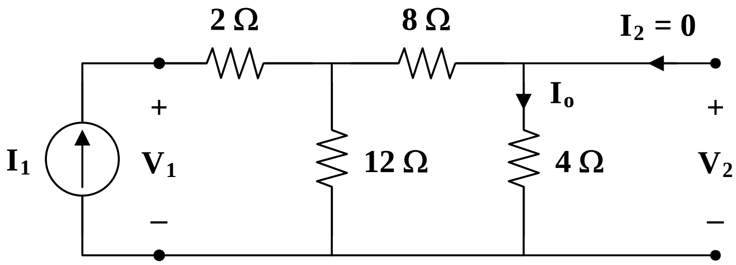
Chapter 19, Solution 1.

To get $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



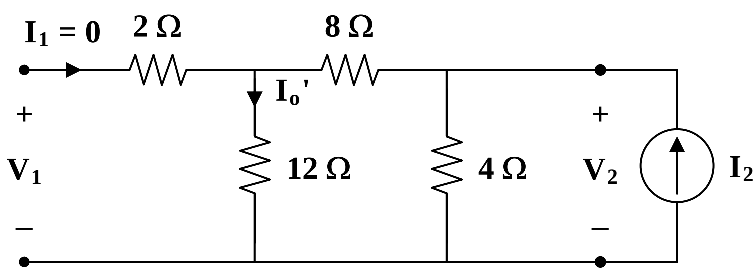
(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2 + 12 \,||\, (8+4) = 8\,\Omega$$

$$\mathbf{I}_o = \frac{1}{2}\mathbf{I}_1, \qquad \mathbf{V}_2 = 4\mathbf{I}_o = \mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = 2\,\Omega$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 4 \,||\, (8+12) = 3.333\,\Omega$$

$$\mathbf{I}_o' = \frac{4}{4+20}\mathbf{I}_2 = \frac{1}{6}\mathbf{I}_2, \qquad \mathbf{V}_1 = 12\mathbf{I}_o' = 2\mathbf{I}_2$$

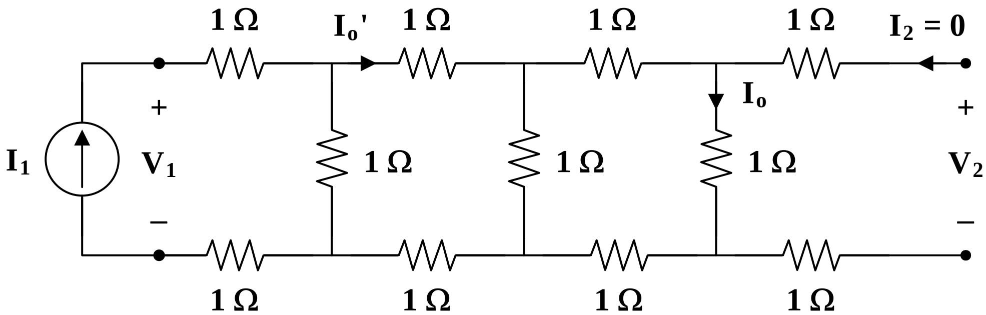
$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = 2\,\Omega$$

Hence,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{8} & \mathbf{2} \\ \mathbf{2} & \mathbf{3.333} \end{bmatrix}\Omega$$

Chapter 19, Solution 2.

Consider the circuit in Fig. (a) to get $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$.



(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2 + 1 \| [2 + 1 \| (2+1)]$$

$$\mathbf{z}_{11} = 2 + 1 \| \left(2 + \frac{3}{4}\right) = 2 + \frac{(1)(11/4)}{1 + 11/4} = 2 + \frac{11}{15} = 2.733$$

$$\mathbf{I}_o = \frac{1}{1+3}\mathbf{I}_o' = \frac{1}{4}\mathbf{I}_o'$$

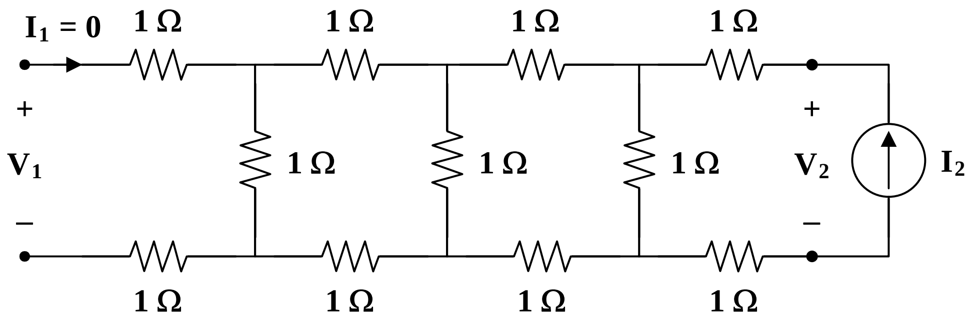
$$\mathbf{I}_o' = \frac{1}{1 + 11/4}\mathbf{I}_1 = \frac{4}{15}\mathbf{I}_1$$

$$\mathbf{I}_o = \frac{1}{4}\cdot\frac{4}{15}\mathbf{I}_1 = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_o = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{15} = \mathbf{z}_{12} = 0.06667$$

To get $\mathbf{z}_{22}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 2 + 1 \| (2 + 1 \| 3) = \mathbf{z}_{11} = 2.733$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{2.733} & \mathbf{0.06667} \\ \mathbf{0.06667} & \mathbf{2.733} \end{bmatrix} \Omega$$

Chapter 19, Solution 3.

We can use Figure 19.5 to determine the z-parameters.

$\mathbf{z}_{12} = \mathbf{j12} = \mathbf{z}_{21}$

$\mathbf{z}_{11} - \mathbf{z}_{12} = 8$ or $\mathbf{z}_{11} = \mathbf{(8+j12)\ \Omega}$

$\mathbf{z}_{22} - \mathbf{z}_{12} = -j20$ or $\mathbf{z}_{22} = \mathbf{(-j8)\ \Omega}$

Chapter 19, Solution 4.

Using Fig. 19.68, design a problem to help other students to better understand how to determine z parameters from an electrical circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Calculate the *z* parameters for the circuit in Fig.19.68.

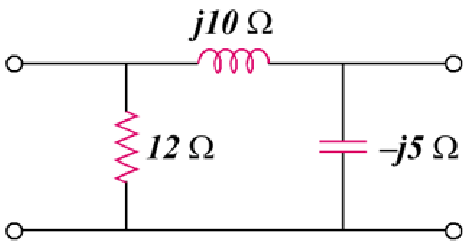
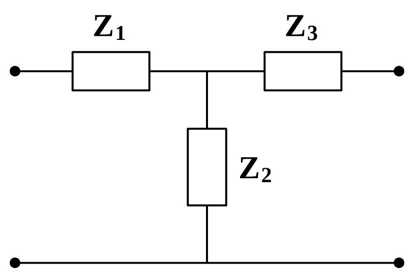


Figure 19.68

Solution

Transform the Π network to a T network.



$$\mathbf{Z}_1 = \frac{(12)(j10)}{12 + j10 - j5} = \frac{j120}{12 + j5}$$

$$\mathbf{Z}_2 = \frac{-j60}{12 + j5}$$

$$\mathbf{Z}_3 = \frac{50}{12 + j5}$$

The z parameters are

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{Z}_2 = \frac{(-j60)(12 - j5)}{144 + 25} = -1.775 - j4.26$$

$$\mathbf{z}_{11} = \mathbf{Z}_1 + \mathbf{z}_{12} = \frac{(j120)(12 - j5)}{169} + \mathbf{z}_{12} = 1.775 + j4.26$$

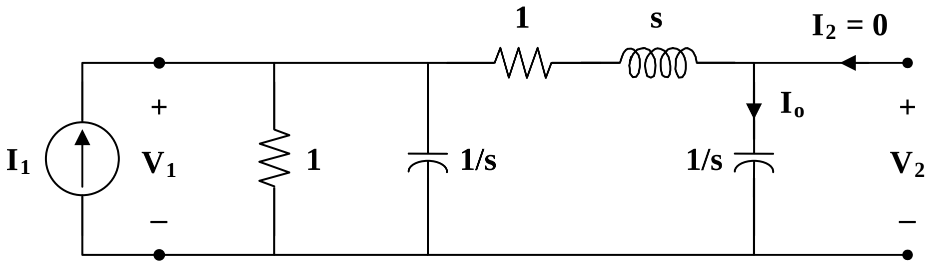
$$\mathbf{z}_{22} = \mathbf{Z}_3 + \mathbf{z}_{21} = \frac{(50)(12 - j5)}{169} + \mathbf{z}_{21} = 1.7758 - j5.739$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{1.775 + j4.26} & \mathbf{-1.775 - j4.26} \\ \mathbf{-1.775 - j4.26} & \mathbf{1.775 - j5.739} \end{bmatrix} \Omega$$

Chapter 19, Solution 5.

Consider the circuit in Fig. (a).



(a)

$$\mathbf{z}_{11} = 1 \| \frac{1}{s} \| \left(1 + s + \frac{1}{s}\right) = \frac{\frac{1}{s}}{1 + \frac{1}{s}} \| \left(1 + s + \frac{1}{s}\right) = \frac{\left(\frac{1}{s+1}\right)\left(1 + s + \frac{1}{s}\right)}{\left(\frac{1}{s+1}\right) + 1 + s + \frac{1}{s}}$$

$$\mathbf{z}_{11} = \frac{s^2 + s + 1}{s^3 + 2s^2 + 3s + 1}$$

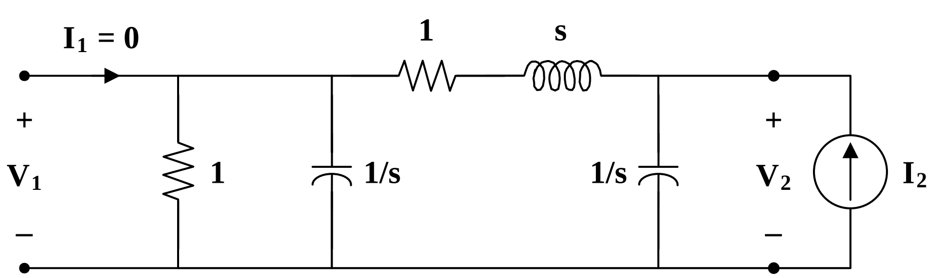
$$\mathbf{I}_o = \frac{1 \| \frac{1}{s}}{1 \| \frac{1}{s} + 1 + s + \frac{1}{s}} \mathbf{I}_1 = \frac{\frac{1}{s+1}}{\frac{1}{s+1} + 1 + s + \frac{1}{s}} \mathbf{I}_1 = \frac{\frac{s}{s+1}}{\frac{s}{s+1} + s^2 + s + 1} \mathbf{I}_1$$

$$\mathbf{I}_o = \frac{s}{s^3 + 2s^2 + 3s + 1} \mathbf{I}_1$$

$$\mathbf{V}_2 = \frac{1}{s} \mathbf{I}_o = \frac{\mathbf{I}_1}{s^3 + 2s^2 + 3s + 1}$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

Consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{s} \parallel \left(1 + s + 1 \parallel \frac{1}{s}\right) = \frac{1}{s} \parallel \left(1 + s + \frac{1}{s+1}\right)$$

$$\mathbf{z}_{22} = \frac{\left(\frac{1}{s}\right)\left(1 + s + \frac{1}{s+1}\right)}{\frac{1}{s} + 1 + s + \frac{1}{s+1}} = \frac{1 + s + \frac{1}{s+1}}{1 + s + s^2 + \frac{s}{s+1}}$$

$$\mathbf{z}_{22} = \frac{s^2 + 2s + 2}{s^3 + 2s^2 + 3s + 1}$$

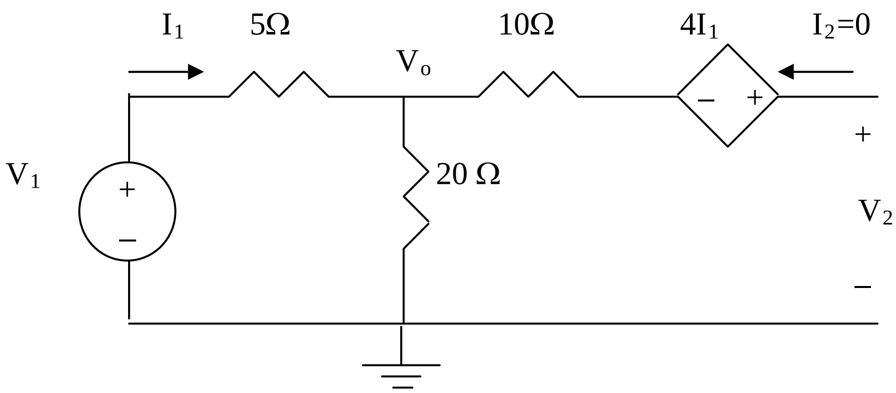
$$\mathbf{z}_{12} = \mathbf{z}_{21}$$

Hence,

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\mathbf{s^2 + s + 1}}{\mathbf{s^3 + 2s^2 + 3s + 1}} & \dfrac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 1}} \\ \dfrac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 1}} & \dfrac{\mathbf{s^2 + 2s + 2}}{\mathbf{s^3 + 2s^2 + 3s + 1}} \end{bmatrix}$$

Chapter 19, Solution 6.

To find z_{11} and z_{21} , consider the circuit below.



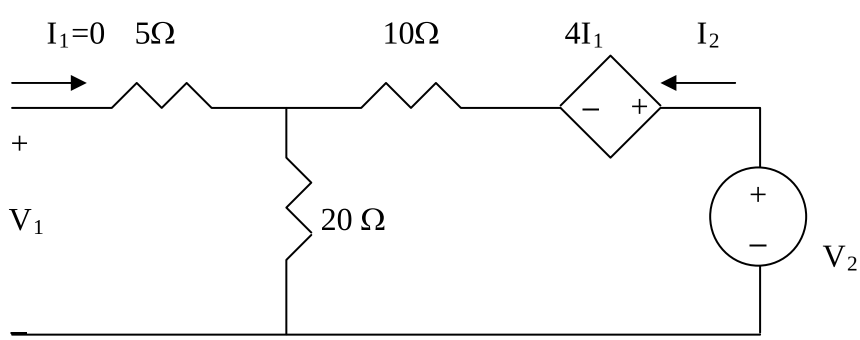
$$z_{11} = \frac{V_1}{I_1} = \frac{(20+5)I_1}{I_1} = 25\ \Omega$$

$$V_o = \frac{20}{25}V_1 = 20I_1$$

$$-V_o - 4I_2 + V_2 = 0 \quad \longrightarrow \quad V_2 = V_o + 4I_1 = 20I_1 + 4I_1 = 24I_1$$

$$z_{21} = \frac{V_2}{I_1} = 24\ \Omega$$

To find z_{12} and z_{22}, consider the circuit below.



$$V_2 = (10+20)I_2 = 30I_2$$

$$z_{22} = \frac{V_2}{I_1} = 30\ \Omega$$

$$V_1 = 20I_2$$

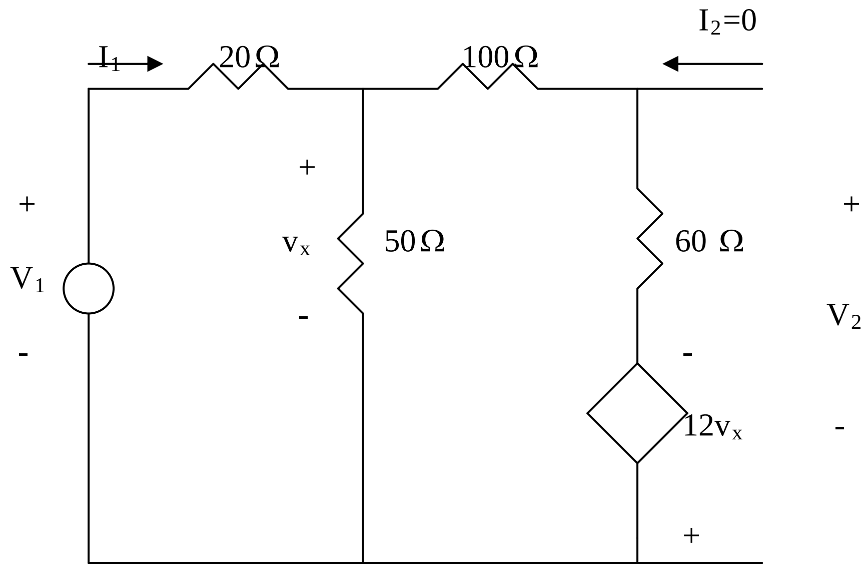
$$z_{12} = \frac{V_1}{I_2} = 20\ \Omega$$

Thus,

$$\underline{[z] = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix} \Omega}$$

Chapter 19, Solution 7.

To get z_{11} and z_{21}, we consider the circuit below.



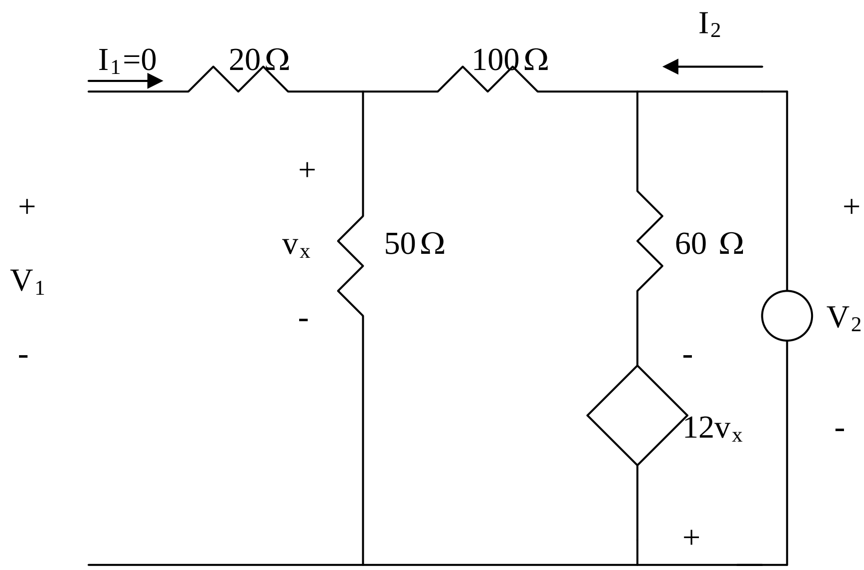
$$\frac{V_1 - V_x}{20} = \frac{V_x}{50} + \frac{V_x + 12V_x}{160} \longrightarrow V_x = \frac{40}{121}V_1$$

$$I_1 = \frac{V_1 - V_x}{20} = \frac{81}{121}\left(\frac{V_1}{20}\right) \longrightarrow z_{11} = \frac{V_1}{I_1} = 29.88$$

$$V_2 = 60\left(\frac{13V_x}{160}\right) - 12V_x = -\frac{57}{8}V_x = -\frac{57}{8}\left(\frac{40}{121}\right)V_1 = -\frac{57}{8}\left(\frac{40}{121}\right)\frac{20x121}{81}I_1$$

$$= -70.37I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = -70.37$$

To get z_{12} and z_{22}, we consider the circuit below.



$$V_x = \frac{50}{100+50}V_2 = \frac{1}{3}V_2, \qquad I_2 = \frac{V_2}{150} + \frac{V_2 + 12V_x}{60} = 0.09V_2$$

$$z_{22} = \frac{V_2}{I_2} = 1/0.09 = 11.11$$

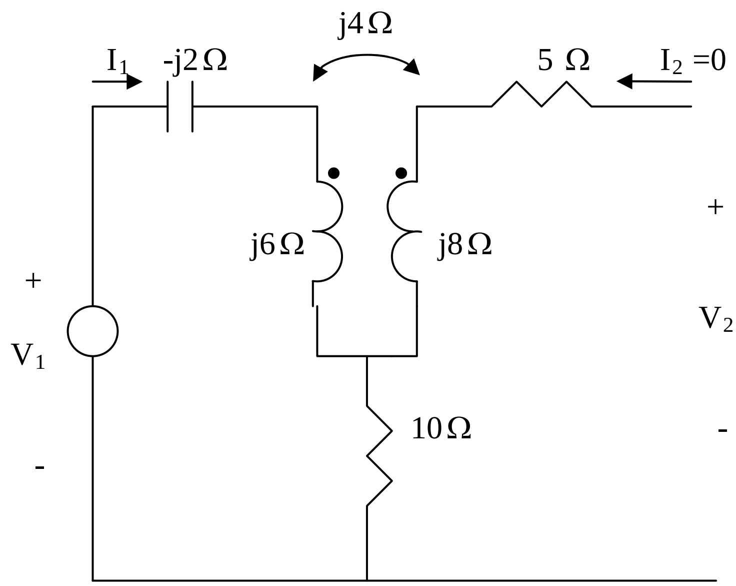
$$V_1 = V_x = \frac{1}{3}V_2 = \frac{11.11}{3}I_2 = 3.704I_2 \quad \longrightarrow \quad z_{12} = \frac{V_1}{I_2} = 3.704$$

Thus,

$$\mathbf{[z]} = \underline{\begin{bmatrix} \mathbf{29.88} & \mathbf{3.704} \\ \mathbf{-70.37} & \mathbf{11.11} \end{bmatrix}\Omega}$$

Chapter 19, Solution 8.

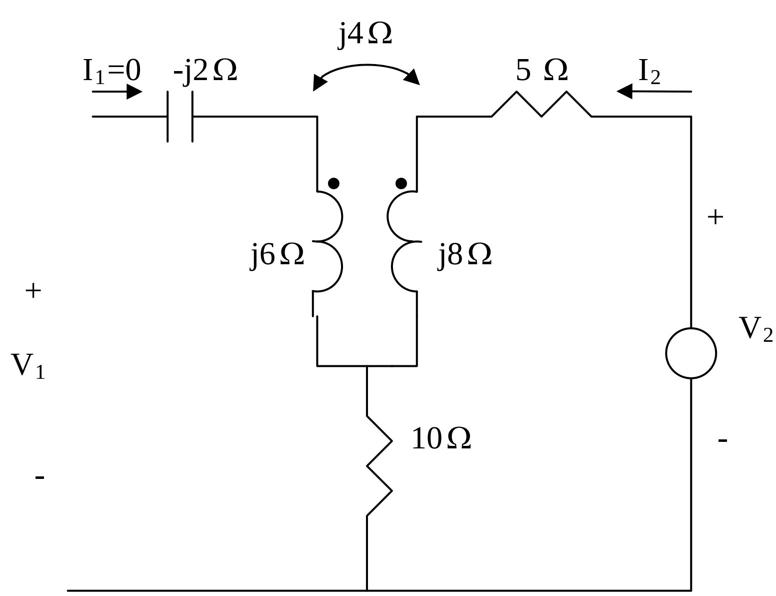
To get z_{11} and z_{21}, consider the circuit below.



$$V_1 = (10 - j2 + j6)I_1 \quad \longrightarrow \quad z_{11} = \frac{V_1}{I_1} = 10 + j4$$

$$V_2 = -10I_1 - j4I_1 \quad \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = -(10 + j4)$$

To get z_{22} and z_{12}, consider the circuit below.



$$V_2 = (5+10+j8)I_2 \longrightarrow z_{22} = \frac{V_2}{I_2} = 15 + j8$$

$$V_1 = -(10+j4)I_2 \longrightarrow z_{12} = \frac{V_1}{I_2} = -(10+j4)$$

Thus,

$$\underline{\mathbf{[z]} = \begin{bmatrix} \mathbf{(10+j4)} & \mathbf{-(10+j4)} \\ \mathbf{-(10+j4)} & \mathbf{(15+j8)} \end{bmatrix} \boldsymbol{\Omega}}$$

Chapter 19, Solution 9.

$$\Delta_y = y_{11}y_{22} - y_{12}y_{21} = 0.5x0.4 - 0.2x0.2 = 0.16$$

$$z_{11} = \frac{y_{22}}{\Delta_y} = \frac{0.4}{0.16} = \mathbf{2.5\ \Omega}$$

$$z_{12} = \frac{-y_{12}}{\Delta_y} = \frac{0.2}{0.16} = \mathbf{1.25\ \Omega} = z_{21}$$

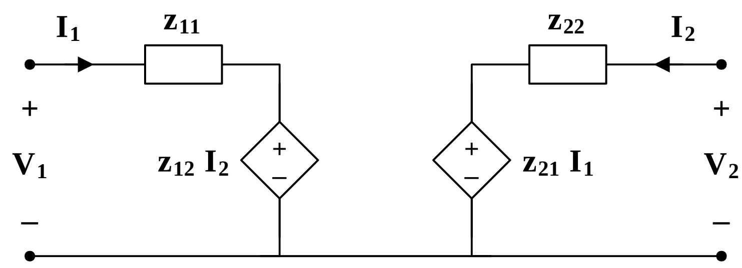
$$z_{22} = \frac{y_{11}}{\Delta_y} = \frac{0.5}{0.16} = \mathbf{3.125\ \Omega}$$

Thus,

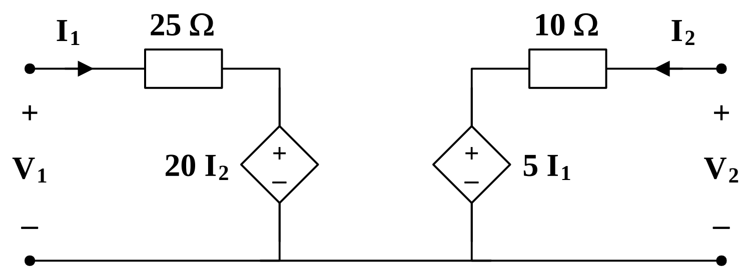
$$\mathbf{Z} = [\mathbf{z}] = \begin{bmatrix} \mathbf{2.5} & \mathbf{1.25} \\ \mathbf{1.25} & \mathbf{3.125} \end{bmatrix}\Omega$$

Chapter 19, Solution 10.

(a) This is a non-reciprocal circuit so that **the two-port looks like the one shown in Figs. (a) and (b)**.

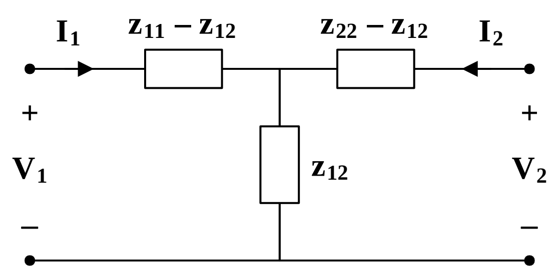


(a)



(b)

(b) This is a reciprocal network and **the two-port look like the one shown in Figs. (c) and (d)**.

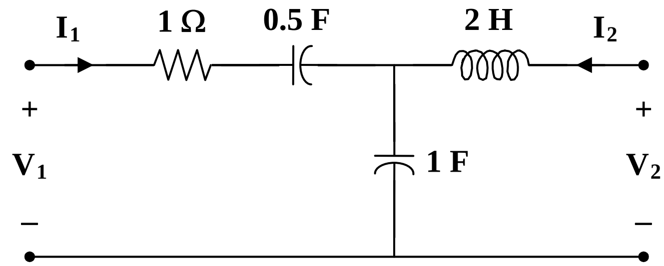


(c)

$$\mathbf{z}_{11} - \mathbf{z}_{12} = 1 + \frac{2}{s} = 1 + \frac{1}{0.5s}$$

$$\mathbf{z}_{22} - \mathbf{z}_{12} = 2s$$

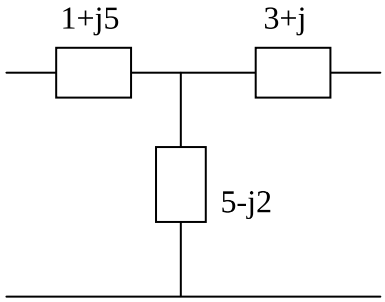
$$\mathbf{z}_{12} = \frac{1}{s}$$

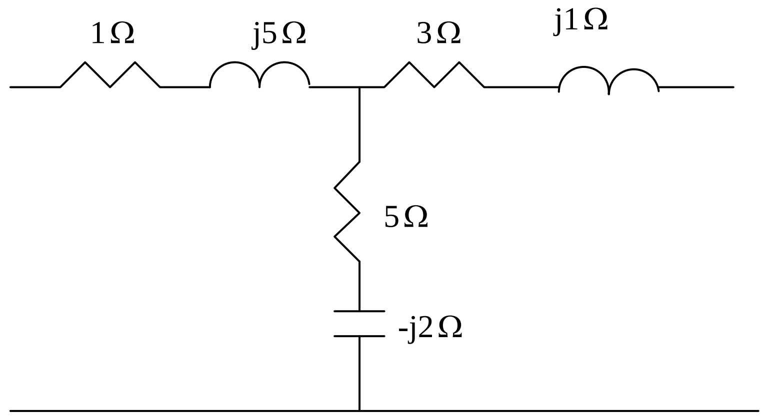


(d)

Chapter 19, Solution 11.

This is a reciprocal network, as shown below.

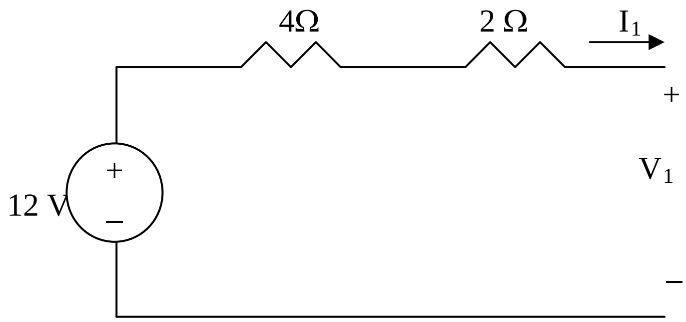




Chapter 19, Solution 12.

$$V_1 = 10I_1 - 6I_2 \qquad (1)$$
$$V_2 = -4I_2 + 12I_2 \qquad (2)$$
$$V_2 = -10I_2 \qquad (3)$$

If we convert the current source to a voltage source, that portion of the circuit becomes what is shown below.



$$-12 + 6I_1 + V_1 = 0 \longrightarrow V_1 = 12 - 6I_1 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we get

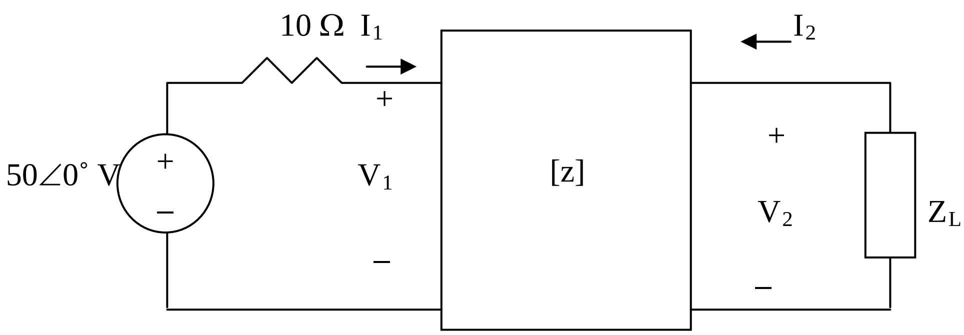
$$12 - 6I_1 = 10I_1 - 6I_2 \longrightarrow 12 = 16I_1 - 6I_2 \qquad (5)$$
$$-10I_2 = -4I_1 + 12I_2 \longrightarrow 0 = -4I_1 + 22I_2 \longrightarrow I_1 = 5.5I_2 \qquad (6)$$

From (5) and (6),

$$12 = 88I_2 - 6I_2 = 82I_2 \longrightarrow I_2 = \underline{0.1463 \text{ A}}$$
$$I_1 = 5.5I_2 = \underline{0.8049 \text{ A}}$$
$$V_2 = -10I_2 = \underline{-1.463 \text{ V}}$$
$$V_1 = 12 - 6I_1 = \underline{7.1706 \text{ V}}$$

Chapter 19, Solution 13.

Consider the circuit as shown below.



$$V_1 = 40I_1 + 60I_2 \qquad (1)$$
$$V_2 = 80I_1 + 100I_2 \qquad (2)$$
$$V_2 = -I_2 Z_L = -I_2(5 + j4) \qquad (3)$$
$$50 = V_1 + 10I_1 \longrightarrow V_1 = 50 - 10I_1 \qquad (4)$$

Substituting (4) in (1)

$$50 - 10I_1 = 40I_1 + 60I_2 \longrightarrow 5 = 5I_1 + 6I_2 \quad (5)$$

Substituting (3) into (2),

$$-I_2(5 + j4) = 80I_1 + 100I_2 \longrightarrow 0 = 80I_1 + (105 + j4)I_2 \qquad (6)$$

Solving (5) and (6) gives

I_2 = –7.423 + j3.299 A

We can check the answer using MATLAB.

First we need to rewrite equations 1-4 as follows,

$$\begin{bmatrix} 1 & 0 & -40 & -60 \\ 0 & 1 & -80 & -100 \\ 0 & 1 & 0 & 5+j4 \\ 1 & 0 & 10 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = A * X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 50 \end{bmatrix} = U$$

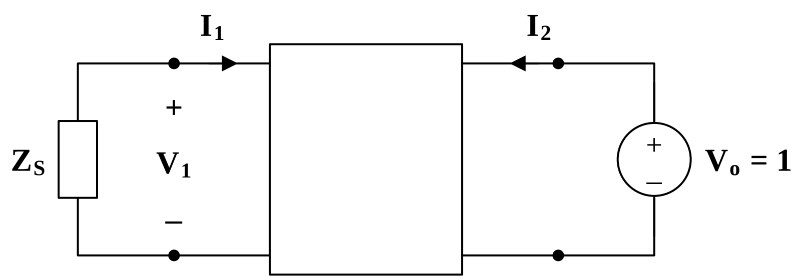
```
>> A=[1,0,-40,-60;0,1,-80,-100;0,1,0,(5+4i);1,0,10,0]
A =
  1.0e+002 *
   0.0100          0       -0.4000       -0.6000
        0     0.0100       -0.8000       -1.0000
        0     0.0100            0     0.0500 + 0.0400i
   0.0100          0        0.1000            0
>> U=[0;0;0;50]
U =
```

```
     0
     0
     0
    50
>> X=inv(A)*U
X =
 -49.0722 +39.5876i
  50.3093 +13.1959i
   9.9072 - 3.9588i
  -7.4227 + 3.2990i
```

$$P = |I_2|^2 5 = \mathbf{329.9\ W}.$$

Chapter 19, Solution 14.

To find $\mathbf{Z}_{Th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{z}_{11}\,\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2 \qquad (1)$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\,\mathbf{I}_1 + \mathbf{z}_{22}\,\mathbf{I}_2 \qquad (2)$$

But

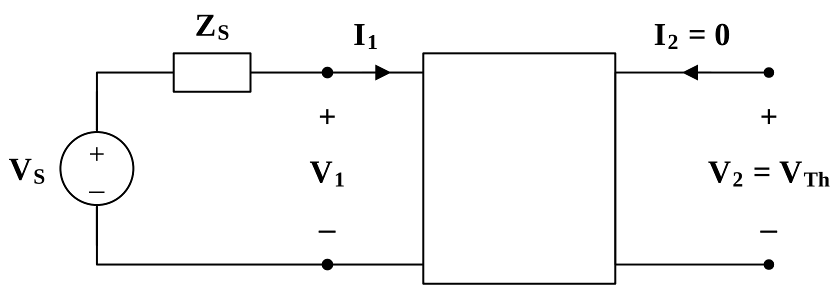
$$\mathbf{V}_2 = 1, \qquad \mathbf{V}_1 = -\mathbf{Z}_s\,\mathbf{I}_1$$

Hence,
$$0 = (\mathbf{z}_{11} + \mathbf{Z}_s)\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2 \longrightarrow \mathbf{I}_1 = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}\mathbf{I}_2$$

$$1 = \left(\frac{-\mathbf{z}_{21}\,\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s} + \mathbf{z}_{22}\right)\mathbf{I}_2$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \underline{\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\,\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}}$$

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = 0, \qquad \mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\,\mathbf{Z}_s$$

Substituting these into (1) and (2),

$$\mathbf{V}_s - \mathbf{I}_1 \mathbf{Z}_s = \mathbf{z}_{11} \mathbf{I}_1 \quad \longrightarrow \quad \mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 = \frac{\mathbf{z}_{21} \mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_2 = \frac{\mathbf{z}_{21} \mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

Chapter 19, Solution 15.

(a) From Prob. 18.12,

$$Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_s} = 120 - \frac{80x60}{40+10} = 24$$

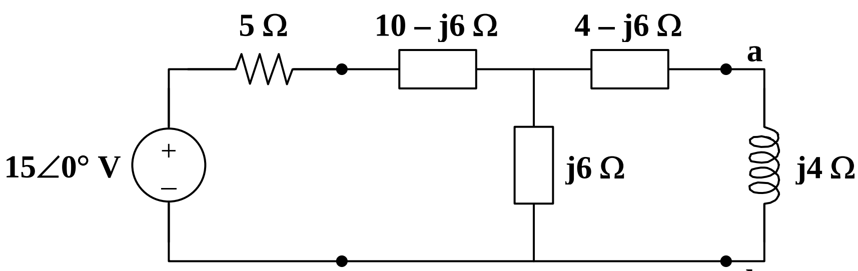
$$\underline{Z_L = Z_{Th} = 24\Omega}$$

(b) $V_{Th} = \frac{z_{21}}{z_{11} + Z_s} V_s = \frac{80}{40+10}(120) = 192$

$$P_{max} = \left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th} = 4^2 x24 = \mathbf{384W}$$

Chapter 19, Solution 16.

As a reciprocal two-port, the given circuit can be represented as shown in Fig. (a).



(a)

At terminals a-b,

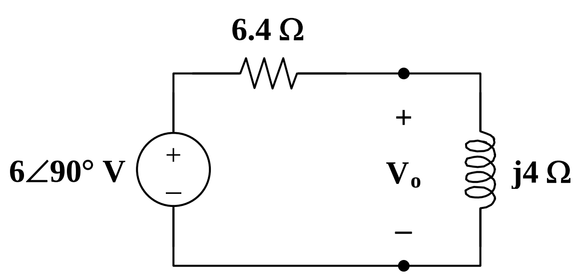
$$\mathbf{Z}_{Th} = (4 - j6) + j6 \,||\, (5 + 10 - j6)$$

$$\mathbf{Z}_{Th} = 4 - j6 + \frac{j6(15 - j6)}{15} = 4 - j6 + 2.4 + j6$$

$$\mathbf{Z}_{Th} = \mathbf{6.4\,\Omega}$$

$$\mathbf{V}_{Th} = \frac{j6}{j6 + 5 + 10 - j6}(15\angle 0°) = j6 = \mathbf{6\angle 90°\ V}$$

The Thevenin equivalent circuit is shown in Fig. (b).



(b)

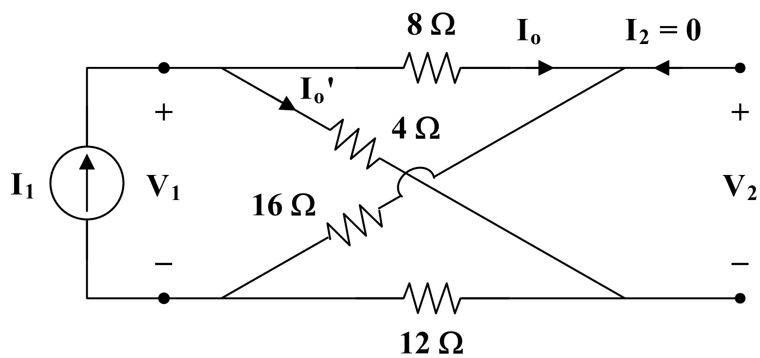
From this,

$$\mathbf{V}_o = \frac{j4}{6.4 + j4}(j6) = 3.18\angle 148°$$

$$v_o(t) = \mathbf{3.18\cos(2t + 148°)\ V}$$

Chapter 19, Solution 17.

To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

In this case, the 8-Ω and 16-Ω resistors are in series, since the same current, $\mathbf{I}_o$, passes through them. Similarly, the 4-Ω and 12-Ω resistors are in series, since the same current, $\mathbf{I}_o^{'}$, passes through them.

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = (8+16) \,||\, (4+12) = 24 \,||\, 16 = \frac{(24)(16)}{40} = \mathbf{9.6\ \Omega}$$

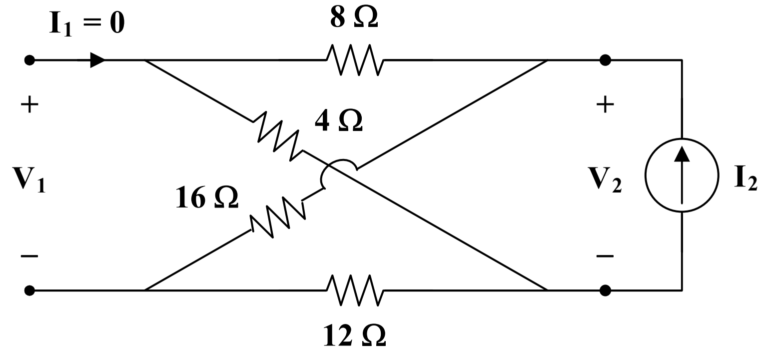
$$\mathbf{I}_o = \frac{16}{16+24}\mathbf{I}_1 = \frac{2}{5}\mathbf{I}_1 \qquad\qquad \mathbf{I}_o^{'} = \frac{3}{5}\mathbf{I}_1$$

But
$$\text{-}\,\mathbf{V}_2 - 8\mathbf{I}_o + 4\mathbf{I}_o^{'} = 0$$

$$\mathbf{V}_2 = \text{-}8\mathbf{I}_o + 4\mathbf{I}_o^{'} = \frac{-16}{5}\mathbf{I}_1 + \frac{12}{5}\mathbf{I}_1 = \frac{-4}{5}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{-4}{5} = \mathbf{-0.8\ \Omega}$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = (8+4)\,||\,(16+12) = 12\,||\,28 = \frac{(12)(28)}{40} = \mathbf{8.4\ \Omega}$$

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{-0.8\ \Omega}$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{9.6} & \mathbf{-0.8} \\ \mathbf{-0.8} & \mathbf{8.4} \end{bmatrix} \mathbf{\Omega}$$

We may take advantage of Table 18.1 to get [**y**] from [**z**].

$$\Delta_z = (9.6)(8.4) - (0.8)^2 = 80$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{8.4}{80} = \mathbf{0.105\ S} \qquad \mathbf{y}_{12} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{0.8}{80} = \mathbf{0.01\ S}$$

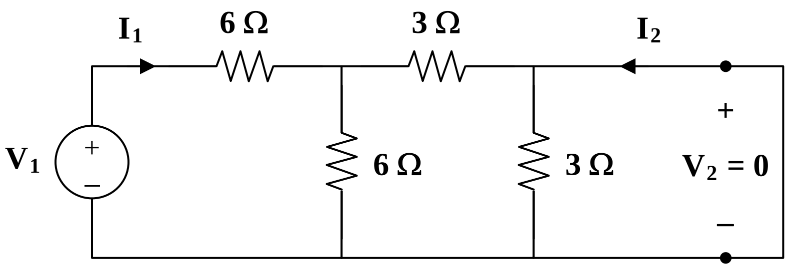
$$\mathbf{y}_{21} = \frac{-\mathbf{z}_{21}}{\Delta_z} = \frac{0.8}{80} = \mathbf{0.01\ S} \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z} = \frac{9.6}{80} = \mathbf{0.12\ S}$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.105} & \mathbf{0.01} \\ \mathbf{0.01} & \mathbf{0.12} \end{bmatrix} \mathbf{S}$$

Chapter 19, Solution 18.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig.(a).



(a)

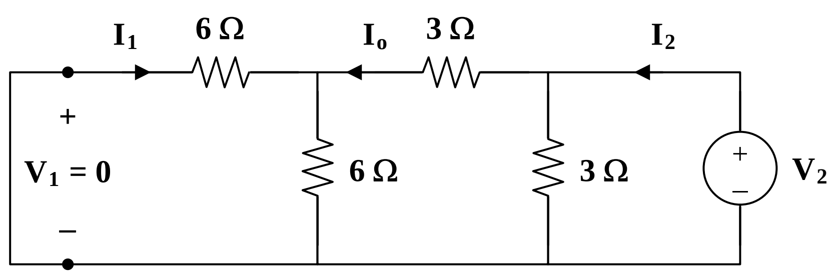
$$\mathbf{V}_1 = (6 + 6 \| 3)\mathbf{I}_1 = 8\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{8}$$

$$\mathbf{I}_2 = \frac{-6}{6+3}\mathbf{I}_1 = \frac{-2}{3}\frac{\mathbf{V}_1}{8} = \frac{-\mathbf{V}_1}{12}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{12}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{3 \| (3 + 6 \| 6)} = \frac{1}{3 \| 6} = \frac{1}{2}$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_o}{2}, \qquad \mathbf{I}_o = \frac{3}{3+6}\mathbf{I}_2 = \frac{1}{3}\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_2}{6} = \left(\frac{-1}{6}\right)\left(\frac{1}{2}\mathbf{V}_2\right) = \frac{-\mathbf{V}_2}{12}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{12} = \mathbf{y}_{21}$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \frac{\mathbf{1}}{\mathbf{8}} & \frac{\mathbf{-1}}{\mathbf{12}} \\ \frac{\mathbf{-1}}{\mathbf{12}} & \frac{\mathbf{1}}{\mathbf{2}} \end{bmatrix} \mathbf{S}$$

Chapter 19, Solution 19.

Using Fig. 19.80, design a problem to help other students to better understand how to find y parameters in the s-domain.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the *y* parameters of the two-port in Fig.19.80 in terms of *s*.

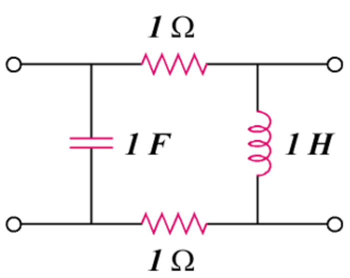
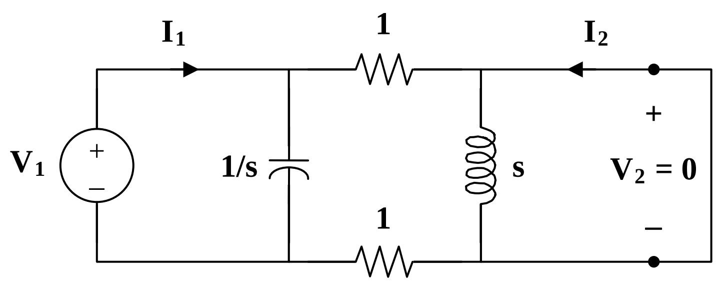


Figure 19.80

Solution

Consider the circuit in Fig.(a) for calculating $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$.



(a)

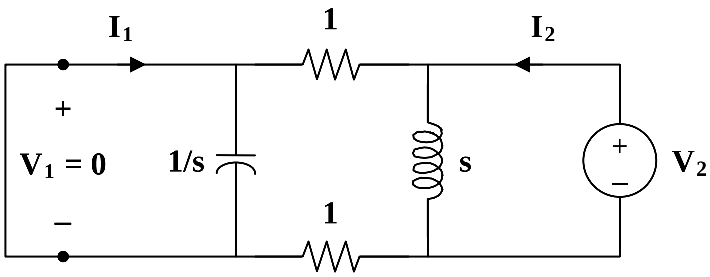
$$\mathbf{V}_1 = \left(\frac{1}{s} \parallel 2\right)\mathbf{I}_1 = \frac{2/s}{2 + (1/s)}\mathbf{I}_1 = \frac{2}{2s+1}\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{2s+1}{2} = s + 0.5$$

$$\mathbf{I}_2 = \frac{(-1/s)}{(1/s)+2}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{2s+1} = \frac{-\mathbf{V}_1}{2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \text{-}0.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig.(b).



(b)

$$\mathbf{V}_2 = (\text{s} \parallel 2)\,\mathbf{I}_2 = \frac{2\text{s}}{\text{s}+2}\mathbf{I}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\text{s}+2}{2\text{s}} = 0.5 + \frac{1}{\text{s}}$$

$$\mathbf{I}_1 = \frac{\text{-}\,\text{s}}{\text{s}+2}\mathbf{I}_2 = \frac{\text{-}\,\text{s}}{\text{s}+2}\cdot\frac{\text{s}+2}{2\text{s}}\mathbf{V}_2 = \frac{\text{-}\,\mathbf{V}_2}{2}$$

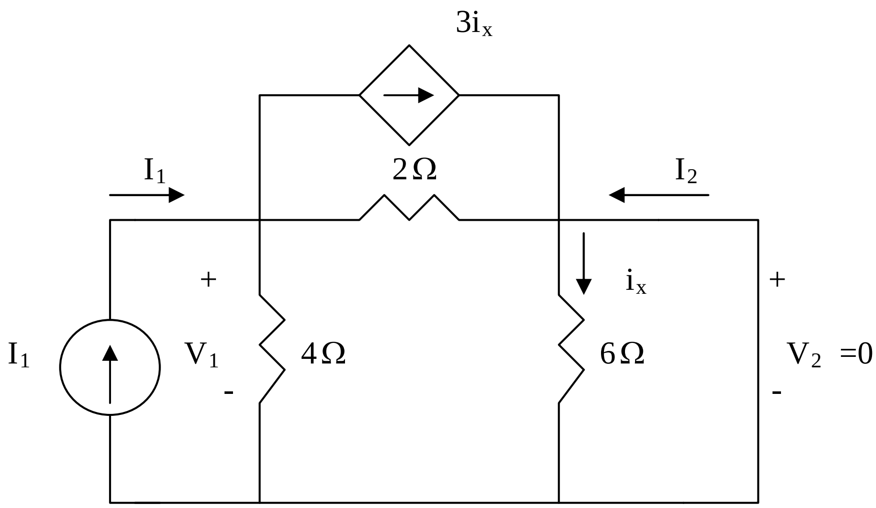
$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \text{-}0.5$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{s+0.5} & \mathbf{-\,0.5} \\ \mathbf{-\,0.5} & \mathbf{0.5+1/s} \end{bmatrix}\mathbf{S}$$

Chapter 19, Solution 20.

To get y_{11} and y_{21}, consider the circuit below.

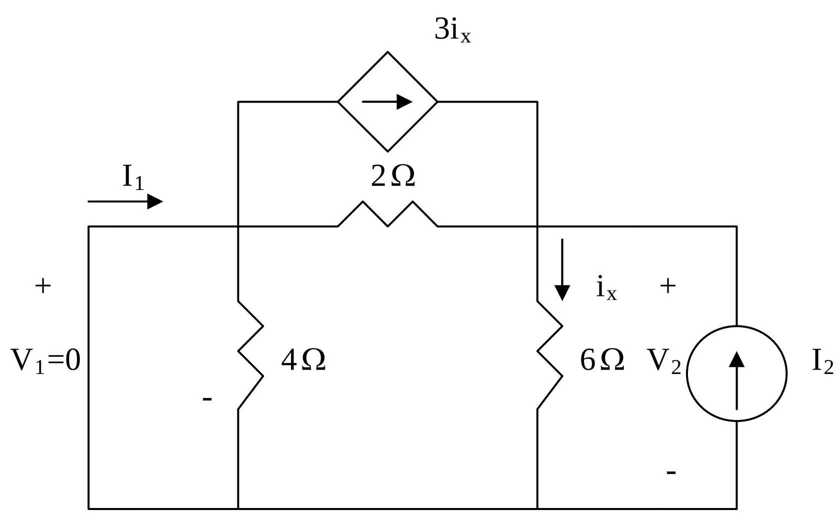


Since 6-ohm resistor is short-circuited, $i_x = 0$

$$V_1 = I_1(4//2) = \frac{8}{6}I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = 0.75$$

$$I_2 = -\frac{4}{4+2}I_1 = -\frac{2}{3}(\frac{6}{8}V_1) = -\frac{1}{2}V_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = -0.5$$

To get y_{22} and y_{12}, consider the circuit below.



$$i_x = \frac{V_2}{6}, \quad I_2 = i_x - 3i_x + \frac{V_2}{2} = \frac{V_2}{6} \longrightarrow y_{22} = \frac{I_2}{V_2} = \frac{1}{6} = 0.1667$$

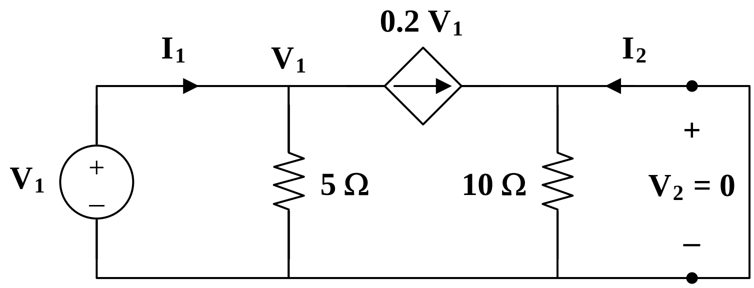
$$I_1 = 3i_x - \frac{V_2}{2} = 0 \longrightarrow y_{12} = \frac{I_1}{V_2} = 0$$

Thus,

$$\underline{\mathbf{[y]} = \begin{bmatrix} \mathbf{0.75} & \mathbf{0} \\ \mathbf{-0.5} & \mathbf{0.1667} \end{bmatrix} \mathbf{S}}$$

Chapter 19, Solution 21.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, refer to Fig. (a).



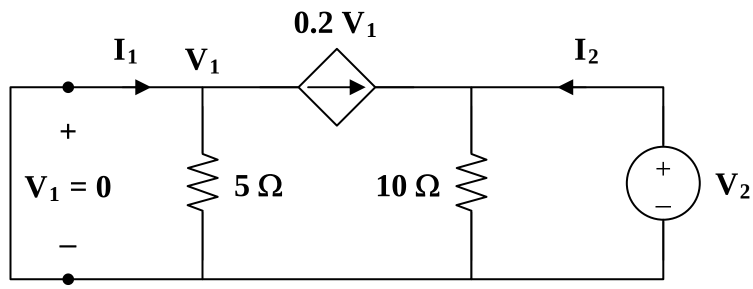
(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{5} + 0.2\mathbf{V}_1 = 0.4\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 0.4$$

$$\mathbf{I}_2 = -0.2\mathbf{V}_1 \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.2$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (b).



(b)

Since $\mathbf{V}_1 = 0$, the dependent current source can be replaced with an open circuit.

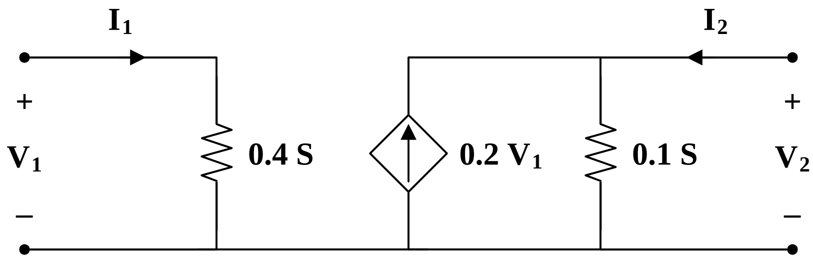
$$\mathbf{V}_2 = 10\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{10} = 0.1$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} 0.4 & 0 \\ -0.2 & 0.1 \end{bmatrix} \text{S}$$

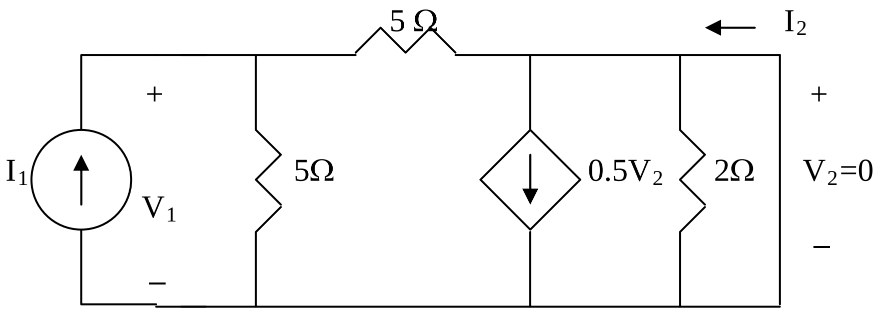
Consequently, **the y parameter equivalent circuit is shown in Fig. (c)**.



(c)

Chapter 19, Solution 22.

To obtain y_{11} and y_{21}, consider the circuit below.

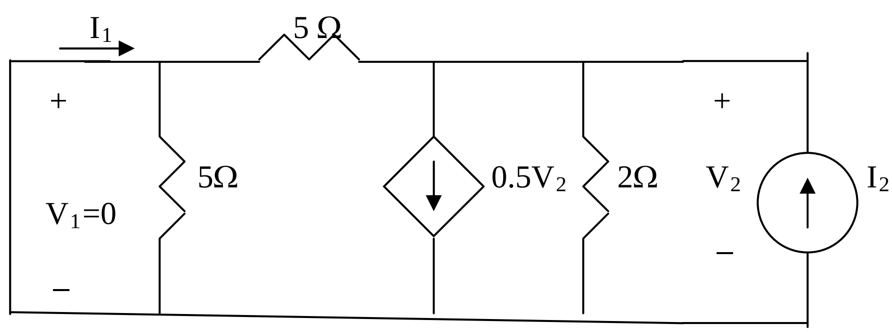


The 2-Ω resistor is short-circuited.

$$V_1 = 5\frac{I_1}{2} \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{2}{5} = 0.4$$

$$I_2 = \frac{1}{2}I_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = \frac{\frac{1}{2}I_1}{2.5I_1} = 0.2$$

To obtain y_{12} and y_{22}, consider the circuit below.



At the top node, KCL gives

$$I_2 = 0.5V_2 + \frac{V_2}{2} + \frac{V_2}{5} = 1.2V_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = 1.2$$

$$I_1 = -\frac{V_2}{5} = -0.2V_2 \longrightarrow y_{12} = \frac{I_1}{V_2} = -0.2$$

Hence,

$$\underline{[y] = \begin{bmatrix} 0.4 & -0.2 \\ 0.2 & 1.2 \end{bmatrix} \text{S}}$$

Chapter 19, Solution 23.

(a)

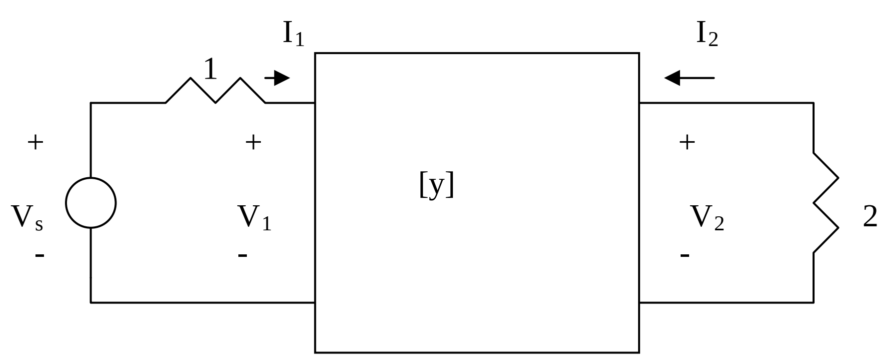
$$1/(-y_{12}) = 1 // \frac{1}{s} = \frac{1}{s+1} \qquad \longrightarrow \qquad y_{12} = -(s+1)$$

$$y_{11} + y_{12} = 1 \qquad \longrightarrow \qquad y_{11} = 1 - y_{12} = 1 + (s+1) = s+2$$

$$y_{22} + y_{12} = s \qquad \longrightarrow \qquad y_{22} = s - y_{12} = \frac{1}{s} + (s+1) = \frac{s^2+s+1}{s}$$

$$\underline{[y] = \begin{bmatrix} s+2 & -(s+1) \\ -(s+1) & \dfrac{s^2+s+1}{s} \end{bmatrix}}$$

(b) Consider the network below.



$$V_s = I_1 + V_1 \qquad (1)$$

$$V_2 = -2I_2 \qquad (2)$$

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad (3)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad (4)$$

From (1) and (3)

$$V_s - V_1 = y_{11}V_1 + y_{12}V_2 \quad \longrightarrow \quad V_s = (1 + y_{11})V_1 + y_{12}V_2 \qquad (5)$$

From (2) and (4),

$$-0.5V_2 = y_{21}V_1 + y_{22}V_2 \quad \longrightarrow \quad V_1 = -\frac{1}{y_{21}}(0.5 + y_{22})V_2 \qquad (6)$$

Substituting (6) into (5),

$$\mathbf{V_s = -\frac{(1+y_{11})(0.5+y_{22})}{y_{21}}V_2 + y_{12}V_2}$$

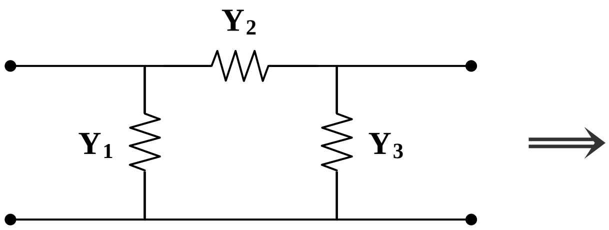
$$\mathbf{= \frac{2}{s} \quad \longrightarrow \quad V_2 = \frac{2/s}{\left[y_{12} - \frac{1}{y_{21}}(1+y_{11})(0.5+y_{22})\right]}}$$

$$\mathbf{V_2 = \frac{2/s}{-(s+1) + \frac{1}{s+1}(1+s+2)\left(0.5 + \frac{s^2+s+1}{s}\right)} = \frac{2/s}{\frac{-s^3 - s^2 - s^2 - s + (s+3)(0.5s + s^2 + s + 1)}{s(s+1)}}}$$

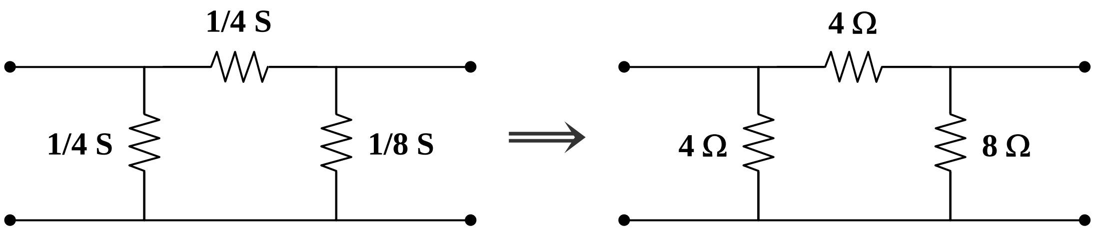
$$\mathbf{= \frac{2(s+1)}{-s^3 - 2s^2 - s + s^3 + 1.5s^2 + s + 3s^2 + 4.5s + 3} = \frac{2(s+1)}{2.5s^2 + 4.5s + 3} = \frac{0.8(s+1)}{s^2 1.8s + 1.2}}$$

Chapter 19, Solution 24.

Since this is a reciprocal network, **a Π network is appropriate, as shown below**.



(a)



(b) (c)

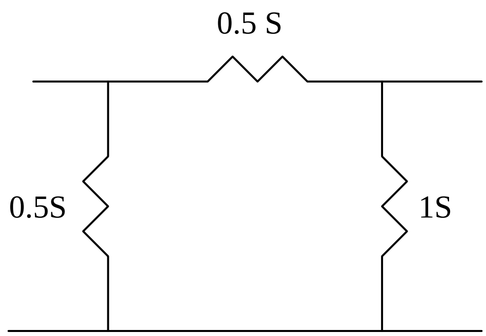
$$\mathbf{Y}_1 = \mathbf{y}_{11} + \mathbf{y}_{12} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\,\mathrm{S}, \qquad \mathbf{Z}_1 = \mathbf{4\,\Omega}$$

$$\mathbf{Y}_2 = -\mathbf{y}_{12} = \frac{1}{4}\,\mathrm{S}, \qquad \mathbf{Z}_2 = \mathbf{4\,\Omega}$$

$$\mathbf{Y}_3 = \mathbf{y}_{22} + \mathbf{y}_{21} = \frac{3}{8} - \frac{1}{4} = \frac{1}{8}\,\mathrm{S}, \qquad \mathbf{Z}_3 = \mathbf{8\,\Omega}$$

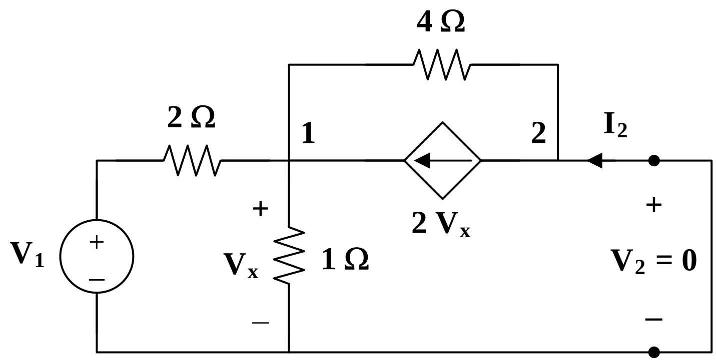
Chapter 19, Solution 25.

This is a reciprocal network and is shown below.



Chapter 19, Solution 26.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).



(a)

At node 1,

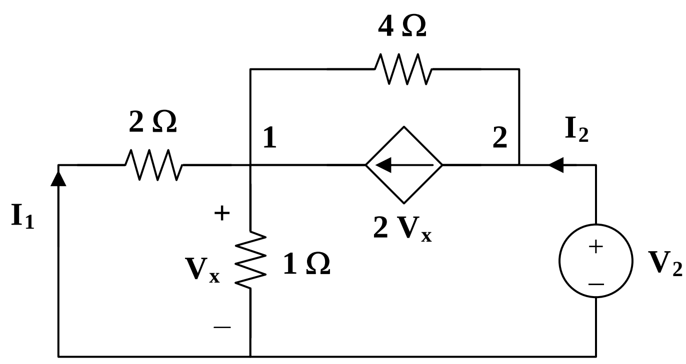
$$\frac{\mathbf{V}_1 - \mathbf{V}_x}{2} + 2\mathbf{V}_x = \frac{\mathbf{V}_x}{1} + \frac{\mathbf{V}_x}{4} \quad \longrightarrow \quad 2\mathbf{V}_1 = \text{-}\mathbf{V}_x \qquad (1)$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{2} = \frac{\mathbf{V}_1 + 2\mathbf{V}_1}{2} = 1.5\mathbf{V}_1 \quad \longrightarrow \quad \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 1.5$$

Also, $$\mathbf{I}_2 + \frac{\mathbf{V}_x}{4} = 2\mathbf{V}_x \quad \longrightarrow \quad \mathbf{I}_2 = 1.75\mathbf{V}_x = \text{-}3.5\mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \text{-}3.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

At node 2,

$$\mathbf{I}_2 = 2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} \qquad (2)$$

At node 1,

$$2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} = \frac{\mathbf{V}_x}{2} + \frac{\mathbf{V}_x}{1} = \frac{3}{2}\mathbf{V}_x \longrightarrow \mathbf{V}_2 = \text{-}\mathbf{V}_x \qquad (3)$$

Substituting (3) into (2) gives

$$\mathbf{I}_2 = 2\mathbf{V}_x - \frac{1}{2}\mathbf{V}_x = 1.5\mathbf{V}_x = \text{-}1.5\mathbf{V}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \text{-}1.5$$

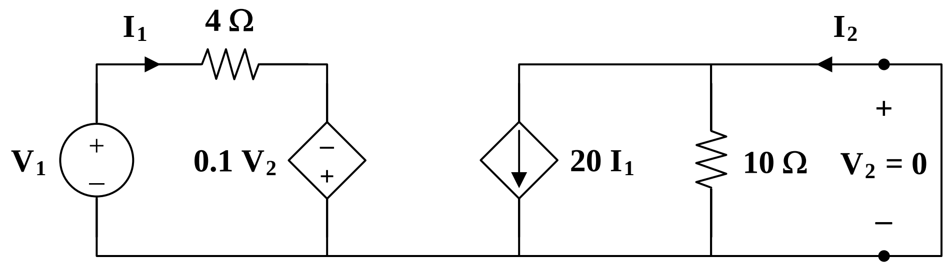
$$\mathbf{I}_1 = \frac{\text{-}\mathbf{V}_x}{2} = \frac{\mathbf{V}_2}{2} \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0.5$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{1.5} & \mathbf{0.5} \\ \mathbf{-3.5} & \mathbf{-1.5} \end{bmatrix} \mathbf{S}$$

Chapter 19, Solution 27.

Consider the circuit in Fig. (a).

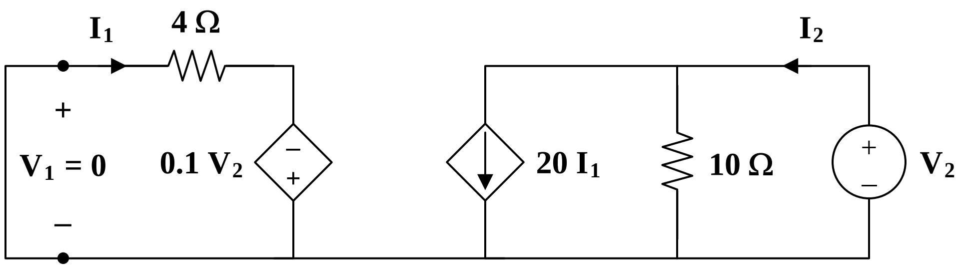


(a)

$$\mathbf{V}_1 = 4\mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{4\mathbf{I}_1} = 0.25$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 = 5\mathbf{V}_1 \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = 5$$

Consider the circuit in Fig. (b).



(b)

$$4\mathbf{I}_1 = 0.1\mathbf{V}_2 \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{0.1}{4} = 0.025$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 + \frac{\mathbf{V}_2}{10} = 0.5\mathbf{V}_2 + 0.1\mathbf{V}_2 = 0.6\mathbf{V}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = 0.6$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.25} & \mathbf{0.025} \\ \mathbf{5} & \mathbf{0.6} \end{bmatrix}\mathbf{S}$$

Alternatively, from the given circuit,

$$\mathbf{V}_1 = 4\mathbf{I}_1 - 0.1\mathbf{V}_2$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 + 0.1\mathbf{V}_2$$

Comparing these with the equations for the h parameters show that

$$\mathbf{h}_{11} = 4, \qquad \mathbf{h}_{12} = -0.1, \qquad \mathbf{h}_{21} = 20, \qquad \mathbf{h}_{22} = 0.1$$

Using Table 18.1,

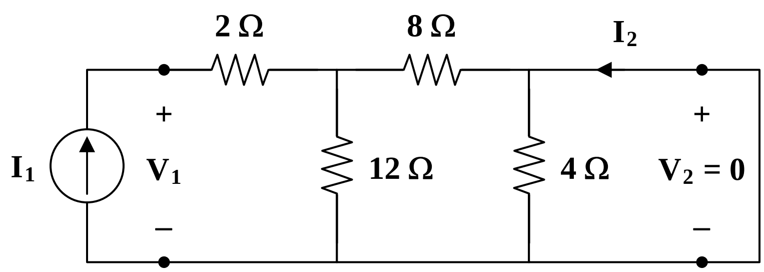
$$\mathbf{y}_{11} = \frac{\mathbf{1}}{\mathbf{h}_{11}} = \frac{\mathbf{1}}{\mathbf{4}} = \mathbf{0.25\,S}, \qquad \mathbf{y}_{12} = \frac{\mathbf{-h}_{12}}{\mathbf{h}_{11}} = \frac{\mathbf{0.1}}{\mathbf{4}} = \mathbf{0.025\,S}$$

$$\mathbf{y}_{21} = \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} = \frac{\mathbf{20}}{\mathbf{4}} = \mathbf{5\,S}, \qquad \mathbf{y}_{22} = \frac{\Delta_{\mathbf{h}}}{\mathbf{h}_{11}} = \frac{\mathbf{0.4+2}}{\mathbf{4}} = \mathbf{0.6\,S}$$

as above.

Chapter 19, Solution 28.

We obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by considering the circuit in Fig.(a).



(a)

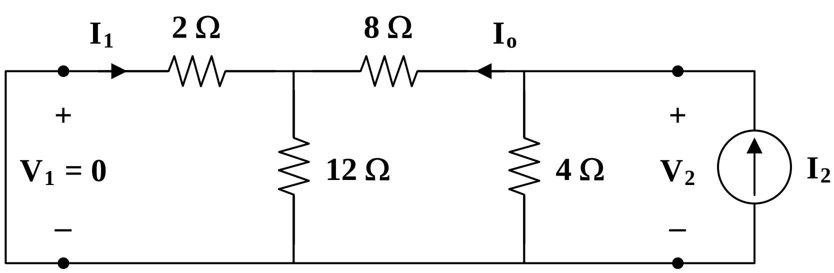
$\mathbf{Z_{in}} = 2 + (12||8) = 6.8\ \Omega$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{\mathbf{Z}_{in}} = \mathbf{147.06\ mS}$$

$\mathbf{I_2} = (-6/10)\mathbf{I_1} = (-0.6)(\mathbf{V_1}/6.8) = -0.08824$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \mathbf{-88.24\ mS}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig. (b).



(b)

$(1/\mathbf{y_{22}}) = [4||(8+(12||2))] = [4||(8+(1.714286))] = 2.833333 = \mathbf{V_2/I_2}$

$\mathbf{y_{22}} = \mathbf{352.9\ mS}$

$\mathbf{I_1} = (-12/14)\mathbf{I_o} = -0.857143\mathbf{I_o}$ and $\mathbf{I_o} = [4/(4+(8+1.714286))]\mathbf{I_2}$
$= 0.29166667\mathbf{I_2} = \mathbf{V_2}/9.714286$

Thus, $\mathbf{I_1} = [(-0.857143)/9.714286]\mathbf{V_2} = -0.088235\mathbf{V_2}$ or

$\mathbf{y_{12} = I_1/V_2} = \mathbf{-88.24\ mS}$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{147.06} & \mathbf{-88.24} \\ \mathbf{-88.24} & \mathbf{352.9} \end{bmatrix} \mathbf{mS}$$

We note that $\mathbf{I} = \mathbf{YV}$, $\mathbf{I_1} = 1$ A, and $-\mathbf{I_2} = \mathbf{V_2}/2$. We now have the following equations,

$1 = 0.14706\mathbf{V_1} - 0.08824\mathbf{V_2}$ and $\mathbf{I_2} = -0.08824\mathbf{V_1} + 0.3529\mathbf{V_2}$ or
$-\mathbf{V_2}/2 = -0.08824\mathbf{V_1} + 0.3529\mathbf{V_2}$ or $0.08824\mathbf{V_1} = 0.8529\mathbf{V_2}$ which leads to
$\mathbf{V_1} = 9.6657\mathbf{V_2}$.

Substituting this into the first equation we get,

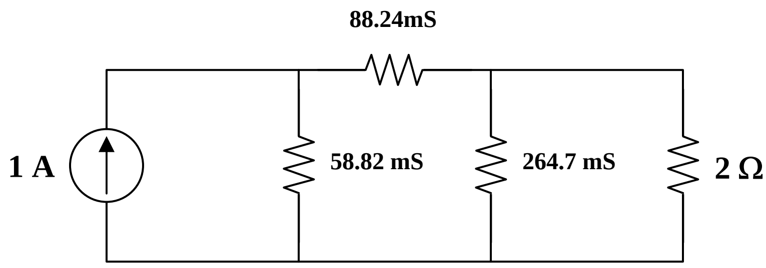
$$1 = (1.42144 - 0.08824)\mathbf{V_2} \text{ or } \mathbf{V_2} = 0.75 \text{ V.}$$

Finally we get,

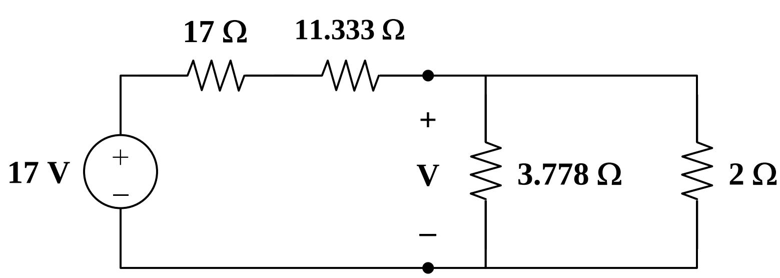
$$P_{2\Omega} = (0.75)^2/2 = \mathbf{281.2\ mW}.$$

Now to check our answer.

The equivalent circuit is shown in Fig. (c). After transforming the current source to a voltage source, we have the circuit in Fig. (d).



(c)



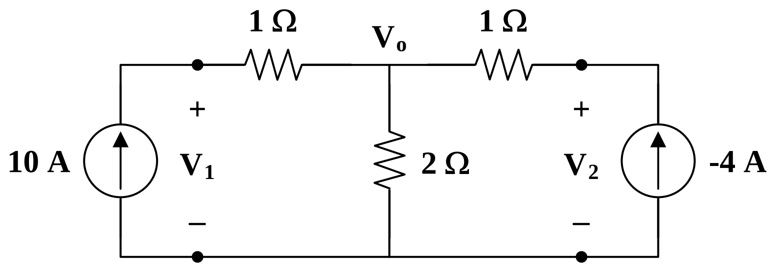
(d)

$$V = \frac{(2 \,||\, 3.778)(17)}{(2 \,||\, 3.778) + 17 + 11.333} = \frac{(1.3077)(17)}{1.3077 + 28.333} = 0.75\,\text{V}$$

$$P = \frac{V^2}{R} = \frac{(0.75)^2}{2} = \mathbf{281.2\ mW}$$

Chapter 19, Solution 29.

(a) Transforming the Δ subnetwork to Y gives the circuit in Fig. (a).



(a)

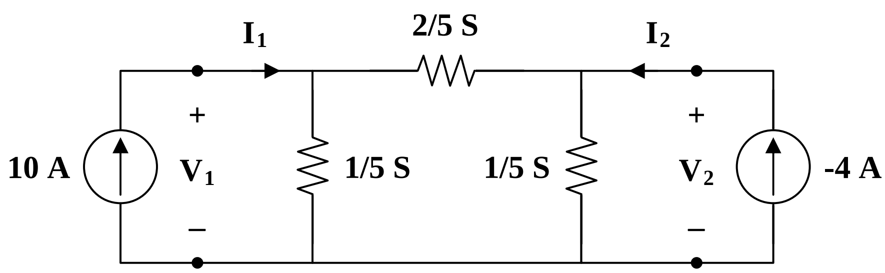
It is easy to get the z parameters

$$\mathbf{z}_{12} = \mathbf{z}_{21} = 2, \quad \mathbf{z}_{11} = 1 + 2 = 3, \quad \mathbf{z}_{22} = 3$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 9 - 4 = 5$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{3}{5} = \mathbf{y}_{22}, \qquad \mathbf{y}_{12} = \mathbf{y}_{21} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{-2}{5}$$

Thus, the equivalent circuit is as shown in Fig. (b).



(b)

$$\mathbf{I}_1 = 10 = \frac{3}{5}\mathbf{V}_1 - \frac{2}{5}\mathbf{V}_2 \longrightarrow 50 = 3\mathbf{V}_1 - 2\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = -4 = \frac{-2}{5}\mathbf{V}_1 + \frac{3}{5}\mathbf{V}_2 \longrightarrow -20 = -2\mathbf{V}_1 + 3\mathbf{V}_2$$

$$10 = \mathbf{V}_1 - 1.5\mathbf{V}_2 \longrightarrow \mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$50 = 30 + 4.5\mathbf{V}_2 - 2\mathbf{V}_2 \longrightarrow \mathbf{V}_2 = \mathbf{8\ V}$$

$$\mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 = \mathbf{22\ V}$$

(b) For direct circuit analysis, consider the circuit in Fig. (a).

For the main non-reference node,

$$10 - 4 = \frac{\mathbf{V}_o}{2} \longrightarrow \mathbf{V}_o = 12$$

$$10 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_1 = 10 + \mathbf{V}_o = \mathbf{22\ V}$$

$$-4 = \frac{\mathbf{V}_2 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_2 = \mathbf{V}_o - 4 = \mathbf{8\ V}$$

Chapter 19, Solution 30.

(a) Convert to z parameters; then, convert to h parameters using Table 18.1.

$$\mathbf{z}_{11} = \mathbf{z}_{12} = \mathbf{z}_{21} = 60\ \Omega, \qquad \mathbf{z}_{22} = 100\ \Omega$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 6000 - 3600 = 2400$$

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}} = \frac{2400}{100} = 24, \qquad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} = \frac{60}{100} = 0.6$$

$$\mathbf{h}_{21} = \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} = -0.6, \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} = 0.01$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{24\ \Omega} & \mathbf{0.6} \\ \mathbf{-0.6} & \mathbf{0.01\ S} \end{bmatrix}$$

(b) Similarly,

$$\mathbf{z}_{11} = 30\ \Omega \qquad \mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{z}_{22} = 20\ \Omega$$

$$\Delta_z = 600 - 400 = 200$$

$$\mathbf{h}_{11} = \frac{200}{20} = 10 \qquad \mathbf{h}_{12} = \frac{20}{20} = 1$$

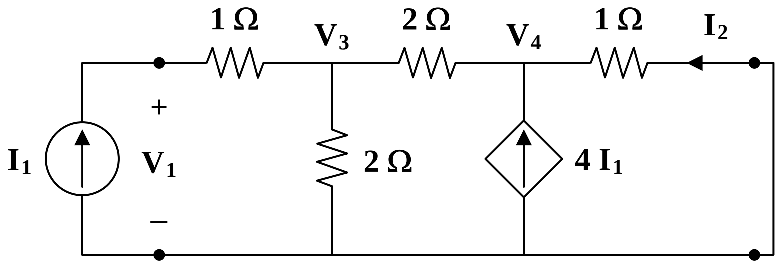
$$\mathbf{h}_{21} = -1 \qquad \mathbf{h}_{22} = \frac{1}{20} = 0.05$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{10\ \Omega} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0.05\ S} \end{bmatrix}$$

Chapter 19, Solution 31.

We get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by considering the circuit in Fig. (a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_3}{2} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{2} \longrightarrow 2\mathbf{I}_1 = 2\mathbf{V}_3 - \mathbf{V}_4 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_3 - \mathbf{V}_4}{2} + 4\mathbf{I}_1 = \frac{\mathbf{V}_4}{1}$$

$$8\mathbf{I}_1 = \text{-}\mathbf{V}_3 + 3\mathbf{V}_4 \longrightarrow 16\mathbf{I}_1 = \text{-}2\mathbf{V}_3 + 6\mathbf{V}_4 \qquad (2)$$

Adding (1) and (2),

$$18\mathbf{I}_1 = 5\mathbf{V}_4 \longrightarrow \mathbf{V}_4 = 3.6\mathbf{I}_1$$

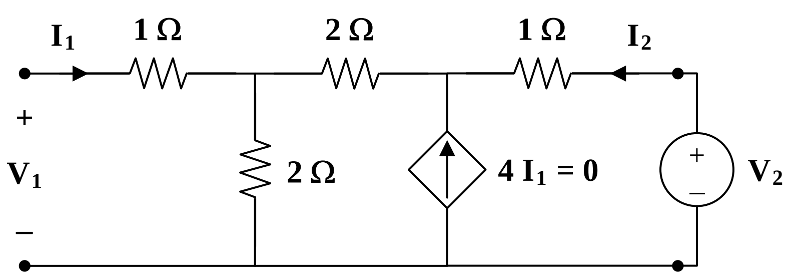
$$\mathbf{V}_3 = 3\mathbf{V}_4 - 8\mathbf{I}_1 = 2.8\mathbf{I}_1$$

$$\mathbf{V}_1 = \mathbf{V}_3 + \mathbf{I}_1 = 3.8\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 3.8\,\Omega$$

$$\mathbf{I}_2 = \frac{\text{-}\mathbf{V}_4}{1} = \text{-}3.6\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \text{-}3.6$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b). The dependent current source can be replaced by an open circuit since $4\mathbf{I}_1 = 0$.



(b)

$$\mathbf{V}_1 = \frac{2}{2+2+1}\mathbf{V}_2 = \frac{2}{5}\mathbf{V}_2 \quad \longrightarrow \quad \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 0.4$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2+2+1} = \frac{\mathbf{V}_2}{5} \quad \longrightarrow \quad \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{5} = 0.2\text{ S}$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{38\,\Omega} & \mathbf{0.4} \\ \mathbf{-3.6} & \mathbf{0.2\ S} \end{bmatrix}$$

Chapter 19, Solution 32.

Using Fig. 19.90, design a problem to help other students to better understand how to find the *h* and *g* parameters for a circuit in the s-domain.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the *h* and *g* parameters of the two-port network in Fig.19.90 as functions of *s*.

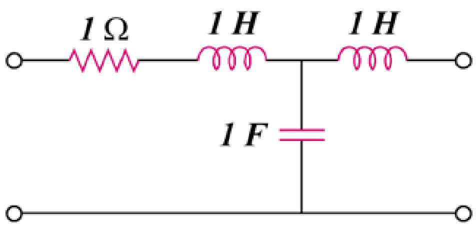
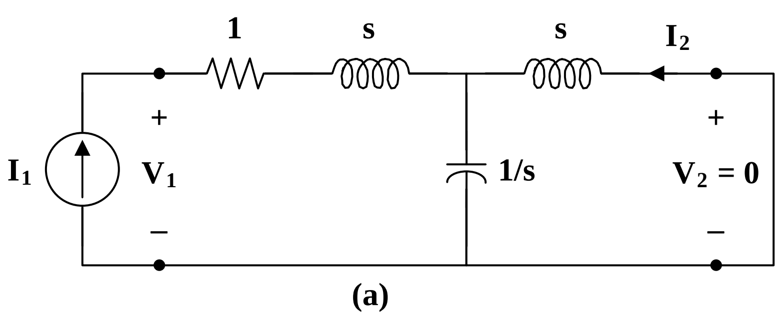


Figure 19.90

Solution

(a) We obtain $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by referring to the circuit in Fig. (a).



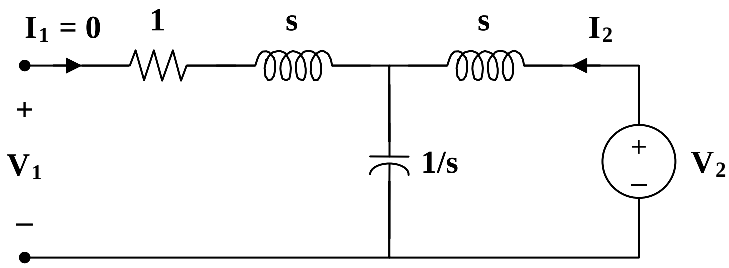
$$\mathbf{V}_1 = \left(1 + s + s \,||\, \frac{1}{s}\right)\mathbf{I}_1 = \left(1 + s + \frac{s}{s^2+1}\right)\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = s + 1 + \frac{s}{s^2+1}$$

By current division,

$$\mathbf{I}_2 = \frac{-1/s}{s + 1/s}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{s+1} \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{s^2+1}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

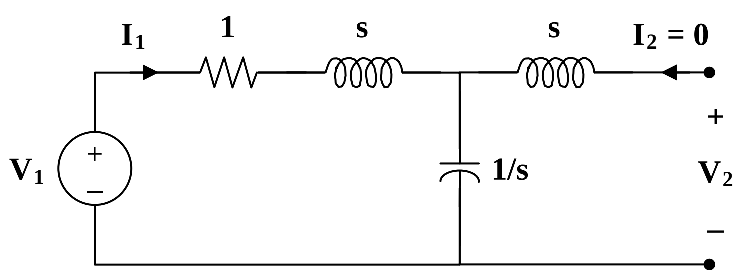
$$\mathbf{V}_1 = \frac{1/s}{s+1/s}\mathbf{V}_2 = \frac{\mathbf{V}_2}{s^2+1} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{s^2+1}$$

$$\mathbf{V}_2 = \left(s+\frac{1}{s}\right)\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{s+1/s} = \frac{s}{s^2+1}$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{s+1+\frac{s}{s^2+1}} & \mathbf{\frac{1}{s^2+1}} \\ \mathbf{\frac{-1}{s^2+1}} & \mathbf{\frac{s}{s^2+1}} \end{bmatrix}$$

(b) To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).

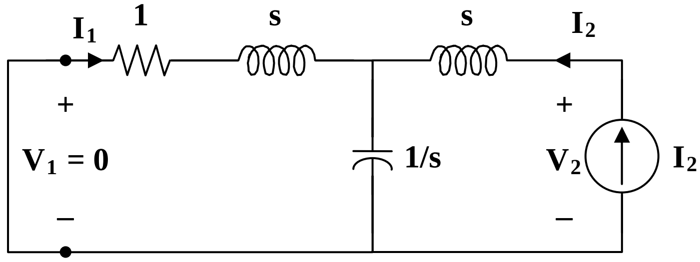


(c)

$$\mathbf{V}_1 = \left(1+s+\frac{1}{s}\right)\mathbf{I}_1 \longrightarrow \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{1+s+1/s} = \frac{s}{s^2+s+1}$$

$$\mathbf{V}_2 = \frac{1/s}{1+s+1/s}\mathbf{V}_1 = \frac{\mathbf{V}_1}{s^2+s+1} \longrightarrow \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s^2+s+1}$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$\mathbf{V}_2 = \left(s + \frac{1}{s} \,\|\, (s+1)\right)\mathbf{I}_2 = \left(s + \frac{(s+1)/s}{1+s+1/s}\right)\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = s + \frac{s+1}{s^2+s+1}$$

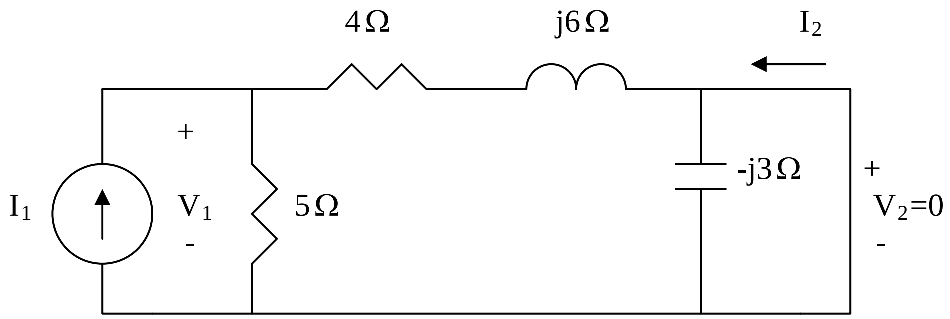
$$\mathbf{I}_1 = \frac{-1/s}{1+s+1/s}\mathbf{I}_2 = \frac{-\mathbf{I}_2}{s^2+s+1} \longrightarrow \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-1}{s^2+s+1}$$

Thus,

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{\mathbf{s}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} & \dfrac{-\mathbf{1}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} \\ \dfrac{\mathbf{1}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} & \mathbf{s}+\dfrac{\mathbf{s}+\mathbf{1}}{\mathbf{s}^2+\mathbf{s}+\mathbf{1}} \end{bmatrix}$$

Chapter 19, Solution 33.

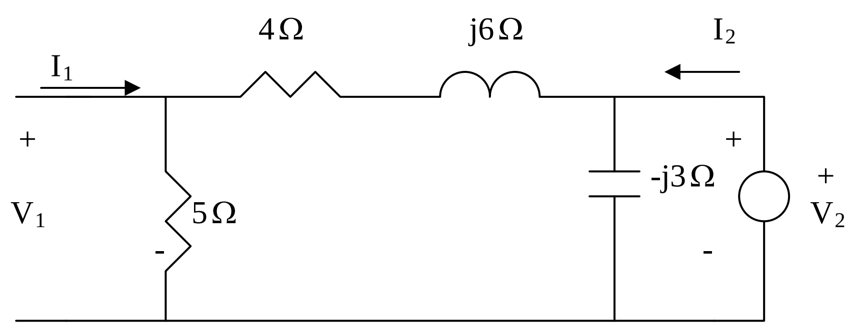
To get h_{11} and h_{21}, consider the circuit below.



$$V_1 = 5//(4+j6)I_1 = \frac{5(4+j6)I_1}{9+j6} \qquad h_{11} = \frac{V_1}{I_1} = 3.0769 + j1.2821$$

Also, $I_2 = -\frac{5}{9+j6} I_1 \longrightarrow h_{21} = \frac{I_2}{I_1} = -0.3846 + j0.2564$

To get h_{22} and h_{12}, consider the circuit below.



$$V_1 = \frac{5}{9+j6} V_2 \longrightarrow h_{12} = \frac{V_1}{V_2} = \frac{5}{9+j6} = 0.3846 - j0.2564$$

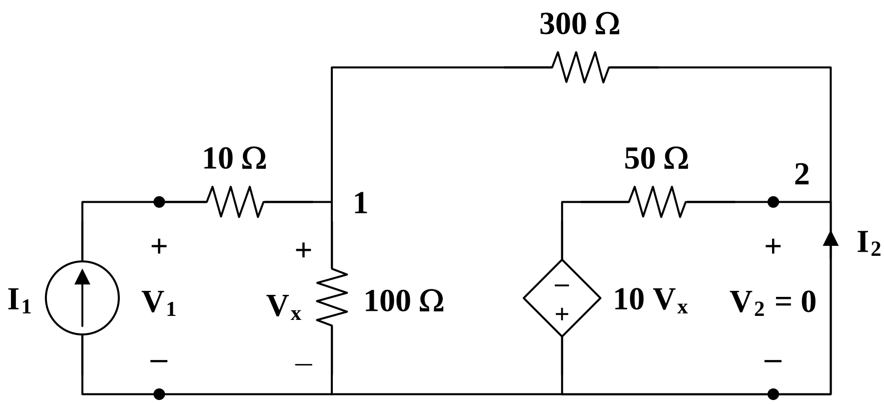
$$V_2 = -j3//(9+j6)I_2 \longrightarrow h_{22} = \frac{I_2}{V_2} = \frac{1}{-j3//(9+j6)} = \frac{9+j3}{-j3(9+j6)}$$
$$= 0.0769 + j0.2821$$

Thus,

$$\mathbf{[h]} = \begin{bmatrix} \mathbf{(3.077 + j1.2821)\ \Omega} & \mathbf{0.3846 - j0.2564} \\ \mathbf{-0.3846 + j0.2564} & \underline{\mathbf{(0.0769 + j0.2821)\ S}} \end{bmatrix}$$

Chapter 19, Solution 34.

Refer to Fig. (a) to get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$.



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x - 0}{300} \longrightarrow 300\mathbf{I}_1 = 4\mathbf{V}_x \qquad (1)$$

$$\mathbf{V}_x = \frac{300}{4}\mathbf{I}_1 = 75\mathbf{I}_1$$

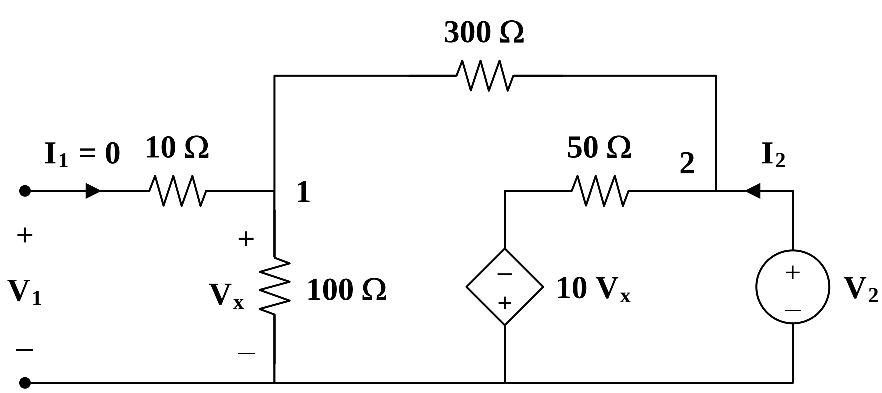
But $\mathbf{V}_1 = 10\mathbf{I}_1 + \mathbf{V}_x = 85\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \dfrac{\mathbf{V}_1}{\mathbf{I}_1} = 85\,\Omega$

At node 2,

$$\mathbf{I}_2 = \frac{0 + 10\mathbf{V}_x}{50} - \frac{\mathbf{V}_x}{300} = \frac{\mathbf{V}_x}{5} - \frac{\mathbf{V}_x}{300} = \frac{75}{5}\mathbf{I}_1 - \frac{75}{300}\mathbf{I}_1 = 14.75\mathbf{I}_1$$

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = 14.75$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

At node 2,

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{400} + \frac{\mathbf{V}_2 + 10\mathbf{V}_x}{50} \longrightarrow 400\mathbf{I}_2 = 9\mathbf{V}_2 + 80\mathbf{V}_x$$

But $\mathbf{V}_x = \frac{100}{400}\mathbf{V}_2 = \frac{\mathbf{V}_2}{4}$

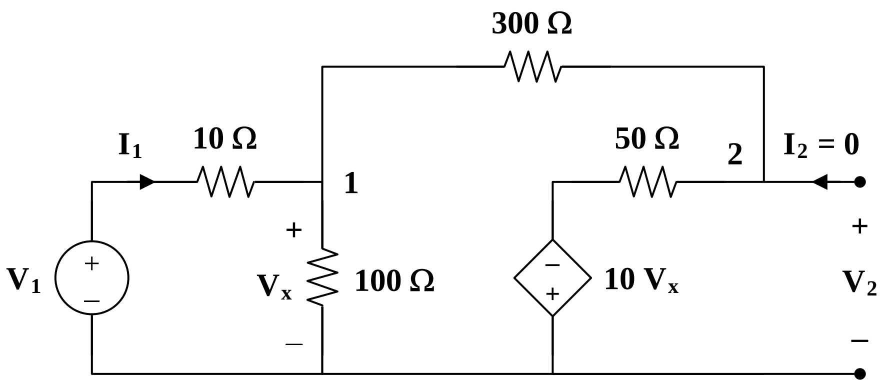
Hence, $400\mathbf{I}_2 = 9\mathbf{V}_2 + 20\mathbf{V}_2 = 29\mathbf{V}_2$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{29}{400} = 0.0725\text{ S}$$

$$\mathbf{V}_1 = \mathbf{V}_x = \frac{\mathbf{V}_2}{4} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{4} = 0.25$$

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{85\,\Omega} & \mathbf{0.25} \\ \mathbf{14.75} & \mathbf{0.0725\ S} \end{bmatrix}$$

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).



(c)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x + 10\mathbf{V}_x}{350} \longrightarrow 350\mathbf{I}_1 = 14.5\mathbf{V}_x \qquad (2)$$

But $\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{10} \longrightarrow 10\mathbf{I}_1 = \mathbf{V}_1 - \mathbf{V}_x$

or $\mathbf{V}_x = \mathbf{V}_1 - 10\mathbf{I}_1 \qquad (3)$

Substituting (3) into (2) gives

$$350\mathbf{I}_1 = 14.5\mathbf{V}_1 - 145\mathbf{I}_1 \longrightarrow 495\mathbf{I}_1 = 14.5\mathbf{V}_1$$

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{14.5}{495} = 0.02929 \text{ S}$$

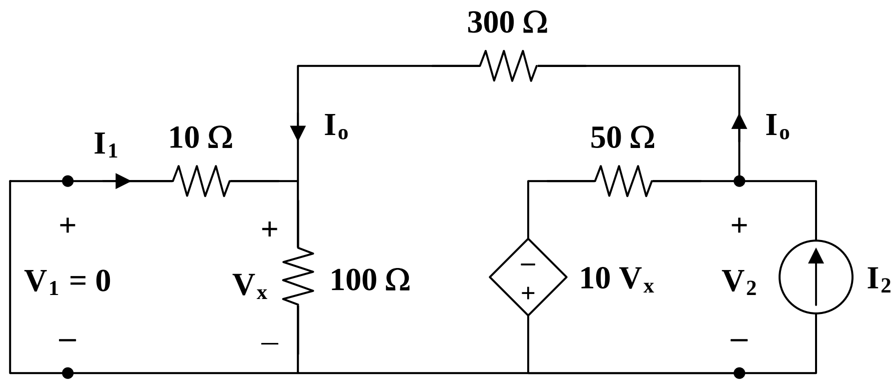
At node 2,

$$\mathbf{V}_2 = (50)\left(\frac{11}{350}\mathbf{V}_x\right) - 10\,\mathbf{V}_x = \text{-}8.4286\,\mathbf{V}_x$$

$$= \text{-}8.4286\,\mathbf{V}_1 + 84.286\,\mathbf{I}_1 = \text{-}8.4286\,\mathbf{V}_1 + (84.286)\left(\frac{14.5}{495}\right)\mathbf{V}_1$$

$$\mathbf{V}_2 = \text{-}5.96\,\mathbf{V}_1 \quad \longrightarrow \quad \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \text{-}5.96$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$10 \,||\, 100 = 9.091$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2 + 10\,\mathbf{V}_x}{50} + \frac{\mathbf{V}_2}{300 + 9.091}$$

$$309.091\,\mathbf{I}_2 = 7.1818\,\mathbf{V}_2 + 61.818\,\mathbf{V}_x \qquad (4)$$

But $$\mathbf{V}_x = \frac{9.091}{309.091}\mathbf{V}_2 = 0.02941\,\mathbf{V}_2 \qquad (5)$$

Substituting (5) into (4) gives

$$309.091\,\mathbf{I}_2 = 9\,\mathbf{V}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 34.34\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{309.091} = \frac{34.34\,\mathbf{I}_2}{309.091}$$

$$\mathbf{I}_1 = \frac{-100}{110}\mathbf{I}_o = \frac{-34.34\,\mathbf{I}_2}{(1.1)(309.091)}$$

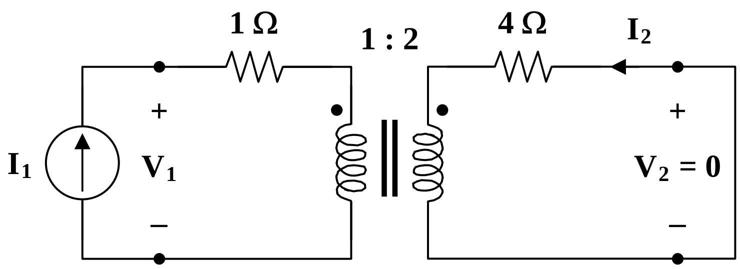
$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = -0.101$$

Thus,

$$[\mathbf{g}] = \begin{bmatrix} \mathbf{0.02929\ S} & \mathbf{-0.101} \\ \mathbf{-5.96} & \mathbf{34.34\,\Omega} \end{bmatrix}$$

Chapter 19, Solution 35.

To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ consider the circuit in Fig. (a).



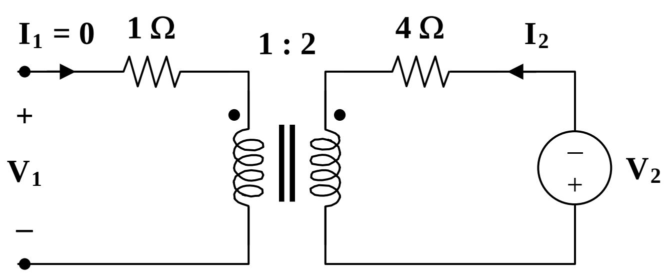
(a)

$$Z_R = \frac{4}{n^2} = \frac{4}{4} = 1$$

$$\mathbf{V}_1 = (1+1)\mathbf{I}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2\,\Omega$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-N_2}{N_1} = -2 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{2} = -0.5$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

Since $\mathbf{I}_1 = 0$, $\mathbf{I}_2 = 0$.

Hence, $\mathbf{h}_{22} = 0$.

At the terminals of the transformer, we have $\mathbf{V}_1$ and $\mathbf{V}_2$ which are related as

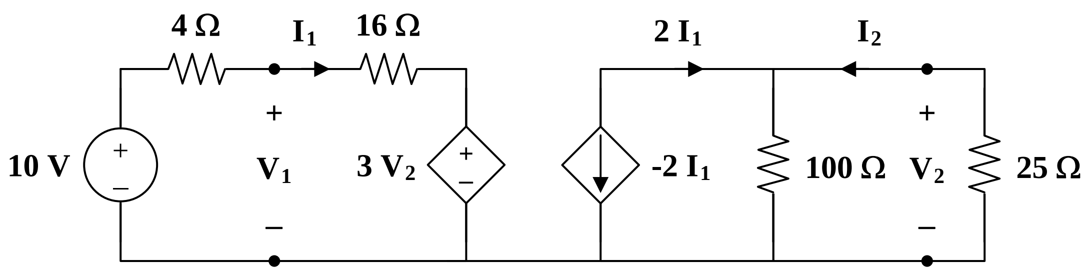
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\mathrm{N}_2}{\mathrm{N}_1} = \mathrm{n} = 2 \quad \longrightarrow \quad \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{2} = 0.5$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{2\,\Omega} & \mathbf{0.5} \\ \mathbf{-\,0.5} & \mathbf{0} \end{bmatrix}$$

Chapter 19, Solution 36.

We replace the two-port by its equivalent circuit as shown below.



$$100 \,||\, 25 = 20\,\Omega$$

$$\mathbf{V}_2 = (20)(2\mathbf{I}_1) = 40\mathbf{I}_1 \qquad (1)$$

$$-10 + 20\mathbf{I}_1 + 3\mathbf{V}_2 = 0$$
$$10 = 20\mathbf{I}_1 + (3)(40\mathbf{I}_1) = 140\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{1}{14}, \qquad \mathbf{V}_2 = \frac{40}{14}$$

$$\mathbf{V}_1 = 16\mathbf{I}_1 + 3\mathbf{V}_2 = \frac{136}{14}$$

$$\mathbf{I}_2 = \left(\frac{100}{125}\right)(2\mathbf{I}_1) = \frac{-8}{70}$$

(a) $$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{40}{136} = \mathbf{0.2941}$$

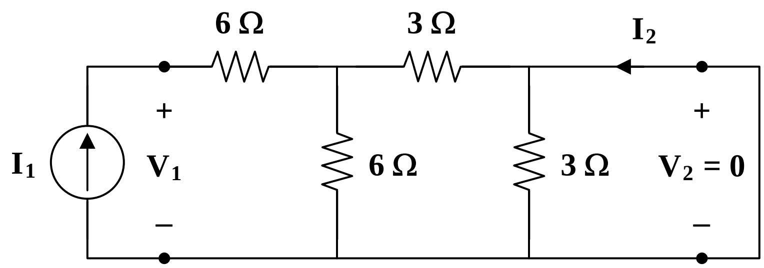
(b) $$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \mathbf{-1.6}$$

(c) $$\frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{136} = \mathbf{7.353 \times 10^{-3}\ S}$$

(d) $$\frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40}{1} = \mathbf{40\,\Omega}$$

Chapter 19, Solution 37.

(a) We first obtain the h parameters. To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ refer to Fig. (a).



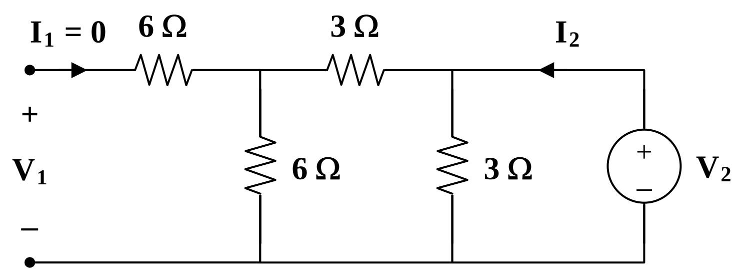
(a)

$$3 \| 6 = 2$$

$$\mathbf{V}_1 = (6+2)\mathbf{I}_1 = 8\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 8\,\Omega$$

$$\mathbf{I}_2 = \frac{-6}{3+6}\mathbf{I}_1 = \frac{-2}{3}\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-2}{3}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b).



(b)

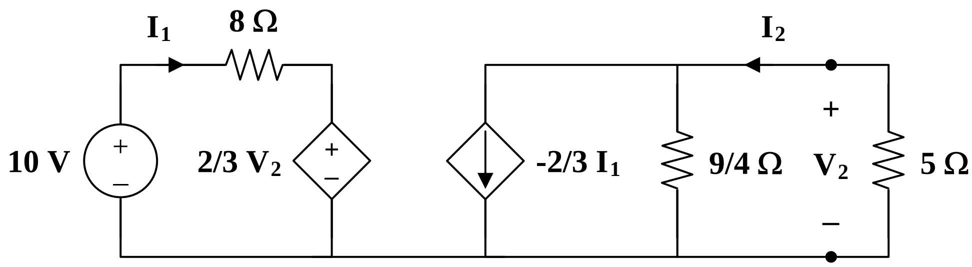
$$3 \| 9 = \frac{9}{4}$$

$$\mathbf{V}_2 = \frac{9}{4}\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{4}{9}$$

$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2 \quad \longrightarrow \quad \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$$

$$[\mathbf{h}] = \begin{bmatrix} 8\,\Omega & \dfrac{2}{3} \\ \dfrac{-2}{3} & \dfrac{4}{9}\,\text{S} \end{bmatrix}$$

The equivalent circuit of the given circuit is shown in Fig. (c).



(c)

$$8\mathbf{I}_1 + \frac{2}{3}\mathbf{V}_2 = 10 \qquad (1)$$

$$\mathbf{V}_2 = \frac{2}{3}\mathbf{I}_1\left(5 \,\|\, \frac{9}{4}\right) = \frac{2}{3}\mathbf{I}_1\left(\frac{45}{29}\right) = \frac{30}{29}\mathbf{I}_1$$

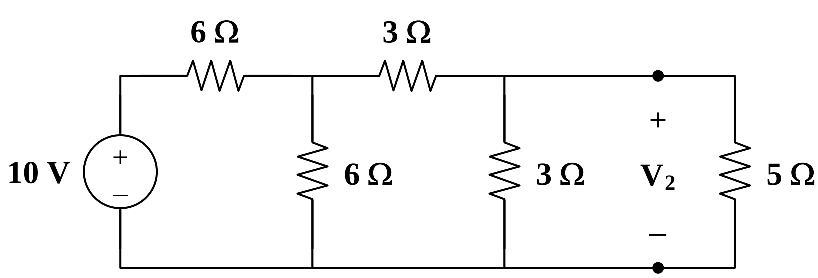
$$\mathbf{I}_1 = \frac{29}{30}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$(8)\left(\frac{29}{30}\right)\mathbf{V}_2 + \frac{2}{3}\mathbf{V}_2 = 10$$

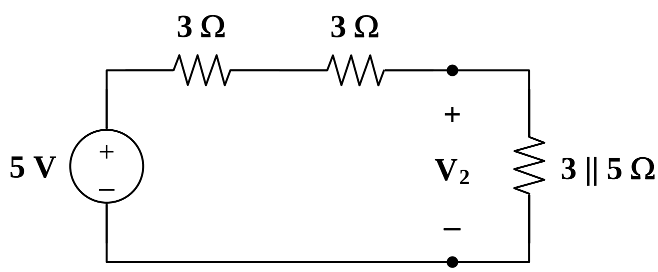
$$\mathbf{V}_2 = \frac{300}{252} = \underline{\mathbf{1.19\ V}}$$

(b) By direct analysis, refer to Fig.(d).



(d)

Transform the 10-V voltage source to a $\frac{10}{6}$-A current source. Since $6 \parallel 6 = 3\,\Omega$, we combine the two 6-Ω resistors in parallel and transform the current source back to $\frac{10}{6} \times 3 = 5$ V voltage source shown in Fig. (e).



(e)

$$3 \parallel 5 = \frac{(3)(5)}{8} = \frac{15}{8}$$

$$\mathbf{V}_2 = \frac{15/8}{6+15/8}(5) = \frac{75}{63} = \mathbf{1.1905\ V}$$

Chapter 19, Solution 38.

From eq. (19.75),

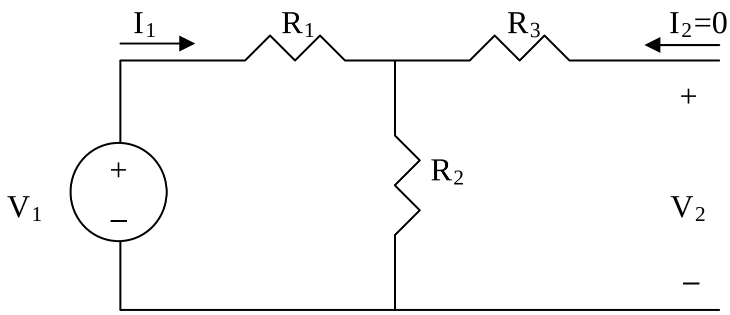
$$Z_{in} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1+h_{oe}R_L} = h_{11} - \frac{h_{12}h_{21}R_L}{1+h_{22}R_L} = 600 - \frac{0.04x30x400}{1+2x10^{-3}x400} = \underline{333.33\ \Omega}$$

From eq. (19.79),

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{0e} - h_{re}h_{fe}} = \frac{R_s + h_{11}}{(R_s + h_{11})h_{22} - h_{21}h_{12}} = \frac{2,000 + 600}{2600x2x10^{-3} - 30x0.04} = \underline{650\ \Omega}$$

Chapter 19, Solution 39.

We obtain g_{11} and g_{21} using the circuit below.

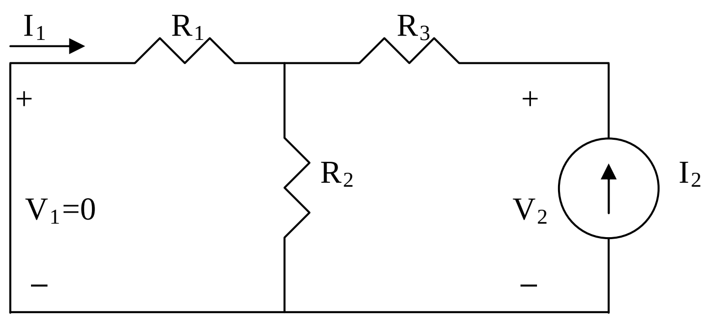


$$I_1 = \frac{V_1}{R_1 + R_2} \quad \longrightarrow \quad g_{11} = \frac{I_1}{V_1} = \underline{\frac{1}{R_1 + R_2}}$$

By voltage division,

$$V_2 = \frac{R_2}{R_1 + R_2} V_1 \quad \longrightarrow \quad g_{21} = \frac{V_2}{V_1} = \underline{\frac{R_2}{R_1 + R_2}}$$

We obtain g_{12} and g_{22} using the circuit below.



By current division,

$$I_1 = -\frac{R_2}{R_1 + R_2} I_2 \quad \longrightarrow \quad g_{12} = \frac{I_1}{I_2} = \underline{-\frac{R_2}{R_1 + R_2}}$$

Also,

$$V_2 = I_2(R_3 + R_1 // R_2) = I_2\left(R_3 + \frac{R_1 R_2}{R_1 + R_2}\right) \quad g_{22} = \frac{V_2}{I_2} = \underline{R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

$$\mathbf{g_{11} = \frac{1}{R_1 + R_2}, g_{12} = -\frac{R_2}{R_1 + R_2}}$$

$$\mathbf{g_{21} = \frac{R_2}{R_1 + R_2}, g_{22} = R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

Chapter 19, Solution 40.

Using Fig. 19.97, design a problem to help other students to better understand how to find *g* parameters in an ac circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the *g* parameters for the circuit in Fig.19.97.

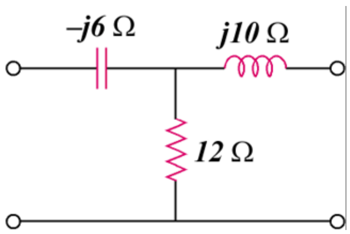
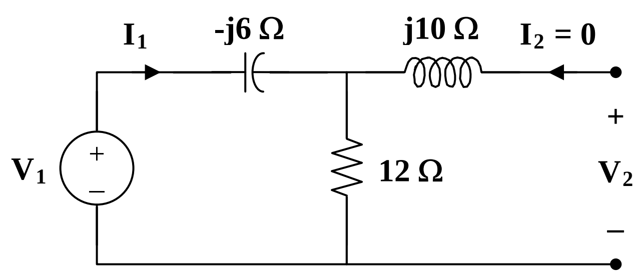


Figure 19.97

Solution

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, consider the circuit in Fig. (a).

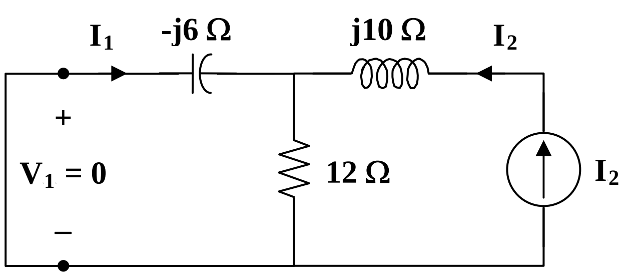


(a)

$$\mathbf{V}_1 = (12 - j6)\,\mathbf{I}_1 \quad \longrightarrow \quad \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{12 - j6} = 0.0667 + j0.0333\text{ S}$$

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{12\,\mathbf{I}_1}{(12 - j6)\,\mathbf{I}_1} = \frac{2}{2 - j} = 0.8 + j0.4$$

To get $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_1 = \frac{-12}{12 - j6}\mathbf{I}_2 \longrightarrow \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-12}{12 - j6} = -\mathbf{g}_{21} = -0.8 - j0.4$$

$$\mathbf{V}_2 = (j10 + 12 \,||\, -j6)\,\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = j10 + \frac{(12)(-j6)}{12 - j6} = 2.4 + j5.2\,\Omega$$

$$[\mathbf{g}] = \begin{bmatrix} \mathbf{0.0667 + j0.0333\ S} & \mathbf{-0.8 - j0.4} \\ \mathbf{0.8 + j0.4} & \mathbf{2.4 + j5.2\,\Omega} \end{bmatrix}$$

Chapter 19, Solution 41.

For the g parameters

$$\mathbf{I}_1 = \mathbf{g}_{11}\,\mathbf{V}_1 + \mathbf{g}_{12}\,\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\,\mathbf{V}_1 + \mathbf{g}_{22}\,\mathbf{I}_2 \qquad (2)$$

But $\mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\,\mathbf{Z}_s$ and

$$\mathbf{V}_2 = -\mathbf{I}_2\,\mathbf{Z}_L = \mathbf{g}_{21}\,\mathbf{V}_1 + \mathbf{g}_{22}\,\mathbf{I}_2$$

$$0 = \mathbf{g}_{21}\,\mathbf{V}_1 + (\mathbf{g}_{22} + \mathbf{Z}_L)\,\mathbf{I}_2$$

or

$$\mathbf{V}_1 = \frac{-(\mathbf{g}_{22} + \mathbf{Z}_L)}{\mathbf{g}_{21}}\,\mathbf{I}_2$$

Substituting this into (1),

$$\mathbf{I}_1 = \frac{(\mathbf{g}_{22}\,\mathbf{g}_{11} + \mathbf{Z}_L\,\mathbf{g}_{11} - \mathbf{g}_{21}\,\mathbf{g}_{12})}{-\mathbf{g}_{21}}\,\mathbf{I}_2$$

or

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-\mathbf{g}_{21}}{\mathbf{g}_{11}\,\mathbf{Z}_L + \Delta_g}$$

Also,

$$\mathbf{V}_2 = \mathbf{g}_{21}\,(\mathbf{V}_s - \mathbf{I}_1\,\mathbf{Z}_s) + \mathbf{g}_{22}\,\mathbf{I}_2$$

$$= \mathbf{g}_{21}\,\mathbf{V}_s - \mathbf{g}_{21}\,\mathbf{Z}_s\,\mathbf{I}_1 + \mathbf{g}_{22}\,\mathbf{I}_2$$

$$= \mathbf{g}_{21}\,\mathbf{V}_s + \mathbf{Z}_s\,(\mathbf{g}_{11}\,\mathbf{Z}_L + \Delta_g)\,\mathbf{I}_2 + \mathbf{g}_{22}\,\mathbf{I}_2$$

But

$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{\mathbf{Z}_L}$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\,\mathbf{V}_s - [\,\mathbf{g}_{11}\,\mathbf{Z}_s\,\mathbf{Z}_L + \Delta_g\,\mathbf{Z}_s + \mathbf{g}_{22}\,]\left[\frac{\mathbf{V}_2}{\mathbf{Z}_L}\right]$$

$$\frac{\mathbf{V}_2\,[\,\mathbf{Z}_L + \mathbf{g}_{11}\,\mathbf{Z}_s\,\mathbf{Z}_L + \Delta_g\,\mathbf{Z}_s + \mathbf{g}_{22}\,]}{\mathbf{Z}_L} = \mathbf{g}_{21}\,\mathbf{V}_s$$

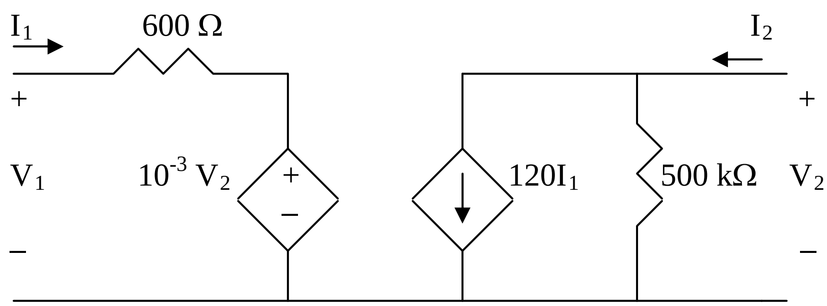
$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\,\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\,\mathbf{Z}_s\,\mathbf{Z}_L + \Delta_g\,\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\,\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\,\mathbf{Z}_s\,\mathbf{Z}_L + \mathbf{g}_{11}\,\mathbf{g}_{22}\,\mathbf{Z}_s - \mathbf{g}_{21}\,\mathbf{g}_{12}\,\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\,\mathbf{Z}_L}{(1 + \mathbf{g}_{11}\,\mathbf{Z}_s)(\mathbf{g}_{22} + \mathbf{Z}_L) - \mathbf{g}_{12}\,\mathbf{g}_{21}\,\mathbf{Z}_s}$$

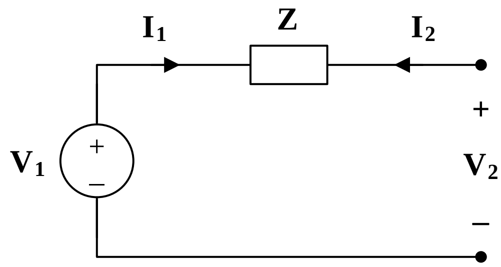
Chapter 19, Solution 42.

With the help of Fig. 19.20, we obtain the circuit model below.



Chapter 19, Solution 43.

(a) To find $\mathbf{A}$ and $\mathbf{C}$, consider the network in Fig. (a).

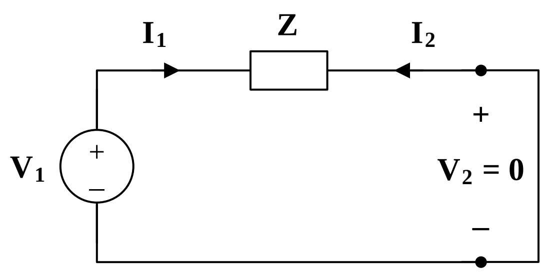


(a)

$$\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{I}_1 = 0 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

To get $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_1 = \mathbf{Z}\mathbf{I}_1, \qquad \mathbf{I}_2 = -\mathbf{I}_1$$

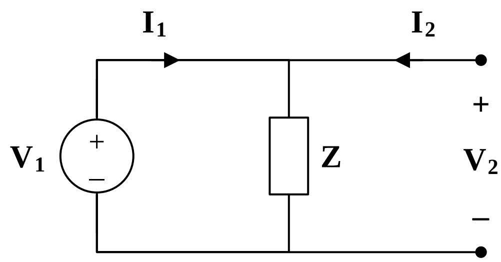
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-\mathbf{Z}\mathbf{I}_1}{-\mathbf{I}_1} = \mathbf{Z}$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Hence,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{1} & \mathbf{Z} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

(b) To find $\mathbf{A}$ and $\mathbf{C}$, consider the circuit in Fig. (c).

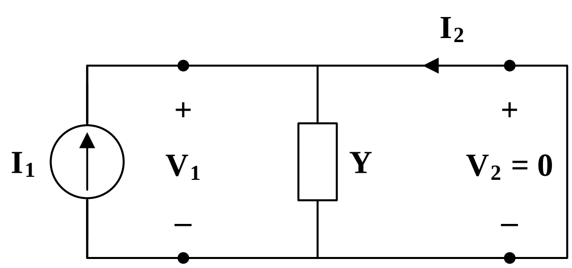


(c)

$$\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{V}_1 = \mathbf{Z}\,\mathbf{I}_1 = \mathbf{V}_2 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{\mathbf{Z}} = \mathbf{Y}$$

To get $\mathbf{B}$ and $\mathbf{D}$, refer to the circuit in Fig.(d).



(d)

$$\mathbf{V}_1 = \mathbf{V}_2 = 0 \qquad \mathbf{I}_2 = -\mathbf{I}_1$$

$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = 0\,, \qquad \mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Thus,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{Y} & \mathbf{1} \end{bmatrix}$$

Chapter 19, Solution 44.

Using Fig. 19.99, design a problem to help other students to better understand how to find the transmission parameters of an ac circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the transmission parameters of the circuit in Fig.19.99.

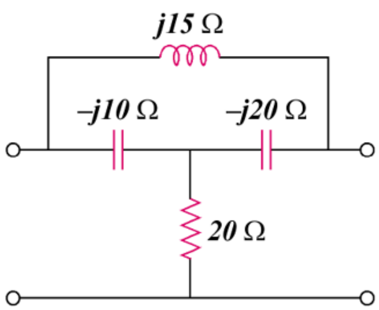
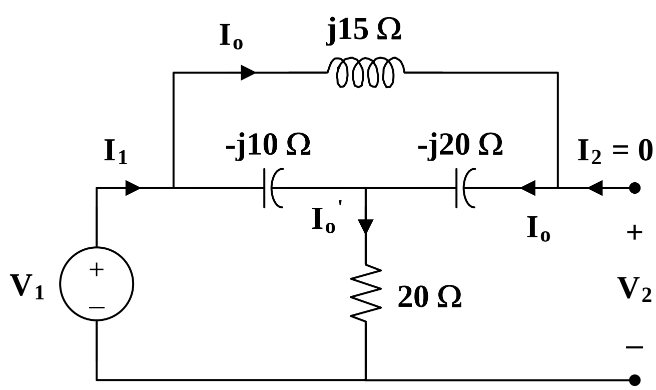


Figure 19.99

Solution

To determine **A** and **C**, consider the circuit in Fig.(a).



(a)

$$\mathbf{V}_1 = [\,20 + (\text{-j}10)\,||\,(\text{j}15 - \text{j}20)\,]\mathbf{I}_1$$

$$\mathbf{V}_1 = \left[\,20 + \frac{(\text{-j}10)(\text{-j}5)}{\text{- j}15}\right]\mathbf{I}_1 = \left[\,20 - \text{j}\frac{10}{3}\right]\mathbf{I}_1$$

$$\mathbf{I}_o^{'} = \mathbf{I}_1$$

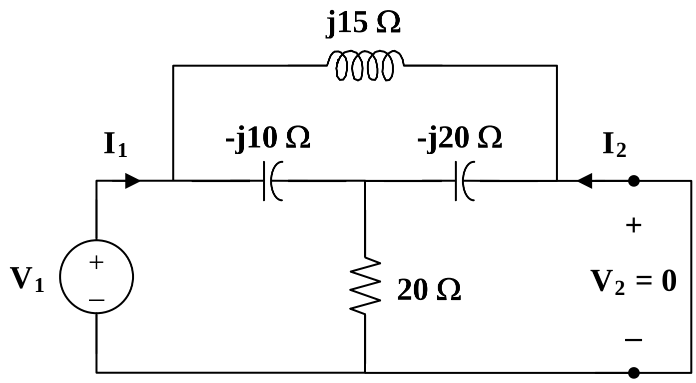
$$\mathbf{I}_o = \left(\frac{-j10}{-j10 - j5}\right)\mathbf{I}_1 = \left(\frac{2}{3}\right)\mathbf{I}_1$$

$$\mathbf{V}_2 = (-j20)\mathbf{I}_o + 20\mathbf{I}_o' = -j\frac{40}{3}I_1 + 20I_1 = \left(20 - j\frac{40}{3}\right)I_1$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{(20 - j10/3)\mathbf{I}_1}{\left(20 - j\frac{40}{3}\right)I_1} = 0.7692 + j0.3461$$

$$\mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{20 - j\frac{40}{3}} = 0.03461 + j0.023$$

To find $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



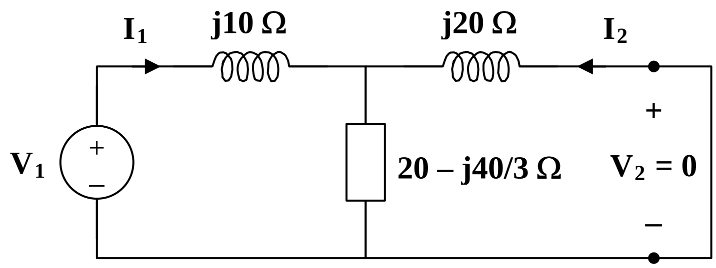
(b)

We may transform the Δ subnetwork to a T as shown in Fig. (c).

$$\mathbf{Z}_1 = \frac{(j15)(-j10)}{j15 - j10 - j20} = j10$$

$$\mathbf{Z}_2 = \frac{(-j10)(-j20)}{-j15} = -j\frac{40}{3}$$

$$\mathbf{Z}_3 = \frac{(j15)(-j20)}{-j15} = j20$$



(c)

$$-\mathbf{I}_2 = \frac{20 - j40/3}{20 - j40/3 + j20}\mathbf{I}_1 = \frac{3 - j2}{3 + j}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{3 + j}{3 - j2} = 0.5385 + j0.6923$$

$$\mathbf{V}_1 = \left[j10 + \frac{(j20)(20 - j40/3)}{20 - j40/3 + j20} \right]\mathbf{I}_1$$

$$\mathbf{V}_1 = [\, j10 + 2\,(9 + j7)\,]\,\mathbf{I}_1 = j\,\mathbf{I}_1\,(24 - j18)$$

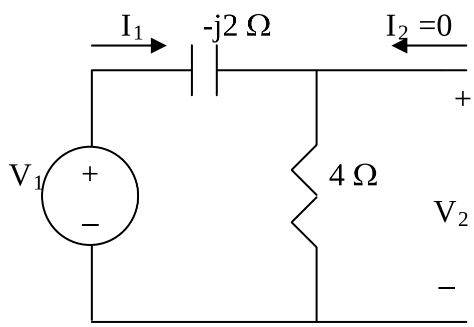
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-j\,\mathbf{I}_1\,(24 - j18)}{\frac{-(3 - j2)}{3 + j}\mathbf{I}_1} = \frac{6}{13}(-15 + j55)$$

$$\mathbf{B} = -6.923 + j25.385\,\Omega$$

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{0.7692 + j0.3461} & \mathbf{(-6.923 + j25.38)\,\Omega} \\ \mathbf{(0.03461 + j0.023)\,S} & \mathbf{0.5385 + j0.6923} \end{bmatrix}$$

Chapter 19, Solution 45.

To determine A and C consider the circuit below.

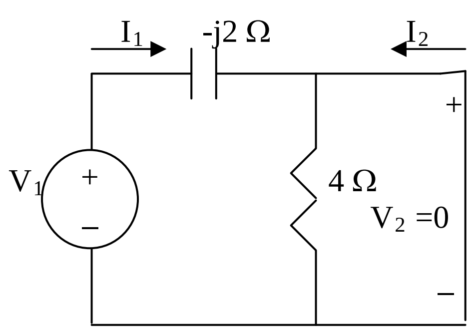


$$V_1 = (4 - j2)I_1, \qquad V_2 = 4I_1$$

$$A = \frac{V_1}{V_2} = \frac{4 - j2}{4} = 1 - j0.5$$

$$C = \frac{I_1}{V_2} = \frac{I_1}{4I_1} = 0.25$$

To determine B and D, consider the circuit below.



The 4-Ω resistor is short-circuited. Hence,

$$I_2 = -I_1, \qquad D = -\frac{I_1}{I_2} = 1$$

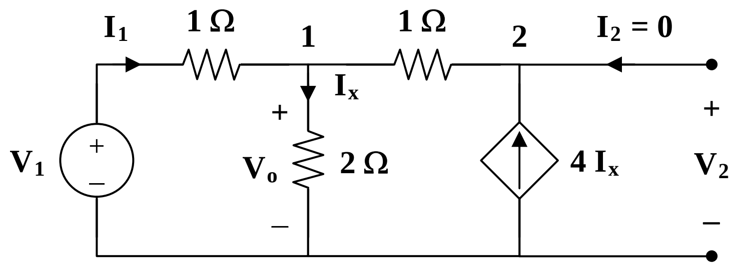
$$V_1 = -j2I_1 = j2I_2 \qquad B = -\frac{V_1}{I_2} = -\frac{j2I_2}{I_2} = -2j\Omega$$

Hence,

$$\textbf{[T]} = \begin{bmatrix} \mathbf{1 - j0.5} & \mathbf{-j2\,\Omega} \\ \mathbf{0.25\,S} & \mathbf{1} \end{bmatrix}$$

Chapter 19, Solution 46.

To get $\mathbf{A}$ and $\mathbf{C}$, refer to the circuit in Fig.(a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{1} \longrightarrow 2\mathbf{I}_1 = 3\mathbf{V}_o - 2\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_o - \mathbf{V}_2}{1} = 4\mathbf{I}_x = \frac{4\mathbf{V}_o}{2} = 2\mathbf{V}_o \longrightarrow \mathbf{V}_o = \text{-}\mathbf{V}_2 \qquad (2)$$

From (1) and (2),

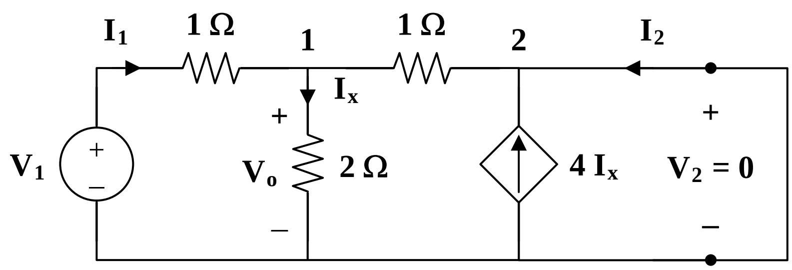
$$2\mathbf{I}_1 = \text{-}5\mathbf{V}_2 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{\text{-}5}{2} = \text{-}2.5\text{ S}$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} = \mathbf{V}_1 + \mathbf{V}_2$$

$$\text{-}2.5\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{V}_2 \longrightarrow \mathbf{V}_1 = \text{-}3.5\mathbf{V}_2$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \text{-}3.5$$

To get $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o}{1} \longrightarrow 2\mathbf{I}_1 = 3\mathbf{V}_o \qquad (3)$$

At node 2,

$$\mathbf{I}_2 + \frac{\mathbf{V}_o}{1} + 4\mathbf{I}_x = 0$$

$$-\mathbf{I}_2 = \mathbf{V}_o + 2\mathbf{V}_o = 0 \longrightarrow \mathbf{I}_2 = -3\mathbf{V}_o \qquad (4)$$

Adding (3) and (4),

$$2\mathbf{I}_1 + \mathbf{I}_2 = 0 \longrightarrow \mathbf{I}_1 = -0.5\mathbf{I}_2 \qquad (5)$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = 0.5$$

But $\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_1 = \mathbf{I}_1 + \mathbf{V}_o \qquad (6)$

Substituting (5) and (4) into (6),

$$\mathbf{V}_1 = \frac{-1}{2}\mathbf{I}_2 + \frac{-1}{3}\mathbf{I}_2 = \frac{-5}{6}\mathbf{I}_2$$

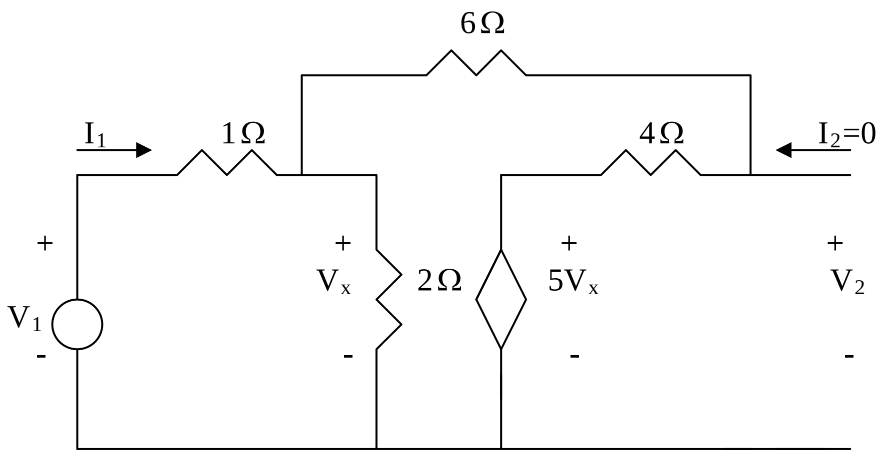
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{5}{6} = 0.8333\,\Omega$$

Thus,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{-3.5} & \mathbf{0.8333\,\Omega} \\ \mathbf{-2.5\ S} & \mathbf{-0.5} \end{bmatrix}$$

Chapter 19, Solution 47.

To get A and C, consider the circuit below.

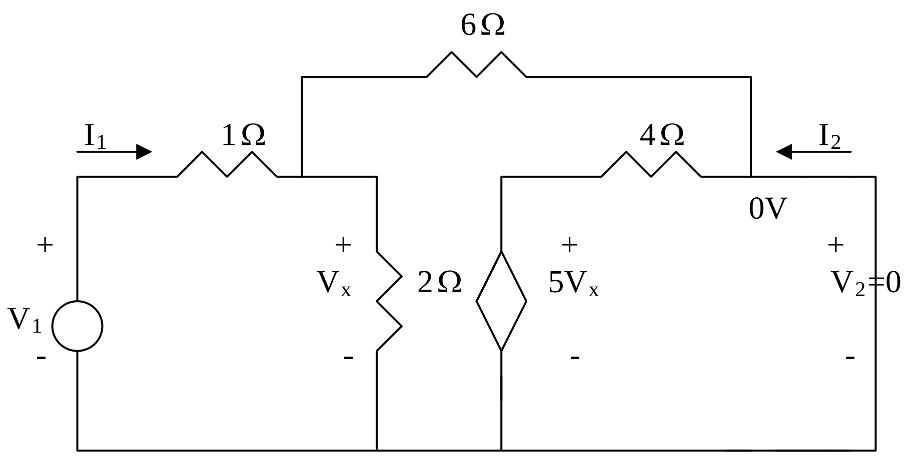


$$\frac{V_1 - V_x}{1} = \frac{V_x}{2} + \frac{V_x - 5V_x}{10} \longrightarrow V_1 = 1.1V_x$$

$$V_2 = 4(-0.4V_x) + 5V_x = 3.4V_x \longrightarrow A = \frac{V_1}{V_2} = 1.1/3.4 = 0.3235$$

$$I_1 = \frac{V_1 - V_x}{1} = 1.1V_x - V_x = 0.1V_x \longrightarrow C = \frac{I_1}{V_2} = 0.1/3.4 = 0.02941$$

To get B and D, consider the circuit below.



$$\frac{V_1 - V_x}{1} = \frac{V_x}{6} + \frac{V_x}{2} \longrightarrow V_1 = \frac{10}{6}V_x \qquad (1)$$

$$I_2 = -\frac{5V_x}{4} - \frac{V_x}{6} = -\frac{17}{12}V_x \qquad (2)$$

$$V_1 = I_1 + V_x \qquad (3)$$

From (1) and (3)

$$I_1 = V_1 - V_x = \frac{4}{6}V_x \longrightarrow D = -\frac{I_1}{I_2} = \frac{4}{6}\left(\frac{12}{17}\right) = 0.4706$$

$$B = -\frac{V_1}{I_2} = \frac{10}{6}\left(\frac{12}{17}\right) = 1.176$$

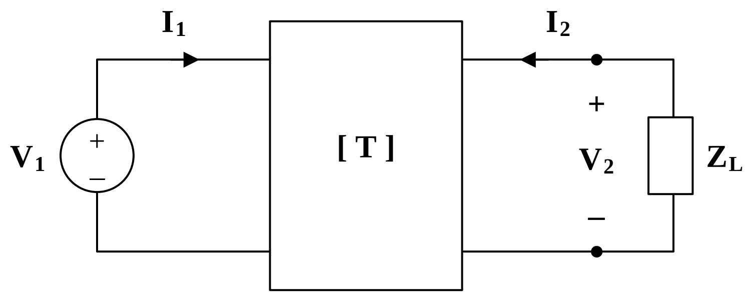
S

Ω

$$\mathbf{[T]} = \underline{\begin{bmatrix} \mathbf{0.3235} & \mathbf{1.176\Omega} \\ \mathbf{0.02941S} & \mathbf{0.4706} \end{bmatrix}}$$

Chapter 19, Solution 48.

(a) Refer to the circuit below.



$$\mathbf{V}_1 = 4\mathbf{V}_2 - 30\mathbf{I}_2 \qquad (1)$$
$$\mathbf{I}_1 = 0.1\mathbf{V}_2 - \mathbf{I}_2 \qquad (2)$$

When the output terminals are shorted, $\mathbf{V}_2 = 0$.
So, (1) and (2) become

$\mathbf{V}_1 = \text{-}30\mathbf{I}_2$ and $\mathbf{I}_1 = \text{-}\mathbf{I}_2$

Hence,

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \mathbf{30\,\Omega}$$

(b) When the output terminals are open-circuited, $\mathbf{I}_2 = 0$.
So, (1) and (2) become

$\mathbf{V}_1 = 4\mathbf{V}_2$

$\mathbf{I}_1 = 0.1\mathbf{V}_2$ or $\mathbf{V}_2 = 10\mathbf{I}_1$

$\mathbf{V}_1 = 40\mathbf{I}_1$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \mathbf{40\,\Omega}$$

(c) When the output port is terminated by a 10-Ω load, $V_2 = \text{-}10I_2$.
So, (1) and (2) become

$\mathbf{V}_1 = \text{-}40\mathbf{I}_2 - 30\mathbf{I}_2 = \text{-}70\mathbf{I}_2$

$\mathbf{I}_1 = \text{-}\mathbf{I}_2 - \mathbf{I}_2 = \text{-}2\mathbf{I}_2$

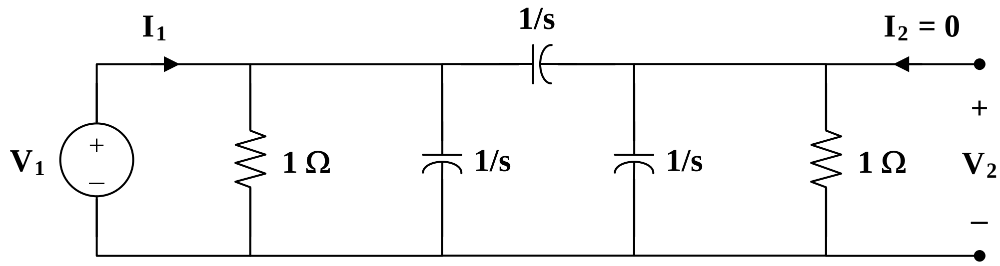
$\mathbf{V}_1 = 35\mathbf{I}_1$

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \mathbf{35\,\Omega}$$

Alternatively, we may use $\mathbf{Z}_{\text{in}} = \dfrac{\mathbf{A}\,\mathbf{Z}_{\text{L}} + \mathbf{B}}{\mathbf{C}\,\mathbf{Z}_{\text{L}} + \mathbf{D}}$

Chapter 19, Solution 49.

To get $\mathbf{A}$ and $\mathbf{C}$, refer to the circuit in Fig.(a).



(a)

$$1 \| \frac{1}{s} = \frac{1/s}{1+1/s} = \frac{1}{s+1}$$

$$\mathbf{V}_2 = \frac{1 \| 1/s}{1/s + 1 \| 1/s}\mathbf{V}_1$$

$$\mathbf{A} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\frac{1}{s+1}}{\frac{1}{s} + \frac{1}{s+1}} = \frac{s}{2s+1}$$

$$\mathbf{V}_1 = \mathbf{I}_1\left(\frac{1}{s+1}\right) \| \left(\frac{1}{s} + \frac{1}{s+1}\right) = \mathbf{I}_1\left(\frac{1}{s+1}\right) \| \left(\frac{2s+1}{s(s+1)}\right)$$

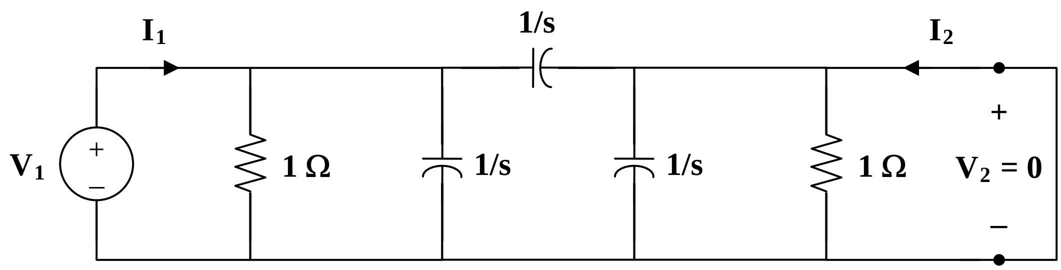
$$\frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\left(\frac{1}{s+1}\right)\cdot\left(\frac{2s+1}{s(s+1)}\right)}{\frac{1}{s+1} + \frac{2s+1}{s(s+1)}} = \frac{2s+1}{(s+1)(3s+1)}$$

But $\quad \mathbf{V}_1 = \mathbf{V}_2 \cdot \dfrac{2s+1}{s}$

Hence, $\quad \dfrac{\mathbf{V}_2}{\mathbf{I}_1} \cdot \dfrac{2s+1}{s} = \dfrac{2s+1}{(s+1)(3s+1)}$

$$\mathbf{C} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{(s+1)(3s+1)}{s}$$

To get $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_1 = \mathbf{I}_1\left(1 \,\|\, \frac{1}{s} \,\|\, \frac{1}{s}\right) = \mathbf{I}_1\left(1 \,\|\, \frac{1}{2s}\right) = \frac{\mathbf{I}_1}{2s+1}$$

$$\mathbf{I}_2 = \frac{\frac{-1}{s+1}\mathbf{I}_1}{\frac{1}{s+1}+\frac{1}{s}} = \frac{-s}{2s+1}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{2s+1}{s} = 2 + \frac{1}{s}$$

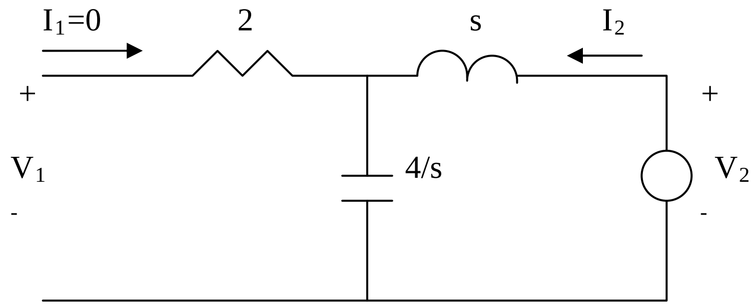
$$\mathbf{V}_1 = \left(\frac{1}{2s+1}\right)\left(\frac{2s+1}{-s}\right)\mathbf{I}_2 = \frac{\mathbf{I}_2}{-s} \quad \longrightarrow \quad \mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{1}{s}$$

Thus,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{\dfrac{2}{2s+1}} & \mathbf{\dfrac{1}{s}} \\ \mathbf{\dfrac{(s+1)(3s+1)}{s}} & \mathbf{2+\dfrac{1}{s}} \end{bmatrix}$$

Chapter 19, Solution 50.

To get a and c, consider the circuit below.

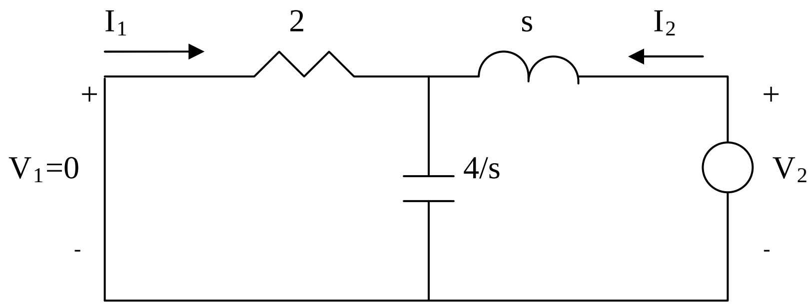


$$V_1 = \frac{4/s}{s+4/s}V_2 = \frac{4}{s^2+4}V_2 \quad \longrightarrow \quad a = V_2/V_1 = 1+0.25s^2$$

$$V_2 = (s+4/s)I_2 \text{ or}$$

$$I_2 = \frac{V_2}{s+4/s} = \frac{(1+0.25s^2)V_1}{s+4/s} \quad \longrightarrow \quad c = \frac{I_2}{V_1} = \frac{s+0.25s^3}{s^2+4}$$

To get b and d, consider the circuit below.



$$I_1 = \frac{-4/s}{2+4/s}I_2 = -\frac{2I_2}{s+2} \quad \longrightarrow \quad d = -\frac{I_2}{I_1} = 1+0.5s$$

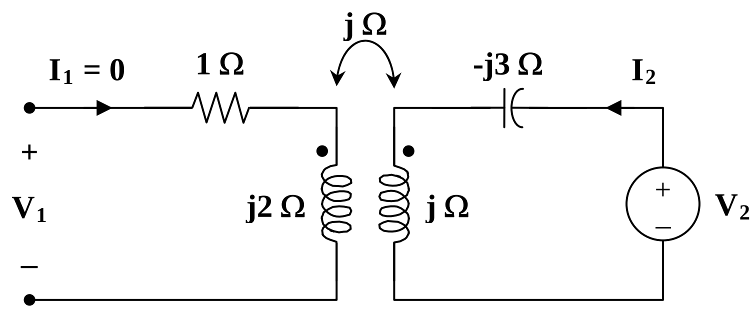
$$V_2 = (s+2//\frac{4}{s})I_2 = \frac{(s^2+2s+4)}{s+2}I_2$$

$$= -\frac{(s^2+2s+4)}{s+2}\frac{(s+2)}{2}I_1 \quad \longrightarrow \quad b = -\frac{V_2}{I_1} = 0.5s^2+s+2$$

$$[t] = \begin{bmatrix} 0.25s^2 + 1 & 0.5s^2 + s + 2 \\ \dfrac{0.25s^2 + s}{s^2 + 4} & 0.5s + 1 \end{bmatrix}$$

Chapter 19, Solution 51.

To get $\mathbf{a}$ and $\mathbf{c}$, consider the circuit in Fig. (a).



(a)

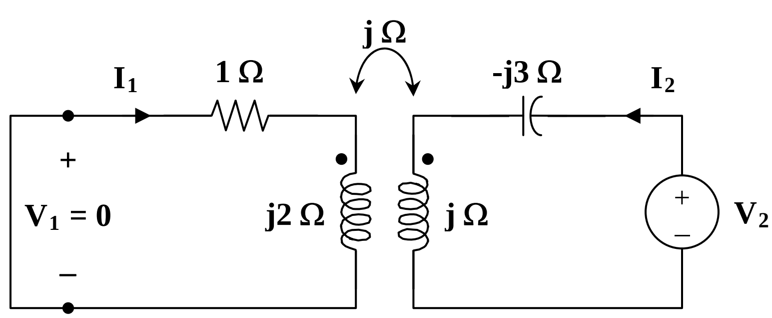
$$\mathbf{V}_2 = \mathbf{I}_2\,(\mathrm{j} - \mathrm{j}3) = \text{-j}2\,\mathbf{I}_2$$

$$\mathbf{V}_1 = \text{-j}\mathbf{I}_2$$

$$\mathbf{a} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\text{- j}2\,\mathbf{I}_2}{\text{- j}\mathbf{I}_2} = 2$$

$$\mathbf{c} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1}{\text{- j}} = \mathrm{j}$$

To get $\mathbf{b}$ and $\mathbf{d}$, consider the circuit in Fig. (b).



(b)

For mesh 1,

$$0 = (1 + \mathrm{j}2)\,\mathbf{I}_1 - \mathrm{j}\,\mathbf{I}_2$$

or

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{1 + \mathrm{j}2}{\mathrm{j}} = 2 - \mathrm{j}$$

$$\mathbf{d} = \frac{\text{-}\,\mathbf{I}_2}{\mathbf{I}_1} = \text{-}2 + \mathrm{j}$$

For mesh 2,

$$\mathbf{V}_2 = \mathbf{I}_2\,(j - j3) - j\,\mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_1\,(2 - j)(-j2) - j\,\mathbf{I}_1 = (-2 - j5)\,\mathbf{I}_1$$

$$\mathbf{b} = \frac{-\mathbf{V}_2}{\mathbf{I}_1} = 2 + j5$$

Thus,

$$[\mathbf{t}] = \begin{bmatrix} \mathbf{2} & \mathbf{2 + j5} \\ \mathbf{j} & \mathbf{-2 + j} \end{bmatrix}$$

Chapter 19, Solution 52.

It is easy to find the z parameters and then transform these to h parameters and T parameters.

$$[\mathbf{z}] = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

$$\Delta_z = (R_1 + R_2)(R_2 + R_3) - R_2^2$$
$$= R_1R_2 + R_2R_3 + R_3R_1$$

(a) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{\Delta_z}{\mathbf{z}_{22}} & \dfrac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \dfrac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \dfrac{1}{\mathbf{z}_{22}} \end{bmatrix} = \begin{bmatrix} \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2 + R_3} & \dfrac{R_2}{R_2 + R_3} \\ \dfrac{-R_2}{R_2 + R_3} & \dfrac{1}{R_2 + R_3} \end{bmatrix}$$

Thus,

$$\mathbf{h}_{11} = \mathbf{R}_1 + \frac{\mathbf{R}_2\mathbf{R}_3}{\mathbf{R}_2 + \mathbf{R}_3}, \quad \mathbf{h}_{12} = \frac{\mathbf{R}_2}{\mathbf{R}_2 + \mathbf{R}_3} = -\mathbf{h}_{21}, \quad \mathbf{h}_{22} = \frac{\mathbf{1}}{\mathbf{R}_2 + \mathbf{R}_3}$$

as required.

(b) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{1}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \\ \dfrac{1}{R_2} & \dfrac{R_2 + R_3}{R_2} \end{bmatrix}$$

Hence,

$$\mathbf{A} = \mathbf{1} + \frac{\mathbf{R}_1}{\mathbf{R}_2}, \quad \mathbf{B} = \mathbf{R}_3 + \frac{\mathbf{R}_1}{\mathbf{R}_2}(\mathbf{R}_2 + \mathbf{R}_3), \quad \mathbf{C} = \frac{\mathbf{1}}{\mathbf{R}_2}, \quad \mathbf{D} = \mathbf{1} + \frac{\mathbf{R}_3}{\mathbf{R}_2}$$

as required.

Chapter 19, Solution 53.

For the z parameters,

$$\mathbf{V}_1 = \mathbf{z}_{11}\,\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2 \tag{1}$$

$$\mathbf{V}_2 = \mathbf{z}_{12}\,\mathbf{I}_1 + \mathbf{z}_{22}\,\mathbf{I}_2 \tag{2}$$

For **ABCD** parameters,

$$\mathbf{V}_1 = \mathbf{A}\,\mathbf{V}_2 - \mathbf{B}\,\mathbf{I}_2 \tag{3}$$

$$\mathbf{I}_1 = \mathbf{C}\,\mathbf{V}_2 - \mathbf{D}\,\mathbf{I}_2 \tag{4}$$

From (4),

$$\mathbf{V}_2 = \frac{\mathbf{I}_1}{\mathbf{C}} + \frac{\mathbf{D}}{\mathbf{C}}\mathbf{I}_2 \tag{5}$$

Comparing (2) and (5),

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}}, \qquad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}}$$

Substituting (5) into (3),

$$\mathbf{V}_1 = \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \left(\frac{\mathbf{AD}}{\mathbf{C}} - \mathbf{B}\right)\mathbf{I}_2$$

$$= \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}}\mathbf{I}_2 \tag{6}$$

Comparing (6) and (1),

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} \qquad \mathbf{z}_{12} = \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} = \frac{\Delta_T}{\mathbf{C}}$$

Thus,

$$[Z] = \begin{bmatrix} \dfrac{\mathbf{A}}{\mathbf{C}} & \dfrac{\mathbf{\Delta_T}}{\mathbf{C}} \\ \dfrac{\mathbf{1}}{\mathbf{C}} & \dfrac{\mathbf{D}}{\mathbf{C}} \end{bmatrix}$$

Chapter 19, Solution 54.

For the y parameters

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \qquad (2)$$

From (2),

$$\mathbf{V}_1 = \frac{\mathbf{I}_2}{\mathbf{y}_{21}} - \frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2$$

or

$$\mathbf{V}_1 = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{12}}\mathbf{V}_2 + \frac{1}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (3)$$

Substituting (3) into (1) gives

$$\mathbf{I}_1 = \frac{-\mathbf{y}_{11}\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2 + \mathbf{y}_{12}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2$$

or

$$\mathbf{I}_1 = \frac{-\Delta_y}{\mathbf{y}_{21}}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

clearly shows that

$$\mathbf{A} = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}}, \quad \mathbf{B} = \frac{-1}{\mathbf{y}_{21}}, \quad \mathbf{C} = \frac{-\Delta_y}{\mathbf{y}_{21}}, \quad \mathbf{D} = \frac{-\mathbf{y}_{11}}{\mathbf{y}_{21}}$$

as required.

Chapter 19, Solution 55.

For the z parameters

$$\mathbf{V}_1 = \mathbf{z}_{11}\,\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\,\mathbf{I}_1 + \mathbf{z}_{22}\,\mathbf{I}_2 \qquad (2)$$

From (1),

$$\mathbf{I}_1 = \frac{1}{\mathbf{z}_{11}}\mathbf{V}_1 - \frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}\mathbf{I}_2 \qquad (3)$$

Substituting (3) into (2) gives

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \left(\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\,\mathbf{z}_{12}}{\mathbf{z}_{11}}\right)\mathbf{I}_2$$

or

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \frac{\Delta_z}{\mathbf{z}_{11}}\mathbf{I}_2 \qquad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{I}_1 = \mathbf{g}_{11}\,\mathbf{V}_1 + \mathbf{g}_{12}\,\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\,\mathbf{V}_1 + \mathbf{g}_{22}\,\mathbf{I}_2$$

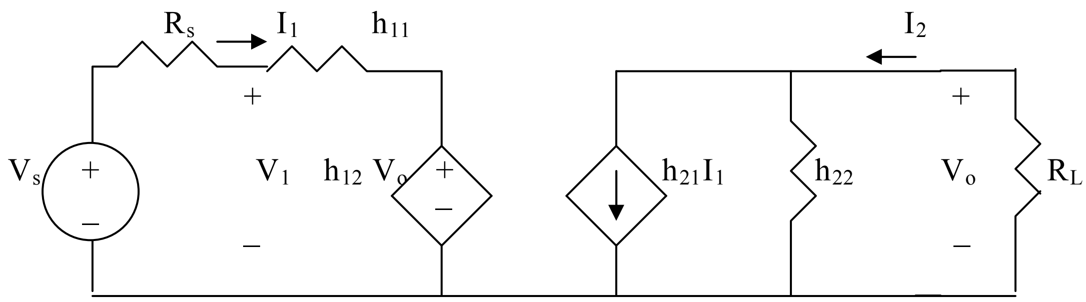
indicates that

$$\mathbf{g}_{11} = \frac{\mathbf{1}}{\mathbf{z}_{11}}, \quad \mathbf{g}_{12} = \frac{-\,\mathbf{z}_{12}}{\mathbf{z}_{11}}, \quad \mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}, \quad \mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}$$

as required.

Chapter 19, Solution 56.

Using Fig. 19.20, we obtain the equivalent circuit as shown below.



We can solve this using MATLAB. First, we generate 4 equations from the given circuit. It may help to let V_s = 10 V.

$-10 + R_sI_1 + V_1 = 0$ or $V_1 + 1000I_1 = 10$
$-10 + R_sI_1 + h_{11}I_1 + h_{12}V_o = 0$ or $0.0001V_s + 1500 = 10$
$I_2 = -V_o/R_L$ or $V_o + 2000I_2 = 0$
$h_{21}I_1 + h_{22}V_o - I_2 = 0$ or $2x10^{-6}V_o + 100I_1 - I_2 = 0$

```
>> A=[1,0,1000,0;0,0.0001,1500,0;0,1,0,2000;0,(2*10^-6),100,-1]
A =
  1.0e+003 *
    0.0010         0    1.0000         0
         0    0.0000    1.5000         0
         0    0.0010         0    2.0000
         0    0.0000    0.1000   -0.0010
>> U=[10;10;0;0]
U =
    10
    10
     0
     0
>> X=inv(A)*U
X =
  1.0e+003 *
    0.0032
   -1.3459
    0.0000
    0.0007
```

Gain = V_o /V_s = –1,345.9/10 = **–134.59**.

There is a second approach we can take to check this problem. First, the resistive value of h_{22} is quite large, 500 kΩ versus R_L so can be ignored. Working on the right side of the circuit we obtain the following,

$I_2 = 100I_1$ which leads to $V_o = -I_2 x2k = -2x10^5 I_1$.

Now the left hand loop equation becomes,

$-V_s + (1000 + 500 + 10^{-4}(-2x10^5))I_1 = 1480I_1$.

Solving for V_o/V_s we get,

$$V_o/V_s = -200{,}000/1480 = \mathbf{-134.14}.$$

Our answer checks!

Chapter 19, Solution 57.

$$\Delta_T = (3)(7) - (20)(1) = 1$$

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\mathbf{A}}{\mathbf{C}} & \dfrac{\Delta_T}{\mathbf{C}} \\ \dfrac{1}{\mathbf{C}} & \dfrac{\mathbf{D}}{\mathbf{C}} \end{bmatrix} = \begin{bmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{7} \end{bmatrix} \Omega$$

$$[\mathbf{y}] = \begin{bmatrix} \dfrac{\mathbf{D}}{\mathbf{B}} & \dfrac{-\Delta_T}{\mathbf{B}} \\ \dfrac{-1}{\mathbf{B}} & \dfrac{\mathbf{A}}{\mathbf{B}} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{7}}{\mathbf{20}} & \dfrac{\mathbf{-1}}{\mathbf{20}} \\ \dfrac{\mathbf{-1}}{\mathbf{20}} & \dfrac{\mathbf{3}}{\mathbf{20}} \end{bmatrix} \mathbf{S}$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{B}}{\mathbf{D}} & \dfrac{\Delta_T}{\mathbf{D}} \\ \dfrac{-1}{\mathbf{D}} & \dfrac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{20}}{\mathbf{7}}\,\Omega & \dfrac{\mathbf{1}}{\mathbf{7}} \\ \dfrac{\mathbf{-1}}{\mathbf{7}} & \dfrac{\mathbf{1}}{\mathbf{7}}\,\mathbf{S} \end{bmatrix}$$

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{\mathbf{C}}{\mathbf{A}} & \dfrac{-\Delta_T}{\mathbf{A}} \\ \dfrac{1}{\mathbf{A}} & \dfrac{\mathbf{B}}{\mathbf{A}} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{1}}{\mathbf{3}}\,\mathbf{S} & \dfrac{\mathbf{-1}}{\mathbf{3}} \\ \dfrac{\mathbf{1}}{\mathbf{3}} & \dfrac{\mathbf{20}}{\mathbf{3}}\,\Omega \end{bmatrix}$$

$$[\mathbf{t}] = \begin{bmatrix} \dfrac{\mathbf{D}}{\Delta_T} & \dfrac{\mathbf{B}}{\Delta_T} \\ \dfrac{\mathbf{C}}{\Delta_T} & \dfrac{\mathbf{A}}{\Delta_T} \end{bmatrix} = \begin{bmatrix} \mathbf{7} & \mathbf{20}\,\Omega \\ \mathbf{1\,S} & \mathbf{3} \end{bmatrix}$$

Chapter 19, Solution 58.

Design a problem to help other students to better understand how to develop the y parameters and transmission parameters, given equations in terms of the hybrid parameters.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A two-port is described by

$$\mathbf{V}_1 = \mathbf{I}_1 + 2\mathbf{V}_2, \quad \mathbf{I}_2 = -2\mathbf{I}_1 + 0.4\mathbf{V}_2$$

Find: (a) the *y* parameters, (b) the transmission parameters.

Solution

The given set of equations is for the h parameters.

$$[\mathbf{h}] = \begin{bmatrix} 1\,\Omega & 2 \\ -2 & 0.4\text{ S} \end{bmatrix} \qquad \Delta_h = (1)(0.4) - (2)(-2) = 4.4$$

(a) $$[\mathbf{y}] = \begin{bmatrix} \dfrac{1}{\mathbf{h}_{11}} & \dfrac{-\mathbf{h}_{12}}{\mathbf{h}_{11}} \\ \dfrac{\mathbf{h}_{21}}{\mathbf{h}_{11}} & \dfrac{\Delta_h}{\mathbf{h}_{11}} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{4.4} \end{bmatrix}\mathbf{S}$$

(b) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{-\Delta_h}{\mathbf{h}_{21}} & \dfrac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \dfrac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \dfrac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \begin{bmatrix} \mathbf{2.2} & \mathbf{0.5\,\Omega} \\ \mathbf{0.2\text{ S}} & \mathbf{0.5} \end{bmatrix}$$

Chapter 19, Solution 59.

$$\Delta_g = (0.06)(2) - (-0.4)(0.2) = 0.12 + 0.08 = 0.2$$

(a) $$[\mathbf{z}] = \begin{bmatrix} \dfrac{1}{\mathbf{g}_{11}} & \dfrac{-\mathbf{g}_{12}}{\mathbf{g}_{11}} \\ \dfrac{\mathbf{g}_{21}}{\mathbf{g}_{11}} & \dfrac{\Delta_g}{\mathbf{g}_{11}} \end{bmatrix} = \begin{bmatrix} \mathbf{16.667} & \mathbf{6.667} \\ \mathbf{3.333} & \mathbf{3.333} \end{bmatrix} \Omega$$

(b) $$[\mathbf{y}] = \begin{bmatrix} \dfrac{\Delta_g}{\mathbf{g}_{22}} & \dfrac{\mathbf{g}_{12}}{\mathbf{g}_{22}} \\ \dfrac{-\mathbf{g}_{21}}{\mathbf{g}_{22}} & \dfrac{1}{\mathbf{g}_{22}} \end{bmatrix} = \begin{bmatrix} \mathbf{0.1} & \mathbf{-0.2} \\ \mathbf{-0.1} & \mathbf{0.5} \end{bmatrix} \mathbf{S}$$

(c) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{g}_{22}}{\Delta_g} & \dfrac{-\mathbf{g}_{12}}{\Delta_g} \\ \dfrac{-\mathbf{g}_{21}}{\Delta_g} & \dfrac{\mathbf{g}_{11}}{\Delta_g} \end{bmatrix} = \begin{bmatrix} \mathbf{10\,\Omega} & \mathbf{2} \\ \mathbf{-1} & \mathbf{0.3\ S} \end{bmatrix}$$

(d) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{1}{\mathbf{g}_{21}} & \dfrac{\mathbf{g}_{22}}{\mathbf{g}_{21}} \\ \dfrac{\mathbf{g}_{11}}{\mathbf{g}_{21}} & \dfrac{\Delta_g}{\mathbf{g}_{21}} \end{bmatrix} = \begin{bmatrix} \mathbf{5} & \mathbf{10\,\Omega} \\ \mathbf{0.3\ S} & \mathbf{1} \end{bmatrix}$$

Chapter 19, Solution 60.

Comparing this with Fig. 19.5,

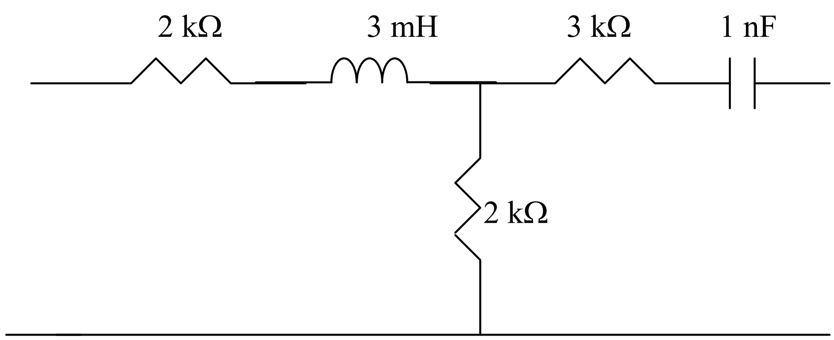
$$z_{11} - z_{12} = 4 + j3 - 2 = 2 + j3k\Omega$$

$z_{22} - z_{12} = 5 - j - 2 = 3 - j$ kΩ

$$X_L = 3x10^3 = \omega L \qquad \longrightarrow \qquad L = \frac{3x10^3}{10^6} = 3mH$$

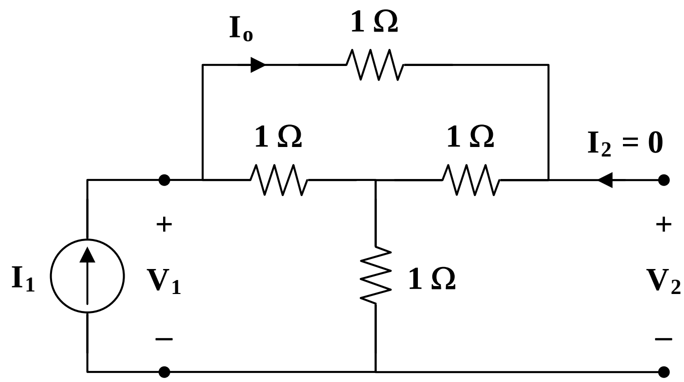
$X_C = 1x10^3 = 1/(\omega C)$ or $C = 1/(10^3 x10^6) = 1$ nF

Hence, the resulting T network is shown below.



Chapter 19, Solution 61.

(a) To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{I}_1[1+1\,\|\,(1+1)] = \mathbf{I}_1\left(1+\frac{2}{3}\right) = \frac{5}{3}\mathbf{I}_1$$

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{5}{3}$$

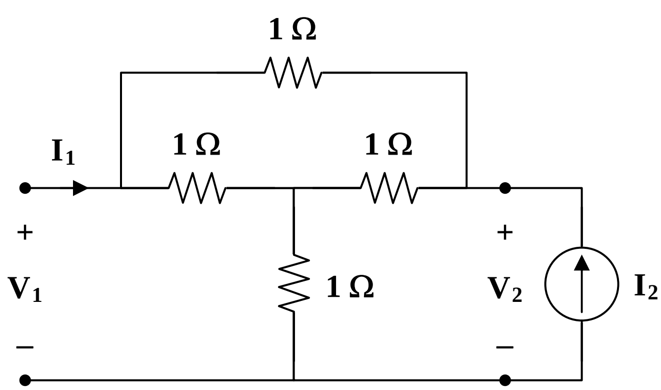
$$\mathbf{I}_o = \frac{1}{1+2}\mathbf{I}_1 = \frac{1}{3}\mathbf{I}_1$$

$$-\mathbf{V}_2 + \mathbf{I}_o + \mathbf{I}_1 = 0$$

$$\mathbf{V}_2 = \frac{1}{3}\mathbf{I}_1 + \mathbf{I}_1 = \frac{4}{3}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{4}{3}$$

To obtain $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

Due to symmetry, this is similar to the circuit in Fig. (a).

$$\mathbf{z}_{22} = \mathbf{z}_{11} = \frac{5}{3}, \qquad \mathbf{z}_{21} = \mathbf{z}_{12} = \frac{4}{3}$$

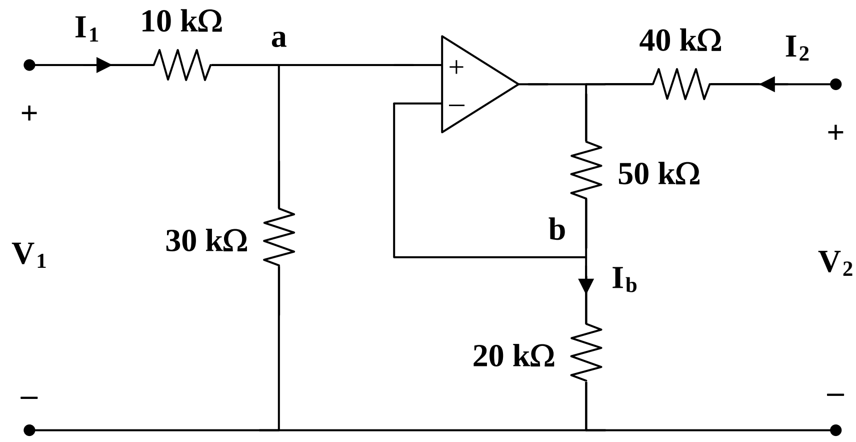
$$[\mathbf{z}] = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix} \Omega$$

(b) $$[\mathbf{h}] = \begin{bmatrix} \frac{\Delta_z}{\mathbf{z}_{22}} & \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \frac{1}{\mathbf{z}_{22}} \end{bmatrix} = \begin{bmatrix} \frac{3}{5}\Omega & \frac{4}{5} \\ \frac{-4}{5} & \frac{3}{5}\mathrm{S} \end{bmatrix}$$

(c) $$[\mathbf{T}] = \begin{bmatrix} \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \frac{\Delta_z}{\mathbf{z}_{21}} \\ \frac{1}{\mathbf{z}_{21}} & \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & \frac{3}{4}\Omega \\ \frac{3}{4}\mathrm{S} & \frac{5}{4} \end{bmatrix}$$

Chapter 19, Solution 62.

Consider the circuit shown below.



Since no current enters the input terminals of the op amp,

$$\mathbf{V}_1 = (10+30)\times 10^3\,\mathbf{I}_1 \qquad (1)$$

But $$\mathbf{V}_a = \mathbf{V}_b = \frac{30}{40}\mathbf{V}_1 = \frac{3}{4}\mathbf{V}_1$$

$$\mathbf{I}_b = \frac{\mathbf{V}_b}{20\times 10^3} = \frac{3}{80\times 10^3}\mathbf{V}_1$$

which is the same current that flows through the 50-kΩ resistor.

Thus, $$\mathbf{V}_2 = 40\times 10^3\,\mathbf{I}_2 + (50+20)\times 10^3\,\mathbf{I}_b$$

$$\mathbf{V}_2 = 40\times 10^3\,\mathbf{I}_2 + 70\times 10^3\cdot\frac{3}{80\times 10^3}\mathbf{V}_1$$

$$\mathbf{V}_2 = \frac{21}{8}\mathbf{V}_1 + 40\times 10^3\,\mathbf{I}_2$$

$$\mathbf{V}_2 = 105\times 10^3\,\mathbf{I}_1 + 40\times 10^3\,\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

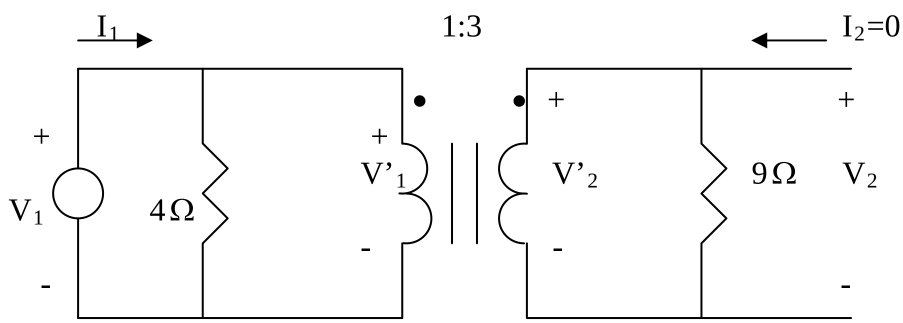
$$[\mathbf{z}] = \begin{bmatrix} \mathbf{40} & \mathbf{0} \\ \mathbf{105} & \mathbf{40} \end{bmatrix}\mathbf{k\Omega}$$

$$\boldsymbol{\Delta_z = z_{11}\,z_{22} - z_{12}\,z_{21} = 16\times 10^8}$$

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{1}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \mathbf{0.381} & \mathbf{15.24\ k\Omega} \\ \mathbf{9.52\ \mu S} & \mathbf{0.381} \end{bmatrix}$$

Chapter 19, Solution 63.

To get z_{11} and z_{21}, consider the circuit below.

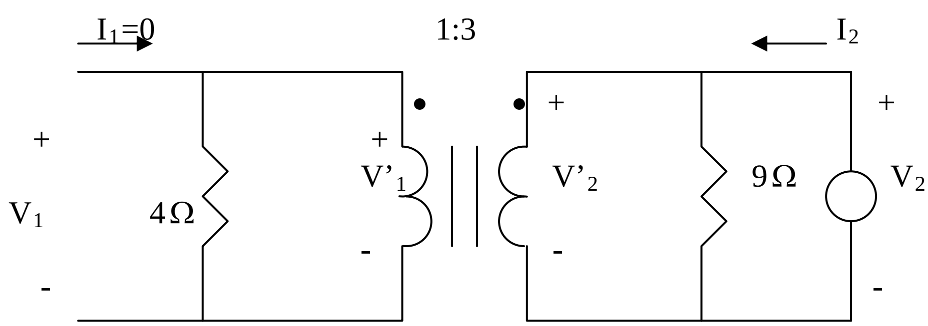


$$Z_R = \frac{9}{n^2} = 1, \qquad n = 3$$

$$V_1 = (4 // Z_R)I_1 = \frac{4}{5}I_1 \longrightarrow z_{11} = \frac{V_1}{I_1} = 0.8$$

$$V_2 = V_2' = nV_1' = nV_1 = 3(4/5)I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = 2.4$$

To get z_{21} and z_{22}, consider the circuit below.



$$Z_R' = n^2(4) = 36, \qquad n = 3$$

$$V_2 = (9 // Z_R')I_2 = \frac{9x36}{45}I_2 \longrightarrow z_{22} = \frac{V_2}{I_2} = 7.2$$

$$V_1 = \frac{V_2}{n} = \frac{V_2}{3} = 2.4I_2 \qquad \longrightarrow \qquad z_{21} = \frac{V_1}{I_2} = 2.4$$

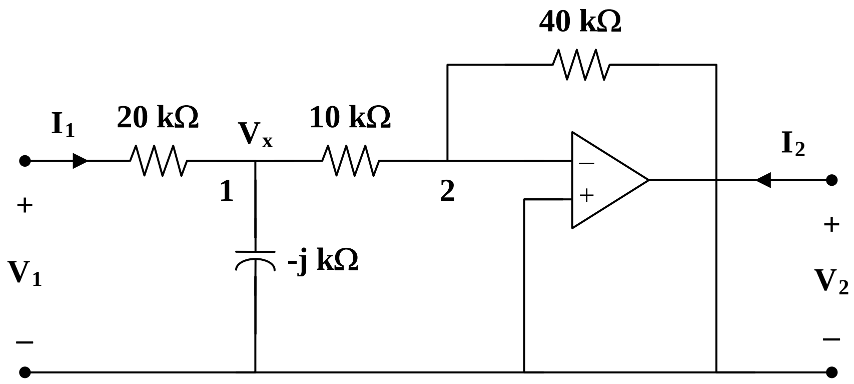
Thus,

$$\underline{\mathbf{[z]} = \begin{bmatrix} \mathbf{0.8} & \mathbf{2.4} \\ \mathbf{2.4} & \mathbf{7.2} \end{bmatrix} \Omega}$$

Chapter 19, Solution 64.

$$1\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{-j}{(10^3)(10^{-6})} = -j\ k\Omega$$

Consider the op amp circuit below.



At node 1,

$$\frac{\mathbf{V}_1 - \mathbf{V}_x}{20} = \frac{\mathbf{V}_x}{-j} + \frac{\mathbf{V}_x - 0}{10}$$

$$\mathbf{V}_1 = (3 + j20)\,\mathbf{V}_x \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_x - 0}{10} = \frac{0 - \mathbf{V}_2}{40} \longrightarrow \mathbf{V}_x = \frac{-1}{4}\mathbf{V}_2 \qquad (2)$$

But $\qquad \mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_x}{20\times 10^3} \qquad (3)$

Substituting (2) into (3) gives

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 + 0.25\,\mathbf{V}_2}{20\times 10^3} = 50\times 10^{-6}\,\mathbf{V}_1 + 12.5\times 10^{-6}\,\mathbf{V}_2 \qquad (4)$$

Substituting (2) into (1) yields

$$\mathbf{V}_1 = \frac{-1}{4}(3 + j20)\,\mathbf{V}_2$$

or $\qquad 0 = \mathbf{V}_1 + (0.75 + j5)\,\mathbf{V}_2 \qquad (5)$

Comparing (4) and (5) with the following equations

$$\mathbf{I}_1 = \mathbf{y}_{11}\,\mathbf{V}_1 + \mathbf{y}_{12}\,\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2$$

indicates that $\mathbf{I}_2 = 0$ and that

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{50 \times 10^{-6}} & \mathbf{12.5 \times 10^{-6}} \\ \mathbf{1} & \mathbf{0.75 + j5} \end{bmatrix} \mathbf{S}$$

$$\Delta_y = \mathbf{(77.5 + j25. - 12.5) \times 10^{-6} = (65 + j250) \times 10^{-6}}$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \begin{bmatrix} \mathbf{2 \times 10^4\ \Omega} & \mathbf{-0.25} \\ \mathbf{2 \times 10^4} & \mathbf{1.3 + j5\ S} \end{bmatrix}$$

Chapter 19, Solution 65.

The network consists of two two-ports in series. It is better to work with z parameters and then convert to y parameters. It is obvious that the upper 1 Ω resistor is shorted out by the top circuit so we are essentially left with 2 Ω connected to 3 Ω. This then produces the Z parameters

$$[\mathbf{z}] = \begin{bmatrix} 5\Omega & 3\Omega \\ 3\Omega & 3\Omega \end{bmatrix}$$

$$\Delta_z = 15 - 9 = 6$$

$$[\mathbf{y}] = \begin{bmatrix} \dfrac{\mathbf{z}_{22}}{\Delta_z} & \dfrac{-\mathbf{z}_{12}}{\Delta_z} \\ \dfrac{-\mathbf{z}_{21}}{\Delta_z} & \dfrac{\mathbf{z}_{11}}{\Delta_z} \end{bmatrix} = \begin{bmatrix} \mathbf{0.5} & \mathbf{-0.5} \\ \mathbf{-0.5} & \dfrac{\mathbf{5}}{\mathbf{6}} \end{bmatrix} \mathbf{S}$$

Chapter 19, Solution 66.

Since we have two two-ports in series, it is better to convert the given y parameters to z parameters.

$$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21} = (2\times10^{-3})(10\times10^{-3}) - 0 = 20\times10^{-6}$$

$$[\mathbf{z}_a] = \begin{bmatrix} \dfrac{\mathbf{y}_{22}}{\Delta_y} & \dfrac{-\mathbf{y}_{12}}{\Delta_y} \\ \dfrac{-\mathbf{y}_{21}}{\Delta_y} & \dfrac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} 500\,\Omega & 0 \\ 0 & 100\,\Omega \end{bmatrix}$$

$$[\mathbf{z}] = \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} = \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$$

i.e.

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

or

$$\mathbf{V}_1 = 600\mathbf{I}_1 + 100\mathbf{I}_2 \qquad (1)$$
$$\mathbf{V}_2 = 100\mathbf{I}_1 + 200\mathbf{I}_2 \qquad (2)$$

But, at the input port,

$$\mathbf{V}_s = \mathbf{V}_1 + 60\mathbf{I}_1 \qquad (3)$$

and at the output port,

$$\mathbf{V}_2 = \mathbf{V}_o = -300\mathbf{I}_2 \qquad (4)$$

From (2) and (4),

$$100\mathbf{I}_1 + 200\mathbf{I}_2 = -300\mathbf{I}_2$$
$$\mathbf{I}_1 = -5\mathbf{I}_2 \qquad (5)$$

Substituting (1) and (5) into (3),

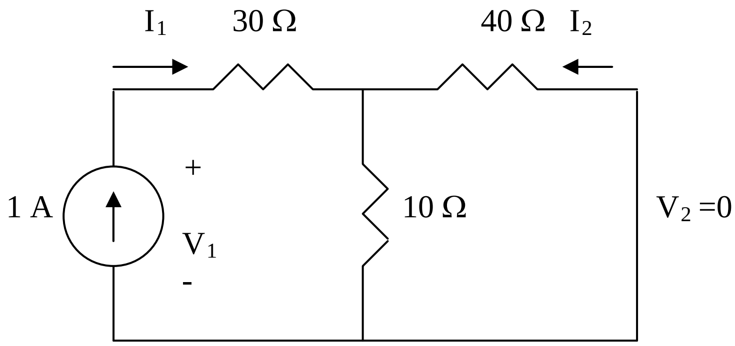
$$\mathbf{V}_s = 600\mathbf{I}_1 + 100\mathbf{I}_2 + 60\mathbf{I}_1$$
$$= (660)(-5)\mathbf{I}_2 + 100\mathbf{I}_2$$
$$= -3200\mathbf{I}_2 \qquad (6)$$

From (4) and (6),

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{-300\mathbf{I}_2}{-3200\mathbf{I}_2} = \mathbf{0.09375}$$

Chapter 19, Solution 67.

We first the y parameters, To find y_{11} and y_{21} consider the circuit below.

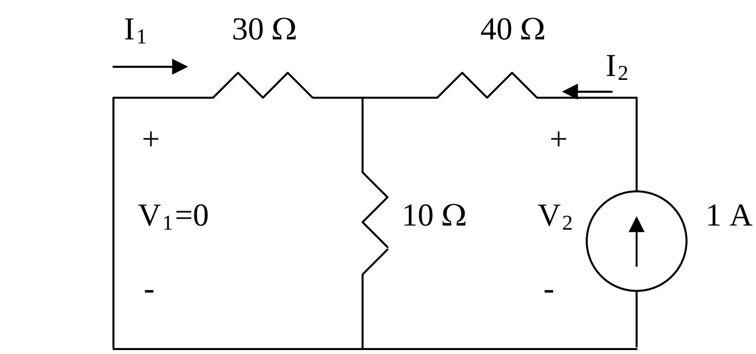


$$V_1 = I_1(30+10//40) = 38I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{1}{38}$$

By current division,

$$I_2 = \frac{-10}{50}I_1 = -0.2I_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = \frac{-0.2I_1}{38I_1} = \frac{-1}{190}$$

To find y_{22} and y_{12} consider the circuit below.



$$V_2 = (40+10//30)I_2 = 47.5I_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = \frac{2}{93}$$ y_{22} = 2/95

By current division,

$$I_1 = -\frac{10}{30+10}I_2 = -\frac{I_2}{4} \longrightarrow y_{12} = \frac{I_1}{V_2} = \frac{-\frac{1}{4}I_2}{47.5I_2} = -\frac{1}{190}$$

$$[y] = \begin{bmatrix} 1/38 & -1/190 \\ -1/190 & 2/95 \end{bmatrix}$$

For three copies cascaded in parallel, we can use MATLAB.

```
>> Y=[1/38,-1/190;-1/190,2/95]
```

```
Y =
    0.0263   -0.0053
   -0.0053    0.0211
>> Y3=3*Y
Y3 =
    0.0789   -0.0158
   -0.0158    0.0632
>> DY=0.0789*0.0632-0.0158*0.158
DY =
    0.0025
>> T=[0.0632/0.0158,1/0.0158;DY/0.0158,0.0789/0.0158]
T =
    4.0000   63.2911
    0.1576    4.9937
```

$$T = \underline{\underline{\begin{bmatrix} \mathbf{4} & \mathbf{63.29\,\Omega} \\ \mathbf{0.1576\,S} & \mathbf{4.994} \end{bmatrix}}}$$

Chapter 19, Solution 68.

For the upper network N_a, $[\mathbf{y}_a] = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$

and for the lower network N_b, $[\mathbf{y}_b] = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

For the overall network,

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$$

$$\Delta_y = 36 - 9 = 27$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \begin{bmatrix} \mathbf{\dfrac{1}{6}\,\Omega} & \mathbf{\dfrac{1}{2}} \\ \mathbf{\dfrac{1}{2}} & \mathbf{\dfrac{9}{2}\,S} \end{bmatrix}$$

Chapter 19, Solution 69.

We first determine the y parameters for the upper network N_a.
To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).

$$n = \frac{1}{2}, \qquad \mathbf{Z}_R = \frac{1/s}{n^2} = \frac{4}{s}$$

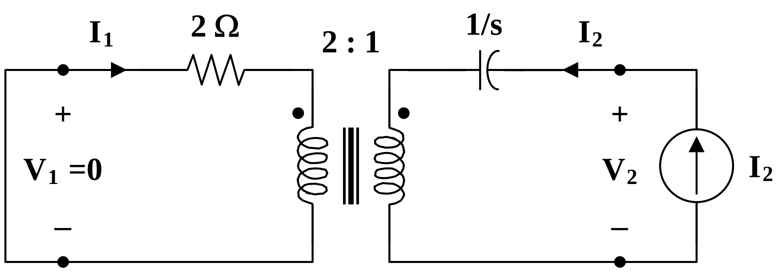
$$\mathbf{V}_1 = (2 + \mathbf{Z}_R)\mathbf{I}_1 = \left(2 + \frac{4}{s}\right)\mathbf{I}_1 = \left(\frac{2s+4}{s}\right)\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{s}{2(s+2)}$$

$$\mathbf{I}_2 = \frac{-\mathbf{I}_1}{n} = -2\mathbf{I}_1 = \frac{-s\mathbf{V}_1}{s+2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-s}{s+2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{Z}_R' = (n^2)(2) = \left(\frac{1}{4}\right)(2) = \frac{1}{2}$$

$$\mathbf{V}_2 = \left(\frac{1}{s} + \mathbf{Z}_R'\right)\mathbf{I}_2 = \left(\frac{1}{s} + \frac{1}{2}\right)\mathbf{I}_2 = \left(\frac{s+2}{2s}\right)\mathbf{I}_2$$

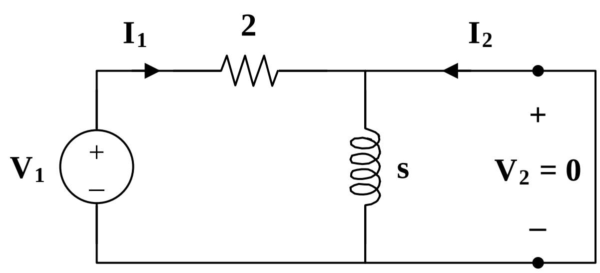
$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{2s}{s+2}$$

$$\mathbf{I}_1 = -n\mathbf{I}_2 = \left(\frac{-1}{2}\right)\left(\frac{2s}{s+2}\right)\mathbf{V}_2 = \left(\frac{-s}{s+2}\right)\mathbf{V}_2$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-s}{s+2}$$

$$[\mathbf{y}_a] = \begin{bmatrix} \frac{s}{2(s+2)} & \frac{-s}{s+2} \\ \frac{-s}{s+2} & \frac{2s}{s+2} \end{bmatrix}$$

For the lower network N_b, we obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by referring to the network in Fig. (c).

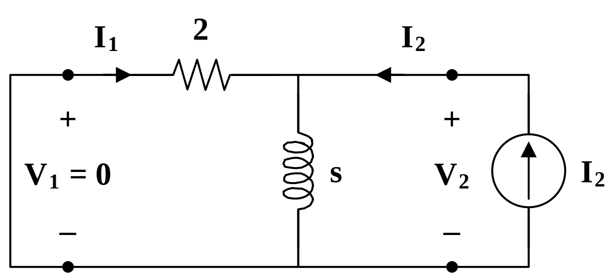


(c)

$$\mathbf{V}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{2}$$

$$\mathbf{I}_2 = -\mathbf{I}_1 = \frac{-\mathbf{V}_1}{2} \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (d).



(d)

$$\mathbf{V}_2 = (s \parallel 2)\mathbf{I}_2 = \frac{2s}{s+2}\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{s+2}{2s}$$

$$\mathbf{I}_1 = -\mathbf{I}_2 \cdot \frac{-s}{s+2} = \left(\frac{-s}{s+2}\right)\left(\frac{s+2}{2s}\right)\mathbf{V}_2 = \frac{-\mathbf{V}_2}{2}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{2}$$

$$[\mathbf{y}_b] = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & (s+2)/2s \end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} \dfrac{\mathbf{s+1}}{\mathbf{s+2}} & \dfrac{\mathbf{-(3s+2)}}{\mathbf{2(s+2)}} \\ \dfrac{\mathbf{-(3s+2)}}{\mathbf{2(s+2)}} & \dfrac{\mathbf{5s^2+4s+4}}{\mathbf{2s(s+2)}} \end{bmatrix}$$

Chapter 19, Solution 70.

We may obtain the g parameters from the given z parameters.

$$[\mathbf{z}_a] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix}, \qquad \Delta_{z_a} = 250 - 100 = 150$$

$$[\mathbf{z}_b] = \begin{bmatrix} 50 & 25 \\ 25 & 30 \end{bmatrix}, \qquad \Delta_{z_b} = 1500 - 625 = 875$$

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{1}{z_{11}} & \dfrac{-z_{12}}{z_{11}} \\ \dfrac{z_{21}}{z_{11}} & \dfrac{\Delta_z}{z_{11}} \end{bmatrix}$$

$$[\mathbf{g}_a] = \begin{bmatrix} 0.04 & -0.8 \\ 0.2 & 6 \end{bmatrix}, \qquad [\mathbf{g}_b] = \begin{bmatrix} 0.02 & -0.5 \\ 0.5 & 17.5 \end{bmatrix}$$

$$[\mathbf{g}] = [\mathbf{g}_a] + [\mathbf{g}_b] = \begin{bmatrix} \mathbf{0.06\ S} & \mathbf{-1.3} \\ \mathbf{0.7} & \mathbf{23.5\ \Omega} \end{bmatrix}$$

Chapter 19, Solution 71.

This is a parallel-series connection of two two-ports. We need to add their g parameters together and obtain z parameters from there.

For the transformer,

$$V_1 = \frac{1}{2}V_2, \quad I_1 = -2I_2$$

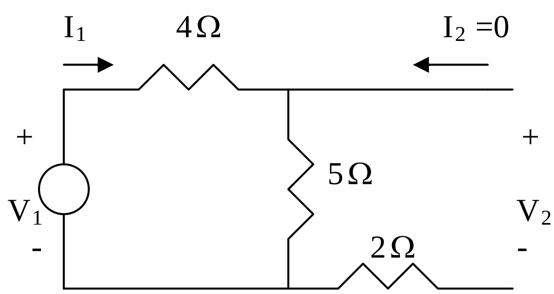
Comparing this with

$$V_1 = AV_2 - BI_2, \qquad I_1 = CV_2 - DI_2$$

shows that

$$[T_{b1}] = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

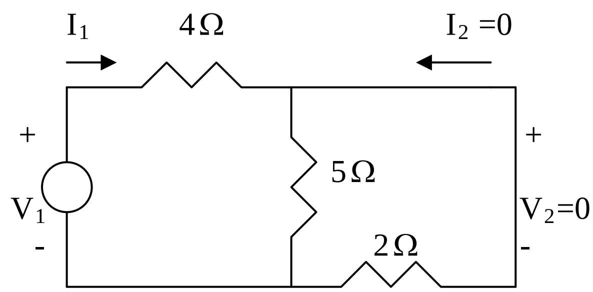
To get A and C for T_{b2} , consider the circuit below.



$$V_1 = 9I_1, \qquad V_2 = 5I_1$$

$$A = \frac{V_1}{V_2} = 9/5 = 1.8, \qquad C = \frac{I_1}{V_2} = 1/5 = 0.2$$

We obtain B and D by looking at the circuit below.



$$I_2 = -\frac{5}{7}I_1 \quad \longrightarrow \quad D = -\frac{I_1}{I_2} = 7/5 = 1.4$$

$$V_1 = 4I_1 - 2I_2 = 4(-\frac{7}{5}I_2) - 2I_2 = -\frac{38}{5}I_2 \quad \longrightarrow \quad B = -\frac{V_1}{I_2} = 7.6$$

$$[T_{b2}] = \begin{bmatrix} 1.8 & 7.6 \\ 0.2 & 1.4 \end{bmatrix}$$

$$[T] = [T_{b1}][T_{b2}] = \begin{bmatrix} 0.9 & 3.8 \\ 0.4 & 2.8 \end{bmatrix}, \qquad \Delta_T = 1$$

$$[g_b] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.4444 & -1.1111 \\ 1.1111 & 4.2222 \end{bmatrix}$$

From Prob. 19.52,

$$[T_a] = \begin{bmatrix} 1.8 & 18.8 \\ 0.1 & 1.6 \end{bmatrix}$$

$$[g_a] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.05555 & -0.5555 \\ 0.5555 & 10.4444 \end{bmatrix}$$

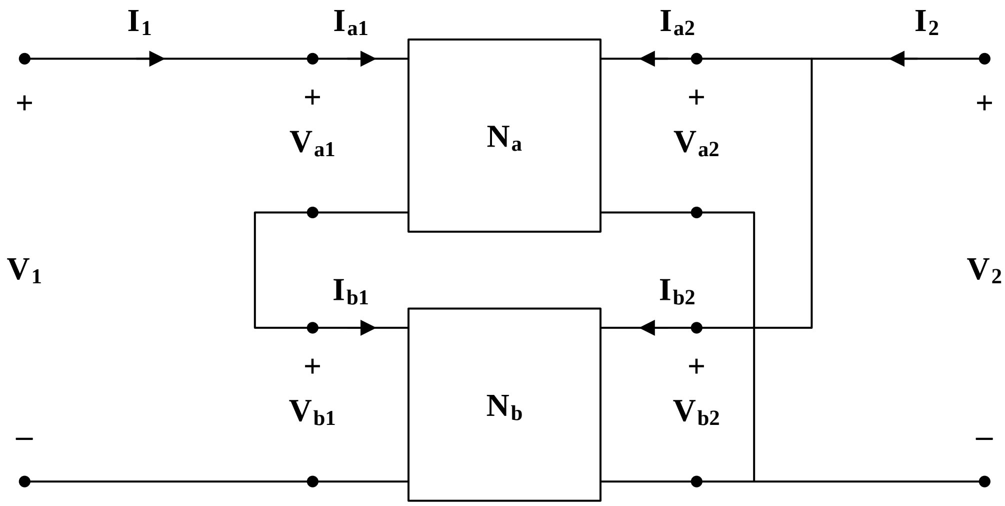
$$[g] = [g_a] + [g_b] = \begin{bmatrix} 0.4999 & -1.6667 \\ 1.6667 & 14.667 \end{bmatrix}$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 1/g_{11} & -g_{21}/g_{11} \\ g_{21}/g_{11} & \Delta_g/g_{11} \end{bmatrix} = \underline{\begin{bmatrix} 2 & -3.334 \\ 3.334 & 20.22 \end{bmatrix} \Omega}$$

Chapter 19, Solution 72.

Consider the network shown below.



$$\mathbf{V}_{a1} = 25\mathbf{I}_{a1} + 4\mathbf{V}_{a2} \quad (1)$$

$$\mathbf{I}_{a2} = -4\mathbf{I}_{a1} + \mathbf{V}_{a2} \quad (2)$$

$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_{b2} \quad (3)$$

$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_{b2} \quad (4)$$

$$\mathbf{V}_1 = \mathbf{V}_{a1} + \mathbf{V}_{b1}$$

$$\mathbf{V}_2 = \mathbf{V}_{a2} = \mathbf{V}_{b2}$$

$$\mathbf{I}_2 = \mathbf{I}_{a2} + \mathbf{I}_{b2}$$

$$\mathbf{I}_1 = \mathbf{I}_{a1}$$

Now, rewrite (1) to (4) in terms of $\mathbf{I}_1$ and $\mathbf{V}_2$

$$\mathbf{V}_{a1} = 25\mathbf{I}_1 + 4\mathbf{V}_2 \quad (5)$$

$$\mathbf{I}_{a2} = -4\mathbf{I}_1 + \mathbf{V}_2 \quad (6)$$

$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_2 \quad (7)$$

$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_2 \quad (8)$$

Adding (5) and (7),

$$\mathbf{V}_1 = 25\mathbf{I}_1 + 16\mathbf{I}_{b1} + 5\mathbf{V}_2 \quad (9)$$

Adding (6) and (8),

$$\mathbf{I}_2 = -4\mathbf{I}_1 - \mathbf{I}_{b1} + 1.5\mathbf{V}_2 \quad (10)$$

$$\mathbf{I}_{b1} = \mathbf{I}_{a1} = \mathbf{I}_1 \quad (11)$$

Because the two networks N_a and N_b are independent,

$$\mathbf{I}_2 = -5\mathbf{I}_1 + 1.5\mathbf{V}_2$$

or

$$\mathbf{V}_2 = 3.333\mathbf{I}_1 + 0.6667\mathbf{I}_2 \tag{12}$$

Substituting (11) and (12) into (9),

$$\mathbf{V}_1 = 41\mathbf{I}_1 + \frac{25}{1.5}\mathbf{I}_1 + \frac{5}{1.5}\mathbf{I}_2$$

$$\mathbf{V}_1 = 57.67\mathbf{I}_1 + 3.333\mathbf{I}_2 \tag{13}$$

Comparing (12) and (13) with the following equations

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

indicates that

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix} \boldsymbol{\Omega}$$

Alternatively,

$$[\mathbf{h}_a] = \begin{bmatrix} 25 & 4 \\ -4 & 1 \end{bmatrix}, \qquad [\mathbf{h}_b] = \begin{bmatrix} 16 & 1 \\ -1 & 0.5 \end{bmatrix}$$

$$[\mathbf{h}] = [\mathbf{h}_a] + [\mathbf{h}_b] = \begin{bmatrix} 41 & 5 \\ -5 & 1.5 \end{bmatrix} \qquad \Delta_h = 61.5 + 25 = 86.5$$

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{22}} & \dfrac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ \dfrac{-\mathbf{h}_{21}}{\mathbf{h}_{22}} & \dfrac{1}{\mathbf{h}_{22}} \end{bmatrix} = \begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix} \boldsymbol{\Omega}$$

as obtained previously.

Chapter 19, Solution 73.

From Problem 19.6,

$$[z] = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix}, \qquad \Delta z = 25x30 - 20x24 = 270$$

$$A = \frac{z_{11}}{z_{21}} = \frac{25}{24}, \qquad B = \frac{\Delta z}{z_{21}} = \frac{270}{24}$$

$$C = \frac{1}{z_{21}} = \frac{1}{24}, \; D = \frac{z_{22}}{z_{21}} = \frac{30}{24}$$

The overall ABCD parameters can be found using MATLAB.

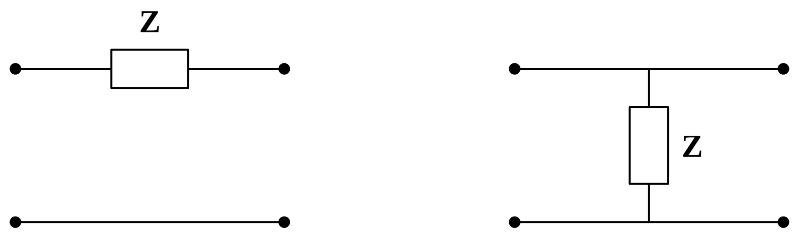
```
 >> T=[25/24,270/24;1/24,30/24]
T =
    1.0417   11.2500
    0.0417    1.2500
>> T3=T*T*T
T3 =
    2.6928   49.7070
    0.1841    3.6133
>> Z=[2.693/0.1841,(2.693*3.613-0.1841*49.71)/0.1841;1/0.1841,3.613/0.1841]
Z =
   14.6279    3.1407
    5.4318   19.6252
```

$$\mathbf{[Z]} = \underline{\begin{bmatrix} \mathbf{14.628} & \mathbf{3.141} \\ \mathbf{5.432} & \mathbf{19.625} \end{bmatrix}}\,\Omega$$

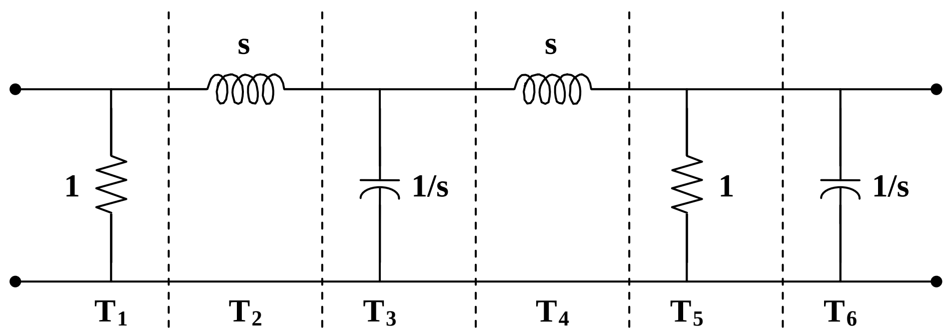
Chapter 19, Solution 74.

From Prob. 18.35, the transmission parameters for the circuit in Figs. (a) and (b) are

$$[\mathbf{T}_a] = \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix}, \qquad [\mathbf{T}_b] = \begin{bmatrix} 1 & 0 \\ 1/\mathbf{Z} & 1 \end{bmatrix}$$



We partition the given circuit into six subcircuits similar to those in Figs. (a) and (b) as shown in Fig. (c) and obtain $[\mathbf{T}]$ for each.



$$[\mathbf{T}_1] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \qquad [\mathbf{T}_2] = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}, \qquad [\mathbf{T}_3] = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

$$[\mathbf{T}_4] = [\mathbf{T}_2], \qquad [\mathbf{T}_5] = [\mathbf{T}_1], \qquad [\mathbf{T}_6] = [\mathbf{T}_3]$$

$$[\mathbf{T}] = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4][\mathbf{T}_5][\mathbf{T}_6] = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

$$= [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix} = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3]\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

$$= [\mathbf{T}_1][\mathbf{T}_2]\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s+1 & 1 \end{bmatrix}$$

$$= [\mathbf{T}_1]\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s^3+s^2+2s+1 & s^2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} s^4+s^3+3s^2+2s+1 & s^3+2s \\ s^3+s^2+2s+1 & s^2+1 \end{bmatrix}$$

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{s^4+s^3+3s^2+2s+1} & \mathbf{s^3+2s} \\ \mathbf{s^4+2s^3+4s^2+4s+2} & \mathbf{s^3+s^2+2s+1} \end{bmatrix}$$

Note that $\mathbf{AB-CD} = 1$ as expected.

Chapter 19, Solution 75.

(a) We convert $[z_a]$ and $[z_b]$ to T-parameters. For N_a, $\Delta_z = 40 - 24 = 16$.

$$[T_a] = \begin{bmatrix} z_{11}/z_{21} & \Delta_z/z_{21} \\ 1/z_{21} & z_{22}/z_{21} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0.25 & 1.25 \end{bmatrix}$$

For N_b, $\Delta_y = 80 + 8 = 88$.

$$[T_b] = \begin{bmatrix} -y_{22}/y_{21} & -1/y_{21} \\ -\Delta_y/y_{21} & -y_{11}/y_{21} \end{bmatrix} = \begin{bmatrix} -5 & -0.5 \\ -44 & -4 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} -186 & -17 \\ -56.25 & -5.125 \end{bmatrix}$$

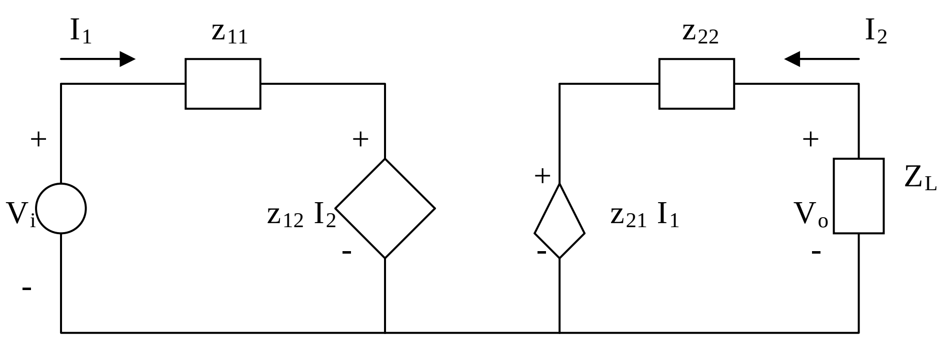
We convert this to y-parameters. $\Delta_T = AD - BC = -3.$

$$\mathbf{[y]} = \begin{bmatrix} \mathbf{D/B} & \mathbf{-\Delta_T/B} \\ \mathbf{-1/B} & \mathbf{A/B} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0.3015} & \mathbf{-0.1765} \\ \mathbf{0.0588} & \mathbf{10.94} \end{bmatrix}\mathbf{S}}$$

(b) The equivalent z-parameters are

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 3.3067 & 0.0533 \\ -0.0178 & 0.0911 \end{bmatrix}$$

Consider the equivalent circuit below.



$$V_i = z_{11}I_1 + z_{12}I_2 \qquad (1)$$

$$V_o = z_{21}I_1 + z_{22}I_2 \qquad (2)$$

But $V_o = -I_2 Z_L \longrightarrow I_2 = -V_o / Z_L \qquad (3)$

From (2) and (3) ,

$$V_o = z_{21}I_1 - z_{22}\frac{V_o}{Z_L} \longrightarrow I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{Z_L z_{21}}\right) \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$\mathbf{\frac{V_i}{V_o} = \left(\frac{z_{11}}{z_{21}} + \frac{z_{11}z_{22}}{z_{21}Z_L}\right) - \frac{z_{12}}{Z_L} = -194.3} \longrightarrow \underline{\mathbf{\frac{V_{o.}}{V_i} = -0.0051}}$$

Chapter 19, Solution 76.

To get z_{11} and z_{21}, we open circuit the output port and let $I_1 = 1A$ so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \qquad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 3.849, \qquad z_{21} = V_2 = 1.122$$

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let $I_2 = 1A$ so that

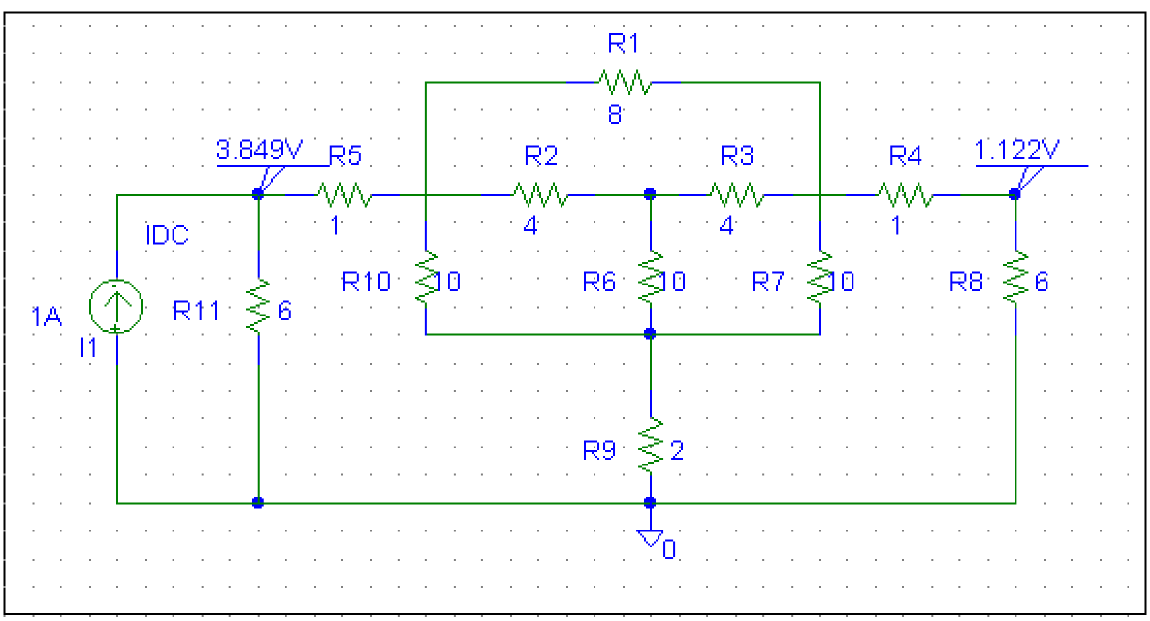
$$z_{12} = \frac{V_1}{I_2} = V_1, \qquad z_{22} = \frac{V_2}{I_2} = V_2$$

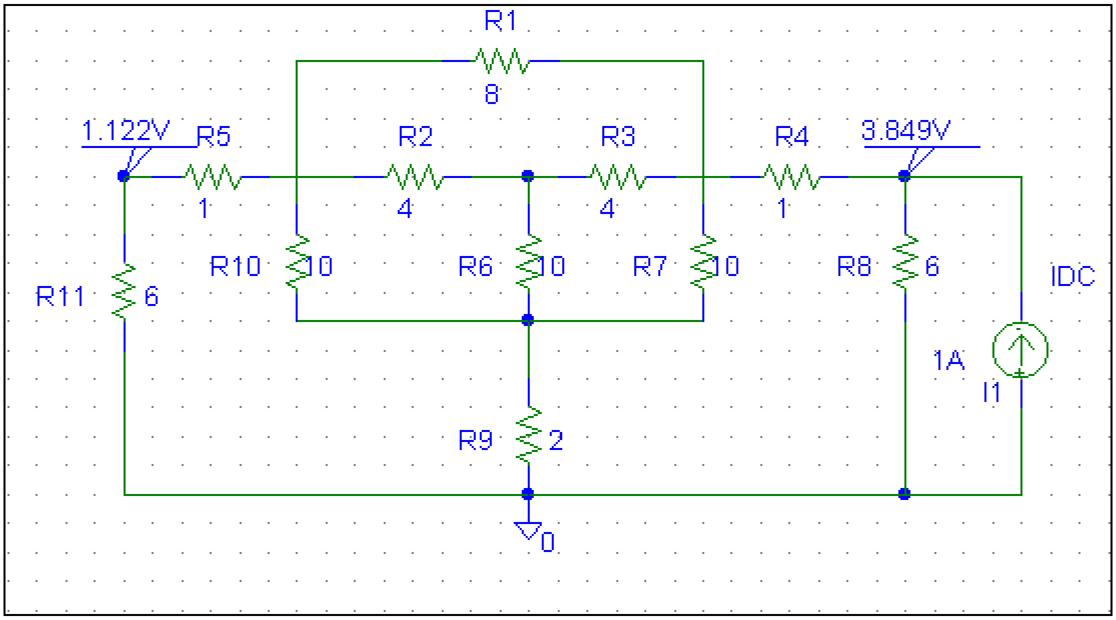
The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 1.122, \qquad z_{22} = V_2 = 3.849$$

Thus,

$$\underline{\mathbf{[z]} = \begin{bmatrix} \mathbf{3.949} & \mathbf{1.122} \\ \mathbf{1.122} & \mathbf{3.849} \end{bmatrix} \Omega}$$




R1
8
1.122V
R5
1
R2
4
R3
4
R4
1
3.849V
R10
10
R6
10
R7
10
R8
6
IDC
R11
6
1A
I1
R9
2
0

Chapter 19, Solution 77.

We follow Example 19.15 except that this is an AC circuit.

(a) We set $V_2 = 0$ and $I_1 = 1$ A. The schematic is shown below. In the AC Sweep Box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

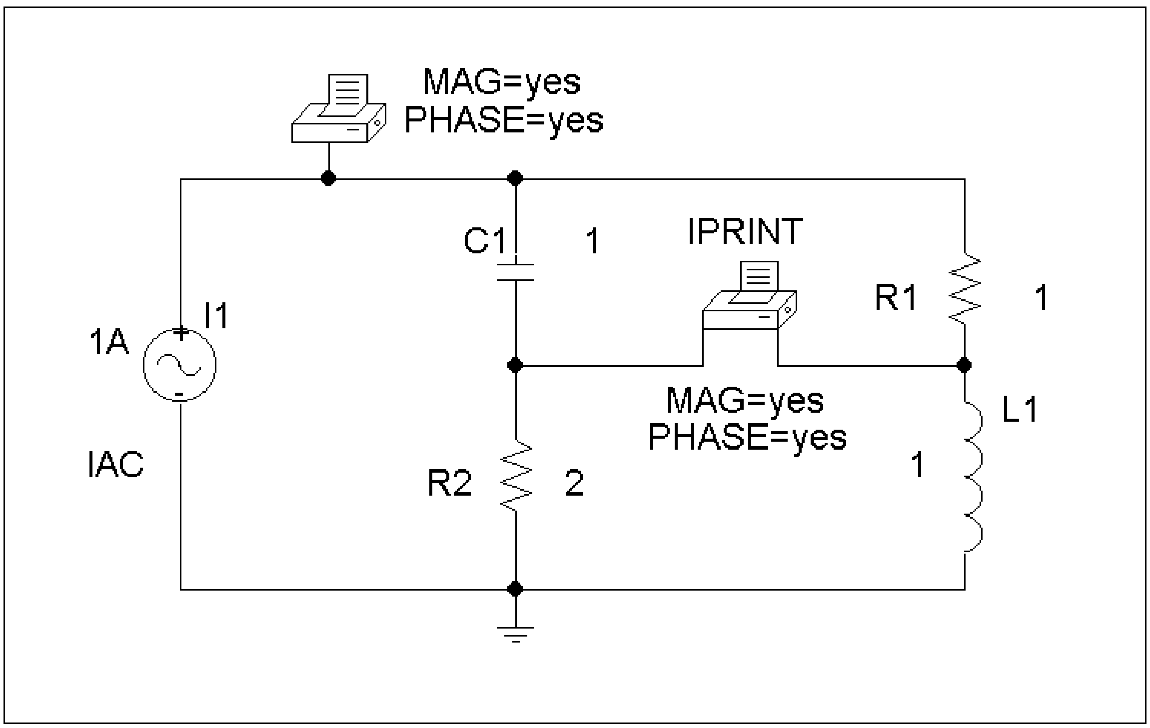
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	3.163 E–.01	–1.616 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	9.488 E–01	–1.616 E+02

From this we obtain

$$h_{11} = V_1/1 = 0.9488\angle{-161.6^\circ}$$

$$h_{21} = I_2/1 = 0.3163\angle{-161.6^\circ}.$$



(b) In this case, we set $I_1 = 0$ and $V_2 = 1V$. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	3.163 E–.01	1.842 E+01

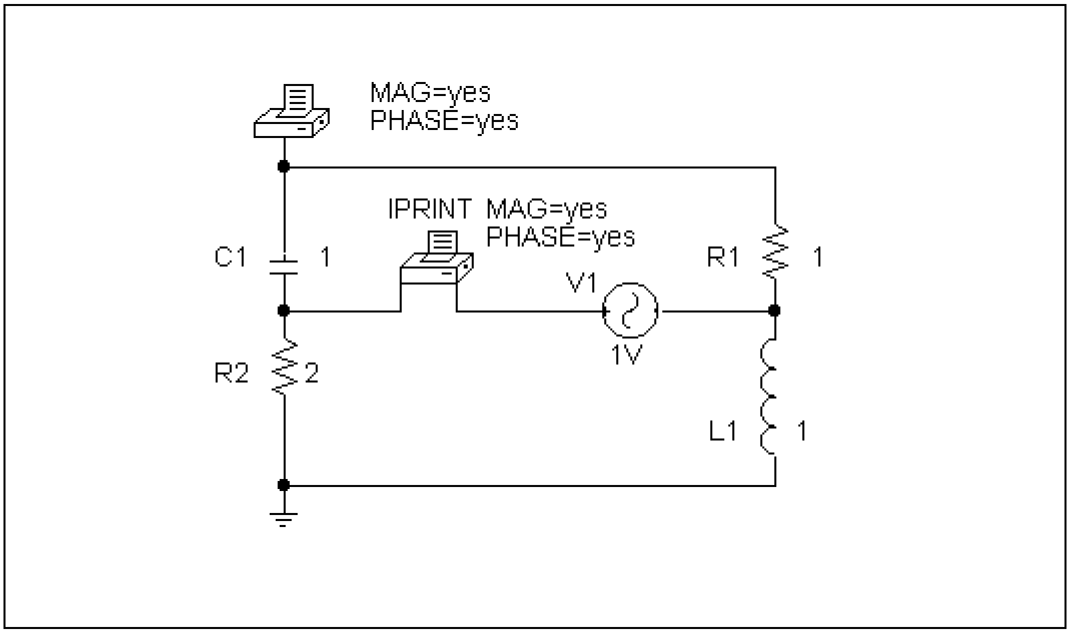
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	9.488 E–01	–1.616 E+02

From this,

$$h_{12} = V_1/1 = 0.3163\angle 18.42°$$

$$h_{21} = I_2/1 = 0.9488\angle -161.6°.$$

Thus, $$[h] = \begin{bmatrix} \mathbf{0.9488\angle -161.6°\,\Omega} & \mathbf{0.3163\angle 18.42°} \\ \mathbf{0.3163\angle -161.6°} & \mathbf{0.9488\angle -161.6°\,S} \end{bmatrix}$$



Chapter 19, Solution 78

For h_{11} and h_{21}, short-circuit the output port and let I_1 = 1A. $f = \omega / 2\pi = 0.6366$. The schematic is shown below. When it is saved and run, the output file contains the following:

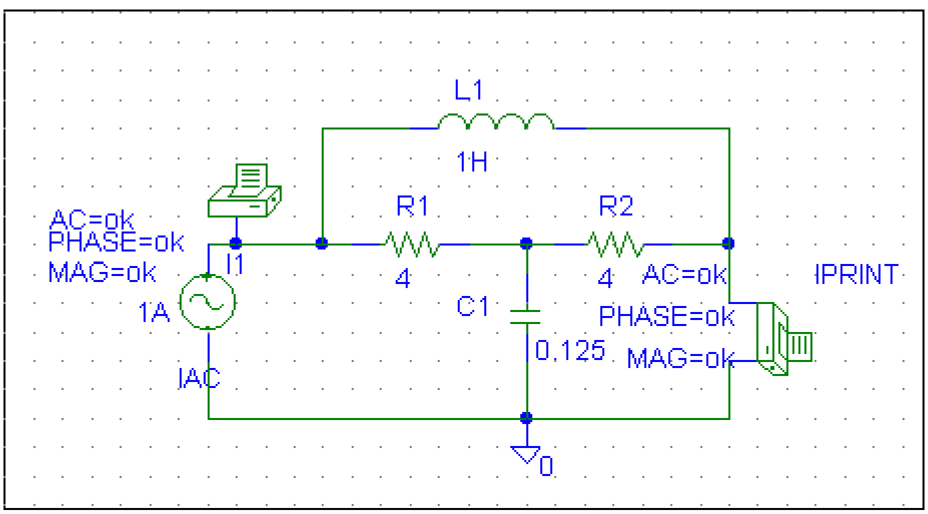
```
FREQ         IM(V_PRINT1)IP(V_PRINT1)

   6.366E-01   1.202E+00   1.463E+02

  FREQ         VM($N_0003) VP($N_0003)

   6.366E-01   3.771E+00  -1.350E+02
```

From the output file, we obtain

$$I_2 = 1.202\angle 146.3^o, \qquad V_1 = 3.771\angle -135^o$$

so that

$$h_{11} = \frac{V_1}{1} = 3.771\angle -135^o, \qquad h_{21} = \frac{I_2}{1} = 1.202\angle 146.3^o$$



For h_{12} and h_{22}, open-circuit the input port and let V_2 = 1V. The schematic is shown below. When it is saved and run, the output file includes:

```
  FREQ         VM($N_0003) VP($N_0003)

   6.366E-01   1.202E+00  -3.369E+01
```

```
  FREQ          IM(V_PRINT1)IP(V_PRINT1)

   6.366E-01   3.727E-01  -1.534E+02
```

From the output file, we obtain

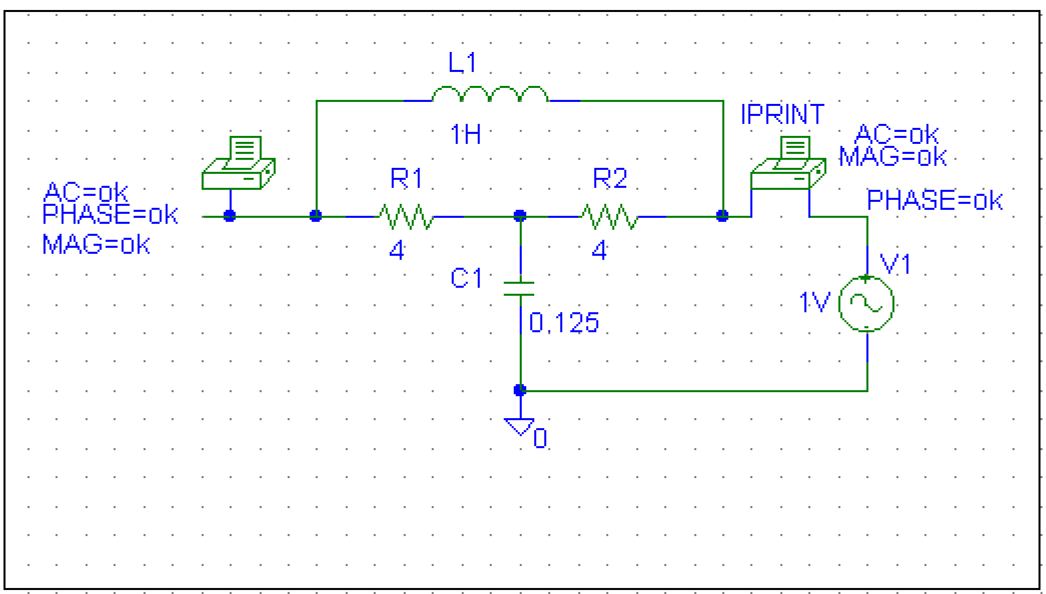
$$I_2 = 0.3727\angle -153.4^o, \qquad V_1 = 1.202\angle -33.69^o$$

so that

$$h_{12} = \frac{V_1}{1} = 1.202\angle -33.69^o, \qquad h_{22} = \frac{I_2}{1} = 0.3727\angle -153.4^o$$

Thus,

$$\mathbf{[h] = \begin{bmatrix} 3.771\angle -135^o\ \Omega & 1.202\angle -33.69^o \\ 1.202\angle 146.3 & 0.3727\angle -153.4^o\ S \end{bmatrix}}$$



Chapter 19, Solution 79

We follow Example 19.16.

(a) We set $I_1 = 1$ A and open-circuit the output-port so that $I_2 = 0$. The schematic is shown below with two VPRINT1s to measure V_1 and V_2. In the AC Sweep box, we enter Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

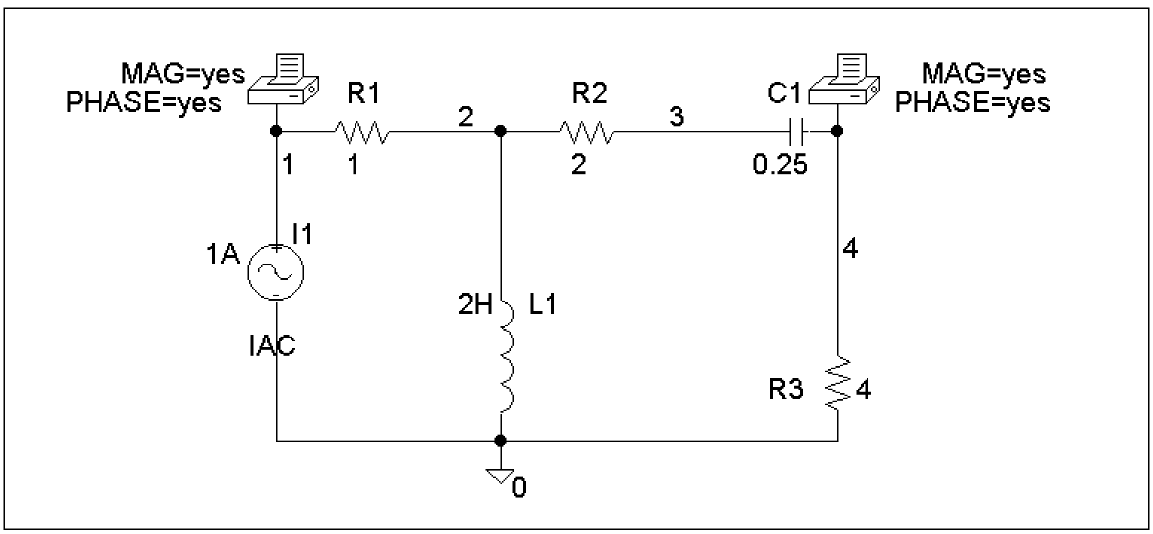
FREQ	VM(1)	VP(1)
3.183 E–01	4.669 E+00	–1.367 E+02

FREQ	VM(4)	VP(4)
3.183 E–01	2.530 E+00	–1.084 E+02

From this,

$$z_{11} = V_1/I_1 = 4.669\angle{-136.7°}/1 = 4.669\angle{-136.7°}$$

$$z_{21} = V_2/I_1 = 2.53\angle{-108.4°}/1 = 2.53\angle{-108.4°}.$$



(b) In this case, we let $I_2 = 1$ A and open-circuit the input port. The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

FREQ	VM(1)	VP(1)
3.183 E–01	2.530 E+00	–1.084 E+02

FREQ	VM(2)	VP(2)
3.183 E–01	1.789 E+00	–1.534 E+02

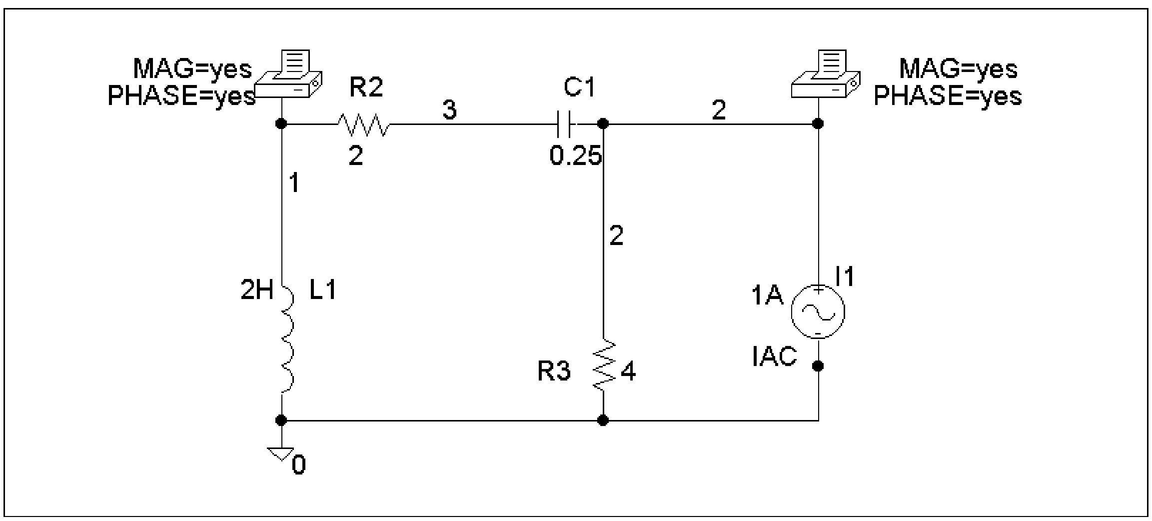
From this,

$$z_{12} = V_1/I_2 = 2.53\angle{-108.4°}/1 = 2.53\angle{-108..4°}$$

$$z_{22} = V_2/I_2 = 1.789\angle{-153.4°}/1 = 1.789\angle{-153.4°}.$$

Thus,

$$[z] = \begin{bmatrix} \mathbf{4.669\angle -136.7°} & \mathbf{2.53\angle -108.4°} \\ \mathbf{2.53\angle -108.4°} & \mathbf{1.789\angle -153.4°} \end{bmatrix} \mathbf{\Omega}$$



Chapter 19, Solution 80

To get z_{11} and z_{21}, we open circuit the output port and let $I_1 = 1A$ so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \qquad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 29.88, \qquad z_{21} = V_2 = -70.37$$

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let $I_2 = 1A$ so that

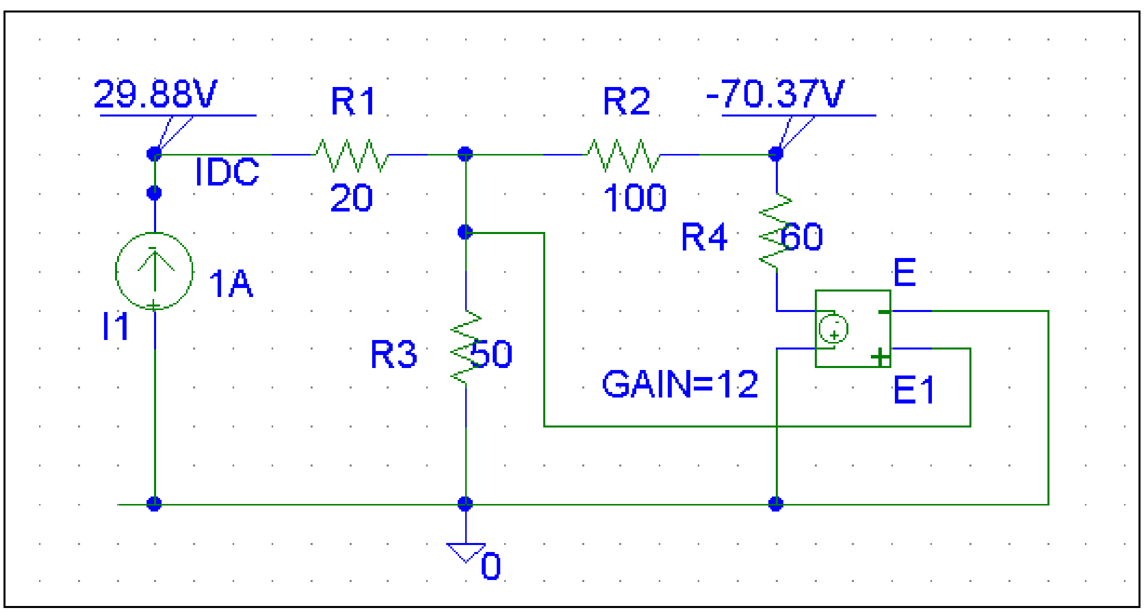
$$z_{12} = \frac{V_1}{I_2} = V_1, \qquad z_{22} = \frac{V_2}{I_2} = V_2$$

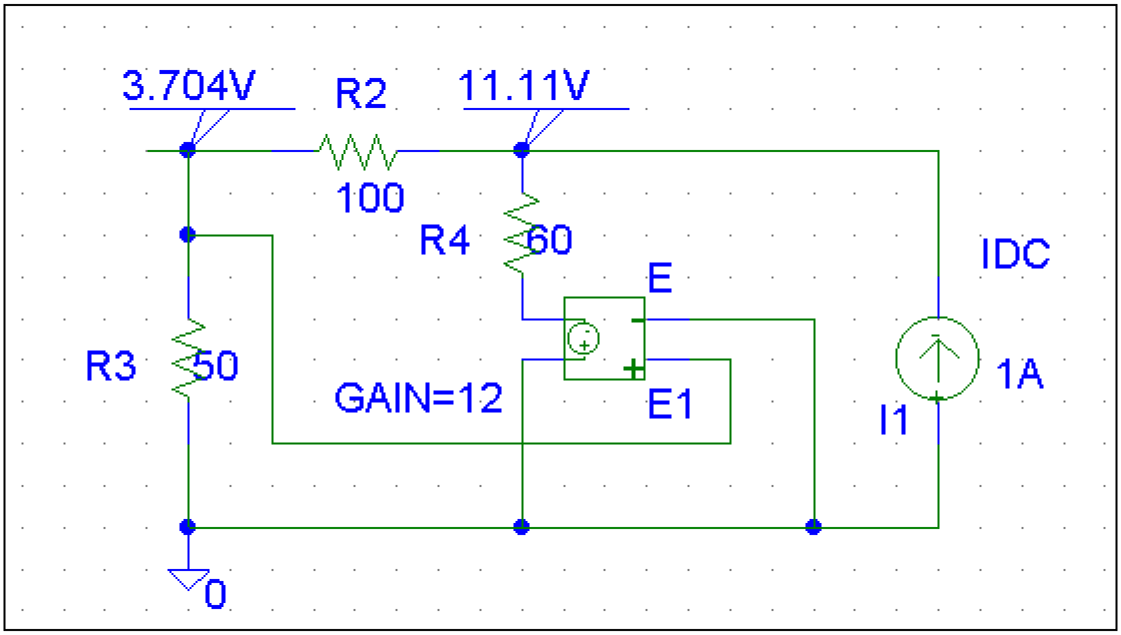
The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 3.704, \qquad z_{22} = V_2 = 11.11$$

Thus,

$$\mathbf{[z]} = \underline{\begin{bmatrix} \mathbf{29.88} & \mathbf{3.704} \\ \mathbf{-70.37} & \mathbf{11.11} \end{bmatrix} \Omega}$$



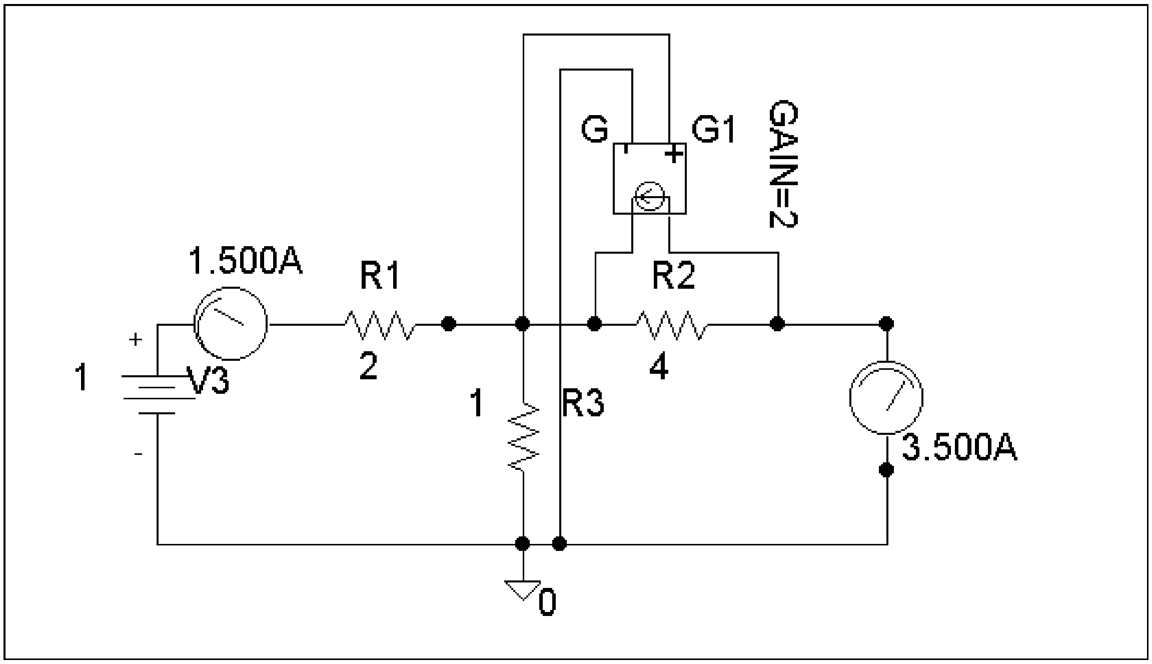


3.704V
R2
11.11V
100
R4
60
IDC
E
R3
50
GAIN=12
E1
1A
I1
0

Chapter 19, Solution 81

(a) We set $V_1 = 1$ and short circuit the output port. The schematic is shown below. After simulation we obtain

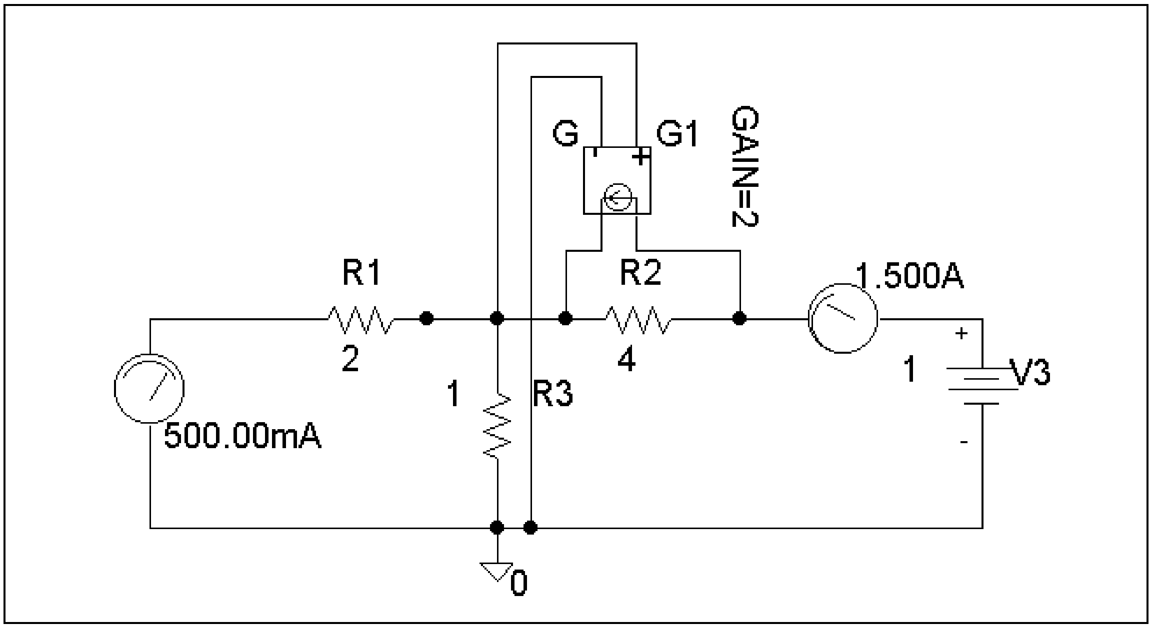
$$y_{11} = I_1 = 1.5, \quad y_{21} = I_2 = 3.5$$



(b) We set $V_2 = 1$ and short-circuit the input port. The schematic is shown below. Upon simulating the circuit, we obtain

$$y_{12} = I_1 = -0.5, \quad y_{22} = I_2 = 1.5$$

$$[Y] = \begin{bmatrix} \mathbf{1.5} & \mathbf{-0.5} \\ \mathbf{3.5} & \mathbf{1.5} \end{bmatrix}\mathbf{S}$$



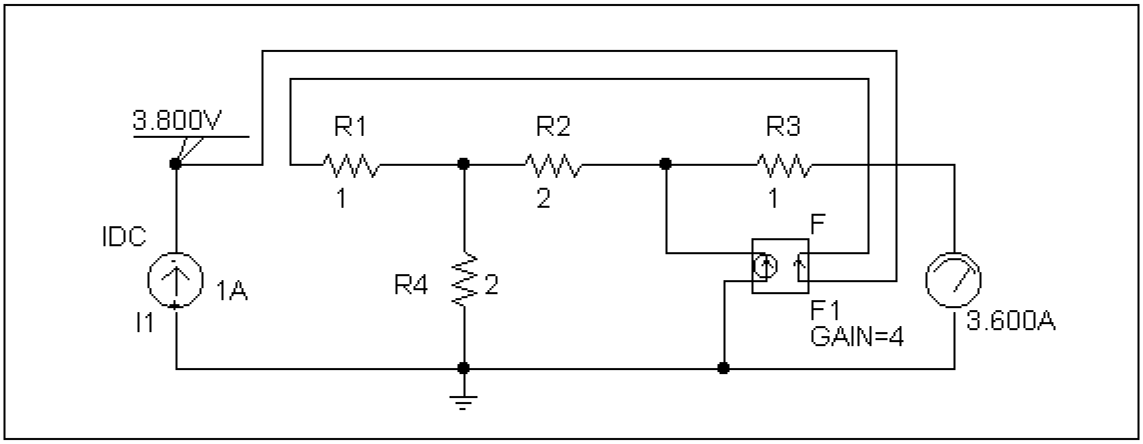
G
G1
GAIN=2
R1
2
R2
4
1.500A
+
1
V3
-
1
R3
500.00mA
0

Chapter 19, Solution 82

We follow Example 19.15.

(a) Set $V_2 = 0$ and $I_1 = 1A$. The schematic is shown below. After simulation, we obtain

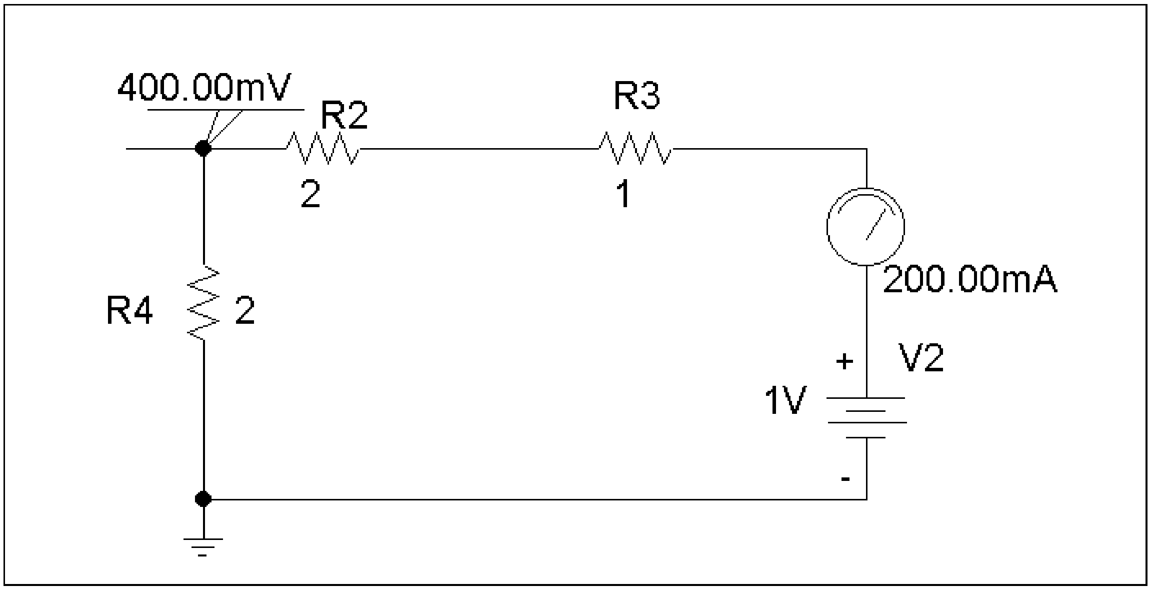
$$h_{11} = V_1/1 = 3.8, \quad h_{21} = I_2/1 = 3.6$$



(b) Set $V_1 = 1$ V and $I_1 = 0$. The schematic is shown below. After simulation, we obtain

$$h_{12} = V_1/1 = 0.4, \quad h_{22} = I_2/1 = 0.25$$

Hence,
$$[h] = \begin{bmatrix} \mathbf{3.8\Omega} & \mathbf{0.4} \\ \mathbf{3.6} & \mathbf{0.25\,S} \end{bmatrix}$$



400.00mV
R2
2
R3
1
200.00mA
R4
2
+
V2
1V
-

Chapter 19, Solution 83

To get A and C, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 11$ and $V_2 = 34$.

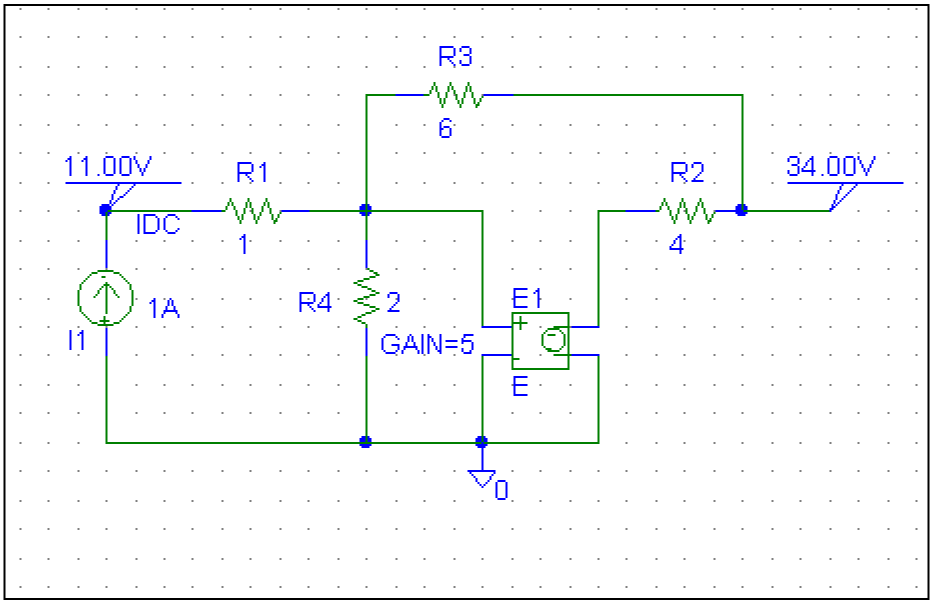
$$A = \frac{V_1}{V_2} = 0.3235, \qquad C = \frac{I_1}{V_2} = \frac{1}{34} = 0.02941$$

Similarly, to get B and D, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 2.5$ and $I_2 = -2.125$.

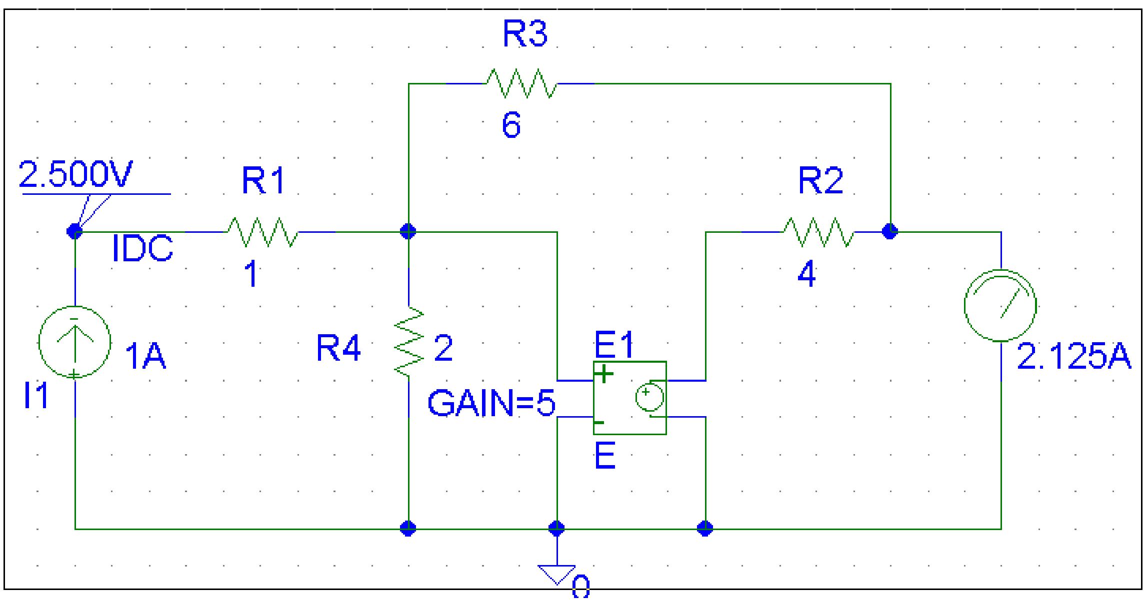
$$B = -\frac{V_1}{I_2} = \frac{2.5}{2.125} = 1.1765, \qquad D = -\frac{I_1}{I_2} = \frac{1}{2.125} = 0.4706$$

Thus,

$$\underline{\mathbf{[T]} = \begin{bmatrix} \mathbf{0.3235} & \mathbf{1.1765\,\Omega} \\ \mathbf{0.02941\,S} & \mathbf{0.4706} \end{bmatrix}}$$



R3
6
11.00V
R1
IDC
1
R2
4
34.00V
R4
2
1A
I1
E1
GAIN=5
E
0



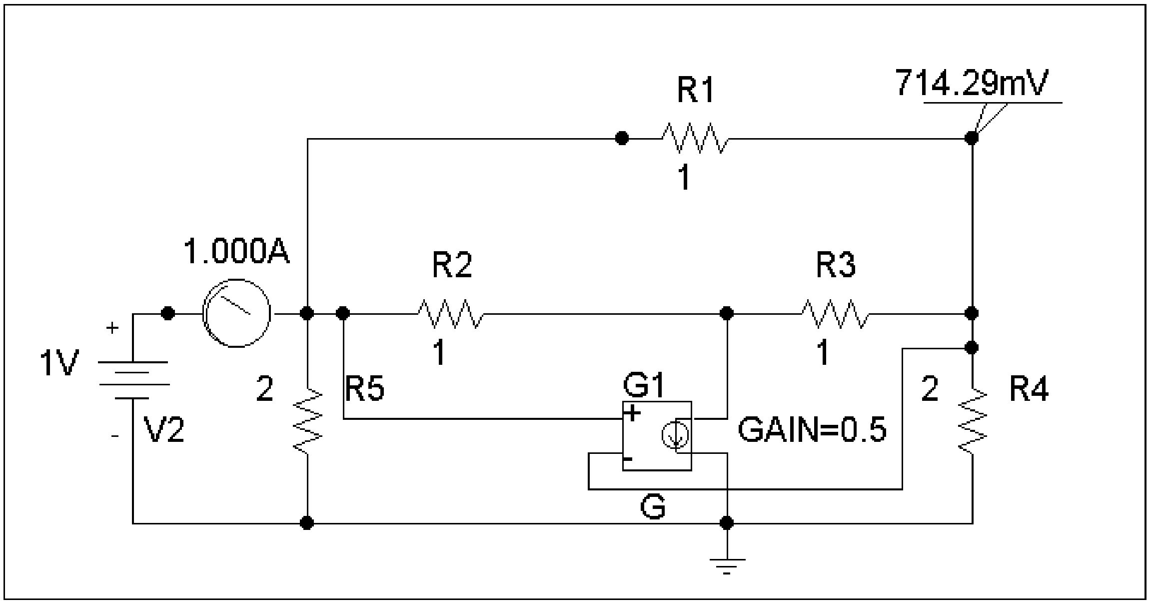
R3
6
2.500V
R1
IDC
1
R2
4
R4
2
1A
I1
E1
GAIN=5
E
2.125A
0

Chapter 19, Solution 84

(a) Since $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$, we open-circuit the output port and let $V_1 = 1$ V. The schematic is as shown below. After simulation, we obtain

$$A = 1/V_2 = 1/0.7143 = 1.4$$

$$C = I_2/V_2 = 1.0/0.7143 = 1.4$$

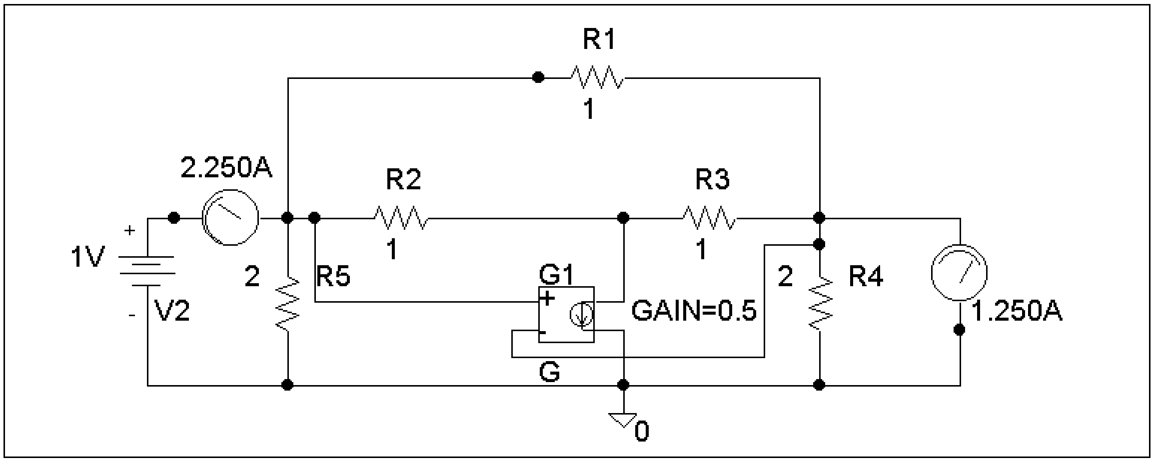


(b) To get B and D, we short-circuit the output port and let $V_1 = 1$. The schematic is shown below. After simulating the circuit, we obtain

$$B = -V_1/I_2 = -1/1.25 = -0.8$$

$$D = -I_1/I_2 = -2.25/1.25 = -1.8$$

Thus $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \mathbf{1.4} & \mathbf{-0.8\Omega} \\ \mathbf{1.4S} & \mathbf{-1.8} \end{bmatrix}$$


R1
1
2.250A
R2
R3
1V
V2
1
1
2
R5
G1
GAIN=0.5
2
R4
1.250A
G
0

Chapter 19, Solution 85

(a) Since $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$, we let $V_1 = 1$ V and open-circuit the output port. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

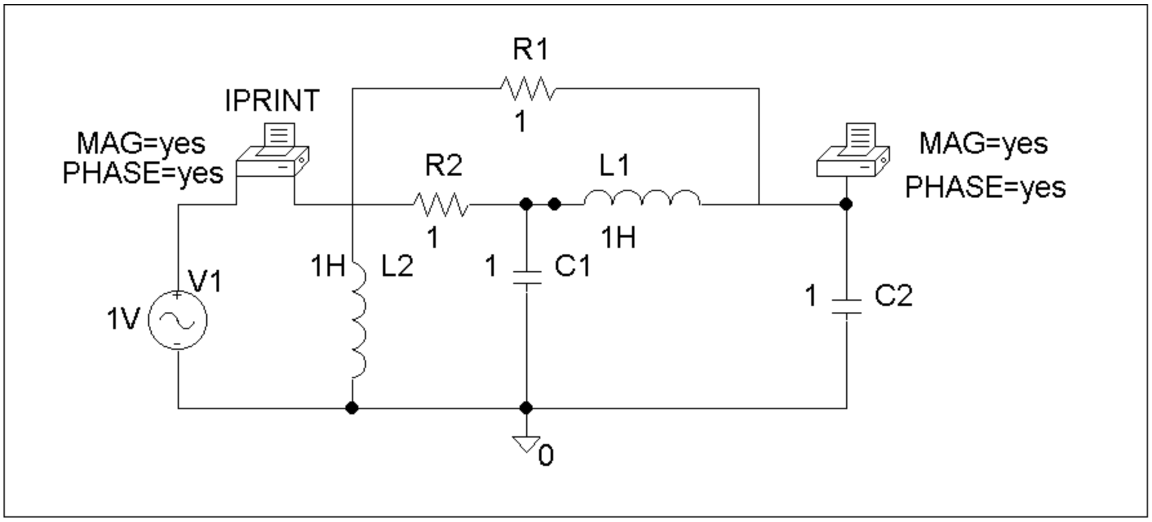
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	6.325 E–01	1.843 E+01

FREQ	VM($N_0002)	VP($N_0002)
1.592 E–01	6.325 E–01	–7.159 E+01

From this, we obtain

$$A = \frac{1}{V_2} = \frac{1}{0.6325\angle -71.59^\circ} = 1.581\angle 71.59^\circ$$

$$C = \frac{I_1}{V_2} = \frac{0.6325\angle 18.43^\circ}{0.6325\angle -71.59^\circ} = 1\angle 90^\circ = j$$



(b) Similarly, since $B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$ and $D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$, we let $V_1 = 1$ V and short-circuit the output port. The schematic is shown below. Again, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. After simulation, we get an output file which includes the following results:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	5.661 E–04	8.997 E+01

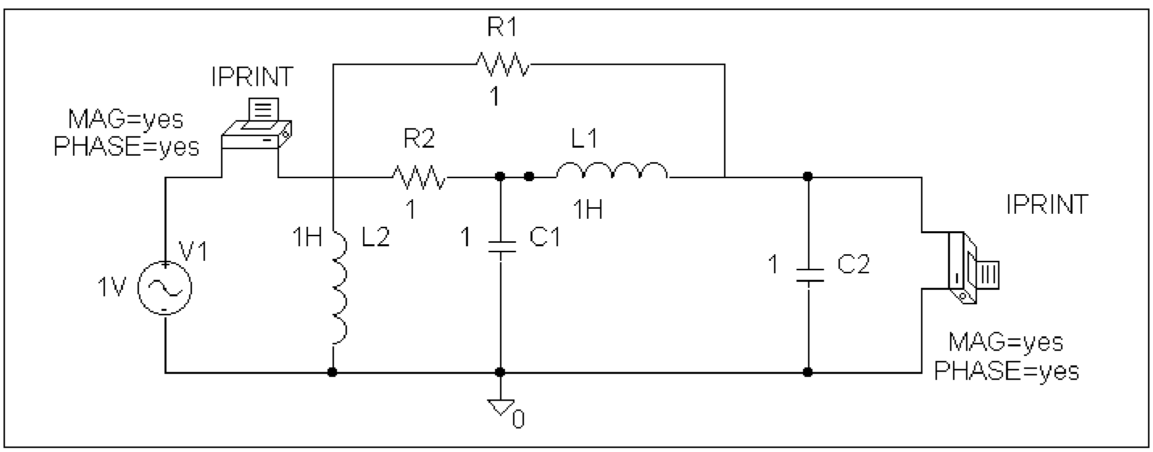
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	9.997 E–01	–9.003 E+01

From this,

$$B = -\frac{1}{I_2} = -\frac{1}{0.9997\angle -90^\circ} = -1\angle 90^\circ = -j$$

$$D = -\frac{I_1}{I_2} = -\frac{5.661\text{x}10^{-4}\angle 89.97^\circ}{0.9997\angle -90^\circ} = 5.561\text{x}10^{-4}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \mathbf{1.581\angle 71.59^\circ} & \mathbf{-j\Omega} \\ \mathbf{jS} & \mathbf{5.661x10^{-4}} \end{bmatrix}$$



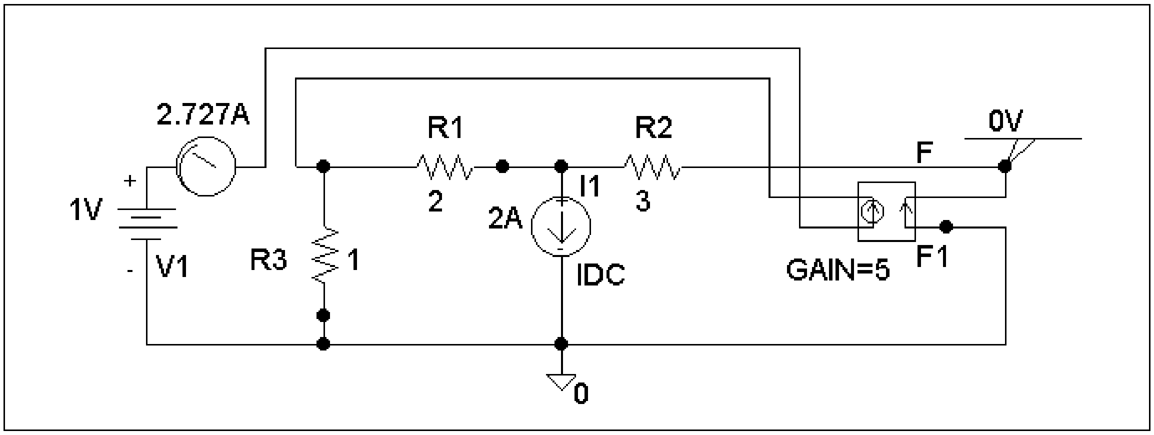
Chapter 19, Solution 86

(a) By definition, $g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$, $g_{21} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$.

We let $V_1 = 1$ V and open-circuit the output port. The schematic is shown below. After simulation, we obtain

$$g_{11} = I_1 = 2.7$$

$$g_{21} = V_2 = 0.0$$



(b) Similarly,

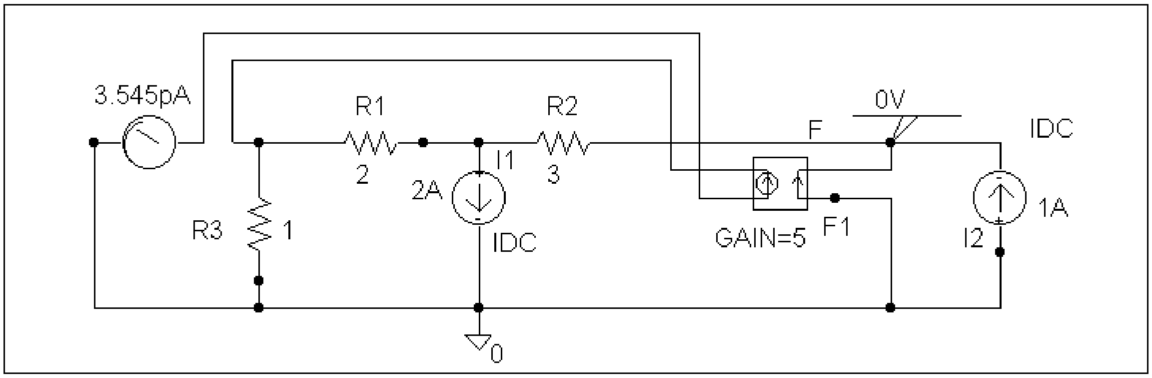
$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}, \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

We let $I_2 = 1$ A and short-circuit the input port. The schematic is shown below. After simulation,

$$g_{12} = I_1 = 0$$

$$g_{22} = V_2 = 0$$

Thus $$[g] = \begin{bmatrix} \mathbf{2.727S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$


3.545pA
R1
2
R2
3
0V
F
IDC
I1
2A
IDC
R3
1
GAIN=5
F1
I2
1A
0

Chapter 19, Solution 87

(a) Since $a = \left.\dfrac{V_2}{V_1}\right|_{I_1=0}$ and $c = \left.\dfrac{I_2}{V_1}\right|_{I_1=0}$,

we open-circuit the input port and let $V_2 = 1$ V. The schematic is shown below. In the AC Sweep box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

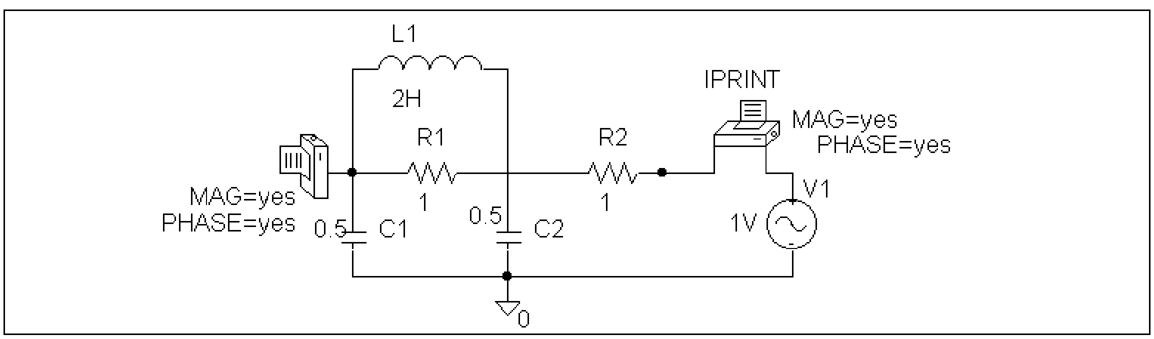
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	5.000 E–01	1.800 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	5.664 E–04	8.997 E+01

From this,

$$a = \frac{1}{5.664\text{x}10^{-4}\angle 89.97^\circ} = 1765\angle -89.97^\circ$$

$$c = \frac{0.5\angle 180^\circ}{5.664\text{x}10^{-4}\angle 89.97^\circ} = -882.28\angle -89.97^\circ$$



(b) Similarly,

$$b = -\left.\frac{V_2}{I_1}\right|_{V_1=0} \quad \text{and} \quad d = -\left.\frac{I_2}{I_1}\right|_{V_1=0}$$

We short-circuit the input port and let $V_2 = 1$ V. The schematic is shown below. After simulation, we obtain an output file which includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)

1.592 E–01	5.000 E–01	1.800 E+02

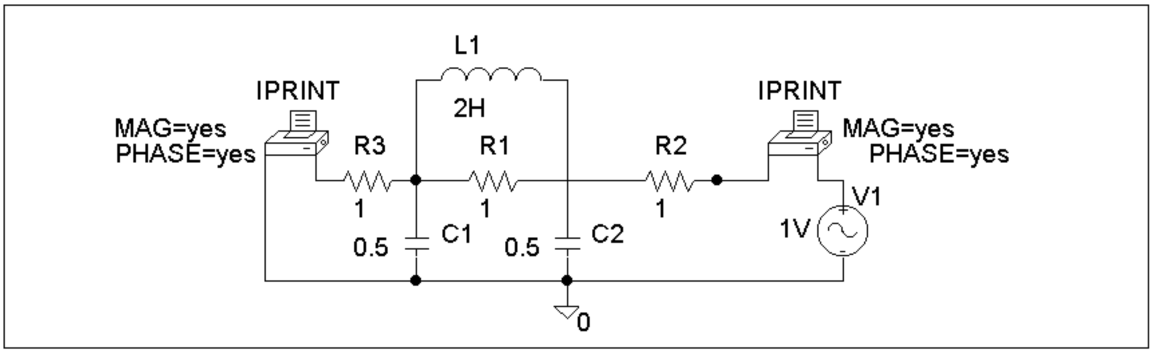
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	5.664 E–04	–9.010 E+01

From this, we get

$$b = -\frac{1}{5.664\text{x}10^{-4}\angle -90.1°} = -j1765$$

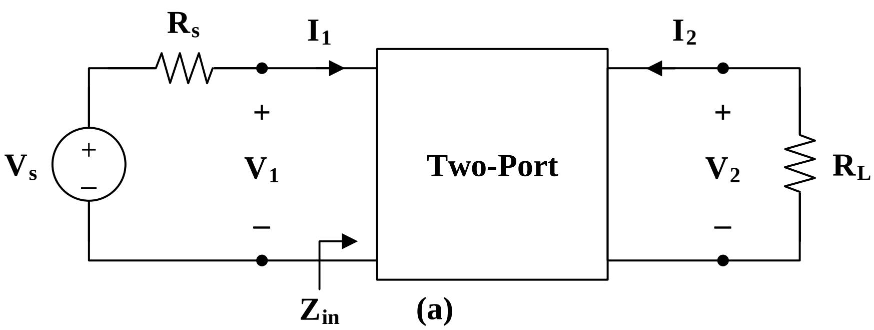
$$d = -\frac{0.5\angle 180°}{5.664\text{x}10^{-4}\angle -90.1°} = j888.28$$

Thus $$\mathbf{[t]} = \begin{bmatrix} \mathbf{-\,j1765} & \mathbf{-\,j1765\,\Omega} \\ \mathbf{j888.2\,S} & \mathbf{j888.2} \end{bmatrix}$$



Chapter 19, Solution 88

To get Z_{in}, consider the network in Fig. (a).



(a)

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \quad (1)$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \quad (2)$$

But
$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{R_L} = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$
$$\mathbf{V}_2 = \frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L} \quad (3)$$

Substituting (3) into (1) yields
$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\cdot\left(\frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L}\right), \qquad \mathbf{Y}_L = \frac{1}{R_L}$$
$$\mathbf{I}_1 = \left(\frac{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)\mathbf{V}_1, \qquad \Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}$$

or
$$Z_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}$$

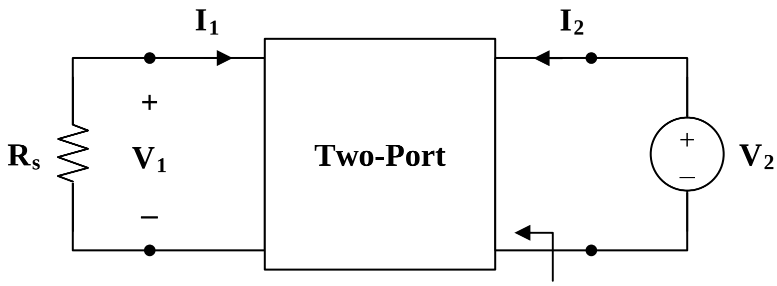
$$A_i = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{\mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2}{\mathbf{I}_1} = \mathbf{y}_{21}Z_{in} + \left(\frac{\mathbf{y}_{22}}{\mathbf{I}_1}\right)\left(\frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$
$$= \mathbf{y}_{21}Z_{in} - \frac{\mathbf{y}_{22}\mathbf{y}_{21}Z_{in}}{\mathbf{y}_{22} + \mathbf{Y}_L} = \left(\frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}\right)\left(\mathbf{y}_{21} - \frac{\mathbf{y}_{22}\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$

$$A_i = \frac{\mathbf{y}_{21}\mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}$$

From (3),

$$A_v = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}$$

To get Z_{out}, consider the circuit in Fig. (b).



(b)

$$Z_{out} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2} \qquad (4)$$

But $\mathbf{V}_1 = -R_s\,\mathbf{I}_1$

Substituting this into (1) yields

$$\mathbf{I}_1 = -\mathbf{y}_{11}\,R_s\,\mathbf{I}_1 + \mathbf{y}_{12}\,\mathbf{V}_2$$

$$(1 + \mathbf{y}_{11}\,R_s)\,\mathbf{I}_1 = \mathbf{y}_{12}\,\mathbf{V}_2$$

$$\mathbf{I}_1 = \frac{\mathbf{y}_{12}\,\mathbf{V}_2}{1 + \mathbf{y}_{11}\,R_s} = \frac{-\mathbf{V}_1}{R_s}$$

or

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{-\mathbf{y}_{12}\,R_s}{1 + \mathbf{y}_{11}\,R_s}$$

Substituting this into (4) gives

$$Z_{out} = \frac{1}{\mathbf{y}_{22} - \dfrac{\mathbf{y}_{12}\,\mathbf{y}_{21}\,R_s}{1 + \mathbf{y}_{11}\,R_s}}$$

$$= \frac{1 + \mathbf{y}_{11}\,R_s}{\mathbf{y}_{22} + \mathbf{y}_{11}\,\mathbf{y}_{22}\,R_s - \mathbf{y}_{21}\,\mathbf{y}_{22}\,R_s}$$

$$Z_{out} = \frac{\mathbf{y}_{11} + \mathbf{Y}_s}{\Delta_y + \mathbf{y}_{22}\,\mathbf{Y}_s}$$

Chapter 19, Solution 89

$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + (2640 \times 16 \times 10^{-6} - 2.6 \times 10^{-4} \times 72) \cdot 10^5}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + 1824} = \mathbf{-1613}$$

$$\text{dc gain} = 20 \log \left| A_v \right| = 20 \log (1613) = \mathbf{64.15\ dB}$$

Chapter 19, Solution 90

(a)
$$Z_{in} = h_{ie} - \frac{h_{re}\,h_{fe}\,R_L}{1 + h_{oe}\,R_L}$$

$$1500 = 2000 - \frac{10^{-4} \times 120\,R_L}{1 + 20 \times 10^{-6}\,R_L}$$

$$500 = \frac{12 \times 10^{-3}}{1 + 2 \times 10^{-5}\,R_L}$$

$$500 + 10^{-2}\,R_L = 12 \times 10^{-3}\,R_L$$

$$500 \times 10^{2} = 0.2\,R_L$$

$$R_L = \mathbf{250\ k\Omega}$$

(b)
$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie}\,h_{oe} - h_{re}\,h_{fe})\,R_L}$$

$$A_v = \frac{-120 \times 250 \times 10^{3}}{2000 + (2000 \times 20 \times 10^{-6} - 120 \times 10^{-4}) \times 250 \times 10^{3}}$$

$$A_v = \frac{-30 \times 10^{6}}{2 \times 10^{3} + 7 \times 10^{3}} = \mathbf{-\ 3333}$$

$$A_i = \frac{h_{fe}}{1 + h_{oe}\,R_L} = \frac{120}{1 + 20 \times 10^{-6} \times 250 \times 10^{3}} = \mathbf{20}$$

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie})\,h_{oe} - h_{re}\,h_{fe}} = \frac{600 + 2000}{(600 + 2000) \times 20 \times 10^{-6} - 10^{-4} \times 120}$$

$$Z_{out} = \frac{2600}{40}\ k\Omega = \mathbf{65\ k\Omega}$$

(c)
$$A_v = \frac{\mathbf{V}_c}{\mathbf{V}_b} = \frac{\mathbf{V}_c}{\mathbf{V}_s} \longrightarrow \mathbf{V}_c = A_v \mathbf{V}_s = -3333 \times 4 \times 10^{-3} = \mathbf{-\ 13.33\ V}$$

Chapter 19, Solution 91

$$R_s = 1.2\ \text{k}\Omega, \qquad R_L = 4\ \text{k}\Omega$$

(a) $$\mathbf{A_t = \frac{-h_{fe}\, R_L}{h_{ie} + (h_{ie}\, h_{oe} - h_{re}\, h_{fe})R_L}}$$

$$\mathbf{A_t = \frac{-80 \times 4 \times 10^3}{1200 + (1200 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80) \times 4 \times 10^3}}$$

$$\mathbf{A_t = \frac{-32000}{1248} = -25.64}$$ This is just the gain for the transistor. If we calculate the gain for the circuit we get $A_t = V_o/V_{be}$ and $V_{be} = V_s[1.2k/(1.2k+2k)] = 0.375$, thus, $\mathbf{V_A}$ = (0.375)(–25.64) = **–9.615.**

(b) $$A_i = \frac{h_{fe}}{1 + h_{oe}\, R_L} = \frac{80}{1 + 20 \times 10^{-6} \times 4 \times 10^3} = \mathbf{74.07}$$

(c) $$Z_{in} = h_{ie} - h_{re}\, A_i$$

$$Z_{in} = 1200 - 1.5 \times 10^{-4} \times 74.074 \cong \mathbf{1.2\ k\Omega}$$

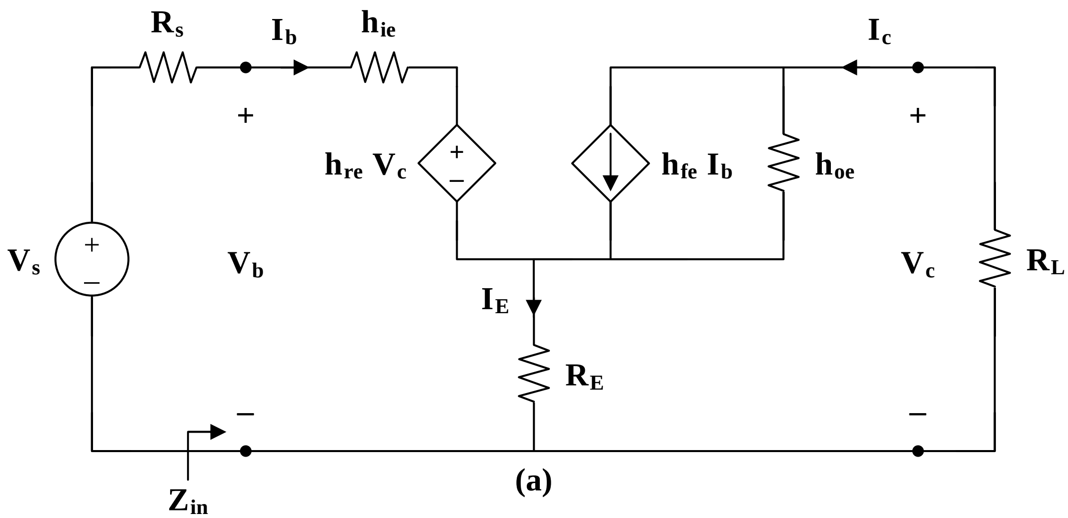
(d) $$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}\, h_{fe}}$$

$$Z_{out} = \frac{1200 + 1200}{2400 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80} = \frac{2400}{0.0468} = \mathbf{51.28\ k\Omega}$$

(a) –25.64 for the transistor and –9.615 for the circuit, (b) 74.07, (c) 1.2 kΩ, (d) 51.28 kΩ

Chapter 19, Solution 92

Due to the resistor $R_E = 240\,\Omega$, we cannot use the formulas in section 18.9.1. We will need to derive our own. Consider the circuit in Fig. (a).



(a)

$$\mathbf{I}_E = \mathbf{I}_b + \mathbf{I}_c \qquad (1)$$

$$\mathbf{V}_b = h_{ie}\,\mathbf{I}_b + h_{re}\,\mathbf{V}_c + (\mathbf{I}_b + \mathbf{I}_c)R_E \qquad (2)$$

$$\mathbf{I}_c = h_{fe}\,\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \qquad (3)$$

But $$\mathbf{V}_c = -\mathbf{I}_c R_L \qquad (4)$$

Substituting (4) into (3),

$$\mathbf{I}_c = h_{fe}\,\mathbf{I}_b - \frac{R_L}{R_E + 1/h_{oe}}\mathbf{I}_c$$

or $$A_i = \frac{\mathbf{I}_c}{\mathbf{I}_b} = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L} \qquad (5)$$

$$A_i = \frac{100(1 + 240\text{x}30\text{x}10^{-6})}{1 + 30\times10^{-6}(4{,}000 + 240)}$$

$$A_i = \underline{\mathbf{79.18}}$$

From (3) and (5),

$$\mathbf{I}_c = \frac{h_{fe}(1+R_E)h_{oe}}{1+h_{oe}(R_L+R_E)}\mathbf{I}_b = h_{fe}\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \qquad (6)$$

Substituting (4) and (6) into (2),

$$\mathbf{V}_b = (h_{ie}+R_E)\mathbf{I}_b + h_{re}\mathbf{V}_c + \mathbf{I}_c R_E$$

$$\mathbf{V}_b = \frac{\mathbf{V}_c(h_{ie}+R_E)}{\left(R_E+\frac{1}{h_{oe}}\right)\left[\frac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)}-h_{fe}\right]} + h_{re}\mathbf{V}_c - \frac{\mathbf{V}_c}{R_L}R_E$$

$$\frac{1}{A_v} = \frac{\mathbf{V}_b}{\mathbf{V}_c} = \frac{(h_{ie}+R_E)}{\left(R_E+\frac{1}{h_{oe}}\right)\left[\frac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)}-h_{fe}\right]} + h_{re} - \frac{R_E}{R_L} \qquad (7)$$

$$\frac{1}{A_v} = \frac{(4000+240)}{\left(240+\frac{1}{30x10^{-6}}\right)\left[\frac{100(1+240x30x10^{-6})}{1+30\times10^{-6}\times4240}-100\right]} + 10^{-4} - \frac{240}{4000}$$

$$\frac{1}{A_v} = -6.06x10^{-3} + 10^{-4} - 0.06 = -0.066$$

$$A_v = \mathbf{-15.15}$$

From (5),

$$\mathbf{I}_c = \frac{h_{fe}}{1+h_{oe}R_L}\mathbf{I}_b$$

We substitute this with (4) into (2) to get

$$\mathbf{V}_b = (h_{ie}+R_E)\mathbf{I}_b + (R_E - h_{re}R_L)\mathbf{I}_c$$

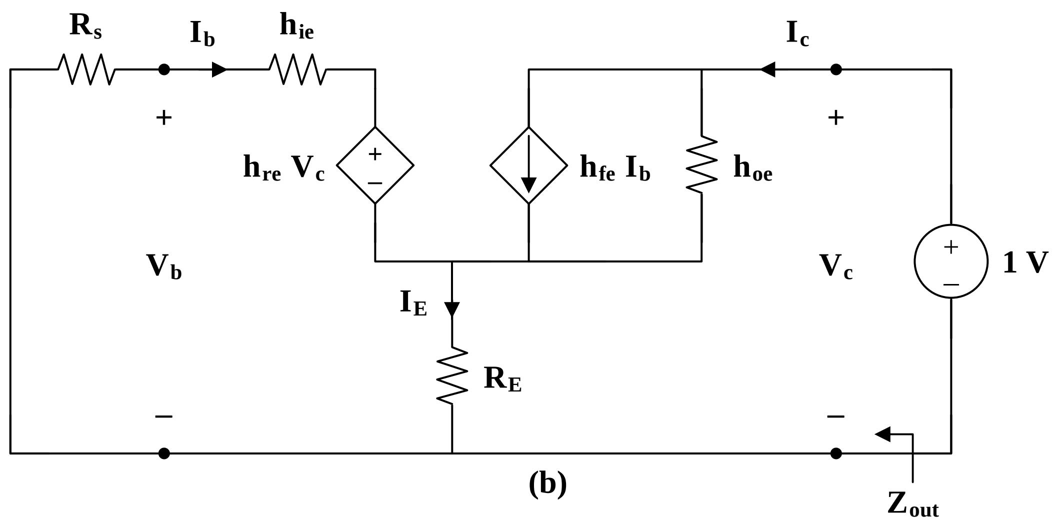
$$\mathbf{V}_b = (h_{ie}+R_E)\mathbf{I}_b + (R_E - h_{re}R_L)\left(\frac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)}\mathbf{I}_b\right)$$

$$Z_{in} = \frac{\mathbf{V}_b}{\mathbf{I}_b} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re}R_L)(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} \qquad (8)$$

$$Z_{in} = 4000 + 240 + \frac{(100)(240\times10^{-4}\times4\times10^{3})(1+240x30x10^{-6})}{1+30\times10^{-6}\times4240}$$

$$Z_{in} = \mathbf{12.818\ k\Omega}$$

To obtain Z_{out}, which is the same as the Thevenin impedance at the output, we introduce a 1-V source as shown in Fig. (b).



(b)

From the input loop,

$$\mathbf{I}_b(R_s + h_{ie}) + h_{re}\mathbf{V}_c + R_E(\mathbf{I}_b + \mathbf{I}_c) = 0$$

But $\qquad \mathbf{V}_c = 1$

So,

$$\mathbf{I}_b(R_s + h_{ie} + R_E) + h_{re} + R_E\mathbf{I}_c = 0 \qquad (9)$$

From the output loop,

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{R_E + \frac{1}{h_{oe}}} + h_{fe}\mathbf{I}_b = \frac{h_{oe}}{R_E h_{oe} + 1} + h_{fe}\mathbf{I}_b$$

or

$$\mathbf{I}_b = \frac{\mathbf{I}_c}{h_{fe}} - \frac{h_{oe}/h_{fe}}{1 + R_E h_{oe}} \qquad (10)$$

Substituting (10) into (9) gives

$$(R_s + R_E + h_{ie})\left(\frac{\mathbf{I}_c}{h_{fe}}\right) + h_{re} + R_E\mathbf{I}_c - \frac{(R_s + R_E + h_{ie})\left(h_{oe}/h_{fe}\right)}{1 + R_E h_{oe}} = 0$$

$$\frac{R_s + R_E + h_{ie}}{h_{fe}}\mathbf{I}_c + R_E\mathbf{I}_c = \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\left(\frac{h_{oe}}{h_{fe}}\right) - h_{re}$$

$$\mathbf{I}_c = \frac{(h_{oe}/h_{fe})\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] - h_{re}}{R_E + (R_s + R_E + h_{ie})/h_{fe}}$$

$$Z_{out} = \frac{1}{\mathbf{I}_c} = \frac{R_E\, h_{fe} + R_s + R_E + h_{ie}}{\left[\dfrac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] h_{oe} - h_{re}\, h_{fe}}$$

$$Z_{out} = \frac{240 \times 100 + (1200 + 240 + 4000)}{\left[\dfrac{1200 + 240 + 4000}{1 + 240 x 30 x 10^{-6}}\right] \times 30 \times 10^{-6} - 10^{-4} \times 100}$$

$$Z_{out} = \frac{24000 + 5440}{0.152} = \mathbf{193.7\ k\Omega}$$

Chapter 19, Solution 93

We apply the same formulas derived in the previous problem.

$$\frac{1}{A_v} = \frac{(h_{ie} + R_E)}{\left(R_E + \frac{1}{h_{oe}}\right)\left[\frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe}\right]} + h_{re} - \frac{R_E}{R_L}$$

$$\frac{1}{A_v} = \frac{(2000 + 200)}{(200 + 10^5)\left[\frac{150(1 + 0.002)}{1 + 0.04} - 150\right]} + 2.5 \times 10^{-4} - \frac{200}{3800}$$

$$\frac{1}{A_v} = -0.004 + 2.5 \times 10^{-4} - 0.05263 = -0.05638$$

$$A_v = \mathbf{-17.74}$$

$$A_i = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} = \frac{150(1 + 200\text{x}10^{-5})}{1 + 10^{-5} \times (200 + 3800)} = \mathbf{144.5}$$

$$Z_{in} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re} R_L)(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}$$

$$Z_{in} = 2000 + 200 + \frac{(150)(200 - 2.5 \times 10^{-4} \times 3.8 \times 10^3)(1.002)}{1.04}$$

$$Z_{in} = 2200 + 28966$$

$$Z_{in} = \mathbf{31.17\ k\Omega}$$

$$Z_{out} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[\frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{200 \times 150 + 1000 + 200 + 2000}{\left[\frac{3200 \times 10^{-5}}{1.002}\right] - 2.5 \times 10^{-4} \times 150} = \frac{33200}{-0.0055}$$

$$Z_{out} = \mathbf{-6.148\ M\Omega}$$

Chapter 19, Solution 94

We first obtain the **ABCD** parameters.

Given $[\mathbf{h}] = \begin{bmatrix} 200 & 0 \\ 100 & 10^{-6} \end{bmatrix}$, $\Delta_h = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21} = 2\times10^{-4}$

$$[\mathbf{T}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{21}} & \dfrac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \dfrac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \dfrac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}$$

The overall **ABCD** parameters for the amplifier are

$$[\mathbf{T}] = \begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}\begin{bmatrix} -2\times10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix} \cong \begin{bmatrix} 2\times10^{-8} & 2\times10^{-2} \\ 10^{-10} & 10^{-4} \end{bmatrix}$$

$$\Delta_T = 2\times10^{-12} - 2\times10^{-12} = 0$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{B}}{\mathbf{D}} & \dfrac{\Delta_T}{\mathbf{D}} \\ \dfrac{-1}{\mathbf{D}} & \dfrac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} 200 & 0 \\ -10^4 & 10^{-6} \end{bmatrix}$$

Thus, $h_{ie} = 200$, $h_{re} = 0$, $h_{fe} = -10^4$, $h_{oe} = 10^{-6}$

$$A_v = \frac{(10^4)(4\times10^3)}{200 + (2\times10^{-4} - 0)\times 4\times10^3} = \mathbf{2\times10^5}$$

$$Z_{in} = h_{ie} - \frac{h_{re}\,h_{fe}\,R_L}{1 + h_{oe}\,R_L} = 200 - 0 = \mathbf{200\,\Omega}$$

Chapter 19, Solution 95

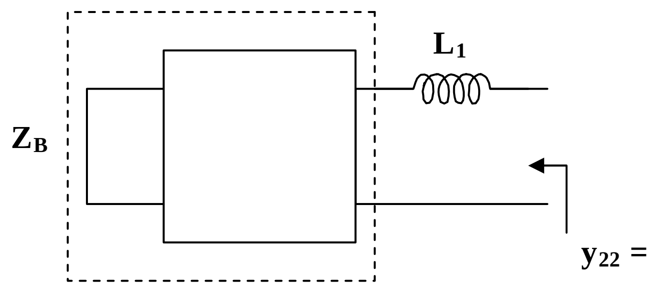
Let $\mathbf{Z}_A = \frac{1}{\mathbf{y}_{22}} = \frac{s^4 + 10s^2 + 8}{s^3 + 5s}$

Using long division,

$$\mathbf{Z}_A = s + \frac{5s^2 + 8}{s^3 + 5s} = sL_1 + \mathbf{Z}_B$$

i.e. $L_1 = 1\,H$ and $\mathbf{Z}_B = \frac{5s^2 + 8}{s^3 + 5s}$

as shown in Fig (a).



(a)

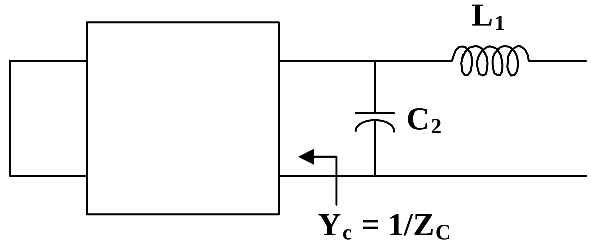
$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{s^3 + 5s}{5s^2 + 8}$$

Using long division,

$$\mathbf{Y}_B = 0.2s + \frac{3.4s}{5s^2 + 8} = sC_2 + \mathbf{Y}_C$$

where $C_2 = 0.2\,F$ and $\mathbf{Y}_C = \frac{3.4s}{5s^2 + 8}$

as shown in Fig. (b).



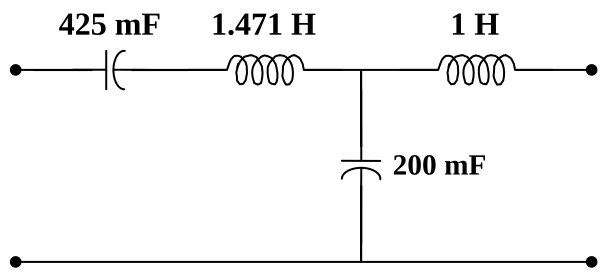
(b)

$$\mathbf{Z}_C = \frac{1}{\mathbf{Y}_C} = \frac{5s^2 + 8}{3.4s} = \frac{5s}{3.4} + \frac{8}{3.4s} = sL_3 + \frac{1}{sC_4}$$

i.e. an inductor in series with a capacitor

$$L_3 = \frac{5}{3.4} = 1.471\text{ H} \quad \text{and} \quad C_4 = \frac{3.4}{8} = 0.425\text{ F}$$

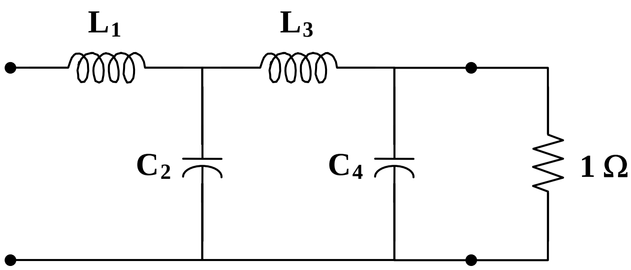
Thus, **the LC network is shown in Fig. (c)**.



(c)

Chapter 19, Solution 96

This is a fourth order network which can be realized with the network shown in Fig. (a).



(a)

$$\Delta(s) = (s^4 + 3.414s^2 + 1) + (2.613s^3 + 2.613s)$$

$$H(s) = \frac{\dfrac{1}{2.613s^3 + 2.613s}}{1 + \dfrac{s^4 + 3.414s^2 + 1}{2.613s^3 + 2.613s}}$$

which indicates that

$$\mathbf{y}_{21} = \frac{-1}{2.613s^3 + 2.613s}$$

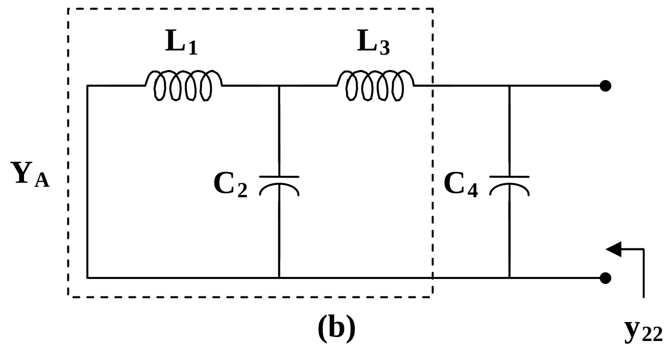
$$\mathbf{y}_{22} = \frac{s^4 + 3.414s + 1}{2.613s^3 + 2.613s}$$

We seek to realize $\mathbf{y}_{22}$. By long division,

$$\mathbf{y}_{22} = 0.383s + \frac{2.414s^2 + 1}{2.613s^3 + 2.613s} = s\,C_4 + \mathbf{Y}_A$$

i.e. $C_4 = 0.383\ \text{F}$ and $\mathbf{Y}_A = \dfrac{2.414s^2 + 1}{2.613s^3 + 2.613s}$

as shown in Fig. (b).



(b)

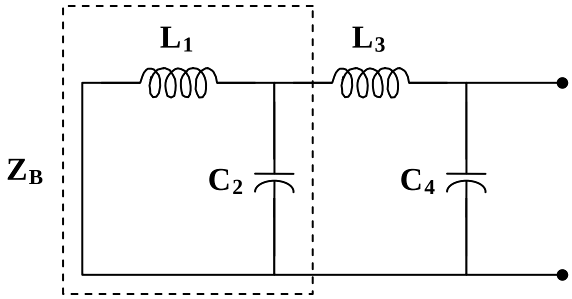
$$\mathbf{Z}_A = \frac{1}{\mathbf{Y}_A} = \frac{2.613s^3 + 2.613s}{2.414s^2 + 1}$$

By long division,

$$\mathbf{Z}_A = 1.082s + \frac{1.531s}{2.414s^2 + 1} = sL_3 + \mathbf{Z}_B$$

i.e. $L_3 = 1.082\ \text{H}$ and $\mathbf{Z}_B = \dfrac{1.531s}{2.414s^2 + 1}$

as shown in Fig.(c).

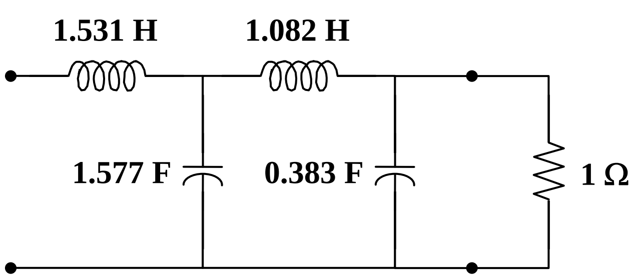


(c)

$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = 1.577s + \frac{1}{1.531s} = sC_2 + \frac{1}{sL_1}$$

i.e. $C_2 = 1.577\ \text{F}$ and $L_1 = 1.531\ \text{H}$

Thus, **the network is shown in Fig. (d)**.



(d)

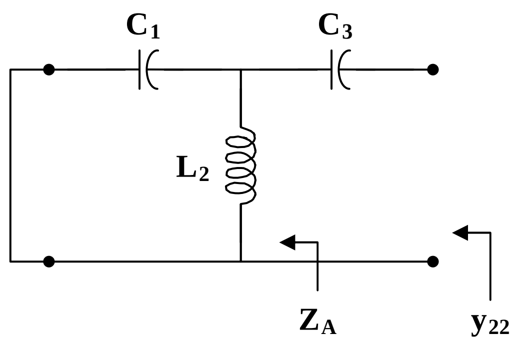
Chapter 19, Solution 97

$$H(s) = \frac{s^3}{(s^3+12s)+(6s^2+24)} = \frac{\dfrac{s^3}{s^3+12s}}{1+\dfrac{6s^2+24}{s^3+12s}}$$

Hence,

$$\mathbf{y}_{22} = \frac{6s^2+24}{s^3+12s} = \frac{1}{sC_3} + \mathbf{Z}_A \qquad (1)$$

where $\mathbf{Z}_A$ is shown in the figure below.



We now obtain C_3 and $\mathbf{Z}_A$ using partial fraction expansion.

Let $$\frac{6s^2+24}{s(s^2+12)} = \frac{A}{s} + \frac{Bs+C}{s^2+12}$$

$$6s^2+24 = A(s^2+12) + Bs^2 + Cs$$

Equating coefficients :

s^0 : $\quad 24 = 12A \longrightarrow A = 2$

s^1: $\quad 0 = C$

s^2 : $\quad 6 = A + B \longrightarrow B = 4$

Thus,

$$\frac{6s^2+24}{s(s^2+12)} = \frac{2}{s} + \frac{4s}{s^2+12} \qquad (2)$$

Comparing (1) and (2),

$$C_3 = \frac{1}{A} = \frac{1}{2}\text{ F}$$

$$\frac{1}{\mathbf{Z}_A} = \frac{s^2 + 12}{4s} = \frac{1}{4}s + \frac{3}{s} \tag{3}$$

But
$$\frac{1}{\mathbf{Z}_A} = sC_1 + \frac{1}{sL_2} \tag{4}$$

Comparing (3) and (4),

$$C_1 = \frac{1}{4}\text{ F} \qquad \text{and} \qquad L_2 = \frac{1}{3}\text{ H}$$

Therefore,

$$C_1 = \mathbf{250\ mF}, \qquad L_2 = \mathbf{333.3\ mH}, \qquad C_3 = \mathbf{500\ mF}$$

Chapter 19, Solution 98

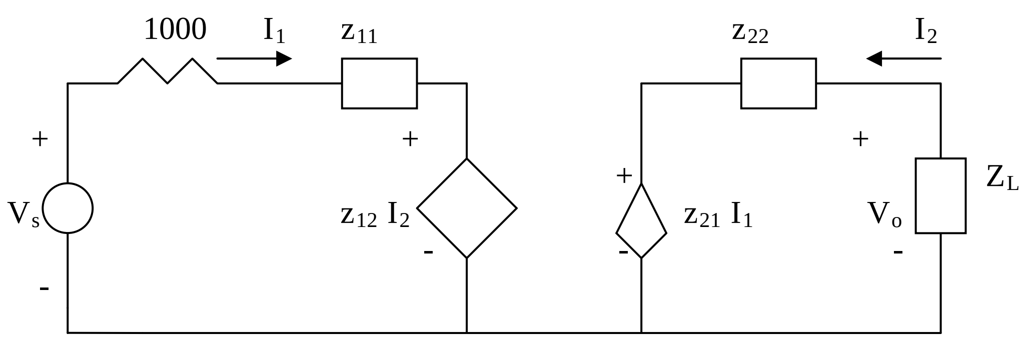
$$\Delta_h = 1 - 0.8 = 0.2$$

$$[T_a] = [T_b] = \begin{bmatrix} -\Delta_h / h_{21} & -h_{11}/h_{21} \\ -h_{22}/h_{21} & -1/h_{21} \end{bmatrix} = \begin{bmatrix} -0.001 & -10 \\ -2.5x10^{-6} & -0.005 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} 2.6x10^{-5} & 0.06 \\ 1.5x10^{-8} & 5x10^{-5} \end{bmatrix}$$

We now convert this to z-parameters

$$[z] = \begin{bmatrix} A/C & \Delta_T / C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 1.733x10^3 & 0.0267 \\ 6.667x10^7 & 3.33x10^3 \end{bmatrix}$$



$$V_s = (1000 + z_{11})I_1 + z_{12}I_2 \qquad (1)$$

$$V_o = z_{22}I_2 + z_{21}I_1 \qquad (2)$$

But $V_o = -I_2Z_L \longrightarrow I_2 = -V_o / Z_L$ (3)

Substituting (3) into (2) gives

$$I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{z_{21}Z_L}\right) \qquad (4)$$

We substitute (3) and (4) into (1)

$$V_s = (1000 + z_{11})\left(\frac{1}{z_{11}} + \frac{z_{22}}{z_{21}Z_L}\right)V_o - \frac{z_{12}}{Z_L}V_o$$

$$= 7.653\text{x}10^{-4} - 2.136\text{x}10^{-5} = \underline{744\mu V}$$

Chapter 19, Solution 99

$$\mathbf{Z}_{ab} = \mathbf{Z}_1 + \mathbf{Z}_3 = \mathbf{Z}_c \parallel (\mathbf{Z}_b + \mathbf{Z}_a)$$

$$\mathbf{Z}_1 + \mathbf{Z}_3 = \frac{\mathbf{Z}_c(\mathbf{Z}_a + \mathbf{Z}_b)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{1}$$

$$\mathbf{Z}_{cd} = \mathbf{Z}_2 + \mathbf{Z}_3 = \mathbf{Z}_a \parallel (\mathbf{Z}_b + \mathbf{Z}_c)$$

$$\mathbf{Z}_2 + \mathbf{Z}_3 = \frac{\mathbf{Z}_a(\mathbf{Z}_b + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{2}$$

$$\mathbf{Z}_{ac} = \mathbf{Z}_1 + \mathbf{Z}_2 = \mathbf{Z}_b \parallel (\mathbf{Z}_a + \mathbf{Z}_c)$$

$$\mathbf{Z}_1 + \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_a + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{3}$$

Subtracting (2) from (1),

$$\mathbf{Z}_1 - \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_c - \mathbf{Z}_a)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{4}$$

Adding (3) and (4),

$$\mathbf{Z}_1 = \underline{\frac{\mathbf{Z_b Z_c}}{\mathbf{Z_a + Z_b + Z_c}}} \tag{5}$$

Subtracting (5) from (3),

$$\mathbf{Z}_2 = \frac{\mathbf{Z_a Z_b}}{\mathbf{Z_a + Z_b + Z_c}} \tag{6}$$

Subtracting (5) from (1),

$$\mathbf{Z}_3 = \frac{\mathbf{Z_c Z_a}}{\mathbf{Z_a + Z_b + Z_c}} \tag{7}$$

Using (5) to (7)

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)}{(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)^2}$$

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \tag{8}$$

Dividing (8) by each of (5), (6), and (7),

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

as required. Note that the formulas above are not exactly the same as those in Chapter 9 because the locations of $\mathbf{Z}_b$ and $\mathbf{Z}_c$ are interchanged in Fig. 18.122.