1500$$
$$S_B = 3000x0.4 - j3000x0.9165 = 1200 - j2749$$
$$S_C = 1000 + j500$$
$$S = S_A + S_B + S_C = 4200 - j749$$

(a) $pf = \dfrac{4200}{\sqrt{4200^2 + 749^2}} =$ **0.9845 leading**

(b) $S = V_{rms}I_{rms}^* \longrightarrow I_{rms}^* = \dfrac{4200 - j749}{120\angle 45°} = 35.55\angle -55.11°$

I_{rms} = **$35.55\angle 55.11°$ A**.

Chapter 11, Solution 53.

$$S = S_A + S_B + S_C = 4000(0.8 - j0.6) + 2400(0.6 + j0.8) + 1000 + j500$$

$$= 5640 + j20 = 5640\angle 0.2^\circ$$

(a) $$\boldsymbol{I_{rms}^* = \frac{S_B}{V_{rms}} + \frac{S_A + S_C}{V_{rms}} = \frac{S}{V_{rms}} = \frac{5640\angle 0.2^\circ}{120\angle 30^\circ} = 47\angle -29.8^\circ}$$

$$\boldsymbol{I = 47\angle 29.8^\circ = \underline{47\angle 29.8^\circ\ A}}$$

(b) pf = $\cos(0.2^\circ)$ **≈ <u>1.0 lagging</u>**.

Chapter 11, Solution 54.

Consider the circuit shown below.

$$\mathbf{I}_1 = \frac{8\angle -20°}{4-j3} = 1.6\angle 16.87°$$

$$\mathbf{I}_2 = \frac{8\angle -20°}{j5} = 1.6\angle -110°$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = (-0.5472 - j1.504) + (1.531 + j0.4643)$$

$$\mathbf{I} = 0.9839 - j1.04 = 1.432\angle -46.58°$$

For the source,

$$\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (8\angle -20°)(1.432\angle 46.58°)$$

$$\mathbf{S} = 11.456\angle 26.58° = \textbf{(10.24+j3.12) VA}$$

For the capacitor,

$$\mathbf{S} = |\mathbf{I}_1|^2\,\mathbf{Z}_c = (1.6)^2(-j3) = \mathbf{-j7.68\ VA}$$

For the resistor,

$$\mathbf{S} = |\mathbf{I}_1|^2\,\mathbf{Z}_R = (1.6)^2(4) = \mathbf{10.24\ VA}$$

For the inductor,

$$\mathbf{S} = |\mathbf{I}_2|^2\,\mathbf{Z}_L = (1.6)^2(j5) = \mathbf{j12.8\ VA}$$

Chapter 11, Solution 55.

Using Fig. 11.74, design a problem to help other students to better understand the conservation of AC power.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the complex power absorbed by each of the five elements in the circuit of Fig. 11.74.

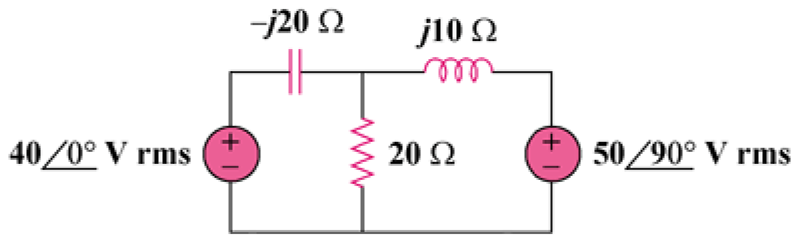
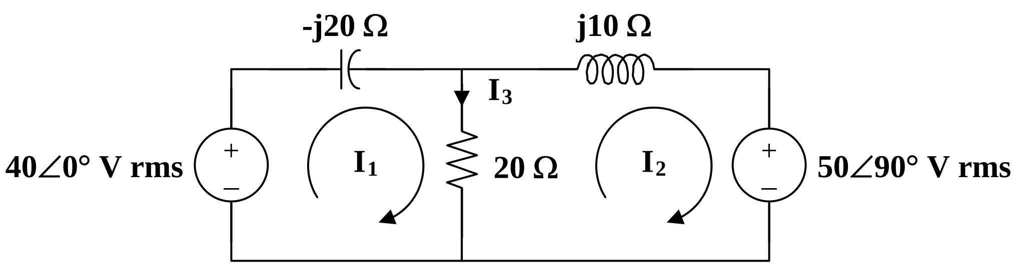


Figure 11.74

Solution

We apply mesh analysis to the following circuit.



For mesh 1,

$$40 = (20 - j20)\,I_1 - 20\,I_2$$

$$2 = (1 - j)\,I_1 - I_2 \qquad (1)$$

For mesh 2,

$$-j50 = (20 + j10)\,I_2 - 20\,I_1$$

$$-j5 = -2\,I_1 + (2 + j)\,I_2 \qquad (2)$$

Putting (1) and (2) in matrix form,

$$\begin{bmatrix} 2 \\ -j5 \end{bmatrix} = \begin{bmatrix} 1-j & -1 \\ -2 & 2+j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 1 - j, \qquad \Delta_1 = 4 - j3, \qquad \Delta_2 = -1 - j5$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4 - j3}{1 - j} = \frac{1}{2}(7 + j) = 3.535\angle 8.13°$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-1 - j5}{1 - j} = 2 - j3 = 3.605\angle -56.31°$$

$$I_3 = I_1 - I_2 = (3.5 + j0.5) - (2 - j3) = 1.5 + j3.5 = 3.808\angle 66.8°$$

For the 40-V source,

$$\mathbf{S} = -\mathbf{V}\,\mathbf{I}_1^* = -(40)\left(\frac{1}{2}\cdot(7 - j)\right) = \mathbf{[-140 + j20]\ VA}$$

For the capacitor,

$$\mathbf{S} = \left|\mathbf{I}_1\right|^2 \mathbf{Z}_c = \mathbf{-\ j250\ VA}$$

For the resistor,

$$\mathbf{S} = \left|\mathbf{I}_3\right|^2 R = \mathbf{290\ VA}$$

For the inductor,

$$\mathbf{S} = \left|\mathbf{I}_2\right|^2 \mathbf{Z}_L = \mathbf{j130\ VA}$$

For the j50-V source,

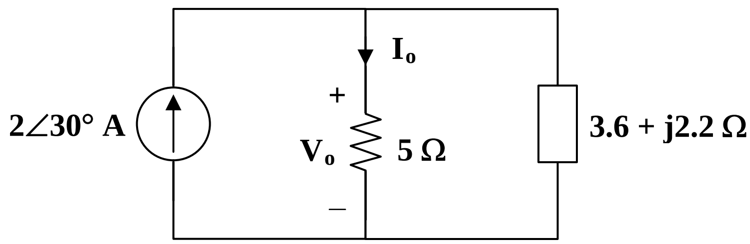
$$\mathbf{S} = \mathbf{V}\,\mathbf{I}_2^* = (j50)(2 + j3) = \mathbf{[-150 + j100]\ VA}$$

Chapter 11, Solution 56.

$$-j2 \,||\, 6 = \frac{(6)(-j2)}{6-j2} = \frac{12\angle-90°}{6.32456\angle-18.435°} = 1.897365\angle-71.565° = 0.6-j1.8$$

$$3+j4+[(-j2)\,||\,6] = 3.6+\mathrm{j}2.2$$

The circuit is reduced to that shown below.



$$\mathbf{I}_o = \frac{3.6+j2.2}{8.6+j2.2}(2\angle30°) = \frac{4.219\angle31.4296°}{8.87694\angle14.3493°}(2\angle30°) = 0.95055\angle47.08°$$

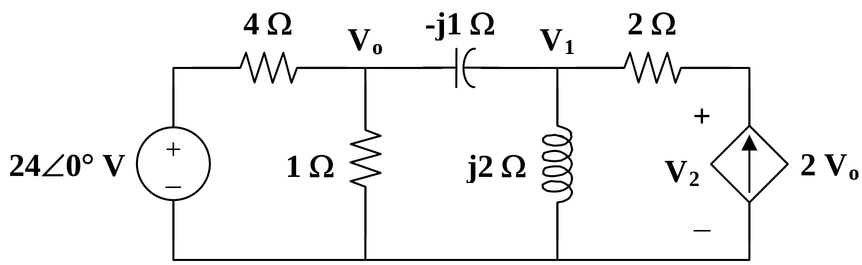
$$\mathbf{V}_o = 5\mathbf{I}_o = 4.75275\angle47.08°$$

$$\mathbf{S} = \mathbf{V}_o\mathbf{I}_s^* = (4.75275\angle47.08°)(2\angle-30°)$$

$$\mathbf{S} = 9.5055\angle17.08° = \mathbf{(9.086+j2.792)\ VA}$$

Chapter 11, Solution 57.

Consider the circuit as shown below.



At node o,

$$\frac{24 - \mathbf{V}_o}{4} = \frac{\mathbf{V}_o}{1} + \frac{\mathbf{V}_o - \mathbf{V}_1}{-j}$$

$$24 = (5 + j4)\,\mathbf{V}_o - j4\,\mathbf{V}_1 \qquad (1)$$

At node 1,

$$\frac{\mathbf{V}_o - \mathbf{V}_1}{-j} + 2\,\mathbf{V}_o = \frac{\mathbf{V}_1}{j2}$$

$$\mathbf{V}_1 = (2 - j4)\,\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1),

$$24 = (5 + j4 - j8 - 16)\,\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{-24}{11 + j4}, \qquad \mathbf{V}_1 = \frac{(-24)(2 - j4)}{11 + j4}$$

The voltage across the dependent source is

$$\mathbf{V}_2 = \mathbf{V}_1 + (2)(2\,\mathbf{V}_o) = \mathbf{V}_1 + 4\,\mathbf{V}_o$$

$$\mathbf{V}_2 = \frac{-24}{11 + j4} \cdot (2 - j4 + 4) = \frac{(-24)(6 - j4)}{11 + j4}$$

$$\mathbf{S} = \mathbf{V}_2\,\mathbf{I}^* = \mathbf{V}_2\,(2\,\mathbf{V}_o^*)$$

$$\mathbf{S} = \frac{(-24)(6 - j4)}{11 + j4} \cdot \frac{-48}{11 - j4} = \left(\frac{1152}{137}\right)(6 - j4)$$

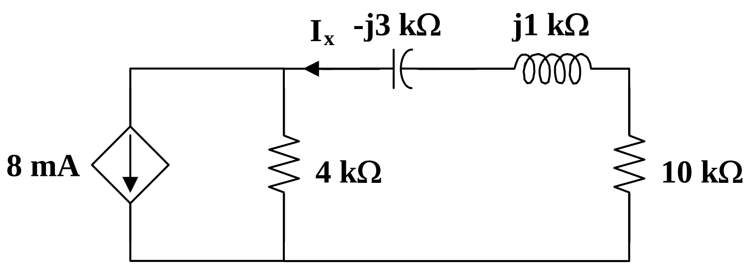
$$\mathbf{S} = \mathbf{(50.45 – j33.64)\ VA}$$

Chapter 11, Solution 58.

From the left portion of the circuit,

$$\mathbf{I}_o = \frac{0.2}{500} = 0.4 \text{ mA}$$

$20\mathbf{I}_o = 8$ mA which then leads to the following circuit,



From the right portion of the circuit,

$$\mathbf{I}_x = \frac{4}{4+10+j-j3}(8 \text{ mA}) = \frac{16}{7-j} \text{ mA}$$

$$\mathbf{S} = \left|\mathbf{I}_x\right|^2 R = \frac{(16\times 10^{-3})^2}{50} \cdot (10\times 10^3)$$

$$\mathbf{S} = \mathbf{51.2\ mVA}$$

It should be noted that the complex power delivered to a resistor is always watts.

Chapter 11, Solution 59.

Let $\mathbf{V}_o$ represent the voltage across the current source and then apply nodal analysis to the circuit and we get:

$$4 + \frac{240 - \mathbf{V}_o}{50} = \frac{\mathbf{V}_o}{-j20} + \frac{\mathbf{V}_o}{40 + j30}$$

$$88 = (0.36 + j0.38)\,\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{88}{0.36 + j0.38} = 168.13\angle -46.55°$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{-j20} = 8.41\angle 43.45°$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o}{40 + j30} = 3.363\angle -83.42°$$

Reactive power in the inductor is

$$\mathbf{S} = |\,\mathbf{I}_2\,|^2\,\mathbf{Z}_L = (3.363)^2(j30) = \mathbf{j339.3\ VAR}$$

Reactive power in the capacitor is

$$\mathbf{S} = |\,\mathbf{I}_1\,|^2\,\mathbf{Z}_c = (8.41)^2(-j20) = \mathbf{-j1.4146\ kVAR}$$

Chapter 11, Solution 60.

$$S_1 = 20 + j\frac{20}{0.8}\sin(\cos^{-1}(0.8)) = 20 + j15$$

$$S_2 = 16 + j\frac{16}{0.9}\sin(\cos^{-1}(0.9)) = 16 + j7.749$$

$$S = S_1 + S_2 = 36 + j22.749 = 42.585\angle 32.29°$$

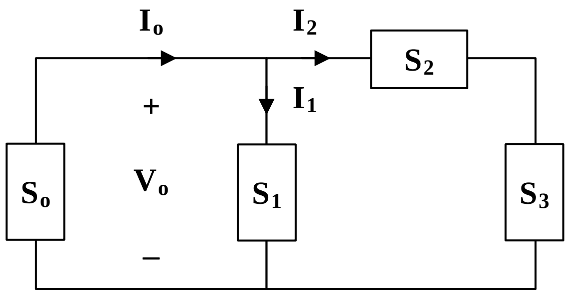
But $\quad S = V_o I^* = 6 V_o$

$$V_o = \frac{S}{6} = \mathbf{7.098\angle 32.29°}$$

$$pf = \cos(32.29°) = \mathbf{0.8454} \quad \textbf{(lagging)}$$

Chapter 11, Solution 61.

Consider the network shown below.



$$\mathbf{S}_2 = 1.2 - j0.8 \text{ kVA}$$

$$\mathbf{S}_3 = 4 + j\frac{4}{0.9}\sin(\cos^{-1}(0.9)) = 4 + j1.937 \text{ kVA}$$

Let $\mathbf{S}_4 = \mathbf{S}_2 + \mathbf{S}_3 = 5.2 + j1.137 \text{ kVA}$

But $\mathbf{S}_4 = \mathbf{V}_o \mathbf{I}_2^*$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_4}{\mathbf{V}_o} = \frac{(5.2 + j1.137)\times 10^3}{100\angle 90°} = 11.37 - j52$$

$$\mathbf{I}_2 = 11.37 + j52$$

Similarly, $\mathbf{S}_1 = \sqrt{2} - j\frac{\sqrt{2}}{0.707}\sin(\cos^{-1}(0.707)) = \sqrt{2}(1 - j) \text{ kVA}$

But $\mathbf{S}_1 = \mathbf{V}_o \mathbf{I}_1^*$

$$\mathbf{I}_1^* = \frac{\mathbf{S}_1}{V_o} = \frac{(1.4142 - j1.4142)\times 10^3}{j100} = -14.142 - j14.142$$

$$\mathbf{I}_1 = -14.142 + j14.142$$

$$\mathbf{I}_o = \mathbf{I}_1 + \mathbf{I}_2 = -2.772 + j66.14 = \mathbf{66.2\angle 92.4°\ A}$$

$$\mathbf{S}_o = V_o I_o^*$$

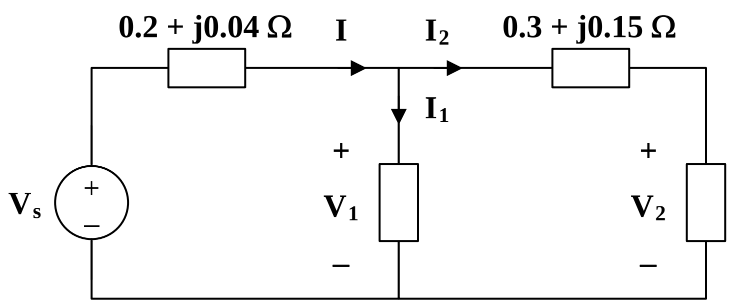
$$\mathbf{S}_o = (100\angle 90°)(66.2\angle -92.4°)\ VA$$

$$\mathbf{S}_o = \mathbf{6.62\angle -2.4°\ kVA}$$

66.2∠92.4° A, 6.62∠–2.4° kVA

Chapter 11, Solution 62.

Consider the circuit below.



$$\mathbf{S}_2 = 15 - j\frac{15}{0.8}\sin(\cos^{-1}(0.8)) = 15 - j11.25$$

But $\mathbf{S}_2 = \mathbf{V}_2 \mathbf{I}_2^*$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_2}{\mathbf{V}_2} = \frac{15 - j11.25}{120}$$

$$\mathbf{I}_2 = 0.125 + j0.09375$$

$$\mathbf{V}_1 = \mathbf{V}_2 + \mathbf{I}_2(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120 + (0.125 + j0.09375)(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120.02 + j0.0469$$

$$\mathbf{S}_1 = 10 + j\frac{10}{0.9}\sin(\cos^{-1}(0.9)) = 10 + j4.843$$

But $\mathbf{S}_1 = \mathbf{V}_1 \mathbf{I}_1^*$

$$\mathbf{I}_1^* = \frac{\mathbf{S}_1}{\mathbf{V}_1} = \frac{11.111\angle 25.84°}{120.02\angle 0.02°}$$

$$\mathbf{I}_1 = 0.093\angle -25.82° = 0.0837 - j0.0405$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.2087 + j0.053$$

$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{I}(0.2 + j0.04)$$

$$\mathbf{V}_s = (120.02 + j0.0469) + (0.2087 + j0.053)(0.2 + j0.04)$$

$$\mathbf{V}_s = 120.06 + j0.0658$$

$$\mathbf{V}_s = \mathbf{120.06\angle 0.03°\ V}$$

Chapter 11, Solution 63.

Let $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$.

$$\mathbf{S}_1 = 12 - \mathrm{j}\frac{12}{0.866}\sin(\cos^{-1}(0.866)) = 12 - \mathrm{j}6.929$$

$$\mathbf{S}_2 = 16 + \mathrm{j}\frac{16}{0.85}\sin(\cos^{-1}(0.85)) = 16 + \mathrm{j}9.916$$

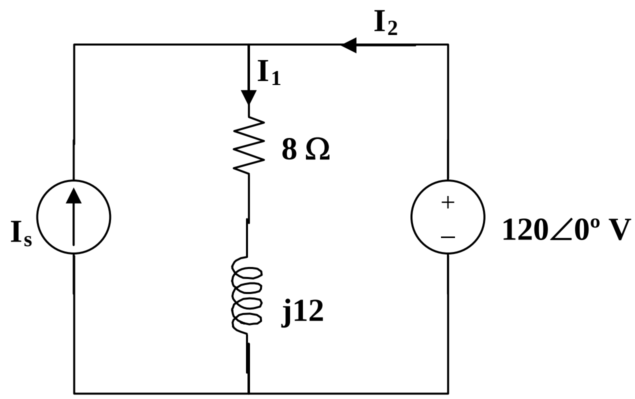
$$\mathbf{S}_3 = \frac{(20)(0.6)}{\sin(\cos^{-1}(0.6))} + j20 = 15 + j20$$

$$\mathbf{S} = 43 + j22.987 = \mathbf{V}\,\mathbf{I}_o^*$$

$$\mathbf{I}_o^* = \frac{\mathbf{S}}{\mathbf{V}} = \frac{(43 + j22.99)x10^3}{220} = 195.45 + j104.5 = 221.6\angle 28.13°$$

$$\mathbf{I}_o = \mathbf{221.6\angle{-28.13}°\ A}$$

Chapter 11, Solution 64.



$I_s + I_2 = I_1$ or $I_s = I_1 - I_2$

$$I_1 = \frac{120}{8 + j12} = 4.615 - j6.923$$

But, $$S = VI_2^* \longrightarrow I_2^* = \frac{S}{V} = \frac{2500 - j400}{120} = 20.83 - j3.333$$

$$\text{or } I_2 = 20.83 + j3.333$$

$I_s = I_1 - I_2 = -16.22 - j10.256 =$ **$19.19\angle{-147.69^\circ}$ A**.

Chapter 11, Solution 65.

$$C = 1\,\text{nF} \longrightarrow \frac{1}{j\omega C} = \frac{-j}{10^4 \times 10^{-9}} = \text{-j100 k}\Omega$$

At the noninverting terminal,

$$\frac{4\angle 0° - \mathbf{V}_o}{100} = \frac{\mathbf{V}_o}{-j100} \longrightarrow \mathbf{V}_o = \frac{4}{1+j}$$

$$\mathbf{V}_o = \frac{4}{\sqrt{2}}\angle -45°$$

$$v_o(t) = \frac{4}{\sqrt{2}}\cos(10^4 t - 45°)$$

$$P = \frac{V_{rms}^2}{R} = \left(\frac{4}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\right)^2 \left(\frac{1}{50 \times 10^3}\right) \text{W}$$

$P =$ **80 μW**

Chapter 11, Solution 66.

As an inverter,

$$\mathbf{V}_o = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}\mathbf{V}_s = \frac{-(2+j4)}{4+j3}\cdot(4\angle 45°)$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{6-j2}\ \text{mA} = \frac{-(2+j4)(4\angle 45°)}{(6-j2)(4+j3)}\ \text{mA}$$

The power absorbed by the 6-kΩ resistor is

$$P = |\mathbf{I}_o|^2 R = \left(\frac{\sqrt{20}\times 4}{\sqrt{40}\times 5}\right)^2 \times 10^{-6}\times 6\times 10^3$$

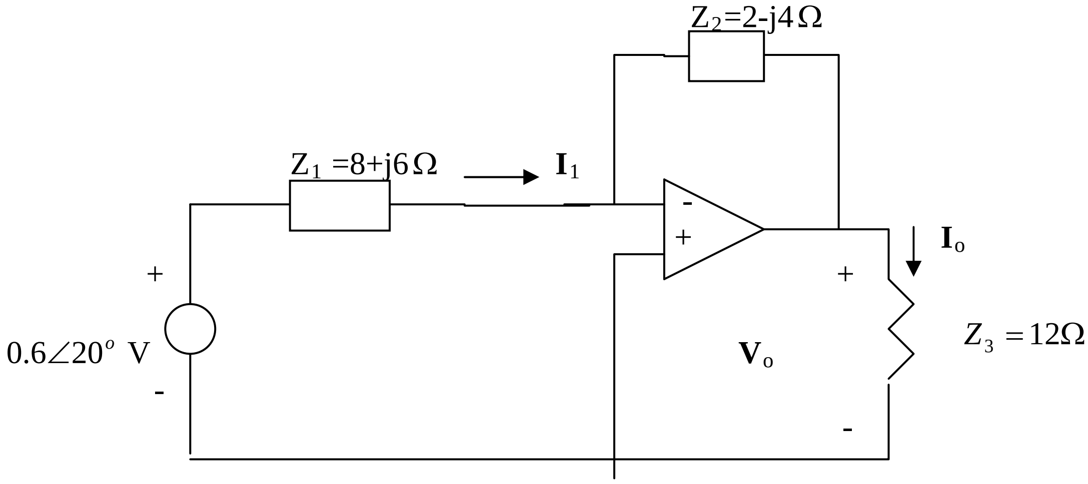
$$\text{P} = \mathbf{1.92\ mW}$$

Chapter 11, Solution 67.

$$\omega = 2, \quad 3H \longrightarrow \quad j\omega L = j6, \quad 0.1F \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j2x0.1} = -j5$$

$$10//(-j5) = \frac{-j50}{10-j5} = 2-j4$$

The frequency-domain version of the circuit is shown below.



(a) $I_1 = \dfrac{0.6\angle 20^o - 0}{8+j6} = \dfrac{0.5638 + j0.2052}{8+j6} = 0.06\angle -16.87^o$

$S = \dfrac{1}{2} V_s I^*{}_1 = (0.3\angle 20^o)(0.06\angle +16.87^o) = \underline{14.4 + j10.8 \text{ mVA}} = \underline{18\angle 36.86^o \text{ mVA}}$

S = (14.4+j10.8) mVA = 18∠36.86° mVA

(b) $V_o = -\dfrac{Z_2}{Z_1} V_s, \qquad I_o = \dfrac{V_o}{Z_3} = -\dfrac{(2-j4)}{12(8+j6)}(0.6\angle 20^o) = 0.0224\angle 99.7^o$

$P = \dfrac{1}{2} |I_o|^2 R = 0.5(0.0224)^2(12) = \underline{2.904 \text{ mW}}$

P = 2.904 mW

(a) 18∠36.86° mVA, (b) 2.904 mW

Chapter 11, Solution 68.

Let $\mathbf{S} = \mathbf{S}_R + \mathbf{S}_L + \mathbf{S}_c$

where

$$\mathbf{S}_R = P_R + jQ_R = \frac{1}{2} I_o^2 R + j0$$

$$\mathbf{S}_L = P_L + jQ_L = 0 + j\frac{1}{2} I_o^2 \omega L$$

$$\mathbf{S}_c = P_c + jQ_c = 0 - j\frac{1}{2} I_o^2 \cdot \frac{1}{\omega C}$$

Hence,

$$\mathbf{S} = \frac{\mathbf{1}}{\mathbf{2}} \mathbf{I}_o^2 \left[\mathbf{R} + \mathbf{j}\left(\omega \mathbf{L} - \frac{\mathbf{1}}{\omega \mathbf{C}}\right)\right]$$

Chapter 11, Solution 69.

(a) Given that $\mathbf{Z} = 10 + j12$

$$\tan\theta = \frac{12}{10} \longrightarrow \theta = 50.19°$$

$$\text{pf} = \cos\theta = \mathbf{0.6402}$$

(b) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{2\mathbf{Z}^*} = \frac{(120)^2}{(2)(10 - j12)} = 295.12 + j354.09$$

The average power absorbed $= P = \text{Re}(\mathbf{S}) = \mathbf{295.1\ W}$

(c) For unity power factor, $\theta_1 = 0°$, which implies that the reactive power due to the capacitor is $Q_c = 354.09$

But $$Q_c = \frac{V^2}{2X_c} = \frac{1}{2}\omega C V^2$$

$$C = \frac{2Q_c}{\omega V^2} = \frac{(2)(354.09)}{(2\pi)(60)(120)^2} = \mathbf{130.4\ \mu F}$$

Chapter 11, Solution 70.

Design a problem to help other students to better understand power factor correction.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

An 880-VA, 220-V, 50-Hz load has a power factor of 0.8 lagging. What value of parallel capacitance will correct the load power factor to unity?

Solution

$$\text{pf} = \cos\theta = 0.8 \quad \longrightarrow \quad \sin\theta = 0.6$$
$$Q = S\sin\theta = (880)(0.6) = 528$$

If the power factor is to be unity, the reactive power due to the capacitor is

$$Q_c = Q = 528 \text{ VAR}$$

$$\text{But} \qquad Q = \frac{V_{rms}^2}{X_c} = \omega C V^2 \quad \longrightarrow \quad C = \frac{Q_c}{\omega V^2}$$

$$C = \frac{(528)}{(2\pi)(50)(220)^2} = \mathbf{34.72\ \mu F}$$

Chapter 11, Solution 71.

(a) For load 1,

$Q_1 = 60$ kVAR, pf = 0.85 or $\theta_1 = 31.79^\circ$

$Q_1 = S_1 \sin\theta_1 = 60k$ or $S_1 = 113.89k$ and $P_1 = 113.89\cos(31.79) = 96.8kW$

$\mathbf{S}_1 = 96.8 + j60$ kVA

For load 2, $\mathbf{S}_2 = 90 - j50$ kVA

For load 3, $\mathbf{S}_3 = 100$ kVA

Hence,

$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \mathbf{S_3} = 286.8 + j10\text{kVA} = 287\angle 2^\circ\text{kVA}$

But $\mathbf{S} = (V_{rms})^2/Z^*$ or $Z^* = 120^2/287\angle 2^\circ k = 0.05017\angle{-2^\circ}$

Thus, $Z = \mathbf{0.05017\angle 2^\circ\Omega}$ or $\mathbf{[50.14 + j1.7509]\ m\Omega}$.

(b) From above, pf = $\cos 2^\circ$ = **0.9994**.

(c) $I_{rms} = V_{rms}/Z = 120/0.05017\angle 2^\circ = \mathbf{2.392\angle{-2^\circ}\ kA}$ or $\mathbf{[2.391 - j0.08348]\ kA}$.

Chapter 11, Solution 72.

(a) $P = S\cos\theta_1 \longrightarrow S = \dfrac{P}{\cos\theta_1} = \dfrac{2.4}{0.8} = 3.0 \text{ kVA}$

$pf = 0.8 = \cos\theta_1 \longrightarrow \theta_1 = 36.87^o$

$Q = S\sin\theta_1 = 3.0\sin 36.87^o = 1.8 \text{ kVAR}$

Hence, S = 2.4 + j1.8 kVA

$S_1 = \dfrac{P_1}{\cos\theta} = \dfrac{1.5}{0.707} = 2.122 \text{ kVA}$

$pf = 0.707 = \cos\theta \longrightarrow \theta = 45^o$

$Q_1 = P_1 = 1.5 \text{ kVAR} \longrightarrow \mathrm{S}_1 = 1.5 + j1.5 \text{ kVA}$

Since, $S = S_1 + S_2 \longrightarrow S_2 = S - S_1 = (2.4 + j1.8) - (1.5 + j1.5) = 0.9 + j0.3 \text{ kVA}$

$S_2 = 0.9497 < 18.43^o$

pf = cos 18.43^o = <u>0.9487</u>

(b) $pf = 0.9 = \cos\theta_2 \longrightarrow \theta_2 = 25.84^o$

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \frac{2400(\tan 36.87 - \tan 25.84)}{2\pi x 60 x (120)^2} = \underline{117.5\ \mu\text{F}}$$

Chapter 11, Solution 73.

(a) $\mathbf{S} = 10 - j15 + j22 = 10 + j7\ \text{kVA}$

$S = |\mathbf{S}| = \sqrt{10^2 + 7^2} = \mathbf{12.21\ kVA}$

(b) $\mathbf{S} = \mathbf{V}\mathbf{I}^* \longrightarrow \mathbf{I}^* = \dfrac{\mathbf{S}}{\mathbf{V}} = \dfrac{10{,}000 + j7{,}000}{240}$

$\mathbf{I} = 41.667 - j29.167 = \mathbf{50.86\angle -35^\circ\ A}$

(c) $\theta_1 = \tan^{-1}\left(\dfrac{7}{10}\right) = 35^\circ$, $\qquad \theta_2 = \cos^{-1}(0.96) = 16.26^\circ$

$Q_c = P_1[\tan\theta_1 - \tan\theta_2] = 10[\tan(35^\circ) - \tan(16.26^\circ)]$

$Q_c = \mathbf{4.083\ kVAR}$

$C = \dfrac{Q_c}{\omega V_{rms}^2} = \dfrac{4083}{(2\pi)(60)(240)^2} = \mathbf{188.03\ \mu F}$

(d) $\mathbf{S}_2 = P_2 + jQ_2$, $\qquad P_2 = P_1 = 10\ \text{kW}$

$Q_2 = Q_1 - Q_c = 7 - 4.083 = 2.917\ \text{kVAR}$

$\mathbf{S}_2 = 10 + j2.917\ \text{kVA}$

But $\quad \mathbf{S}_2 = \mathbf{V}\mathbf{I}_2^*$

$\mathbf{I}_2^* = \dfrac{\mathbf{S}_2}{\mathbf{V}} = \dfrac{10{,}000 + j2917}{240}$

$\mathbf{I}_2 = 41.667 - j12.154 = \mathbf{43.4\angle -16.26^\circ\ A}$

Chapter 11, Solution 74.

(a) $\theta_1 = \cos^{-1}(0.8) = 36.87°$

$$S_1 = \frac{P_1}{\cos\theta_1} = \frac{24}{0.8} = 30 \text{ kVA}$$

$$Q_1 = S_1 \sin\theta_1 = (30)(0.6) = 18 \text{ kVAR}$$

$$\mathbf{S}_1 = 24 + j18 \text{ kVA}$$

$$\theta_2 = \cos^{-1}(0.95) = 18.19°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{40}{0.95} = 42.105 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 13.144 \text{ kVAR}$$

$$\mathbf{S}_2 = 40 + j13.144 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 64 + j31.144 \text{ kVA}$$

$$\theta = \tan^{-1}\left(\frac{31.144}{64}\right) = 25.95°$$

$$\text{pf} = \cos\theta = \mathbf{0.8992}$$

(b) $\theta_2 = 25.95°$, $\qquad \theta_1 = 0°$

$$Q_c = P[\tan\theta_2 - \tan\theta_1] = 64[\tan(25.95°) - 0] = 31.144 \text{ kVAR}$$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{31{,}144}{(2\pi)(60)(120)^2} = \mathbf{5.74\ mF}$$

Chapter 11, Solution 75.

(a) $$\mathbf{S}_1 = \frac{|\mathbf{V}|^2}{\mathbf{Z}_1^*} = \frac{(240)^2}{80 + j50} = \frac{5760}{8 + j5} = 517.75 - j323.59 \text{ VA}$$

$$\mathbf{S}_2 = \frac{(240)^2}{120 - j70} = \frac{5760}{12 - j7} = 358.13 + j208.91 \text{ VA}$$

$$\mathbf{S}_3 = \frac{(240)^2}{60} = 960 \text{ VA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = \mathbf{[1.8359 - \mathit{j}0.11468]}\ \boldsymbol{kVA}$$

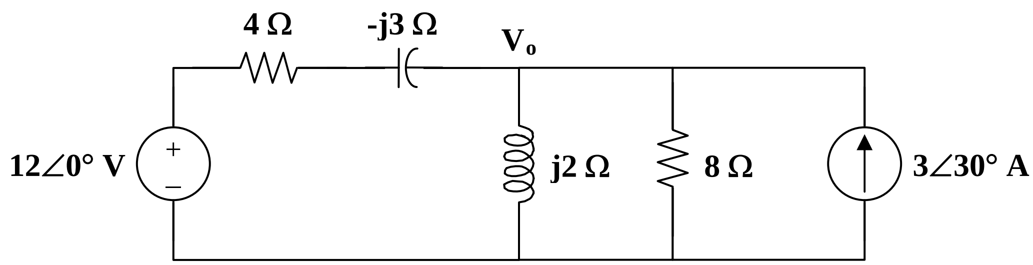
(b) $$\theta = \tan^{-1}\left(\frac{114.68}{1835.88}\right) = 3.574°$$

$$\text{pf} = \cos\theta = \mathbf{0.998}\ \{\text{leading}\}$$

(c) Since the circuit already has a leading power factor, near unity, no compensation is necessary.

Chapter 11, Solution 76.

The wattmeter reads the real power supplied by the current source. Consider the circuit below.



$$3\angle 30° + \frac{12 - \mathbf{V}_o}{4 - j3} = \frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o}{8}$$

$$\mathbf{V}_o = \frac{36.14 + j23.52}{2.28 - j3.04} = 0.7547 + j11.322 = 11.347\angle 86.19°$$

$$\mathbf{S} = \mathbf{V}_o\,\mathbf{I}_o^* = (11.347\angle 86.19°)(3\angle -30°)$$

$$\mathbf{S} = 34.04\angle 56.19° \text{ VA}$$

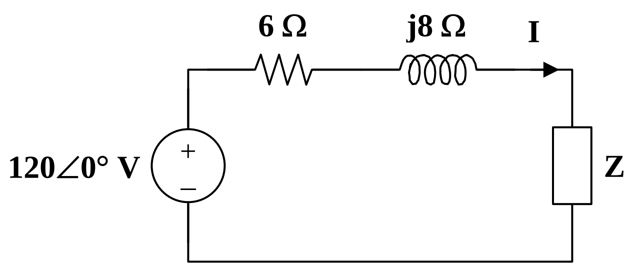
$$\text{P} = \text{Re}(\mathbf{S}) = \mathbf{18.942\ W}$$

Chapter 11, Solution 77.

The wattmeter measures the power absorbed by the parallel combination of 0.1 F and 150 Ω.

$$120\cos(2t) \longrightarrow 120\angle 0°, \qquad \omega = 2$$
$$4\text{ H} \longrightarrow j\omega L = j8$$
$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j5$$

Consider the following circuit.



$$\mathbf{Z} = 15 \,\|\, (\text{-}j5) = \frac{(15)(\text{-}j5)}{15 - j5} = 1.5 - j4.5$$

$$\mathbf{I} = \frac{120}{(6 + j8) + (1.5 - j4.5)} = 14.5\angle\text{-}25.02°$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}\left|\mathbf{I}\right|^2 \mathbf{Z} = \frac{1}{2}\cdot(14.5)^2(1.5 - j4.5)$$
$$\mathbf{S} = 157.69 - j473.06 \text{ VA}$$

The wattmeter reads

$$P = \text{Re}(\mathbf{S}) = \mathbf{157.69\ W}$$

Chapter 11, Solution 78.

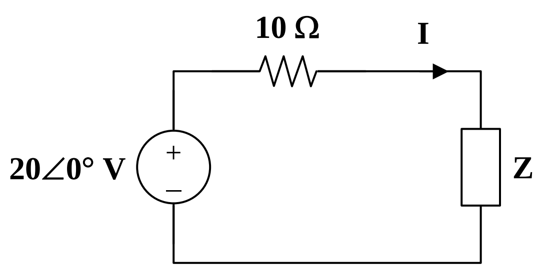
The wattmeter reads the power absorbed by the element to its right side.

$$2\cos(4t) \longrightarrow 2\angle 0°, \qquad \omega = 4$$

$$1\text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j3$$

Consider the following circuit.



$$\mathbf{Z} = 5 + j4 + 4 \,||\, \text{-}j3 = 5 + j4 + \frac{(4)(\text{-}j3)}{4 - j3}$$

$$\mathbf{Z} = 6.44 + j2.08$$

$$\mathbf{I} = \frac{20}{16.44 + j2.08} = 1.207\angle\text{-}7.21°$$

$$\mathbf{S} = \frac{1}{2}\left|\mathbf{I}\right|^2 \mathbf{Z} = \frac{1}{2}\cdot(1.207)^2(6.44 + j2.08)$$

$$\text{P} = \text{Re}(\mathbf{S}) = \mathbf{4.691\ W}$$

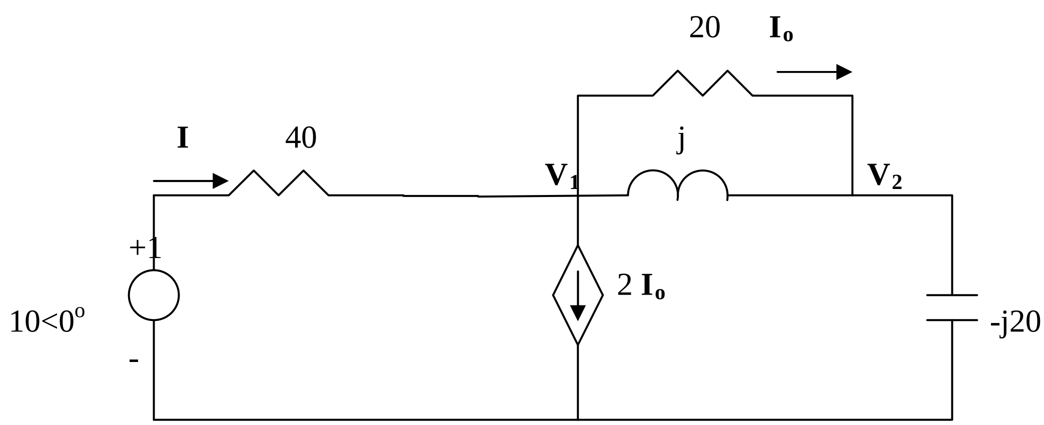
Chapter 11, Solution 79.

The wattmeter reads the power supplied by the source and partly absorbed by the 40-Ω resistor.

$\omega = 100$,

$$10\,\text{mH} \longrightarrow j100 \text{x} 10 \text{x} 10^{-3} = j, \quad 500\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j100\text{x}500\text{x}10^{-6}} = -j20$$

The frequency-domain circuit is shown below.



At node 1,

$$\frac{10 - V_1}{40} = 2I_o + \frac{V_1 - V_2}{j} + \frac{V_1 - V_2}{20} = \frac{3(V_1 - V_2)}{20} + \frac{V_1 - V_2}{j} \longrightarrow \quad (1)$$

$$10 = (7 - j40)V_1 + (-6 + j40)V_2$$

At node 2,

$$\frac{V_1 - V_2}{j} + \frac{V_1 - V_2}{20} = \frac{V_2}{-j20} \longrightarrow 0 = (20 + j)V_1 - (19 + j)V_2 \quad (2)$$

Solving (1) and (2) yields $V_1 = 1.5568 - j4.1405$

$$I = \frac{10 - V_1}{40} = 0.2111 + j0.1035, \quad S = \frac{1}{2}V_1 I^* = -0.04993 - j0.5176$$

P = Re(S) = **50 mW**.

Chapter 11, Solution 80.

(a) $$|\mathbf{I}| = \frac{|\mathbf{V}|}{|\mathbf{Z}|} = \frac{110}{6.4} = \mathbf{17.19\ A}$$

(b) $$|\mathbf{S}| = \frac{|\mathbf{V}|^2}{|\mathbf{Z}|} = \frac{(110)^2}{6.4} = 1890.6$$

$$\cos\theta = \text{pf} = 0.825 \longrightarrow \theta = 34.41°$$

$$P = S\cos\theta = 1559.8 \cong \mathbf{1.6\ kW}$$

Chapter 11, Solution 81.

Design a problem to help other students to better understand how to correct power factor to values other than unity.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A 120-V rms, 60-Hz electric hair dryer consumes 600 W at a lagging pf of 0.92. Calculate the rms-valued current drawn by the dryer.

How would you power factor correct this to a value of 0.95?

Solution

P = 600 W, $pf = 0.92 \longrightarrow \theta = 23.074^o$

$$P = S\cos\theta \longrightarrow S = \frac{P}{0.92} = 652.17 \text{ VA}$$

S = P+jQ = 600 + j652.17sin23.09° = 600 +j255.6

But $S = V_{rms}I^*_{rms}$.

$$I^*_{rms} = \frac{S}{V_{rms}} = \frac{600 + j255.6}{120}$$

$$I_{rms} = 5 - j2.13 = \mathbf{5.435\angle{-23.07^\circ}\,A}.$$

To correct this to a pf = 0.95, I would add a capacitor in parallel with the hair dryer (remember, series compensation will increase the power delivered to the load and probably burn out the hair dryer.

pf = 0.95 = 600/S or S = 631.6 VA and θ = 18.19° and VARs = 197.17

Thus,

$VARs_{cap} = 255.6 - 197.17 = 58.43 = 120xI_C$ or $I_C = 58.43/120 = 0.4869A$

Next,

$X_C = 120/0.4869 = 246.46 = 1/(377xC)$ or C = **10.762 μF**

Chapter 11, Solution 82.

(a) $P_1 = 5,000, \qquad Q_1 = 0$

$$P_2 = 30,000x0.82 = 24,600, \qquad Q_2 = 30,000\sin(\cos^{-1} 0.82) = 17,171$$

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = (P_1 + P_2) + j(Q_1 + Q_2) = 29,600 + j17,171$$

$$\boldsymbol{S = |\overline{S}| = \underline{34.22\,\text{kV}}\text{A}}$$

(b) Q = **17.171 kVAR**

(c) $pf = \dfrac{P}{S} = \dfrac{29,600}{34,220} = 0.865$

$$\boldsymbol{Q_c = P(\tan\theta_1 - \tan\theta_2)}$$
$$\boldsymbol{= 29,600\left[\tan(\cos^{-1} 0.865) - \tan(\cos^{-1} 0.9)\right] = \underline{2833\ \text{VAR}}}$$

(c) $$\boldsymbol{C = \frac{Q_c}{\omega V^2_{rms}} = \frac{2833}{2\pi x 60 x 240^2} = \underline{130.46\mu\,\text{F}}}$$

Chapter 11, Solution 83.

(a) $\overline{S} = \frac{1}{2}VI^* = \frac{1}{2}(210\angle 60^o)(8\angle -25^o) = 840\angle 35^o$

$$\boldsymbol{P = S\cos\theta = 840\cos 35^o = \underline{688.1\ \text{W}}}$$

(b) S = **<u>840 VA</u>**

(c) $\boldsymbol{Q = S\sin\theta = 840\sin 35^o = \underline{481.8\ \text{VAR}}}$

(d) $\boldsymbol{pf = P/S = \cos 35^o = \underline{0.8191\ \text{(lagging)}}}$

Chapter 11, Solution 84.

(a) Maximum demand charge $= 2{,}400 \times 30 = \$72{,}000$

Energy cost $= \$0.04 \times 1{,}200 \times 10^3 = \$48{,}000$

Total charge = **$120,000**

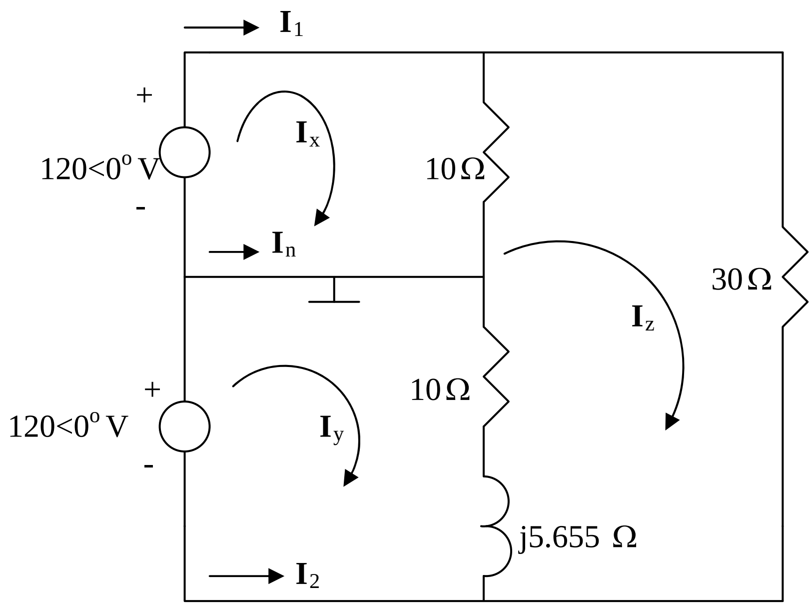
(b) To obtain $120,000 from 1,200 MWh will require a flat rate of

$$\frac{\$120{,}000}{1{,}200 \times 10^3} \text{ per kWh} = \mathbf{\$0.10 \text{ per kWh}}$$

Chapter 11, Solution 85.

(a) 15 mH $\longrightarrow$ $j2\pi x60x15x10^{-3} = j5.655$

We apply mesh analysis as shown below.



For mesh x,

$120 = 10\,\mathbf{I}_x - 10\,\mathbf{I}_z$ (1)

For mesh y,

$120 = (10+j5.655)\,\mathbf{I}_y - (10+j5.655)\,\mathbf{I}_z$ (2)

For mesh z,

$0 = -10\,\mathbf{I}_x - (10+j5.655)\,\mathbf{I}_y + (50+j5.655)\,\mathbf{I}_z$ (3)

Solving (1) to (3) gives

$\mathbf{I}_x = 20$, $\mathbf{I}_y = 17.09 - j5.142$, $\mathbf{I}_z = 8$

Thus,

$\mathbf{I}_1 = \mathbf{I}_x = \mathbf{20\ A}$

$\mathbf{I}_2 = -\mathbf{I}_y = -17.09 + j5.142 = \underline{\mathbf{17.85\angle 163.26^o\ \ A}}$

$\mathbf{I}_n = \mathbf{I}_y - \mathbf{I}_x = -2.91 - j5.142 = \underline{\mathbf{5.907\angle -119.5^o\ \ A}}$

(b) $\overline{S_1} = (120)I^{\bullet}{}_x = 120x20 = 2400$, $\overline{S_2} = (120)I^{\bullet}{}_y = 2051 + j617$

$\overline{\boldsymbol{S}} = \overline{\boldsymbol{S}_1} + \overline{\boldsymbol{S}_2} = \underline{\mathbf{[4.451 + \boldsymbol{j}0.617]kVA}}$

(c) pf = P/S = 4451/4494 = **0.9904** (lagging)

Chapter 11, Solution 86.

For maximum power transfer

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* \longrightarrow \mathbf{Z}_i = \mathbf{Z}_{Th} = \mathbf{Z}_L^*$$

$$\mathbf{Z}_L = R + j\omega L = 75 + j(2\pi)(4.12\times10^6)(4\times10^{-6})$$
$$\mathbf{Z}_L = 75 + j103.55\,\Omega$$

$$\mathbf{Z}_i = \mathbf{[75 - \mathit{j}103.55]\,\Omega}$$

Chapter 11, Solution 87.

$$\mathbf{Z} = R \pm jX$$

$$\mathbf{V}_R = \mathbf{I}\,R \quad\longrightarrow\quad R = \frac{\mathbf{V}_R}{\mathbf{I}} = \frac{80}{50 \times 10^{-3}} = 1.6 \text{ k}\Omega$$

$$\left|\mathbf{Z}\right|^2 = R^2 + X^2 \quad\longrightarrow\quad X^2 = \left|\mathbf{Z}\right|^2 - R^2 = (3)^2 - (1.6)^2$$

$$X = 2.5377 \text{ k}\Omega$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{2.5377}{1.6}\right) = 57.77°$$

$$\text{pf} = \cos\theta = \mathbf{0.5333}$$

Chapter 11, Solution 88.

(a) $\mathbf{S} = (110)(2\angle 55°) = 220\angle 55°$

$$P = S\cos\theta = 220\cos(55°) = \mathbf{126.2\ W}$$

(b) $S = |\mathbf{S}| = \mathbf{220\ VA}$

Chapter 11, Solution 89.

(a) Apparent power $= S =$ **12 kVA**

$$P = S\cos\theta = (12)(0.78) = 9.36 \text{ kW}$$
$$Q = S\sin\theta = 12\sin(\cos^{-1}(0.78)) = 7.51 \text{ kVAR}$$

$$\mathbf{S} = P + jQ = \mathbf{[9.36 + \mathit{j}7.51]\ \mathit{kVA}}$$

(b) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} \longrightarrow \mathbf{Z}^* = \frac{|\mathbf{V}|^2}{\mathbf{S}} = \frac{(210)^2}{(9.36 + j7.51)\times 10^3} = 2.866 - j2.3$$

$$\mathbf{Z} = \mathbf{[2.866 + j2.3]\ \Omega}$$

Chapter 11, Solution 90

Original load :

$$P_1 = 2000 \text{ kW}, \qquad \cos\theta_1 = 0.85 \longrightarrow \theta_1 = 31.79°$$

$$S_1 = \frac{P_1}{\cos\theta_1} = 2352.94 \text{ kVA}$$

$$Q_1 = S_1 \sin\theta_1 = 1239.5 \text{ kVAR}$$

Additional load :

$$P_2 = 300 \text{ kW}, \qquad \cos\theta_2 = 0.8 \longrightarrow \theta_2 = 36.87°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 375 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 225 \text{ kVAR}$$

Total load :

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = (P_1 + P_2) + j(Q_1 + Q_2) = P + jQ$$

$$P = 2000 + 300 = 2300 \text{ kW}$$

$$Q = 1239.5 + 225 = 1464.5 \text{ kVAR}$$

The minimum operating pf for a 2300 kW load and not exceeding the kVA rating of the generator is

$$\cos\theta = \frac{P}{S_1} = \frac{2300}{2352.94} = 0.9775$$

or $\qquad \theta = 12.177°$

The maximum load kVAR for this condition is

$$Q_m = S_1 \sin\theta = 2352.94 \sin(12.177°)$$

$$Q_m = 496.313 \text{ kVAR}$$

The capacitor must supply the difference between the total load kVAR (i.e. Q) and the permissible generator kVAR (i.e. Q_m). Thus,

$$Q_c = Q - Q_m = \mathbf{968.2 \text{ kVAR}}$$

Chapter 11, Solution 91

The nameplate of an electric motor has the following information:

Line voltage: 220 V rms
Line current: 15 A rms
Line frequency: 60 Hz
Power: 2700 W

Determine the power factor (lagging) of the motor. Find the value of the capacitance *C* that must be connected across the motor to raise the pf to unity.

Solution

I = V/Z which leads to $Z = [220/15]\angle\theta = 14.6667\angle\theta$, $S = (220)(15)\angle\theta = 3.3\angle\theta$

kVA, where $\cos^{-1}(2700/3300) = \cos^{-1}(0.818182) = 35.097°$, and $X_L = 3300\sin(35.097°) = 1897.38 = X_C$. This leads to $C = 1/[377(1897.38)] =$ **1.398 μF**.

pf = **0.8182** (lagging)

C = **1.398 μF**

0.8182 (lagging), **1.398 μF**

Chapter 11, Solution 92

(a) Apparent power drawn by the motor is

$$S_m = \frac{P}{\cos\theta} = \frac{60}{0.75} = 80 \text{ kVA}$$

$$Q_m = \sqrt{S^2 - P^2} = \sqrt{(80)^2 - (60)^2} = 52.915 \text{ kVAR}$$

Total real power

$$P = P_m + P_c + P_L = 60 + 0 + 20 = 80 \text{ kW}$$

Total reactive power

$$Q = Q_m + Q_c + Q_L = 52.915 - 20 + 0 = \mathbf{32.91\ kVAR}$$

Total apparent power

$$S = \sqrt{P^2 + Q^2} = \mathbf{86.51\ kVA}$$

(b) $$\text{pf} = \frac{P}{S} = \frac{80}{86.51} = \mathbf{0.9248}$$

(c) $$I = \frac{S}{V} = \frac{86510}{550} = \mathbf{157.3\ A}$$

Chapter 11, Solution 93

(a) $P_1 = (5)(0.7457) = 3.7285 \text{ kW}$

$$S_1 = \frac{P_1}{pf} = \frac{3.7285}{0.8} = 4.661 \text{ kVA}$$

$$Q_1 = S_1 \sin(\cos^{-1}(0.8)) = 2.796 \text{ kVAR}$$

$$\mathbf{S}_1 = 3.7285 + j2.796 \text{ kVA}$$

$P_2 = 1.2 \text{ kW}$, $\quad Q_2 = 0 \text{ VAR}$

$$\mathbf{S}_2 = 1.2 + j0 \text{ kVA}$$

$P_3 = (10)(120) = 1.2 \text{ kW}$, $\quad Q_3 = 0 \text{ VAR}$

$$\mathbf{S}_3 = 1.2 + j0 \text{ kVA}$$

$Q_4 = 1.6 \text{ kVAR}$, $\quad \cos\theta_4 = 0.6 \longrightarrow \sin\theta_4 = 0.8$

$$S_4 = \frac{Q_4}{\sin\theta_4} = 2 \text{ kVA}$$

$$P_4 = S_4 \cos\theta_4 = (2)(0.6) = 1.2 \text{ kW}$$

$$\mathbf{S}_4 = 1.2 - j1.6 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$

$$\mathbf{S} = 7.3285 + j1.196 \text{ kVA}$$

Total real power = **7.3285 kW**

Total reactive power = **1.196 kVAR**

(b) $$\theta = \tan^{-1}\left(\frac{1.196}{7.3285}\right) = 9.27^\circ$$

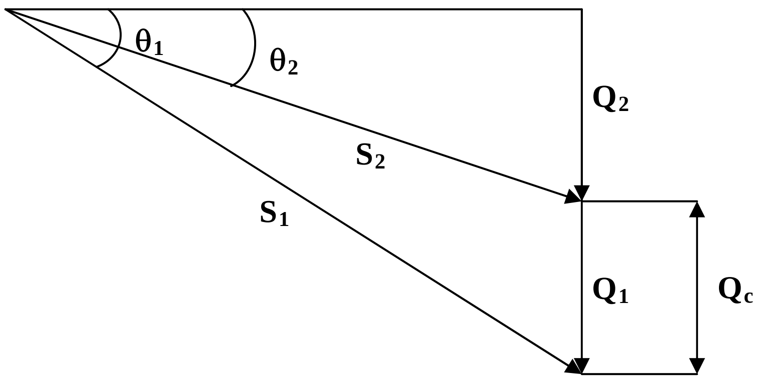
pf $= \cos\theta =$ **0.987**

Chapter 11, Solution 94

$$\cos\theta_1 = 0.7 \longrightarrow \theta_1 = 45.57°$$
$$S_1 = 1\text{ MVA} = 1000\text{ kVA}$$
$$P_1 = S_1\cos\theta_1 = 700\text{ kW}$$
$$Q_1 = S_1\sin\theta_1 = 714.14\text{ kVAR}$$

For improved pf,

$$\cos\theta_2 = 0.95 \longrightarrow \theta_2 = 18.19°$$
$$P_2 = P_1 = 700\text{ kW}$$
$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{700}{0.95} = 736.84\text{ kVA}$$
$$Q_2 = S_2\sin\theta_2 = 230.08\text{ kVAR}$$



(a) Reactive power across the capacitor

$$Q_c = Q_1 - Q_2 = 714.14 - 230.08 = 484.06\text{ kVAR}$$

Cost of installing capacitors $= \$30 \times 484.06 =$ **\$14,521.80**

(b) Substation capacity released $= S_1 - S_2$

$$= 1000 - 736.84 = 263.16\text{ kVA}$$

Saving in cost of substation and distribution facilities

$= \$120 \times 263.16 =$ **\$31,579.20**

(c) **Yes**, because (a) is greater than (b). Additional system capacity obtained by using capacitors costs only 46% as much as new substation and distribution facilities.

Chapter 11, Solution 95

(a) Source impedance $\mathbf{Z}_s = R_s - jX_c$

Load impedance $\mathbf{Z}_L = R_L + jX_2$

For maximum load transfer

$$\mathbf{Z}_L = \mathbf{Z}_s^* \longrightarrow R_s = R_L, \quad X_c = X_L$$

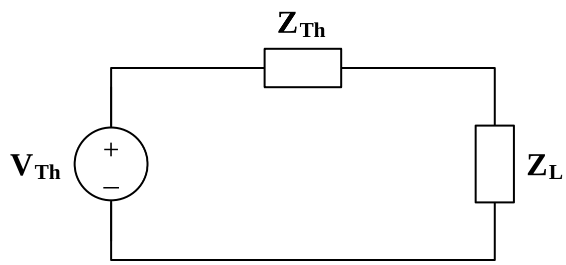
$$X_c = X_L \longrightarrow \frac{1}{\omega C} = \omega L$$

or $$\omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(80\times10^{-3})(40\times10^{-9})}} = \mathbf{2.814\ kHz}$$

(b) $$P = \left(\frac{V_S}{(10+4)}\right)^2 4 = \left(\frac{4.6}{14}\right)^2 4 = \mathbf{431.8\ mW} \quad \text{(since } V_s \text{ is in rms)}$$

Chapter 11, Solution 96



(a) $V_{Th} = 146 \text{ V}, \quad 300 \text{ Hz}$

$Z_{Th} = 40 + j8\,\Omega$

$Z_L = Z_{Th}^* = \mathbf{[40 - \mathit{j}8]\,\Omega}$

(b) $$P = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{(146)^2}{(8)(40)} = \mathbf{66.61\ W}$$

Chapter 11, Solution 97

$$Z_T = (2)(0.1 + j) + (100 + j20) = 100.2 + j22\ \Omega$$

$$I = \frac{V_s}{Z_T} = \frac{240}{100.2 + j22}$$

$$P = \left| I \right|^2 R_L = 100\left| I \right|^2 = \frac{(100)(240)^2}{(100.2)^2 + (22)^2} = \mathbf{547.3\ W}$$