Chapter 18, Solution 1.

$$f'(t) = \delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2)$$

$$j\omega F(\omega) = e^{j2\omega} - e^{j\omega} - e^{-j\omega} + e^{-j\omega 2}$$

$$= 2\cos 2\omega - 2\cos\omega$$

$$F(\omega) = \mathbf{\frac{2[\cos 2\omega - \cos\omega]}{j\omega}}$$

Chapter 18, Solution 2.

Using Fig. 18.27, design a problem to help other students to better understand the Fouier transform given a wave shape.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

What is the Fourier transform of the triangular pulse in Fig. 18.27?

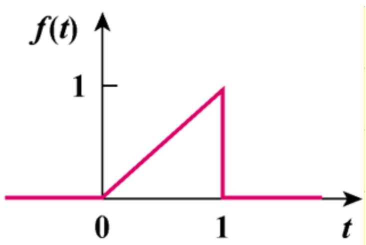
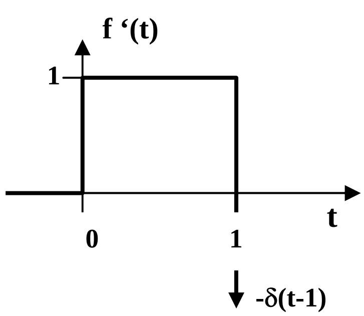
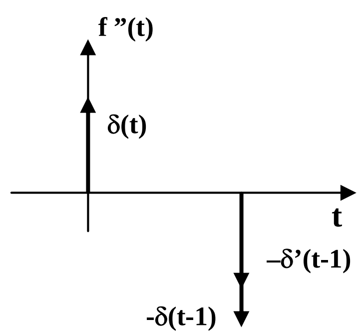


Figure 18.27

Solution

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$





$f''(t) = \delta(t) - \delta(t - 1) - \delta'(t - 1)$

Taking the Fourier transform gives

$$-\omega^2 F(\omega) = 1 - e^{-j\omega} - j\omega e^{-j\omega}$$

$$F(\omega) = \frac{(1 + j\omega)e^{-j\omega} - 1}{\omega^2}$$

or $F(\omega)=\int_0^1 t\,e^{-j\omega t}\,dt$

But $\int x\,e^{ax}dx=\frac{e^{ax}}{a^2}(ax-1)+c$

$$F(\omega)=\frac{e^{-j\omega}}{(-j\omega)^2}(-j\omega t-1)\Big|_0^1=\boldsymbol{\frac{1}{\omega^2}\left[(1+j\omega)e^{-j\omega}-1\right]}$$

Chapter 18, Solution 3.

$$f(t) = \frac{1}{2}t, \; -2 < t < 2, \qquad f'(t) = \frac{1}{2}, \; -2 < t < 2$$

$$F(\omega) = \int_{-2}^{2} \frac{1}{2} t\, e^{j\omega t} dt = \frac{e^{-j\omega t}}{2(-j\omega)^2}(-j\omega t - 1)\Big|_{-2}^{2}$$

$$= -\frac{1}{2\omega^2}\left[e^{-j\omega 2}(-j\omega 2 - 1) - e^{j\omega 2}(j\omega 2 - 1)\right]$$

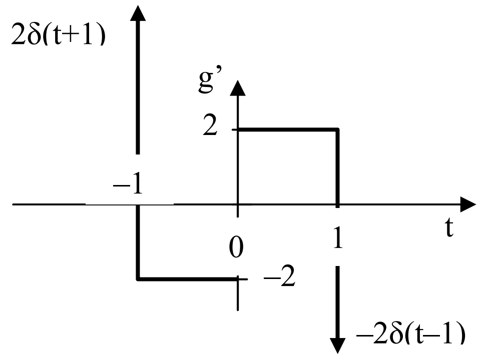
$$= -\frac{1}{2\omega^2}\left[-j\omega 2\left(e^{-j\omega 2} + e^{j\omega 2}\right) + e^{j\omega 2} - e^{-j\omega 2}\right]$$

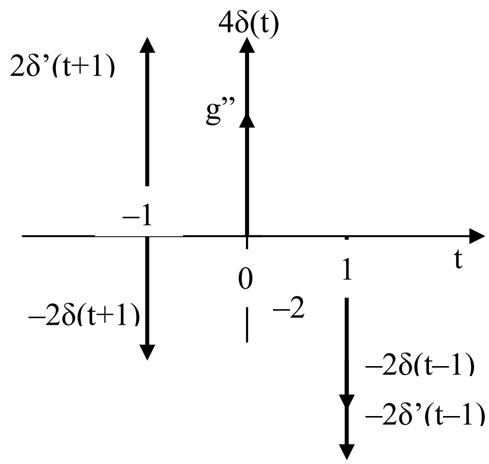
$$= -\frac{1}{2\omega^2}\left(-j\omega 4\cos 2\omega + j2\sin 2\omega\right)$$

$$F(\omega) = \boldsymbol{\frac{j}{\omega^2}(2\omega\cos 2\omega - \sin 2\omega)}$$

Chapter 18, Solution 4.

We can solve the problem by following the approach demonstrated in Example 18.5.



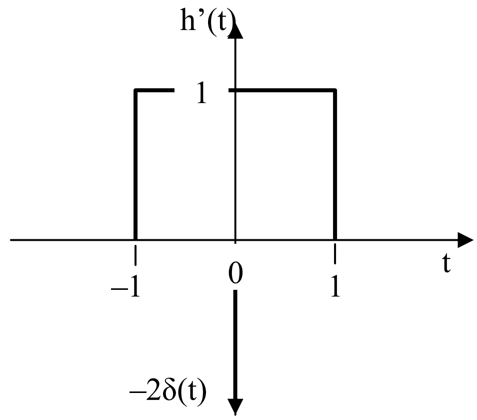


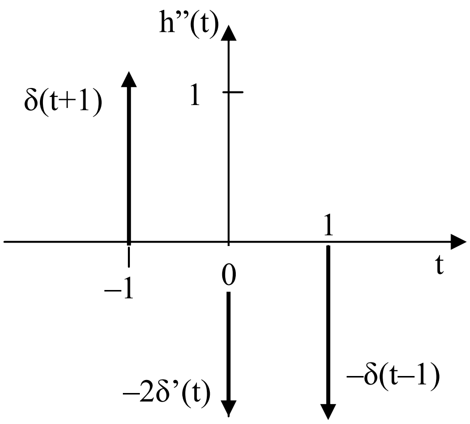
$$g'' = -2\delta(t+1) + 2\delta'(t+1) + 4\delta(t) - 2\delta(t-1) - 2\delta'(t-1)$$

$$\begin{aligned}(j\omega)^2 G(\omega) &= -2e^{j\omega} + 2j\omega e^{j\omega} + 4 - 2e^{-j\omega} - 2j\omega e^{-j\omega} \\ &= -4\cos\omega - 4\omega\sin\omega + 4\end{aligned}$$

$$\underline{\mathbf{G(\omega) = \frac{4}{\omega^2}(\cos\omega + \omega\sin\omega - 1)}}$$

Chapter 18, Solution 5.





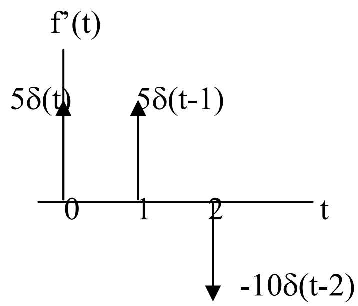
$$h''(t) = \delta(t+1) - \delta(t-1) - 2\delta'(t)$$

$$(j\omega)^2 H(\omega) = e^{j\omega} - e^{-j\omega} - 2j\omega = 2j\sin\omega - 2j\omega$$

$$H(\omega) = \mathbf{\frac{2j}{\omega} - \frac{2j}{\omega^2}\sin\omega}$$

Chapter 18, Solution 6.

(a) The derivative of f(t) is shown below.



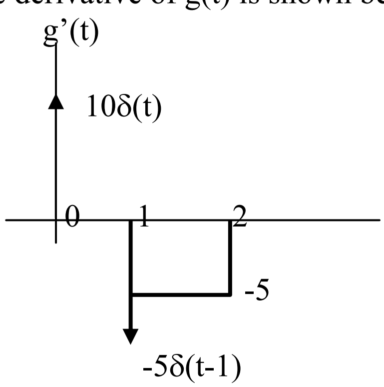
$$f'(t) = 5\delta(t) + 5\delta(t-1) - 10\delta(t-2)$$

Taking the Fourier transform of each term,

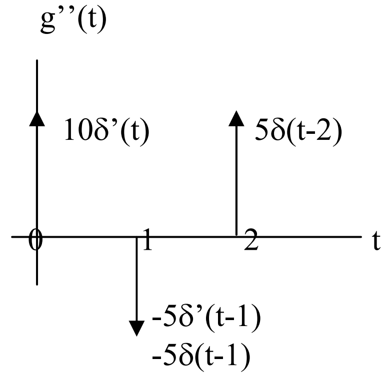
$$j\omega F(\omega) = 5 + 5e^{-j\omega} - 10e^{-j2\omega}$$

$$\underline{F(\omega) = \frac{5 + 5e^{-j\omega} - 10e^{-j2\omega}}{j\omega}}$$

(b) The derivative of g(t) is shown below.



The second derivative of g(t) is shown below.



$$g''(t) = 10\delta'(t) - 5\delta'(t-1) - 5\delta(t-1) + 5\delta(t-2)$$

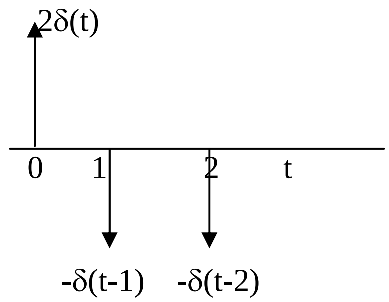
Take the Fourier transform of each term.

$$(j\omega)^2 G(j\omega) = 10j\omega - 5j\omega e^{-j\omega} - 5e^{-j\omega} + 5e^{-j2\omega}$$ which leads to

$$G(j\omega) = \mathbf{(-10j\omega + 5j\omega e^{-j\omega} + 5e^{-j\omega} - 5e^{-j2\omega})/\omega^2}$$

Chapter 18, Solution 7.

(a) Take the derivative of $f_1(t)$ and obtain $f_1'(t)$ as shown below.



$$f_1'(t) = 2\delta(t) - \delta'(t-1) - \delta(t-2)$$

Take the Fourier transform of each term,

$$j\omega F_1(\omega) = 2 - e^{-j\omega} - e^{-j2\omega}$$

$$F_1(\omega) = \frac{2 - e^{-j\omega} - e^{-j2\omega}}{j\omega}$$

(b) $f_2(t) = 5t$

$$F_2(\omega) = \int_{-\infty}^{\infty} f_2(t) e^{-j\omega} dt = \int_0^2 5te^{-j\omega} dt = \frac{5}{(-j\omega)^2} e^{-j\omega t}(-j\omega - 1)\Big|_0^2$$

$$F_2(\omega) = \frac{5e^{-j2\omega}}{\omega^2}(1 + j\omega 2) - \frac{5}{\omega^2}$$

Chapter 18, Solution 8.

(a)

$$F(\omega) = \int_0^1 2e^{-j\omega t}dt + \int_1^2 (4-2t)e^{-j\omega t}dt$$

$$= \frac{2}{-j\omega}e^{-j\omega t}\Big|_0^1 + \frac{4}{-j\omega}e^{-j\omega t}\Big|_1^2 - \frac{2}{-\omega^2}e^{-j\omega t}(-j\omega t-1)\Big|_1^2$$

$$\mathbf{F(\omega) = \underline{\frac{2}{\omega^2} + \frac{2}{j\omega}e^{-j\omega} + \frac{2}{j\omega} - \frac{4}{j\omega}e^{-j2\omega} - \frac{2}{\omega^2}(1+j2\omega)e^{-j2\omega}}}$$

(b) g(t) = 2[u(t+2) – u(t-2)] - [u(t+1) – u(t-1)]

$$\mathbf{G(\omega) = \underline{\frac{4\sin 2\omega}{\omega} - \frac{2\sin\omega}{\omega}}}$$

Chapter 18, Solution 9.

(a) $y(t) = u(t+2) - u(t-2) + 2[\, u(t+1) - u(t-1)\,]$

$$\underline{\mathbf{Y}(\omega) = \frac{2}{\omega}\sin 2\omega + \frac{4}{\omega}\sin\omega}$$

(b) $$\mathbf{Z}(\omega) = \int_0^1 (-2t)e^{-j\omega t}dt = \frac{-2e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_0^1 = \underline{\frac{2}{\omega^2} - \frac{2e^{-j\omega}}{\omega^2}(1 + j\omega)}$$

Chapter 18, Solution 10.

(a) $x(t) = e^{2t}u(t)$

$X(\omega) =$ **$1/(-2 + j\omega)$**

(b) $e^{-(t)} = \begin{bmatrix} e^{-t}, & t > 0 \\ e^{t}, & t < 0 \end{bmatrix}$

$$Y(\omega) = \int_{-1}^{1} y(t)e^{j\omega t}\,dt = \int_{-1}^{0} e^{t}\,e^{j\omega t}\,dt + \int_{0}^{1} e^{-t}\,e^{-j\omega t}\,dt$$

$$= \frac{e^{(1-j\omega)t}}{1-j\omega}\bigg|_{-1}^{0} + \frac{e^{-(1+j\omega)t}}{-(1+j\omega)}\bigg|_{0}^{1}$$

$$= \frac{2}{1+\omega^2} - e^{-1}\left[\frac{\cos\omega + j\sin\omega}{1-j\omega} + \frac{\cos\omega - j\sin\omega}{1+j\omega}\right]$$

$$Y(\omega) = \boldsymbol{\frac{2}{1+\omega^2}\left[1 - e^{-1}(\cos\omega - \omega\sin\omega)\right]}$$

Chapter 18, Solution 11.

$$f(t) = \sin \pi t\,[u(t) - u(t-2)]$$

$$F(\omega) = \int_0^2 \sin \pi t\, e^{-j\omega t}\, dt = \frac{1}{2j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right) e^{-j\omega t}\, dt$$

$$= \frac{1}{2j}\left[\int_0^2 (e^{+j(-\omega+\pi)t} + e^{-j(\omega+\pi)t})dt\right]$$

$$= \frac{1}{2j}\left[\frac{1}{-j(\omega-\pi)} e^{-j(\omega-\pi)t}\Big|_0^2 + \frac{e^{-j(\omega+\pi)t}}{-j(\omega+\pi)}\Big|_0^2\right]$$

$$= \frac{1}{2}\left(\frac{1-e^{-j2\omega}}{\pi-\omega} + \frac{1-e^{-j2\omega}}{\pi+\omega}\right)$$

$$= \frac{1}{2(\pi^2-\omega^2)}\left(2\pi + 2\pi e^{-j2\omega}\right)$$

$$F(\omega) = \boldsymbol{\frac{\pi}{\omega^2-\pi^2}\left(e^{-j\omega 2} - 1\right)}$$

Chapter 18, Solution 12.

(a) $F_1(\omega) = \dfrac{10}{(3 + j\omega)^2 + 100}$

(b) $F_2(\omega) = \dfrac{4 + j\omega}{(4 + j\omega)^2 + 100}$

Chapter 18, Solution 13.

(a) We know that $\mathcal{F}[\cos at] = \pi[\delta(\omega - a) + \delta(\omega + a)]$.

Using the time shifting property,

$$\mathbf{\mathcal{F}[\cos a(t - \pi/3a)] = \pi e^{-j\omega\pi/3a}[\delta(\omega - a) + \delta(\omega + a)] = \underline{\pi e^{-j\pi/3}\delta(\omega - a) + \pi e^{j\pi/3}\delta(\omega + a)}}$$

(b) $\sin\pi(t+1) = \sin\pi t\cos\pi + \cos\pi t\sin\pi = -\sin\pi t$

g(t) = -u(t+1) sin (t+1)

Let x(t) = u(t)sin t, then $X(\omega) = \dfrac{1}{(j\omega)^2 + 1} = \dfrac{1}{1 - \omega^2}$

Using the time shifting property,

$$\mathbf{G(\omega) = -\frac{1}{1 - \omega^2}e^{j\omega} = \underline{\frac{e^{j\omega}}{\omega^2 - 1}}}$$

(c) Let y(t) = 1 + Asin at, then $Y(\omega) = 2\pi\delta(\omega) + j\pi A[\delta(\omega + a) - \delta(\omega - a)]$

h(t) = y(t) cos bt

Using the modulation property,

$$H(\omega) = \frac{1}{2}[Y(\omega + b) + Y(\omega - b)]$$

$$\mathbf{H(\omega) = \underline{\pi[\delta(\omega + b) + \delta(\omega - b)] + \frac{j\pi A}{2}[\delta(\omega + a + b) - \delta(\omega - a + b) + \delta(\omega + a - b) - \delta(\omega - a - b)]}}$$

(d) $$\mathbf{I(\omega) = \int_0^4 (1 - t)e^{-j\omega t}dt = \frac{e^{-j\omega t}}{-j\omega} - \frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_0^4 = \underline{\frac{1}{\omega^2} - \frac{e^{-j4\omega}}{j\omega} - \frac{e^{-j4\omega}}{\omega^2}(j4\omega + 1)}}$$

Chapter 18, Solution 14.

Design a problem to help other students to better understand finding the Fourier transform of a variety of time varying functions (do at least three).

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Fourier transforms of these functions:

(a) $f(t) = e^{-t} \cos(3t + \pi)\, u(t)$
(b) $g(t) = \sin \pi t\, [\, u(t + 1) - u(t-1)]$
(c) $h(t) = e^{-2t} \cos \pi t\, u(t-1)$
(d) $p(t) = e^{-2t} \sin 4t\, u(-t)$
(e) $q(t) = 4 \operatorname{sgn}(t - 2) + 3\delta(t) - 2\, u(t - 2)$

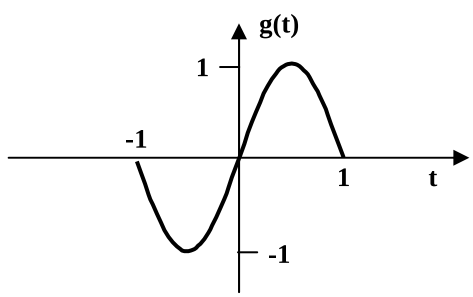
Solution

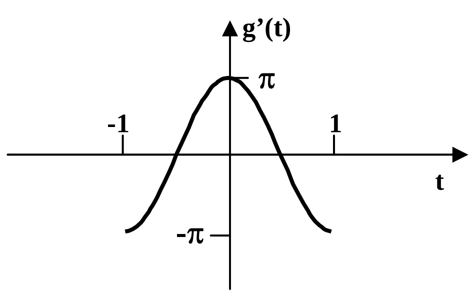
(a) $\cos(3t + \pi) = \cos 3t \cos \pi - \sin 3t \sin \pi = \cos 3t(-1) - \sin 3t(0) = -\cos(3t)$

$$f(t) = -e^{-t} \cos 3t\, u(t)$$

$$F(\omega) = \frac{-(1 + j\omega)}{(1 + j\omega)^2 + 9}$$

(b)





$$g'(t) = \pi \cos \pi t[u(t-1) - u(t-1)]$$
$$g''(t) = -\pi^2 g(t) - \pi\delta(t+1) + \pi\delta(t-1)$$
$$-\omega^2 G(\omega) = -\pi^2 G(\omega) - \pi e^{j\omega} + \pi e^{-j\omega}$$
$$(\pi^2 - \omega^2)G(\omega) = -\pi(e^{j\omega} - e^{-j\omega}) = -2j\pi \sin\omega$$
$$G(\omega) = \mathbf{\frac{2j\pi\sin\omega}{\omega^2 - \pi^2}}$$

Alternatively, we compare this with Prob. 17.7

f(t) = g(t - 1)
F(ω) = G(ω)e$^{-j\omega}$

$$G(\omega) = F(\omega)e^{j\omega} = \frac{\pi}{\omega^2 - \pi^2}\left(e^{-j\omega} - e^{j\omega}\right)$$
$$= \frac{-j2\pi\sin\omega}{\omega^2 - \pi^2}$$
$$G(\omega) = \mathbf{\frac{2j\pi\sin\omega}{\pi^2 - \omega^2}}$$

(c) $\cos\pi(t-1) = \cos\pi t\cos\pi + \sin\pi t\sin\pi = \cos\pi t(-1) + \sin\pi t(0) = -\cos\pi t$

Let $x(t) = e^{-2(t-1)}\cos\pi(t-1)u(t-1) = -e^2 h(t)$

and $y(t) = e^{-2t}\cos(\pi t)u(t)$
$$Y(\omega) = \frac{2 + j\omega}{(2 + j\omega)^2 + \pi^2}$$
$$y(t) = x(t-1)$$
$$Y(\omega) = X(\omega)e^{-j\omega}$$
$$X(\omega) = \frac{(2 + j\omega)e^{j\omega}}{(2 + j\omega)^2 + \pi^2}$$
$$X(\omega) = -e^2 H(\omega)$$
$$H(\omega) = -e^{-2}X(\omega)$$
$$= \mathbf{\frac{-(2 + j\omega)e^{j\omega - 2}}{(2 + j\omega)^2 + \pi^2}}$$

(d) Let $x(t) = e^{-2t}\sin(-4t)u(-t) = y(-t)$
$$p(t) = -x(t)$$
where $y(t) = e^{2t}\sin 4t\, u(t)$
$$Y(\omega) = \frac{2 + j\omega}{(2 + j\omega)^2 + 4^2}$$
$$X(\omega) = Y(-\omega) = \frac{2 - j\omega}{(2 - j\omega)^2 + 16}$$

$$p(\omega) = -X(\omega) = \frac{\mathbf{j\omega - 2}}{\mathbf{(j\omega - 2)^2 + 16}}$$

(e) $$Q(\omega) = \frac{8}{j\omega} e^{-j\omega 2} + 3 - 2\left(\pi\delta(\omega) + \frac{1}{j\omega}\right) e^{-j\omega 2}$$

$$Q(\omega) = \frac{\mathbf{6}}{\mathbf{j\omega}} \mathbf{e^{j\omega 2} + 3 - 2\pi\delta(\omega) e^{-j\omega 2}}$$

Chapter 18, Solution 15.

(a) $F(\omega) = e^{j3\omega} - e^{-j\omega 3} = \mathbf{2j\sin 3\omega}$

(b) Let $g(t) = 2\delta(t-1)$, $G(\omega) = 2e^{-j\omega}$

$$F(\omega) = \mathcal{F}\left(\int_{-\infty}^{t} g(t)\,dt\right)$$

$$= \frac{G(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

$$= \frac{2e^{-j\omega}}{j\omega} + 2\pi\delta(-1)\delta(\omega)$$

$$= \boldsymbol{\frac{2e^{-j\omega}}{j\omega}}$$

(c) $\mathcal{F}\left[\delta(2t)\right] = \frac{1}{2}\cdot 1$

$$F(\omega) = \frac{1}{3}\cdot 1 - \frac{1}{2}j\omega = \boldsymbol{\frac{1}{3} - \frac{j\omega}{2}}$$

Chapter 18, Solution 16.

(a) Using duality properly

$$|t| \rightarrow \frac{-2}{\omega^2}$$

$$\frac{-2}{t^2} \rightarrow 2\pi|\omega|$$

or $$\frac{4}{t^2} \rightarrow -4\pi|\omega|$$

$$F(\omega) = \mathcal{F}\left(\frac{4}{t^2}\right) = \mathbf{-4\pi|\omega|}$$

(b) $$e^{-|a|t} \longrightarrow \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+t^2} \longrightarrow 2\pi e^{-a|\omega|}$$

$$\frac{8}{a^2+t^2} \longrightarrow 4\pi e^{-2|\omega|}$$

$$G(\omega) = \mathcal{F}\left(\frac{8}{4+t^2}\right) = \mathbf{4\pi e^{-2|\omega|}}$$

Chapter 18, Solution 17.

(a) Since $H(\omega) = \mathcal{F}\left(\cos\omega_0 t\ f(t)\right) = \frac{1}{2}\left[F(\omega+\omega_0) + F(\omega-\omega_0)\right]$

where $F(\omega) = \mathcal{F}\left[u(t)\right] = \pi\delta(\omega) + \frac{1}{j\omega}$, $\omega_0 = 2$

$$H(\omega) = \frac{1}{2}\left[\pi\delta(\omega+2) + \frac{1}{j(\omega+2)} + \pi\delta(\omega-2) + \frac{1}{j(\omega-2)}\right]$$

$$= \frac{\pi}{2}\left[\delta(\omega+2) + \delta(\omega-2)\right] - \frac{j}{2}\left[\frac{\omega+2+\omega-2}{(\omega+2)(\omega-2)}\right]$$

$$H(\omega) = \boldsymbol{\frac{\pi}{2}\left[\delta(\omega+2) + \delta(\omega-2)\right] - \frac{j\omega}{\omega^2-4}}$$

(b) $G(\omega) = \mathcal{F}\left[\sin\omega_0 t\ f(t)\right] = \frac{j}{2}\left[F(\omega+\omega_0) - F(\omega-\omega_0)\right]$

where $F(\omega) = \mathcal{F}\left[u(t)\right] = \pi\delta(\omega) + \frac{1}{j\omega}$

$$G(\omega) = \frac{j}{2}\left[\pi\delta(\omega+10) + \frac{1}{j(\omega+10)} - \pi\delta(\omega-10) - \frac{1}{j(\omega-10)}\right]$$

$$= \frac{j\pi}{2}\left[\delta(\omega+10) - \delta(\omega-10)\right] + \frac{j}{2}\left[\frac{j}{\omega-10} - \frac{j}{\omega+10}\right]$$

$$= \boldsymbol{\frac{j\pi}{2}\left[\delta(\omega+10) - \delta(\omega-10)\right] - \frac{10}{\omega^2-100}}$$

Chapter 18, Solution 18.

(a) $F[f(t-t_o)] = \int_{-\infty}^{\infty} f(t-t_o)e^{-j\omega t}dt$

Let $t-t_o = \lambda \longrightarrow t = \lambda + t_o, \quad dt = d\lambda$

$$F[f(t-t_o)] = \int_{-\infty}^{\infty} f(\lambda)e^{-j\omega\lambda}e^{-j\omega t_o}d\lambda = e^{-j\omega t_o}F(\omega)$$

(b) Given that $f(t) = F^{-1}[F(\omega)] = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$

$$f'(t) = \frac{j\omega}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}dt = j\omega F^{-1}[F(\omega)]$$

or

$$F[f'(t)] = j\omega F(\omega)$$

(c) This is a special case of the time scaling property when a = –1. Hence,

$$F[f(-t)] = \frac{1}{|-1|}F(-\omega) = F(-\omega)$$

(d) $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$

Differentiating both sides respect to ω and multiplying by t yields

$$j\frac{dF(\omega)}{d\omega} = j\int_{-\infty}^{\infty}(-jt)f(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} tf(t)e^{-j\omega t}dt$$

Hence,

$$j\frac{dF(\omega)}{d\omega} = F[tf(t)]$$

Chapter 18, Solution 19.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{j\omega t}\,dt = \frac{1}{2}\int_0^1 \left(e^{j2\pi t} + e^{-j2\pi t}\right)e^{-j\omega t}\,dt$$

$$F(\omega) = \frac{1}{2}\int_0^1 \left[e^{-j(\omega+2\pi)t} + e^{-j(\omega-2\pi)t}\right]dt$$

$$= \frac{1}{2}\left[\frac{1}{-j(\omega+2\pi)}e^{-j(\omega+2\pi)t} + \frac{1}{-j(\omega-2\pi)}e^{-j(\omega-2\pi)t}\right]_0^1$$

$$= -\frac{1}{2}\left[\frac{e^{-j(\omega+2\pi)}-1}{j(\omega+2\pi)} + \frac{e^{-j(\omega-2\pi)}-1}{j(\omega-2\pi)}\right]$$

But $\quad e^{j2\pi} = \cos 2\pi + j\sin 2\pi = 1 = e^{-j2\pi}$

$$F(\omega) = -\frac{1}{2}\left(\frac{e^{-j\omega}-1}{j}\right)\left(\frac{1}{\omega+2\pi} + \frac{1}{\omega-2\pi}\right)$$

$$= \boldsymbol{\frac{j\omega}{\omega^2 - 4\pi^2}\left(e^{-j\omega} - 1\right)}$$

Chapter 18, Solution 20.

(a) $F(c_n) = c_n\delta(\omega)$

$$F\left(c_n e^{jn\omega_o t}\right) = c_n\delta(\omega - n\omega_o)$$

$$F\left(\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}\right) = \sum_{n=-\infty}^{\infty} \mathbf{c_n\delta(\omega - n\omega_o)}$$

(b) $T = 2\pi \longrightarrow \omega_o = \dfrac{2\pi}{T} = 1$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}\,dt = \frac{1}{2\pi}\left(\int_0^\pi 1\cdot e^{-jnt}\,dt + 0\right)$$

$$= \frac{1}{2\pi}\left(-\frac{1}{jn}e^{jnt}\Big|_0^\pi\right) = \frac{j}{2\pi n}\left(e^{-jn\pi} - 1\right)$$

But $e^{-jn\pi} = \cos n\pi + j\sin n\pi = \cos n\pi = (-1)^n$

$$c_n = \frac{j}{2n\pi}\left[(-1)^n - 1\right] = \begin{cases} 0, & n=\text{even} \\ \frac{-j}{n\pi}, & n=\text{odd}, n\neq 0 \end{cases}$$

for n = 0

$$c_n = \frac{1}{2\pi}\int_0^\pi 1\,dt = \frac{1}{2}$$

Hence

$$f(t) = \frac{1}{2} - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=\text{odd}}}^{\infty} \frac{j}{n\pi}e^{jnt}$$

$$F(\omega) = \mathbf{\frac{1}{2}\delta\omega - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=odd}}^{\infty} \frac{j}{n\pi}\delta(\omega - n)}$$

Chapter 18, Solution 21.

Using Parseval's theorem,

$$\int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega$$

If f(t) = u(t+a) – u(t+a), then

$$\int_{-\infty}^{\infty} f^2(t)dt = \int_{-a}^{a}(1)^2 dt = 2a = \frac{1}{2\pi}\int_{-\infty}^{\infty} 4a^2\left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega$$

or

$$\int_{-\infty}^{\infty}\left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega = \frac{4\pi a}{4a^2} = \frac{\pi}{a} \quad \text{as required.}$$

Chapter 18, Solution 22.

$$\mathcal{F}\left[f(t)\sin\omega_o t\right] = \int_{-\infty}^{\infty} f(t)\frac{\left(e^{j\omega_o t} - e^{-j\omega_o t}\right)}{2j} e^{-j\omega t}\,dt$$

$$= \frac{1}{2j}\left[\int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_o)t}\,dt - \int_{-\infty}^{\infty} e^{-j(\omega+\omega_o)t}\,dt\right]$$

$$= \mathbf{\frac{1}{2j}\left[F(\omega-\omega_o) - F(\omega+\omega_o)\right]}$$

Chapter 18, Solution 23.

(a) f(3t) leads to $\frac{1}{3} \cdot \frac{10}{(2+j\omega/3)(5+j\omega/3)} = \frac{30}{(6+j\omega)(15+j\omega)}$

$F[f(-3t)] = \mathbf{\frac{30}{(6-j\omega)(15-j\omega)}}$

(b) f(2t) $\longrightarrow$ $\frac{1}{2} \cdot \frac{10}{(2+j\omega/2)(15+j\omega/2)} = \frac{20}{(4+j\omega)(10+j\omega)}$

f(2t-1) = f [2(t-1/2)] $\longrightarrow$ $\mathbf{\frac{20e^{-j\omega/2}}{(4+j\omega)(10+j\omega)}}$

(c) f(t) cos 2t $\longrightarrow$ $\frac{1}{2}F(\omega+2) + \frac{1}{2}F(\omega+2)$

$= \mathbf{\frac{5}{[2+j(\omega+2)][5+j(\omega+2)]} + \frac{5}{[2+j(\omega-2)[5+j(\omega-2)]]}}$

(d) $F[\mathbf{f'(t)}] = \mathbf{j\omega F(\omega)} = \mathbf{\frac{j\omega 10}{(2+j\omega)(5+j\omega)}}$

(e) $\int_{-\infty}^{t} f(t)dt \longrightarrow \frac{F(\omega)}{j(\omega)} + \pi F(0)\delta(\omega)$

$= \frac{10}{j\omega(2+j\omega)(5+j\omega)} + \pi\delta(\omega)\frac{x10}{2x5}$

$= \mathbf{\frac{10}{j\omega(2+j\omega)(5+j\omega)} + \pi\delta(\omega)}$

Chapter 18, Solution 24.

(a) $X(\omega) = F(\omega) + \mathcal{F}[3]$

$$= \mathbf{6\pi\delta(\omega) + \frac{j}{\omega}\left(e^{-j\omega} - 1\right)}$$

(b) $y(t) = f(t-2)$

$$Y(\omega) = e^{-j\omega 2}F(\omega) = \mathbf{\frac{je^{-j2\omega}}{\omega}\left(e^{-j\omega} - 1\right)}$$

(c) If $h(t) = f\,'(t)$

$$H(\omega) = j\omega F(\omega) = j\omega\frac{j}{\omega}\left(e^{-j\omega} - 1\right) = \mathbf{1 - e^{-j\omega}}$$

(d) $g(t) = 4f\left(\frac{2}{3}t\right) + 10f\left(\frac{5}{3}t\right)$, $G(\omega) = 4x\frac{3}{2}F\left(\frac{3}{2}\omega\right) + 10x\frac{3}{5}F\left(\frac{3}{5}\omega\right)$

$$= 6\cdot\frac{j}{\frac{3}{2}\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{6j}{\frac{3}{5}\omega}\left(e^{-j3\omega/5} - 1\right)$$

$$= \mathbf{\frac{j4}{\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{j10}{\omega}\left(e^{-j3\omega/5} - 1\right)}$$

Chapter 18, Solution 25.

(a) g(t) = $\mathbf{5e^{2t}u(t)}$

(b) h(t) = $\mathbf{6e^{-2t}}$

(c) $X(\omega) = \frac{A}{s-1} + \frac{B}{s-2}, \qquad s = j\omega$

$$A = \frac{10}{1-2} = -10, \qquad B = \frac{10}{2-1} = 10$$

$$X(\omega) = \frac{-10}{j\omega - 1} + \frac{10}{j\omega - 2}$$

x(t) = $\mathbf{(-10e^{t}u(t)+10e^{2t})u(t)}$

(a) $\mathbf{5e^{2t}u(t)}$, **(b)** $\mathbf{6e^{-2t}}$ **, (c)** $\mathbf{(-10e^{t}u(t)+10e^{2t})u(t)}$

Chapter 18, Solution 26.

(a) $\underline{f(t) = e^{-(t-2)}u(t)}$

(b) $\underline{h(t) = te^{-4t}u(t)}$

(c) If $\quad x(t) = u(t+1) - u(t-1) \quad \longrightarrow \quad X(\omega) = 2\dfrac{\sin\omega}{\omega}$

By using duality property,

$$G(\omega) = 2u(\omega+1) - 2u(\omega-1) \quad \longrightarrow \quad \underline{g(t) = \frac{2\sin t}{\pi t}}$$

Chapter 18, Solution 27.

(a) Let $F(s) = \dfrac{100}{s(s+10)} = \dfrac{A}{s} + \dfrac{B}{s+10}, \quad s = j\omega$

$$A = \frac{100}{10} = 10, \; B = \frac{100}{-10} = -10$$

$$F(\omega) = \frac{10}{j\omega} - \frac{10}{j\omega + 10}$$

f(t) $=$ $\mathbf{(5\,sgn(t) - 10e^{-10t})u(t)}$

(b) $G(s) = \dfrac{10s}{(2-s)(3+s)} = \dfrac{A}{2-s} + \dfrac{B}{s+3}, \quad s = j\omega$

$$A = \frac{20}{5} = 4, \quad B = \frac{-30}{5} = -6$$

$$G(\omega) = \frac{4}{= -j\omega + 2} - \frac{6}{j\omega + 3}$$

g(t) $=$ $\mathbf{4e^{2t}u(-t) - 6e^{-3t}u(t)}$

(c) $H(\omega) = \dfrac{60}{(j\omega)^2 + j40\omega + 1300} = \dfrac{60}{(j\omega + 20)^2 + 900}$

h(t) $=$ $\mathbf{2e^{-20t}\,sin(30t)\,u(t)}$

(d) $y(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \dfrac{\delta(\omega)e^{j\omega t}d\omega}{(2 + j\omega)(j\omega + 1)} = \dfrac{1}{2}\pi \cdot \dfrac{1}{2} = \mathbf{\dfrac{1}{4}\pi}$

Chapter 18, Solution 28.

(a) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi\delta(\omega)\,e^{j\omega t}}{(5+j\omega)(2+j\omega)}d\omega$$

$$= \frac{1}{2}\,\frac{1}{(5)(2)} = \frac{1}{20} = \mathbf{0.05}$$

(b) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{10\delta(\omega+2)}{j\omega(j\omega+1)}e^{j\omega t}\,d\omega = \frac{10}{2\pi}\frac{e^{-j2t}}{(-j2)(-j2+1)}$$

$$= \frac{j5}{2\pi}\frac{e^{-j2t}}{1-j2} = \boldsymbol{\frac{(-2+j)e^{-j2t}}{2\pi}}$$

(c) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{20\delta(\omega-1)e^{j\omega t}}{(2+j\omega)(3+5\omega)}d\omega = \frac{20}{2\pi}\frac{e^{jt}}{(2+j)(3+j)}$$

$$= \frac{20e^{jt}}{2\pi(5+5j)} = \boldsymbol{\frac{(1-j)e^{jt}}{\pi}}$$

(d) Let $$F(\omega) = \frac{5\pi\delta(\omega)}{(5+j\omega)} + \frac{5}{j\omega(5+j\omega)} = F_1(\omega) + F_2(\omega)$$

$$f_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{5\pi\delta(\omega)}{5+j\omega}e^{j\omega t}\,d\omega = \frac{5\pi}{2\pi}\cdot\frac{1}{5} = 0.5$$

$$F_2(s) = \frac{5}{s(5+s)} = \frac{A}{s} + \frac{B}{s+5}\ ,\ A = 1, B = -1$$

$$F_2(\omega) = \frac{1}{j\omega} - \frac{1}{j\omega+5}$$

$$f_2(t) = \frac{1}{2}\mathrm{sgn}(t) - e^{-5t} = -\frac{1}{2} + u(t) - e^{5t}$$

$$f(t) = f_1(t) + f_2(t) = \mathbf{u(t) - e^{-5t}}$$

Chapter 18, Solution 29.

(a) $f(t) = F^{-1}[\delta(\omega)] + F^{-1}\left[4\delta(\omega+3) + 4\delta(\omega-3)\right]$

$$= \frac{1}{2\pi} + \frac{4\cos 3t}{\pi} = \mathbf{\frac{1}{2\pi}\left(1 + 8\cos 3t\right)}$$

(b) If $h(t) = u(t+2) - u(t-2)$

$$H(\omega) = \frac{2\sin 2\omega}{\omega}$$

$$G(\omega) = 4H(\omega) \quad \longrightarrow \quad g(t) = \frac{1}{2\pi}\cdot\frac{8\sin 2t}{t}$$

$$g(t) = \mathbf{\frac{4\sin 2t}{\pi t}}$$

(c) Since

$$\cos(at) \;\square\; \pi\delta(\omega + a) + \pi\delta(\omega - a)$$

Using the reversal property,

$$2\pi\cos 2\omega \leftrightarrow \pi\delta(t+2) + \pi\delta(t-2)$$

or $F^{-1}\left[6\cos 2\omega\right] = \mathbf{3\delta(t+2) + 3\delta(t-2)}$

Chapter 18, Solution 30.

(a) $y(t) = \text{sgn}(t) \longrightarrow Y(\omega) = \frac{2}{j\omega}, \qquad X(\omega) = \frac{1}{a + j\omega}$

$$\mathbf{H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2(a + j\omega)}{j\omega} = 2 + \frac{2a}{j\omega}} \longrightarrow \underline{\mathbf{h(t) = 2\delta(t) + a[u(t) - u(-t)]}}$$

(b) $X(\omega) = \frac{1}{1 + j\omega}, \qquad Y(\omega) = \frac{1}{2 + j\omega}$

$$\mathbf{H(\omega) = \frac{1 + j\omega}{2 + j\omega} = 1 - \frac{1}{2 + j\omega}} \longrightarrow \underline{\mathbf{h(t) = \delta(t) - e^{-2t}u(t)}}$$

(c) In this case, by definition, $\underline{\mathbf{h(t) = \quad y(t) = e^{-at} \sin bt\, u(t)}}$

Chapter 18, Solution 31.

(a) $Y(\omega) = \dfrac{1}{(a + j\omega)^2}, \quad H(\omega) = \dfrac{1}{a + j\omega}$

$$\mathbf{X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{1}{a + j\omega} \quad \longrightarrow \quad \underline{x(t) = e^{-at}u(t)}}$$

(b) By definition, $\underline{\mathbf{x(t) = y(t) = u(t+1) - u(t-1)}}$

(c) $Y(\omega) = \dfrac{1}{(a + j\omega)}, \quad H(\omega) = \dfrac{2}{j\omega}$

$$\mathbf{X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{j\omega}{2(a + j\omega)} = \frac{1}{2} - \frac{a}{2(a + j\omega)} \quad \longrightarrow \quad \underline{x(t) = \frac{1}{2}\delta(t) - \frac{a}{2}e^{-at}u(t)}}$$

Chapter 18, Solution 32.

(a) Since $\dfrac{e^{-j\omega}}{j\omega+1}$ $\qquad$ $e^{-(t-1)}u(t-1)$

and $F(-\omega)$ $\longrightarrow$ f(-t)

$$F_1(\omega) = \frac{e^{j\omega}}{-j\omega+1} \longrightarrow f_1(t) = e^{-(-t-1)}u(-t-1)$$

$f_1(t) = \mathbf{e^{(t+1)}u(-t-1)}$

(b) From Section 17.3,

$$\frac{2}{t^2+1} \longrightarrow 2\pi e^{-|\omega|}$$

If $F_2(\omega) = 2e^{-|\omega|}$, then

$$f_2(t) = \mathbf{\frac{2}{\pi(t^2+1)}}$$

(b) By partial fractions

$$F_3(\omega) = \frac{1}{(j\omega+1)^2(j\omega-1)^2} = \frac{\frac{1}{4}}{(j\omega+1)^2} + \frac{\frac{1}{4}}{(j\omega+1)} + \frac{\frac{1}{4}}{(j\omega-1)^2} - \frac{\frac{1}{4}}{j\omega-1}$$

Hence $f_3(t) = \dfrac{1}{4}\left(te^{-t} + e^{-t} + te^{t} - e^{t}\right)u(t)$

$= \mathbf{\dfrac{1}{4}(t+1)e^{-t}u(t) + \dfrac{1}{4}(t-1)e^{t}u(t)}$

(d) $$f_4(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\delta(\omega)e^{j\omega t}}{1+j2\omega} = \mathbf{\frac{1}{2\pi}}$$

Chapter 18, Solution 33.

(a) Let $x(t) = 2\sin\pi t\left[u(t+1) - u(t-1)\right]$

From Problem 17.9(b),

$$X(\omega) = \frac{4j\pi\sin\omega}{\pi^2 - \omega^2}$$

Applying duality property,

$$f(t) = \frac{1}{2\pi}X(-t) = \frac{2j\sin(-t)}{\pi^2 - t^2}$$

$$f(t) = \mathbf{\frac{2j\sin t}{t^2 - \pi^2}}$$

(b) $F(\omega) = \frac{j}{\omega}(\cos 2\omega - j\sin 2\omega) - \frac{j}{\omega}(\cos\omega - j\sin\omega)$

$$= \frac{j}{\omega}\left(e^{j2\omega} - e^{-j\omega}\right) = \frac{e^{-j\omega}}{j\omega} - \frac{e^{j2\omega}}{j\omega}$$

$$f(t) = \frac{1}{2}\mathrm{sgn}(t-1) - \frac{1}{2}\mathrm{sgn}(t-2)$$

But $\mathrm{sgn}(t) = 2u(t) - 1$

$$f(t) = u(t-1) - \frac{1}{2} - u(t-2) + \frac{1}{2}$$

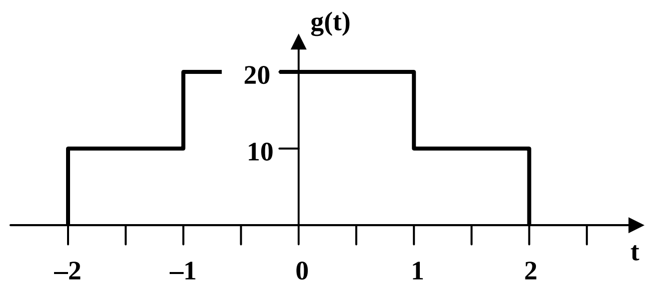
$$= \mathbf{u(t-1) - u(t-2)}$$

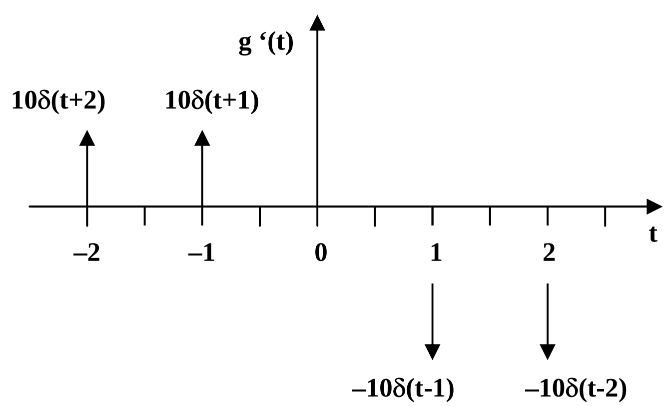
Chapter 18, Solution 34.

First, we find G(ω) for g(t) shown below.

$$g(t) = 10[u(t+2) - u(t-2)] + 10[u(t+1) - u(t-1)]$$
$$g'(t) = 10[\delta(t+2) - \delta(t-2)] + 10[\delta(t+1) - \delta(t-1)]$$

The Fourier transform of each term gives





$$j\omega G(\omega) = 10\left(e^{j\omega 2} - e^{-j\omega 2}\right) + 10\left(e^{j\omega} - e^{-j\omega}\right)$$
$$= 20j\sin 2\omega + 20j\sin\omega$$
$$G(\omega) = \frac{20\sin 2\omega}{\omega} + \frac{20\sin\omega}{\omega} = 40 \text{ sinc}(2\omega) + 20 \text{ sinc}(\omega)$$

Note that G(ω) = G(-ω).

$$F(\omega) = 2\pi G(-\omega)$$
$$f(t) = \frac{1}{2\pi} G(t)$$
$$= \mathbf{(20/\pi)sinc(2t) + (10/\pi)sinc(t)}$$

Chapter 18, Solution 35.

(a) x(t) = f[3(t-1/3)]. Using the scaling and time shifting properties,

$$\mathbf{X(\omega) = \frac{1}{3}\frac{1}{2 + j\omega/3}e^{-j\omega/3} = \underline{\frac{e^{-j\omega/3}}{(6 + j\omega)}}}$$

(b) Using the modulation property,

$$\mathbf{Y(\omega) = \frac{1}{2}[F(\omega + 5) + F(\omega - 5)] = \underline{\frac{1}{2}\left[\frac{1}{2 + j(\omega + 5)} + \frac{1}{2 + j(\omega - 5)}\right]}}$$

(c) $$\mathbf{Z(\omega) = j\omega F(\omega) = \underline{\frac{j\omega}{2 + j\omega}}}$$

(d) $$\mathbf{H(\omega) = F(\omega)F(\omega) = \underline{\frac{1}{(2 + j\omega)^2}}}$$

(e) $$\mathbf{I(\omega) = j\frac{d}{d\omega}F(\omega) = j\frac{(0 - j)}{(2 + j\omega)^2} = \underline{\frac{1}{(2 + j\omega)^2}}}$$

Chapter 18, Solution 36.

$$H(\omega)=\frac{Y(\omega)}{X(\omega)} \longrightarrow Y(\omega)=H(\omega)X(\omega)$$

$$x(t)=v_s(t)=e^{-4t}u(t) \longrightarrow X(\omega)=\frac{1}{4+j\omega}$$

$$Y(\omega)=\frac{2}{(j\omega+2)(4+j\omega)}=\frac{2}{(s+2)(s+4)}, \qquad s=j\omega$$

$$Y(s)=\frac{A}{s+2}+\frac{B}{s+4}$$

$$A=\frac{2}{-2+4}=1, \qquad B=\frac{2}{-4+2}=-1$$

$$Y(s)=\frac{1}{s+2}-\frac{1}{s+4}$$

$$y(t)=\underline{\left(e^{-2t}-e^{-4t}\right)u(t)}$$

Please note, the units are not known since the transfer function does not give them. If the transfer function was a voltage gain then the units on y(t) would be volts.

Chapter 18, Solution 37.

$$2 \| j\omega = \frac{j2\omega}{2 + j\omega}$$

By current division,

$$H(\omega) = \frac{I_o(\omega)}{I_s(\omega)} = \frac{\dfrac{j2\omega}{2 + j\omega}}{4 + \dfrac{j2\omega}{2 + j\omega}} = \frac{j2\omega}{j2\omega + 8 + j4\omega}$$

$$H(\omega) = \boldsymbol{\frac{j\omega}{4 + j3\omega}}$$

Chapter 18, Solution 38.

Using Fig. 18.40, design a problem to help other students to better understand using Fourier transforms to do circuit analysis.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Suppose $v_s(t) = u(t)$ for t>0. Determine i(t) in the circuit of Fig. 18.40 using Fourier transform.

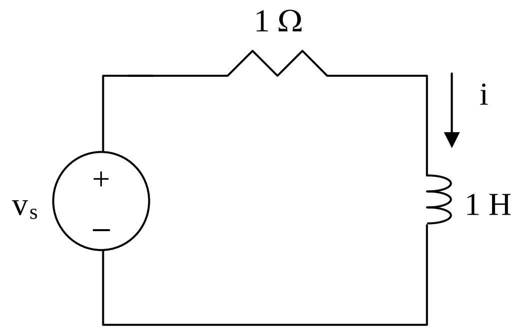


Figure 18.40 For Prob. 18.38.

Solution

$$V_s = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$I(\omega) = \frac{V_s}{1+j\omega} = \frac{1}{1+j\omega}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

Let $I(\omega) = I_1(\omega) + I_2(\omega) = \dfrac{\pi\delta(\omega)}{1+j\omega} + \dfrac{1}{j\omega(1+j\omega)}$

$$I_2(\omega) = \frac{1}{j\omega(1+j\omega)} = \frac{A}{s} + \frac{B}{s+1}, \qquad s = j\omega$$

where $A = \dfrac{1}{1} = 1, \qquad B = \dfrac{1}{-1} = -1$

$$I_2(\omega) = \frac{1}{j\omega} + \frac{-1}{j\omega+1} \quad \longrightarrow \quad i_2(t) = \frac{1}{2}\mathrm{sgn}(t) - e^{-t}$$

$$I_1(\omega) = \frac{\pi\delta(\omega)}{1+j\omega}$$

$$i_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi\delta(\omega)}{1+j\omega} e^{j\omega t} d\omega = \frac{1}{2}\frac{e^{j\omega t}}{1+j\omega}\bigg|_{\omega=0} = \frac{1}{2}$$

Hence,

$$i(t) = i_1(t) + i_2(t) = \underline{\frac{1}{2} + \frac{1}{2}\mathrm{sgn}(t) - e^{-t}}$$

Chapter 18, Solution 39.

$$V_s(\omega) = \int_{-\infty}^{\infty} (1-t)e^{-j\omega t}dt = \frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2}e^{-j\omega}$$

$$\mathbf{I}(\boldsymbol{\omega}) = \frac{\mathbf{V_s(\omega)}}{\mathbf{10^3 + j\omega x10^{-3}}} = \underline{\frac{\mathbf{10^3}}{\mathbf{10^6 + j\omega}}\left(\frac{\mathbf{1}}{\mathbf{j\omega}} + \frac{\mathbf{1}}{\boldsymbol{\omega}^2} - \frac{\mathbf{1}}{\boldsymbol{\omega}^2}\mathbf{e}^{-j\omega}\right)}$$

Chapter 18, Solution 40.

$$\ddot{v}(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$$

$$-\omega^2 V(\omega) = 1 - 2e^{-j\omega} + e^{j\omega 2}$$

$$V(\omega) = \frac{1 - 2e^{-j\omega} + e^{-j\omega 2}}{-\omega^2}$$

Now $$Z(\omega) = 2 + \frac{1}{j\omega} = \frac{1 + j2\omega}{j\omega}$$

$$I = \frac{V(\omega)}{Z(\omega)} = \frac{2e^{j\omega} - e^{j\omega 2} - 1}{\omega^2} \cdot \frac{j\omega}{1 + j2\omega}$$

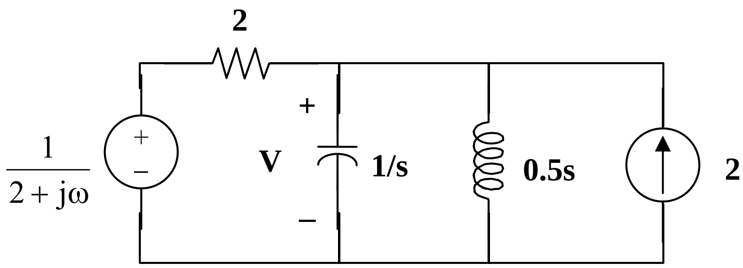
$$= \frac{1}{j\omega(0.5 + j\omega)}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

But $$\frac{1}{s(s+0.5)} = \frac{A}{s} + \frac{B}{s+0.5} \longrightarrow A = 2,\ B = -2$$

$$I(\omega) = \frac{2}{j\omega}\left(0.5 + 0.5e^{j\omega 2} - e^{-j\omega}\right) - \frac{2}{0.5 + j\omega}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

i(t) = $$\mathbf{\frac{1}{2}sgn(t) + \frac{1}{2}sgn(t-2) - sgn(t-1) - e^{-0.5t}u(t) - e^{-0.5(t-2)}u(t-2) - 2e^{-0.5(t-1)}u(t-1)}$$

Chapter 18, Solution 41.



$$\frac{V - \frac{1}{2+j\omega}}{2} + j\omega V + \frac{2V}{j\omega} - 2 = 0$$

$$\left(j\omega - 2\omega^2 + 4\right)V = j4\omega + \frac{j\omega}{2+j\omega} = \frac{-4\omega^2 + j9\omega}{2+j\omega}$$

$$V(\omega) = \frac{\mathbf{2j\omega(4.5 + j2\omega)}}{\mathbf{(2 + j\omega)(4 - 2\omega^2 + j\omega)}}$$

Chapter 18, Solution 42.

By current division, $I_o = \dfrac{2}{2 + j\omega} \cdot I(\omega)$

(a) For i(t) = 5 sgn (t),

$$I(\omega) = \frac{10}{j\omega}$$

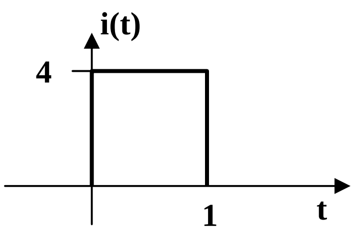
$$I_o = \frac{2}{2 + j\omega} \cdot \frac{10}{j\omega} = \frac{20}{j\omega(2 + j\omega)}$$

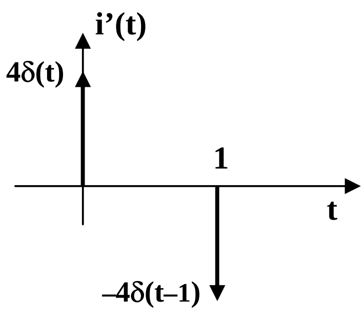
Let $I_o = \dfrac{20}{s(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+2}$, $A = 10$, $B = -10$

$$I_o(\omega) = \frac{10}{j\omega} - \frac{10}{2 + j\omega}$$

$i_o(t)$ = $\mathbf{5\,sgn(t) - 10e^{-2t}u(t)A}$

(b)





$$i'(t) = 4\delta(t) - 4\delta(t-1)$$

$$j\omega I(\omega) = 4 - 4e^{-j\omega}$$

$$I(\omega) = \frac{4\left(1 - e^{-j\omega}\right)}{j\omega}$$

$$I_o = \frac{8\left(1 - e^{-j\omega}\right)}{j\omega(2 + j\omega)} = 4\left(\frac{1}{j\omega} - \frac{1}{2 + j\omega}\right)\left(1 - e^{-j\omega}\right)$$

$$= \frac{4}{j\omega} - \frac{4}{2 + j\omega} - \frac{4e^{-j\omega}}{j\omega} + \frac{4e^{-j\omega}}{2 + j\omega}$$

$i_o(t)$ = $\mathbf{2\,sgn(t) - 2\,sgn(t-1) - 4e^{-2t}u(t) + 4e^{-2(t-1)}u(t-1)A}$

Chapter 18, Solution 43.

$$20\,\text{mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j20x10^{-3}\omega} = \frac{50}{j\omega}, \qquad i_s = 5e^{-t} \longrightarrow I_s = \frac{5}{1+j\omega}$$

$$V_o = \frac{40}{40+\frac{50}{j\omega}} I_s \bullet \frac{50}{j\omega} = \frac{250}{(s+1)(s+1.25)}, \qquad s = j\omega$$

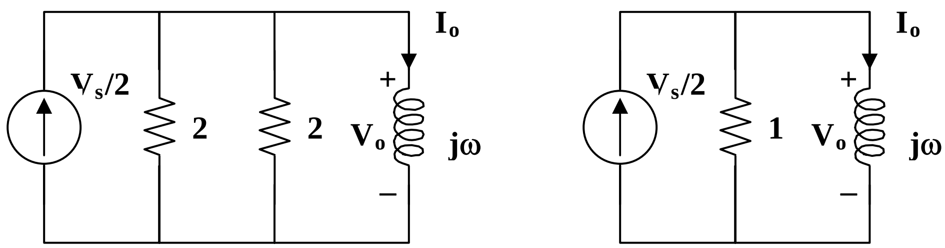
$$V_o = \frac{A}{s+1} + \frac{B}{s+1.25} = 1000\left[\frac{1}{s+1} - \frac{1}{s+1.25}\right]$$

$$\mathbf{v_o(t) = \underline{1(e^{-1t} - e^{-1.25t})u(t)\ kV}}$$

Chapter 18, Solution 44.

$$1\text{H} \longrightarrow j\omega$$

We transform the voltage source to a current source as shown in Fig. (a) and then combine the two parallel 2Ω resistors, as shown in Fig. (b).



(a) (b)

$$2\|2 = 1\Omega,\quad I_o = \frac{1}{1+j\omega}\cdot\frac{V_s}{2}$$

$$V_o = j\omega I_o = \frac{j\omega V_s}{2(1+j\omega)}$$

$$\ddot{v}_s(t) = 10\delta(t) - 10\delta(t-2)$$

$$j\omega V_s(\omega) = 10 - 10e^{-j2\omega}$$

$$V_s(\omega) = \frac{10\left(1-e^{-j2\omega}\right)}{j\omega}$$

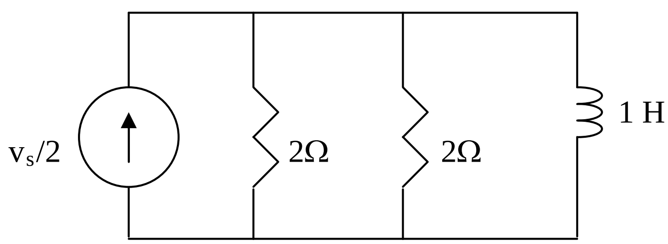
Hence $V_o = \dfrac{5\left(1-e^{-j2\omega}\right)}{1+j\omega} = \dfrac{5}{1+j\omega} - \dfrac{5}{1+j\omega}e^{-j2\omega}$

$$v_o(t) = 5e^{-t}u(t) - 5e^{-(t-2)}u(t-2)$$

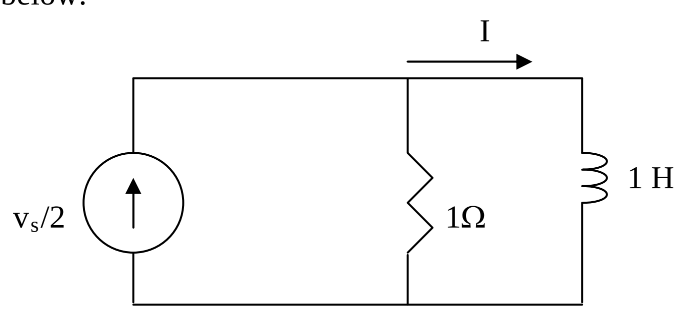
$$v_o(1) = 5e^{-1} - 0 = \mathbf{1.839\ V}$$

Chapter 18, Solution 45.

We may convert the voltage source to a current source as shown below.



Combining the two 2-Ω resistors gives 1 Ω. The circuit now becomes that shown below.



$$I = \frac{1}{1+j\omega} \bullet \frac{V_s}{2} = \frac{1}{1+j\omega} \bullet \frac{5}{2+j\omega} = \frac{5}{(s+1)(s+2)}, \qquad s = j\omega$$

$$= \frac{A}{s+1} + \frac{B}{s+2}$$

where $A = 5/1 = 5, \qquad B = 5/-1 = -5$

$$I = \frac{5}{s+1} - \frac{5}{s+2}$$

$$i(t) = \underline{5(e^{-t} - e^{-2t})u(t)}\,\text{A}$$

Chapter 18, Solution 46.

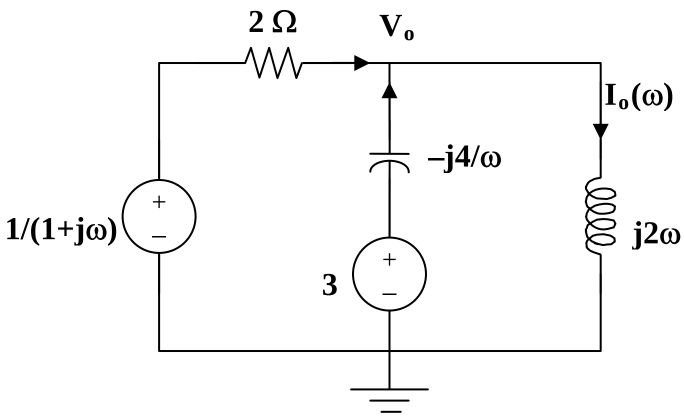
$$\frac{1}{4}\text{F} \longrightarrow \frac{1}{j\omega\frac{1}{4}} = \frac{-j4}{\omega}$$

$$2\text{H} \longrightarrow j\omega 2$$

$$3\delta(t) \longrightarrow 3$$

$$e^{-t}u(t) \longrightarrow \frac{1}{1+j\omega}$$

The circuit in the frequency domain is shown below:



At node V_o, KCL gives

$$\frac{\frac{1}{1+j\omega} - V_o}{2} + \frac{3 - V_o}{\frac{-j4}{\omega}} = \frac{V_o}{j2\omega}$$

$$\frac{2}{1+j\omega} - 2V_o + j\omega 3 - j\omega V_o = -\frac{j2V_o}{\omega}$$

$$V_o = \frac{\frac{2}{1+j\omega} + j\omega 3}{2 + j\omega - \frac{j2}{\omega}}$$

$$I_o(\omega) = \frac{V_o}{j2\omega} = \frac{\frac{2 + j\omega 3 - 3\omega^2}{1+j\omega}}{j2\omega\left(2 + j\omega - \frac{j2}{\omega}\right)}$$

$$I_o(\omega) = \mathbf{\frac{2 + j\omega^2 - 3\omega^2}{4 - 6\omega^2 + j(8\omega - 2\omega^3)}}$$

Chapter 18, Solution 47.

$$\frac{1}{2}F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{2}{j\omega}$$

$$I_o = \frac{1}{1+\frac{2}{j\omega}} I_s$$

$$V_o = \frac{2}{j\omega} I_o = \frac{\frac{2}{j\omega}}{1+\frac{2}{j\omega}} I_s = \frac{2}{2+j\omega}\frac{8}{1+j\omega}$$

$$= \frac{16}{(s+1)(s+2)}, \; s = j\omega$$

$$= \frac{A}{s+1} + \frac{B}{s+2}$$

where $A = 16/1 = 16$, $B = 16/(-1) = -16$

Thus,

$$v_o(t) = \mathbf{16(e^{-t} - e^{-2t})u(t)\ V.}$$

Chapter 18, Solution 48.

$$0.2\text{F} \longrightarrow \frac{1}{j\omega C} = -\frac{j5}{\omega}$$

As an integrator,

$$RC = 20\,x\,10^{3}\ x\ 20\,x\,10^{-6} = 0.4$$

$$v_o = -\frac{1}{RC}\int_o^t v_i dt$$

$$V_o = -\frac{1}{RC}\left[\frac{V_i}{j\omega} + \pi V_i(0)\delta(\omega)\right]$$

$$= -\frac{1}{0.4}\left[\frac{2}{j\omega(2+j\omega)} + \pi\delta(\omega)\right]$$

$$I_o = \frac{V_o}{20}\text{mA} = -0.125\left[\frac{2}{j\omega(2+j\omega)} + \pi\delta(\omega)\right]$$

$$= -\frac{0.125}{j\omega} + \frac{0.125}{2+j\omega} - 0.125\pi\delta(\omega)$$

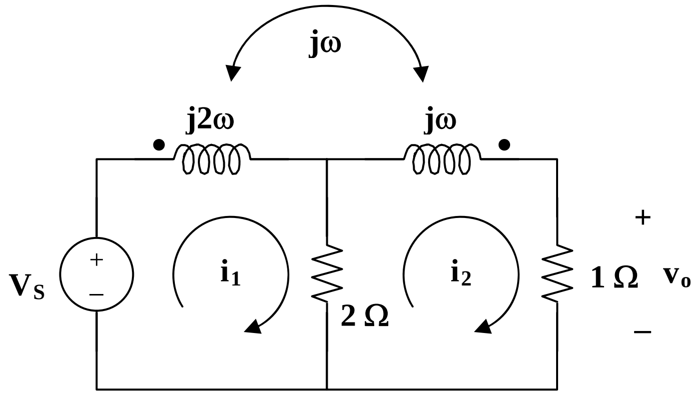
$$i_o(t) = -0.125\,\text{sgn}(t) + 0.125e^{-2t}u(t) - \frac{0.125}{2\pi}\int \pi\delta(\omega)e^{j\omega t}dt$$

$$= 0.125 + 0.25u(t) + 0.125e^{-2t}u(t) - \frac{0.125}{2}$$

$$i_o(t) = \mathbf{[0.625 - 0.25u(t) + 0.125e^{-2t}u(t)]mA}$$

Chapter 18, Solution 49.

Consider the circuit shown below:



$$V_s = \pi[\delta(\omega+1)+\delta(\omega-2)]$$

For mesh 1, $-V_s + (2 + j2\omega)I_1 - 2I_2 - j\omega I_2 = 0$

$$V_s = 2(1 + j\omega)I_1 - (2 + j\omega)I_2 \qquad (1)$$

For mesh 2, $\quad 0 = (3 + j\omega)I_2 - 2I_1 - j\omega I_1$

$$I_1 = \frac{(3+\omega)I_2}{(2+\omega)} \qquad (2)$$

Substituting (2) into (1) gives

$$V_s = 2\frac{2(1+j\omega)(3+j\omega)I_2}{2+j\omega} - (2+j\omega)I_2$$

$$V_s(2+\omega) = [2(3+j4\omega-\omega^2) - (4+j4\omega-\omega^2)]I_2$$

$$= I_2(2+j4\omega-\omega^2)$$

$$I_2 = \frac{(s+2)V_s}{s^2+4s+2}, \ s = j\omega$$

$$V_o = I_2 = \frac{(j\omega+2)\pi[\delta(\omega+1)+\delta(\omega-1)]}{(j\omega)^2 + j\omega 4 + 2}$$

$$v_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_o(\omega)e^{j\omega t}d\omega$$

$$= \int_{-\infty}^{\infty} \frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega+1)d\omega}{(j\omega)^2 + j\omega 4 + 2} + \frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega-1)d\omega}{(j\omega)^2 + j\omega 4 + 2}$$

$$= \frac{\frac{1}{2}(-j+2)e^{jt}}{-1-j4+2} + \frac{\frac{1}{2}(j+2)e^{jt}}{-1+j4+2}$$

$$v_o(t) = \frac{\frac{1}{2}(2-j)(1+j4)}{17} e^{jt} + \frac{\frac{1}{2}(2-j)(1-j4)e^{jt}}{17}$$

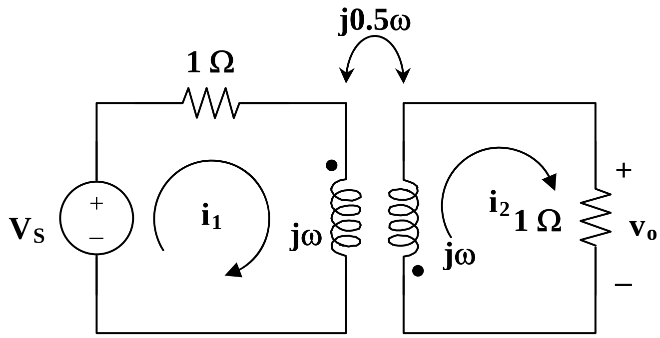
$$= \frac{1}{34}(6+j7)e^{jt} + \frac{1}{34}(6-j7)e^{jt}$$

$$= 0.271e^{-j(t-13.64°)} + 0.271e^{j(t-13.64°)}$$

$$v_o(t) = \mathbf{542\cos(t-13.64°)\,mV}$$

Chapter 18, Solution 50.

Consider the circuit shown below:



For loop 1,

$$-2+(1+j\omega)I_1 + j0.5\omega I_2 = 0 \qquad (1)$$

For loop 2,

$$(1+j\omega)I_2 + j0.5\omega I_1 = 0 \qquad (2)$$

From (2),

$$I_1 = \frac{(1+j\omega)I_2}{-j0.5\omega} = -2\frac{(1+j\omega)I_2}{j\omega}$$

Substituting this into (1),

$$2 = \frac{-2(1+j\omega)I_2}{j\omega} + \frac{j\omega}{2}I_2$$

$$2j\omega = -\left(4 + j4\omega - \frac{3}{2}\omega^2\right)I_2$$

$$I_2 = \frac{2j\omega}{4 + j4\omega - 1.5\omega^2}$$

$$V_o = I_2 = \frac{-2j\omega}{4 + j4\omega + 1.5(j\omega)^2}$$

$$V_o = \frac{\frac{4}{3}j\omega}{\frac{8}{3} + j\frac{8\omega}{3} + (j\omega)^2}$$

$$= \frac{-4\left(\frac{4}{3} + j\omega\right)}{\left(\frac{4}{3} + j\omega\right)^2 + \left(\frac{\sqrt{8}}{3}\right)^2} + \frac{\frac{16}{3}}{\left(\frac{4}{3} + j\omega\right)^2 + \left(\frac{\sqrt{8}}{3}\right)^2}$$

$$V_o(t) = \mathbf{-4e^{-4t/3}\cos\left(\frac{\sqrt{8}}{3}t\right)u(t) + 5.657e^{-4t/3}\sin\left(\frac{\sqrt{8}}{3}t\right)u(t)\,V}$$

Chapter 18, Solution 51.

In the frequency domain, the voltage across the 2-Ω resistor is

$$V(\omega) = \frac{2}{2+j\omega}V_s = \frac{2}{2+j\omega}\bullet\frac{10}{1+j\omega} = \frac{20}{(s+1)(s+2)}, \qquad s = j\omega$$

$$V(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = 20/1 = 20, \qquad B = 20/-1 = -20$$

$$V(\omega) = \frac{20}{j\omega+1} - \frac{20}{j\omega+2}$$

$$v(t) = \left(20e^{-t} - 20e^{-2t}\right)u(t)$$

$$W = \frac{1}{2}\int_0^\infty v^2(t)dt = 0.5\int 400\left(e^{-2t} + e^{-4t} - 3e^{-3t}\right)dt$$

$$= 200\left(\frac{e^{-2t}}{-2} + \frac{e^{-4t}}{-4} - \frac{2e^{-3t}}{-3}\right)\Bigg|_0^\infty = \mathbf{16.667\ J}.$$

Chapter 18, Solution 52.

$$J = 2\int_0^{\infty} f^2(t)\,dt = \frac{1}{\pi}\int_0^{\infty} \left|F(\omega)\right|^2 d\omega$$

$$= \frac{1}{\pi}\int_0^{\infty} \frac{1}{9^2+\omega^2}\,d\omega = \frac{1}{3\pi}\tan^{-1}(\omega/3)\Big|_0^{\infty} = \frac{1}{3\pi}\,\frac{\pi}{2} = \mathbf{(1/6)}$$

Chapter 18, Solution 53.

If $f(t) = e^{-2|t|}$, find $J = \int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega$.

$$J = \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} f^2(t)\, dt$$

$$f(t) = \begin{cases} e^{2t}, & t < 0 \\ e^{-2t}, & t > 0 \end{cases}$$

$$J = 2\pi\left[\int_{-\infty}^{0} e^{4t} dt + \int_{0}^{\infty} e^{-4t} dt\right] = 2\pi\left[\left.\frac{e^{4t}}{4}\right|_{-\infty}^{0} + \left.\frac{e^{-4t}}{-4}\right|_{0}^{\infty}\right] = 2\pi[(1/4) + (1/4)] = \pi$$

Chapter 18, Solution 54.

Design a problem to help other students better understand finding the total energy in a given signal.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Given the signal $f(t) = 4\,e^{-t}\,u(t)$, what is the total energy in $f(t)$?

Solution

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)\,dt = 16\int_0^{\infty} e^{-2t}dt = -8e^{-2t}\Big|_0^{\infty} = \mathbf{8\ J}$$

Chapter 18, Solution 55.

$f(t) = 5e^{2}e^{-t}u(t)$

$F(\omega) = 5e^{2}/(1 + j\omega)$, $|F(\omega)|^{2} = 25e^{4}/(1 + \omega^{2})$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^\infty |F(\omega)|^2 d\omega = \frac{25e^4}{\pi}\int_0^\infty \frac{1}{1+\omega^2} d\omega = \frac{25e^4}{\pi}\tan^{-1}(\omega)\Big|_0^\infty$$

$= 12.5e^{4}$ = **682.5 J**

or $$W_{1\Omega} = \int_{-\infty}^\infty f^2(t)\,dt = 25e^4\int_0^\infty e^{-2t} dt = 12.5e^4 = \mathbf{682.5\ J}$$

Chapter 18, Solution 56.

(a) $W = \int_{-\infty}^{\infty} V^2(t)dt = \int_0^{\infty} t^2 e^{-4t} dt = \frac{e^{-4t}}{(-4)^3}(16t^2 + 8t + 2)\Big|_0^{\infty} = \frac{2}{64} = \underline{0.0313 \text{ J}}$

(b) In the frequency domain,

$$V(\omega) = \frac{1}{(2 + j\omega)^2}$$

$$|V(\omega)|^2 = V(\omega)V^*(\omega) = \frac{1}{(4 + j\omega)^2}$$

$$W_o = \frac{1}{2\pi}\int_{-2}^{2} |V(\omega)|^2 d\omega = \frac{2}{2\pi}\int_0^2 \frac{1}{(4+\omega^2)^2} d\omega$$

$$= \frac{1}{\pi}\frac{1}{2\text{x}4}\left(\frac{\omega}{\omega^2 + 4} + 0.5\tan^{-1}(0.5\omega)\right)\Bigg|_0^2 = \frac{1}{32\pi} + \frac{1}{64} = 0.0256$$

$$\text{Fraction} = \frac{W_o}{W} = \frac{0.0256}{0.0313} = \underline{81.79\%}$$

Chapter 18, Solution 57.

$$W_{1\Omega} = \int_{-\infty}^{\infty} i^2(t)\,dt = \int_{-\infty}^{0} 4e^{2t}\,dt = 2e^{2t}\Big|_{-\infty}^{0} = \mathbf{2\ J} \quad \text{or}$$

$$I(\omega) = 2/(1 - j\omega), \quad |I(\omega)|^2 = 4/(1 + \omega^2)$$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |I(\omega)|^2\,d\omega = \frac{4}{2\pi}\int_{-\infty}^{\infty} \frac{1}{(1+\omega^2)}\,d\omega = \frac{4}{\pi}\tan^{-1}(\omega)\Big|_{0}^{\infty} = \frac{4}{\pi}\frac{\pi}{2} = \mathbf{2\ J}$$

In the frequency range, $-5 < \omega < 5$,

$$W = \frac{4}{\pi}\tan^{-1}\omega\Big|_{0}^{5} = \frac{4}{\pi}\tan^{-1}(5) = \frac{4}{\pi}(1.373) = 1.7487$$

$$W/ W_{1\Omega} = 1.7487/2 = 0.8743 \quad \text{or}$$

87.43%

Chapter 18, Solution 58.

$\omega_m = 200\pi = 2\pi f_m$ which leads to $f_m = 100$ Hz

(a) $\omega_c = \pi x10^4 = 2\pi f_c$ which leads to $f_c = 10^4/2 =$ **5 kHz**

(b) lsb $= f_c - f_m = 5{,}000 - 100 =$ **4,900 Hz**

(c) usb $= f_c + f_m = 5{,}000 + 100 =$ **5,100 Hz**

Chapter 18, Solution 59.

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{\dfrac{10}{2+j\omega} - \dfrac{6}{4+j\omega}}{2} = \frac{5}{2+j\omega} - \frac{3}{4+j\omega}$$

$$V_o(\omega) = H(\omega)V_i(\omega) = \left(\frac{5}{2+j\omega} - \frac{3}{4+j\omega}\right)\frac{4}{1+j\omega}$$

$$= \frac{20}{(s+1)(s+2)} - \frac{12}{(s+1)(s+4)}, \qquad s = j\omega$$

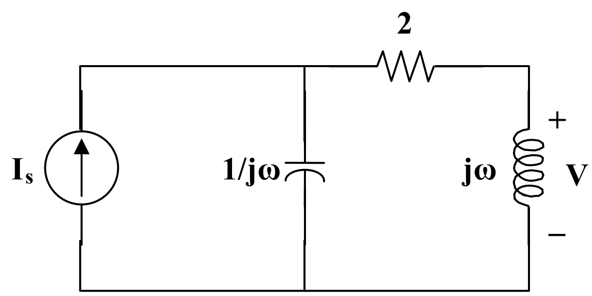
Using partial fraction,

$$V_o(\omega) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+1} + \frac{D}{s+4} = \frac{16}{1+j\omega} - \frac{20}{2+j\omega} + \frac{4}{4+j\omega}$$

Thus,

$$\mathbf{v_o(t) = \underline{\left(16e^{-t} - 20e^{-2t} + 4e^{-4t}\right)u(t)\,V}}$$

Chapter 18, Solution 60.



$$V = j\omega I_s \frac{\frac{1}{j\omega}}{\frac{1}{j\omega} + 2 + j\omega} = \frac{j\omega I_s}{1 - \omega^2 + j2\omega}$$

Since the voltage appears across the inductor, there is no DC component.

$$V_1 = \frac{2\pi\angle 90°8}{1 - 4\pi^2 + j4\pi} = \frac{50.27\angle 90°}{-38.48 + j12.566} = 1.2418\angle -71.92°$$

$$V_2 = \frac{4\pi\angle 90°5}{1 - 16\pi^2 + j8\pi} = \frac{62.83\angle 90°}{-156.91 + j25.13} = 0.3954\angle -80.9°$$

$$\mathbf{v(t) = \underline{1.2418\cos(2\pi t - 41.92°) + 0.3954\cos(4\pi t + 129.1°)\,mV}}$$

Chapter 18, Solution 61.

$$y(t) = (2 + \cos \omega_o t)x(t)$$

We apply the Fourier Transform

$$Y(\omega) = \mathbf{2X(\omega) + 0.5X(\omega+\omega_o) + 0.5X(\omega-\omega_o)}.$$

Chapter 18, Solution 62.

For the lower sideband, the frequencies range from

10,000,000 – 3,500 Hz = **9,996,500 Hz** to
10,000,000 – 400 Hz = **9,999,600 Hz**

For the upper sideband, the frequencies range from

10,000,000 + 400 Hz = **10,000,400 Hz** to
10,000,000 + 3,500 Hz = **10,003,500 Hz**

Chapter 18, Solution 63.

Since $f_n = 5$ kHz, $2f_n = 10$ kHz

i.e. the stations must be spaced 10 kHz apart to avoid interference.

$$\Delta f = 1600 - 540 = 1060 \text{ kHz}$$

The number of stations $= \Delta f / 10$ kHz $=$ **106 stations**

Chapter 18, Solution 64.

$\Delta f = 108 - 88 \text{ MHz} = 20 \text{ MHz}$

The number of stations = 20 MHz/0.2 MHz = **100 stations**

Chapter 18, Solution 65.

$\omega = 3.4$ kHz

$$f_s = 2\omega = \mathbf{6.8\ kHz}$$

Chapter 18, Solution 66.

$\omega = 4.5$ MHz

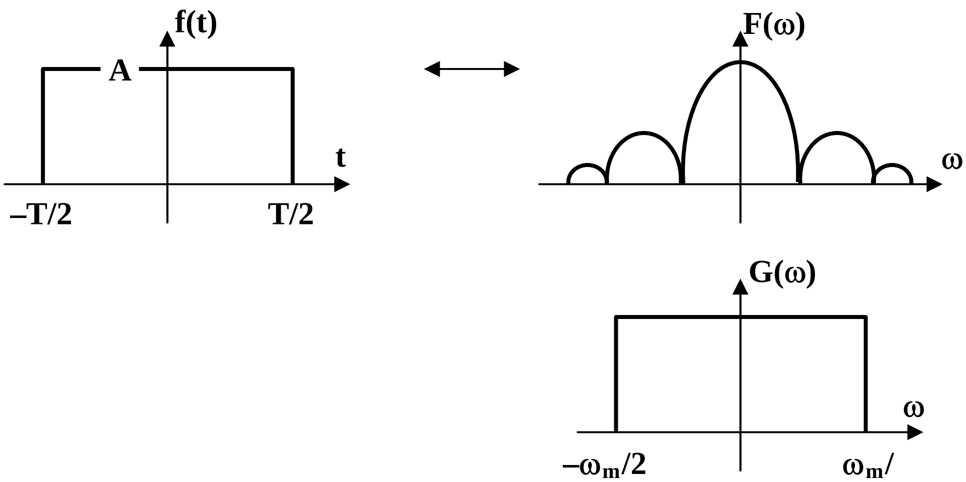
$f_c = 2\omega = 9$ MHz

$T_s = 1/f_c = 1/(9x10^6) = 1.11x10^{-7} =$ **111 ns**

Chapter 18, Solution 67.

We first find the Fourier transform of g(t). We use the results of Example 17.2 in conjunction with the duality property. Let Arect(t) be a rectangular pulse of height A and width T as shown below.

$$\text{Arect}(t) \text{ transforms to } \text{Atsinc}(\omega^2/2)$$



According to the duality property,

$$A\tau\text{sinc}(\tau t/2) \quad \text{becomes} \quad 2\pi A\text{rect}(\tau)$$

$$g(t) = \text{sinc}(200\pi t) \text{ becomes } 2\pi A\text{rect}(\tau)$$

where $A\tau = 1$ and $\tau/2 = 200\pi$ or $T = 400\pi$

i.e. the upper frequency $\omega_u = 400\pi = 2\pi f_u$ or $f_u = 200$ Hz

The Nyquist rate $= f_s =$ **200 Hz**

The Nyquist interval $= 1/f_s = 1/200 =$ **5 ms**

Chapter 18, Solution 68.

The total energy is

$$W_T = \int_{-\infty}^{\infty} v^2(t)\,dt$$

Since v(t) is an even function,

$$W_T = \int_0^{\infty} 2500e^{-4t}dt = 5000\left.\frac{e^{-4t}}{-4}\right|_0^{\infty} = 1250 \text{ J}$$

$V(\omega) = 50x4/(4 + \omega^2)$

$$W = \frac{1}{2\pi}\int_1^5 |V(\omega)|^2\ d\omega = \frac{1}{2\pi}\int_1^5 \frac{(200)^2}{(4+\omega^2)^2}d\omega$$

But $$\int \frac{1}{(a^2+x^2)^2}dx = \frac{1}{2a^2}\left[\frac{x}{x^2+a^2} + \frac{1}{a}\tan^{-1}(x/a)\right] + C$$

$$W = \frac{2x10^4}{\pi}\frac{1}{8}\left.\left[\frac{\omega}{4+\omega^2} + \frac{1}{2}\tan^{-1}(\omega/2)\right]\right|_1^5$$

$= (2500/\pi)[(5/29) + 0.5\tan^{-1}(5/2) - (1/5) - 0.5\tan^{-1}(1/2) = 267.19$

$W/W_T = 267.19/1250 = 0.2137$ or **21.37%**

Chapter 18, Solution 69.

The total energy is

$$W_T = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|F(\omega)\right|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{400}{4^2+\omega^2}d\omega$$

$$= \frac{400}{\pi}\left[(1/4)\tan^{-1}(\omega/4)\right]_0^{\infty} = \frac{100}{\pi}\frac{\pi}{2} = 50$$

$$W = \frac{1}{2\pi}\int_{0}^{2}\left|F(\omega)\right|^2 d\omega = \frac{400}{2\pi}\left[(1/4)\tan^{-1}(\omega/4)\right]_0^{2}$$

$$= [100/(2\pi)]\tan^{-1}(2) = (50/\pi)(1.107) = 17.6187$$

$W/W_T = 17.6187/50 = 0.3524$ or **35.24%**