Chapter 12, Solution 1.

(a) If $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle -30^\circ = \mathbf{231\angle -30^\circ\ V}$$

$$\mathbf{V}_{bn} = \mathbf{231\angle -150^\circ\ V}$$

$$\mathbf{V}_{cn} = \mathbf{231\angle -270^\circ\ V}$$

(b) For the acb sequence,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0^\circ - V_p\angle 120^\circ$$

$$\mathbf{V}_{ab} = V_p\left(1 + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = V_p\sqrt{3}\angle -30^\circ$$

i.e. in the acb sequence, $\mathbf{V}_{ab}$ lags $\mathbf{V}_{an}$ by 30°.

Hence, if $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle 30^\circ = \mathbf{231\angle 30^\circ\ V}$$

$$\mathbf{V}_{bn} = \mathbf{231\angle 150^\circ\ V}$$

$$\mathbf{V}_{cn} = \mathbf{231\angle -90^\circ\ V}$$

Chapter 12, Solution 2.

Since phase c lags phase a by 120°, this is an **acb sequence**.

$$\mathbf{V}_{bn} = 120\angle(30° + 120°) = \mathbf{120\angle 150°}\ \boldsymbol{V}$$

Chapter 12, Solution 3.

Since $\mathbf{V}_{bn}$ leads $\mathbf{V}_{cn}$ by 120°, this is an **abc sequence**.

$\mathbf{V}_{an} = 440\angle(130° + 120°) = \mathbf{440\angle{-110°}}$ **V**.

Chapter 12, Solution 4.

Knowing the line-to-line voltages we can calculate the wye voltages and can let the value of V_a be a reference with a phase shift of zero degrees.

$V_L = 440 = \sqrt{3} V_p$ or $V_p = 440/1.7321 = 254$ V or $\mathbf{V}_{an} = 254\angle 0°$ V which

determines, using abc roataion, both $\mathbf{V}_{bn} = 254\angle -120°$ and $\mathbf{V}_{cn} = 254\angle 120°$.

$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y = 254/(40\angle 30°) = \mathbf{6.35\angle -30° \; A}$

$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{6.35\angle -150° \; A}$

$\mathbf{I}_c = \mathbf{I}_a\angle +120° = \mathbf{6.35\angle 90° \; A}$

Chapter 12, Solution 5.

$\mathbf{V}_{AB} = 1.7321x\mathbf{V}_{AN}\angle{+30^\circ} = 207.8\angle(32^\circ{+}30^\circ) = 207.8\angle 62^\circ$ V or

$$v_{AB} = \mathbf{207.8cos(\omega t+62^\circ)\ V}$$

which also leads to,

$$v_{BC} = \mathbf{207.8cos(\omega t-58^\circ)\ V}$$

and

$$v_{CA} = \mathbf{207.8cos(\omega t+182^\circ)\ V}$$

207.8cos(ωt+62°) V, 207.8cos(ωt–58°) V, 207.8cos(ωt+182°) V

Chapter 12, Solution 6.

Using Fig. 12.41, design a problem to help other students to better understand balanced wye-wye connected circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the Y-Y circuit of Fig. 12.41, find the line currents, the line\ voltages, and the load voltages.

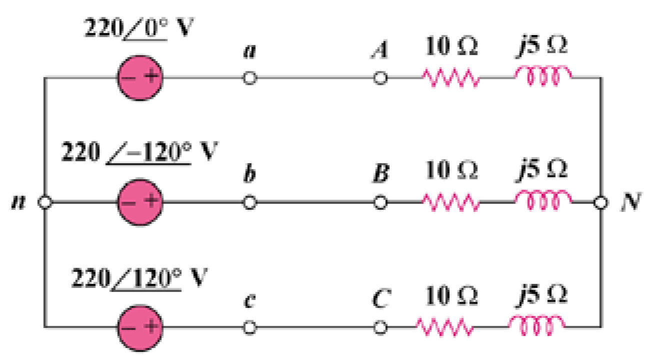


Figure 12.41

Solution

$$\mathbf{Z}_Y = 10 + j5 = 11.18\angle 26.56°$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{220\angle 0°}{11.18\angle 26.56°} = \mathbf{19.68\angle -26.56°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{19.68\angle -146.56°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{19.68\angle 93.44°\ A}$$

The line voltages are

$$\mathbf{V}_{ab} = 220\sqrt{3}\angle 30° = \mathbf{381\angle 30°\ V}$$

$$\mathbf{V}_{bc} = \mathbf{381\angle -90°\ V}$$

$$\mathbf{V}_{ca} = \mathbf{381\angle -210°\ V}$$

The load voltages are

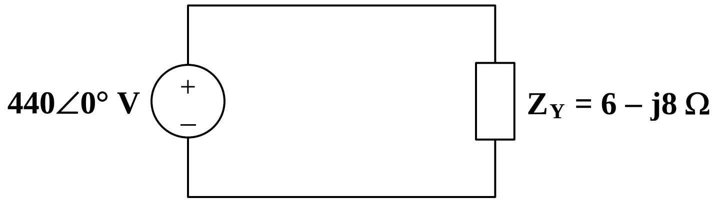
$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \mathbf{V}_{an} = \mathbf{220\angle 0°\ V}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{bn} = \mathbf{220\angle -120°\ V}$$

$$\mathbf{V}_{CN} = \mathbf{V}_{cn} = \mathbf{220\angle 120°\ V}$$

Chapter 12, Solution 7.

This is a balanced Y-Y system.



Using the per-phase circuit shown above,

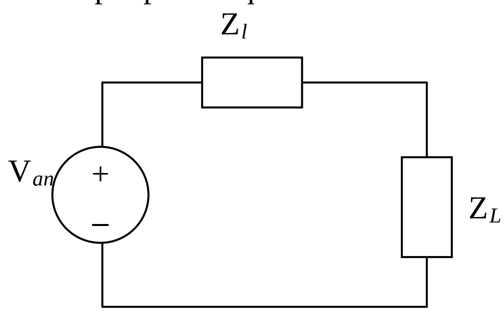
$$\mathbf{I}_a = \frac{440\angle 0°}{6 - j8} = \mathbf{44\angle 53.13°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120° = \mathbf{44\angle -66.87°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \mathbf{44\angle 173.13°\ A}$$

Chapter 12, Solution 8.

Consider the per phase equivalent circuit shown below.



5.396∠–35.1˚ A

$\mathbf{I_a} = \mathbf{V_{an}}/(\mathbf{Z}_l + \mathbf{Z}_L) = (100\angle 20°)/(10.6+j15.2) = (100\angle 20°)/(18.531\angle 55.11°)$

$= \mathbf{5.396\angle{-35.11°}}$ **amps**.

$\mathbf{I_b} = \mathbf{I_a}\ \angle{-120°} = \mathbf{5.396\angle{-155.11°}}$ **amps**.

$\mathbf{I_c} = \mathbf{I_a}\ \angle{+120°} = \mathbf{5.396\angle 84.89°}$ **amps**.

$\mathbf{V}_{La} = \mathbf{I_a Z}_L = (4.414–j3.103)(10+j14) = (5.396\angle{-35.11°})(17.205\angle 54.46°)$

$$= \mathbf{92.84\angle 19.35° \text{ volts}}.$$

$$\mathbf{V}_{\boldsymbol{L}\mathbf{b}} = \mathbf{V}_{\boldsymbol{L}\mathbf{a}} \angle{-120°} = \mathbf{94.84\angle{-100.65°} \text{ volts}}.$$

$$\mathbf{V}_{\boldsymbol{L}\mathbf{c}} = \mathbf{V}_{\boldsymbol{L}\mathbf{a}} \angle{+120°} = \mathbf{94.84\angle 139.35° \text{ volts}}.$$

Chapter 12, Solution 9.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}_Y} = \frac{120\angle 0°}{20 + j15} = \mathbf{4.8\angle -36.87°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120° = \mathbf{4.8\angle -156.87°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \mathbf{4.8\angle 83.13°\ A}$$

As a balanced system, $\mathbf{I}_n = \mathbf{0\ A}$

Chapter 12, Solution 10.

Since the neutral line is present, we can solve this problem on a per-phase basis.

For phase a,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_A + 2} = \frac{440\angle 0°}{27 - j10} = \frac{440}{28.79\angle -20.32°} = 15.283\angle 20.32°$$

For phase b,

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_B + 2} = \frac{440\angle \text{-}120°}{22} = 20\angle \text{-}120°$$

For phase c,

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_C + 2} = \frac{440\angle 120°}{12 + j5} = \frac{440\angle 120°}{13\angle 22.62°} = 33.85\angle 97.38°$$

The current in the neutral line is

$$\mathbf{I}_\mathrm{n} = \text{-}(\mathbf{I}_\mathrm{a} + \mathbf{I}_\mathrm{b} + \mathbf{I}_\mathrm{c}) \text{ or } \text{-}\mathbf{I}_\mathrm{n} = \mathbf{I}_\mathrm{a} + \mathbf{I}_\mathrm{b} + \mathbf{I}_\mathrm{c}$$

$$\text{-}\mathbf{I}_\mathrm{n} = (14.332 + j5.308) + (-10 - j17.321) + (-4.346 + j33.57)$$

$$\mathbf{I}_n = 0.014 - j21.56 = \mathbf{21.56\angle{-89.96}°A}$$

Chapter 12, Solution 11.

Given that $V_p = 240$ and that the system is balanced, $V_L = 1.7321V_p = 415.7$ V.

$I_p = V_L/|2–j3| = 415.7/3.606 = 115.29$ A and

$$I_L = 1.7321x115.29 = \mathbf{199.69\ A}.$$

Chapter 12, Solution 12.

Using Fig. 12.45, design a problem to help other students to better understand wye-delta connected circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve for the line currents in the **Y-Δ** circuit of Fig. 12.45. Take $\mathbf{Z}_\Delta = 60\angle 45°\Omega$.

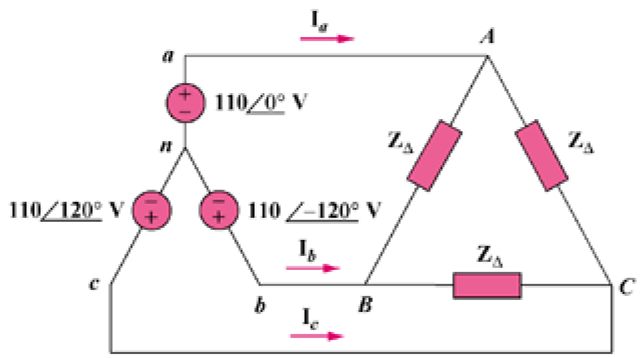
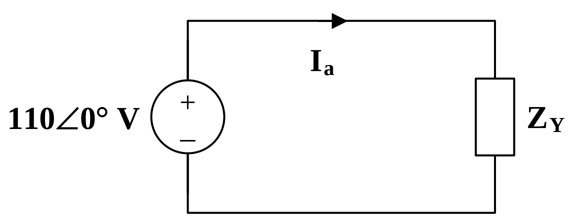


Figure 12.45

Solution

Convert the delta-load to a wye-load and apply per-phase analysis.



$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 45°\,\Omega$$

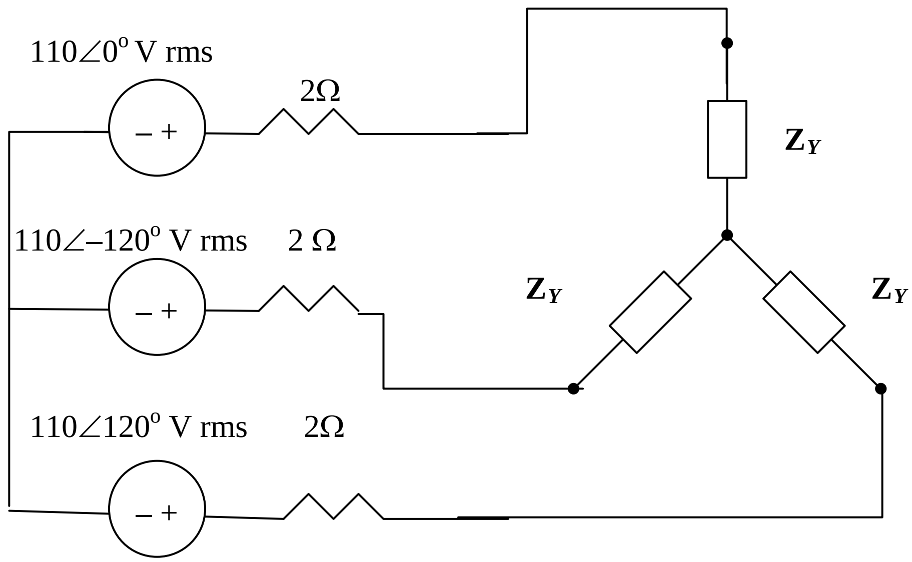
$$\mathbf{I}_a = \frac{110\angle 0°}{20\angle 45°} = \mathbf{5.5\angle -45°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120° = \mathbf{5.5\angle -165°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \mathbf{5.5\angle 75°\ A}$$

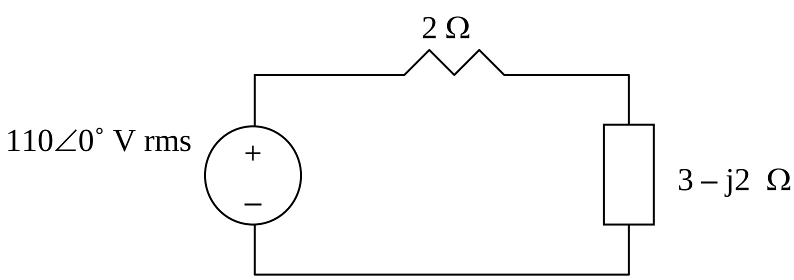
Chapter 12, Solution 13.

Convert the delta load to wye as shown below.



$$Z_Y = \frac{1}{3} Z_\Delta = 3 - j2\ \Omega$$

We consider the single phase equivalent shown below.



$\mathbf{I_a}$ = 110/(2 + 3 – j2) = 20.43∠21.8° A

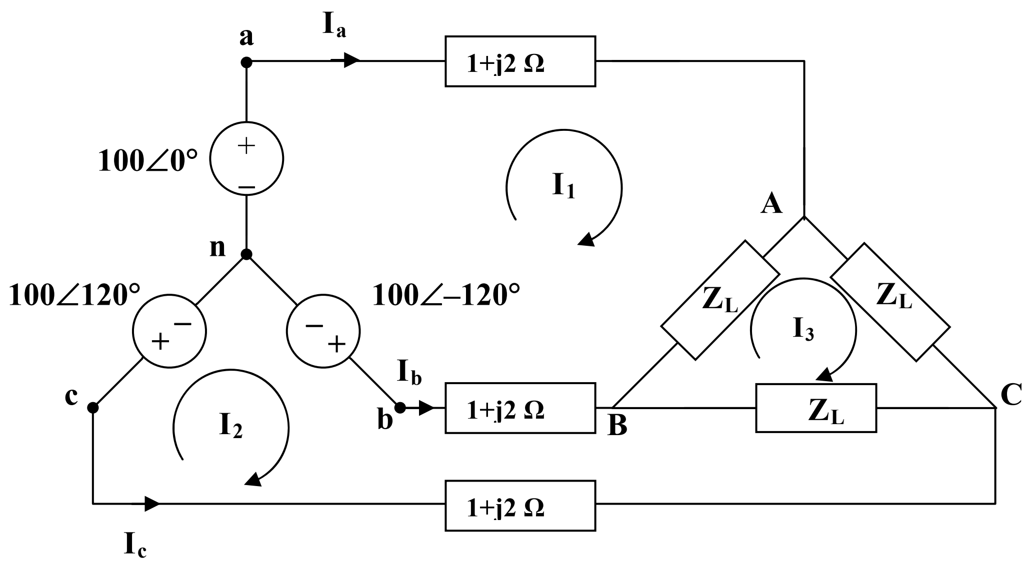
$I_L = |\mathbf{I_a}|$ = **20.43 A**

S = $3|\mathbf{I_a}|^2 Z_Y = 3(20.43)^2(3–j2)$ = 4514∠–33.96˚ = 3744 – j2522

P = Re(S) = **3.744 kW**.

Chapter 12, Solution 14.

We apply mesh analysis with $Z_L = (12+j12)\ \Omega$.



For mesh 1,

$$-100+100\angle-120^o + I_1(14+j16)-(1+j2)I_2-(12+j12)I_3 = 0 \text{ or}$$

$$(14+j16)I_1-(1+j2)I_2-(12+j12)I_3 = 100+50-j86.6 = 150+j86.6 \qquad (1)$$

For mesh 2,

$$100\angle120^o - 100\angle-120^o - I_1(1+j2)-(12+j12)I_3+(14+j16)I_2 = 0 \text{ or}$$

$$-(1+j2)I_1+(14+j16)I_2-(12+j12)I_3 = -50-j86.6+50-j86.6 = -j173.2 \qquad (2)$$

For mesh 3,

$$-(12+j12)I_1-(12+j12)I_2+(36+j36)I_3 = 0 \text{ or } \mathbf{I_3} = \mathbf{I_1} + \mathbf{I_2} \qquad (3)$$

Solving for $\mathbf{I_1}$ and $\mathbf{I_2}$ using (1) to (3) gives

$\mathbf{I_1} = 12.804\angle-50.19°$ A = (8.198 – j9.836) A and
$\mathbf{I_2} = 12.804\angle-110.19°$ A = (–4.419 – j12.018) A

$$\mathbf{I_a} = \mathbf{I_1} = \mathbf{12.804\angle-50.19°\ A}$$

$$\mathbf{I_b} = \mathbf{I_2} - \mathbf{I_1} = \mathbf{12.804\angle-170.19°\ A}$$

$$\mathbf{I_c} = -\mathbf{I_2} = \mathbf{12.804\angle69.81°\ A}$$

As a check we can convert the delta into a wye circuit. Thus,

$\mathbf{Z_Y}$ = (12+j12)/3 = 4+j4 and $\mathbf{I_a}$ = 100/(1+j2+4+j4) = 100/(5+j6)

= 100/(7.8102∠50.19°) =

12.804 ∠–50.19° A.

So, the answer does check.

Chapter 12, Solution 15.

Convert the delta load, $\mathbf{Z}_\Delta$, to its equivalent wye load.

$$\mathbf{Z}_{Ye} = \frac{\mathbf{Z}_\Delta}{3} = 8 - j10$$

$$\mathbf{Z}_p = \mathbf{Z}_Y \parallel \mathbf{Z}_{Ye} = \frac{(12 + j5)(8 - j10)}{20 - j5} = 8.076\angle -14.68°$$

$$\mathbf{Z}_p = 7.812 - j2.047$$

$$\mathbf{Z}_T = \mathbf{Z}_p + \mathbf{Z}_L = 8.812 - j1.047$$

$$\mathbf{Z}_T = 8.874\angle -6.78°$$

We now use the per-phase equivalent circuit.

$$\mathbf{I}_a = \frac{V_p}{\mathbf{Z}_p + \mathbf{Z}_L}, \qquad \text{where} \quad V_p = \frac{210}{\sqrt{3}}$$

$$\mathbf{I}_a = \frac{210}{\sqrt{3}\,(8.874\angle -6.78°)} = 13.66\angle 6.78°$$

$$\mathbf{I}_L = \left|\mathbf{I}_a\right| = \mathbf{13.66\ A}$$

Chapter 12, Solution 16.

(a) $\mathbf{I}_{CA} = -\mathbf{I}_{\mathrm{AC}} = 5\angle(-30° + 180°) = 5\angle 150°$

This implies that

$$\mathbf{I}_{AB} = 5\angle 30°$$
$$\mathbf{I}_{BC} = 5\angle -90°$$

$\mathbf{I}_{\mathrm{a}} = \mathbf{I}_{\mathrm{AB}}\sqrt{3}\angle -30° = \mathbf{8.66\angle 0°\ A}$

$\mathbf{I}_{\mathrm{b}} = \mathbf{8.66\angle{-120}°\ A}$

$\mathbf{I}_{\mathrm{c}} = \mathbf{8.66\angle 120°\ A}$

(b) $\mathbf{Z}_{\Delta} = \dfrac{\mathbf{V}_{AB}}{\mathbf{I}_{AB}} = \dfrac{110\angle 0°}{5\angle 30°} = \mathbf{22\angle{-30}°\ \Omega}.$

Chapter 12, Solution 17.

$\mathbf{I}_a = 1.7321\text{x}\mathbf{I}_{AB}\angle{-30^\circ}$ or

$\mathbf{I}_{AB} = \mathbf{I}_a/(1.7321\angle{-30^\circ}) = 2.887\angle(-25^\circ+30^\circ) = \mathbf{2.887\angle 5^\circ\ A}$

$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle{-120^\circ} = \mathbf{2.887\angle{-115^\circ}\ A}$

$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle{+120^\circ} = \mathbf{2.887\angle 125^\circ\ A}$

$2.887\angle 5^\circ$ A, $2.887\angle{-115^\circ}$ A, $2.887\angle 125^\circ$ A

Chapter 12, Solution 18.

$$\mathbf{V}_{AB} = \mathbf{V}_{an}\sqrt{3}\angle 30° = (220\angle 60°)(\sqrt{3}\angle 30°) = 381.1\angle 90°$$

$$\mathbf{Z}_{\Delta} = 12 + j9 = 15\angle 36.87°$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{381.1\angle 90°}{15\angle 36.87°} = \mathbf{25.4\angle 53.13^\circ\ A}$$

$$\mathbf{I}_{\mathrm{BC}} = \mathbf{I}_{\mathrm{AB}}\angle -120° = \mathbf{25.4\angle{-66.87}^\circ\ A}$$

$$\mathbf{I}_{\mathrm{CA}} = \mathbf{I}_{\mathrm{AB}}\angle 120° = \mathbf{25.4\angle 173.13^\circ\ A}$$

Chapter 12, Solution 19.

$$\mathbf{Z}_\Delta = 30 + j10 = 31.62\angle 18.43°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{173\angle 0°}{31.62\angle 18.43°} = \mathbf{5.47\angle -18.43°\ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \mathbf{5.47\angle -138.43°\ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{5.47\angle 101.57°\ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle -30°$$

$$\mathbf{I}_a = 5.47\sqrt{3}\angle -48.43° = \mathbf{9.474\angle -48.43°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{9.474\angle -168.43°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{9.474\angle 71.57°\ A}$$

Chapter 12, Solution 20.

Using Fig. 12.51, design a problem to help other students to better understand balanced delta-delta connected circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Refer to the Δ-Δ circuit in Fig. 12.51. Find the line and phase currents. Assume that the load impedance is 12 + j9Ω per phase.

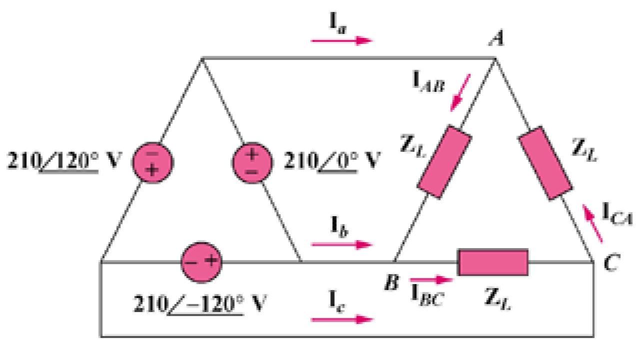


Figure 12.51

Solution

$$\mathbf{Z}_\Delta = 12 + j9 = 15\angle 36.87°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{210\angle 0°}{15\angle 36.87°} = \mathbf{14\angle -36.87°\ A}$$
$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \mathbf{14\angle -156.87°\ A}$$
$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{14\angle 83.13°\ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \mathbf{24.25\angle -66.87°\ A}$$
$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{24.25\angle -186.87°\ A}$$
$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{24.25\angle 53.13°\ A}$$

Chapter 12, Solution 21.

(a) $$\mathbf{I_{AC}} = \frac{-230\angle 120^\circ}{10 + j8} = \frac{-230\angle 120^\circ}{12.806\angle 38.66^\circ} = \underline{17.96\angle -98.66^\circ\ A}$$

$$\mathbf{I_{AC} = 17.96\angle{-98.66^\circ}\ A}$$

(b)
$$I_{bB} = I_{BC} + I_{BA} = I_{BC} - I_{AB} = \frac{230\angle -120}{10 + j8} - \frac{230\angle 0^\circ}{10 + j8}$$
$$= 17.96\angle -158.66^\circ - 17.96\angle -38.66^\circ$$
$$= -16.729 - j6.536 - 14.024 + j11.220 = -30.75 + j4.684$$

$$\mathbf{I_{bB} = 31.1\angle 171.34^\circ\ A}.$$

Chapter 12, Solution 22.

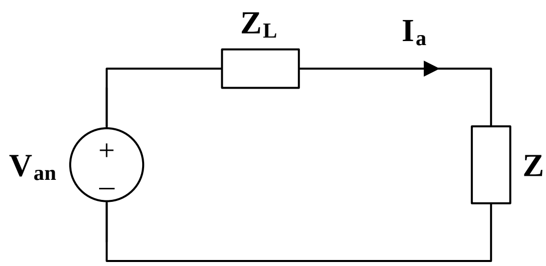
Convert the Δ-connected source to a Y-connected source.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}} \angle -30° = \frac{440}{\sqrt{3}} \angle -30° = 254 \angle -30°$$

Convert the Δ-connected load to a Y-connected load.

$$\mathbf{Z} = \mathbf{Z}_{\text{Y}} \parallel \frac{\mathbf{Z}_{\Delta}}{3} = (4 + \text{j}6) \parallel (4 - \text{j}5) = \frac{(4 + \text{j}6)(4 - \text{j}5)}{8 + \text{j}}$$

$$\mathbf{Z} = 5.723 - \text{j}0.2153$$



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}} = \frac{254 \angle -30°}{7.723 - j0.2153} = \mathbf{32.88 \angle –28.4° \text{ A}}$$

$$\mathbf{I}_{\text{b}} = \mathbf{I}_{\text{a}} \angle -120° = \mathbf{32.88 \angle –148.4° \text{ A}}$$

$$\mathbf{I}_{\text{c}} = \mathbf{I}_{\text{a}} \angle 120° = \mathbf{32.88 \angle 91.6° \text{ A}}$$

Chapter 12, Solution 23.

(a) $$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{202}{25\angle 60^o}$$

$$I_a = I_{AB}\sqrt{3}\angle -30^o = \frac{202\sqrt{3}\angle -30^o}{25\angle 60^o} = 13.995\angle -90^o$$

$$I_L = |I_a| = 13.995 \text{ A}$$

(b)

$$P = P_1 + P_2 = \sqrt{3}V_L I_L \cos\theta = \sqrt{3}(202)\left(\frac{202\sqrt{3}}{25}\right)\cos 60^o$$

$$= \mathbf{2.448 \text{ kW}}$$

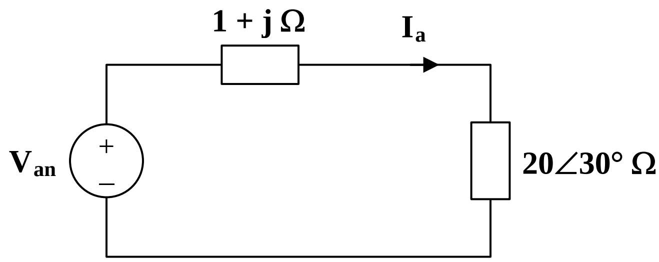
Chapter 12, Solution 24.

Convert both the source and the load to their wye equivalents.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 30° = 17.32 + j10$$

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30° = 240.2\angle 0°$$

We now use per-phase analysis.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{(1+j)+(17.32+j10)} = \frac{240.2}{21.37\angle 31°} = \mathbf{11.24\angle -31°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{11.24\angle -151°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{11.24\angle 89°\ A}$$

But $\quad \mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$

$$\mathbf{I}_{AB} = \frac{11.24\angle -31°}{\sqrt{3}\angle -30°} = \mathbf{6.489\angle -1°\ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \mathbf{6.489\angle -121°\ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{6.489\angle 119°\ A}$$

Chapter 12, Solution 25.

Convert the delta-connected source to an equivalent wye-connected source and consider the single-phase equivalent.

$$\mathbf{I}_a = \frac{440\angle(10^\circ - 30^\circ)}{\sqrt{3}\,\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = 3 + j2 + 10 - j8 = 13 - j6 = 14.318\angle -24.78^\circ$

$$\mathbf{I}_a = \frac{440\angle -20^\circ}{\sqrt{3}(14.318\angle -24.78^\circ)}$$ \= **17.742∠4.78° amps**.

$\mathbf{I_b} = \mathbf{I_a}\angle\mathbf{-120^\circ} = \mathbf{17.742\angle -115.22^\circ}$ **amps**.

$\mathbf{I_c} = \mathbf{I_a}\angle\mathbf{+120^\circ} = \mathbf{17.742\angle 124.78^\circ}$ **amps**.

Chapter 12, Solution 26.

Using Fig. 12.55, design a problem to help other students to better understand balanced delta connected sources delivering power to balanced wye connected loads.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the balanced circuit in Fig. 12.55, $\mathbf{V}_{ab} = 125\angle 0°$ V. Find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

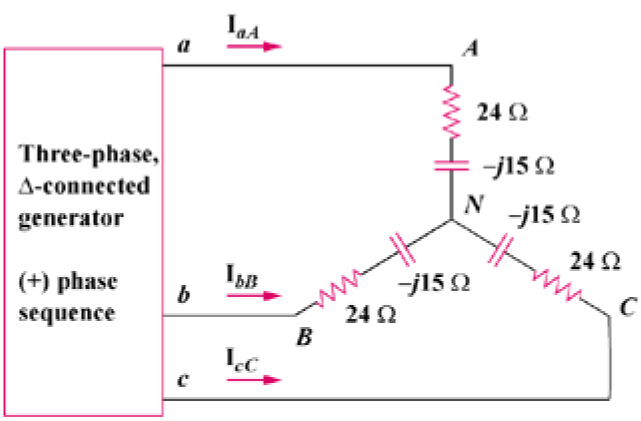


Figure 12.55

Solution

Transform the source to its wye equivalent.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30° = 72.17\angle -30°$$

Now, use the per-phase equivalent circuit.

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}}, \qquad \mathbf{Z} = 24 - j15 = 28.3\angle -32°$$

$$\mathbf{I}_{aA} = \frac{72.17\angle -30°}{28.3\angle -32°} = \mathbf{2.55\angle 2°\ A}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{aA}\angle -120° = \mathbf{2.55\angle -118°\ A}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{aA}\angle 120° = \mathbf{2.55\angle 122°\ A}$$

Chapter 12, Solution 27.

Since Z_L and Z_ℓ are in series, we can lump them together so that

$$Z_Y = 2 + j + 6 + j4 = 8 + j5$$

$$I_a = \frac{\frac{V_P}{\sqrt{3}} < -30^o}{Z_Y} = \frac{208 < -30^o}{\sqrt{3}(8 + j5)}$$

$$V_L = (6 + j4)I_a = \frac{208(0.866 - j0.5)(6 + j4)}{\sqrt{3}(8 + j5)} = 80.81 - j43.54$$

$|V_L|$ = **91.79 V**

Chapter 12, Solution 28.

$$V_L = |V_{ab}| = 440 = \sqrt{3}V_P \quad \text{or} \quad V_P = 440/1.7321 = 254$$

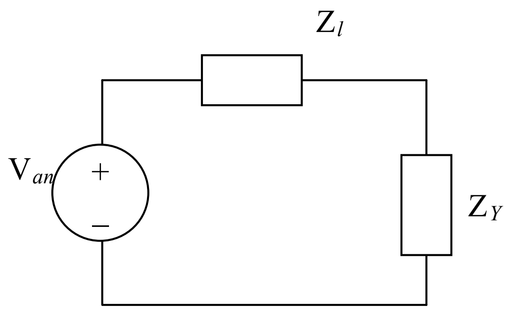
For reference, let V_{AN} = $\mathbf{254\angle 0^\circ}$ **V** which leads to
V_{BN} = $\mathbf{254\angle{-120^\circ}}$ **V** and V_{CN} = $\mathbf{254\angle 120^\circ}$ **V**.

The line currents are found as follows,

$I_a = V_{AN}/Z_Y = 254/25\angle 30^\circ$ = $\mathbf{10.16\angle{-30^\circ}}$ **A**.
This leads to, I_b = $\mathbf{10.16\angle{-150^\circ}}$ **A** and I_c = $\mathbf{10.16\angle 90^\circ}$ **A**.

Chapter 12, Solution 29.

We can replace the delta load with a wye load, $Z_Y = Z_\Delta/3 = 17+j15\Omega$.
The per-phase equivalent circuit is shown below.



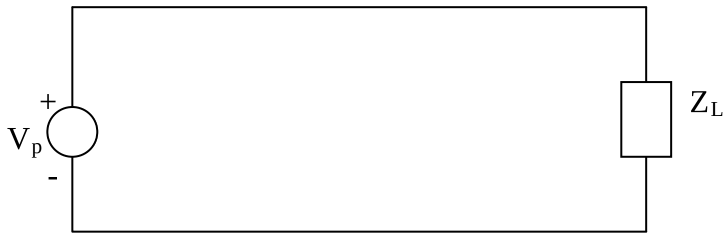
$\mathbf{I_a} = \mathbf{V_{an}}/|\mathbf{Z}_Y + \mathbf{Z}_l| = 240/|17+j15+0.4+j1.2| = 240/|17.4+j16.2| = 240/23.77 = 10.095$

$\mathbf{S} = 3[(I_a)^2(17+j15)] = 3x101.91(17+j15)$

$= \mathbf{[5.197+j4.586]\ kVA}$.

Chapter 12, Solution 30.

Since this a balanced system, we can replace it by a per-phase equivalent, as shown below.



$$\overline{S} = 3\overline{S}_p = \frac{3V^2{}_p}{Z^*{}_p}, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$\overline{S} = \frac{V^2{}_L}{Z^*{}_p} = \frac{(208)^2}{30\angle -45^o} = 1.4421\angle 45^o \text{ kVA}$$

$$\boldsymbol{P = S\cos\theta = \underline{1.02\,kW}}$$

Chapter 12, Solution 31.

(a)

$$P_p = 6{,}000, \qquad \cos\theta = 0.8, \qquad S_p = \frac{P_P}{\cos\theta} = 6/0.8 = 7.5\,\text{kVA}$$

$$Q_p = S_P \sin\theta = 4.5\,\text{kVAR}$$

$$\overline{S} = 3\overline{S}_p = 3(6 + j4.5) = 18 + j13.5\,\text{kVA}$$

For delta-connected load, $V_p = V_L = 240$ (rms). But

$$\overline{S} = \frac{3V^2{}_p}{Z^*{}_p} \quad\longrightarrow\quad Z^*{}_p = \frac{3V^2{}_p}{S} = \frac{3(240)^2}{(18 + j13.5)x10^3}, \qquad \underline{Z_P = [6.144 + j4.608]\Omega}$$

(b) $$P_p = \sqrt{3}V_L I_L \cos\theta \quad\longrightarrow\quad I_L = \frac{6000}{\sqrt{3}x240x0.8} = \underline{18.04\text{ A}}$$

(c) We find C to bring the power factor to unity

$$Q_c = Q_p = 4.5\,\text{kVA} \quad\longrightarrow\quad C = \frac{Q_c}{\omega V^2{}_{rms}} = \frac{4500}{2\pi x 60 x 240^2} = \underline{207.2\ \mu\text{F}}$$

Chapter 12, Solution 32.

Design a problem to help other students to better understand power in a balanced three-phase system.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A balanced wye load is connected to a 60-Hz three-phase source with $V_{ab} = 240\angle 0^\circ$V. The load has lagging pf =0.5 and each phase draws 5 kW. (a) Determine the load impedance Z_Y. (b) Find I_a, I_b, and I_c.

Solution

(a) $|V_{ab}| = \sqrt{3}V_p = 240 \quad \longrightarrow \quad V_p = \dfrac{240}{\sqrt{3}} = 138.56$

$$V_{an} = V_p < -30^o$$

$$pf = 0.5 = \cos\theta \quad \longrightarrow \quad \theta = 60^o$$

$$P = S\cos\theta \quad \longrightarrow \quad S = \frac{P}{\cos\theta} = \frac{5}{0.5} = 10 \text{ kVA}$$

$$Q = S\sin\theta = 10\sin 60 = 8.66$$

$$S_p = 5 + j8.66 \text{ kVA}$$

But

$$S_P = \frac{V_p^2}{Z_p^*} \quad \longrightarrow \quad Z_p^* = \frac{V_p^2}{S_p} = \frac{138.56^2}{(5 + j8.66)x10^3} = 0.96 - j1.663$$

$\mathbf{Z_p = [0.96 + j1.663]\ \Omega}$

(b) $I_a = \dfrac{V_{an}}{Z_Y} = \dfrac{138.56 < -30^o}{0.96 + j1.6627} = \underline{72.17 < -90^o \text{ A}}$ = **72.17∠–90° A**

$I_b = I_a < -120^o = \underline{72.17 < -210^o \text{ A}}$ = **72.17∠150° A**

$I_c = I_a < +120^o = \underline{72.17 < 30^o \text{ A}}$ = **72.17∠30° A**

Chapter 12, Solution 33.

$$\mathbf{S} = \sqrt{3}\,V_L I_L \angle\theta$$

$$S = \left|\mathbf{S}\right| = \sqrt{3}\,V_L I_L$$

For a Y-connected load,

$$I_L = I_p, \qquad V_L = \sqrt{3}\,V_p$$

$$S = 3 V_p I_p$$

$$I_L = I_p = \frac{S}{3V_p} = \frac{4800}{(3)(208)} = \mathbf{7.69\ A}$$

$$V_L = \sqrt{3}\,V_p = \sqrt{3} \times 208 = \mathbf{360.3\ V}$$

Chapter 12, Solution 34.

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}}$$

$$\mathbf{I}_a = \frac{V_p}{\mathbf{Z}_Y} = \frac{220}{\sqrt{3}(10 - j16)} = \frac{127.02}{18.868\angle -58^\circ} = 6.732\angle 58^\circ$$

$$I_L = I_p = \mathbf{6.732A}$$

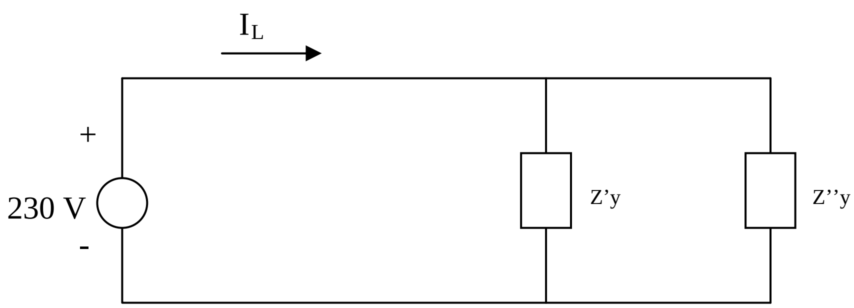
$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta = \sqrt{3} \times 220 \times 6.732\angle -58^\circ = 2565\angle -58^\circ$$

S = [1.3592–j2.175] kVA

Chapter 12, Solution 35.

(a) This is a balanced three-phase system and we can use per phase equivalent circuit. The delta-connected load is converted to its wye-connected equivalent

$$Z''_y = \frac{1}{3} Z_\Delta = (60 + j30)/3 = 20 + j10$$



$$Z_y = Z'_y \,//\, Z''_y = (40 + j10)//(20 + j10) = 13.5 + j5.5$$

$$\mathbf{I_L = \frac{230}{13.5 + j5.5} = [14.61 - j5.953]\ A}$$

(b) $\mathbf{S = 3V_s I^*_L = [10.081 + j4.108]\ kVA}$

(c) pf = P/S = **0.9261**

Chapter 12, Solution 36.

(a) S = 1 [0.75 + sin($\cos^{-1}$0.75)] = **0.75 + j0.6614 MVA**

(b) $\overline{S} = 3V_p I^*{}_p \longrightarrow I^*{}_p = \dfrac{S}{3V_p} = \dfrac{(0.75 + j0.6614)x10^6}{3x4200} = 59.52 + j52.49$

$$\boldsymbol{P_L = |I_p|^2 R_l = (79.36)^2 (4) = \underline{25.19\,\text{kW}}}$$

(c) $\boldsymbol{V_s = V_L + I_p(4 + j) = 4.4381 - j0.21\,\text{kV} = \underline{4.443\angle\text{-}2.709^o\ \text{kV}}}$

Chapter 12, Solution 37.

$$S = \frac{P}{pf} = \frac{12}{0.6} = 20$$

$$\mathbf{S} = S\angle\theta = 20\angle\theta = 12 - j16 \text{ kVA}$$

But $\quad \mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta$

$$I_L = \frac{20 \times 10^3}{\sqrt{3} \times 208} = \mathbf{55.51\ A}$$

$$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$$

For a Y-connected load, $I_L = I_p$.

$$\mathbf{Z}_p = \frac{\mathbf{S}}{3\left|I_L\right|^2} = \frac{(12 - j16) \times 10^3}{(3)(55.51)^2}$$

$$\mathbf{Z}_p = \mathbf{[1.298 - \mathit{j}1.731]\ \Omega}$$

Chapter 12, Solution 38.

As a balanced three-phase system, we can use the per-phase equivalent shown below.

$$\mathbf{I}_a = \frac{110\angle 0^\circ}{(1+j2)+(9+j12)} = \frac{110\angle 0^\circ}{10+j14}$$

$$\mathbf{S}_p = |\mathbf{I}_a|^2 \mathbf{Z}_Y = \frac{(110)^2}{(10^2+14^2)} \cdot (9+j12)$$

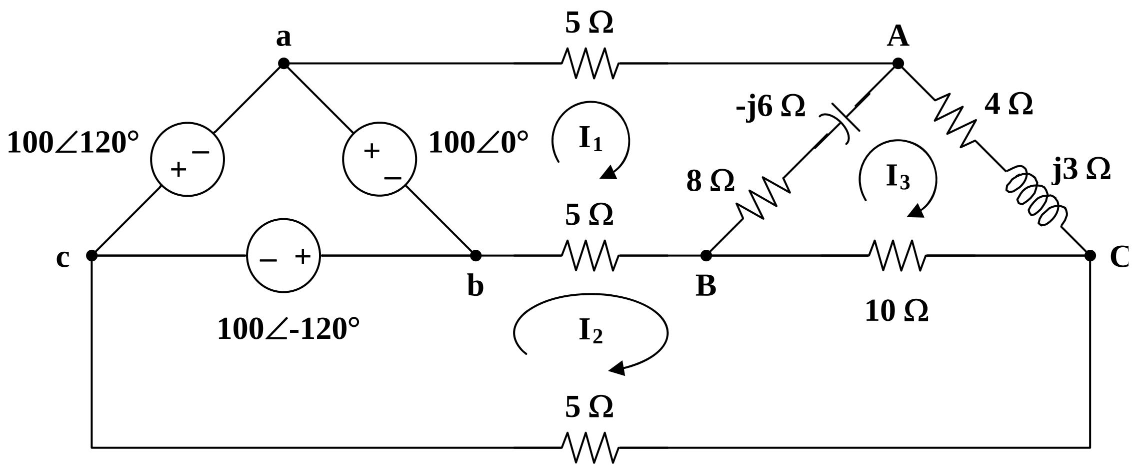
The complex power is

$$\mathbf{S} = 3\mathbf{S}_p = 3\frac{(110)^2}{296} \cdot (9+j12)$$

$$\mathbf{S} = \mathbf{(1.1037+j1.4716)\ kVA}$$

Chapter 12, Solution 39.

Consider the system shown below.



For mesh 1,

$$100 = (18 - j6)\mathbf{I}_1 - 5\mathbf{I}_2 - (8 - j6)\mathbf{I}_3 \qquad (1)$$

For mesh 2,

$$100\angle\text{-}120° = 20\mathbf{I}_2 - 5\mathbf{I}_1 - 10\mathbf{I}_3$$

$$20\angle\text{-}120° = \text{-}\mathbf{I}_1 + 4\mathbf{I}_2 - 2\mathbf{I}_3 \qquad (2)$$

For mesh 3,

$$0 = \text{-}(8 - j6)\mathbf{I}_1 - 10\mathbf{I}_2 + (22 - j3)\mathbf{I}_3 \qquad (3)$$

To eliminate $\mathbf{I}_2$, start by multiplying (1) by 2,

$$200 = (36 - j12)\mathbf{I}_1 - 10\mathbf{I}_2 - (16 - j12)\mathbf{I}_3 \qquad (4)$$

Subtracting (3) from (4),

$$200 = (44 - j18)\mathbf{I}_1 - (38 - j15)\mathbf{I}_3 \qquad (5)$$

Multiplying (2) by $5/4$,

$$25\angle\text{-}120° = \text{-}1.25\mathbf{I}_1 + 5\mathbf{I}_2 - 2.5\mathbf{I}_3 \qquad (6)$$

Adding (1) and (6),

$$87.5 - j21.65 = (16.75 - j6)\mathbf{I}_1 - (10.5 - j6)\mathbf{I}_3 \qquad (7)$$

In matrix form, (5) and (7) become

$$\begin{bmatrix} 200 \\ 87.5 - j12.65 \end{bmatrix} = \begin{bmatrix} 44 - j18 & -38 + j15 \\ 16.75 - j6 & -10.5 + j6 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix}$$

$$\Delta = 192.5 - j26.25, \qquad \Delta_1 = 900.25 - j935.2, \qquad \Delta_3 = 110.3 - j1327.6$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{1298.1\angle -46.09°}{194.28\angle -7.76°} = 6.682\angle -38.33° = 5.242 - j4.144$$

$$\mathbf{I}_3 = \frac{\Delta_3}{\Delta} = \frac{1332.2\angle -85.25°}{194.28\angle -7.76°} = 6.857\angle -77.49° = 1.485 - j6.694$$

We obtain $\mathbf{I}_2$ from (6),

$$\mathbf{I}_2 = 5\angle -120° + \frac{1}{4}\mathbf{I}_1 + \frac{1}{2}\mathbf{I}_3$$

$$\mathbf{I}_2 = (-2.5 - j4.33) + (1.3104 - j1.0359) + (0.7425 - j3.347)$$

$$\mathbf{I}_2 = -0.4471 - j8.713$$

The average power absorbed by the 8-Ω resistor is

$$P_1 = \left| \mathbf{I}_1 - \mathbf{I}_3 \right|^2 (8) = \left| 3.756 + j2.551 \right|^2 (8) = 164.89 \text{ W}$$

The average power absorbed by the 4-Ω resistor is

$$P_2 = \left| \mathbf{I}_3 \right|^2 (4) = (6.8571)^2 (4) = 188.1 \text{ W}$$

The average power absorbed by the 10-Ω resistor is

$$P_3 = \left| \mathbf{I}_2 - \mathbf{I}_3 \right|^2 (10) = \left| -1.9321 - j2.019 \right|^2 (10) = 78.12 \text{ W}$$

Thus, the total real power absorbed by the load is

$$P = P_1 + P_2 + P_3 = \mathbf{431.1\ W}$$

Chapter 12, Solution 40.

Transform the delta-connected load to its wye equivalent.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 7 + j8$$

Using the per-phase equivalent circuit above,

$$\mathbf{I}_a = \frac{100\angle 0°}{(1 + j0.5) + (7 + j8)} = 8.567\angle -46.75°$$

For a wye-connected load,

$$I_p = I_a = |\mathbf{I}_a| = 8.567$$

$$\mathbf{S} = 3|\mathbf{I}_p|^2 \mathbf{Z}_p = (3)(8.567)^2 (7 + j8)$$

$$P = \text{Re}(\mathbf{S}) = (3)(8.567)^2 (7) = \mathbf{1.541\ kW}$$

Chapter 12, Solution 41.

$$S = \frac{P}{pf} = \frac{5\text{ kW}}{0.8} = 6.25\text{ kVA}$$

But $\quad S = \sqrt{3}\,V_L I_L$

$$I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{6.25 \times 10^3}{\sqrt{3} \times 400} = \mathbf{9.021\ A}$$

Chapter 12, Solution 42.

The load determines the power factor.

$$\tan\theta = \frac{40}{30} = 1.333 \quad \longrightarrow \quad \theta = -53.13°$$

$$\text{pf} = \cos\theta = 0.6 \quad \text{(leading)}$$

$$\mathbf{S} = 7.2 - j\left(\frac{7.2}{0.6}\right)(0.8) = 7.2 - j9.6 \text{ kVA}$$

But $\quad \mathbf{S} = 3\left|\,\mathbf{I}_p\,\right|^2 \mathbf{Z}_p$

$$\left|\,\mathbf{I}_p\,\right|^2 = \frac{\mathbf{S}}{3\mathbf{Z}_p} = \frac{(7.2 - j9.6)\times 10^3}{(3)(30 - j40)} = 80$$

$$I_p = 8.944 \text{ A}$$

$$I_L = I_p = \mathbf{8.944\ A}$$

$$V_L = \frac{S}{\sqrt{3}\, I_L} = \frac{12\times 10^3}{\sqrt{3}\,(8.944)} = \mathbf{774.6\ V}$$

Chapter 12, Solution 43.

$$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p, \qquad I_p = I_L \text{ for Y-connected loads}$$

$$\mathbf{S} = (3)(13.66)^2(7.812 - j2.047)$$

$$\mathbf{S = [4.373 - \mathit{j}1.145]\ \mathit{kVA}}$$

Chapter 12, Solution 44.

For a Δ-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\, I_p$$

$$S = \sqrt{3}\, V_L I_L$$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{\sqrt{(12^2 + 5^2)} \times 10^3}{\sqrt{3}\,(240)} = 31.273$$

At the source,

$$\mathbf{V_L'} = \mathbf{V_L} + \mathbf{I_L}\,\mathbf{Z_l} + \mathbf{I_L Z_l}$$

$$\mathbf{V_L'} = 240\angle 0° + 2(31.273)(1 + j3) = 240+62.546+j187.638$$

$$\mathbf{V_L'} = 302.546+j187.638 = 356\angle 31.81°$$

$$\left|\mathbf{V_L'}\right| = \mathbf{356\ V}$$

Also, at the source,

S' = $3(31.273)^2(1+j3) + (12,000+j5,000) = 2,934+12,000+j(8,802+5,000)$

= $14,934+j13,802 = 20,335\angle 42.744°$ thus, $\theta = 42.744°$.

pf = cos(42.744°) = **0.7344**

Checking, $V_Y = 240/1.73205 = 138.564$, $\mathbf{S} = 3(138.564)^2/(Z_Y)^* = 12,000+l5,000$, and Z_Y

$= 57,600/(12,000–j5,000) = 57.6/(13\angle –22.62°) = 4.4308\angle 22.62° = 4.09+j1.70416$. The

total load seen by the source is $1+j3+4.09+j1.70416 = 5.09+j4.70416 = 6.9309\angle 42.74°$

per phase. This leads to $\theta = \text{Tan}^{-1}(4.70416/5.09) = \text{Tan}^{-1}(0.9242) = 42.744°$. Clearly, the answer checks. $I_l = 138.564/4.4308 = 31.273$ A. Again the answer checks. Finally,

$3(31.273)^2(5.09+j4.70416) = 2{,}934(6.9309\angle 42.74°) = 20{,}335\angle 42.74°$, the same as we calculated above.

Chapter 12, Solution 45.

$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta$$

$$I_L = \frac{|\mathbf{S}|\angle -\theta}{\sqrt{3}\, V_L}, \qquad |\mathbf{S}| = \frac{P}{pf} = \frac{450\times10^3}{0.708} = 635.6 \text{ kVA}$$

$$\mathbf{I}_L = \frac{(635.6)\angle -\theta}{\sqrt{3}\times 440} = 834\angle -45° \text{ A}$$

At the source,

$$\mathbf{V}_L = 440\angle 0° + \mathbf{I}_L\,(0.5 + j2)$$
$$\mathbf{V}_L = 440 + (834\angle -45°)(2.062\angle 76°)$$
$$\mathbf{V}_L = 440 + 1719.7\angle 31°$$
$$\mathbf{V}_L = 1914.1 + j885.7$$

$$\mathbf{V}_L = \mathbf{2.109\angle 24.83°\ V}$$

Chapter 12, Solution 46.

For the wye-connected load,

$$I_L = I_p, \qquad V_L = \sqrt{3}\,V_p \qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p\mathbf{I}_p^* = \frac{3|\mathbf{V}_p|^2}{\mathbf{Z}^*} = \frac{3|\mathbf{V}_L/\sqrt{3}|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{|\mathbf{V}_L|^2}{\mathbf{Z}^*} = \frac{(110)^2}{100} = 121 \text{ W}$$

For the delta-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\,I_p, \qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p\mathbf{I}_p^* = \frac{3|\mathbf{V}_p|^2}{\mathbf{Z}^*} = \frac{3|\mathbf{V}_L|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{(3)(110)^2}{100} = 363 \text{ W}$$

This shows that the **delta-connected load** will absorb three times more average power than the wye-connected load using the same elements.. This is also evident from $\mathbf{Z}_Y = \dfrac{\mathbf{Z}_\Delta}{3}$.

Chapter 12, Solution 47.

$$\text{pf} = 0.8 \quad \text{(lagging)} \quad \longrightarrow \quad \theta = \cos^{-1}(0.8) = 36.87°$$
$$\mathbf{S}_1 = 250\angle 36.87° = 200 + j150 \text{ kVA}$$

$$\text{pf} = 0.95 \quad \text{(leading)} \quad \longrightarrow \quad \theta = \cos^{-1}(0.95) = -18.19°$$
$$\mathbf{S}_2 = 300\angle -18.19° = 285 - j93.65 \text{ kVA}$$

$$\text{pf} = 1.0 \quad \longrightarrow \quad \theta = \cos^{-1}(1) = 0°$$
$$\mathbf{S}_3 = 450 \text{ kVA}$$

$$\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 935 + j56.35 = 936.7\angle 3.45° \text{ kVA}$$

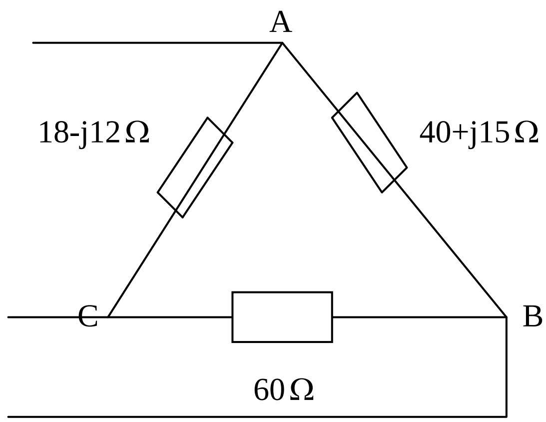
$$\left|\mathbf{S}_T\right| = \sqrt{3}\, V_L I_L$$

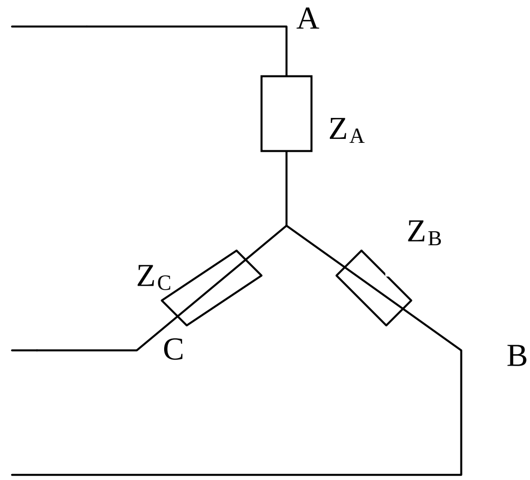
$$I_L = \frac{936.7 \times 10^3}{\sqrt{3}\,(13.8 \times 10^3)} = \mathbf{39.19 \text{ A rms}}$$

$$\text{pf} = \cos\theta = \cos(3.45°) = \mathbf{0.9982} \quad \textbf{(lagging)}$$

Chapter 12, Solution 48.

(a) We first convert the delta load to its equivalent wye load, as shown below.



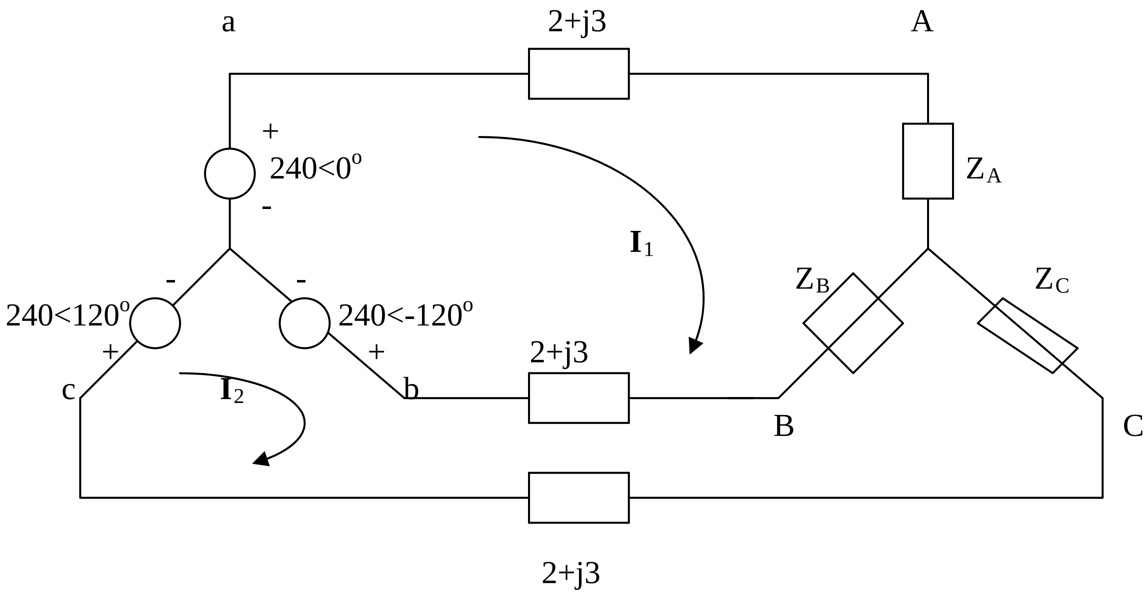


$$Z_A = \frac{(40+j15)(18-j12)}{118+j3} = 7.577 - j1.923$$

$$Z_B = \frac{60(40+j15).}{118+j3} = 20.52 + j7.105$$

$$Z_C = \frac{60(18-j12)}{118+j3} = 8.992 - j6.3303$$

The system becomes that shown below.



We apply KVL to the loops. For mesh 1,

$$-240+240\angle-120^o + I_1(2Z_l + Z_A + Z_B) - I_2(Z_B + Z_l) = 0$$

or

$$(32.097 + j11.13)I_1 - (22.52 + j10.105)I_2 = 360 + j207.85 \qquad (1)$$

For mesh 2,

$$240\angle120^o - 240\angle-120^o - I_1(Z_B + Z_l) + I_2(2Z_l + Z_B + Z_C) = 0$$

or

$$-(22.52 + j10.105)I_1 + (33.51 + j6.775)I_2 = -j415.69 \qquad (2)$$

Solving (1) and (2) gives

$I_1 = 23.75 - j5.328, \qquad I_2 = 15.165 - j11.89$

$\boldsymbol{I_{aA} = I_1 = \underline{24.34\angle-12.64^o \text{ A}}, \qquad I_{bB} = I_2 - I_1 = \underline{10.81\angle-142.6^o \text{ A}}}$

$\boldsymbol{I_{cC} = -I_2 = \underline{19.27\angle141.9^o \text{ A}}}$

(b) $\mathbf{S_a = (240\angle0^o)(24.34\angle12.64^o) = 5841.6\angle12.64^o}$

$\mathbf{S_b = (240\angle-120^o)(10.81\angle142.6^o) = 2594.4\angle22.6^o}$

$\mathbf{S_c = (240\angle120^o)(19.27\angle-141.9^o) = 4624.8\angle-21.9^o}$

$\mathbf{S = S_a + S_b + S_c = 12.386 + j0.55 \text{ kVA} = \underline{12.4\angle2.54^o \text{ kVA}}}$

Chapter 12, Solution 49.

(a) For the delta-connected load, $Z_p = 20 + j10\Omega$, $V_p = V_L = 220$ (rms),

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x220^2}{(20 - j10)} = 5808 + j2904 = \underline{6.943\angle 26.56^o \text{ kVA}}$$

P = **5.808 kW**

(b) For the wye-connected load, $Z_p = 20 + j10\Omega$, $V_p = V_L / \sqrt{3}$,

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x220^2}{3(20 - j10)} = \underline{2.164\angle 26.56^o \text{ kVA}}$$

P = **1.9356 kW**

Chapter 12, Solution 50.

$\overline{S} = \overline{S}_1 + \overline{S}_2 = 8(0.6 + j0.8) = 4.8 + j6.4\,\text{kVA}, \qquad \overline{S}_1 = 3\,\text{kVA}$

Hence,

$\overline{S}_2 = \overline{S} - \overline{S}_1 = 1.8 + j6.4\,\text{kVA}$

But $\overline{S}_2 = \dfrac{3V^2{}_p}{Z^*{}_p}, \quad V_p = \dfrac{V_L}{\sqrt{3}} \quad \longrightarrow \quad \overline{S}_2 = \dfrac{.V^2{}_L}{Z^*{}_p}$

$$\boldsymbol{Z^*{}_p = \frac{V^*{}_L}{\overline{S}_2} = \frac{240^2}{(1.8 + j6.4)x10^3} \quad \longrightarrow \quad \underline{Z_p = 2.346 + j8.34\Omega}}$$

Chapter 12, Solution 51.

This is an unbalanced system.

$$I_{AB} = \frac{240 < 0^o}{Z_1} = \frac{240 < 0^o}{8 + j6} = \underline{19.2\text{-j}14.4 \ \ \text{A}}$$

$$\text{I}_{\text{BC}} = \frac{240\angle 120°}{\text{Z}_2} = \frac{240\angle 120°}{4.7413\angle -27.65} = 50.62\angle 147.65^\circ = \mathbf{[–42.76+j27.09]\ A}$$

$$\text{I}_{\text{CA}} = \frac{240\angle -120°}{\text{Z}_3} = \frac{240\angle -120°}{10} = \mathbf{[–12–j20.78]\ A}$$

At node A,

$$I_{aA} = I_{AB} - I_{CA} = (19.2 - j14.4) - (-12 - j20.78) = \underline{31.2 + j6.38\ \text{A}}$$

$$\text{I}_{bB} = I_{BC} - I_{AB} = (-42.76 + j27.08) - (19.2 - j14.4) = \underline{-61.96 + j41.48\ \text{A}}$$

$$\text{I}_{cC} = I_{CA} - I_{BC} = (-12 - j20.78) - (-42.76 + j27.08) = \underline{30.76 - j47.86\ \ \text{A}}$$

Chapter 12, Solution 52.

Since the neutral line is present, we can solve this problem on a per-phase basis.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{AN}} = \frac{120\angle 120°}{20\angle 60°} = 6\angle 60°$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_{BN}} = \frac{120\angle 0°}{30\angle 0°} = 4\angle 0°$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_{CN}} = \frac{120\angle -120°}{40\angle 30°} = 3\angle -150°$$

Thus,

$$-\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_n = 6\angle 60° + 4\angle 0° + 3\angle -150°$$

$$-\mathbf{I}_n = (3 + j5.196) + (4) + (-2.598 - j1.5)$$

$$-\mathbf{I}_n = 4.405 + j3.696 = 5.75\angle 40°$$

$$\mathbf{I}_n = \mathbf{5.75\angle 220°\ A}$$

Chapter 12, Solution 53.

Using Fig. 12.61, design a problem that will help other students to better understand unbalanced three-phase systems.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the wye-wye system shown in Fig. 12.61, loads connected to the source are unbalanced. (a) Calculate I_a, I_b , and I_c. (b) Find the total power delivered to the load. Take V_P = 240 V rms.

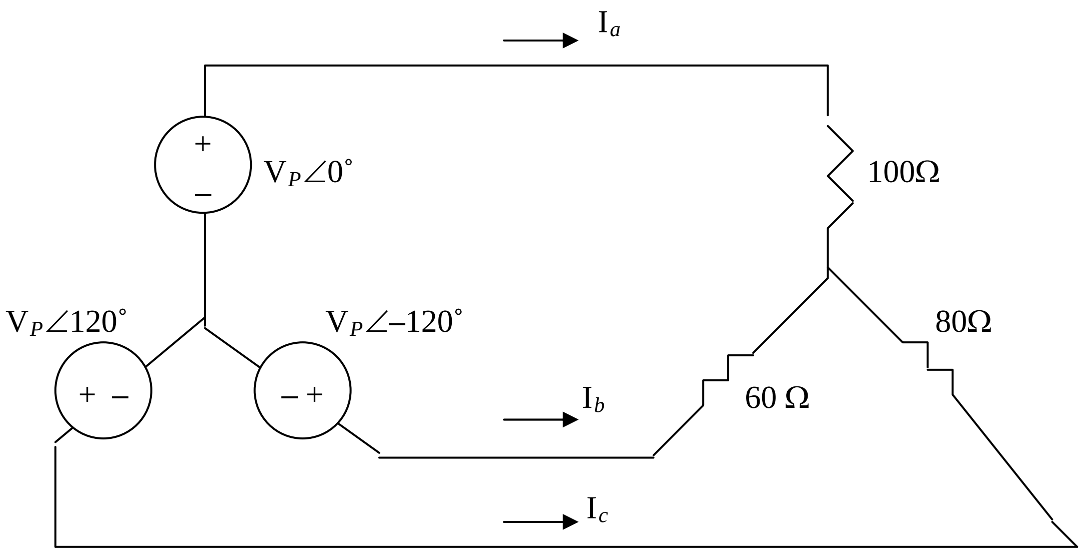
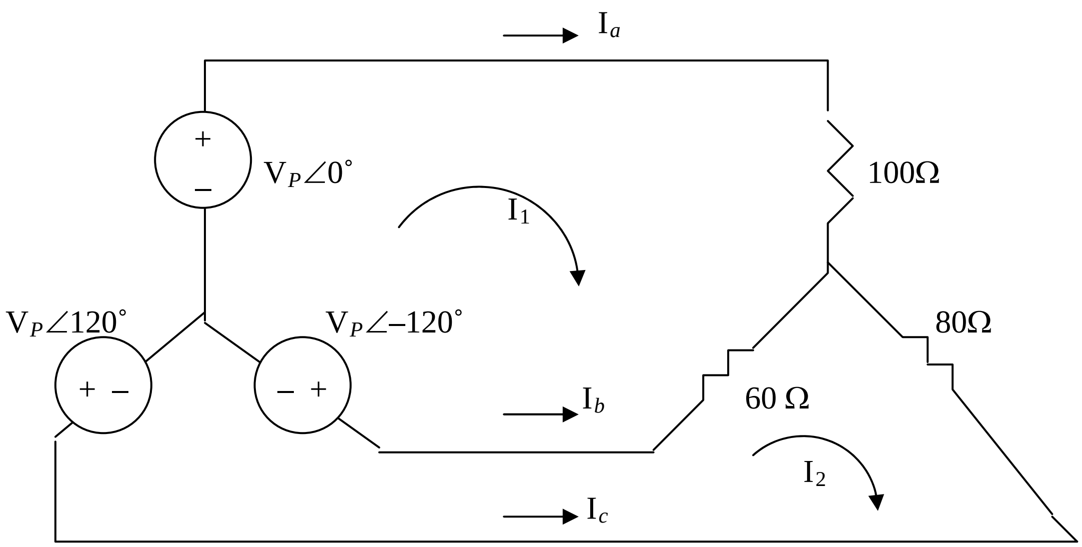


Figure 12.61 For Prob. 12.53.

Solution

Applying mesh analysis as shown below, we get.



$240\angle{-120^\circ} - 240 + 160\mathbf{I_1} - 60\mathbf{I_2} = 0$ or $160\mathbf{I_1} - 60\mathbf{I_2} = 360+j207.84$ (1)

$240\angle{120^\circ} - 240\angle{-120^\circ} - 60\mathbf{I_1} + 140\mathbf{I_2} = 0$ or $-60\mathbf{I_1} + 140\mathbf{I_2} = -j415.7$ (2)

In matrix form, (1) and (2) become

$$\begin{bmatrix} 160 & -60 \\ -60 & 140 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 360 + j207.84 \\ -j415.7 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Z=[160,-60;-60,140]
Z =
   160   -60
   -60   140
>> V=[(360+207.8i);-415.7i]
V =
  1.0e+002 *
   3.6000 + 2.0780i
        0 - 4.1570i
>> I=inv(Z)*V
I =
   2.6809 + 0.2207i
   1.1489 - 2.8747i
```

$I_1 = 2.681 + j0.2207$ and $I_2 = 1.1489 - j2.875$

$$I_a = I_1 = \mathbf{2.69\angle 4.71^\circ\ A}$$

$$I_b = I_2 - I_1 = -1.5321 - j3.096 = \mathbf{3.454\angle -116.33^\circ\ A}$$

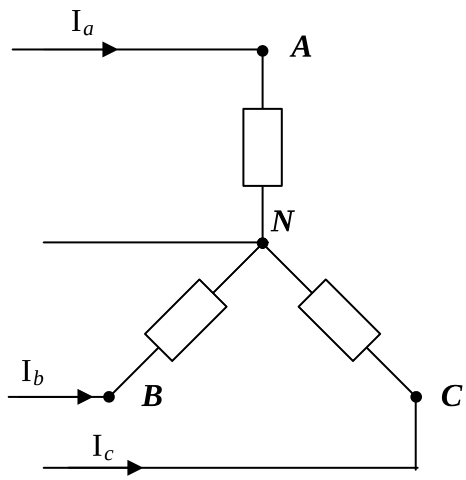
$$I_c = -I_2 = \mathbf{3.096\angle 111.78^\circ\ A}$$

$$S_a = |I_a|^2 Z_a = (2.69)^2 x100 = 723.61$$
$$S_b = |I_b|^2 Z_b = (3.454)^2 x60 = 715.81$$
$$S_c = |I_c|^2 Z_c = (3.0957)^2 x80 = 766.67$$

$$S = S_a + S_b + S_c = \underline{2.205\ \text{kVA}}$$

Chapter 12, Solution 54.

Consider the load as shown below.



$$I_a = \frac{210 < 0^o}{80} = \underline{2.625 \text{ A}}$$

$$I_b = \frac{210\angle 0°}{60 + j90} = \frac{210}{108.17\angle 56.31°} = \mathbf{1.9414\angle{-56.31}^\circ\ A}$$

$$I_c = \frac{210 < 0^o}{j80} = \underline{2.625 < -90^o \text{ A}}$$

$$S_a = VI_a^* = 210x2.625 = 551.25$$

$$S_b = VI_b^* = \frac{|V|^2}{Z_b^*} = \frac{210^2}{60 - j90} = 226.15 + j339.2$$

$$S_c = \frac{|V|^2}{Z_c^*} = \frac{210^2}{-j80} = j551.25$$

$$S = S_a + S_b + S_c = \underline{777.4 + j890.45 \text{ VA}}$$

Chapter 12, Solution 55.

The phase currents are:

$$I_{AB} = 240/j25 = \mathbf{9.6\angle{-90^\circ}\ A}$$

$$I_{CA} = 240\angle 120^\circ/40 = \mathbf{6\angle 120^\circ\ A}$$

$$I_{BC} = 240\angle{-120^\circ}/30\angle 30^\circ = \mathbf{8\angle{-150^\circ}\ A}$$

The complex power in each phase is:

$$S_{AB} = |I_{AB}|^2 Z_{AB} = (9.6)^2 j25 = j2304$$

$$S_{AC} = |I_{AC}|^2 Z_{AC} = (6)^2 40 < 0^o = 1440$$

$$S_{BC} = |I_{BC}|^2 Z_{BC} = (8)^2 30 < 30^o = 1662.77 + j960$$

The total complex power is

$$S = S_{AB} + S_{AC} + S_{BC} = \underline{3102.77 + j3264\ \text{VA}}$$

$$= \mathbf{[3.103+j3.264]\ kVA}$$

Chapter 12, Solution 56.

Using Fig. 12.63, design a problem to help other students to better understand unbalanced three-phase systems.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Refer to the unbalanced circuit of Fig. 12.63. Calculate:

(a) the line currents

(b) the real power absorbed by the load

(c) the total complex power supplied by the source

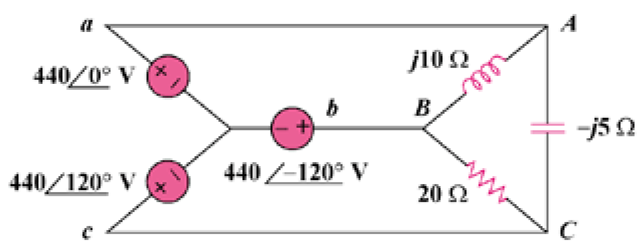
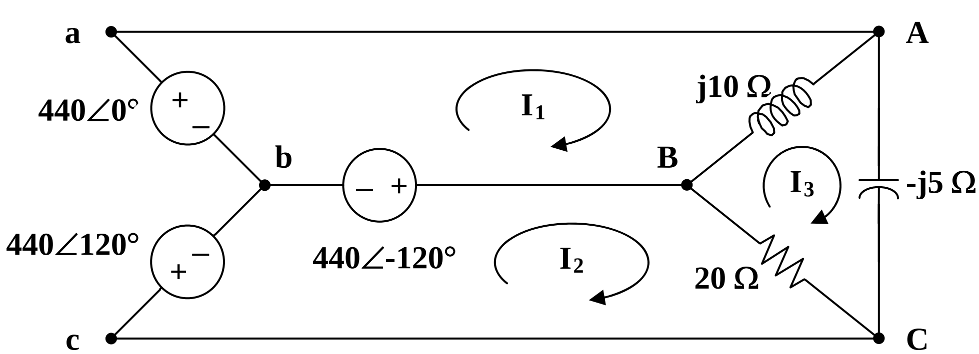


Figure 12.63

Solution

(a) Consider the circuit below.



For mesh 1,

$$440\angle -120° - 440\angle 0° + j10(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_1 - \mathbf{I}_3 = \frac{(440)(1.5 + j0.866)}{j10} = 76.21\angle -60° \quad (1)$$

For mesh 2,

$$440\angle 120° - 440\angle -120° + 20(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_3 - \mathbf{I}_2 = \frac{(440)(j1.732)}{20} = j38.1 \quad (2)$$

For mesh 3,

$$j10(\mathbf{I}_3 - \mathbf{I}_1) + 20(\mathbf{I}_3 - \mathbf{I}_2) - j5\mathbf{I}_3 = 0$$

Substituting (1) and (2) into the equation for mesh 3 gives,

$$\mathbf{I}_3 = \frac{(440)(-1.5 + j0.866)}{j5} = 152.42\angle 60° \quad (3)$$

From (1),

$$\mathbf{I}_1 = \mathbf{I}_3 + 76.21\angle -60° = 114.315 + j66 = 132\angle 30°$$

From (2),

$$\mathbf{I}_2 = \mathbf{I}_3 - j38.1 = 76.21 + j93.9 = 120.93\angle 50.94°$$

$$\mathbf{I}_a = \mathbf{I}_1 = \mathbf{132\angle 30°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = -38.105 + j27.9 = \mathbf{47.23\angle 143.8°\ A}$$

$$\mathbf{I}_c = -\mathbf{I}_2 = \mathbf{120.9\angle 230.9°\ A}$$

(b)

$$\mathbf{S}_{AB} = |\mathbf{I}_1 - \mathbf{I}_3|^2 (j10) = j58.08 \text{ kVA}$$

$$\mathbf{S}_{BC} = |\mathbf{I}_2 - \mathbf{I}_3|^2 (20) = 29.04 \text{ kVA}$$

$$\mathbf{S}_{CA} = |\mathbf{I}_3|^2 (-j5) = (152.42)^2 (-j5) = -j116.16 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_{AB} + \mathbf{S}_{BC} + \mathbf{S}_{CA} = 29.04 - j58.08 \text{ kVA}$$

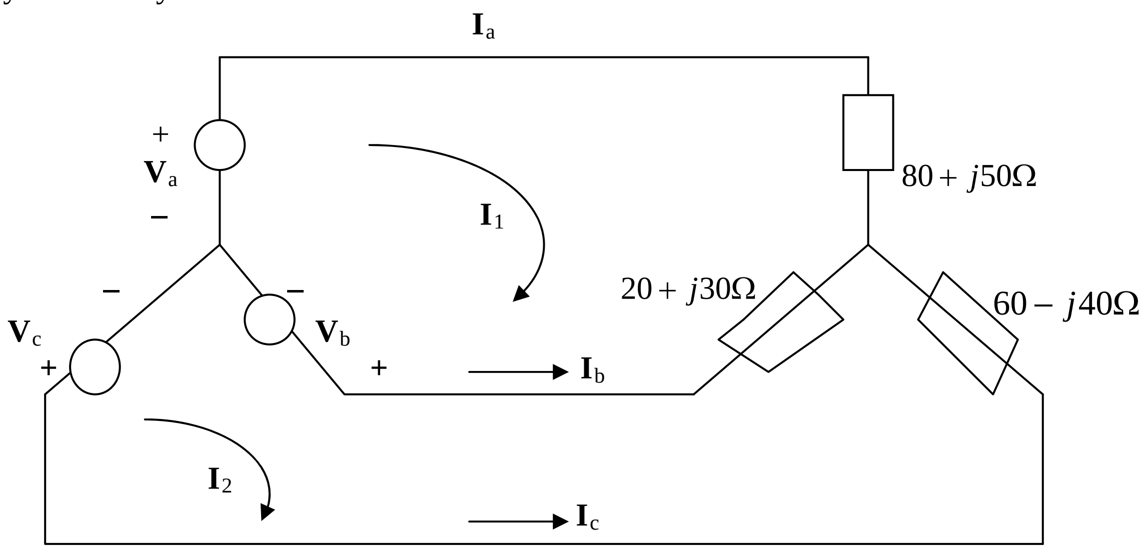
Real power absorbed = **29.04 kW**

(c) Total complex supplied by the source is

$$\mathbf{S} = \mathbf{29.04 - j58.08\ kVA}$$

Chapter 12, Solution 57.

We apply mesh analysis to the circuit shown below.



$$(100 + j80)I_1 - (20 + j30)I_2 = V_a - V_b = 165 + j95.263 \qquad (1)$$

$$-(20 + j30)I_1 + (80 - j10)I_2 = V_b - V_c = -j190.53 \qquad (2)$$

Solving (1) and (2) gives $I_1 = 1.8616 - j0.6084$, $\quad I_2 = 0.9088 - j1.722$.

$$\boldsymbol{I_a = I_1 = \underline{1.9585\angle -18.1^o \text{ A}}, \qquad I_b = I_2 - I_1 = -0.528 - j1.1136 = \underline{1.4656\angle -130.55^o \text{ A}}}$$

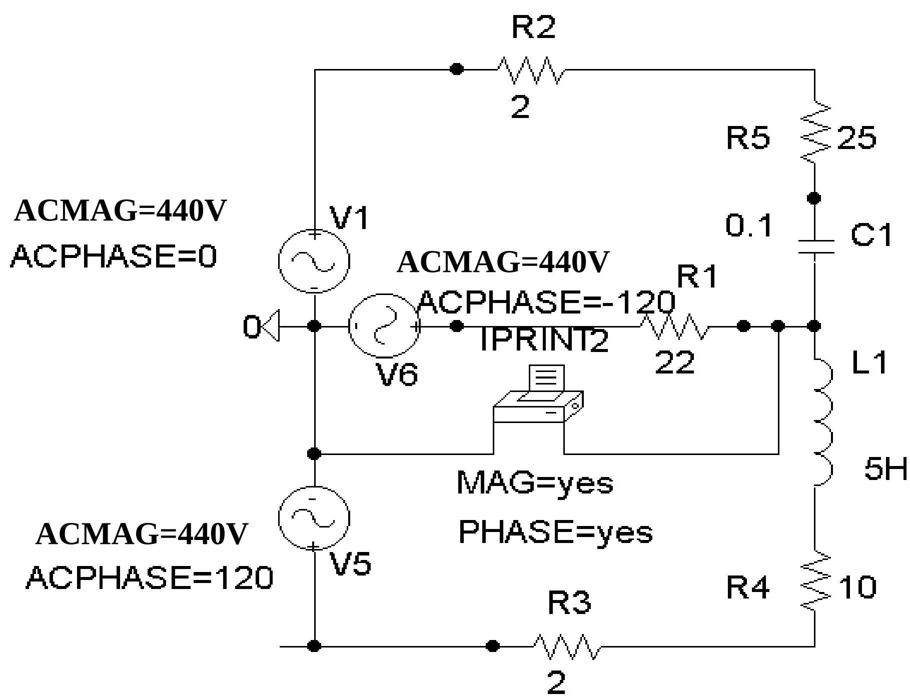
$$\boldsymbol{I_c = -I_2 = \underline{1.947\angle 117.8^o \text{ A}}}$$

Chapter 12, Solution 58.

The schematic is shown below. IPRINT is inserted in the neutral line to measure the current through the line. In the AC Sweep box, we select Total Ptss = 1, Start Freq. = 0.1592, and End Freq. = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	2.156 E+01	–8.997 E+01

i.e. $I_n = \mathbf{21.56\angle{-89.97^\circ}\ A}$

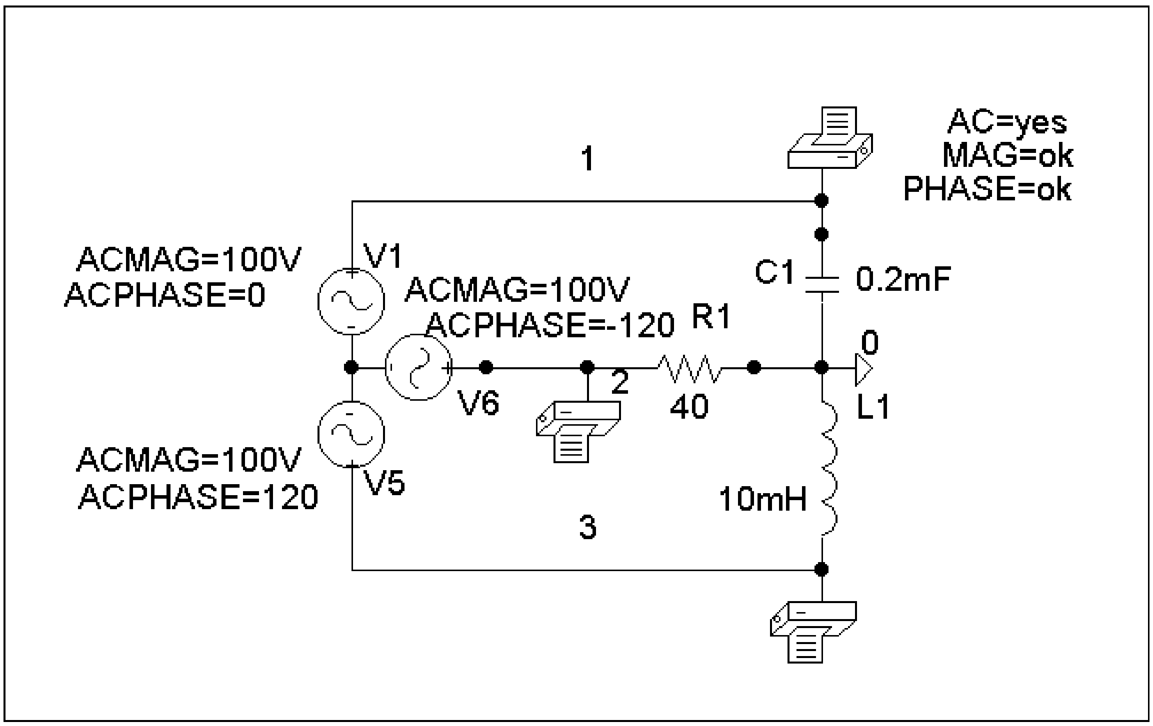


Chapter 12, Solution 59.

The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, we obtain an output file which includes

FREQ	VM(1)	VP(1)
6.000 E+01	2.206 E+02	–3.456 E+01
FREQ	VM(2)	VP(2)
6.000 E+01	2.141 E+02	–8.149 E+01
FREQ	VM(3)	VP(3)
6.000 E+01	4.991 E+01	–5.059 E+01

i.e. V_{AN} = $\mathbf{220.6\angle{-34.56^\circ}}$, V_{BN} = $\mathbf{214.1\angle{-81.49^\circ}}$, V_{CN} = $\mathbf{49.91\angle{-50.59^\circ}}$ **V**

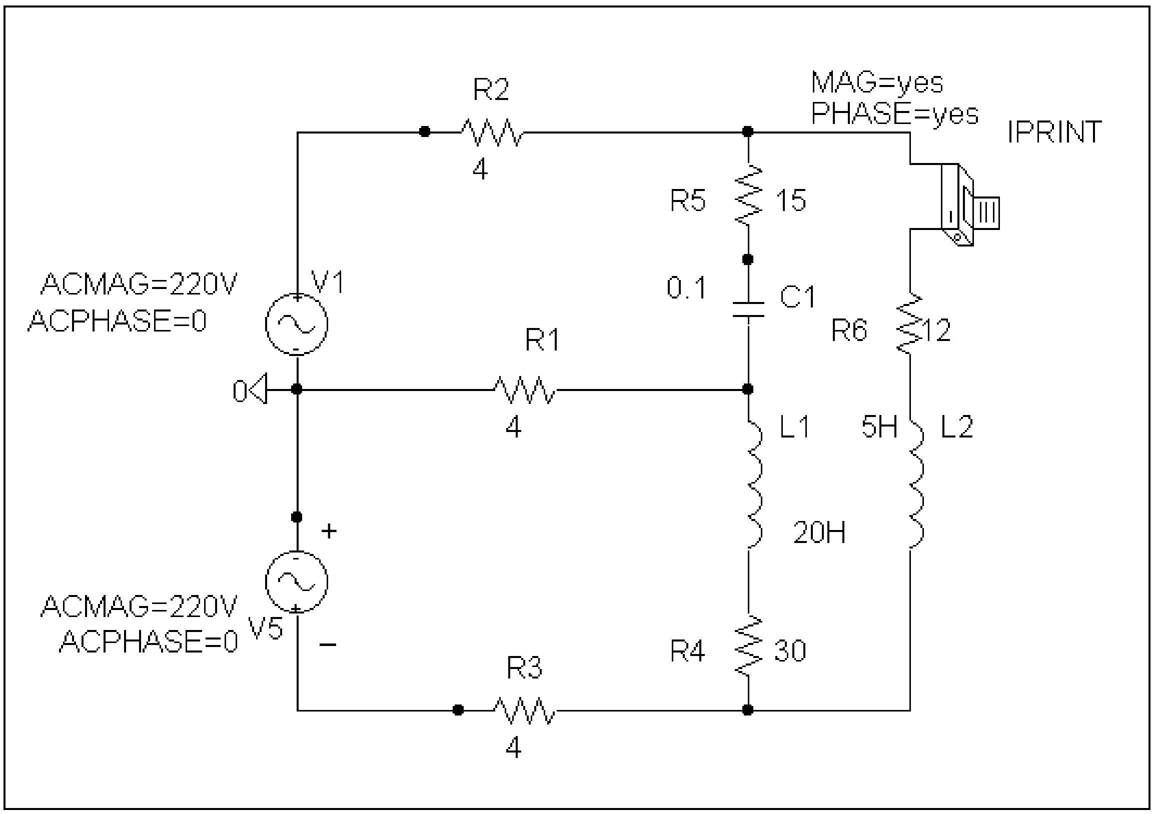


Chapter 12, Solution 60.

The schematic is shown below. IPRINT is inserted to give I_o. We select Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. Upon simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	1.953 E+01	–1.517 E+01

from which, $I_o = \mathbf{19.53\angle -15.17^\circ\ A}$



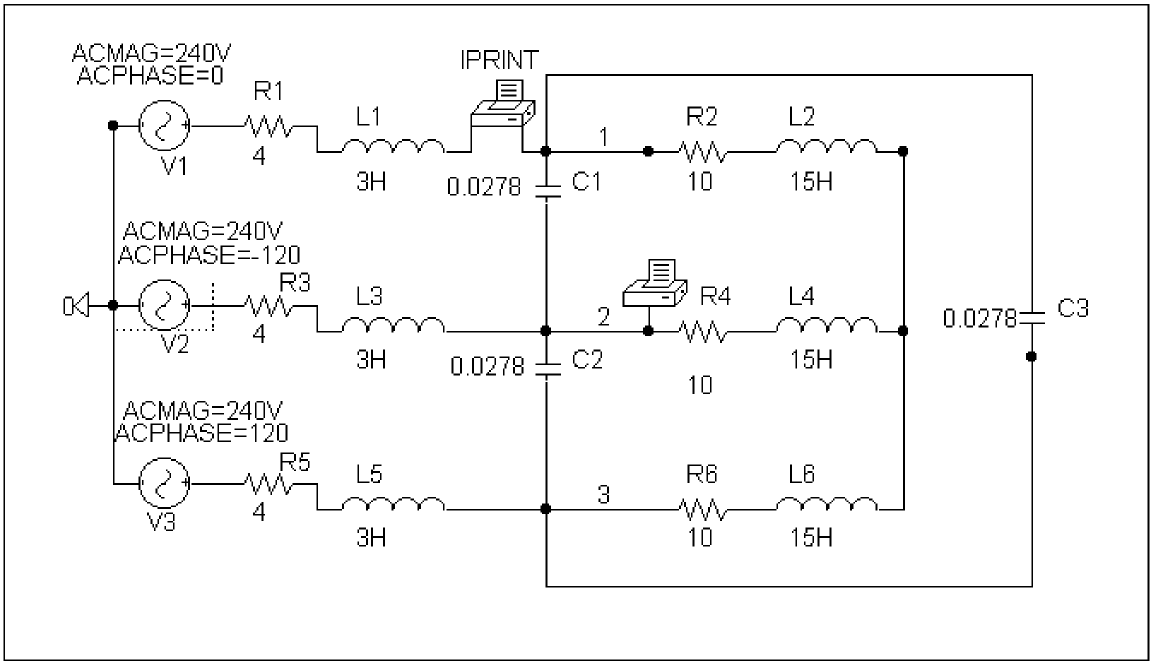
Chapter 12, Solution 61.

The schematic is shown below. Pseudocomponents IPRINT and PRINT are inserted to measure I_{aA} and V_{BN}. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. Once the circuit is simulated, we get an output file which includes

FREQ	VM(2)	VP(2)
1.592 E–01	2.308 E+02	–1.334 E+02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.115 E+01	3.699 E+01

from which

$$I_{aA} = \mathbf{11.15\angle 37^\circ\ A}, \quad V_{BN} = \mathbf{230.8\angle -133.4^\circ\ V}$$



Chapter 12, Solution 62.

Using Fig. 12.68, design a problem to help other students to better understand how to use *PSpice* to analyze three-phase circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The circuit in Fig. 12.68 operates at 60 Hz. Use *PSpice* to find the source current $\mathbf{I}_{ab}$ and the line current $\mathbf{I}_{bB}$.

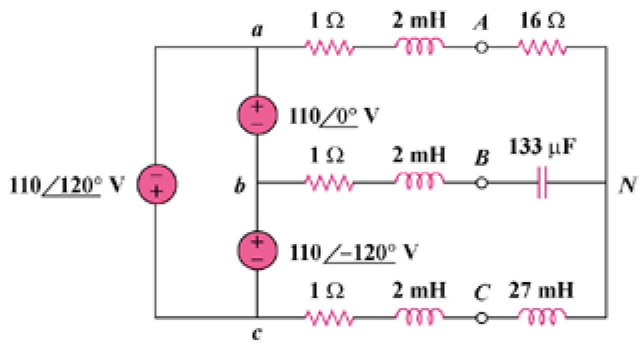


Figure 12.68

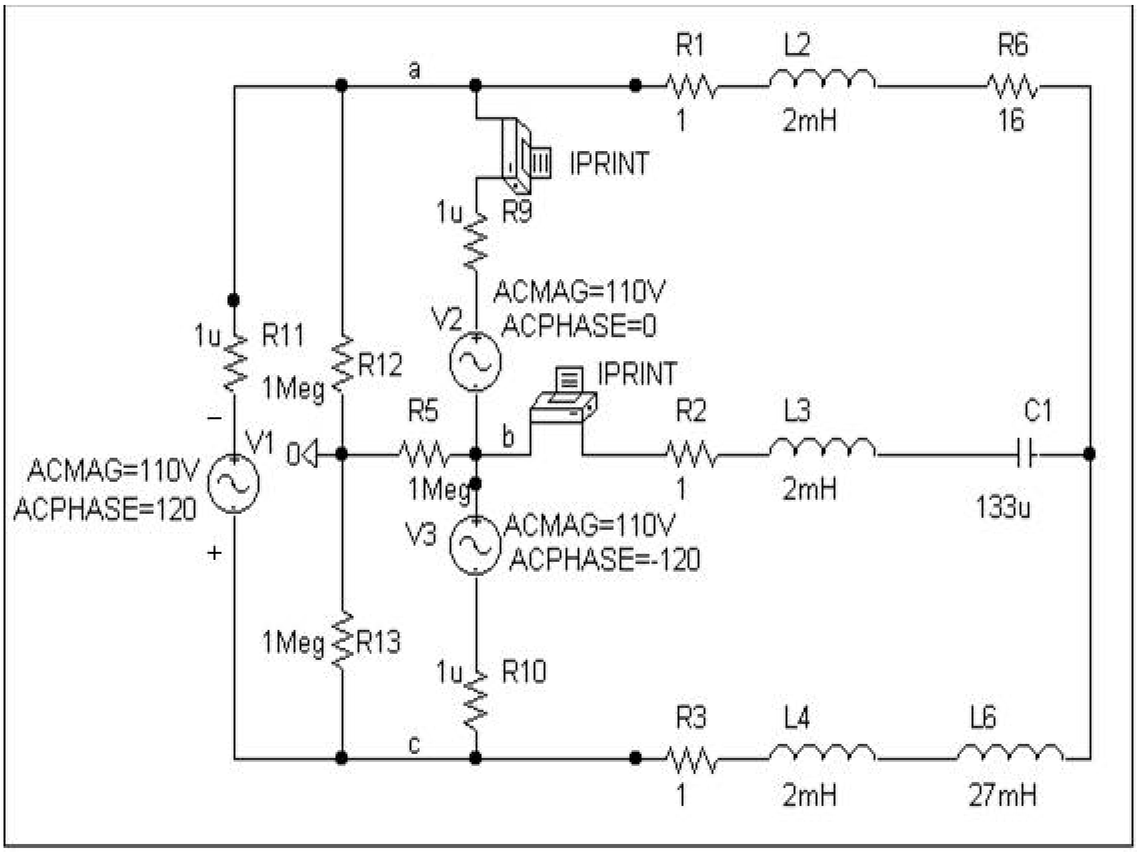
Solution

Because of the delta-connected source involved, we follow Example 12.12. In the AC Sweep box, we type Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, the output file includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
6.000 E+01	5.960 E+00	–9.141 E+01
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.000 E+01	7.333 E+07	1.200 E+02

From which

$$I_{ab} = \mathbf{3.432\angle{-46.31^\circ}\ A},\quad I_{bB} = \mathbf{10.39\angle{-78.4^\circ}\ A}$$

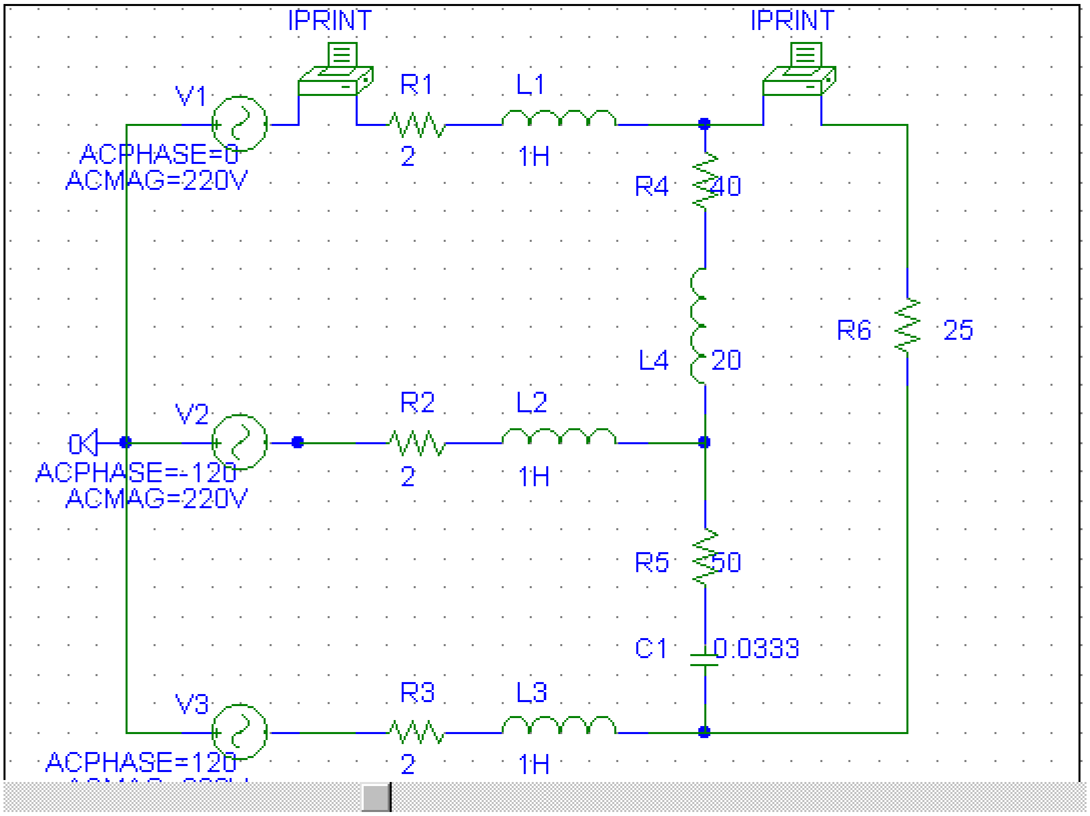


R1
L2
R6
a
1
2mH
16
IPRINT
1u
R9
ACMAG=110V
ACPHASE=0
V2
1u
R11
R12
1Meg
IPRINT
R5
R2
L3
C1
V1
0
b
ACMAG=110V
ACPHASE=120
1Meg
1
2mH
133u
V3
ACMAG=110V
ACPHASE=-120
1Meg
R13
1u
R10
R3
L4
L6
c
1
2mH
27mH

Chapter 12, Solution 63.

Let $\omega = 1$ so that $L = X/\omega = 20\,\text{H}$, and $C = \dfrac{1}{\omega X} = 0.0333\,\text{F}$

The schematic is shown below..



When the file is saved and run, we obtain an output file which includes the following:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	1.867E+01	1.589E+02

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E-01	1.238E+01	1.441E+02

From the output file, the required currents are:

$$\underline{\boldsymbol{I_{aA} = 18.67\angle 158.9^o \text{ A}, \quad I_{AC} = 12.38\angle 144.1^o \text{ A}}}$$

Chapter 12, Solution 64.

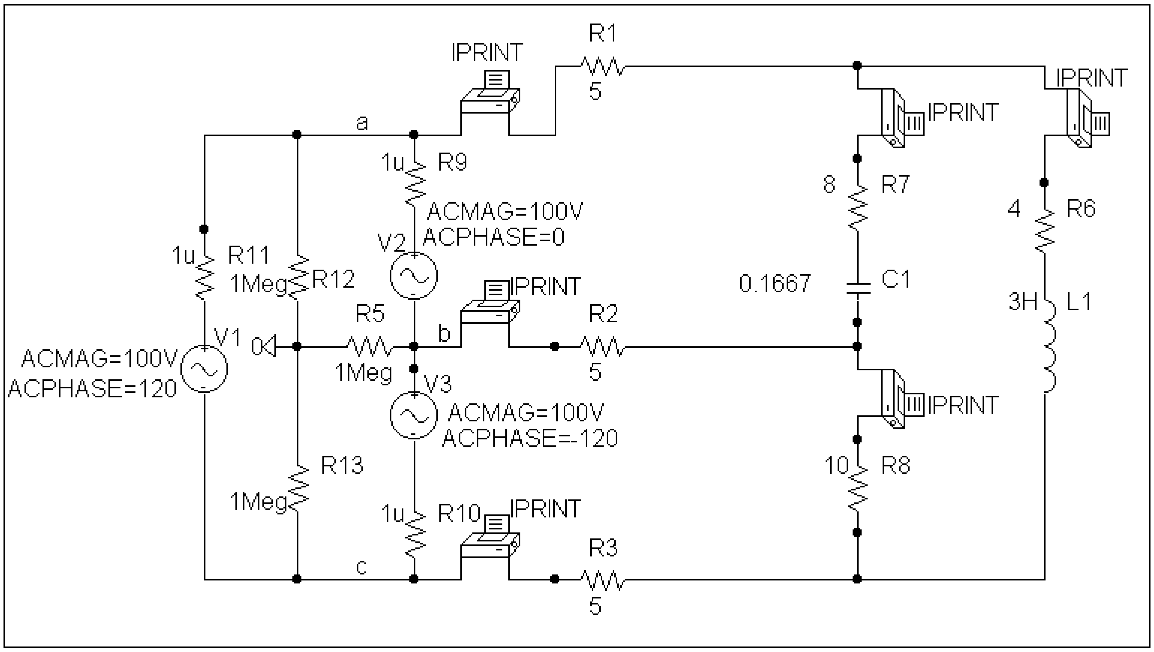
We follow Example 12.12. In the AC Sweep box we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.710 E+00	7.138 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.781 E+07	–1.426 E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	3.898 E+00	–5.076 E+00
FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	3.547 E+00	6.157 E+01
FREQ	IM(V_PRINT5)	IP(V_PRINT5)
1.592 E–01	1.357 E+00	9.781 E+01
FREQ	IM(V_PRINT6)	IP(V_PRINT6)
1.592 E–01	3.831 E+00	–1.649 E+02

from this we obtain

$$I_{aA} = \mathbf{4.71\angle 71.38^\circ\ A},\quad I_{bB} = \mathbf{6.781\angle -142.6^\circ\ A},\quad I_{cC} = \mathbf{3.898\angle -5.08^\circ\ A}$$

$$I_{AB} = \mathbf{3.547\angle 61.57^\circ\ A},\ I_{AC} = \mathbf{1.357\angle 97.81^\circ\ A},\quad I_{BC} = \mathbf{3.831\angle -164.9^\circ\ A}$$



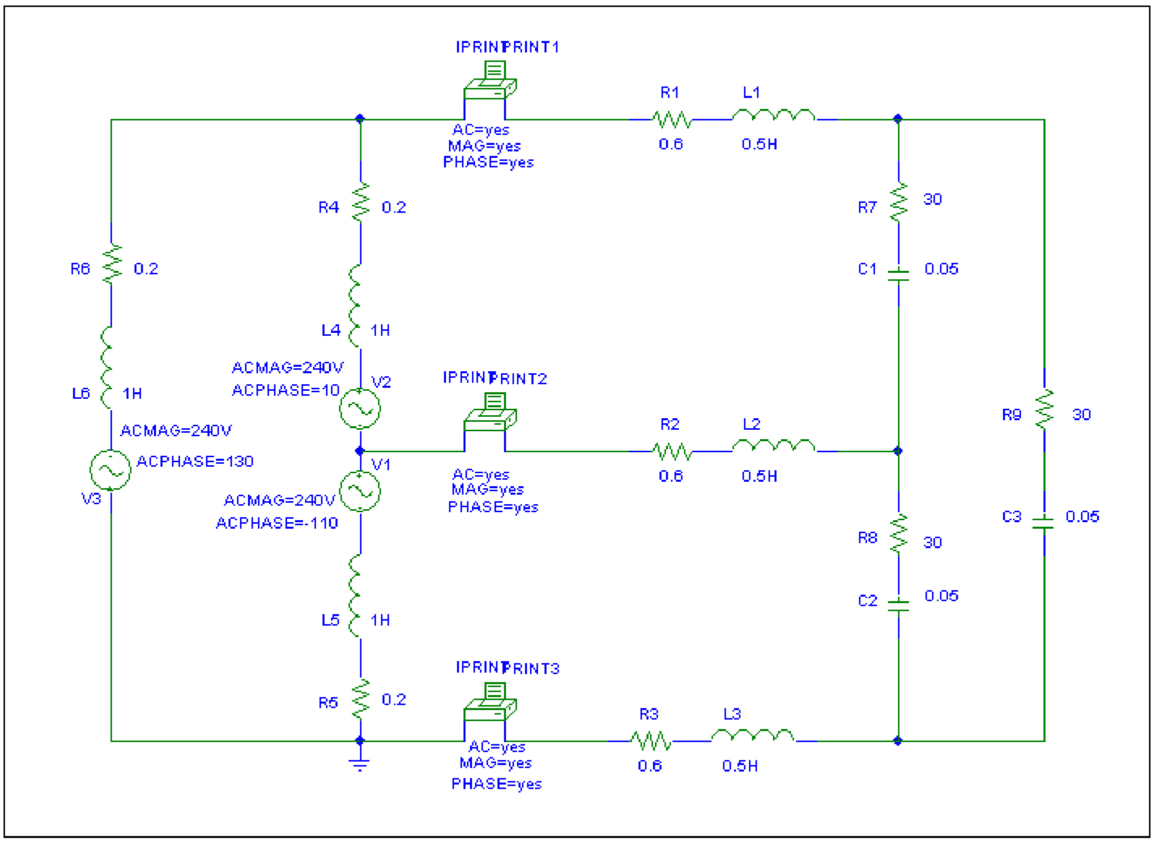
IPRINT
R1
5
IPRINT
IPRINT
a
1u
R9
8
R7
4
R6
ACMAG=100V
ACPHASE=0
V2
1u
R11
1Meg
R12
IPRINT
0.1667
C1
3H
L1
R5
R2
V1
0
b
ACMAG=100V
ACPHASE=120
1Meg
5
V3
IPRINT
ACMAG=100V
ACPHASE=-120
R13
10
R8
1Meg
1u
R10
IPRINT
R3
c
5

Chapter 12, Solution 65.

Due to the delta-connected source, we follow Example 12.12. We type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. The schematic is shown below. After it is saved and simulated, we obtain an output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	1.140E+01	8.664E+00
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E-01	1.140E+01	-1.113E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592E-01	1.140E+01	1.287E+02

Thus, I_{aA} = $\mathbf{11.02\angle 12° \ A}$, I_{bB} = $\mathbf{11.02\angle{-108}° \ A}$, I_{cC} = $\mathbf{11.02\angle 132° \ A}$



Since this is a balanced circuit, we can perform a quick check. The load resistance is large compared to the line and source impedances so we will ignore them (although it would not be difficult to include them).

Converting the sources to a Y configuration we get:

$$V_{an} = 138.56\ \angle{-20^\circ}\ \text{Vrms}$$

and

$$Z_Y = 10 - j6.667 = 12.019\angle{-33.69^\circ}$$

Now we can calculate,

$$I_{aA} = (138.56\ \angle{-20^\circ})/(12.019\angle{-33.69^\circ}) = 11.528\angle 13.69^\circ$$

Clearly, we have a good approximation which is very close to what we really have.

Chapter 12, Solution 66.

(a) $V_p = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = \mathbf{120\ V}$

(b) Because the load is unbalanced, we have an unbalanced three-phase system. Assuming an abc sequence,

$$\mathbf{I}_1 = \frac{120\angle 0°}{48} = 2.5\angle 0°\ \text{A}$$

$$\mathbf{I}_2 = \frac{120\angle -120°}{40} = 3\angle -120°\ \text{A}$$

$$\mathbf{I}_3 = \frac{120\angle 120°}{60} = 2\angle 120°\ \text{A}$$

$$-\mathbf{I}_N = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 2.5 + (3)\left(-0.5 - j\frac{\sqrt{3}}{2}\right) + (2)\left(-0.5 + j\frac{\sqrt{3}}{2}\right)$$

$$\mathbf{I}_N = j\frac{\sqrt{3}}{2} = j0.866 = 0.866\angle 90°\ \text{A}$$

Hence,

$I_1 = \mathbf{2.5\ A}$, $I_2 = \mathbf{3\ A}$, $I_3 = \mathbf{2\ A}$, $I_N = \mathbf{0.866\ A}$

(c) $P_1 = I_1^2 R_1 = (2.5)^2(48) = \mathbf{300\ W}$

$P_2 = I_2^2 R_2 = (3)^2(40) = \mathbf{360\ W}$

$P_3 = I_3^2 R_3 = (2)^2(60) = \mathbf{240\ W}$

(d) $P_T = P_1 + P_2 + P_3 = \mathbf{900\ W}$

Chapter 12, Solution 67.

(a) The power to the motor is

$$P_T = S\cos\theta = (260)(0.85) = 221\text{ kW}$$

The motor power per phase is

$$P_p = \frac{1}{3}P_T = 73.67\text{ kW}$$

Hence, the wattmeter readings are as follows:

$$W_a = 73.67 + 24 = \mathbf{97.67\ kW}$$

$$W_b = 73.67 + 15 = \mathbf{88.67\ kW}$$

$$W_c = 73.67 + 9 = \mathbf{82.67\ kW}$$

(b) The motor load is balanced so that $I_N = 0$.

For the lighting loads,

$$I_a = \frac{24{,}000}{120} = 200\text{ A}$$

$$I_b = \frac{15{,}000}{120} = 125\text{ A}$$

$$I_c = \frac{9{,}000}{120} = 75\text{ A}$$

If we let

$$\mathbf{I}_a = I_a\angle 0° = 200\angle 0°\text{ A}$$

$$\mathbf{I}_b = 125\angle -120°\text{ A}$$

$$\mathbf{I}_c = 75\angle 120°\text{ A}$$

Then,

$$-\mathbf{I}_N = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_N = 200 + (125)\left(-0.5 - j\frac{\sqrt{3}}{2}\right) + (75)\left(-0.5 + j\frac{\sqrt{3}}{2}\right)$$

$$-\mathbf{I}_N = 100 - j43.3\text{ A}$$

$$\left|\mathbf{I}_N\right| = \mathbf{108.97\ A}$$

Chapter 12, Solution 68.

(a) $S = \sqrt{3}\, V_L I_L = \sqrt{3}\,(330)(8.4) = \mathbf{4801\ VA}$

(b) $P = S\cos\theta \longrightarrow \text{pf} = \cos\theta = \dfrac{P}{S}$

$$\text{pf} = \frac{4500}{4801.24} = \mathbf{0.9372}$$

(c) For a wye-connected load,

$$I_p = I_L = \mathbf{8.4\ A}$$

(d) $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{330}{\sqrt{3}} = \mathbf{190.53\ V}$

Chapter 12, Solution 69.

For load 1,

$$\overline{S}_1 = S_1 \cos\theta_1 + jS_1 \sin\theta_1$$

$$pf = 0.85 = \cos\theta_1 \longrightarrow \theta_1 = 31.79^o$$

$$\overline{S}_1 = 13.6 + j8.43 \text{ kVA}$$

For load 2,

$$\overline{S_2} = 12x0.6 + j12x0.8 = 7.2 + j9.6 \text{ kVA}$$

For load 3,

$$\overline{S}_3 = 8 + j0 \text{ kVA}$$

Therefore,

$$\mathbf{S = S_1 + S_2 + S_3 = [28.8+j18.03]\ kVA}$$

Although we can solve this using a delta load, it will be easier to assume our load is wye connected. We also need the wye voltages and will assume that the phase angle on V_{an} = 208/1.73205 = 120.089 is –30 degrees.

Since **S =** 3**VI*** or **I*** = **S**/(3**V**) = (33,978∠32.048°)/[3(120.089)∠–30°] =

94.31∠62.05° A.

$\mathbf{I_a}$ = **94.31∠–62.05° A**, $\mathbf{I_b}$ = **94.31∠177.95° A**, $\mathbf{I_c}$ = **94.31∠57.95° A**

I = 138.46 – j86.68 = **163.35∠-32˚ A**.

Chapter 12, Solution 70.

$$P_T = P_1 + P_2 = 1200 - 400 = 800$$

$$Q_T = P_2 - P_1 = -400 - 1200 = -1600$$

$$\tan\theta = \frac{Q_T}{P_T} = \frac{-1600}{800} = -2 \quad \longrightarrow \quad \theta = -63.43°$$

$$\text{pf} = \cos\theta = \mathbf{0.4472} \quad \textbf{(leading)}$$

$$Z_p = \frac{V_L}{I_L} = \frac{240}{6} = 40$$

$$\mathbf{Z_p = 40\angle -63.43°\ \Omega}$$

Chapter 12, Solution 71.

(a) If $\mathbf{V}_{ab} = 208\angle 0°$, $\mathbf{V}_{bc} = 208\angle -120°$, $\mathbf{V}_{ca} = 208\angle 120°$,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{Ab}} = \frac{208\angle 0°}{20} = 10.4\angle 0°$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{BC}} = \frac{208\angle -120°}{10\sqrt{2}\angle -45°} = 14.708\angle -75°$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{CA}} = \frac{208\angle 120°}{13\angle 22.62°} = 16\angle 97.38°$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 10.4\angle 0° - 16\angle 97.38°$$
$$\mathbf{I}_{aA} = 10.4 + 2.055 - j15.867$$
$$\mathbf{I}_{aA} = 20.171\angle -51.87°$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = 16\angle 97.83° - 14.708\angle -75°$$
$$\mathbf{I}_{cC} = 30.64\angle 101.03°$$

$$P_1 = |\mathbf{V}_{ab}||\mathbf{I}_{aA}|\cos(\theta_{V_{ab}} - \theta_{I_{aA}})$$
$$P_1 = (208)(20.171)\cos(0° + 51.87°) = \mathbf{2.590\ \textit{kW}}$$

$$P_2 = |\mathbf{V}_{cb}||\mathbf{I}_{cC}|\cos(\theta_{V_{cb}} - \theta_{I_{cC}})$$

But $\mathbf{V}_{cb} = -\mathbf{V}_{bc} = 208\angle 60°$

$$P_2 = (208)(30.64)\cos(60° - 101.03°) = \mathbf{4.808\ \textit{kW}}$$

(b)
$$P_T = P_1 + P_2 = 7398.17\ \text{W}$$
$$Q_T = \sqrt{3}(P_2 - P_1) = 3840.25\ \text{VAR}$$
$$\mathbf{S}_T = P_T + jQ_T = 7398.17 + j3840.25\ \text{VA}$$
$$S_T = |\mathbf{S}_T| = \mathbf{8.335\ \textit{kVA}}$$

Chapter 12, Solution 72.

From Problem 12.11,

$$\mathbf{V}_{AB} = 220\angle 130° \text{ V} \quad \text{and} \quad \mathbf{I}_{aA} = 30\angle 180° \text{ A}$$

$$P_1 = (220)(30)\cos(130° - 180°) = \mathbf{4.242\ \mathit{kW}}$$

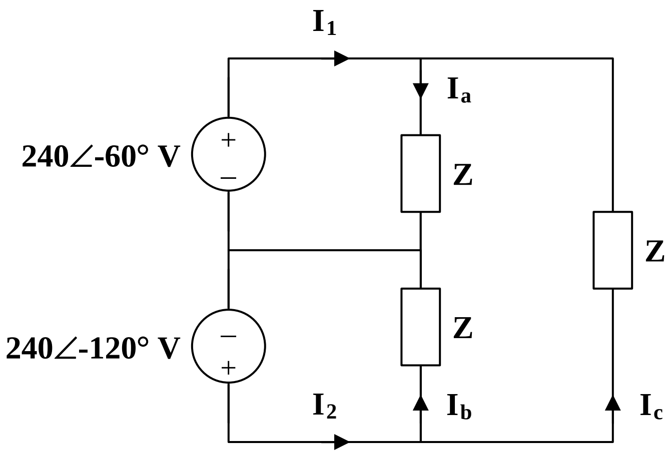
$$\mathbf{V}_{CB} = -\mathbf{V}_{BC} = 220\angle 190°$$

$$\mathbf{I}_{cC} = 30\angle -60°$$

$$P_2 = (220)(30)\cos(190° + 60°) = \mathbf{-\ 2.257\ \mathit{k}W}$$

Chapter 12, Solution 73.

Consider the circuit as shown below.



$$\mathbf{Z} = 10 + j30 = 31.62\angle 71.57°$$

$$\mathbf{I}_a = \frac{240\angle -60°}{31.62\angle 71.57°} = 7.59\angle -131.57°$$

$$\mathbf{I}_b = \frac{240\angle -120°}{31.62\angle 71.57°} = 7.59\angle -191.57°$$

$$\mathbf{I}_c\,\mathbf{Z} + 240\angle -60° - 240\angle -120° = 0$$

$$\mathbf{I}_c = \frac{-240}{31.62\angle 71.57°} = 7.59\angle 108.43°$$

$$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_c = 13.146\angle -101.57°$$

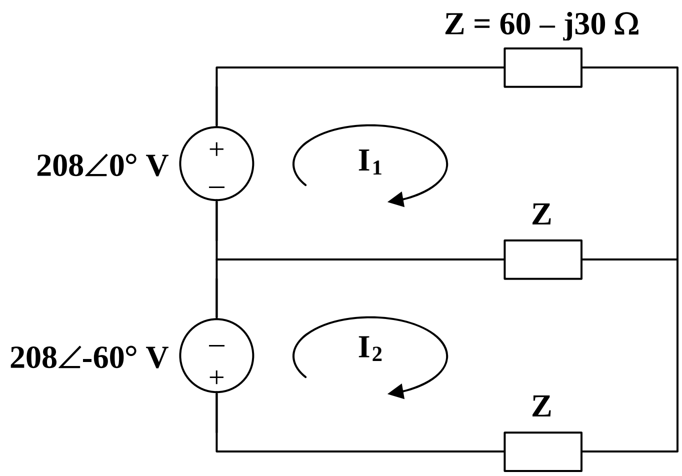
$$\mathbf{I}_2 = \mathbf{I}_b + \mathbf{I}_c = 13.146\angle 138.43°$$

$$P_1 = \text{Re}\left[\mathbf{V}_1\,\mathbf{I}_1^*\right] = \text{Re}\left[(240\angle -60°)(13.146\angle 101.57°)\right] = \mathbf{2.360}\ \boldsymbol{kW}$$

$$P_2 = \text{Re}\left[\mathbf{V}_2\,\mathbf{I}_2^*\right] = \text{Re}\left[(240\angle -120°)(13.146\angle -138.43°)\right] = \mathbf{-632.8\ W}$$

Chapter 12, Solution 74.

Consider the circuit shown below.



For mesh 1,

$$208 = 2\mathbf{Z}\mathbf{I}_1 - \mathbf{Z}\mathbf{I}_2$$

For mesh 2,

$$-208\angle -60° = -\mathbf{Z}\mathbf{I}_1 + 2\mathbf{Z}\mathbf{I}_2$$

In matrix form,

$$\begin{bmatrix} 208 \\ -208\angle -60° \end{bmatrix} = \begin{bmatrix} 2\mathbf{Z} & -\mathbf{Z} \\ -\mathbf{Z} & 2\mathbf{Z} \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 3\mathbf{Z}^2, \quad \Delta_1 = (208)(1.5 + j0.866)\mathbf{Z}, \quad \Delta_2 = (208)(j1.732)\mathbf{Z}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{(208)(1.5 + j0.866)}{(3)(60 - j30)} = 1.789\angle 56.56°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{(208)(j1.732)}{(3)(60 - j30)} = 1.79\angle 116.56°$$

$$P_1 = \mathrm{Re}\left[\mathbf{V}_1\mathbf{I}_1^*\right] = \mathrm{Re}\left[(208)(1.789\angle -56.56°)\right] = \mathbf{208.98\ W}$$

$$P_2 = \mathrm{Re}\left[\mathbf{V}_2(-\mathbf{I}_2)^*\right] = \mathrm{Re}\left[(208\angle -60°))(1.79\angle 63.44°)\right] = \mathbf{371.65\ W}$$

Chapter 12, Solution 75.

(a) $I = \dfrac{V}{R} = \dfrac{12}{600} = \mathbf{20\ mA}$

(b) $I = \dfrac{V}{R} = \dfrac{120}{600} = \mathbf{200\ mA}$

Chapter 12, Solution 76.

If both appliances have the same power rating, P,

$$I = \frac{P}{V_s}$$

For the 120-V appliance, $\quad I_1 = \frac{P}{120}$.

For the 240-V appliance, $\quad I_2 = \frac{P}{240}$.

$$\text{Power loss} = I^2 R = \begin{cases} \dfrac{P^2 R}{120^2} & \text{for the 120-V appliance} \\ \dfrac{P^2 R}{240^2} & \text{for the 240-V appliance} \end{cases}$$

Since $\frac{1}{120^2} > \frac{1}{240^2}$, **the losses in the 120-V appliance are higher**.

Chapter 12, Solution 77.

$$P_g = P_T - P_{load} - P_{line}, \qquad pf = 0.85$$

But $\quad P_T = 3600\cos\theta = 3600 \times pf = 3060$

$$P_g = 3060 - 2500 - (3)(80) = \mathbf{320\ W}$$

Chapter 12, Solution 78.

$$\cos\theta_1 = \frac{51}{60} = 0.85 \quad \longrightarrow \quad \theta_1 = 31.79°$$
$$Q_1 = S_1 \sin\theta_1 = (60)(0.5268) = 31.61\ \text{kVAR}$$

$$P_2 = P_1 = 51\ \text{kW}$$
$$\cos\theta_2 = 0.95 \quad \longrightarrow \quad \theta_2 = 18.19°$$
$$S_2 = \frac{P_2}{\cos\theta_2} = 53.68\ \text{kVA}$$
$$Q_2 = S_2 \sin\theta_2 = 16.759\ \text{kVAR}$$

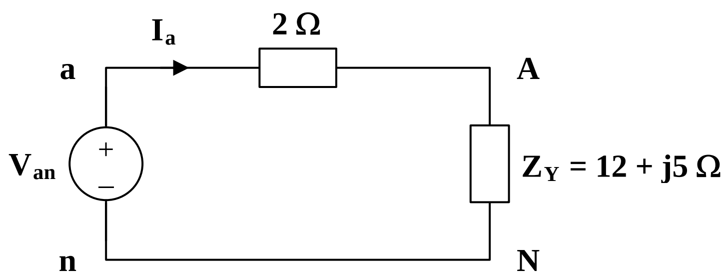
$$Q_c = Q_1 - Q_2 = 3.61 - 16.759 = 14.851\ \text{kVAR}$$

For each load,

$$Q_{c1} = \frac{Q_c}{3} = 4.95\ \text{kVAR}$$
$$C = \frac{Q_{c1}}{\omega V^2} = \frac{4950}{(2\pi)(60)(440)^2} = \mathbf{67.82\ \mu F}$$

Chapter 12, Solution 79.

Consider the per-phase equivalent circuit below.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y + 2} = \frac{255\angle 0°}{14 + j5} = \mathbf{17.15\angle -19.65°\ A}$$

Thus,

$$\mathbf{I}_b = \mathbf{I}_a \angle -120° = \mathbf{17.15\angle -139.65°\ A}$$
$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \mathbf{17.15\angle 100.35°\ A}$$

$$\mathbf{V}_{AN} = \mathbf{I}_a\,\mathbf{Z}_Y = (17.15\angle -19.65°)(13\angle 22.62°) = \mathbf{223\angle 2.97°\ V}$$

Thus,

$$\mathbf{V}_{BN} = \mathbf{V}_{AN} \angle -120° = \mathbf{223\angle -117.63°\ V}$$
$$\mathbf{V}_{CN} = \mathbf{V}_{AN} \angle 120° = \mathbf{223\angle 122.97°\ V}$$

Chapter 12, Solution 80.

$$S = S_1 + S_2 + S_3 = 6[0.83 + j\sin(\cos^{-1} 0.83)] + S_2 + 8(0.7071 - j0.7071)$$

$$S = 10.6368 - j2.31 + S_2 \text{ kVA} \qquad (1)$$

But

$$S = \sqrt{3} V_L I_L \angle\theta = \sqrt{3}(208)(84.6)(0.8 + j0.6) \text{ VA} = 24.383 + \text{j}18.287 \text{ kVA} \qquad (2)$$

From (1) and (2),

$$S_2 = 13.746 + j20.6 = 24.76\angle 56.28 \text{ kVA}$$

Thus, the unknown load is **24.76 kVA at 0.5551 pf lagging**.

Chapter 12, Solution 81.

$$\text{pf} = 0.8 \quad \text{(leading)} \quad \longrightarrow \quad \theta_1 = -36.87°$$
$$\mathbf{S}_1 = 150\angle -36.87° \text{ kVA}$$

$$\text{pf} = 1.0 \quad \longrightarrow \quad \theta_2 = 0°$$
$$\mathbf{S}_2 = 100\angle 0° \text{ kVA}$$

$$\text{pf} = 0.6 \quad \text{(lagging)} \quad \longrightarrow \quad \theta_3 = 53.13°$$
$$\mathbf{S}_3 = 200\angle 53.13° \text{ kVA}$$

$$\mathbf{S}_4 = 80 + j95 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$
$$\mathbf{S} = 420 + j165 = 451.2\angle 21.45° \text{ kVA}$$

$$S = \sqrt{3}\, V_L I_L$$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{451.2 \times 10^3}{\sqrt{3} \times 480} = 542.7 \text{ A}$$

For the line,

$$\mathbf{S}_L = 3 I_L^2 \mathbf{Z}_L = (3)(542.7)^2 (0.02 + j0.05)$$
$$\mathbf{S}_L = 17.67 + j44.18 \text{ kVA}$$

At the source,

$$\mathbf{S}_T = \mathbf{S} + \mathbf{S}_L = 437.7 + j209.2$$
$$\mathbf{S}_T = 485.1\angle 25.55° \text{ kVA}$$

$$V_T = \frac{S_T}{\sqrt{3}\, I_L} = \frac{485.1 \times 10^3}{\sqrt{3} \times 542.7} = \mathbf{516\ V}$$

Chapter 12, Solution 82.

$$\overline{S}_1 = 400(0.8 + j0.6) = 320 + j240 \text{ kVA}, \qquad \overline{S}_2 = 3\frac{V_p^2}{Z_p^*}$$

For the delta-connected load, $V_L = V_p$

$$\overline{S}_2 = 3x\frac{(2400)^2}{10 - j8} = 1053.7 + j842.93 \text{ kVA}$$

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = 1.3737 + j1.0829 \text{ MVA}$$

Let $I = I_1 + I_2$ be the total line current. For I_1,

$$S_1 = 3V_p I_1^*, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$I_1^* = \frac{S_1}{\sqrt{3}V_L} = \frac{(320 + j240)x10^3}{\sqrt{3}(2400)}, \qquad I_1 = 76.98 - j57.735$$

For I_2, convert the load to wye.

$$I_2 = I_p\sqrt{3}\angle -30^o = \frac{2400}{10 + j8}\sqrt{3}\angle -30^o = 273.1 - j289.76$$

$$I = I_1 + I_2 = 350 - j347.5$$

$$\boldsymbol{V_s = V_L + V_{line} = 2400 + I(3 + j6) = 5.185 + j1.405 \text{ kV}} \qquad \longrightarrow \qquad |\boldsymbol{V_s}| = \underline{\mathbf{5.372 \text{ kV}}}$$

Chapter 12, Solution 83.

$$S_1 = 120x746x0.95(0.707 + j0.707) = 60.135 + j60.135 \text{ kVA}, \qquad \text{S}_2 = 80 \text{ kVA}$$

$$S = S_1 + S_2 = 140.135 + j60.135 \text{ kVA}$$

But $|S| = \sqrt{3}V_L I_L \quad \longrightarrow \quad I_L = \dfrac{|S|}{\sqrt{3}V_L} = \dfrac{152.49x10^3}{\sqrt{3}x480} = \underline{183.42 \text{ A}}$

Chapter 12, Solution 84.

We first find the magnitude of the various currents.

For the motor,

$$I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{4000}{440\sqrt{3}} = 5.248 \text{ A}$$

For the capacitor,

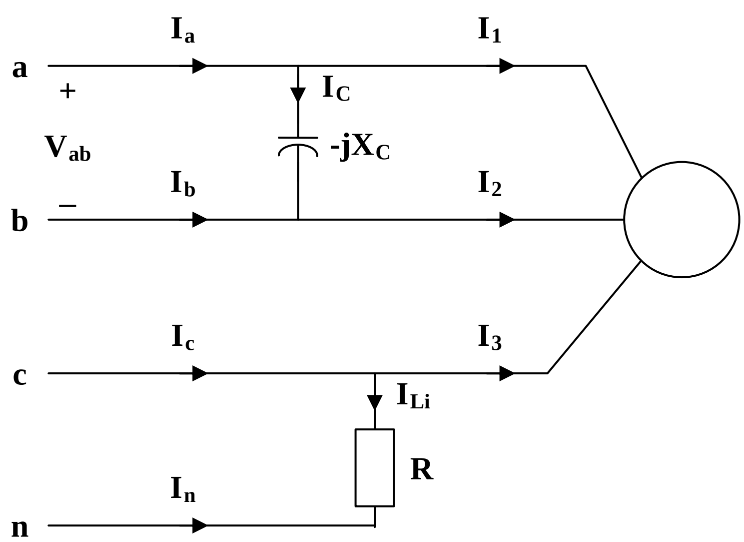
$$I_C = \frac{Q_c}{V_L} = \frac{1800}{440} = 4.091 \text{ A}$$

For the lighting,

$$V_p = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

$$I_{Li} = \frac{P_{Li}}{V_p} = \frac{800}{254} = 3.15 \text{ A}$$

Consider the figure below.



If $\mathbf{V}_{an} = V_p\angle 0°$, $\quad \mathbf{V}_{ab} = \sqrt{3}\,V_p\angle 30°$

$$\mathbf{V}_{cn} = V_p\angle 120°$$

$$\mathbf{I}_C = \frac{\mathbf{V}_{ab}}{-jX_C} = 4.091\angle 120°$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = 4.091\angle(\theta + 30°)$$

where $\theta = \cos^{-1}(0.72) = 43.95°$

$$\mathbf{I}_1 = 5.249\angle 73.95°$$

$$\mathbf{I}_2 = 5.249\angle -46.05°$$

$$\mathbf{I}_3 = 5.249\angle 193.95°$$

$$\mathbf{I}_{Li} = \frac{\mathbf{V}_{cn}}{R} = 3.15\angle 120°$$

Thus,

$$\mathbf{I}_a = \mathbf{I}_1 + \mathbf{I}_C = 5.249\angle 73.95° + 4.091\angle 120°$$

$$\mathbf{I}_a = \mathbf{8.608\angle 93.96°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_C = 5.249\angle -46.05° - 4.091\angle 120°$$

$$\mathbf{I}_b = \mathbf{9.271\angle -52.16°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_3 + \mathbf{I}_{Li} = 5.249\angle 193.95° + 3.15\angle 120°$$

$$\mathbf{I}_c = \mathbf{6.827\angle 167.6°\ A}$$

$$\mathbf{I}_n = -\mathbf{I}_{Li} = \mathbf{3.15\angle -60°\ A}$$

Chapter 12, Solution 85.

Let $\quad Z_Y = R$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{240}{\sqrt{3}} = 138.56 \text{ V}$$

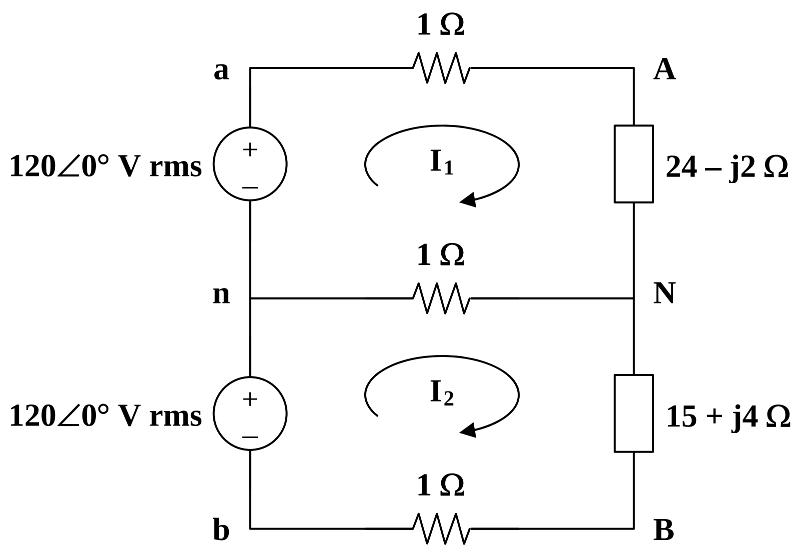
$$P = V_p I_p = \frac{27}{2} = 9 \text{ kW} = \frac{V_p^2}{R}$$

$$R = \frac{V_p^2}{P} = \frac{(138.56)^2}{9000} = 2.133\,\Omega$$

Thus, $\quad Z_Y = \mathbf{2.133\,\Omega}$

Chapter 12, Solution 86.

Consider the circuit shown below.



For the two meshes,

$$120 = (26 - j2)\mathbf{I}_1 - \mathbf{I}_2 \qquad (1)$$

$$120 = (17 + j4)\mathbf{I}_2 - \mathbf{I}_1 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 120 \\ 120 \end{bmatrix} = \begin{bmatrix} 26 - j2 & -1 \\ -1 & 17 + j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 449 + j70, \qquad \Delta_1 = (120)(18 + j4), \qquad \Delta_2 = (120)(27 - j2)$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{120 \times 18.44\angle 12.53°}{454.42\angle 8.86°} = 4.87\angle 3.67°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{120 \times 27.07\angle -4.24°}{454.42\angle 8.86°} = 7.15\angle -13.1°$$

$$\mathbf{I}_{aA} = \mathbf{I}_1 = \mathbf{4.87\angle 3.67°\ A}$$

$$\mathbf{I}_{bB} = -\mathbf{I}_2 = \mathbf{7.15\angle 166.9°\ A}$$

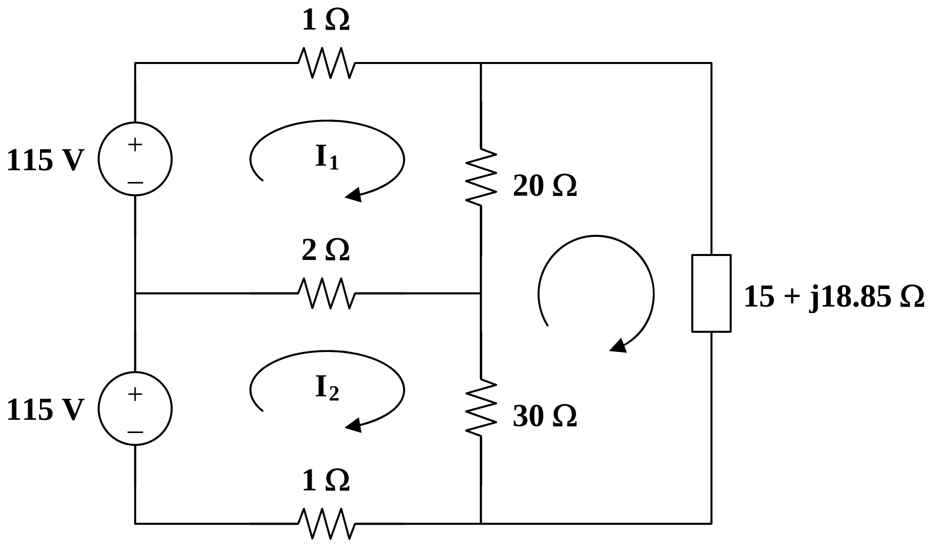
$$\mathbf{I}_{nN} = \mathbf{I}_2 - \mathbf{I}_1 = \frac{\Delta_2 - \Delta_1}{\Delta}$$

$$\mathbf{I}_{nN} = \frac{(120)(9 - j6)}{449 + j70} = \mathbf{2.856\angle -42.55°\ A}$$

Chapter 12, Solution 87.

$$L = 50\text{ mH} \longrightarrow j\omega L = j(2\pi)(60)(5010^{-3}) = j18.85$$

Consider the circuit below.



Applying KVl to the three meshes, we obtain

$$23\mathbf{I}_1 - 2\mathbf{I}_2 - 20\mathbf{I}_3 = 115 \qquad (1)$$

$$-2\mathbf{I}_1 + 33\mathbf{I}_2 - 30\mathbf{I}_3 = 115 \qquad (2)$$

$$-20\mathbf{I}_1 - 30\mathbf{I}_2 + (65 + j18.85)\mathbf{I}_3 = 0 \qquad (3)$$

In matrix form,

$$\begin{bmatrix} 23 & -2 & -20 \\ -2 & 33 & -30 \\ -20 & -30 & 65 + j18.85 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 115 \\ 115 \\ 0 \end{bmatrix}$$

$\Delta = 12{,}775 + j14{,}232$, $\quad \Delta_1 = (115)(1975 + j659.8)$

$\Delta_2 = (115)(1825 + j471.3)$, $\quad \Delta_3 = (115)(1450)$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{115 \times 2082\angle 18.47°}{19214\angle 48.09°} = 12.52\angle -29.62°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{115 \times 1884.9\angle 14.48°}{19124\angle 48.09°} = 11.33\angle -33.61°$$

$$\mathbf{I}_n = \mathbf{I}_2 - \mathbf{I}_1 = \frac{\Delta_2 - \Delta_1}{\Delta} = \frac{(115)(-150 - j188.5)}{12{,}775 + j14{,}231.75} = \mathbf{1.448\angle -176.6°\ A}$$

$$\mathbf{S}_1 = \mathbf{V}_1 \mathbf{I}_1^* = \mathbf{(115)(12.52\angle 29.62°) = [1.252 + \mathit{j}0.7116]\ \mathit{kVA}}$$

$$\mathbf{S}_2 = \mathbf{V}_2 \mathbf{I}_2^* = (115)(1.33\angle 33.61°) = \mathbf{[1.085 + \mathit{j}0.7212]\ \mathit{kVA}}$$