Chapter16, Solution 1.

The current in an *RLC* circuit is described by

$$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 25i = 0$$

If $i(0) = 2$ and $di(0)/dt = 0$, find $i(t)$ for $t > 0$.

Solution

Step 1. Transform the equation into the s-domain and solve for I(s).

$s^2I(s) – (di(0^–)/dt) – si(0^–) + 10sI(s) – 10i(0^–) +25\ I(s) = 0$

$(s^2+10s+25)I(s) + [–(di(0^–)/dt)–si(0^–)–10i(0^–)] = 0$

$(s^2+10s+25)I(s) + [–2s–20] = 0$ or $(s^2+10s+25)I(s) = 2(s+10)$ or

$I(s) = 2(s+10)/ (s^2+10s+25)$

Step 2. Perform a partial fraction expansion and then solve for i(t) in the time domain.

$s^2 + 10s + 25 = 0$, thus $s_{1,2} = \dfrac{-10 \pm \sqrt{10-10}}{2} = –5$, repeated roots.

$I(s) = 2(s+10)/(s+5)^2 = A/(s+5) + B/(s+5)^2 = (As+A5+B)/(s+5)^2$ or

A = 2 and 5A+B = 20 or B = 20 – 10 = 10 or

$I(s) = 2/(s+5) + 10/(s+5)^2$ or

i(t) = $\mathbf{[(2+10t)e^{–5t}]u(t)\ A}$

Chapter 16, Solution 2.

The differential equation that describes the voltage in an *RLC* network is

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 4v = 0$$

Given that $v(0) = 0$, $dv(0)/dt = 5$, obtain $v(t)$.

Solution

Step 1. Transform the equation into the s-domain and solve for V(s).

$s^2V(s) – (dv(0^–)/dt) – sv(0^–) + 5sV(s) – 5v(0^–) +4V(s) = 0$ or

$(s^2 + 5s +4)V(s) – 5 = 0$ or $V(s) = 5/(s^2+5s+4)$

Step 2. Perform a partial fraction expansion of V(s) and then solve for v(t).

$s^2 + 5s + 4 = 0$, thus $s_{1,2} = \dfrac{-5 \pm \sqrt{25-16}}{2} = –4, –1.$

Thus, $V(s) = 5/[(s+1)(s+4)] = A/(s+1) + B/(s+4)$ where $A = 5/3$ and $B = –5/3$

Thus,

$$v(t) = \mathbf{[(5/3)e^{–t} – (5/3)e^{–4t}]u(t)\ V}$$

Chapter 16, Solution 3.

The natural response of an *RLC* circuit is described by the differential equation

$$\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + v = 0$$

for which the initial conditions are v(0) = 20 V and dv(0)/dt = 0. Solve for v(t).

Solution

Step 1. Transform the equation into the s-domain and solve for v(t).

$s^2V(s) – (dv(0^–)/dt) – sv(0^–) + 2sV(s) – 2v(0^–) + V(s) = 0$ or

$(s^2+2s+1)V(s) – 20s – 40 = 0$ or $V(s) = 20(s+2)/(s^2+2s+1)$

Step 2. Perform a partial fraction expansion and solve for V(s). Inverse transform into the time-domain and solve for v(t).

$s^2 + 2s + 1 = 0$, thus $s_{1,2} = \dfrac{-2 \pm \sqrt{4-4}}{2} = -1$, repeated roots.

$V(s) = 20(s+2)/(s^2+2s+1) = 20(s+2)/(s+1)^2 = A/(s+1) + B/(s+1)^2$

As+A+B = 20s+40 or A = 20 and A+B = 40 = 20 + B or B = 40 – 20 = 20

Thus,

$$v(t) = \mathbf{[(20 + 20t)e^{-t}]u(t)\ V}$$

Chapter 16, Solution 4.

If R = 20 Ω, L = 0.6 H, what value of C will make an RLC series circuit:

(a) overdamped,
(b) critically damped,
(c) underdamped?

Solution

Step 1. Since we are working with a series RLC circuit, we can express our values in terms of I(s) and the s equation that multiplies it in the s-domain. From here we can easily find the values that produce over damped, critically damped, and underdamped conditions.

Equating the mesh equation we get, RI(s) + LsI(s) + (1/C)I(s)/s – V(s) = 0 or

(0.6s + 20 + 1/(Cs))I(s) = V(s) or $[s^2$+(20/0.6)+(1/(0.6Cs)]I(s) = V(s)/0.6 or

$[s^s$+(20/0.6)s+1/(0.6C)]I(s) = sV(s)/0.6

The roots for the denominator are $s_{1,2} = \dfrac{-(20/0.6) \pm \sqrt{(400/0.36) - 4/(0.6C)}}{2}$.

Step 2. To find the values of our roots that produces overdamped, critically damped, and underdamped conditions, we note that when s_1 and s_2 values that produces these values,

overdamped is when s_1 and s_2 are real with no complex values
critically damped is when $s_1 = s_2$
underdamped is when both s_1 and s_2 have complex roots and $s_1 = s_2^*$

Now all we need to do is to solve for these conditions.

(a) Overdamped is when [4/(0.6C)] is less than 400/0.36 or C > 4x0.36/(400x0.6) = $6\text{x}10^{-3}$, or C > **6 mF**

(b) Critically damped is when [4/(0.6C)] is equal to 400/0.36 or C = 4x0.36/(400x0.6) = $6\text{x}10^{-3}$ = **6 mF**

(c) Underdamped is when [4/(0.6C)] is greater than 400/0.36 or C < 4x0.36/(400x0.6) = $6\text{x}10^{-3}$ or C < **6mF**

Chapter 16, Solution 5.

The responses of a series *RLC* circuit are

$v_C(t) = [30 – 10e^{–20t} + 30e^{–10t}]u(t)$ V

$i_L(t) = [40e^{–20t} – 60e^{–10t}]u(t)$ mA

where $v_C(t)$ and $i_L(t)$ are the capacitor voltage and inductor current, respectively. Determine the values of *R*, *L*, and *C*.

Solution

Step 1. We can start with the generalized mesh equation for a series RLC network. We can lump the initial conditions ($v_C(0)$ = 30–10+30 = 50 volts and $i_L(0)$ = 40–60 = 20 amps) with the source in the loop since all we are currently after are the values of R, L, and C.

RI(s) + Ls(I(s)+20/s) + (1/C)I(s)/s – 50/s – V(s) = 0 or $[s^2 + (R/L)s + 1/(LC)]I(s)$ = (V(s)/L) – 20 + 50/(Ls)

Step 2. The values of R, L, and C will come from the roots of the denominator s equation. We already know that they are equal to –10 and –20. We note however, that this will give us only two equations. Obviously we need a third, and that will come from knowing the current through the capacitor and the voltage across it.

$$s_{1,2} = \frac{-(R/L) \pm \sqrt{(R/L)^2 - 4/(LC)}}{2} = -10, -20$$

We can simplify our effort by noting that $s_1 + s_2$ = –R/L[(1/2)+(1/2)] = –30 or R = 30L.

Next, $s_1 - s_2 = \dfrac{2\sqrt{(R/L)^2 - 4/(LC)}}{2}$ = 10 or $(R/L)^2$ – 4/(LC) = 100. Since (R/L) = 30, we then get 900 – 100 = 4/(LC) or LC = 4/800 = 1/200.

Now we work with $i_C(t) = Cdv_C(t)/dt$ or $40e^{–20t} – 60e^{–10t}$ mA = $C[200e^{–20t} – 300e^{–10t}]$ V or C = $0.2x10^{–3}$ = **200 µF**. Since LC = 1/200 then L = $1/(200x200x10^{–6})$ = 1/0.04 = **25 H**. Finally R = 30L = 30x25 = **750 Ω**.

750 Ω, 25 H, 200 µF

Chapter 16, Solution 6.

Design a parallel RLC circuit that has the characteristic equation

$$.s^2+100s+10^6 = 0.$$

Solution

Step 1. Develop a general equation for a parallel RLC circuit with initial conditions lumped into a parallel current source i(t).

$$[Cs + (1/R) + (1/(Ls))]V(s) – I(s) = 0 \text{ or } [s^2+(1/(RC))s+1/(LC)]V(s) = sI(s)/C$$

Step 2. The next step is to equate the unknowns to the parameters in the characteristic equation. This does become a design problem in that we have two equations with three unknowns. We need to pick one of the values so that the other values are realistic.

$1/(RC) = 100$ and $1/(LC) = 10^6$ or $RC = 0.01$ and $LC = 10^{-6}$. We can start with some values of R and see what happens to the values of L and C.

R	**L**	**C**
1 Ω	100 μH	10 mF
10 Ω	1 mH	1 mF
100 Ω	10 mH	100 μF
1 k Ω	100 mH	10 μF
10 k Ω	1 H	1 μF
100 k Ω	10 H	0.1 μF

We now need to pick reasonable values, R = **10 kΩ**, L = **1 H**, and C = **1 μF** represents an acceptable set since their values are relatively common and inexpensive.

Chapter 16, Solution 7.

The step response of an *RLC* circuit is given by

$$\frac{d^2 i}{dt^2} + 2\frac{di}{dt} + 5i = 10$$

Given that $i(0) = 6$ and $di(0)/dt = 12$, solve for $i(t)$.

Solution

Step 1. We start by transforming the equation into the s-domain. We then solve for I(s).

$s^2I(s) - (di(0^-)/dt) - si(0^-) + 2sI(s) - 2i(0^-) + 5\ I(s) = 10/s$ or

$s^2I(s) - (12) - 6s + 2sI(s) - 2x6 + 5I(s) = 10/s = (s^2+2s+5)I(s) - 6(s+4)$ or

$(s^2+2s+5)I(s) = [6(s^2+4s)+10]/s$ or $I(s) = [6(s^2+4s)+10]/[s(s^2+2s+5)]$

Step 2. We need to find the roots of (s^2+2s+5) and then perform a partial fraction expansion and then transform back into the time domain and realize i(t).

$s^2 + 2s + 5$, has the roots $s_{1,2} = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1\pm j2$

$I(s) = [6(s^2+4s)+10]/[s(s+1+j2)(s+1-j2)] = [A/s] + [B/(s+1+j2)] + [C/(s+1-j2)]$

$A = 10/5 = 2$; $B = [6(1+j4-4-4-j8)+10]/[(-1-j2)(-j4)] = [6(-7-j4)+10]/[-8+j4] =$

$(-32-j24)/[4(-2+j)] = 4(-8-j6)/[4(-2+j)] = 2(-4-j3)/(-2+j) =$

$[2(-4-j3)(-2-j)]/[(-2+j)(-2-j)] = 2(8-3+j6+j4)/5 = 2(1+j2)$; $C =$

$[6(1-j4-4-4+j8)+10]/[(-1+j2)(j4)] = [6(-7+j4)+10]/[-8-j4] =$

$(-32+j24)/[4(-2-j)] = 4(-8+j6)/[4(-2-j)] = 2(-4+j3)/(-2-j) =$

$[2(-4+j3)(-2+j)]/[(-2-j)(-2+j)] = 2(8-3-j6-j4)/5 = 2(1-j2)$.

$I(s) = [2/s] + [(2+j4)/(s+1+j2)] + [(2-j4)/(s+1-j2)]$ or

$i(t) = \mathbf{[2+4e^{-t}(\cos(2t)+2\sin(2t))]u(t)\ A}$

Chapter 16, Solution 8.

A branch voltage in an *RLC* circuit is described by

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 48$$

If the initial conditions are $v(0) = 0 = dv(0)/dt$, find $v(t)$.

Solution

Step 1. First we transform the equation into the s-domain. Then we solve for V(s).

$$s^2V(s) – (dv(0^–)/dt) – sv(0^–) + 4sV(s) – 4v(0^–) + 8V(s) = 48/s \text{ or}$$

$$s^2V(s) + 4sV(s) + 8V(s) = 48/s = (s^2+5s+8)V(s) \text{ or}$$

$$V(s) = 48/[s(s^2+4s+8)]$$

Step 2. Now we need to solve for the roots of the denominator and perform a partial fraction expansion. Then we can inverse transform the answer back into the time domain.

$s^2 + 4s + 8$ has the roots $s_{1,2} = \dfrac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$ thus,

$V(s) = 48/[s(s+2+j2)(s+2–j2)] = [A/s] + [B/(s+2+j2)] + [C/(s+2–j2)]$

where $A = 48/4 = 6$; $B = 48/[(–2–j2)(–j4)] = 48/(–8+j8) = 48(–1–j)/[8(–1+j)(–1–j)] = 6(–1–j)/2 = 3(–1–j)$; and $C = 48/[(–2+j2)(j4)] = 48/(–8–j8) = 48(–1+j)/[8(–1–j)(–1+j)] = 6(–1+j)/2 = 3(–1+j)$.

Therefore, $V(s) = [8/s] + [3(–1–j)/(s+2+j2)] + [3(–1+j)/(s+2–j2)]$

$v(t) =$ $\mathbf{[6 – 6e^{-2t}(\cos 2t + \sin 2t)]u(t)}$ **volts**

Chapter 16, Solution 9.

A series RLC circuit is described by

$$L\frac{d^2 i(t)}{dt} + R\frac{di(t)}{dt} + \frac{i(t)}{C} = 2$$

Find the response when L = 0.5 H, R = 4 Ω, and C = 0.2 F. Let $i(0^-)$ = 1 A and $[di(0^-)/dt] = 0$.

Solution

Step 1. First transform the equation into the s-domain. Then solve for I(s).

$0.5s^2I(s) – 0.5(di(0^-)/dt) – 0.5si(0^-) + 4sI(s) – 4i(0^-) + 5I(s) = 2/s$ or
$s^2I(s) – s + 8sI(s) – 8 + 10I(s) = 4/s$ or
$(s^2+8s+10)I(s) = s+8+4/s = (s^2+8s+4)/s$ or
$I(s) = (s^2+8s+4)/[s(s^2+8s+10)]$

Step 2. Next we need to find the roots of $(s^2+8s+10)$ and then perform a partial fraction expansion and then inverse transform back into the time domain.

$s_{1,2}$ = –1.5505 and –6.45

$(s^2+8s+2)/[s(s^2+8s+10)] = [A/s] + [B/(s+1.5505)] + [C/(s+6.45)]$

A = 0.4; B = 0.7898; and C = –0.1898 thus,

I(s) = [0.4/s] + [0.7898/(s+1.5505)] + [–0.1898/(s+6.45)] and

i(t) = $\mathbf{[400+789.8e^{–1.5505t}–189.8e^{–6.45t}]}$ **mA**.

Chapter 16, Solution 10.

The step responses of a series RLC circuit are

$$v_c = 40 - 10e^{-2000t} - 10e^{-4000t} \text{ V}, \ \ t>0$$
$$i_L(t) = 3e^{-2000t} + 6e^{-4000t} \text{ mA}, \ \ t>0$$

(a) Find C. (b) Determine what type of damping exhibited by the circuit.

Solution

(a) $$i_L(t) = i_C(t) = C\frac{dv_o}{dt} \qquad (1)$$

$$\frac{dv}{dt} = 2000x10e^{-2000t} + 4000x10e^{-4000t} = 2x10^4(e^{-2000t} + 2e^{-4000t}) \qquad (2)$$

But $i_L(t) = 3[e^{-2000t} + 2e^{-4000t}]x10^{-3}$ (3)

Substituting (2) and (3) into (1), we get

$$2x10^4 xC = 3x10^{-3} \longrightarrow C = 1.5x10^{-7} = \underline{150 \text{ nF}}$$

(b) Since s_1 = - 2000 and s_2 = - 4000 are real and negative, it is an **overdamped** case.

Chapter 16, Solution 11.

The step response of a parallel RLC circuit is

$$v = 10 + 20e^{-300t}(\cos 400t - 2\sin 400t) \text{ V}, \quad t \geq 0$$

when the inductor is 50 mH. Find R and C.

Solution

Step 1. There are different ways to approach this problem so, we will convert everything into the s-domain and then solve for the unknowns. We should also note that the steady-state voltage is 10 volts, then the circuit is a step input voltage across a parallel combination of a capacitor and an inductor all in series with an output resistor.

The nodal equation for this circuit is given by,

[(V–10/s)/R] + [(V–0)/(0.05s)] + [(V–0)/(1/sC)] + = 0 or

[(1/R)+(1/(0.05s))+sC]V = 10/(Rs) = [(20R+RCs^2+s)/(Rs)]V or

V = [10/(Rs)][Rs/(RCs^2+s+20R)] = 10/[(RCs)(s^2+(1/(RC))s+(20/C))]

Step 2. From the value of v(t) we can determine the value of the roots of the polynomial (s^2+(1/(RC))s+(20/C)) = (s+300+j400)(s+300–j400) thus, 20/C = 300^2+400^2 = 90,000 + 160,000 = 250,000 or

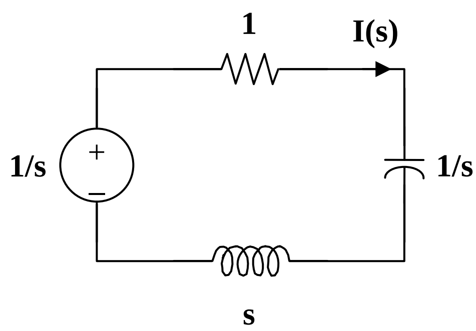
C = 20/250,000 = **80 μF**

and 1/(RC) = 600 or

R = 1/(600x80x10^{-6}) = **20.83 Ω**.

Chapter 16, Solution 12.

Consider the s-domain form of the circuit which is shown below.



$$I(s) = \frac{1/s}{1 + s + 1/s} = \frac{1}{s^2 + s + 1} = \frac{1}{(s + 1/2)^2 + (\sqrt{3}/2)^2}$$

$$i(t) = \frac{2}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2} t\right) u(t) \text{ A}$$

$$i(t) = \mathbf{1.155\, e^{-0.5t} \sin(0.866t) u(t) \ A}$$

Chapter 16, Solution 13.

Using Fig. 16.36, design a problem to help other students to better understand circuit analysis using Laplace transforms.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find v_x in the circuit shown in Fig. 16.36 given $v_s = 4u(t)$ V.

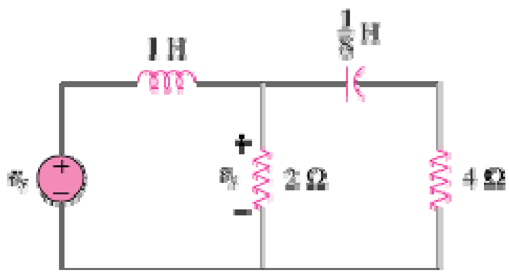
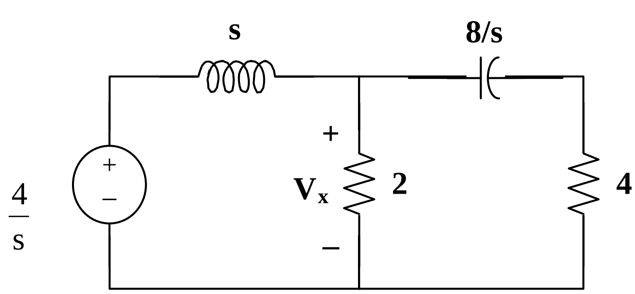


Figure 16.36
For Prob. 16.13.

Solution



$$\frac{V_x - \frac{4}{s}}{s} + \frac{V_x - 0}{2} + \frac{V_x - 0}{4 + \frac{8}{s}} = V_x(4s+8) - \frac{(16s+32)}{s} + (2s^2 + 4s)V_x + s^2 V_x = 0$$

$$V_x(3s^2 + 8s + 8) = \frac{16s+32}{s}$$

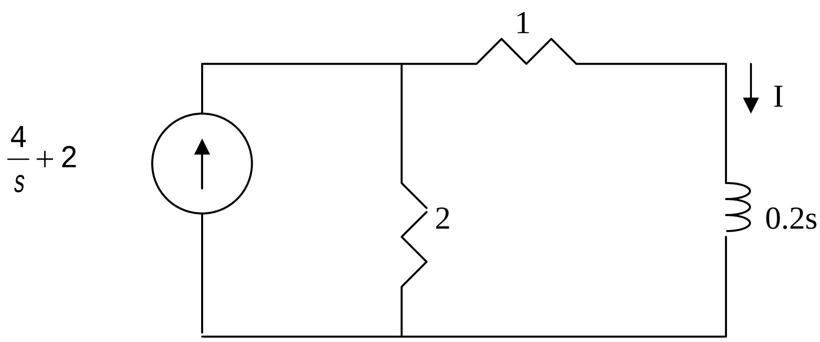
$$V_x = 16\frac{s+2}{s(3s^2+8s+8)} = 16\left(\frac{0.25}{s} + \frac{-0.125}{s + \frac{4}{3} + j\frac{\sqrt{8}}{3}} + \frac{-0.125}{s + \frac{4}{3} - j\frac{\sqrt{8}}{3}}\right)$$

$$v_x = \underline{(4 - 2e^{-(1.3333 + j0.9428)t} - 2e^{-(1.3333 - j0.9428)t})u(t)\,V}$$

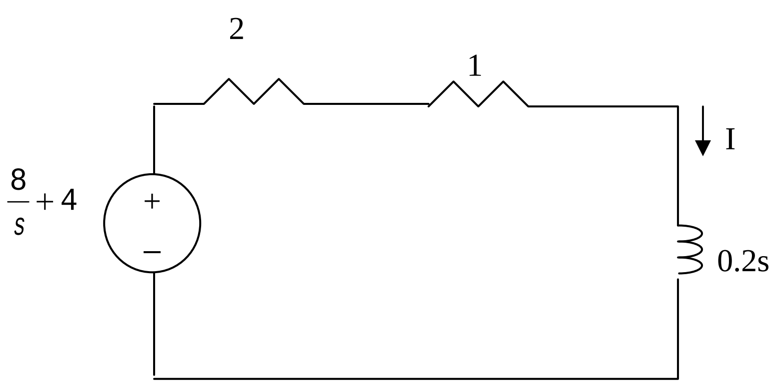
$$v_x = \left[\mathbf{4 - 4e^{-4t/3}\cos\left(\frac{2\sqrt{2}}{3}t\right)}\right]\mathbf{u(t)V}$$

Chapter 16, Solution 14.

In the s-domain, the circuit becomes that shown below.



We transform the current source to a voltage source and obtain the circuit shown below.



$$I = \frac{\frac{8}{s}+4}{3+0.2s} = \frac{20s+40}{s(s+15)} = \frac{A}{s} + \frac{B}{s+15}$$

$$A = \frac{40}{15} = \frac{8}{3}, \qquad B = \frac{-15x20+40}{-15} = \frac{52}{3}$$

$$I = \frac{8/3}{s} + \frac{52/3}{s+15}$$

i(t) = **[(2.667+17.333e^{-15t}]u(t) A**

Chapter 16, Solution 15.

For the circuit in Fig. 16.38, calculate the value of *R* needed to have a critically damped response.

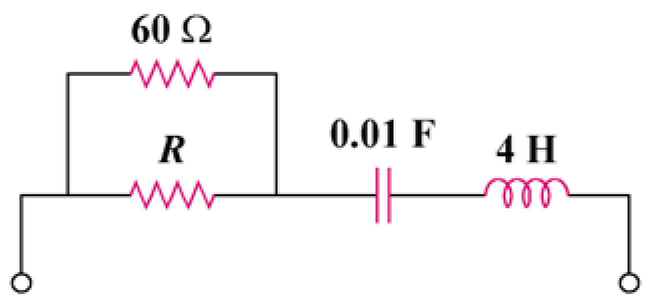


Figure 16.38
For Prob. 16.15.

8.13

Solution

Step 1. Let R||60 = R_o. Next, convert the circuit into the s-domain and solve for T(s) = R_o + [1/(0.01s)] + 4s = R_o + (100/s) + 4s = [(4s^2+R_os+100)/s]. Now to solve for the roots that represent a critically damped system.

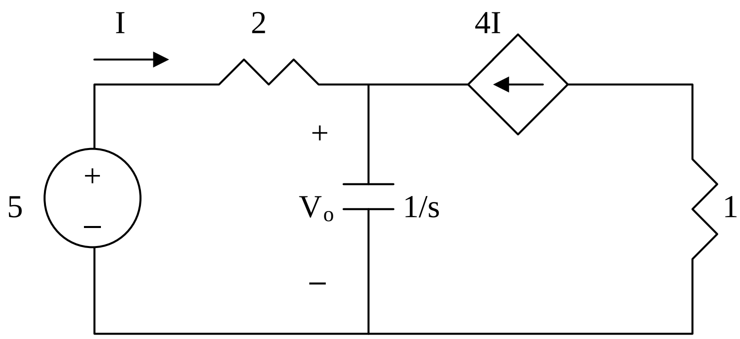
$s_{1,2} = \{-R_o \pm [(R_o)^2-4(400)]^{0.5}\}/2.$

The system is critically damped when $[(R_o)^2-4(400)] = 0$.

Step 2. $(R_o)^2 = 1600$ or $R_o = 40$. Since $R_o = [Rx60/(R+60)] = 40$ or 60R = 40R+2400 or 20R = 2400 or R = **120 Ω**.

Chapter 16, Solution 16.

The circuit in the s-domain is shown below.



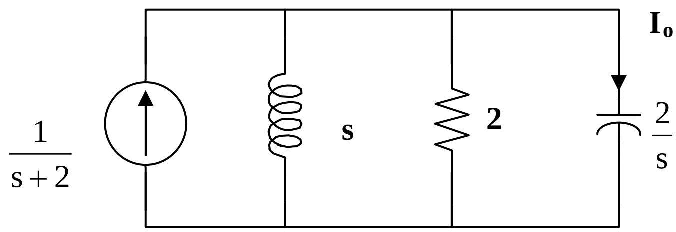
$$I + 4I = \frac{V_o}{1/s} \quad \longrightarrow \quad 5I = sV_o$$

But $I = \dfrac{5 - V_o}{2}$

$$5\left(\frac{5 - V_o}{2}\right) = sV_o \quad \longrightarrow \quad V_o = \frac{12.5}{s + 5/2}$$

$v_o(t) =$ $\mathbf{12.5e^{-2.5t}u(t)\ V}$

Chapter 16, Solution 17.



(-1+-sqrt(1-8))/2 = (-1+-(j2.646))/2 = -0.5+-j(1.3229)

$$V = \frac{1}{s+2}\left(\frac{1}{\frac{1}{s}+\frac{1}{2}+\frac{s}{2}}\right) = \frac{1}{s+2}\left(\frac{2s}{s^2+s+2}\right) = \frac{2s}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$I_o = \frac{Vs}{2} = \frac{s^2}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$= \frac{1}{s+2} + \frac{\frac{(-0.5-j1.3229)^2}{(1.5-j1.3229)(-j2.646)}}{s+0.5+j1.3229} + \frac{\frac{(-0.5+j1.3229)^2}{(1.5+j1.3229)(+j2.646)}}{s+0.5-j1.3229}$$

$$i_o(t) = \underline{\left(e^{-2t} + 0.3779e^{-90°}e^{-t/2}e^{-j1.3229t} + 0.3779e^{90°}e^{-t/2}e^{j1.3229t}\right)u(t)\,A}$$

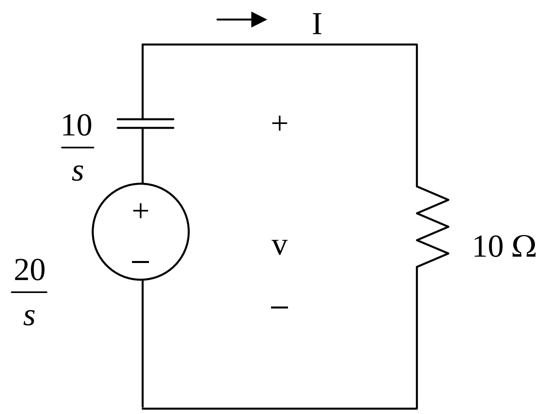
or

$$\underline{= \left(e^{-2t} - 0.7559e^{-0.5t}\sin 1.3229t\right)u(t)\,A}$$

$$\text{or } i_o(t) = \left(\mathbf{e^{-2t} - \frac{2}{\sqrt{7}}e^{-0.5t}\sin\left(\frac{\sqrt{7}}{2}t\right)}\right)\mathbf{u(t)A}$$

Chapter 16, Solution 18.

For $t<0$, $v(0) = v_s = 20$ V
For $t>0$, the circuit in the s-domain is as shown below.



$$100mF = 0.1F \longrightarrow \frac{1}{sC} = \frac{10}{s}$$

$$I = \frac{20/s}{10 + 10/s} = \frac{2}{s+1}$$

$$V = 10I = \frac{20}{s+1}$$

$$v(t) = \underline{20e^{-t}u(t)}$$

Chapter 16, Solution 19.

The switch in Fig. 16.42 moves from position A to position B at t=0 (please note that the switch must connect to point B before it breaks the connection at A, a make before break switch). Find v(t) for t >0.

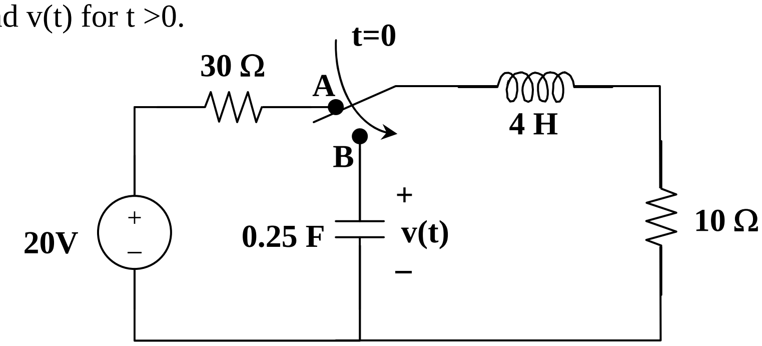
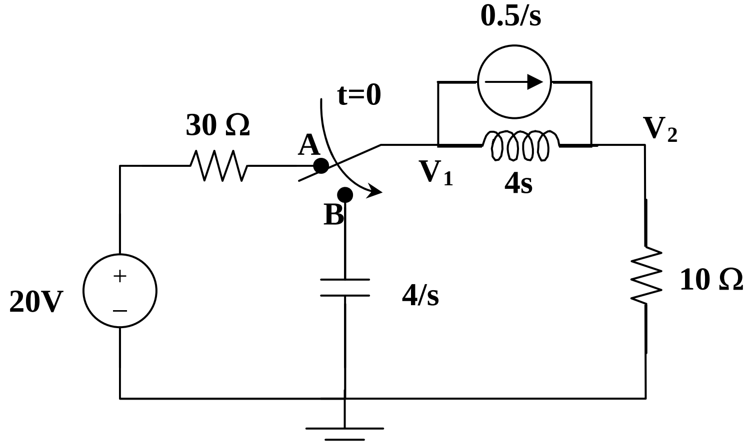


Figure 16.42
For Prob. 16.19.

Solution

Step 1. First find all the initial conditions and then transform into the s-domain. Since the capacitor is not connected to a circuit, we do not know its initial condition so we can assume it is zero (v(0) = 0). We can find $i_L(0)$ by letting the inductor be a short and $i_L(0) = 20/40 = 0.5$ amp.



$[(V_1–0)/(4/s)] + [(V_1–V_2)/(4s)] + (0.5/s) = 0$ and $[(V_2–V_1)/(4s)] + (–0.5/s) + [(V_2–0)/10] = 0$ where $V = V_1$. Next, add these together, $[sV_1/4] + [V_2/10] = 0$ or $V_2 = –2.5sV_1$. Now we can solve for V_1 and V.

Step 2. $[(s/4)+(1/(4s))+(2.5s/(4s))]V_1 = –0.5/s$
$= [(s^2+2.5s+1)/(4s)]V_1$ or $V_1 = –0.5(4)/(s^2+2.5s+1) = –2/[(s+0.5)(s+2)]$
$= [–1.3333/(s+0.5)]+[1.3333/(s+2)]$ or

$$v(t) = \mathbf{[–1.3333e^{–t/2}+1.3333e^{–2t}]u(t)\ volts}.$$

Chapter 16, Solution 20.

Find $i(t)$ for $t > 0$ in the circuit of Fig. 8.43.

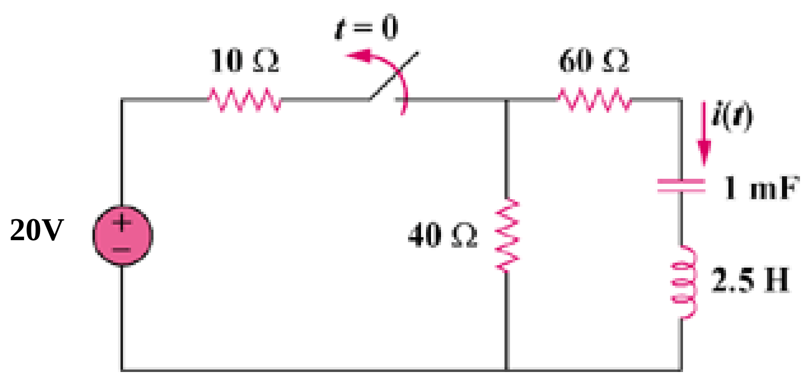
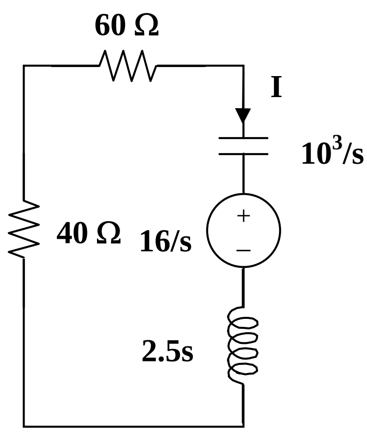


Figure 16.43
For Prob. 16.20.

8.16

Step 1. Convert the circuit into the s-domain and write one loop equation noting that i(0) = 0 and $v_c(0) = 16$ volts.



$[(1000/s)]I + [16/s] + [2.5s]I + [40+60]I = 0$ or
$[(2.5s^2+100s+1000)/s]I = -16/s$ or $I = -16/[2.5(s^2+40s+400)] = -.64/(s+20)^2$

Step 2. $I = [A/(s+20)] + [B/(s+20)^2]$ where $B = -64$ so $A = 0$. Thus,

$$i(t) = \mathbf{6.4te^{-20t}u(t)\ A}.$$

Chapter 16, Solution 21.

In the circuit of Fig. 16.44, the switch moves (make before break switch) from position *A* to *B* at $t = 0$. Find $v(t)$ for all $t \geq 0$.

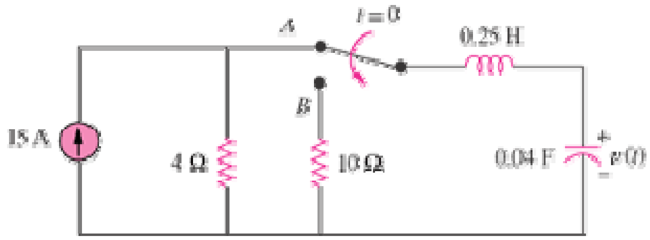
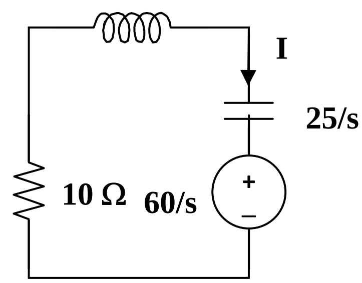


Figure 16.44
For Prob. 16.21.

Solution

Step 1. First we need to find our initial conditions, clearly $i(0) = 0$ and $v(0) = 4 \times 15 = 60$ volts. Next we convert the circuit into the s-domain. We can then write a mesh equation and solve for $v(t)$.



$[10+(s/4)+(25/s)]I + 60/s = 0$ and $V = (25/s)I + 60/s$

Step 2. $I = -(60/s)/[\,10+(s/4)+(25/s)] = -(60/s)\{4s/[s^2+40s+100]$
$= -240/[(s+2.679)(s+37.32)] = [A/(s+2.679)]+[B/(s+37.32)]$ where
$A = -240/(-2.679+37.32) = -6.928$ and $B = -240/(-37.32+2.679) = 6.928$.

This now leads to $V = (25/s)I+60/s$
$= \{(25)(-6.928)/[s(s+2.679)]\}+\{(25)(6.928)/[s(s+37.32)]\}+60/s$
$= \{-173.2/[s(s+2.679)]\}+\{173.2/[s(s+37.32)]\}+60/s$
$= [a/s]+[b/(s+2.679)]+[c/(s+37.32)]$ where
$a = [-173.2/2.679]+[173.2/37.32]+60 = -64.65+4.641+60 = -0.009$ (In practice and theoretically, this term must be equal to be zero since there will be no energy in the circuit at $t = \infty$!);
$b = [-173.2/(-2.679)] = 64.65$; and $c = 173.2/(-37.32) = -4.641$ or -4.65 if we correct the rounding errors. Thus,

$$v(t) = \mathbf{[64.65e^{-2.679t} - 4.65e^{-37.32t}]u(t)\ volts}.$$

Chapter 16, Solution 22.

Find the voltage across the capacitor as a function of time for $t > 0$ for the circuit in Fig. 16.45. Assume steady-state conditions exist at $t = 0^-$.

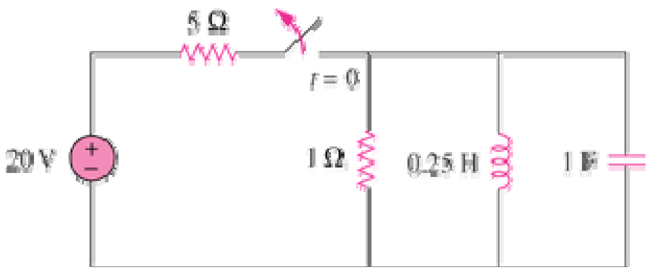
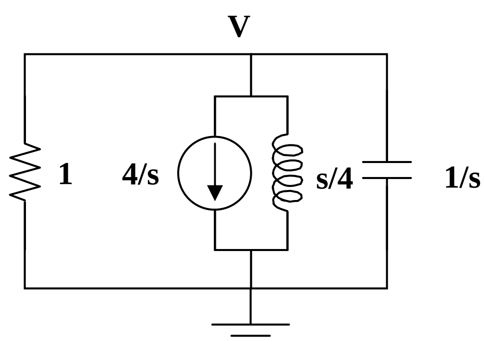


Figure 16.45
For Prob. 16.22.

Solution

Step 1. First we need to calculate the initial conditions, $v_C(0) = 0$ and $i_L(0) = 20/5 = 4$ amps. Next we need to convert the circuit into the s-domain and solve for the node voltage $V = V_C$. Convert this back into the time domain and obtain $v_C(t)$.



$[(V–0)/1]+[4/s]+[(V–0)/(s/4)]+[(V–0)/(1/s)] = 0$ then solve for V, next complete a partial fraction expansion, and then convert back into the time domain.

Step 2. $[1+(4/s)+s]V = [(s^2+s+4)/s]V = –4/s$ or
$V = –4/[(s+0.5+j1.9365)(s+0.5–j1.9365)]$
$= [A/(s+0.5+j1.9365)] + [B/(s+0.5–j1.9365)]$ where $A = –4/(–j3.873)$
$= 1.0328\angle–90^\circ$ and $B = –4/(j3.873) = 1.0328\angle90^\circ$. Thus,

$v_C(t) = 1.0328e^{–t/2}[e^{–(j1.9365t+90^\circ)}+e^{(j1.9365t+90^\circ)}]u(t)$

$=$ **$2.066e^{–t/2}\cos(1.9365t+90^\circ)u(t)$ volts**.

Chapter 16, Solution 23.

Obtain $v(t)$ for $t > 0$ in the circuit of Fig. 16.46.

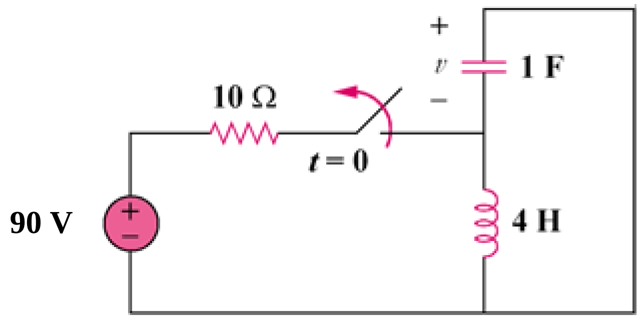
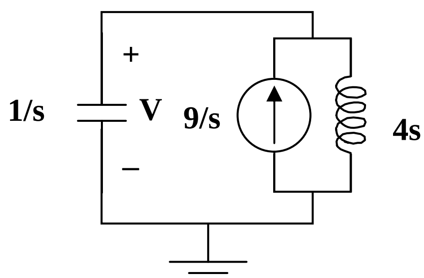


Figure 16.46
For Prob. 16.23.

Solution

Step 1. First we need to calculate the initial conditions. Clearly since the inductor looks like a short, v(0) = 0 and $i_L(0) = 90/10 = 9$ amps. Next we convert the circuit into the s-domain and solve for V and then obtain the partial fraction expansion and convert back into the time domain.



$[(V–0)/(1/s)]+(–9/s)+[(V–0)/4s] = 0$

Step 2.
$[s+(1/(4s))]V = 9/s = [(s^2+0.25)/(4s)]V$ or $V = 36/[(s+j0.5)(s–j0.5)]$
$= [A/(s+j0.5)] + [B/(s–j0.5)]$ where $A = 36/(–j) = 36\angle 90^\circ$ and
$B = 36/(j) = 36\angle –90^\circ$. Thus,
$v(t) = 36[e^{–(j0.5t–90^\circ)}+e^{(j0.5t–90^\circ)}]u(t)$

$=$ **$18\cos(0.5t–90^\circ)u(t)$ volts**.

Chapter 8, Solution 24.

The switch in the circuit of Fig. 16.47 has been closed for a long time but is opened at $t = 0$. Determine $i(t)$ for $t > 0$.

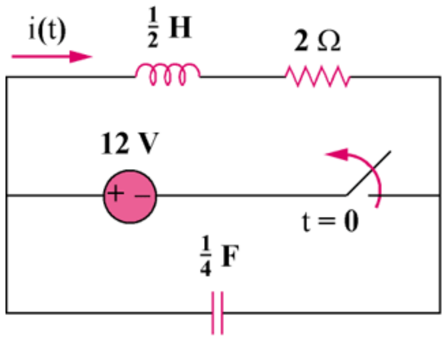
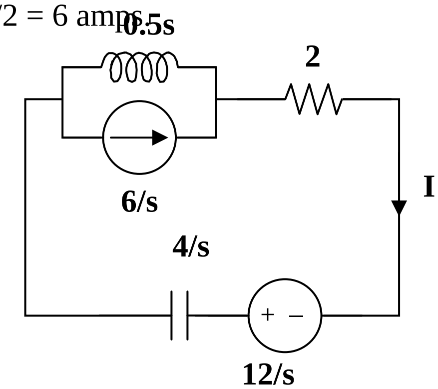


Figure 16.47
For Prob. 16.24.

8.20

Solution

Step 1. First we solve for the initial conditions and then convert the circuit into the s-domain and then solve for I, perform a partial fraction expansion, then convert back into the time domain. We recognize that the capacitor becomes an open circuit and the inductor becomes a short circuit at t = 0^{-}. Therefore, v(0) = 12 volts and i(0) = 12/2 = 6 amps.



We can use mesh analysis, $-(12/s) + (4/s)I + (0.5s)(I-6/s) + 2I = 0$.

Step 2. $[(4/s)+0.5s+2]I = (12/s)+(3) = (3s+12)/s = [(s^2+4s+8)/(2s)]I$ or
$I = (6s+24)/[(s+2+j2)(s+2-j2)] = [A/(s+2+j2)] + [B/(s+2-j2)]$ where
$A = (-12-j12+24)/(-j4) = 16.97\angle{-45^\circ}/4\angle{-90^\circ} = 4.243\angle 45^\circ$ and
$B = (-12+j12+24)/(j4) = 4.243\angle{-45^\circ}$. Thus,
$i(t) = 4.243e^{-2t}[e^{-(j2t-45^\circ)}+e^{(j2t-45^\circ)}]u(t)$

$= \mathbf{8.486e^{-2t}\cos(2t-45^\circ)}$ **amps**.

Chapter 16, Problem 25.

Calculate $v(t)$ for $t > 0$ in the circuit of Fig. 16.48.

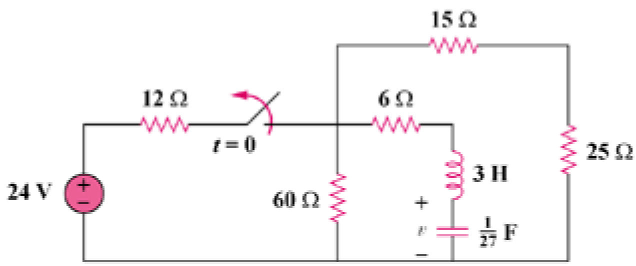
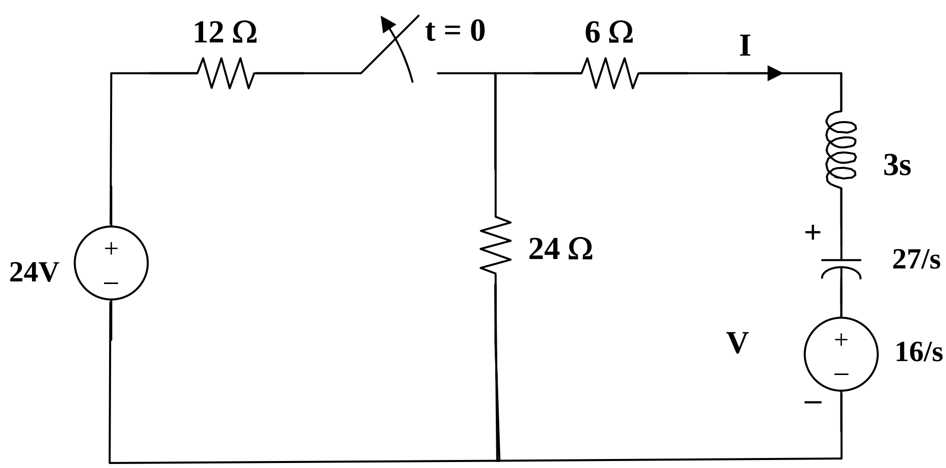


Figure 16.48
For Prob. 16.25.

Step 1. First solve for the initial conditions. Then simplify the circuit and then convert it into the s-domain and then solve for v(t). Since the capacitor becomes an open circuit, $i_L(0) = 0$ and $v(0) = (24)24/36 = 16$ volts.



We can now use mesh analysis to solve for V(s). $(30+3s+27/s)I + 16/s = 0$ or $[3(s^2+10s+9)/s]I = -16/s$ or $I = -(16/3)/[(s+1)(s+9)]$ and $v = (27/s)I + 16/s$.

Step 2. $V = -(16/3)(27)/[s(s+1)(s+9)] + 16/s = -144/[s(s+1)(s+9)] + 16/s$. Thus,

$V = [A/s]+[B/(s+1)]+[C/(s+9)]$ where $A = -(144/9)+16 = 0$ (as expected) and $B = -144/[(-1)(-1+9)] = 144/8 = 18$ and $C = -144/[(-9)(-9+1)] = -144/72 = -2$.

$v(t) =$ **$[18e^{-t}-2e^{-9t}]u(t)$ volts**.

Chapter 16, Problem 26.

The switch in Fig. 16.49 moves from position A to position B at t=0 (please note that the switch must connect to point B before it breaks the connection at A, a make before break switch). Determine i(t) for t >0. Also assume that the initial voltage on the capacitor is zero.

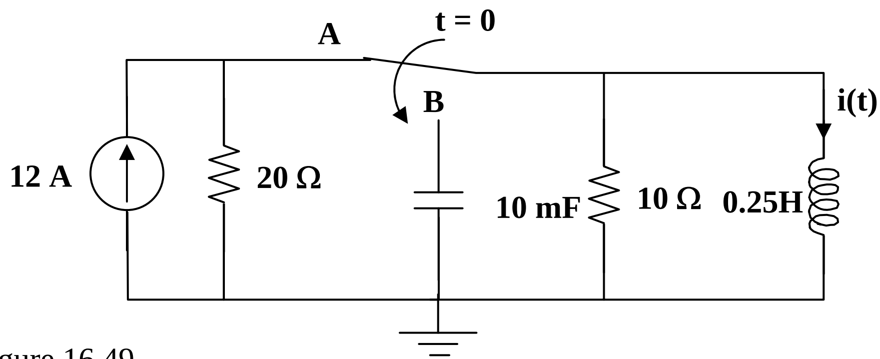
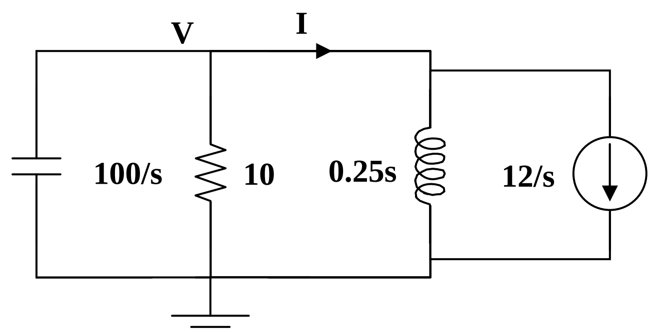


Figure 16.49
For Prob. 16.26.

Solution

Step 1. Determine the initial conditions and then convert the circuit into the s-domain. Then solve for V and then find I. Convert it into the time domain. It is clear from the circuit that $i_L(0) = 12$ A.



Applying nodal analysis we get,
$[(V–0)/(100/s)]+[(V–0)/10]+[(V–0)/(0.25s)]+12/s = 0$ where
$I = [(V–0)/(0.25s)]+12/s$.

Step 2. $[(s^2+10s+400)/(100s)]V = –12/s$ or
$V = –1200/[(s+5+j19.365)(s+5–j19.365)]$ or
$I = –\{4800/[s(s+5+j19.365)(s+5–j19.365)]\}+12/s$
$= [A/s]+[B/(s+5+j19.365)]+[C/(s+5–j19.365)]$ where $A = –12+12 = 0$, as to be expected; $B = –4800/[(–5–j19.365)(–j38.73)]$
$= 4800\angle 180^\circ/[(20\angle –104.48)(38.73\angle –90^\circ)] = 6.197\angle 14.48^\circ$; and
$C = –4800/[(–5+j19.365)(j38.73)] = 4800\angle 180^\circ/[(20\angle 104.48)(38.73\angle 90^\circ)]$
$= 6.197\angle –14.48^\circ$.

$i(t) =$ $\mathbf{[12.394e^{–5t}\cos(19.365t+14.48^\circ)]u(t)}$ **amps**.

Chapter 16, Problem 27.

Find $v(t)$ for $t > 0$ in the circuit in Fig. 16.50.

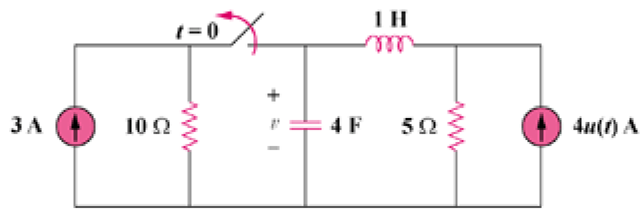


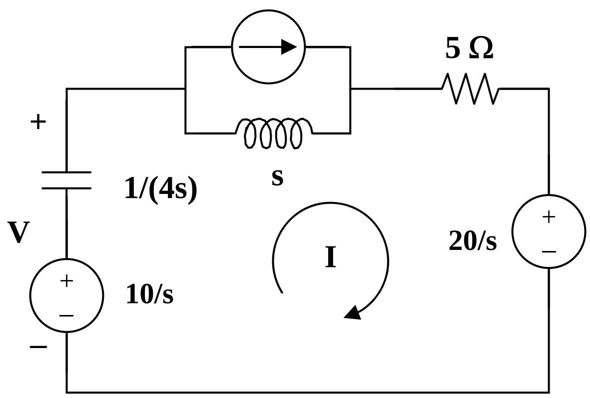
Figure 16.50
For Problem 16.27.

Solution

Step 1. First we need to determine the initial conditions. We note that the source on the right is equal to zero until the switch opens. So, all initial conditions come from the 3-amp source on the left. Since the capacitor looks like an open and the inductor looks like a short we get,

v(0) = 3[5x10/(5+10)] = 10 volts and $i_L(0)$ = 10/5 = 2 amps.

Next we convert the circuit (t > 0) into the s-domain with initial conditions. Then we can solve for V, perform a partial fraction expansion and solve for v(t).



–[10/s]+[1/(4s)]I+[s(I–(2/s))]+5I+[20/s] = 0 and V = [1/(4s)](–I) + [10/s]

Step 2. {[1/(4s)]+s+5}I = {[s^2+5s+0.25]/(s)}I = 2–10/s = 2(s–5)/s or
I = 2(s–5)/[(s+0.05051)(s+4.949)] and
V = {–2(s–5)/[(4s)(s+0.05051)(s+4.949)]}+10/s
= {–0.5(s–5)/[s(s+0.05051)(s+4.949)]}+10/s

V = [A/s] + [B/(s+0.05051)] + [C/(s+4.949)] where

A = [2.5/[(0.05051)(4.949)]]+10 = 20;
B = –0.5(–0.05051–5)/[(–0.05051)(–0.05051+4.949)]
= 2.52525/(–0.24742) = –10.206; and
C = –0.5(–4.949–5)/[(–4.949)(–4.949+0.05051)] = 4.9745/24.243 = 0.2052.

$$v(t) = \mathbf{[20-10.206e^{-0.05051t}+0.2052e^{-4.949t}]u(t)\ volts}.$$

dv/dt = –10.206(–0.05051)+0.2052(–4.949) = 0.5155–1.0155 = –0.5 or
Cdv/dt = –4x0.5 = –2 amps, the answer checks!

Chapter 16, Problem 28.

For the circuit in Fig. 16.51, find $v(t)$ for $t > 0$.

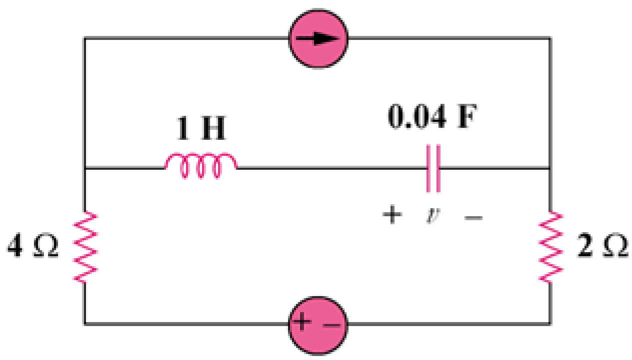
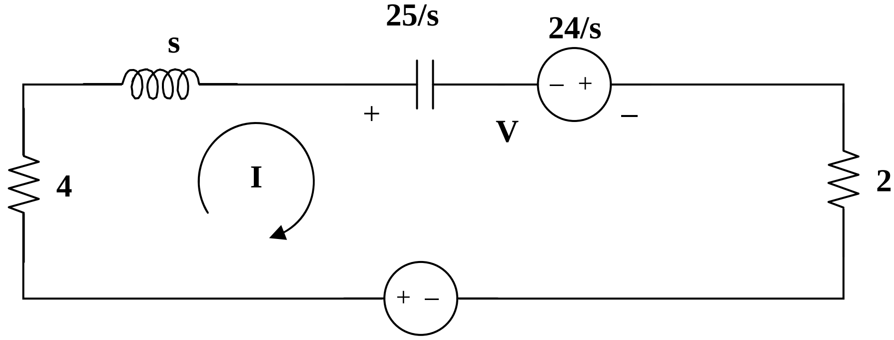


Figure 16.51
For Prob. 16.28.

Solution

Step 1. Determine the initial conditions (at t = 0, the 4 amp current source turns off and the 100 volt voltage source becomes active). Since the capacitor becomes an open circuit, $i_L(0) = 0$ and $v(0) = –4x6 = –24$ volts. Now convert the circuit into the s-domain and solve for V and then convert it into the time domain to obtain v(t),



Now for the mesh equation, $[4+s+(25/s)+2]I–(24/s)–(100/s) = 0$. $V = (25/s)I–24/s$.

Step 2. $[(s^2+6s+25)/s]I = 124/s$ or $I = 124/(s^2+6s+25) = 124/[(s+3+j4)(s+3–j4)]$ thus,

$V = \{3100/[s(s+3+j4)(s+3–j4)]\}–24/s = [A/s]+[B/(s+3+j4)]+[C/(s+3–j4)]$ where
$A = (3100/25)–24 = 124–24 = 100$; $B = 3100/[(–3–j4)(–j8)]$
$= 3100/[(5\angle–126.87^\circ)(8\angle–90^\circ)] = 77.5\angle–143.13^\circ$; and
$C = 3100/[(j8)(–3+j4)] = 3100/[(5\angle126.87^\circ)(8\angle90^\circ)] = 77.5\angle143.13^\circ$.

$v(t) =$ **$[100+155e^{–3t}\cos(4t+143.13^\circ)]u(t)$ volts**.

Chapter 16, Problem 29.

Calculate $i(t)$ for $t > 0$ in the circuit in Fig. 16.52.

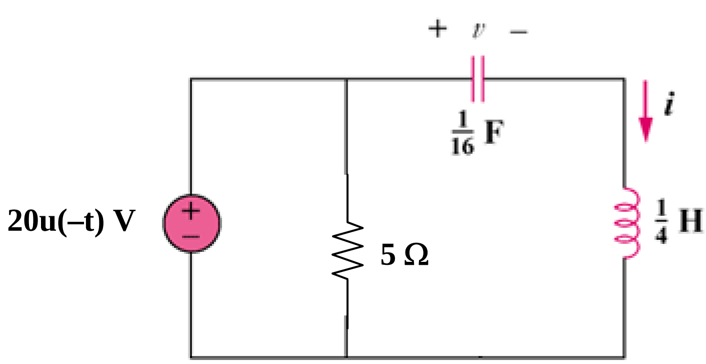
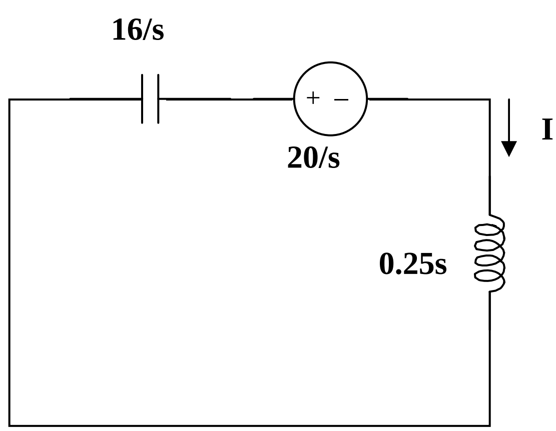


Figure 16.52
For Prob. 16.29.

Solution

Step 1. Calculate the initial conditions and then convert the above circuit into the s-domain. Then solve for I, perform a partial fraction expansion, and convert into the time domain. v(0) = 20 volts and i(0) = 0.



$[16/s]I + [20/s] + 0.25sI = 0.$

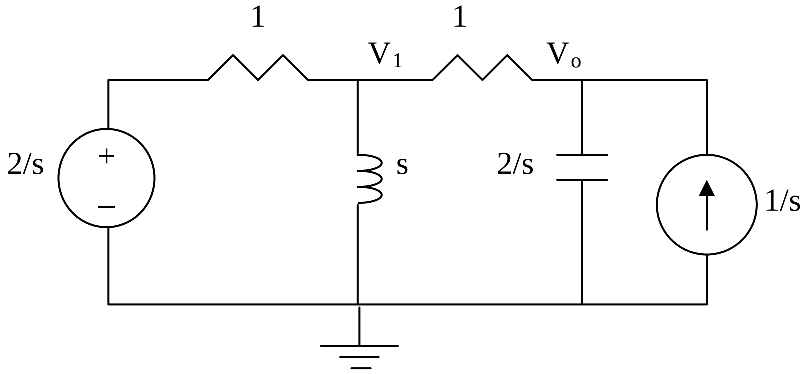
Step 2. $\{[16/s]+0.25s\}I = -20/s = \{[s^2+64]/(4s)\}I$ or $I = -80/[(s+j8)(s-j8)]$ or

$I = [A/(s+j8)] + [B/(s-j8)]$ where $A = -80/(-j16) = 5\angle-90^\circ$ and
$B = -80/(j16) = 5\angle 90^\circ$ thus,

$$i(t) = [5e^{-j8t-90^\circ}+5e^{j8t+90^\circ}]u(t) = \mathbf{10cos(8t+90^\circ)u(t)\ amps}.$$

Chapter 16, Solution 30.

The circuit in the s-domain is shown below. Please note, $i_L(0) = 0$ and $v_o(0) = o$ because both sources were equal to zero for all t<0.



At node 1

$[(V_1–2/s)/1] + [(V_1–0)/s] + [(V_1–V_2)/1] = 0$ or $[1+(1/s)+1]V_1–V_2 = 2/s$ or
$[(2s+1)/s]V_1–V_2 = 2/s$

At node o

$[(V_o–V_1)/1] + [(V_o–0)/(2/s)] – (1/s) = 0$ or
$–V_1+[(s+2)/2]V_o = 1/s$

In matrix form we get,

$$\begin{bmatrix} \frac{2s+1}{s} & -1 \\ -1 & \frac{s+2}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \frac{2}{s} \\ \frac{1}{s} \end{bmatrix}$$ or

$$\begin{bmatrix} V_1 \\ V_o \end{bmatrix} = \frac{\begin{bmatrix} \frac{s+2}{2} & 1 \\ 1 & \frac{2s+1}{s} \end{bmatrix}}{\frac{(2s+1)(s+2)}{2s} - (-1)(-1)} \begin{bmatrix} \frac{2}{s} \\ \frac{1}{s} \end{bmatrix} = \frac{\begin{bmatrix} \frac{s+2}{2} & 1 \\ 1 & \frac{2s+1}{s} \end{bmatrix}}{\frac{s^2+1.5s+1}{s}} \begin{bmatrix} \frac{2}{s} \\ \frac{1}{s} \end{bmatrix}$$

$s^2+1.5s+1 = (s+0.75+j0.6614)(s+0.75–j0.6614)$

$V_o = s[(2/s)+(2s+1)/s^2]/[(s+0.75+j0.6614)(s+0.75–j0.6614)]$
$= (4s+1)/[s(s+0.75+j0.6614)(s+0.75–j0.6614)]$
$= [A/s] + [B/(s+0.75+j0.6614)] + [C/(s+0.75–j0.6614]$ where $A = 1$;
$B = [4(–0.75–j0.6614)+1]/[(–0.75–j0.6614)(–j1.3228)]$
$= [–3–j2.6456+1]/[(1\angle–138.59^\circ)(1.3228\angle–90^\circ)]$
$= (3.3165\angle–127.09^\circ)/[(1\angle–138.59^\circ)(1.3228\angle–90^\circ)] = 2.507\angle101.5^\circ$
$C = [4(–0.75+j0.6614)+1]/[(–0.75+j0.6614)(j1.3228)]$
$= [–3+j2.6456+1]/[(1\angle138.59^\circ)(1.3228\angle90^\circ)]$
$= (3.3165\angle127.09^\circ)/[(1\angle138.59^\circ)(1.3228\angle90^\circ)] = 2.507\angle–101.5^\circ$

Therefore,

$$v_o(t) = [1+2.507e^{-0.75t}e^{-(j0.6614t-101.5^\circ)}+2.507e^{-0.75t}e^{(j0.6614t-101.5^\circ)}]u(t) \text{ volts or}$$

$$= \mathbf{[1+5.014e^{-0.75t}\cos(0.6614t-101.5^\circ)]u(t)\ volts}.$$

An alternate solution is,

$$V_o = \frac{(4s+1)}{s(s^2+1.5s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1.5s+1}$$

$$4s+1 = A(s^2+1.5s+1)+Bs^2+Cs$$

We equate coefficients.

s^2 : 0 = A+ B or B = –A

s: 4=1.5A + C

constant: 1 = A, B=–1, C = 4–1.5A = 2.5

$$V_o = \frac{1}{s} + \frac{-s+2.5}{s^2+1.5s+1} = \frac{1}{s} - \frac{s+3/4}{(s+3/4)^2+\left(\frac{\sqrt{7}}{4}\right)^2} + \frac{\frac{3.25}{\frac{\sqrt{7}}{4}}x\frac{\sqrt{7}}{4}}{(s+3/4)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$$ where $\frac{\sqrt{7}}{4}$ = 0.6614.

This now leads to,

$$v_o(t) = \mathbf{[1-e^{-3t/4}\cos(0.6614t)+4.914e^{-3t/4}\sin(0.6614t)]u(t)\ volts}.$$

Chapter 16, Solution 31.

Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit in Fig. 16.54.

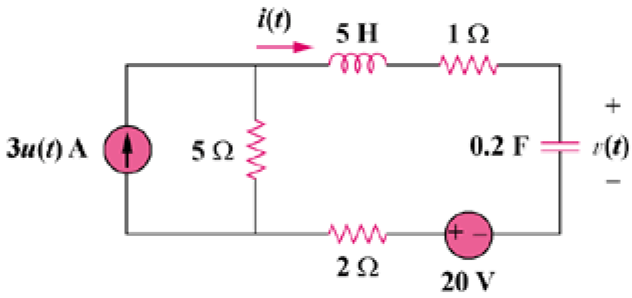
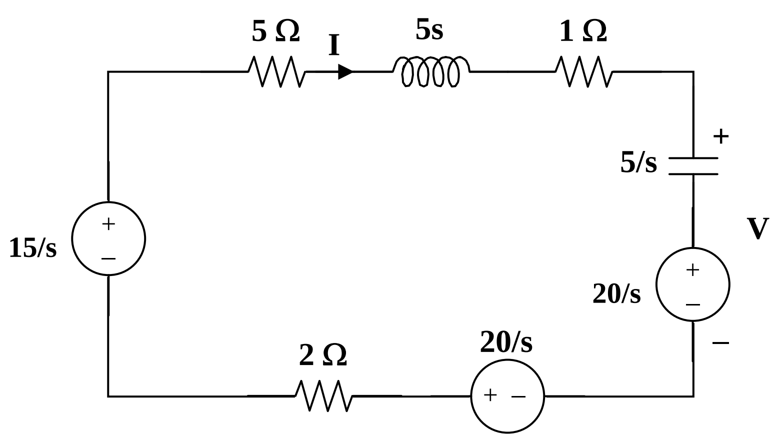


Figure 16.54
For Prob. 16.31.

Solution

Step 1. First, find the initial conditions and then transform the above circuit into the s-domain after converting the current source in parallel with the 5-ohm resistor into a 15 volts voltage source in series with a 5-ohm resistor. Then solve for V and I, perform a partial fraction expansion on each and then convert back into the time domain. The steady state the values are i(0) = 0 and v(0) = 20 volts.



–[15/s]+5I+(5s)I+1I+[5/s]I+[20/s]–[20/s]+2I = 0 and V = {[5/s]I+[20/s].

Step 2. {5+[5s]+1+[5/s]+2}I = [15/s]–[20/s]+[20/s] = 15/s or

$\{5[s^2+1.6s+1]/s\}I$ = 15/s or I = 3/[(s+0.8+j0.6)(s+0.8–j0.6)] and
V = {[5/s]I+[20/s] = 15/[s(s+0.8+j0.6)(s+0.8–j0.6)] + 20/s
= [A/s] + [B/(s+0.8+j0.6)] + [C/(s+0.8–j0.6)] where A = [15/(0.64+0.36)]+20 = 35; B = 15/[(–0.8–j0.6)(–j1.2)] = 12.5∠90˚/1∠–143.13˚ = 12.5∠–126.87˚; and C = 15/[(–0.8+j0.6)(j1.2)] = 12.5∠–90˚/(1∠143.13˚ = 12.5∠126.87˚.

$v(t) = [35+12.5e^{-0.8t-j0.6t-126.87^\circ}+12.\ 5e^{-0.8t+j0.6t+126.87^\circ}]u(t)$ volts

$$= \mathbf{[35+25e^{-0.8t}\cos(0.6t+126.87^\circ)]u(t)\ volts}.$$

$I = 3/[(s+0.8+j0.6)(s+0.8-j0.6)] = [A/(s+0.8+j0.6)] + [B/(s+0.8-j0.6)]$ where $A = 3/(-j1.2) = 2.5\angle 90^\circ$ and $B = 3/(j1.2) = 2.5\angle -90^\circ$. Thus,

$i(t) = 2.5e^{-0.8t}[e^{-j0.6t+90^\circ}+e^{j0.6t-90^\circ}]u(t)$

$$= \mathbf{5e^{-0.8t}[\cos(0.6t-90^\circ)]u(t)\ amps}.$$

Chapter 16, Solution 32.

For the network in Fig. 16.55, solve for $i(t)$ for $t > 0$.

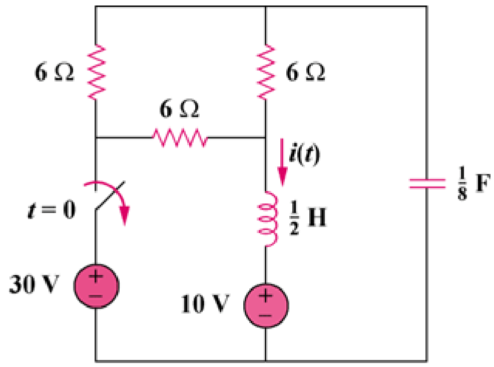
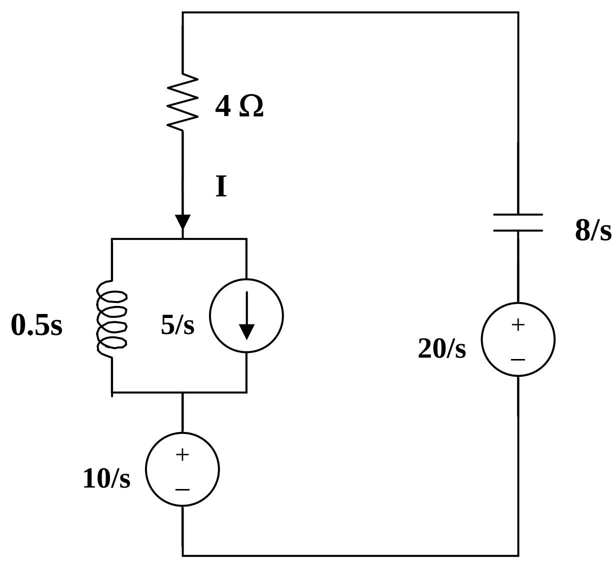


Figure 16.55
For Prob. 16.32.

Solution

Step 1. First we need to find all the initial conditions. Then we need to transform the circuit into the s-domain and solve for I. We then perform a partial fraction expansion and convert the results into the time domain. The inductor becomes a short and the capacitor becomes an open circuit. Thus, $i(0) = [20/6] + [20/12] = 5$ amps and $v_C(0) = 10 + 10 = 20$ volts.



Loop equation, $-[10/s]-0.5s(I-5/s)-4I-[8/s]I+[20/s] = 0$.

Step 2. $[0.5s+4+8/s]I = [(s^2+8s+16)/(2s)]I = -[10/s]+2.5+20/s = (s+4)/(0.4s)$ or

$I = 5(s+4)/[(s+4)^2] = [A/(s+4)]+[B/(s+4)^2]$ where $A(s+4)+B = 5(s+4)$ or $A = 5$ and $B = 0$. Therefore,

$$i(t) = \mathbf{[5e^{-4t}]u(t)\ amps}.$$

Chapter 16, Solution 33.

Using Fig. 16.56, design a problem to help other students to better understand how to use Thevenin's theorem (in the s-domain) to aid in circuit analysis.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use Thevenin's theorem to determine $v_o(t)$, t>0 in the circuit of Fig. 16.56.

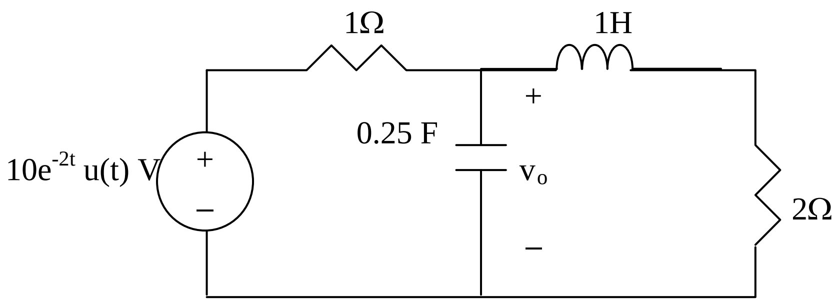


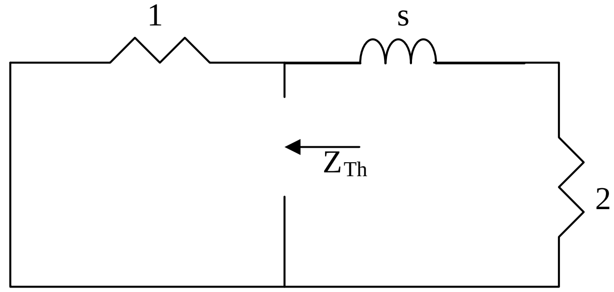
Figure 16.56 For Prob. 16.33.

Solution

$1H \longrightarrow 1s$ and $i_L(0) = 0$ (the sources is zero for all t<0).

$\frac{1}{4}F \longrightarrow \frac{1}{sC} = \frac{4}{s}$ and $v_C(0) = 0$ (again, there are no source contributions for all t<0).

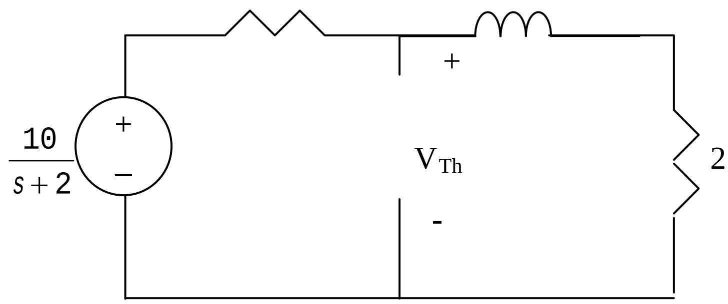
To find Z_{Th}, consider the circuit below.



$$Z_{Th} = 1//(s+2) = \frac{s+2}{s+3}$$

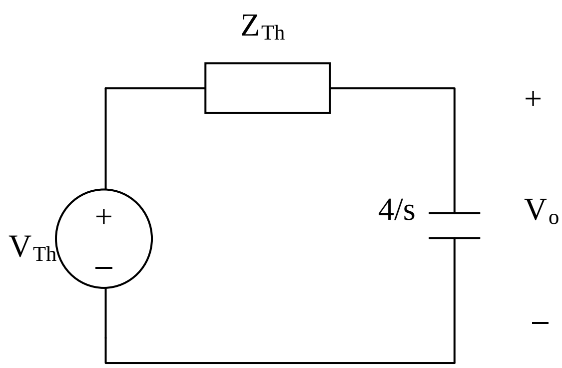
To find V_{Th}, consider the circuit below.

1 s



$$V_{Th} = \frac{s+2}{s+3} \bullet \frac{10}{s+2} = \frac{10}{s+3}$$

The Thevenin equivalent circuit is shown below

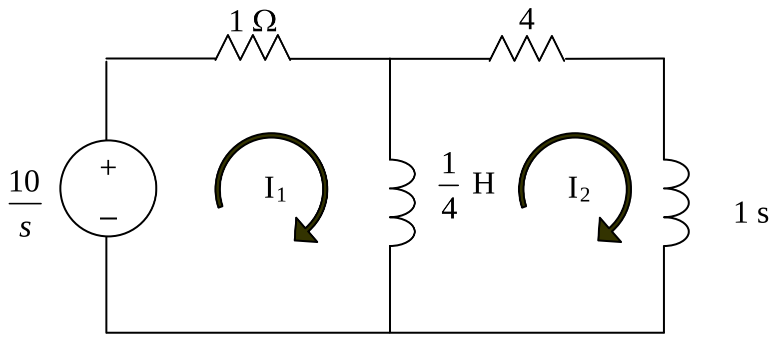


$$V_o = \frac{\frac{4}{s}}{\frac{4}{s}+Z_{Th}} V_{Th} = \frac{\frac{4}{s}}{\frac{4}{s}+\frac{s+2}{s+3}} \bullet \frac{10}{s+3} = \frac{40}{s^2+6s+12} = \frac{\frac{40}{\sqrt{3}}\sqrt{3}}{(s+3)^3+(\sqrt{3})^2}\,.$$

$$v_o(t) = \underline{23.094e^{-3t}\sin\sqrt{3}t}\ \text{V}$$

Chapter 16, Solution 34.

In the s-domain, the circuit is as shown below.



$$\frac{10}{s} = (1+\frac{s}{4})I_1 - \frac{1}{4}sI_2 \qquad (1)$$

$$-\frac{1}{4}sI_1 + I_2(4+\frac{5}{4}s) = 0 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} \frac{10}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} 1+\frac{s}{4} & -\frac{1}{4}s \\ -\frac{1}{4}s & 4+\frac{5}{4}s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \Delta = \frac{1}{4}s^2 + \frac{9}{4}s + 4$$

$$\Delta_1 = \begin{vmatrix} \frac{10}{s} & -\frac{1}{4}s \\ 0 & 4+\frac{5}{4}s \end{vmatrix} = \frac{40}{s} + \frac{50}{4} \quad \Delta_2 = \begin{vmatrix} 1+\frac{s}{4} & \frac{10}{s} \\ -\frac{1}{4}s & 0 \end{vmatrix} = \frac{5}{2}$$

$$\mathbf{I_1} = \frac{\Delta_1}{\Delta} = \frac{\frac{40}{s} + \frac{25}{2}}{0.25s^2 + 2.25s + 4} = \underline{\frac{50s + 160}{s(s^2 + 9s + 16)}}$$

$$\mathbf{I_2} = \frac{\Delta_2}{\Delta} = \frac{2.5}{0.25s^2 + 2.25s + 4} = \underline{\frac{10}{s^2 + 9s + 16}}$$

Chapter 16, Solution 35.

Find $v_o(t)$ in the circuit in Fig. 16.58.

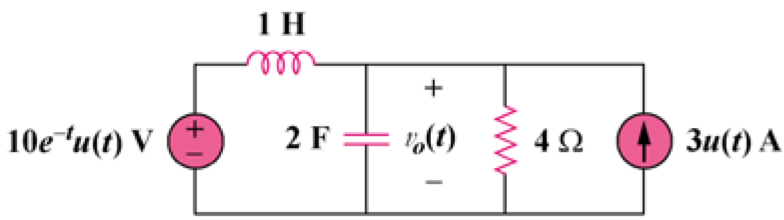
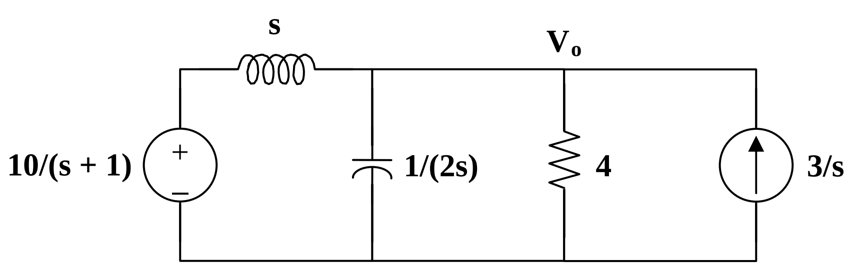


Figure 16.58
For Prob. 16.35.

Solution

Step 1. First we note that the initial condition on the capacitor and inductor must be equal to zero since the circuit is unexcited until t = 0. Next we transform the circuit into the s-domain.



We then can solve for V_o using nodal analysis.

$$\frac{V_o - \frac{10}{s+1}}{s} + \frac{2s(V_o - 0)}{1} + \frac{V_o - 0}{4} - \frac{3}{s} = 0$$

$$\left(\frac{1}{s} + 2s + 0.25\right)V_o = \frac{10}{s(s+1)} + \frac{3}{s}$$

Finally we solve for V_o, perform a partial fraction expansion and then convert into the time-domain.

Step 2.

$$2\left(\frac{s^2 + 0.125s + 0.5}{s}\right)V_o = \frac{3s + 13}{s(s+1)} \text{ or } V_o = \frac{1.5s + 6.5}{(s^2 + 0.125s + 0.5)(s+1)}$$

Next

$$s_{1,2} = \frac{-0.125 \pm \sqrt{0.015625 - 2}}{2} = -0.0625 \pm \frac{\sqrt{-1.984375}}{2} = -0.0625 \pm j0.70435$$

$$V_o = \frac{1.5s + 6.5}{(s+1)(s+0.0625+j0.70435)(s+0.0625-j0.70435)}$$
$$= \frac{A}{s+1} + \frac{B}{s+0.0625+j0.70435} + \frac{C}{s+0.0625-j0.70435}$$

where A = $\frac{-1.5+6.5}{1-0.125+0.5} = \frac{5}{1.375} = 3.636$

B =
$$\frac{1.5(-0.0625-j0.70435)+6.5}{(-0.0625-j0.70435+1)(-j1.4087)} = \frac{6.40625-j1.056525}{(0.9375-j0.70435)(-j1.4087)}$$

$$= (6.49279\angle-9.36497°))/(1.17261\angle-36.9178°)(1.4087\angle-90°) = 3.9306\angle117.553°$$

C =
$$\frac{1.5(-0.0625+j0.70435)+6.5}{(-0.0625+j0.70435+1)(j1.4087)} = \frac{6.40625+j1.056525}{(0.9375+j0.70435)(j1.4087)}$$

$= \frac{6.49279\angle9.36497°}{(1.17261\angle36.9178°)(1.4087\angle90°)}$ $=3.9306\angle-117.553°$

Thus,

$v_o(t) =$

$$[3.636e^{-t} + 3.931e^{-0.0625}(e^{117.55°}e^{-j0.7044t} + e^{-117.55°}e^{j0.7044t})]u(t)\text{volts}$$

or

$$[3.636e^{-t} + 7.862e^{-0.0625t}\cos(0.7044t - 117.55°)]u(t)\text{volts}.$$

Chapter 16, Solution 36.

Refer to the circuit in Fig. 16.59. Calculate $i(t)$ for $t > 0$.

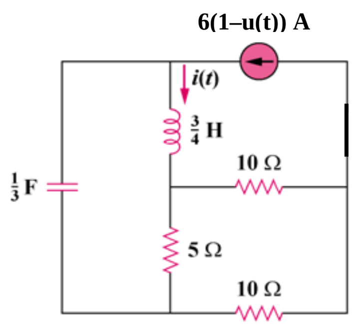
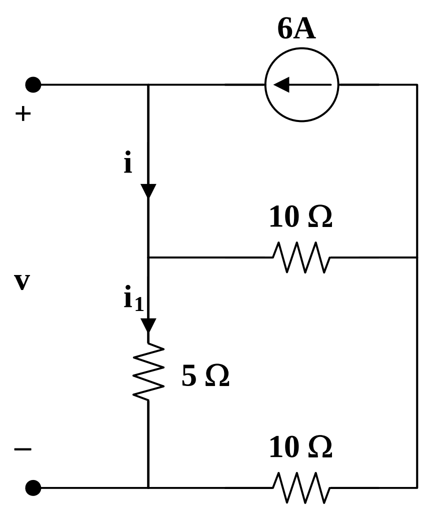


Figure 16.59
For Prob. 16.36.

Solution

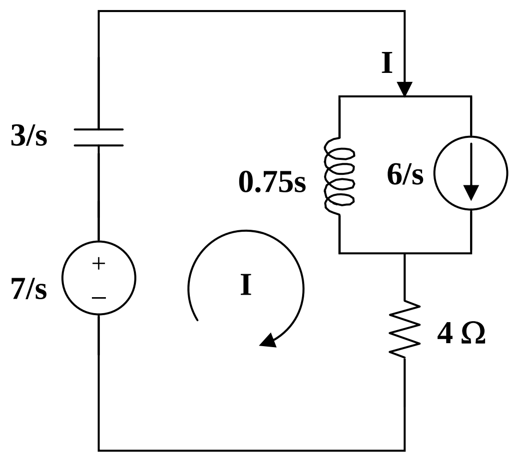
Step 1. First we need to determine the initial conditions and then transform the circuit into the s-domain.



Clearly i = 6 A. The current then travels through the parallel combination of the 10 ohm resistor and the combined 15 ohm resistance. $i_1 = 6[(15)(10)/(15+10)]/(15) = 2.4$ A. Therefore, v(0) = 5x2.4 = 12 V and i(0) = 6 A. We also note that the two 10 ohm resistors are in series and the combination is in parallel with the 5 ohm resistor resulting in a 100/25 = 4 ohm resistor.

The circuit in the s-domain is shown below.

$-[12/s] + [3/s]I + [0.75s](I-6/s) + 4I = 0.$



Step 2. $[(s^2+5.333s+4)/(4s/3)]I = 4.5+12/s = 4.5(s+2.667)/s$ or

$I = 6(s+2.667)/[(s+0.903)(s+4.43)] = [A/(s+0.903)]+[B/(s+4.43)]$ where

$A = 6(-0.903+2.667)/(-0.903+4.43) = 6x1.764/3.527 = 3.001$ and
$B = 6(-4.43+2.667)/(-4.43+0.903) = 6x(-1.763)/(-3.527) = 2.999$ or

$i(t) = \mathbf{[3.001e^{-0.903t}+2.999e^{-4.43t}]u(t)}$ **amps**.

Chapter 16, Solution 37.

Determine $v(t)$ for $t > 0$ in the circuit in Fig. 16.60.

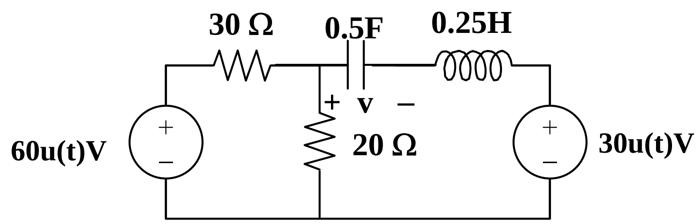
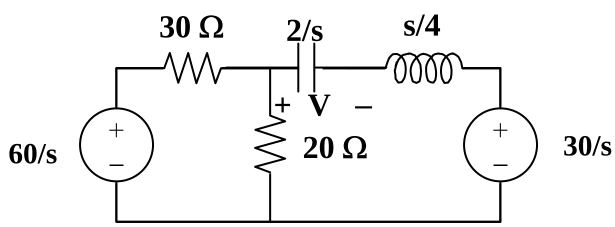


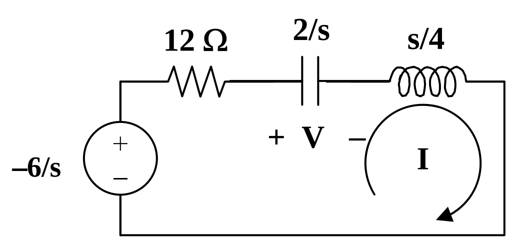
Figure 16.60
For Prob. 16.37.

Solution

Step 1. First we need to determine the initial conditions for this circuit. Since both sources were zero (shorts) until t = 0, the initial conditions for this circuit are equal to zero ($v(0) = 0$ and $i_L(0) = 0$). Next we transform the circuit into the s-domain. Then we can write node equations and then solve for V. Then perform a partial fraction expansion and convert back into the time domain.



From this circuit there are different ways of solving for v(t). Perhaps the easiest is to replace the circuit seen by the capacitor and inductor with a Thevenin equivalent circuit. $V_{Thev} = [(60/s)/(30+20)]20 – 30/s = (24/s)–(30/s) = –6/s$ and $R_{eq} = 20x30/(20+30) = 12\ \Omega$. Thus we now have the following circuit where we can now find I. Once we have I we can find V and then perform a partial fraction expansion and then convert into the time domain to solve for v(t).



$–[–6/s] + 12I + [2/s]I + [s/4]I = 0$ and $V = [2/s]I$.

Step 2. $[(s/4)+12+(2/s)]I = –6/s = [(s^2+48s+8)/(4s)]I$ or

$I = (–6/s)(4s)/ [(s^2+48s+8)] = –24/[(s+0.165)(s+47.84)]$ and
$V = –48/[s(s+0.1672)(s+47.84)] = [A/s]+[B/(s+0.1672)]+[C/(s+47.84)]$ where
$A = –48/[0.1672x47.84] = 6$; $B = –48/[–0.1672(–0.1672+47.84)] = 6.022$; and
$C = –48/[–47.84(–47.84+0.1672)] = –0.021$

Therefore,

$$v(t) = \mathbf{[–6+6.022e^{–0.1672t}–0.021e^{–47.84t}]u(t)\ volts}.$$

Chapter 16, Solution 38.

The switch in the circuit of Fig. 16.61 is moved from position *a* to *b* (a make before break switch) at $t = 0$. Determine $i(t)$ for $t > 0$.

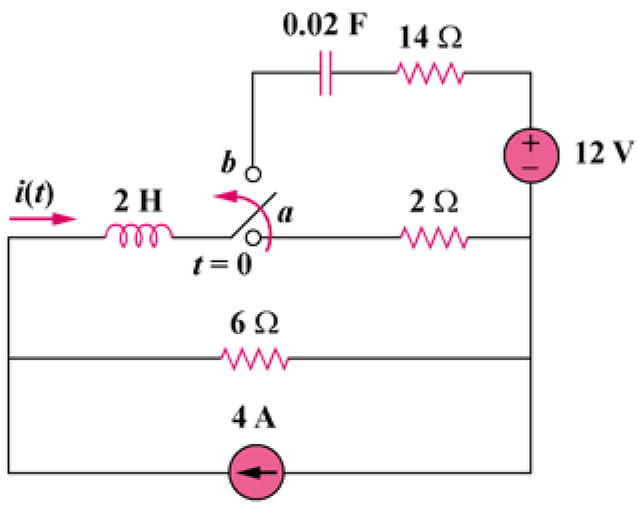
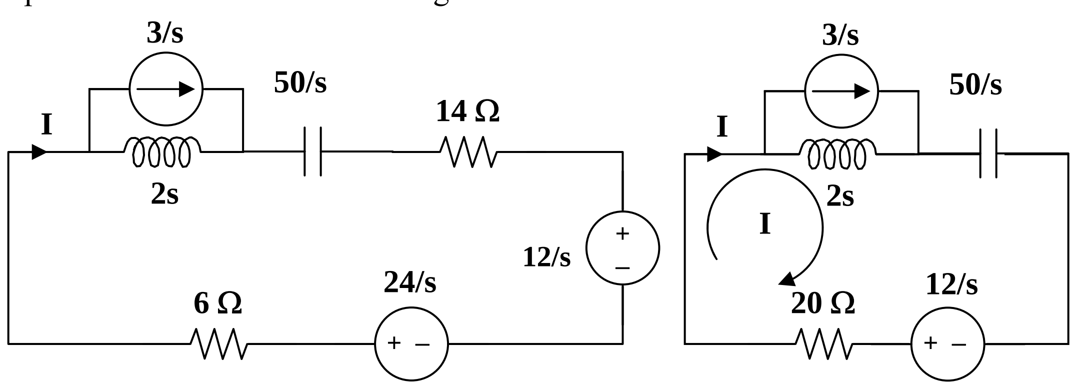


Figure 16.61
For Prob. 16.38.

Solution

Step 1. We first determine the initial conditions. We assume that $v_C(0) = 0$ since we are not given otherwise. $i(0) = [4(2x6)/(2+6)]/2 = 3$ amps. Next we need to convert the circuit for $t > 0$ into the s-domain converting the current source in parallel with the 6Ω into a voltage source in series with 6Ω.



Using the simplified circuit on the right, $2s(I–3/s) + [50/s]I –(12/s) +20I = 0$. Now we solve for I, perform a partial fraction expansion, and then convert into the time domain.

Step 2. $[2s+(50/s)+20]I = 6+12/s = [(s^2+10s+25)/(0.5s)]I = 6(s+2)/s$ or $I = [3(s+2)/(s+5)^2] = [A/(s+5)] + [B/(s+5)^2]$ where $As+5A+B = 3s+6$ or $A = 3$ and $B = –5A+6 = –15+6 = –9$. Thus,

$$i(t) = \mathbf{[(3–9t)e^{–5t}]u(t)\ amps}.$$

Chapter 16, Solution 39.

For the network in Fig. 16.62, find $i(t)$ for $t > 0$.

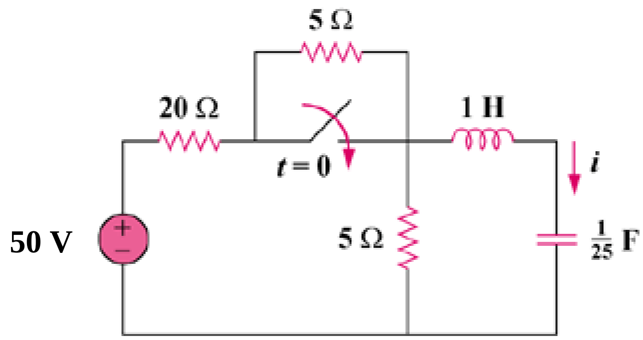
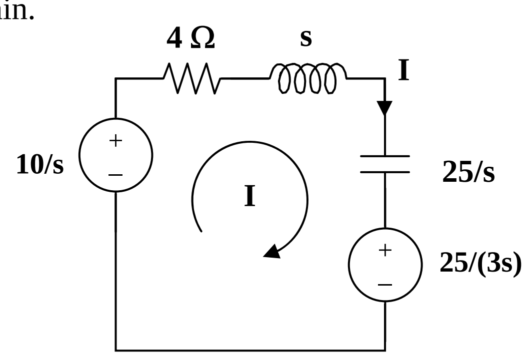


Figure 16.62
For Prob. 16.39.

Solution

Step 1. First determine the initial conditions at t = 0. Clearly i(0) = 0 and v(0) = [50/(20+5+5)]5 = 25/3 volts. Next simplify and convert the circuit for t > 0 into the s-domain.



–[10/s] + [4+s+(25/s)]I + [25/(3s)] = 0 Now we need to solve for I, perform a partial fraction expansion, and then convert into the time domain.

Step 2. $[(s^2+4s+25)/s]I = [10/s] – 25/(3s) = [5/(3s)]$ or
I = 1.6667/[(s+2+j4.583)(s+2–j4.583)] = [A/(s+2+j4.583)]+[B/(s+2–j4.583)]
where A = 1.6667/(–j9.166) = 0.18182∠90˚ and B = 0.18182∠–90˚. Therefore,

$i(t) = [0.18182e^{-2t}(e^{-j4.583t+90^\circ}+e^{j4.583t-90^\circ})]u(t)$ amps or

$= \mathbf{[363.6e^{-2t}\cos(4.583t-90^\circ)]u(t)\ mA}$.

Chapter 16, Solution 40.

Given the network in Fig. 16.63, find $v(t)$ for $t > 0$.

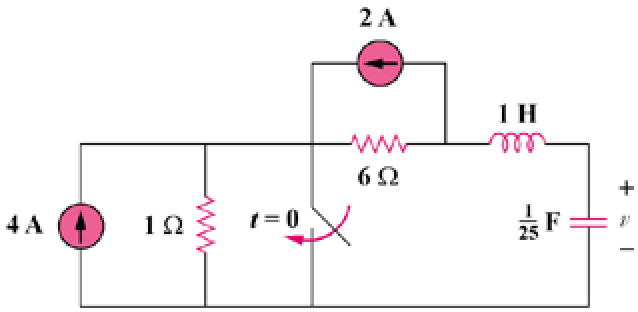
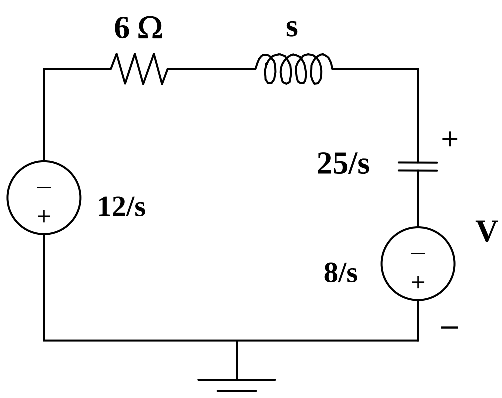


Figure 16.63
For Prob. 16.40.

Solution

Step 1. First we determine initial conditions and then simplify the circuit and then transform it into the s-domain. Just before the switch closes, the capacitor is an open circuit ($i_L(0) = 0$) with v(0) = 4 –12 = –8 volts.



We can write a node equation at V and then solve for V. Then we perform a partial fraction expansion and then solve for v(t).

[(V–(–12/s))/(s+6)]+[(V–(–8/s))/(25/s)] = 0.

Step 2. [(1/(s+6))+s/25]V = [(s^2+6s+25)/(25(s+6))]V
= –[12/(s(s+6))]–[8/25] = –[(12+0.32s^2+1.92s)/(s(s+6))]
= –0.32[(s^2+6s+37.5)/(s(s+6))] or
V = –8[(s^2+6s+37.5)/(s(s+3+j4)(s+3–j4))]
= [A/s]+[B/(s+3+j4)]+[C/(s+3–j4)] where A = –8[37.5/25] = –12;
B = –8[((–3–j4)^2+6(–3–j4)+37.5)/((–3–j4)(–j8))]
= –8[(–7+j24–18–j24+37.5)/(–32+j24)]
= –8[(12.5)/(40∠143.13°)] = 2.5∠36.87°; and

$C = –8[((–3+j4)^2+6(–3+j4)+37.5)/((–3+j4)(j8)]$
$= –8[(–7–j24–18+j24+37.5)/(–32–j24)]$
$= –8[(12.5)/(40\angle–143.13^\circ)] = 2.5\angle–36.87^\circ$
$V = [–12/s] + [2.5\angle36.87^\circ/(s+3+j4)] + [2.5\angle–36.87^\circ/(s+3–j4)]$ or

$v(t) = [–12+2.5e^{–3t}(e^{–j4t+36.87^\circ}+e^{j4–36.87^\circ})]u(t)$ amps

$= \mathbf{[–12+5e^{–3t}(\cos(4t–36.87^\circ)]u(t)}$ **volts**.

Chapter 16, Solution 41.

Find the output voltage $v_o(t)$ in the circuit of Fig. 16.64.

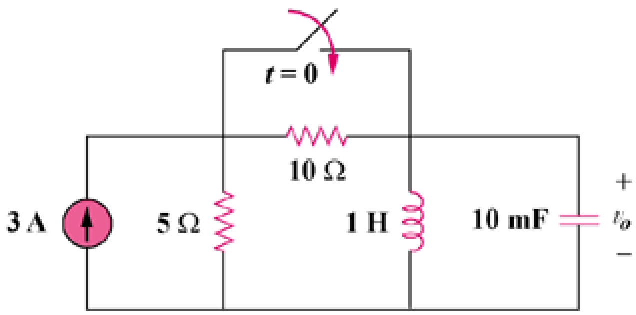
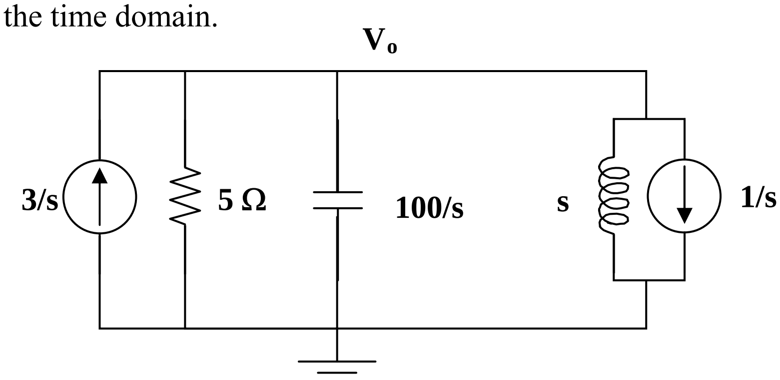


Figure 16.64
For Prob. 16.41.

Solution

Step 1. First we need to determine the initial conditions. We see that $v_o(0) = 0$ since the inductor becomes a short. We also note that the initial current through the inductor is the same as the current through the 10 Ω resistor or $i_L(0) = [3(5x10)/(5+10)]/10 = 1$ amp. Then we simplify the circuit and convert it into the s-domain and solve for V_o. We then perform a partial fraction expansion and convert into the time domain.



$-[3/s] + [(V_o-0)/5] + [(V_o-0)/(100/s)] + [(V_o-0)/s] + [1/s] = 0.$

Step 2. $[0.2+(s/100)+(1/s)]V_o = 2/s = [(s^2+20s+100)/(100s)]V_o$ or

$V_o = 200/[(s+10)^2]$ and

$v_o(t) =$ **$[200te^{-10t}]u(t)$ volts**.

Chapter 16, Solution 42.

Given the circuit in Fig. 16.65, find *i*(*t*) and *v*(*t*) for *t* > 0.

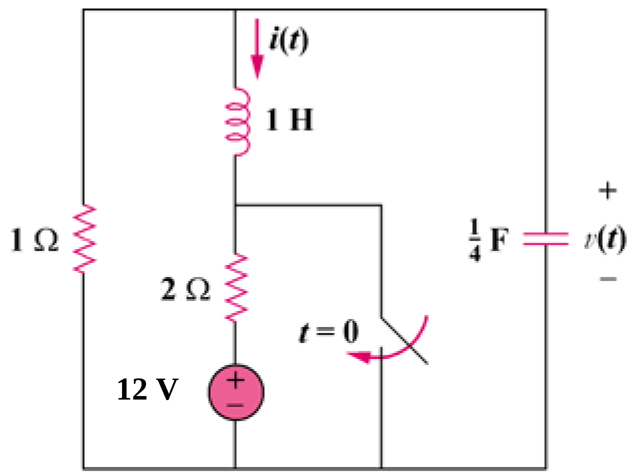
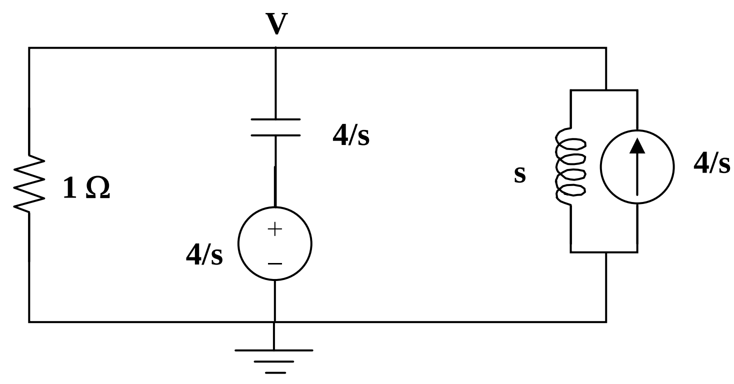


Figure 16.65
For Prob. 16.42.

Solution

Step 1. First we need to find the initial conditions. Since the inductor becomes a short and the capacitor becomes an open circuit, all the current flows through the 1 Ω and 2 Ω resistors or i(0) = –12/3 = –4 amps and v(0) = 4x1 = 4 volts. Next we need to convert the circuit into the s-domain and solve for V and I. Once we have done that, we can perform partial fraction expansions and convert back into the time domain.



[(V–0)/1] + [(V–4/s)/(4/s)] + [(V–0)/s] – [4/s] = 0 and I = [(V–0)/s] – [4/s].

Step 2. $[(1+(s/4)+(1/s)]V = [(s^2+4s+4)/(4s)]V = 1+4/s = (s+4)/s$ or $V = (4s+16)/[(s+2)^2] = [A/(s+2)]+[B/(s+2)^2]$ where As+2A + B = 4s+16 and A = 4 and B = 16–2A =8. $I = [4/(s(s+2))]+[8/(s(s+2)^2)]–4/s$

The partial fraction expansion is straight forward for the first and third terms, but the second term takes a little work. $8/(s(s+2)^2 = [a/s]+[b/(s+2)]+[c/(s+2)^2]$ or $as^2+a4s+a4+bs^2+b2s+cs = 8$ or $a = 2$, $b = –2$, and $c = –4$.

Thus, $I = [2/s]+[–2/(s+2)]+[2/s]+[–2/(s+2)]+[–4/(s+2)^2]–4/s = –[4/(s+2)] – [4/(s+2)^2]$ and we finally get,

$v(t) =$ **$[4e^{–2t} + 8te^{–2t}]u(t)$ volts** and

$i(t) =$ **$[–4e^{–2t} – 4te^{–2t}]u(t)$ amps**.

Chapter 16, Solution 43.

Determine $i(t)$ for $t > 0$ in the circuit of Fig. 16.66.

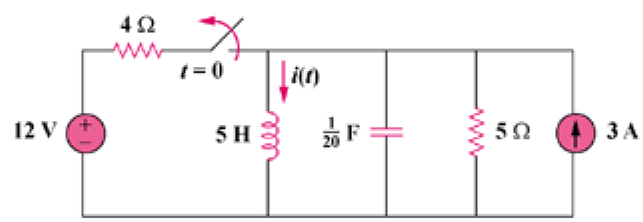
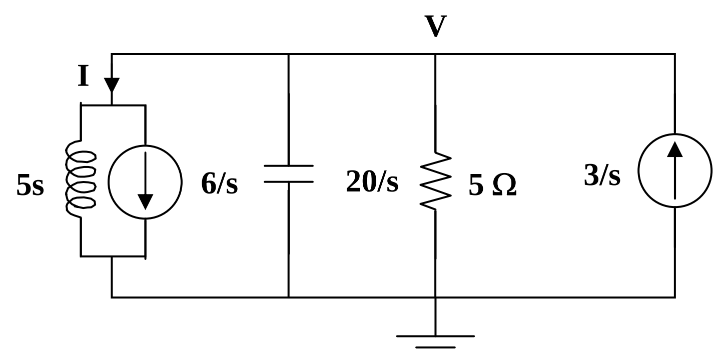


Figure 16.66
For Prob. 16.43.

Solution

Step 1. First we need to determine the initial conditions. Then we need to transform the circuit into the s-domain. Once in the s-domain we can calculate V and I. We then perform a partial fraction expansion on I and convert back into the time domain. Since the inductor looks like a short just before the switch opens, $v_C(0) = 0$ and $i(0) = (12/4)+3 = 6$ amps.



$[(V–0)/(5s)] + [6/s] + [(V–0)/(20/s)] + [(V–0)/5] – [3/s] = 0$ and $I = [(V–0)/(5s)] + [6/s]$.

Step 2. $[(1/(5s))+(s/20)+(1/5)]V = [(s^2+4s+4)/(20s)]V = –3/s$ or $V = –60/(s+2)^2$ and $I = –[12/(s(s+2)^2)] + 6/s = [A/s] + [B/(s+2)] + [C/(s+2)^2] + (6/s)$ where $A = –3$ and $A(s^2+4s+4) + B(s^2+2s) + Cs = –12$ $= –3s^2 – 12s –12 + Bs^2 + B(2s) + Cs$ or $–3 B = 0$ or $B = 3$ and $–12 + 6 + C = 0$ or $C = 6$.

$i(t) =$ **$[3+3e^{–2t}+6te^{–2t}]u(t)$ amps**.

Chapter 16, Solution 44.

For the circuit in Fig. 16.67, find $i(t)$ for $t > 0$.

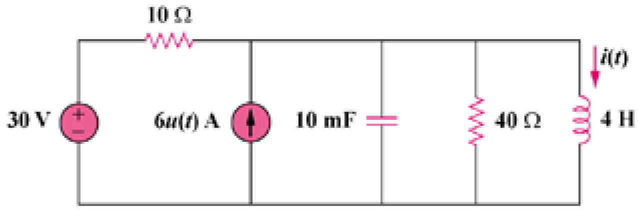
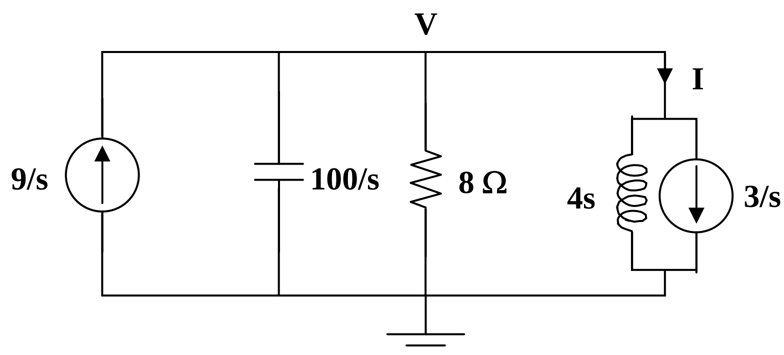


Figure 16.67
For Prob. 16.44.

Solution

Step 1. First we identify the initial conditions. Then we simplify the circuit (for t > 0) and then transform it into the s-domain. We then solve for the node voltage, V, and then find I. Finally we perform a partial fraction expansion and convert the answer into the time domain. For t < 0, the inductor looks like a short circuit producing $v_C(0) = 0$ and $i(0) = 30/10 = 3$ amps.



$-[9/s] + [(V-0)/(100/s)] + [(V-0)/8] + [(V-0)/(4s)] + [3/s] = 0$ and
$I = [(V-0)/(4s)] + [3/s]$.

Step 2. $[(s/100)+(1/8)+1/(4s)]V = [(s^2+12.5s+25)/(100s)]V = 6/s$ or
$V = 600/[(s+2.5)(s+10)]$ and $I = 150/[s(s+2.5)(s+10)]+[3/s]$
$= [A/s] + [B/(s+2.5)] + [C/(s+10)]$ where $A = 6+3 = 9$; $B = 150/[-2.5(-2.5+10)]$
$= -8$; and $C = 150/[-10(-10+2.5)] = 2$.

$$i(t) = \mathbf{[9-8e^{-2.5t}+2e^{-10t}]u(t)\ amps}.$$

Chapter 16, Solution 45.

Find $v(t)$ for $t > 0$ in the circuit in Fig. 16.68.

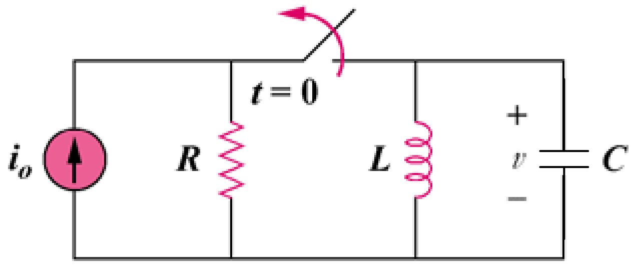
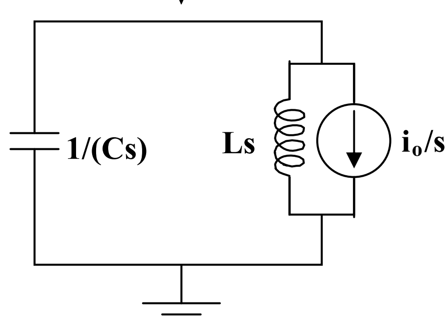


Figure 16.68
For Prob. 16.45.

Solution

Step 1. First, determine the initial conditions. Next convert the circuit into the s-domain and solve for V. Perform a partial fraction expansion and convert back into the time domain. For t < 0, the inductor looks like a short circuit so that v(0) = 0 and $i_L(0) = i_o$.



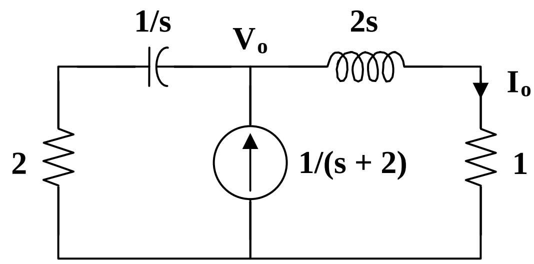
$[(V–0)/(1/(Cs))] + [(V–0)/(Ls)] + [i_o/s] = 0.$

Step 2. $[Cs+(1/(Ls))]V = [C\{s^2+(1/(LC))\}/s]V = –i_o/s$ or
$V = –(i_o/C)/[(s^2+1/(LC))]$. If we let $\boldsymbol{\omega^2 = 1/(LC)}$ then we get,
$V = –(i_o/C)/[(s^2+\omega^2)] = –(i_o/C)/[(s+j\omega)(s–j\omega)] = [A/(s+j\omega)]+[B/(s–j\omega)]$ where
$A = –(i_o/C)/(–j2\omega) = [i_o/(2\omega C)]\angle–90^\circ$ and $B = –(i_o/C)/(j2\omega) = [i_o/(2\omega C)]\angle–90^\circ$.
Thus,

$v(t) = [i_o/(2\omega C)][e^{–j\omega t–90^\circ}+e^{j\omega t+90^\circ}] =$ $\boldsymbol{[i_o/(\omega C)]\cos(\omega t+90^\circ)u(t)}$ **volts**.

Chapter 16, Solution 46.

Consider the following circuit.



Applying KCL at node o,

$$\frac{1}{s+2} = \frac{V_o}{2s+1} + \frac{V_o}{2+1/s} = \frac{s+1}{2s+1} V_o$$

$$V_o = \frac{2s+1}{(s+1)(s+2)}$$

$$I_o = \frac{V_o}{2s+1} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

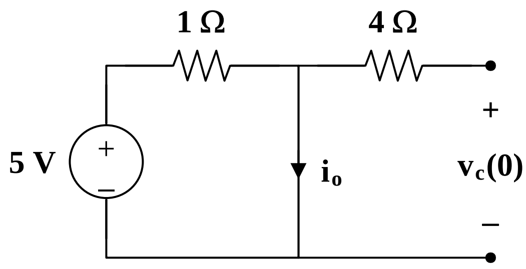
$A = 1, \qquad B = -1$

$$I_o = \frac{1}{s+1} - \frac{1}{s+2}$$

$$i_o(t) = \left(\mathbf{e^{-t} - e^{-2t}}\right)\mathbf{u(t)\ A}$$

Chapter 16, Solution 47.

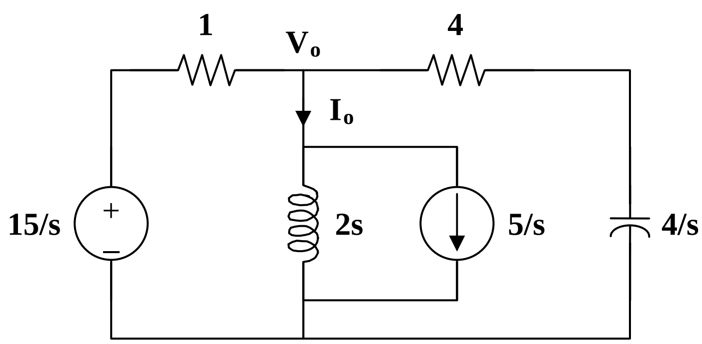
We first find the initial conditions from the circuit in Fig. (a).



(a)

$$i_o(0^-) = 5\text{ A}, \quad v_c(0^-) = 0\text{ V}$$

We now incorporate these conditions in the s-domain circuit as shown in Fig.(b).



(b)

At node o,

$$\frac{V_o - 15/s}{1} + \frac{V_o}{2s} + \frac{5}{s} + \frac{V_o - 0}{4 + 4/s} = 0$$

$$\frac{15}{s} - \frac{5}{s} = \left(1 + \frac{1}{2s} + \frac{s}{4(s+1)}\right)V_o$$

$$\frac{10}{s} = \frac{4s^2 + 4s + 2s + 2 + s^2}{4s(s+1)}V_o = \frac{5s^2 + 6s + 2}{4s(s+1)}V_o$$

$$V_o = \frac{40(s+1)}{5s^2 + 6s + 2}$$

$$I_o = \frac{V_o}{2s} + \frac{5}{s} = \frac{4(s+1)}{s(s^2 + 1.2s + 0.4)} + \frac{5}{s}$$

$$I_o = \frac{5}{s} + \frac{A}{s} + \frac{Bs + C}{s^2 + 1.2s + 0.4}$$

$$4(s+1) = A(s^2 + 1.2s + 0.4) + Bs^s + Cs$$

Equating coefficients :

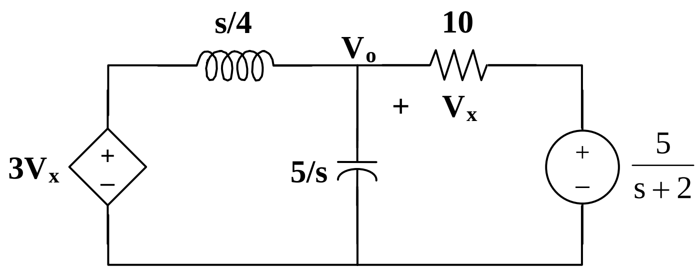
$$s^0: \quad 4 = 0.4A \longrightarrow A = 10$$
$$s^1: \quad 4 = 1.2A + C \longrightarrow C = -1.2A + 4 = -8$$
$$s^2: \quad 0 = A + B \longrightarrow B = -A = -10$$

$$I_o = \frac{5}{s} + \frac{10}{s} - \frac{10s + 8}{s^2 + 1.2s + 0.4}$$
$$I_o = \frac{15}{s} - \frac{10(s+0.6)}{(s+0.6)^2 + 0.2^2} - \frac{10(0.2)}{(s+0.6)^2 + 0.2^2}$$

$$i_o(t) = \left[\mathbf{15 - 10\,e^{-0.6t}\left(cos(0.2\,t) - sin(0.2\,t)\right)}\right] \mathbf{u(t)\ A}$$

Chapter 16, Solution 48.

First we need to transform the circuit into the s-domain.



$$\frac{V_o - 3V_x}{s/4} + \frac{V_o - 0}{5/s} + \frac{V_o - \frac{5}{s+2}}{10} = 0$$

$$40V_o - 120V_x + 2s^2V_o + sV_o - \frac{5s}{s+2} = 0 = (2s^2 + s + 40)V_o - 120V_x - \frac{5s}{s+2}$$

But, $V_x = V_o - \frac{5}{s+2} \rightarrow V_o = V_x + \frac{5}{s+2}$

We can now solve for V_x.

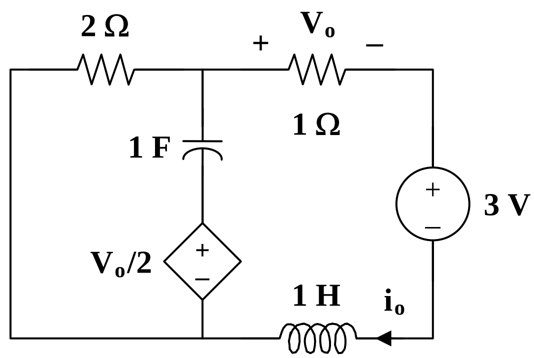
$$(2s^2 + s + 40)\left(V_x + \frac{5}{s+2}\right) - 120V_x - \frac{5s}{s+2} = 0$$

$$2(s^2 + 0.5s - 40)V_x = -10\frac{(s^2 + 20)}{s+2}$$

$$\underline{V_x = -5\frac{(s^2 + 20)}{(s+2)(s^2 + 0.5s - 40)}}$$

Chapter 16, Solution 49.

We first need to find the initial conditions. For $t < 0$, the circuit is shown in Fig. (a).



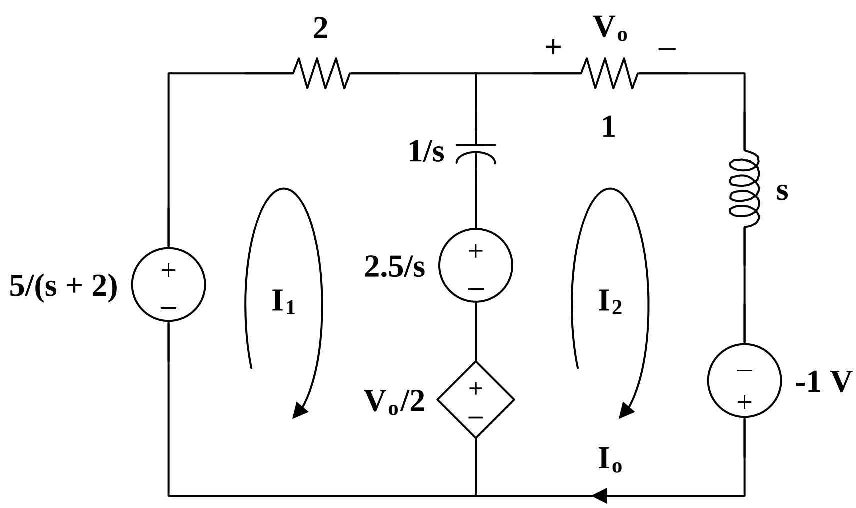
(a)

To dc, the capacitor acts like an open circuit and the inductor acts like a short circuit. Hence,

$$i_L(0) = i_o = \frac{-3}{3} = -1\text{ A}, \qquad v_o = -1\text{ V}$$

$$v_c(0) = -(2)(-1) - \left(\frac{-1}{2}\right) = 2.5\text{ V}$$

We now incorporate the initial conditions for $t > 0$ as shown in Fig. (b).



(b)

For mesh 1,

$$\frac{-5}{s+2} + \left(2 + \frac{1}{s}\right)I_1 - \frac{1}{s}I_2 + \frac{2.5}{s} + \frac{V_o}{2} = 0$$

But, $V_o = I_o = I_2$

$$\left(2+\frac{1}{s}\right)I_1 + \left(\frac{1}{2}-\frac{1}{s}\right)I_2 = \frac{5}{s+2} - \frac{2.5}{s} \qquad (1)$$

For mesh 2,

$$\left(1+s+\frac{1}{s}\right)I_2 - \frac{1}{s}I_1 + 1 - \frac{V_o}{2} - \frac{2.5}{s} = 0$$

$$-\frac{1}{s}I_1 + \left(\frac{1}{2}+s+\frac{1}{s}\right)I_2 = \frac{2.5}{s} - 1 \qquad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 2+\frac{1}{s} & \frac{1}{2}-\frac{1}{s} \\ \frac{-1}{s} & \frac{1}{2}+s+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{s+2} - \frac{2.5}{s} \\ \frac{2.5}{s} - 1 \end{bmatrix}$$

$$\Delta = 2s + 2 + \frac{3}{s}, \qquad \Delta_2 = -2 + \frac{4}{s} + \frac{5}{s(s+2)}$$

$$I_o = I_2 = \frac{\Delta_2}{\Delta} = \frac{-2s^2 + 13}{(s+2)(2s^2+2s+3)} = \frac{A}{s+2} + \frac{Bs + C}{2s^2+2s+3}$$

$$-2s^2 + 13 = A(2s^2+2s+3) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

s^2: $\quad -2 = 2A + B$

s^1: $\quad 0 = 2A + 2B + C$

s^0: $\quad 13 = 3A + 2C$

Solving these equations leads to
$A = 0.7143, \quad B = -3.429, \quad C = 5.429$

$$I_o = \frac{0.7143}{s+2} - \frac{3.429s - 5.429}{2s^2+2s+3} = \frac{0.7143}{s+2} - \frac{1.7145s - 2.714}{s^2+s+1.5}$$

$$I_o = \frac{0.7143}{s+2} - \frac{1.7145(s+0.5)}{(s+0.5)^2+1.25} + \frac{(3.194)(\sqrt{1.25})}{(s+0.5)^2+1.25}$$

$$i_o(t) = \left[\mathbf{0.7143\,e^{-2t} - 1.7145\,e^{-0.5t}\cos(1.25t) + 3.194\,e^{-0.5t}\sin(1.25t)}\right]\mathbf{u(t)\ A}$$

Chapter 16, Solution 50.

For the circuit in Fig. 16.73, find $v(t)$ for $t > 0$. Assume that $v(0^+) = 4$ V and $i(0^+) = 2$ A.

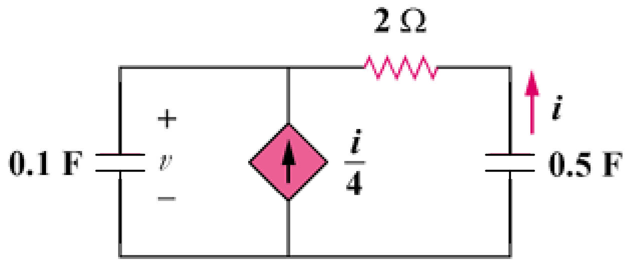
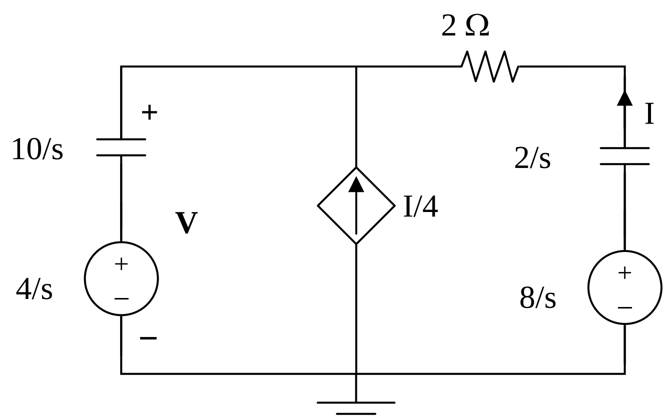


Figure 16.73
For Prob. 16.50.

Solution

Step 1. Determine the initial condition of the second capacitor and then convert the circuit into the s-domain. Finally, solve for V, perform a partial fraction expansion and convert the answer back into the time domain. Since v(0) = 4 volts and i(0) = 2 amps then $-4-2(2)+v_2(0) =$ or $v_2(0) = 8$.



$[(V-4/s)/(10/s)]-[I/4]+[(V-8/s)/(2+2/s)] = 0$ and $I = [((8/s)-V)/(2+2/s)]$
$= [4/(s+1)]-0.5sV/(s+1)$

Step 2. $[(V-4/s)/(10/s)]+[0.125sV/(s+1)]-[1/(s+1)]+[(V-8/s)/(2+2/s)]$
$[(s/10)+(0.125s/(s+1))+(0.5s/(s+1))]V =$
$[0.4+4/(s+1)]+[1/(s+1)] = (0.4s+5.4)/(s+1)$
$= [(s^2+s+6.25s)/(10(s+1))]V = [s(s+7.25)/(10(s+1))]V$ or
$V = 4(s+13.5)/[s(s+7.25)] = [A/s]+[B/(s+7.25)]$ where $A = 4(13.5)/7.25 = 7.748$ and $B = 4(-7.25+13.5)/(-7.25) = -3.448$ or

$$v(t) = \mathbf{[7.748-3.448e^{-7.25t}]u(t)\ volts}.$$

Chapter 16, Solution 51.

In the circuit of Fig. 16.74, find $i(t)$ for $t > 0$.

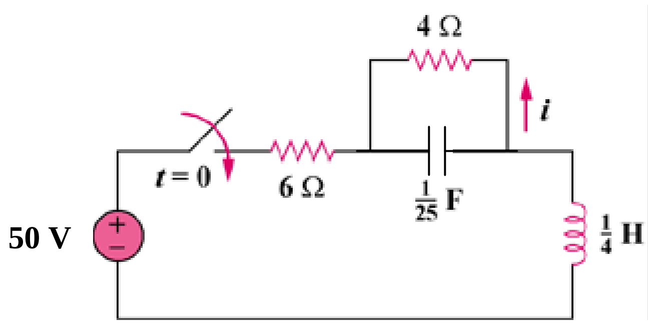
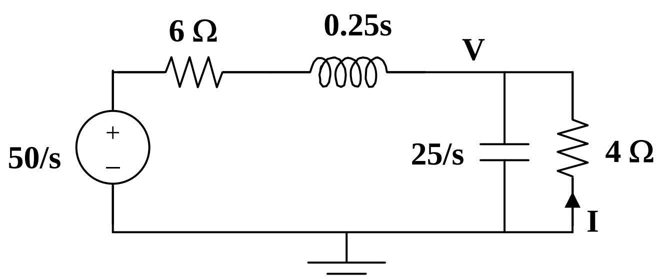


Figure 16.74
For Prob. 16.51.

Solution

Step 1. First we note that the initial conditions for the capacitor and inductor have to be equal to zero. Next we simplify the circuit and then convert the circuit into the s-domain and solve for V. Then we can solve for I and then perform a partial fraction expansion and convert I back into the time domain.



$[(V–50/s)/(0.25(s+24))]+[s(V–0)/25]+[(V–0)/4] = 0$ and $I = [(0–V)/4] = –V/4$.

Step 2. $[(4/(s+24))+(s/25)+0.25]V = [(s^2+24s+6.25s+100+150)/(25(s+24))]V$
$= [(s^2+30.25s+250)/(25(s+24))]V$
$= [\{(s+15.125+j4.608)(s+15.125–j4.608)\}/(25(s+24))]V = [200/(s(s+24))]$ or
$V = 5{,}000/[s(s+15.125+j4.608)(s+15.125–j4.608)]$ and
$I = –1250/[s(s+15.125+j4.608)(s+15.125–j4.608)] = [A/s]+[B/(s+15.125+j4.608)]$
$+[C/(s+15.125–j4.608)]$ where $A = –1250/250 = –5$;
$B = –1250/[(–15.125–j4.608)(–j9.216)] = 1250\angle 180^\circ/[(15.811\angle –163.06^\circ)(9.216\angle –90^\circ)]$
$= 8.578\angle 73.06^\circ$; and $C = 1250\angle 180^\circ/[(15.811\angle 163.06^\circ)(9.216\angle 90^\circ)] = 8.578\angle –73.06^\circ$.
Thus, $i(t) = [–5+8.578e^{–15.125t}(e^{–j4.608t+73.06^\circ}+e^{j4.608–73.06^\circ})]u(t)$ amps

$i(t) =$ **$[–5+17.156e^{–15.125t}\cos(4.608t–73.06^\circ)]u(t)$ amps**.

Chapter 16, Solution 52.

If the switch in Fig. 16.75 has been closed for a long time before $t = 0$ but is opened at $t = 0$, determine i_x and v_R for $t > 0$.

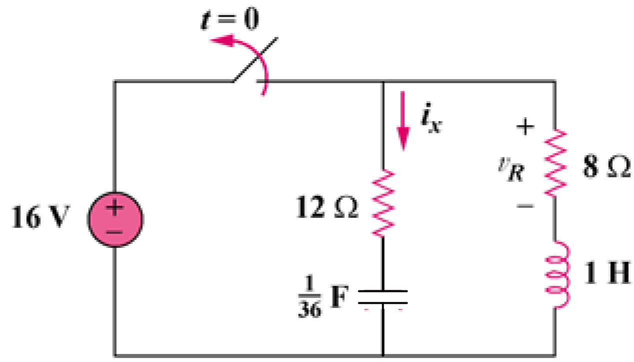
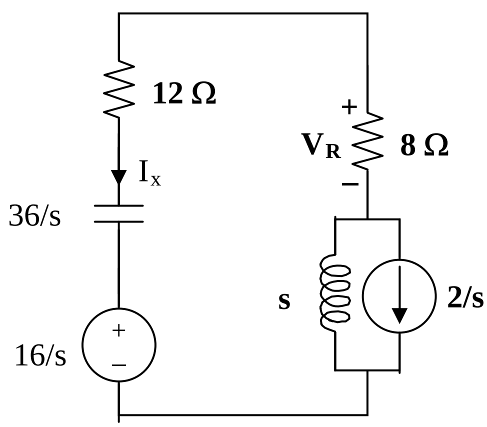


Figure 16.75
For Prob. 16.52.

Solution

Step 1. Fist we need to determine the initial conditions. Just before the switch opens, $v_C(0) = 16$ volts and $i_L(0) = 2$ amps. Next we convert the circuit into the s-domain.



We can now write a mesh equation (this time going in the counter-clockwise direction). $[s(I_x+2/s)]+[8I_x]+[12I_x]+[(36/s)I_x]+(16/s) = 0$ and $V_R = -8I_x$.

Step 2. $[s+8+12+(36/s)]I_x = [(s^2+20s+36)/s]I_x = -2-16/s = -[2(s+8)/s]$ or $I_x = -2(s+8)/[(s+2)(s+18)] = [A/(s+2)]+[B/(s+18)]$ where $A = -2(-2+8)/(-2+18) = -2\text{x}6/16 = -0.75$ and $B = -2(-18+8)/(-18+2) = -1.25$ thus,

$i_x(t) = \mathbf{[-0.75e^{-2t}-1.25e^{-18t}]u(t)\ amps}$ and
$v_R(t) = -8i_x(t) = \mathbf{[6e^{-2t}+10e^{-18t}]u(t)\ volts}$.

Chapter 16, Solution 53.

In the circuit of Fig. 16.76, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$. Find:

(a) $v(0^+)$, $dv(0^+)/dt$
(b) $v(t)$ for $t \geq 0$.

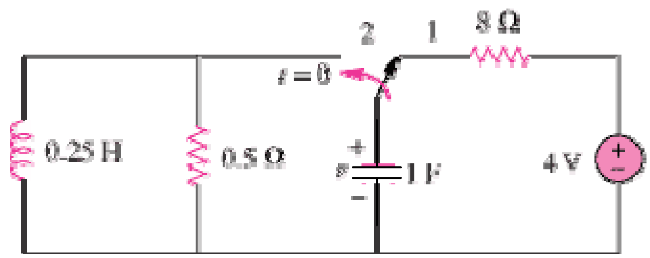
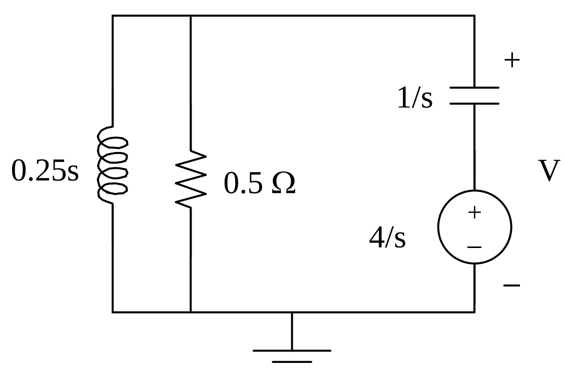


Figure 16.76
For Prob. 16.53.

Solution

Step 1. Clearly $i_L(0) = 0$ and $v(0) =$ **4 volts**. When the switch moves to 2, $i_C(0^+) = Cdv(0)/dt = -4/0.5 =$ **–8 volts/second** $= 1dv(0)/dt$. Next we convert the circuit into the s-domain and solve for V. Then we perform a partial fraction expansion and then convert back into the time domain.



$[(V-0)/(0.25s)]+[(V-0)/0.5]+[(V-4/s)s/1] = 0.$

Step 2. $[(4/s)+2+s]V = [(s^2+2s+4)/s]V = 4$ or $V = 4s/[(s+1+j1.7321)(s+1-j1.7321)]$
$= [A/(s+1+j1.7321)] + [B/(s+1-j1.7321)]$ where
$A = 4(-1-j1.7321)/(3.464\angle-90^\circ) = 4(2\angle-120^\circ)/(3.464\angle-90^\circ) = 2.309\angle-30^\circ$ and
$B = 4(2\angle120^\circ)/(3.464\angle90^\circ) = 2.309\angle30^\circ$ or
$v(t) = 2.309e^{-t}[e^{-j1.7321t-30^\circ}+e^{j1.7321t+30^\circ}]u(t)$ volts or

$v(t) =$ **$[4.618e^{-t}\cos(1.7321t+30^\circ)]u(t)$ volts**.

Chapter 16, Solution 54.

The switch in Fig. 16.77 has been in position 1 for $t < 0$. At $t = 0$, it is moved from position 1 to the top of the capacitor at t = 0. Please note that the switch is a make before break switch, it stays in contact with position 1 until it makes contact with the top of the capacitor and then breaks the contact at position 1. Determine $v(t)$.

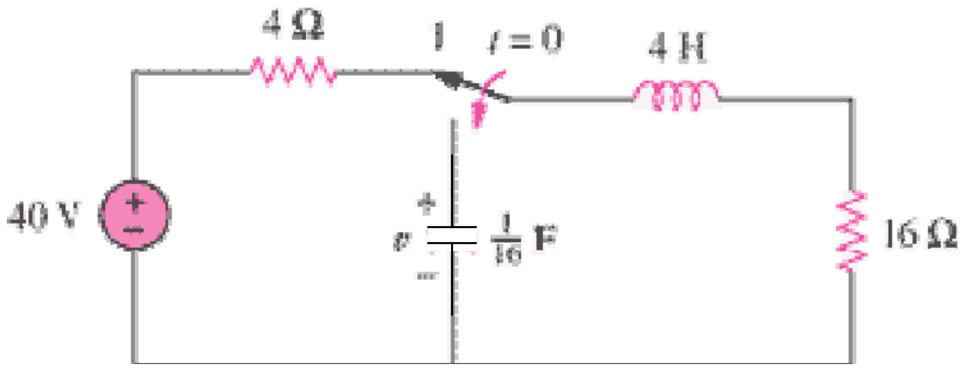
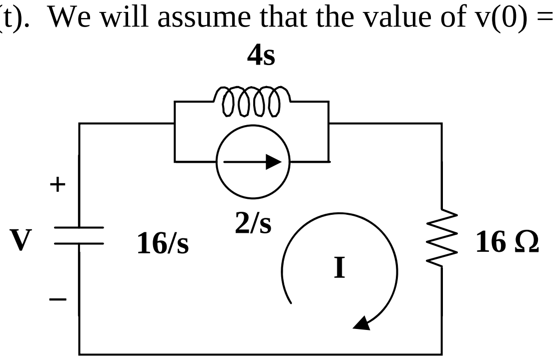


Figure 16.77
For Prob. 16.54.

Solution

Step 1. First determine the initial conditions and then transform the circuit into the s-domain and solve for V. Then perform a partial fraction expansion and then find v(t). We will assume that the value of v(0) = 0. $i_L(0) = 40/20 = 2$ amps.



$[16/s]I + [4s](I–2/s) + 16I = 0$ and $V = [16/s](–I)$.

Step 2. $[(16/s)+4s+16]I = [4(s^2+4s+4)/s]I = 8$ or
$I = 8s/[4(s+2)^2] = 2s/[(s+2)^2]$ and $V = –32/[(s+2)^2]$

$v(t) =$ **$[–32te^{–2t}]u(t)$ volts**.

Chapter 16, Solution 55.

Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 16.78.

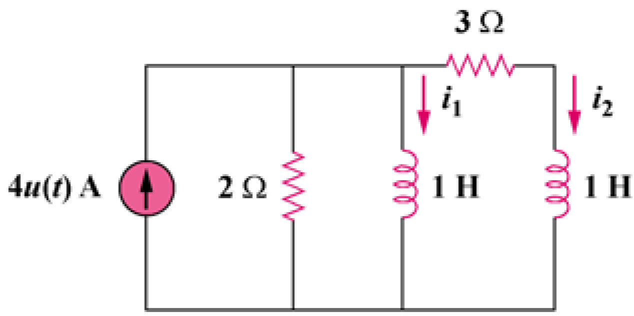
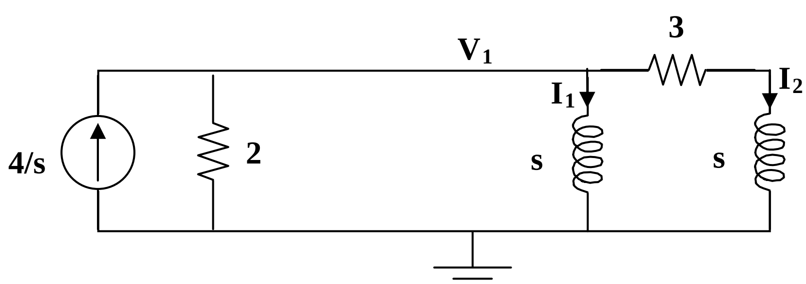


Figure 16.78
For Prob. 16.55.

Solution

Step 1. The first thing we do is to determine the initial conditions. Since there is no excitation of the circuit before t = 0, all initial conditions must be zero. Next we convert the circuit into the s-domain. Then use nodal analysis and eventually solve for I_1 and I_2, then perform a partial fraction expansion and convert back into the time domain.



$-[4/s]+[(V_1-0)/2]+[(V_1-0)/s]+[(V_1-0)/(s+3)] = 0$ and $I_1 = [(V_1-0)/s]$ and $I_2 = [(V_1-0)/(s+3)]$.

Step 2. $\{[1/2]+[1/s]+[1/(s+3)]\}V_1 = 4/s = \{[s^2+3s+2s+6+2s]/[2s(s+3)]\}V_1$ or

$V_1 = 8(s+3)/[s^2+7s+6] = 8(s+3)/[(s+1)(s+6)]$ and $I_1 = 8(s+3)/[s(s+1)(s+6)]$ $= [A/s]+[B/(s+1)]+[C/(s+6)]$ where $A = 8x3/6 = 4$; $B = 8(-1+3)/[(-1)(-1+6)]$ $= -16/5 = -3.2$; $C = 8(-6+3)/[(-6)(-6+1)] = -24/30 = -0.8$. Thus,

$i_1(t) =$ **$[4-3.2e^{-t}-0.8e^{-6t}]u(t)$ amps**.

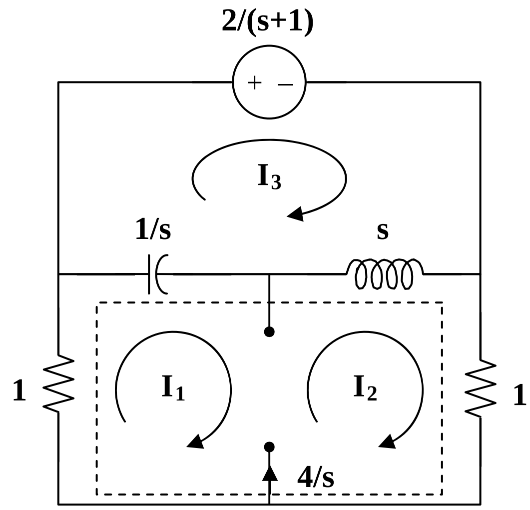
$I_2 = [(V_1-0)/(s+3)] = 8/[(s+1)(s+6)] = [A/(s+1)] + [B/(s+6)]$ where $A = 8/5 = 1.6$ and $B = 8/(-6+1) = -1.6$. Thus,

$i_2(t) =$ **$[1.6e^{-t}-1.6e^{-6t}]u(t)$ amps**.

$[4-3.2e^{-t}-0.8e^{-6t}]u(t)$ amps, $[1.6e^{-t}-1.6e^{-6t}]u(t)$ amps

Chapter 16, Solution 56.

We apply mesh analysis to the s-domain form of the circuit as shown below.



For mesh 3,

$$\frac{2}{s+1} + \left(s + \frac{1}{s}\right) I_3 - \frac{1}{s} I_1 - s\, I_2 = 0 \qquad (1)$$

For the supermesh,

$$\left(1 + \frac{1}{s}\right) I_1 + (1+s)\, I_2 - \left(\frac{1}{s} + s\right) I_3 = 0 \qquad (2)$$

Adding (1) and (2) we get, $I_1 + I_2 = -2/(s+1)$ (3)

But $-I_1 + I_2 = 4/s$ (4)

Adding (3) and (4) we get, $I_2 = (2/s) - 1/(s+1)$ (5)

Substituting (5) into (4) yields, $I_1 = -(2/s) - (1/(s+1))$ (6)

Substituting (5) and (6) into (1) we get,

$$\frac{2}{s^2} + \frac{1}{s(s+1)} - 2 + \frac{s}{s+1} + \left(\frac{s^2+1}{s}\right) I_3 = -\frac{2}{s+1}$$

$$I_3 = -\frac{2}{s} + \frac{1.5 - 0.5j}{s+j} + \frac{1.5 + 0.5j}{s-j}$$

Substituting (3) into (1) and (2) leads to

$$-\left(s+\frac{1}{s}\right)I_2+\left(s+\frac{1}{s}\right)I_3=\frac{2(-s^2+2s+2)}{s^2(s+1)} \qquad (4)$$

$$\left(2+s+\frac{1}{s}\right)I_2-\left(s+\frac{1}{s}\right)I_3=-\frac{4(s+1)}{s^2} \qquad (5)$$

We can now solve for I_o.

$$I_o = I_2 - I_3 = (4/s) - (1/(s+1)) + ((-1.5+0.5j)/(s+j)) + ((-1.5-0.5)/(s-j))$$

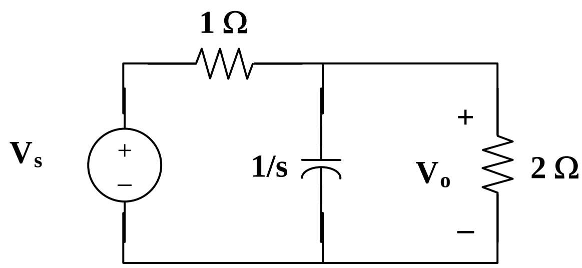
or

$$i_o(t) = \mathbf{[4 - e^{-t} + 1.5811e^{-jt+161.57^\circ} + 1.5811e^{jt-161.57^\circ}]u(t)A}$$

This is a challenging problem. I did check it with using a Thevenin equivalent circuit and got the same exact answer.

Chapter 16, Solution 57.

$v_s(t) = 3u(t) – 3u(t–1)$ or $V_s = \dfrac{3}{s} - \dfrac{e^{-s}}{s} = \dfrac{3}{s}(1 - e^{-s})$



$$\frac{V_o - V_s}{1} + sV_o + \frac{V_o}{2} = 0 \rightarrow (s + 1.5)V_o = V_s$$

$$V_o = \frac{3}{s(s+1.5)}(1 - e^{-s}) = \left(\frac{2}{s} - \frac{2}{s+1.5}\right)(1 - e^{-s})$$

$$v_o(t) = \underline{[(2 - 2e^{-1.5t})u(t) - (2 - 2e^{-1.5(t-1)})u(t-1)]V}$$

(a) $(3/s)[1–e^{–s}]$, (b) $[(2–2e^{–1.5t})u(t) – (2–2e^{–1.5(t–1)})u(t–1)]$ V

Chapter 16, Solution 58.

Using Fig. 16.81, design a problem to help other students to better understand circuit analysis in the s-domain with circuits that have dependent sources.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit of Fig. 16.81, let $i(0) = 1$ A, $v_o(0) = 2$ V, and $v_s = 4\,e^{-2t}\,u(t)$ V. Find $v_o(t)$ for $t > 0$.

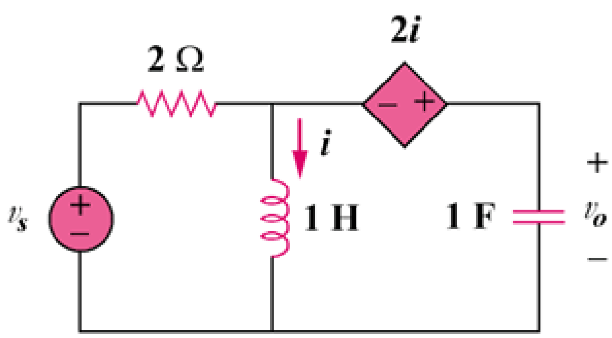
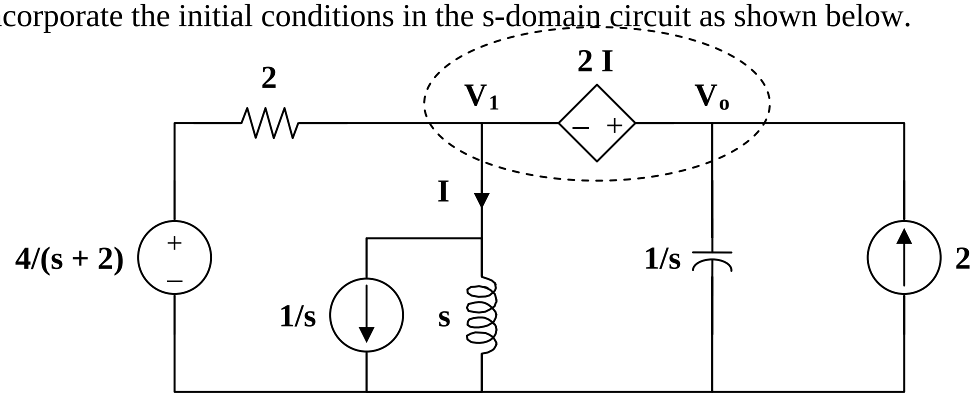


Figure 16.81
For Prob. 16.58.

Solution

We incorporate the initial conditions in the s-domain circuit as shown below.



At the supernode,

$$\frac{(4/(s+2)) - V_1}{2} + 2 = \frac{V_1}{s} + \frac{1}{s} + sV_o$$

$$\frac{2}{s+2} + 2 = \left(\frac{1}{2} + \frac{1}{s}\right)V_1 + \frac{1}{s} + s\,V_o \qquad (1)$$

But $\quad V_o = V_1 + 2I \quad$ and $\quad I = \dfrac{V_1 + 1}{s}$

$$V_o = V_1 + \frac{2(V_1 + 1)}{s} \quad \longrightarrow \quad V_1 = \frac{V_o - 2/s}{(s+2)/s} = \frac{sV_o - 2}{s+2} \qquad (2)$$

Substituting (2) into (1)

$$\frac{2}{s+2} + 2 - \frac{1}{s} = \left(\frac{s+2}{2s}\right)\left[\left(\frac{s}{s+2}\right)V_o - \frac{2}{s+2}\right] + sV_o$$

$$\frac{2}{s+2} + 2 - \frac{1}{s} + \frac{1}{s} = \left[\left(\frac{1}{2}\right) + s\right]V_o$$

$$\frac{2s+4+2}{(s+2)} = \frac{2s+6}{s+2} = (s + 1/2)V_o$$

$$V_o = \frac{2s+6}{(s+2)(s+1/2)} = \frac{A}{s+1/2} + \frac{B}{s+2}$$

$A = (-1+6)/(-0.5+2) = 3.333, \quad B = (-4+6)/(-2+1/2) = -1.3333$

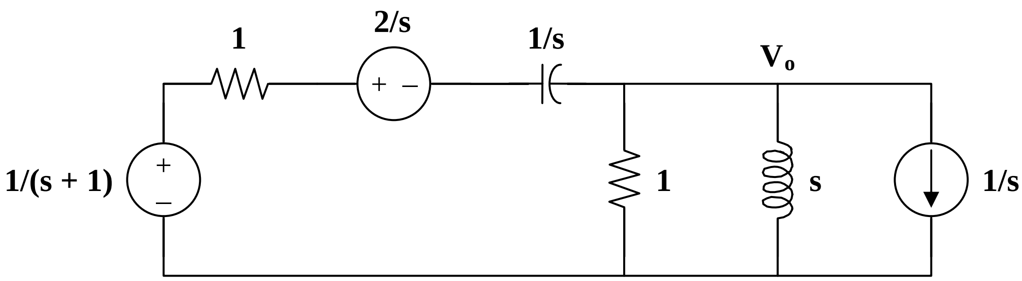
$$V_o = \frac{3.333}{s+1/2} - \frac{1.3333}{s+2}$$

Therefore,

$$v_o(t) = \underline{\mathbf{(3.333e^{-t/2} - 1.3333e^{-2t})u(t)\ V}}$$

Chapter 16, Solution 59.

We incorporate the initial conditions and transform the current source to a voltage source as shown.



At the main non-reference node, KCL gives

$$\frac{1/(s+1) - 2/s - V_o}{1 + 1/s} = \frac{V_o}{1} + \frac{V_o}{s} + \frac{1}{s}$$

$$\frac{s}{s+1} - 2 - s\,V_o = (s+1)(1+1/s)\,V_o + \frac{s+1}{s}$$

$$\frac{s}{s+1} - \frac{s+1}{s} - 2 = (2s + 2 + 1/s)\,V_o$$

$$V_o = \frac{-2s^2 - 4s - 1}{(s+1)(2s^2 + 2s + 1)}$$

$$V_o = \frac{-s - 2s - 0.5}{(s+1)(s^2 + s + 0.5)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + s + 0.5}$$

$$A = (s+1)\,V_o\Big|_{s=-1} = 1$$

$$-s^2 - 2s - 0.5 = A\,(s^2 + s + 0.5) + B\,(s^2 + s) + C\,(s+1)$$

Equating coefficients :

$$s^2: \quad -1 = A + B \longrightarrow B = -2$$

$$s^1: \quad -2 = A + B + C \longrightarrow C = -1$$

$$s^0: \quad -0.5 = 0.5A + C = 0.5 - 1 = -0.5$$

$$V_o = \frac{1}{s+1} - \frac{2s+1}{s^2 + s + 0.5} = \frac{1}{s+1} - \frac{2\,(s+0.5)}{(s+0.5)^2 + (0.5)^2}$$

$$v_o(t) = \left[\mathbf{e^{-t} - 2\,e^{-t/2}\cos(t/2)}\right]\mathbf{u(t)\ V}$$

Chapter 16, Solution 60.

Find the response $v_R(t)$ for $t > 0$ in the circuit in Fig. 16.83. Let $R = 3\ \Omega$, $L = 2$ H, and $C = 1/18$ F.

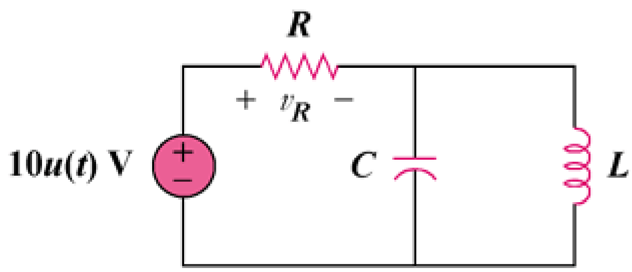
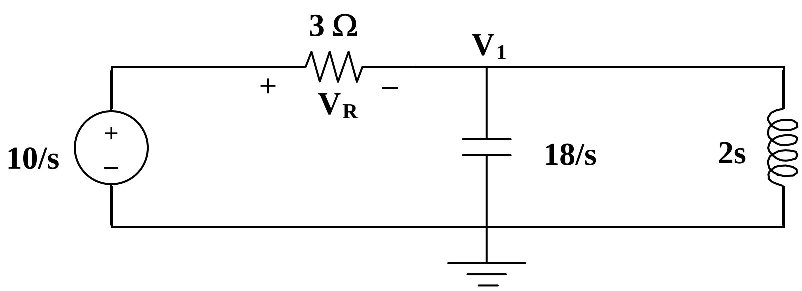


Figure 16.83
For Prob. 16.60.

Solution

Step 1. First convert the circuit into the s-domain. Then use nodal analysis and eventually solve for V_R, then perform a partial fraction expansion and convert back into the time domain.



$[(V_1\text{-}10/s)/3]+[(V_1\text{-}0)/(18/s)]+[(V_1\text{-}0)/(2s)] = 0$ and $V_R = (10/s)\text{-}V_1$.

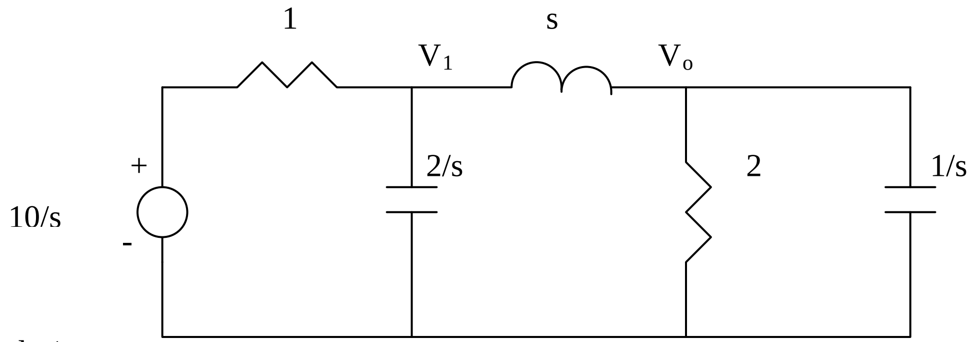
Step2. $[(1/3)+(s/18)+1/(2s)]V_1 = 3.333/s = [(s^2+6s+9)/(18s)]V_1$ or $V_1 = 60/[(s+3)^2]$ and $V_R = (10/s)\text{-}60/[(s+3)^2]$.

Thus,

$v_R(t) = \mathbf{[10\text{-}60te^{-3t}]u(t)}$ **volts**.

Chapter 16, Solution 61.

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{V_1 - \frac{10}{s}}{1} + \frac{V_1 - V_o}{s} + \frac{s}{2}(V_1 - 0) = 0 \quad \longrightarrow \quad \left(\frac{s^2}{2} + s + 1\right)V_1 + (-1)V_o = 10 \qquad (1)$$

At node 2,

$$\frac{V_o - V_1}{s} + \frac{V_o - 0}{2} + s(V_o - 0) = 0 \quad \longrightarrow \quad V_1 = (s^2 + 0.5s + 1)V_o \qquad (2)$$

Substituting (2) into (1) gives

$10 = [0.5(s^2 + 2s + 2)(s^2 + 0.5s + 1)V_o - V_o = 0.5(s^4 + 2.5s^3 + 4s^2 + 3s + 2 - 2)V_o$

$$V_o = \frac{20}{s(s^3 + 2.5s^2 + 4s + 3)}$$

Use MATLAB to find the roots.

```
>> p=[1 2.5 4 3]

p =

    1.0000    2.5000    4.0000    3.0000

>> r=roots(p)

r =

  -0.6347 + 1.4265i
  -0.6347 - 1.4265i
  -1.2306
```

Thus,

$$V_o = \frac{20}{s(s+1.2306)(s+0.6347+j1.4265)(s+0.6347-j1.4265)}$$

$$= \frac{A}{s} + \frac{B}{(s+1.2306)} + \frac{C}{(s+0.6347+j1.4265)} + \frac{D}{(s+0.6347-j1.4265)}$$

Where A = 20/3 = 6.667; B =

$$\frac{20}{(-1.2306)(-1.2306+0.6347+j1.4265)(-1.2306+0.6347-j1.4265)}$$

$$= \frac{-16.252}{(0.3551+2.035)} = -6.8$$

$$C = \frac{20}{(-0.6347-j1.4265)(-0.6347-j1.4265+1.2306)(-j2.853)}$$

$$= \frac{20}{(1.5613\angle -113.99°)(1.546\angle -67.33°)(2.853\angle -90°)} = \frac{20}{6.886\angle 88.68°} = 2.904\angle -88.68°$$

$$D = \frac{20}{(-0.6347+j1.4265)(-0.6347+j1.4265+1.2306)(j2.853)}$$

$$= \frac{20}{(1.5613\angle 113.99°)(1.546\angle 67.33°)(2.853\angle 90°)} = \frac{20}{6.886\angle -88.68°} = 2.904\angle 88.68°$$

$$V_o = \frac{6.667}{s} + \frac{-6.8}{(s+1.2306)} + \frac{2.904\angle -88.68°}{(s+0.6347+j1.4265)} + \frac{2.904\angle 88.68°}{(s+0.6347-j1.4265)} \text{ or}$$

$v_o(t) = [6.667–6.8e^{–1.2306t}+2.904e^{–0.6347t}(e^{–(1.4265t+88.68^\circ)}+e^{(1.4265t+88.68^\circ)})]u(t)$ volts or

$$= \mathbf{[6.667–6.8e^{–1.2306t}+5.808e^{–0.6347t}\cos(1.4265t+88.68^\circ)]u(t)\ V}.$$

Answer does check for initial values and final values.

Chapter 16, Solution 62.

Using Fig. 16.85, design a problem to help other students better understand solving for node voltages by working in the s-domain.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the node voltages v_1 and v_2 in the circuit of Fig. 16.85 using Laplace transform technique. Assume that $i_s = 12e^{-t}\,u(t)$ A and that all initial conditions are zero.

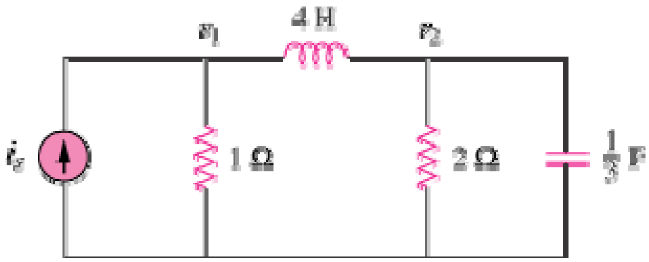
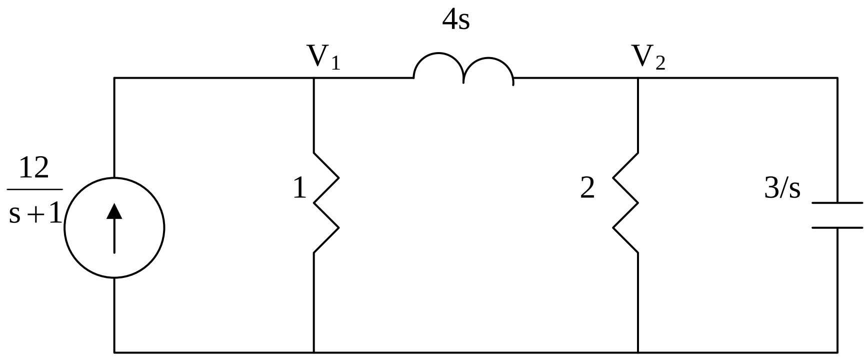


Figure 16.85
For Prob. 16.62.

Solution

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{12}{s+1} = \frac{V_1}{1} + \frac{V_1 - V_2}{4s} \longrightarrow \frac{12}{s+1} = V_1\left(1 + \frac{1}{4s}\right) - \frac{V_2}{4s} \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{4s} = \frac{V_2}{2} + \frac{s}{3}V_2 \quad \longrightarrow \quad V_1 = V_2\left(\frac{4}{3}s^2 + 2s + 1\right) \qquad (2)$$

Substituting (2) into (1),

$$\frac{12}{s+1} = V_2\left[\left(\frac{4}{3}s^2 + 2s + 1\right)\left(1 + \frac{1}{4s}\right) - \frac{1}{4s}\right] = \left(\frac{4}{3}s^2 + \frac{7}{3}s + \frac{3}{2}\right)V_2$$

$$V_2 = \frac{9}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{A}{(s+1)} + \frac{Bs + C}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9 = A(s^2 + \frac{7}{4}s + \frac{9}{8}) + B(s^2 + s) + C(s+1)$$

Equating coefficients:

$$\begin{array}{lll} s^2: & 0 = A + B & \\ s: & 0 = \frac{7}{4}A + B + C = \frac{3}{4}A + C & \longrightarrow \quad C = -\frac{3}{4}A \\ \text{constant}: & 9 = \frac{9}{8}A + C = \frac{3}{8}A & \longrightarrow \quad A = 24,\ B = \text{-}24,\ C = \text{-}18 \end{array}$$

$$V_2 = \frac{24}{(s+1)} - \frac{24s + 18}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{24}{(s+1)} - \frac{24(s + 7/8)}{(s + \frac{7}{8})^2 + \frac{23}{64}} + \frac{3}{(s + \frac{7}{8})^2 + \frac{23}{64}}$$

Taking the inverse of this produces:

$$\underline{\mathbf{v_2(t) = \left[24e^{-t} - 24e^{-0.875t}\cos(0.5995t) + 5.004e^{-0.875t}\sin(0.5995t)\right]u(t)V}}$$

Similarly,

$$V_1 = \frac{9\left(\frac{4}{3}s^2 + 2s + 1\right)}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{D}{(s+1)} + \frac{Es + F}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9\left(\frac{4}{3}s^2 + 2s + 1\right) = D(s^2 + \frac{7}{4}s + \frac{9}{8}) + E(s^2 + s) + F(s+1)$$

Equating coefficients:

$$s^2: \qquad 12 = D + E$$

$$s: \qquad 18 = \frac{7}{4}D + E + F \text{ or } 6 = \frac{3}{4}D + F \quad \longrightarrow \quad F = 6 - \frac{3}{4}D$$

$$\text{constant}: \qquad 9 = \frac{9}{8}D + F \text{ or } \ 3 = \frac{3}{8}D \quad \longrightarrow \quad D = 8,\ E = 4,\ F = 0$$

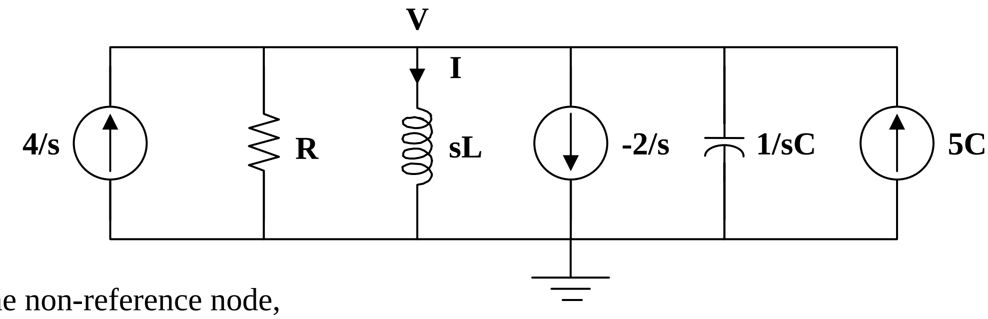
$$V_1 = \frac{8}{(s+1)} + \frac{4s}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{8}{(s+1)} + \frac{4(s+7/8)}{(s+\frac{7}{8})^2 + \frac{23}{64}} - \frac{7/2}{(s+\frac{7}{8})^2 + \frac{23}{64}}$$

Thus,

$$\underline{\mathbf{v_1(t) = \left[8e^{-t} + 4e^{-0.875t}\cos(0.5995t) - 5.838e^{-0.875t}\sin(0.5995t)\right]u(t)V}}$$

Chapter 16, Solution 63.

The s-domain form of the circuit with the initial conditions is shown below.



At the non-reference node,

$$\frac{4}{s}+\frac{2}{s}+5C=\frac{V}{R}+\frac{V}{sL}+sCV$$

$$\frac{6+5sC}{s}=\frac{CV}{s}\left(s^2+\frac{s}{RC}+\frac{1}{LC}\right)$$

$$V=\frac{5s+6/C}{s^2+(s/RC)+(1/LC)}$$

But $\quad \frac{1}{RC}=\frac{1}{10/80}=8, \qquad \frac{1}{LC}=\frac{1}{4/80}=20$

$$V=\frac{5s+480}{s^2+8s+20}=\frac{5(s+4)}{(s+4)^2+2^2}+\frac{(230)(2)}{(s+4)^2+2^2}$$

$$v(t)=\mathbf{[5\,e^{-4t}\ \cos(2t)+230\,e^{-4t}\ \sin(2t)]u(t)\ V}$$

$$I=\frac{V}{sL}=\frac{5s+480}{4s(s^2+8s+20)}$$

$$I=\frac{1.25s+120}{s(s^2+8s+20)}=\frac{A}{s}+\frac{Bs+C}{s^2+8s+20}$$

$A=6, \qquad B=-6, C=-46.75$

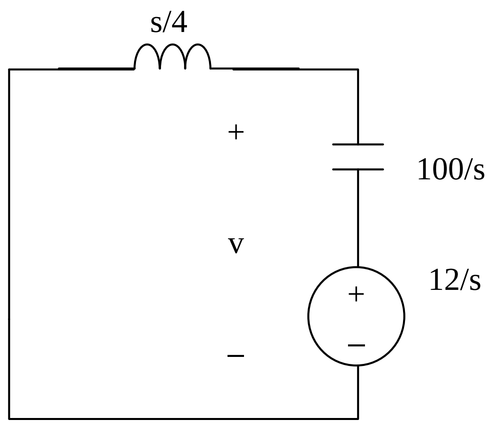
$$I=\frac{6}{s}-\frac{6s+46.75}{s^2+8s+20}=\frac{6}{s}-\frac{6(s+4)}{(s+4)^2+2^2}-\frac{(11.375)(2)}{(s+4)^2+2^2}$$

$$i(t)=\mathbf{[6-6\,e^{-4t}\ \cos(2t)-11.375\,e^{-4t}\ \sin(2t)]u(t)\ A}$$

Checking, Ldi/dt = 4{24 e^{-4t}cos(2t) + 12 e^{-4t}sin(2t) + 45.5 e^{-4t}sin(2t) – 22.75 e^{-4t}cos(2t)}u(t) = [5 e^{-4t}cos(2t) + 230 e^{-4t}sin(2t)]u(t). Answer checks.

Chapter 16, Solution 64.

When the switch is position 1, v(0)=12, and $i_L(0) = 0$. When the switch is in position 2, we have the circuit as shown below.



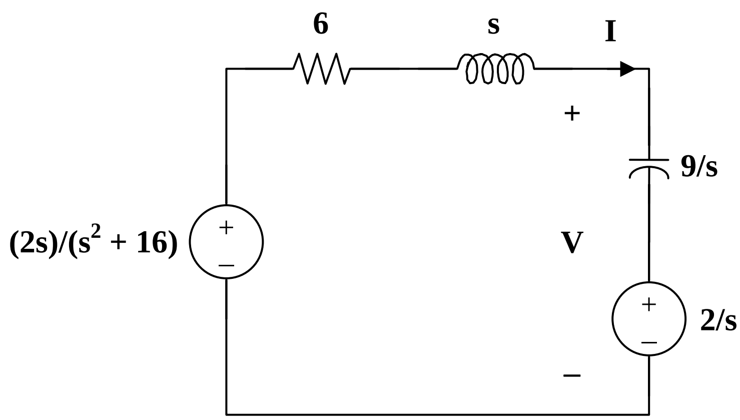
$$10mF = 0.01F \quad \longrightarrow \quad \frac{1}{sC} = \frac{100}{s}$$

$$I = \frac{12/s}{s/4 + 100/s} = \frac{48}{s^2 + 400}\ , \qquad V = sLI = \frac{s}{4}I = \frac{12s}{s^2 + 400}$$

v(t) = **[12cos(20t)]u(t) V**

Chapter 16, Solution 65.

For $t > 0$, the circuit in the s-domain is shown below.



Applying KVL,

$$\frac{-2s}{s^2+16} + \left(6 + s + \frac{9}{s}\right)I + \frac{2}{s} = 0$$

$$I = \frac{-32}{(s^2+6s+9)(s^2+16)}$$

$$V = \frac{9}{s}I + \frac{2}{s} = \frac{2}{s} + \frac{-288}{s\,(s+3)^2(s^2+16)}$$

$$= \frac{2}{s} + \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+3)^2} + \frac{Ds+E}{s^2+16}$$

$$-288 = A\,(s^4 + 6s^3 + 25s^2 + 96s + 144) + B\,(s^4 + 3s^3 + 16s^2 + 48s)$$
$$+ C\,(s^3 + 16s) + D\,(s^4 + 6s^3 + 9s^2) + E\,(s^3 + 6s^2 + 9s)$$

Equating coefficients :

s^0: $\quad -288 = 144A \qquad (1)$

s^1: $\quad 0 = 96A + 48B + 16C + 9E \qquad (2)$

s^2: $\quad 0 = 25A + 16B + 9D + 6E \qquad (3)$

s^3: $\quad 0 = 6A + 3B + C + 6D + E \qquad (4)$

s^4: $\quad 0 = A + B + D \qquad (5)$

Solving equations (1), (2), (3), (4) and (5) gives

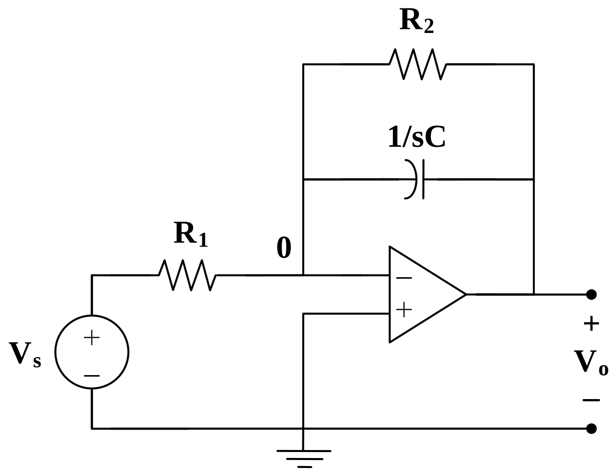
$A = -2$, $\quad B = 2.202$, $\quad C = 3.84$, $\quad D = -0.202$, $\quad E = 2.766$

$$V(s) = \frac{2.202}{s+3} + \frac{3.84}{(s+3)^2} - \frac{0.202\,s}{s^2+16} + \frac{(0.6915)(4)}{s^2+16}$$

$v(t) =$ $\mathbf{\{2.202e^{-3t} + 3.84te^{-3t} - 0.202\cos(4t) + 0.6915\sin(4t)\}u(t)\ V}$

Chapter 16, Solution 66.

Consider the op-amp circuit below where R_1 = 20 kΩ, R_2 = 10 kΩ, C = 50 μF, and $v_s(t) = [3e^{-5t}]u(t)$ V.



At node 0,

$$\frac{V_s - 0}{R_1} = \frac{0 - V_o}{R_2} + (0 - V_o)sC$$

$$V_s = R_1\left(\frac{1}{R_2} + sC\right)(-V_o)$$

$$\frac{V_o}{V_s} = \frac{-1}{sR_1C + R_1/R_2}$$

But $\quad \frac{R_1}{R_2} = \frac{20}{10} = 2, \qquad R_1C = (20 \times 10^3)(50 \times 10^{-6}) = 1$

So, $\quad \frac{V_o}{V_s} = \frac{-1}{s+2}$

$$v_s(t) = 3e^{-5t} \longrightarrow V_s = 3/(s+5)$$

$$V_o = \frac{-3}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5} \text{ where } A = -1 \text{ and } B = 1.$$

$$V_o = \frac{1}{s+5} - \frac{1}{s+2}$$

$$v_o(t) = \left(\mathbf{e^{-5t} - e^{-2t}}\right)\mathbf{u(t)\ V}.$$

Chapter 16, Solution 67.

Given the op amp circuit in Fig. 16.90. If $v_1(0^+) = 2$ V and $v_2(0^+) = 0$ V, find v_o for $t > 0$. Let $R = 100$ kΩ and C = 1 μF.

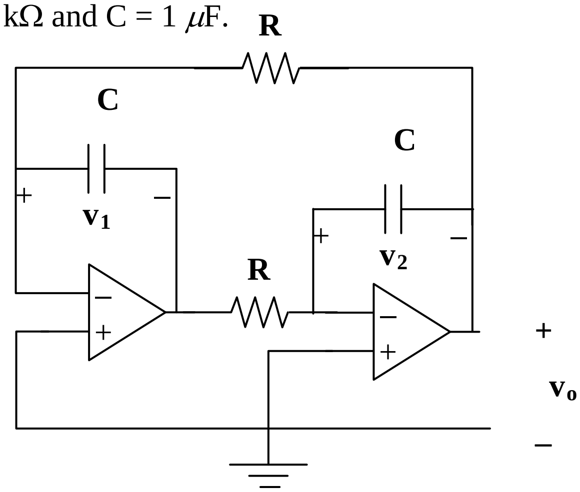
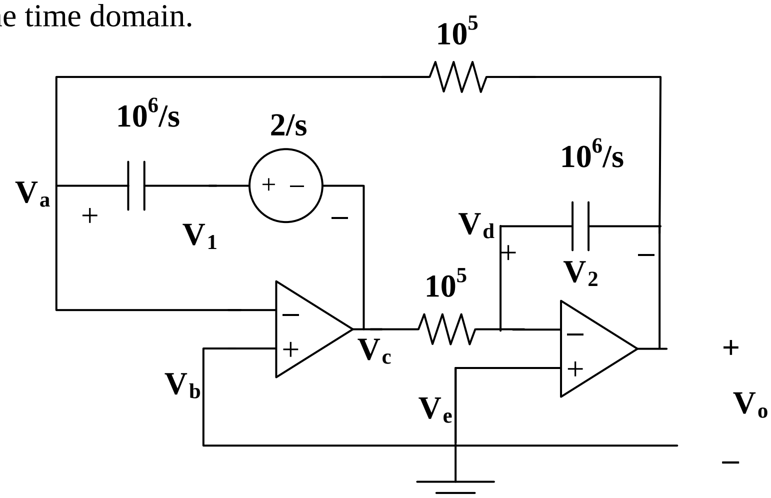


Figure 16.90
For Prob. 16.67.

Solution

Step 1. Convert the circuit into the s-domain and insert initial conditions. Next, solve for $V_o(s)$, then obtain the partial fraction expansion and convert back into the time domain.



$[(V_a-(V_c+2/s))/(10^6/s)]+[(V_a-V_o)/10^5]+0 = 0$; $V_a = V_b = 0$ and
$[(V_d-V_c)/10^5]+[(V_d-V_o)/(10^6/s)]+0 = 0$; $V_d = V_e = 0$.

Step 2. $sV_c + 10V_o = -2$ and $10V_c + sV_o = 0$ or $V_c = -0.1sV_o$ thus,

$(-0.1s^2+10)V_o = -2$ or $V_o = 20/(s^2-100) = [A/(s-10)]+[B/(s+10)]$ where $A = 20/(10+10) = 1$ and $B = 20/(-10-10) = -1$. This now leads to

$$v_o(t) = \mathbf{[e^{10t}-e^{-10t}]u(t)\ volts}.$$

It should be noted that this is an unstable circuit!

Chapter 16, Solution 68.

Obtain V_o/V_s in the op amp circuit in Fig. 8.91.

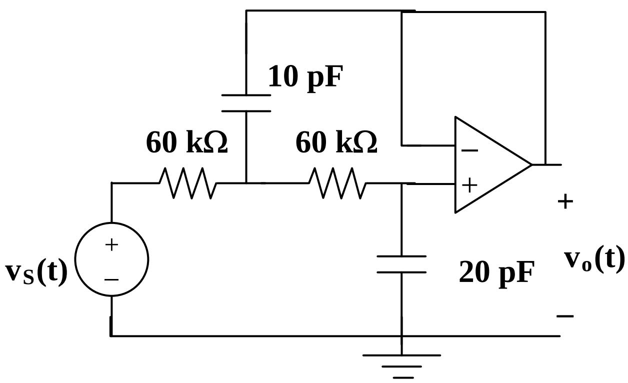
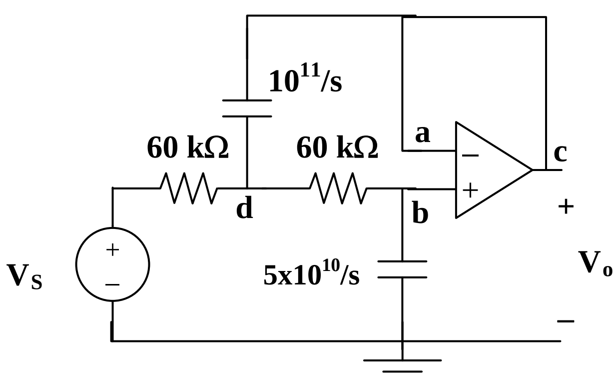


Figure 8.91
For Prob. 8.68.

Solution

Step 1. Convert the circuit into the s-domain and then solve for $V_o(s)$ in terms of $V_s(s)$. Then solve for $V_o/V_s = T(s)$.



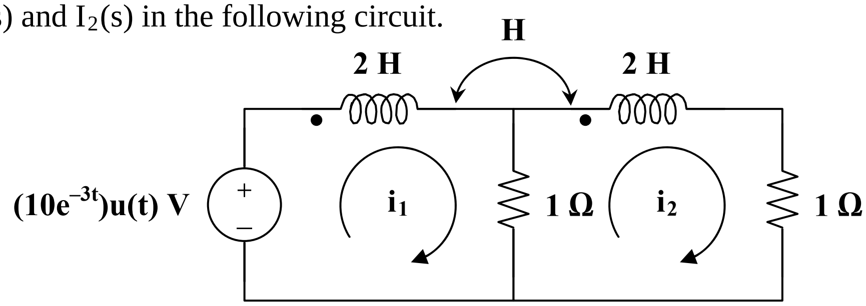
At a, $V_a = V_b = V_c = V_o$. At b, $[(V_b–V_d)/60k]+[(V_b–0)/(5\text{x}10^{10}/s)]+0 = 0$ or $[(V_o–V_d)/60k]+[(V_o–0)/(5\text{x}10^{10}/s)] = 0$ or $[(1/60k)V_d = [(1/60k)+(s/(5\text{x}10^{10}))]V_o$ or $V_d = [(1.2\text{x}10^{-6})s+1]V_o$.

At d, $[(V_d–V_s)/60k]+[(V_d–V_c)/(10^{11}/s)]+(V_d–V_b)/60k] = 0$ or
$[(2/60k)+(s/10^{11})]V_d–(s/10^{11})V_o–(1/60k)V_o = (1/60k)V_s$ or
$[(2/60k)+(s/10^{11})] [(1.2\text{x}10^{-6})s+1]V_o–(s/10^{11})V_o–(1/60k)V_o = (1/60k)V_s$ or
$[2+(6\text{x}10^{-7})s] [(1.2\text{x}10^{-6})s+1]V_o–(6\text{x}10^{-7})sV_o–V_o = V_s$ or
$[7.2\text{x}10^{-13}s^2+(2.4\text{x}10^{-6}+0.6\text{x}10^{-6}–0.6\text{x}10^{-6})s+(2–1)]V_o = V_s$ or

$$T(s) = V_o/V_s = \mathbf{1/[(7.2\text{x}10^{-13})s^2+(2.4\text{x}10^{-6})s+1]}.$$

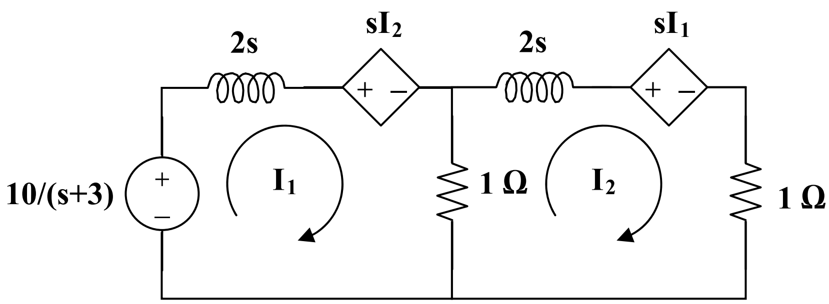
Chapter 16, Solution 69.

Find $I_1(s)$ and $I_2(s)$ in the following circuit.



Solution

Step 1. We note that the initial conditions in this case are equal to zero. Next, we need to convert the circuit into the s-domain and use the model for mutually coupled circuits. Then we can write the mesh equations and solve for I_1 and I_2.



Step 2. $-[10/(s+3)] + 2sI_1 + sI_2 + 1(I_1-I_2) = 0$ and $1(I_2-I_1) + 2sI_2 + sI_1 + 1I_2 = 0$. Simplifying we get,

$(2s+1)I_1 + (s-1)I_2 = 10/(s+3)$ and $(s-1)I_1 + (2s+1)I_2 = 0$.

We can solve this directly using substitution or use matrices. Let us use matrices.

$$\begin{bmatrix} 2s+1 & s-1 \\ s-1 & 2s+1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{10}{(s+3)} \\ 0 \end{bmatrix}$$ The matrix inverse

$$\begin{bmatrix} 2s+1 & s-1 \\ s-1 & 2s+1 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} 2s+1 & -s+1 \\ -s+1 & 2s+1 \end{bmatrix}}{4s^2+4s+1-s^2+2s-1} = \frac{\begin{bmatrix} 2s+1 & -s+1 \\ -s+1 & 2s+1 \end{bmatrix}}{3s(s+2)}$$

Therefore,

$I_1 =$ **6.667(s+0.5)/[s(s+2)(s+3)]** and
$I_2 =$ **–3.333(s–1)/[s(s+2)(s+3)]**

6.667(s+0.5)/[s(s+2)(s+3)], –3.333(s–1)/[s(s+2)(s+3)]

Chapter 16, Solution 70.

Using Fig. 16.93, design a problem to help other students better understand how to do circuit analysis with circuits that have mutually coupled elements by working in the s-domain.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 16.93, find $v_o(t)$ for $t > 0$.

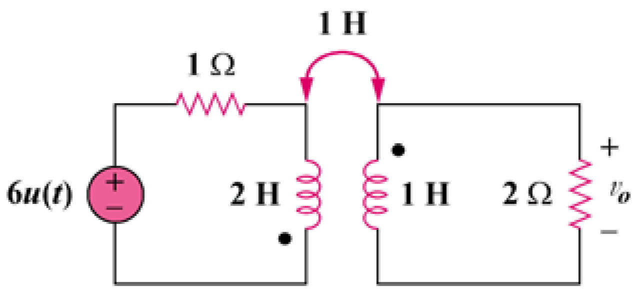
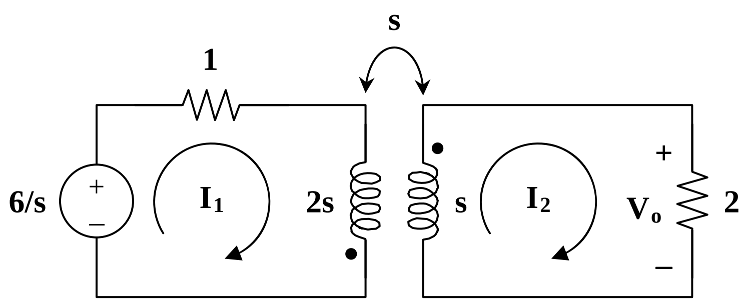


Figure 16.93
For Prob. 16.70.

Solution

Consider the circuit shown below.



For mesh 1,

$$\frac{6}{s} = (1+2s)\,I_1 + s\,I_2 \qquad (1)$$

For mesh 2,

$$0 = s\,I_1 + (2+s)\,I_2$$

$$I_1 = -\left(1+\frac{2}{s}\right)I_2 \qquad (2)$$

Substituting (2) into (1) gives

$$\frac{6}{s} = -(1+2s)\left(1+\frac{2}{s}\right)I_2 + s\,I_2 = \frac{-(s^2+5s+2)}{s}I_2$$

or $$I_2 = \frac{-6}{s^2+5s+2}$$

$$V_o = 2I_2 = \frac{-12}{s^2+5s+2} = \frac{-12}{(s+0.438)(s+4.561)}$$

Since the roots of $s^2+5s+2=0$ are -0.438 and -4.561,

$$V_o = \frac{A}{s+0.438} + \frac{B}{s+4.561}$$

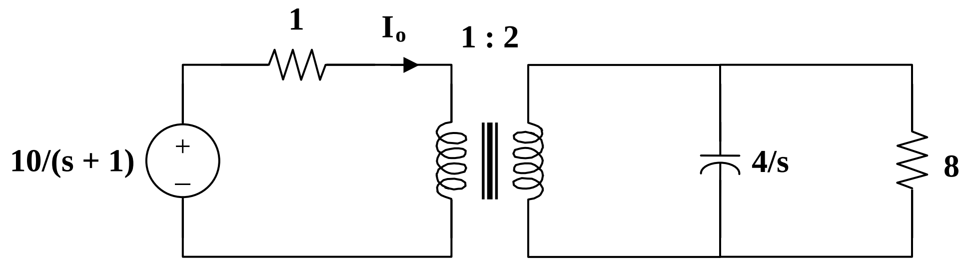
$$A = \frac{-12}{4.123} = -2.91, \qquad B = \frac{-12}{-4.123} = 2.91$$

$$V_o(s) = \frac{-2.91}{s+0.438} + \frac{2.91}{s+4.561}$$

$$v_o(t) = \underline{\mathbf{2.91\left[e^{-4.561t} - e^{0.438t}\right]u(t)\ V}}$$

Chapter 16, Solution 71.

Consider the following circuit.



Let $\quad Z_L = 8 \,\|\, \frac{4}{s} = \frac{(8)(4/s)}{8 + 4/s} = \frac{8}{2s+1}$

When this is reflected to the primary side,

$$Z_{in} = 1 + \frac{Z_L}{n^2}, \quad n = 2$$

$$Z_{in} = 1 + \frac{2}{2s+1} = \frac{2s+3}{2s+1}$$

$$I_o = \frac{10}{s+1} \cdot \frac{1}{Z_{in}} = \frac{10}{s+1} \cdot \frac{2s+1}{2s+3}$$

$$I_o = \frac{10s+5}{(s+1)(s+1.5)} = \frac{A}{s+1} + \frac{B}{s+1.5}$$

$$A = -10, \qquad B = 20$$

$$I_o(s) = \frac{-10}{s+1} + \frac{20}{s+1.5}$$

$$i_o(t) = \mathbf{10\left[2e^{-1.5t} - e^{-t}\right]u(t)\ A}$$

Chapter 16, Solution 72.

$$Y(s) = H(s)\,X(s)\,, \qquad X(s) = \frac{4}{s+1/3} = \frac{12}{3s+1}$$

$$Y(s) = \frac{12\,s^2}{(3s+1)^2} = \frac{4}{3} - \frac{8s+4/3}{(3s+1)^2}$$

$$Y(s) = \frac{4}{3} - \frac{8}{9}\cdot\frac{s}{(s+1/3)^2} - \frac{4}{27}\cdot\frac{1}{(s+1/3)^2}$$

Let $G(s) = \frac{-8}{9}\cdot\frac{s}{(s+1/3)^2}$

Using the time differentiation property,

$$g(t) = \frac{-8}{9}\cdot\frac{d}{dt}(t\,e^{-t/3}) = \frac{-8}{9}\left(\frac{-1}{3}t\,e^{-t/3} + e^{-t/3}\right)$$

$$g(t) = \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3}$$

Hence,

$$y(t) = \left[\frac{4}{3} + \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3} - \frac{4}{27}t\,e^{-t/3}\right]u(t)$$

$$y(t) = \boldsymbol{\left[\frac{4}{3} - \frac{8}{9}e^{-t/3} + \frac{4}{27}t\,e^{-t/3}\right]u(t)}$$

Chapter 16, Solution 73.

$$x(t) = u(t) \quad \longrightarrow \quad X(s) = \frac{1}{s}$$

$$y(t) = 10\cos(2t) \quad \longrightarrow \quad Y(s) = \frac{10s}{s^2 + 4}$$

$$H(s) = \frac{Y(s)}{X(s)} = \mathbf{\frac{10s^2}{s^2 + 4}}$$

Chapter 16, Solution 74.

Design a problem to help other students to better understand how to find outputs when given a transfer function and an input.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A circuit is known to have its transfer function as

$$H(s) = \frac{s+3}{s^2+4s+5}$$

Find its output when:

(a) the input is a unit step function

(b) the input is $6te^{-2t}\,u(t)$.

Solution

(a) $\quad Y(s) = H(s)\,X(s)$

$$= \frac{s+3}{s^2+4s+5} \cdot \frac{1}{s}$$

$$= \frac{s+3}{s\,(s^2+4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+5}$$

$$s+3 = A\,(s^2+4s+5) + Bs^2 + Cs$$

Equating coefficients :

$s^0: \quad 3 = 5A \longrightarrow A = 3/5$

$s^1: \quad 1 = 4A + C \longrightarrow C = 1 - 4A = -7/5$

$s^2: \quad 0 = A + B \longrightarrow B = -A = -3/5$

$$Y(s) = \frac{3/5}{s} - \frac{1}{5} \cdot \frac{3s+7}{s^2+4s+5}$$

$$Y(s) = \frac{0.6}{s} - \frac{1}{5} \cdot \frac{3(s+2)+1}{(s+2)^2+1}$$

$$y(t) = \left[\mathbf{0.6 - 0.6\,e^{-2t}\cos(t) - 0.2\,e^{-2t}\sin(t)}\right]\mathbf{u(t)}$$

(b) $x(t) = 6\,t\,e^{-2t} \longrightarrow X(s) = \dfrac{6}{(s+2)^2}$

$$Y(s) = H(s)\,X(s) = \frac{s+3}{s^2+4s+5} \cdot \frac{6}{(s+2)^2}$$

$$Y(s) = \frac{6\,(s+3)}{(s+2)^2(s^2+4s+5)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+4s+5}$$

Equating coefficients :

s^3: $0 = A + C \longrightarrow C = -A$ (1)

s^2: $0 = 6A + B + 4C + D = 2A + B + D$ (2)

s^1: $6 = 13A + 4B + 4C + 4D = 9A + 4B + 4D$ (3)

s^0: $18 = 10A + 5B + 4D = 2A + B$ (4)

Solving (1), (2), (3), and (4) gives

$A = 6$, $B = 6$, $C = -6$, $D = -18$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6s+18}{(s+2)^2+1}$$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6\,(s+2)}{(s+2)^2+1} - \frac{6}{(s+2)^2+1}$$

$$y(t) = \left[\mathbf{6\,e^{-2t} + 6\,t\,e^{-2t} - 6\,e^{-2t}\cos(t) - 6\,e^{-2t}\sin(t)}\right]\mathbf{u(t)}$$

Chapter 16, Solution 75.

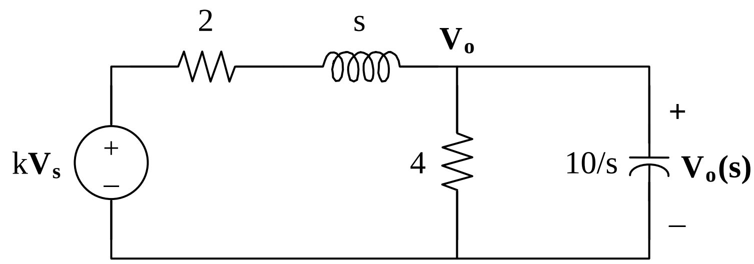
$$H(s) = \frac{Y(s)}{X(s)}, \qquad X(s) = \frac{1}{s}$$

$$Y(s) = \frac{4}{s} + \frac{1}{2(s+3)} - \frac{2s}{(s+2)^2+16} - \frac{(3)(4)}{(s+2)^2+16}$$

$$H(s) = s\,Y(s) = 4 + \frac{s}{2(s+3)} - \frac{2s(s+2)}{s^2+4s+20} - \frac{12s}{s^2+4s+20}$$

Chapter 16, Solution 76.

Consider the following circuit.



Using nodal analysis,

$$\frac{kV_s - V_o}{s+2} = \frac{V_o}{4} + \frac{V_o}{10/s}$$

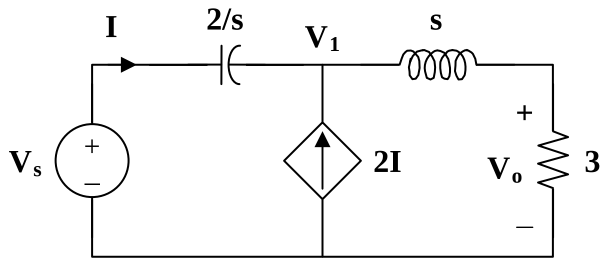
$$V_s = (1/k)(s+2)\left(\frac{1}{s+2} + \frac{1}{4} + \frac{s}{10}\right)V_o = (1/k)\left(1 + \frac{1}{4}(s+2) + \frac{1}{10}(s^2 + 2s)\right)V_o$$

$$V_s = \frac{1}{20k}\left(2s^2 + 9s + 30\right)V_o$$

$$\frac{V_o}{V_s} = \mathbf{10k/(s^2+4.5s+15)}$$

Chapter 16, Solution 77.

Consider the following circuit.



At node 1,

$$2I + I = \frac{V_1}{s+3}, \qquad \text{where} \quad I = \frac{V_s - V_1}{2/s}$$

$$3 \cdot \frac{V_s - V_1}{2/s} = \frac{V_1}{s+3}$$

$$\frac{V_1}{s+3} = \frac{3s}{2} V_s - \frac{3s}{2} V_1$$

$$\left(\frac{1}{s+3} + \frac{3s}{2} \right) V_1 = \frac{3s}{2} V_s$$

$$V_1 = \frac{3s\,(s+3)}{3s^2 + 9s + 2} V_s$$

$$V_o = \frac{3}{s+3} V_1 = \frac{9s}{3s^2 + 9s + 2} V_s$$

$$H(s) = \frac{V_o}{V_s} = \mathbf{\frac{9s}{3s^2 + 9s + 2}}$$

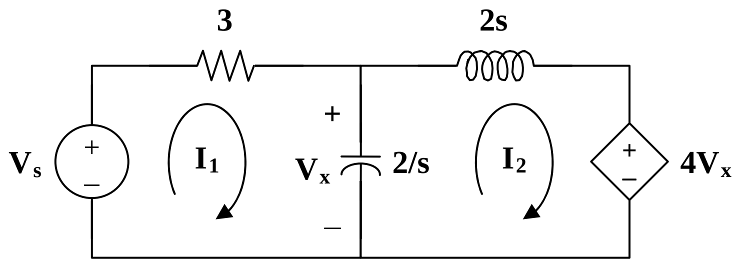
Chapter 16, Solution 78.

Taking the inverse Laplace transform of each term gives

$$h(t) = \underline{\left(5e^{-t} - 3e^{-2t} + 6e^{-4t}\right)u(t)}$$

Chapter 16, Solution 79.

(a) Consider the circuit shown below.



For loop 1,

$$V_s = \left(3 + \frac{2}{s}\right) I_1 - \frac{2}{s} I_2 \qquad (1)$$

For loop 2,

$$4V_x + \left(2s + \frac{2}{s}\right) I_2 - \frac{2}{s} I_1 = 0$$

But, $V_x = (I_1 - I_2)\left(\dfrac{2}{s}\right)$

So, $$\frac{8}{s}(I_1 - I_2) + \left(2s + \frac{2}{s}\right) I_2 - \frac{2}{s} I_1 = 0$$

$$0 = \frac{-6}{s} I_1 + \left(\frac{6}{s} - 2s\right) I_2 \qquad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} 3 + 2/s & -2/s \\ -6/s & 6/s - 2s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \left(3 + \frac{2}{s}\right)\left(\frac{6}{s} - 2s\right) - \left(\frac{6}{s}\right)\left(\frac{2}{s}\right)$$

$$\Delta = \frac{18}{s} - 6s - 4$$

$$\Delta_1 = \left(\frac{6}{s} - 2s\right) V_s, \qquad \Delta_2 = \frac{6}{s} V_s$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{(6/s - 2s)}{18/s - 4 - 6s} V_s$$

$$\frac{I_1}{V_s} = \frac{3/s - s}{9/s - 2 - 3} = \mathbf{\frac{s^2 - 3}{3s^2 + 2s - 9}}$$

(b) $$I_2 = \frac{\Delta_2}{\Delta}$$

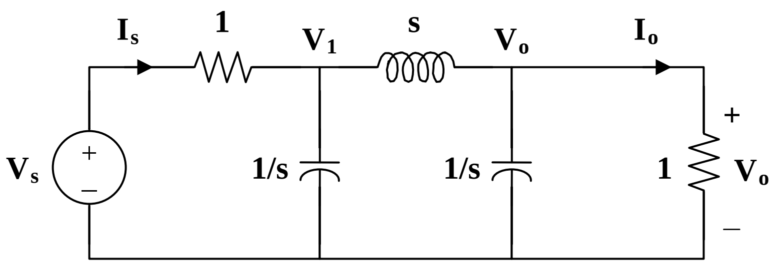
$$V_x = \frac{2}{s}(I_1 - I_2) = \frac{2}{s}\left(\frac{\Delta_1 - \Delta_2}{\Delta}\right)$$

$$V_x = \frac{2/s\, V_s\, (6/s - 2s - 6/s)}{\Delta} = \frac{-4V_s}{\Delta}$$

$$\frac{I_2}{V_x} = \frac{6/s\, V_s}{-4V_s} = \mathbf{\frac{-3}{2s}}$$

Chapter 16, Solution 80.

(a) Consider the following circuit.



At node 1,

$$\frac{V_s - V_1}{1} = s\,V_1 + \frac{V_1 - V_o}{s}$$

$$V_s = \left(1 + s + \frac{1}{s}\right)V_1 - \frac{1}{s}V_o \qquad (1)$$

At node o,

$$\frac{V_1 - V_o}{s} = s\,V_o + V_o = (s+1)\,V_o$$

$$V_1 = (s^2 + s + 1)\,V_o \qquad (2)$$

Substituting (2) into (1)

$$V_s = (s + 1 + 1/s)(s^2 + s + 1)V_o - 1/s\,V_o$$

$$V_s = (s^3 + 2s^2 + 3s + 2)V_o$$

$$H_1(s) = \frac{V_o}{V_s} = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 2}}}$$

(b) $I_s = V_s - V_1 = (s^3 + 2s^2 + 3s + 2)V_o - (s^2 + s + 1)V_o$

$I_s = (s^3 + s^2 + 2s + 1)V_o$

$$H_2(s) = \frac{V_o}{I_s} = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + s^2 + 2s + 1}}}$$

(c) $I_o = \dfrac{V_o}{1}$

$$H_3(s) = \frac{I_o}{I_s} = \frac{V_o}{I_s} = H_2(s) = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + s^2 + 2s + 1}}}$$

(d) $$H_4(s) = \frac{I_o}{V_s} = \frac{V_o}{V_s} = H_1(s) = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 2}}}$$

Chapter 16, Solution 81.

For the op-amp circuit in Fig. 16.99, find the transfer function, $T(s) = I_o(s)/V_s(s)$. Assume all initial conditions are zero.

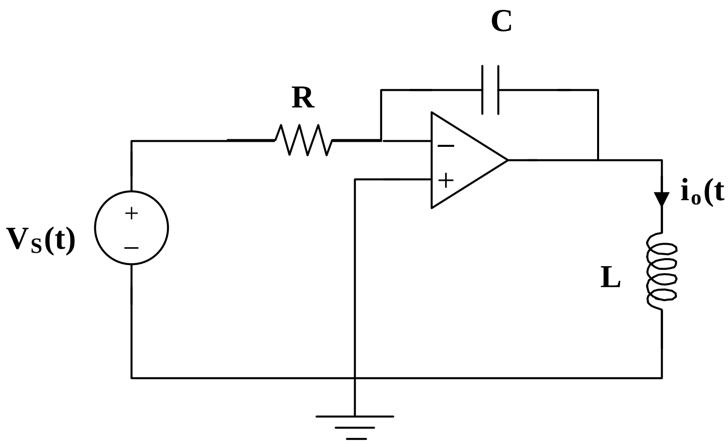
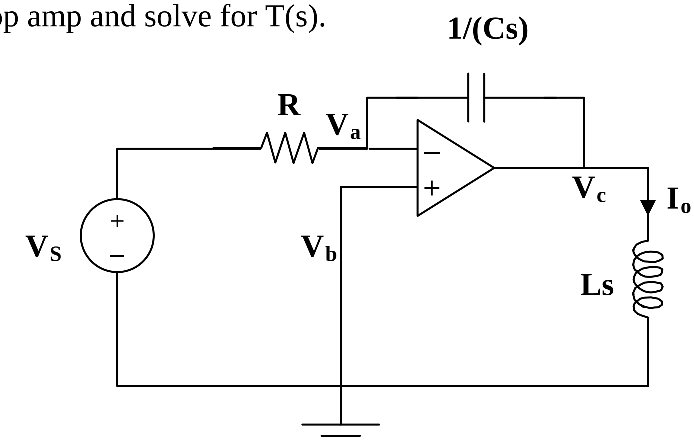


Figure 16.99
For Prob. 16.81.

Solution

Step 1. Convert the circuit into the s-domain. The write the node equations at the input to the op amp and solve for T(s).



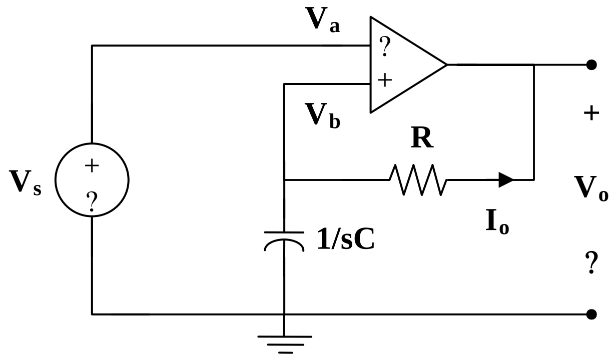
$[(V_a–V_s)/R]+[(V_a–V_c)/(1/(Cs))]+0 = 0$; $V_a = V_b = 0$ and $I_o = (V_c–0)/(Ls)$.

Step 2. $CsV_c = –V_s/R$ or $V_c = –V_s/(RCs)$ and $I_o = –V_s/(RLCs^2)$ or

$$T(s) = \mathbf{–1/(RLCs^2)}.$$

Chapter 16, Solution 82.

Consider the circuit below.



Since no current enters the op amp, I_o flows through both R and C.

$$V_o = -I_o\left(R + \frac{1}{sC}\right)$$

$$V_a = V_b = V_s = \frac{-I_o}{sC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{R + 1/sC}{1/sC} = \mathbf{sRC + 1}$$

Chapter 16, Solution 83.

(a) $$H(s) = \frac{V_o}{V_s} = \frac{R}{R + sL} = \frac{R/L}{s + R/L}$$

$$h(t) = \mathbf{\frac{R}{L} e^{-Rt/L}\ u(t)}$$

(b) $$v_s(t) = u(t) \quad \longrightarrow \quad V_s(s) = 1/s$$

$$V_o = \frac{R/L}{s + R/L} V_s = \frac{R/L}{s(s + R/L)} = \frac{A}{s} + \frac{B}{s + R/L}$$

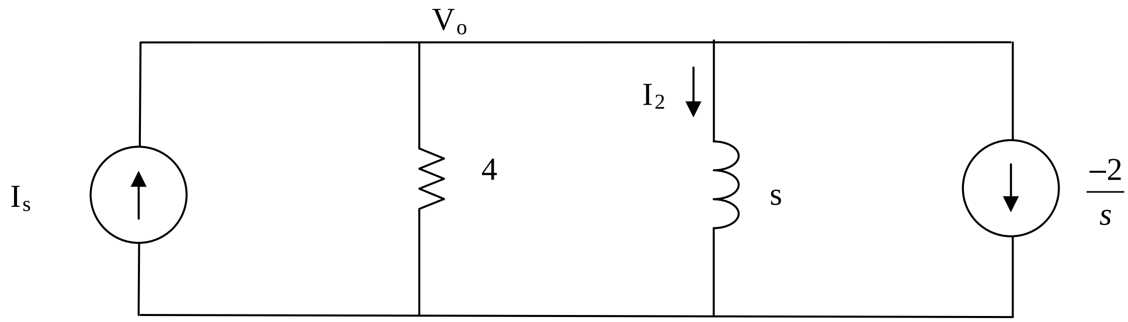
$A = 1$, $\qquad B = -1$

$$V_o = \frac{1}{s} - \frac{1}{s + R/L}$$

$$v_o(t) = u(t) - e^{-Rt/L}\,u(t) = \mathbf{(1 - e^{-Rt/L})\,u(t)}$$

Chapter 16, Solution 84.

Consider the circuit as shown below.



$$I_s = \frac{V_o}{4} + \frac{V_o}{s} - \frac{2}{s}$$

But $\quad I_S = \dfrac{2}{s+1}$

$$\frac{2}{s+1} = V_o\left(\frac{1}{4} + \frac{1}{s}\right) - \frac{2}{s} \quad \longrightarrow \quad V_o(\frac{s+4}{4s}) = \frac{2}{s+1} + \frac{2}{s} = \frac{4s+2}{s(s+1)}$$

$$V_o = \frac{8(2s+1)}{(s+1)(s+4)}$$

$$I_L = \frac{V_o}{s} = \frac{8(2s+1)}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = \frac{8(1)}{(1)(4)} = 2, \qquad B = \frac{8(-2+1)}{(-1)(2)} = 8/3, \qquad C = \frac{8(-8+1)}{(-4)(-3)} = -14/3$$

$$I_L = \frac{V_o}{s} = \frac{2}{s} + \frac{8/3}{s+1} + \frac{-14/3}{s+4}$$

$$i_L(t) = \underline{\left(2 + \frac{8}{3}e^{-t} - \frac{14}{3}e^{-4t}\right)u(t)}$$

Chapter 16, Solution 85.

$$H(s) = \frac{s+4}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$s+4 = A(s+2)^2 + B(s+1)(s+2) + C(s+1) = A(s^2+2s+4) + B(s^2+3s+2) + C(s+1)$$

We equate coefficients.

s^2 : 0=A+B or B=-A

s: 1=4A+3B+C=B+C

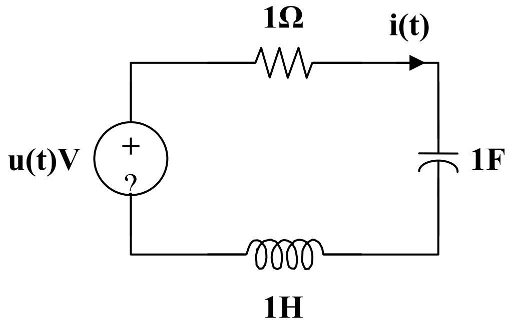
constant: 4=4A+2B+C =2A+C

Solving these gives A=3, B=-3, C=-2

$$H(s) = \frac{3}{s+1} - \frac{3}{s+2} - \frac{2}{(s+2)^2}$$

$$h(t) = \underline{(3e^{-t} - 3e^{-2t} - 2te^{-2t})u(t)}$$

Chapter 16, Solution 86.



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KVL we get:

$$-u(t) + i + v_C + i' = 0;\ \ i = v_C'$$

Thus,

$$\begin{aligned}\dot{v}_C &= i \\ \dot{i} &= -v_C - i + u(t)\end{aligned}$$

Finally we get,

$$\begin{bmatrix}\dot{v}_C \\ \dot{i}\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -1 & -1\end{bmatrix}\begin{bmatrix}v_C \\ i\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}u(t);\ i(t) = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}v_C \\ i\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}u(t)$$

Chapter 16, Solution 87.

Develop the state equations for the problem you designed in Prob. 16.13.

Although there is no correct way to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Develop the state equations for Problem 16.13.

Chapter 16, Problem 13.

Find v_x in the circuit shown in Fig. 16.36 given $v_s = 4u(t)$ V.

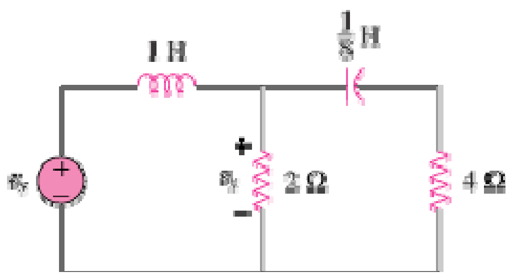
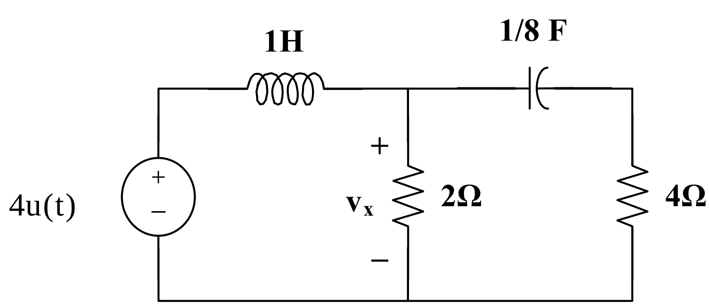


Figure 16.36

Solution



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KCL we get:

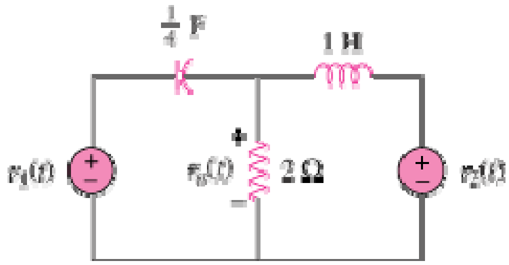
$$-i_L + \frac{v_x}{2} + \frac{\dot{v}_C}{8} = 0; \quad or \quad \dot{v}_C = 8i_L - 4v_x$$
$$\dot{i}_L = 4u(t) - v_x$$
$$v_x = v_C + 4\frac{\dot{v}_C}{8} = v_C + \frac{\dot{v}_C}{2} = v_C + 4i_L - 2v_x; \ or \ v_x = 0.3333v_C + 1.3333i_L$$

$$\dot{v}_C = 8i_L - 1.3333v_C - 5.333i_L = -1.3333v_C + 2.666i_L$$
$$\dot{i}_L = 4u(t) - 0.3333v_C - 1.3333i_L$$

Now we can write the state equations.

$$\begin{bmatrix} \dot{\mathbf{v}}_{\mathbf{C}} \\ \dot{\mathbf{i}}_{\mathbf{L}} \end{bmatrix} = \underline{\begin{bmatrix} -1.3333 & 2.666 \\ -0.3333 & -1.3333 \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\mathbf{C}} \\ \mathbf{i}_{\mathbf{L}} \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} \mathbf{u(t)};} \ \ \mathbf{v}_{\mathbf{x}} = \underline{[0.3333 \quad 1.3333] \begin{bmatrix} \mathbf{v}_{\mathbf{C}} \\ \mathbf{i}_{\mathbf{L}} \end{bmatrix}}$$

Chapter 16, Solution 88.



First select the inductor current i_L (current flowing left to right) and the capacitor voltage v_C (voltage positive on the left and negative on the right) to be the state variables.

Applying KCL we get:

$$-\frac{\dot{v}_C}{4}+\frac{v_o}{2}+i_L=0 \text{ or } \dot{v}_C=4i_L+2v_o$$

$$\dot{i}_L=v_o-v_2$$

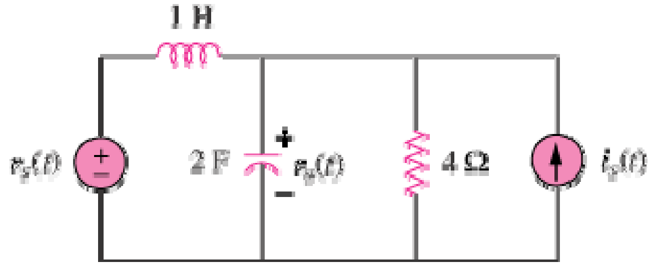
$$v_o=-v_C+v_1$$

$$\dot{v}_C=4i_L-2v_C+2v_1$$

$$\dot{i}_L=-v_C+v_1-v_2$$

$$\begin{bmatrix}\dot{\mathbf{i}}_L\\ \dot{\mathbf{v}}_C\end{bmatrix}=\underline{\begin{bmatrix}0 & -1\\ 4 & -2\end{bmatrix}\begin{bmatrix}\mathbf{i}_L\\ \mathbf{v}_C\end{bmatrix}+\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_1(\mathbf{t})\\ \mathbf{v}_2(\mathbf{t})\end{bmatrix}};\ \mathbf{v}_o(\mathbf{t})=\underline{\begin{bmatrix}0 & -1\end{bmatrix}\begin{bmatrix}\mathbf{i}_L\\ \mathbf{v}_C\end{bmatrix}+\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_1(\mathbf{t})\\ \mathbf{v}_2(\mathbf{t})\end{bmatrix}}$$

Chapter 16, Solution 89.



First select the inductor current i_L (left to right) and the capacitor voltage v_C to be the state variables.

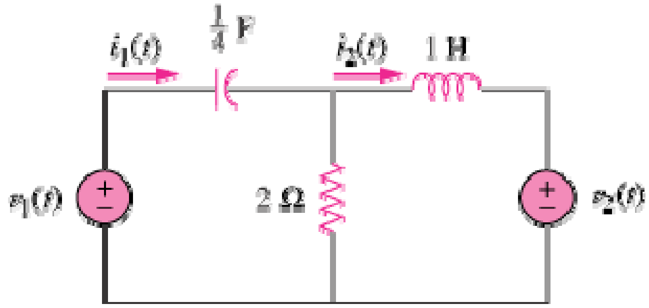
Letting $v_o = v_C$ and applying KCL we get:

$$-i_L + \dot{v}_C + \frac{v_C}{4} - i_s = 0 \ or \ \dot{v}_C = -0.25v_C + i_L + i_s$$
$$\dot{i}_L = -v_C + v_s$$

Thus,

$$\begin{bmatrix} \dot{\mathbf{v}}_C \\ \dot{\mathbf{i}}_L \end{bmatrix} = \underline{\begin{bmatrix} -0.25 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_C \\ \mathbf{i}_L \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_s \\ \mathbf{i}_s \end{bmatrix}}; \ \mathbf{v}_o(\mathbf{t}) = \underline{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_C \\ \mathbf{i}_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{v}_s \\ \mathbf{i}_s \end{bmatrix}}$$

Chapter 16, Solution 90.



First select the inductor current i_L (left to right) and the capacitor voltage v_C (+ on the left) to be the state variables.

Letting $i_1 = \frac{\dot{v}_C}{4}$ and $i_2 = i_L$ and applying KVL we get:

Loop 1:

$$-v_1 + v_C + 2\left(\frac{\dot{v}_C}{4} - i_L\right) = 0 \ or \ \dot{v}_C = 4i_L - 2v_C + 2v_1$$

Loop 2:

$$2\left(i_L - \frac{\dot{v}_C}{4}\right) + \dot{i}_L + v_2 = 0 \ or$$

$$\dot{i}_L = -2i_L + \frac{4i_L - 2v_C + 2v_1}{2} - v_2 = -v_C + v_1 - v_2$$

$$i_1 = \frac{4i_L - 2v_C + 2v_1}{4} = i_L - 0.5v_C + 0.5v_1$$

$$\begin{bmatrix} \dot{\mathbf{i}}_L \\ \dot{\mathbf{v}}_C \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{4} & \mathbf{-2} \end{bmatrix}\begin{bmatrix} \mathbf{i}_L \\ \mathbf{v}_C \end{bmatrix} + \begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{v}_1(\mathbf{t}) \\ \mathbf{v}_2(\mathbf{t}) \end{bmatrix}}; \begin{bmatrix} \mathbf{i}_1(\mathbf{t}) \\ \mathbf{i}_2(\mathbf{t}) \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{-0.5} \\ \mathbf{1} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{i}_L \\ \mathbf{v}_C \end{bmatrix} + \begin{bmatrix} \mathbf{0.5} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{v}_1(\mathbf{t}) \\ \mathbf{v}_2(\mathbf{t}) \end{bmatrix}}$$

Chapter 16, Solution 91.

Let $x_1 = y(t)$. Thus, $\dot{x}_1 = \dot{y} = x_2$ *and* $\dot{x}_2 = \ddot{y} = -3x_1 - 4x_2 + z(t)$

This gives our state equations.

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{3} & -\mathbf{4} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix} \mathbf{z(t)}};\quad \mathbf{y(t)} = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \end{bmatrix} \mathbf{z(t)}}$$

Chapter 16, Solution 92.

Let $x_1 = y(t)$ *and* $x_2 = \dot{x}_1 - z = \dot{y} - z$ *or* $\dot{y} = x_2 + z$

Thus,

$$\dot{x}_2 = \ddot{y} - \dot{z} = -9x_1 - 7(x_2 + z) + \dot{z} + 2z - \dot{z} = -9x_1 - 7x_2 - 5z$$

This now leads to our state equations,

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{9} & -\mathbf{7} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{1} \\ -\mathbf{5} \end{bmatrix} \mathbf{z(t)};} \; \mathbf{y(t)} = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \end{bmatrix} \mathbf{z(t)}}$$

Chapter 16, Solution 93.

Let $x_1 = y(t)$, $x_2 = \dot{x}_1$, *and* $x_3 = \dot{x}_2$.

Thus,

$$\ddot{x}_3 = -6x_1 - 11x_2 - 6x_3 + z(t)$$

We can now write our state equations.

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \\ \dot{\mathbf{x}}_3 \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{-6} & \mathbf{-11} & \mathbf{-6} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix} \mathbf{z(t)};}\ \ \mathbf{y(t)} = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \end{bmatrix} \mathbf{z(t)}}$$

Chapter 16, Solution 94.

We transform the state equations into the s-domain and solve using Laplace transforms.

$$sX(s) - x(0) = AX(s) + B\left(\frac{1}{s}\right)$$

Assume the initial conditions are zero.

$$(sI - A)X(s) = B\left(\frac{1}{s}\right)$$

$$X(s) = \begin{bmatrix} s+4 & -4 \\ 2 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 2 \end{bmatrix} \left(\frac{1}{s}\right) = \frac{1}{s^2 + 4s + 8} \begin{bmatrix} s & 4 \\ 2 & s+4 \end{bmatrix} \begin{bmatrix} 0 \\ (2/s) \end{bmatrix}$$

$$Y(s) = X_1(s) = \frac{8}{s(s^2 + 4s + 8)} = \frac{1}{s} + \frac{-s-4}{s^2 + 4s + 8}$$

$$= \frac{1}{s} + \frac{-s-4}{(s+2)^2 + 2^2} = \frac{1}{s} + \frac{-(s+2)}{(s+2)^2 + 2^2} + \frac{-2}{(s+2)^2 + 2^2}$$

$$\mathbf{y(t) = \left(1 - e^{-2t}(\cos 2t + \sin 2t)\right)u(t)}$$

Chapter 16, Solution 95.

Assume that the initial conditions are zero. Using Laplace transforms we get,

$$X(s) = \begin{bmatrix} s+2 & 1 \\ -2 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1/s \\ 2/s \end{bmatrix} = \frac{1}{s^2+6s+10} \begin{bmatrix} s+4 & -1 \\ 2 & s+2 \end{bmatrix} \begin{bmatrix} 3/s \\ 4/s \end{bmatrix}$$

$$X_1 = \frac{3s+8}{s((s+3)^2+1^2)} = \frac{0.8}{s} + \frac{-0.8s-1.8}{(s+3)^2+1^2}$$

$$= \frac{0.8}{s} - 0.8\frac{s+3}{(s+3)^2+1^2} + .6\frac{1}{(s+3)^2+1^2}$$

$$x_1(t) = (0.8 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)$$

$$X_2 = \frac{4s+14}{s((s+3)^2+1^2} = \frac{1.4}{s} + \frac{-1.4s-4.4}{(s+3)^2+1^2}$$

$$= \frac{1.4}{s} - 1.4\frac{s+3}{(s+3)^2+1^2} - 0.2\frac{1}{(s+3)^2+1^2}$$

$$x_2(t) = (1.4 - 1.4e^{-3t}\cos t - 0.2e^{-3t}\sin t)u(t)$$

$$\mathbf{y_1(t) = -2x_1(t) - 2x_2(t) + 2u(t)}$$
$$\mathbf{= \underline{(-2.4 + 4.4e^{-3t}\cos t - 0.8e^{-3t}\sin t)u(t)}}$$

$$\mathbf{y_2(t) = x_1(t) - 2u(t) = \underline{(-1.2 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)}}$$

[–2.4 + 4.4e^{-3t}cos(t) – 0.8e^{-3t}sin(t)]u(t), [–1.2 – 0.8e^{-3t}cos(t) + 0.6e^{-3t}sin(t)]u(t)

Chapter 16, Solution 96.

If V_o is the voltage across R, applying KCL at the non-reference node gives

$$I_s = \frac{V_o}{R} + sC\,V_o + \frac{V_o}{sL} = \left(\frac{1}{R} + sC + \frac{1}{sL}\right)V_o$$

$$V_o = \frac{I_s}{\dfrac{1}{R} + sC + \dfrac{1}{sL}} = \frac{sRL\,I_s}{sL + R + s^2RLC}$$

$$I_o = \frac{V_o}{R} = \frac{sL\,I_s}{s^2RLC + sL + R}$$

$$H(s) = \frac{I_o}{I_s} = \frac{sL}{s^2RLC + sL + R} = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

The roots

$$s_{1,2} = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

both lie in the left half plane since R, L, and C are positive quantities.

Thus, **the circuit is stable**.

Chapter 16, Solution 97.

(a) $$H_1(s) = \frac{3}{s+1}, \qquad H_2(s) = \frac{1}{s+4}$$

$$H(s) = H_1(s)H_2(s) = \frac{3}{(s+1)(s+4)}$$

$$h(t) = L^{-1}\left[H(s)\right] = L^{-1}\left[\frac{A}{s+1} + \frac{B}{s+4}\right]$$

$$A = 1, \qquad B = -1$$

$$h(t) = \mathbf{(e^{-t} - e^{-4t})u(t)}$$

(b) Since the poles of H(s) all lie in the left half s-plane, **the system is stable**.

Chapter 16, Solution 98.

Let V_{o1} be the voltage at the output of the first op amp.

$$\frac{V_{o1}}{V_s} = \frac{-1/sC}{R} = \frac{-1}{sRC}, \qquad \frac{V_o}{V_{o1}} = \frac{-1}{sRC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{1}{s^2R^2C^2}$$

$$h(t) = \frac{t}{R^2C^2}$$

$\lim_{t\to\infty} h(t) = \infty$, i.e. the output is unbounded.

Hence, **the circuit is unstable**.

Chapter 16, Solution 99.

$$sL \parallel \frac{1}{sC} = \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{1 + s^2LC}$$

$$\frac{V_2}{V_1} = \frac{\frac{sL}{1+s^2LC}}{R + \frac{sL}{1+s^2LC}} = \frac{sL}{s^2RLC + sL + R}$$

$$\frac{V_2}{V_1} = \frac{s \cdot \frac{1}{RC}}{s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}}$$

Comparing this with the given transfer function,

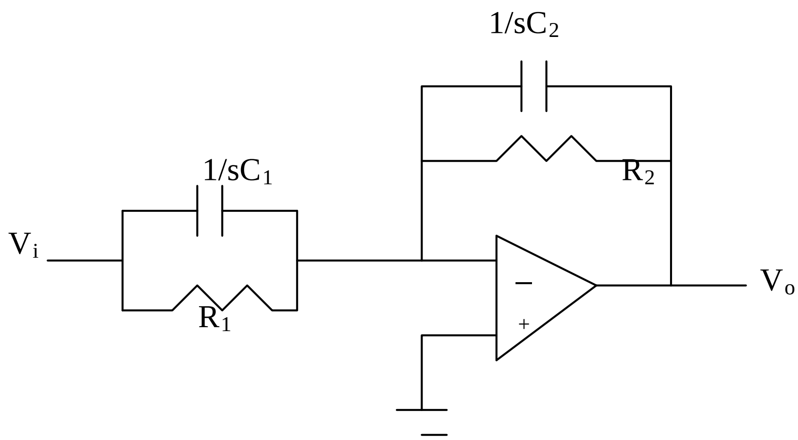
$$2 = \frac{1}{RC}, \qquad 6 = \frac{1}{LC}$$

If $R = 1\,k\Omega$, $C = \frac{1}{2R} =$ **500 μF**

$L = \frac{1}{6C} =$ **333.3 H**

Chapter 16, Solution 100.

The circuit is transformed in the s-domain as shown below.



Let $Z_1 = R_1 // \dfrac{1}{sC_1} = \dfrac{R_1 \dfrac{1}{sC_1}}{R_1 + \dfrac{1}{sC_1}} = \dfrac{R_1}{1 + sR_1C_1}$

$$Z_2 = R_2 // \frac{1}{sC_2} = \frac{R_2 \dfrac{1}{sC_2}}{R_2 + \dfrac{1}{sC_2}} = \frac{R_2}{1 + sR_2C_2}$$

This is an inverting amplifier.

$$H(s) = V_o / V_i = -\frac{Z_2}{Z_1} = \frac{-\dfrac{R_2}{1+sR_2C_2}}{\dfrac{R_1}{1+sR_1C_1}} = -\frac{R_2}{R_1}\frac{R_1C_1}{R_2C_2}\left[\frac{s + \dfrac{1}{R_1C_1}}{s + \dfrac{1}{R_2C_2}}\right] = \frac{-C_1}{C_2}\left[\frac{s + \dfrac{1}{R_1C_1}}{s + \dfrac{1}{R_2C_2}}\right]$$

Comparing this with

$$H(s) = -\frac{(s+1000)}{2(s+4000)}$$

we obtain:

$$\frac{C_1}{C_2} = 1/2 \quad \longrightarrow \quad C_2 = 2C_1 = \underline{20\mu F}$$

$$\frac{1}{R_1C_1} = 1000 \quad \longrightarrow \quad R_1 = \frac{1}{1000C_1} = \frac{1}{10^3 x 10 x 10^{-6}} = \underline{100\Omega}$$

$$\frac{1}{R_2C_2} = 4000 \quad \longrightarrow \quad R_2 = \frac{1}{4000C_2} = \frac{1}{4x10^3 x 20 x 10^{-6}} = \underline{\mathbf{12.5\Omega}}$$

Chapter 16, Solution 101.

We apply KCL at the noninverting terminal at the op amp.

$$(V_s - 0)\,Y_3 = (0 - V_o)(Y_1 - Y_2)$$

$$Y_3\,V_s = -(Y_1 + Y_2)V_o$$

$$\frac{V_o}{V_s} = \frac{-Y_3}{Y_1 + Y_2}$$

Let $Y_1 = sC_1,\qquad Y_2 = 1/R_1\,,\qquad Y_3 = sC_2$

$$\frac{V_o}{V_s} = \frac{-sC_2}{sC_1 + 1/R_1} = \frac{-sC_2/C_1}{s + 1/R_1C_1}$$

Comparing this with the given transfer function,

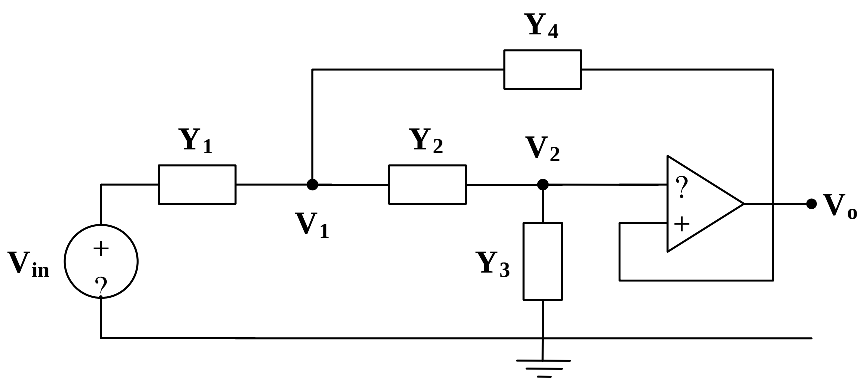
$$\frac{C_2}{C_1} = 1,\qquad \frac{1}{R_1C_1} = 10$$

If $R_1 = 1\,k\Omega$,

$$C_1 = C_2 = \frac{1}{10^4} = \mathbf{100\ \mu F}$$

Chapter 16, Solution 102.

Consider the circuit shown below. We notice that $V_3 = V_o$ and $V_2 = V_3 = V_o$.



At node 1,

$$(V_{in} - V_1)\,Y_1 = (V_1 - V_o)\,Y_2 + (V_1 - V_o)\,Y_4$$

$$V_{in}\,Y_1 = V_1(Y_1 + Y_2 + Y_4) - V_o\,(Y_2 + Y_4) \qquad (1)$$

At node 2,

$$(V_1 - V_o)\,Y_2 = (V_o - 0)\,Y_3$$

$$V_1\,Y_2 = (Y_2 + Y_3)\,V_o$$

$$V_1 = \frac{Y_2 + Y_3}{Y_2}\,V_o \qquad (2)$$

Substituting (2) into (1),

$$V_{in}\,Y_1 = \frac{Y_2 + Y_3}{Y_2}\cdot(Y_1 + Y_2 + Y_4)\,V_o - V_o\,(Y_2 + Y_4)$$

$$V_{in}Y_1Y_2 = V_o(Y_1Y_2 + Y_2^2 + Y_2Y_4 + Y_1Y_3 + Y_2Y_3 + Y_3Y_4 - Y_2^2 - Y_2Y_4)$$

$$\frac{V_o}{V_{in}} = \frac{Y_1Y_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3 + Y_3Y_4}$$

Y_1 and Y_2 must be resistive, while Y_3 and Y_4 must be capacitive.

Let $\quad Y_1 = \dfrac{1}{R_1}, \qquad Y_2 = \dfrac{1}{R_2}, \qquad Y_3 = sC_1, \qquad Y_4 = sC_2$

$$\frac{V_o}{V_{in}} = \frac{\dfrac{1}{R_1R_2}}{\dfrac{1}{R_1R_2} + \dfrac{sC_1}{R_1} + \dfrac{sC_1}{R_2} + s^2C_1C_2}$$

$$\frac{V_o}{V_{in}} = \frac{\dfrac{1}{R_1R_2C_1C_2}}{s^2 + s\cdot\left(\dfrac{R_1+R_2}{R_1R_2C_2}\right) + \dfrac{1}{R_1R_2C_1C_2}}$$

Choose $R_1 = 1\,k\Omega$, then

$$\frac{1}{R_1R_2C_1C_2} = 10^6 \qquad \text{and} \qquad \frac{R_1+R_2}{R_1R_2C_2} = 100$$

We have three equations and four unknowns. Thus, there is a family of solutions. One such solution is

$$R_2 = \mathbf{1\,k\Omega}, \quad C_1 = \mathbf{50\,nF}, \quad C_2 = \mathbf{20\,\mu F}$$

Chapter 16, Solution 103.

Using the result of Practice Problem 16.14,

$$\frac{V_o}{V_i} = \frac{-Y_1 Y_2}{Y_2 Y_3 + Y_4 (Y_1 + Y_2 + Y_3)}$$

When $Y_1 = sC_1$, $\quad C_1 = 0.5\ \mu F$

$$Y_2 = \frac{1}{R_1}, \qquad R_1 = 10\ k\Omega$$

$$Y_3 = Y_2, \qquad Y_4 = sC_2, \qquad C_2 = 1\ \mu F$$

$$\frac{V_o}{V_i} = \frac{-sC_1/R_1}{1/R_1^2 + sC_2 (sC_1 + 2/R_1)} = \frac{-sC_1 R_1}{1 + sC_2 R_1 (2 + sC_1 R_1)}$$

$$\frac{V_o}{V_i} = \frac{-sC_1 R_1}{s^2 C_1 C_2 R_1^2 + s \cdot 2C_2 R_1 + 1}$$

$$\frac{V_o}{V_i} = \frac{-s(0.5\times10^{-6})(10\times10^3)}{s^2 (0.5\times10^{-6})(1\times10^{-6})(10\times10^3)^2 + s(2)(1\times10^{-6})(10\times10^3) + 1}$$

$$\frac{V_o}{V_i} = \frac{-100s}{s^2 + 400s + 2\times10^4}$$

Therefore,

$$a = \mathbf{-100}, \qquad b = \mathbf{400}, \qquad c = \mathbf{2\times10^4}$$

Chapter 16, Solution 104.

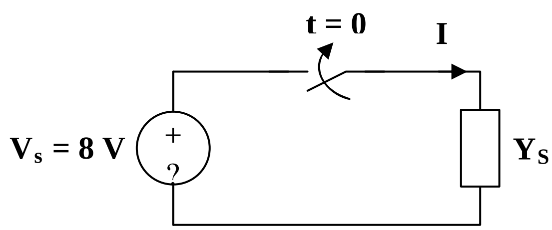
(a) Let $Y(s) = \frac{K(s+1)}{s+3}$

$$Y(\infty) = \lim_{s\to\infty} \frac{K(s+1)}{s+3} = \lim_{s\to\infty} \frac{K(1+1/s)}{1+3/s} = K$$

i.e. $0.25 = K$.

Hence, $Y(s) = \mathbf{\frac{s+1}{4(s+3)}}$

(b) Consider the circuit shown below.



$$V_s = 8u(t) \longrightarrow V_s = 8/s$$

$$I = \frac{V_s}{Z} = Y(s)V_s(s) = \frac{8}{4s} \cdot \frac{s+1}{s+3} = \frac{2(s+1)}{s(s+3)}$$

$$I = \frac{A}{s} + \frac{B}{s+3}$$

$A = 2/3$, $B = 2(-3+1)/(-3) = 4/3$

$$i(t) = \mathbf{\frac{1}{3}\left[2 + 4e^{-3t}\right]u(t)\ A}$$

Chapter 16, Solution 105.

The gyrator is equivalent to two cascaded inverting amplifiers. Let V_1 be the voltage at the output of the first op amp.

$$V_1 = \frac{-R}{R} V_i = -V_i$$

$$V_o = \frac{-1/sC}{R} V_1 = \frac{1}{sCR} V_i$$

$$I_o = \frac{V_o}{R} = \frac{V_o}{sR^2C}$$

$$\frac{V_o}{I_o} = sR^2C$$

$$\frac{\mathbf{V_o}}{\mathbf{I_o}} = \mathbf{sL}, \quad \text{when } \mathbf{L = R^2C}$$, so **if you let L = R^2C then V_o/I_o = sL**.