전기회로 (가, 나)

Chapter 3 : Methods of Analysis

2017. 1학기

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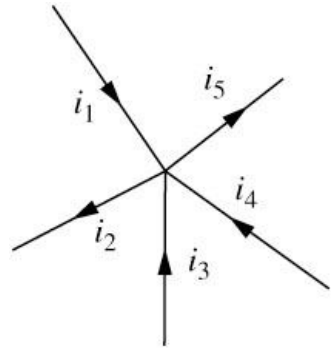
Learning Goals

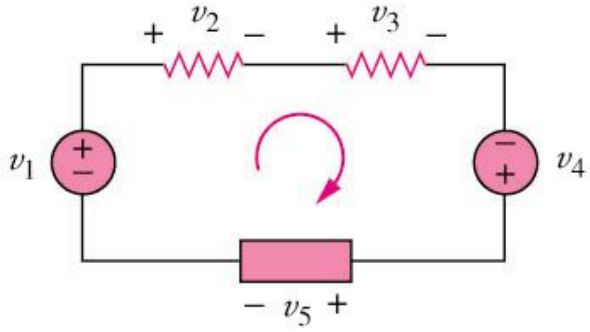
- Nodal analysis
  - without voltage sources
  - with voltage sources
- Mesh analysis
  - without current sources
  - with current sources

3.1 Introduction

- 옴의 법칙
  - 전자 회로 전 분야에서 가장 기본적이며 강력한 법칙
- 키르히호프의 전류 법칙
  - 한 개의 노드를 (들어가는 전류의 합) = (나오는 전류의 합)
  - 절점 해석 (Nodal Analysis)에 이용



- 키르히호프의 전압 법칙
  - 폐 경로에서 (전압 강하의 합) = (전압 상승의 합)
  - 망로 해석 (Mesh Analysis)에 이용

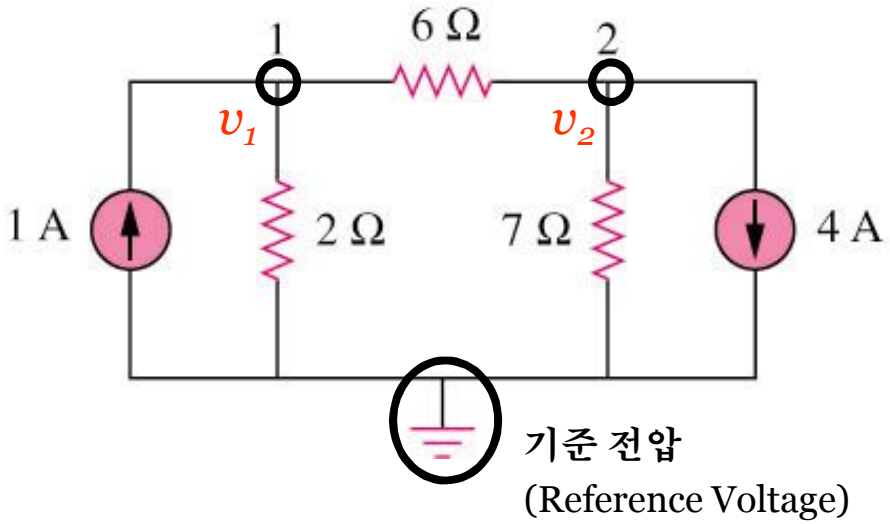


3.2 Nodal Analysis

Nodal Analysis

- 식의 표현 : **KCL** 이용 ($\sum I_k = 0$)
- 각 branch에 흐르는 **전류** I_k : Ohm의 법칙($I_k = v/R$) 을 이용하여 계산

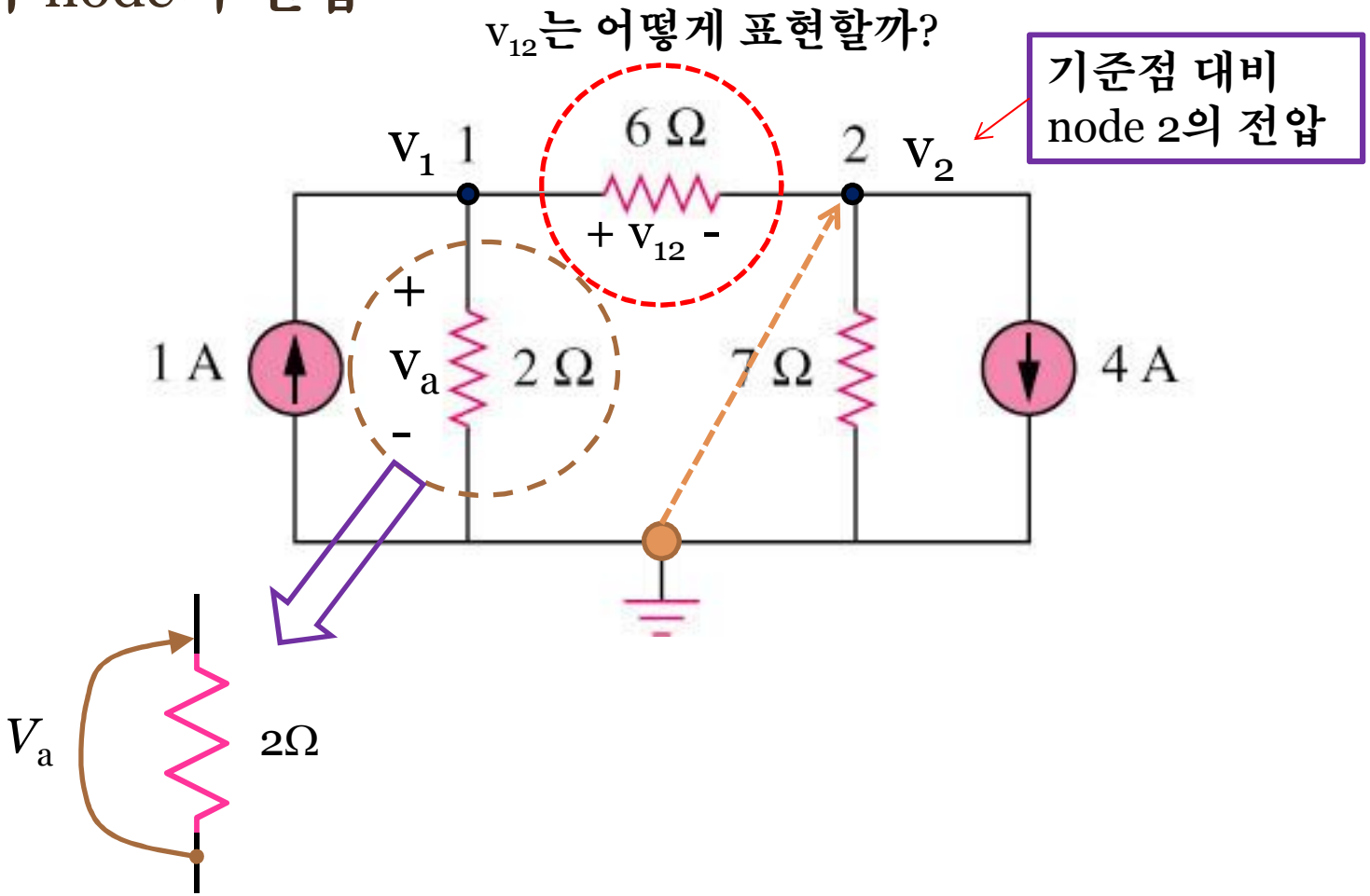
  → 각 node 의 **전압**을 변수로 사용



기준 전압 대비 각 노드의 전압

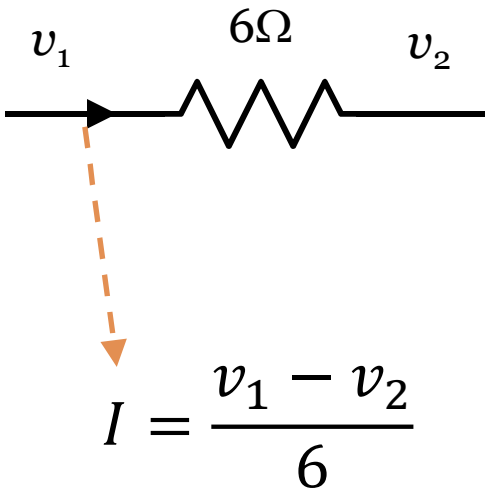
3.2 Nodal Analysis

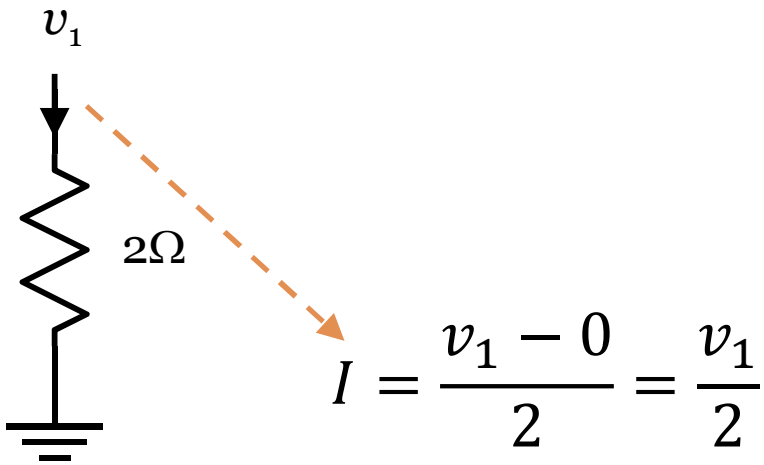
- 각 node의 전압



3.2 Nodal Analysis

- 전류와 node 전압과의 관계

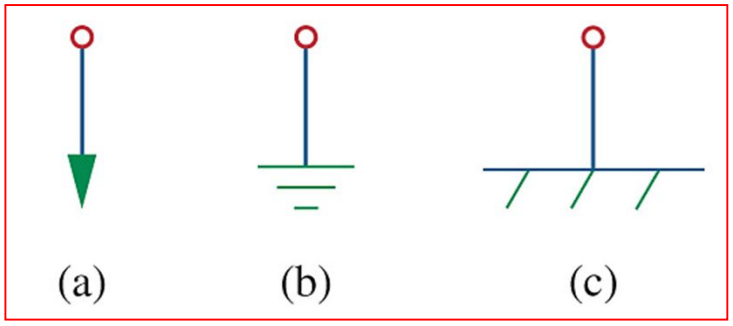




3.2 Nodal Analysis without voltage sources

Nodal Analysis

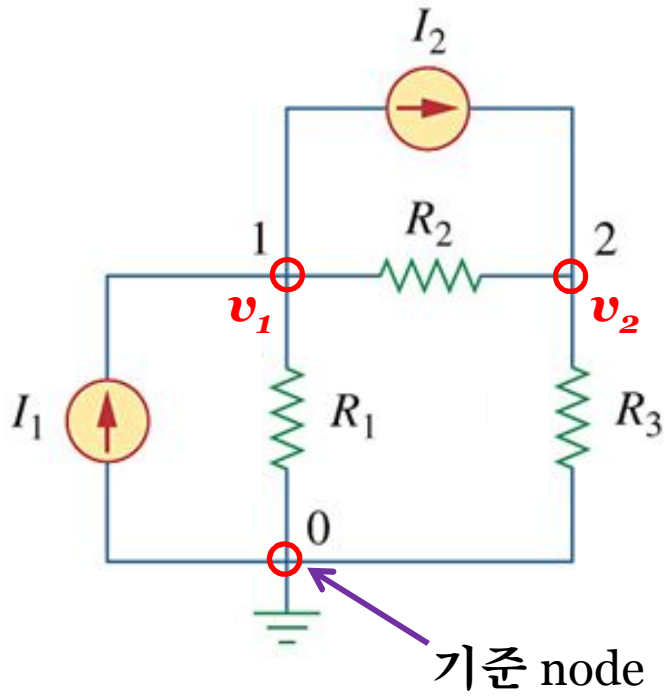
- Step 1 :
  기준 node (예: GND)의 선택
  나머지 node는 $v_1, v_2, \ldots v_{n-1}$으로 표시
- Step 2 :
  기준 node가 아닌 node에 KCL 적용
  전류는 $v_1, v_2, \ldots v_{n-1}$과 저항으로 표현 (옴의 법칙)
- Step 3 :
  연립 방정식 계산



일반적으로 기준 node는 0V의 전위로 가정
→ 접지 (Ground, GND)
(b)의 기호가 가장 흔히 사용 됨

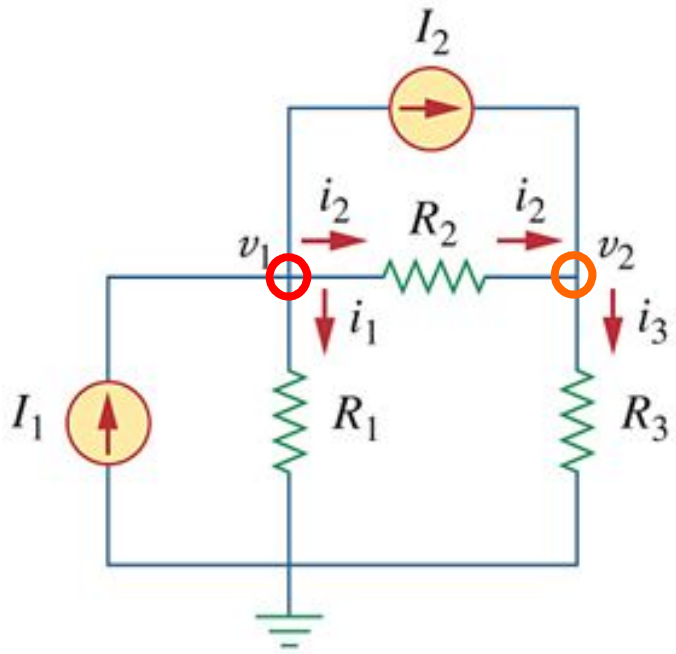
3.2 Nodal Analysis without voltage sources

- Step 1
  - 기준 node의 선택
  - 나머지 node에 $v_1, v_2, \ldots, v_{n-1}$ 할당 (기준 node에 대한 상대전압)



3.2 Nodal Analysis without voltage sources

- Step 2
  - 각 node에 KCL 적용
  - 전류는 node전압과 저항을 이용하여 계산 ← Ohm's law



at node #1 :

$$I_1 = I_2 + i_1 + i_2$$

$$I_1 = I_2 + (v_1\text{-}0)/R_1 + (v_1\text{-}v_2)/R_2$$

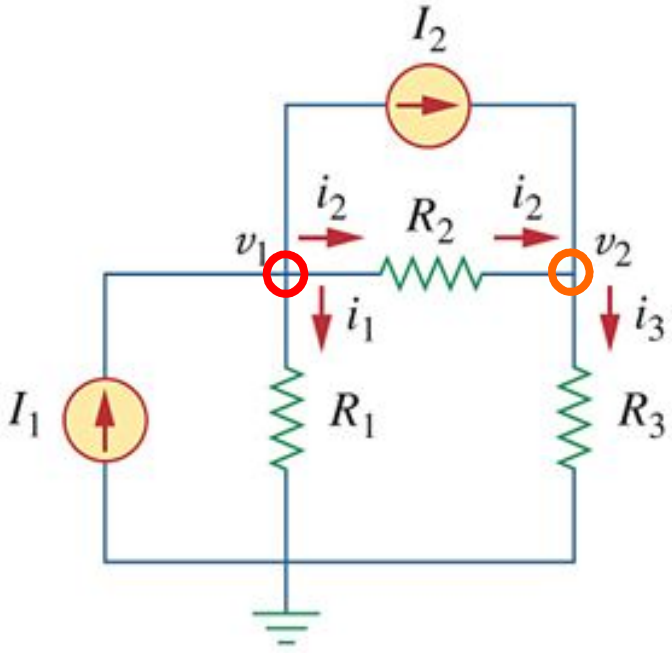
at node #2 :

$$I_2 + i_2 = i_3$$

$$I_2 + (v_1\text{-}v_2)/R_2 = (v_2\text{-}0)/R_3$$

3.2 Nodal Analysis without voltage sources

- Step 3
  - 연립방정식 풀이



$$I_1 = I_2 + (v_1\text{-}o)/R_1 + (v_1\text{-}v_2)/R_2$$

$$I_2 + (v_1\text{-}v_2)/R_2 = (v_2\text{-}o)/R_3$$

or

$$I_1 = I_2 + v_1G_1 + (v_1\text{-}v_2)G_2$$

$$I_2 + (v_1\text{-}v_2)G_2 = v_2G_3$$

$$\begin{bmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

연립방정식 풀이 방법 – Cramer's rule

[2원 1차 연립방정식]

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}\begin{bmatrix} u \\ v \end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta},\ y = \frac{\Delta_2}{\Delta}$$

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix},\quad \Delta_1 = \begin{vmatrix} u & b \\ v & d \end{vmatrix},\quad \Delta_2 = \begin{vmatrix} a & u \\ c & v \end{vmatrix}$$

$ad - bc$ $\qquad$ $ud - bv$ $\qquad$ $av - uc$

연립방정식 풀이 방법 – Cramer's rule

[3원 1차 연립방정식]

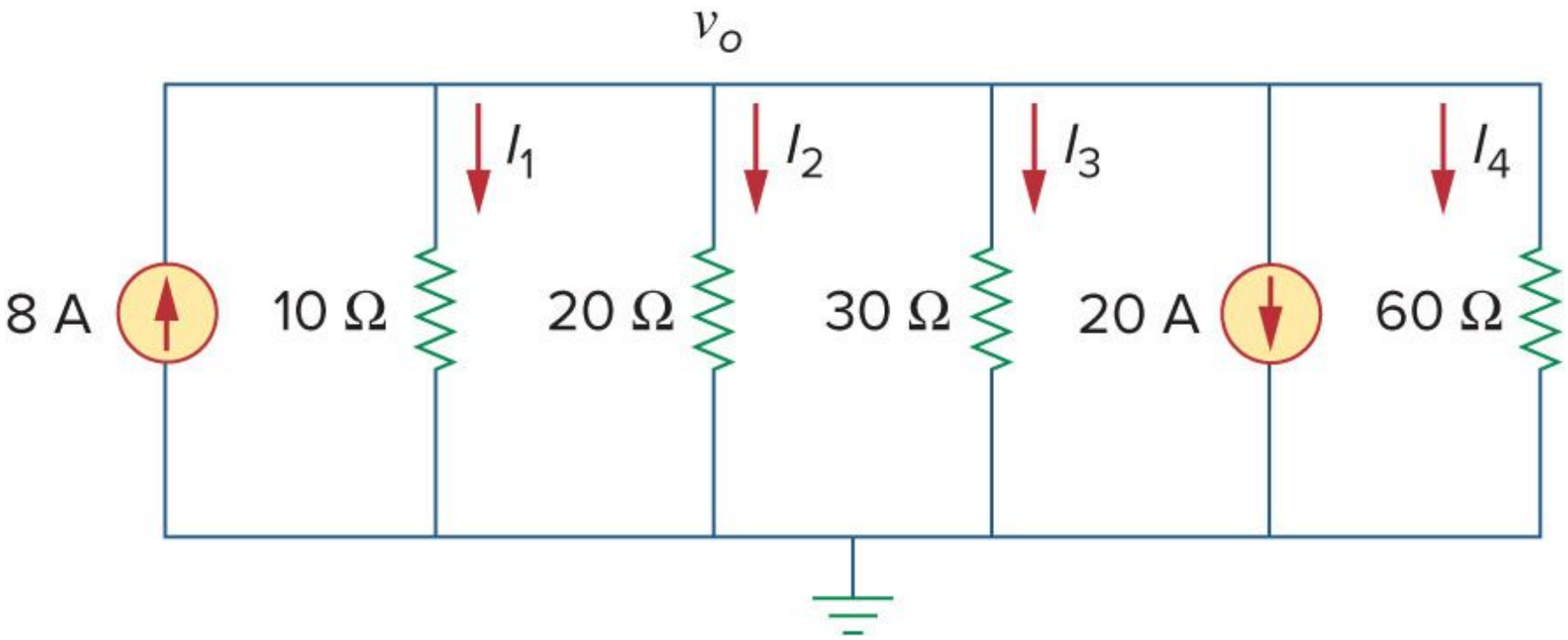
$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1}\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta},\ y = \frac{\Delta_2}{\Delta},\ z = \frac{\Delta_3}{\Delta}$$

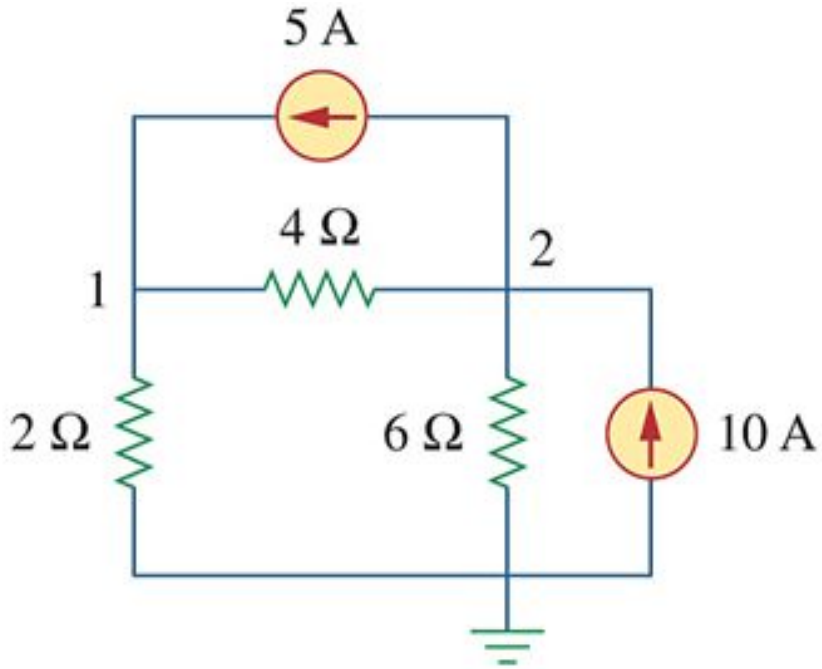
$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix},\quad \Delta_1 = \begin{vmatrix} u & b & c \\ v & e & f \\ w & h & i \end{vmatrix},\quad \Delta_2 = \begin{vmatrix} a & u & c \\ d & v & f \\ g & w & i \end{vmatrix},\quad \Delta_3 = \begin{vmatrix} a & b & u \\ d & e & v \\ g & f & w \end{vmatrix}$$

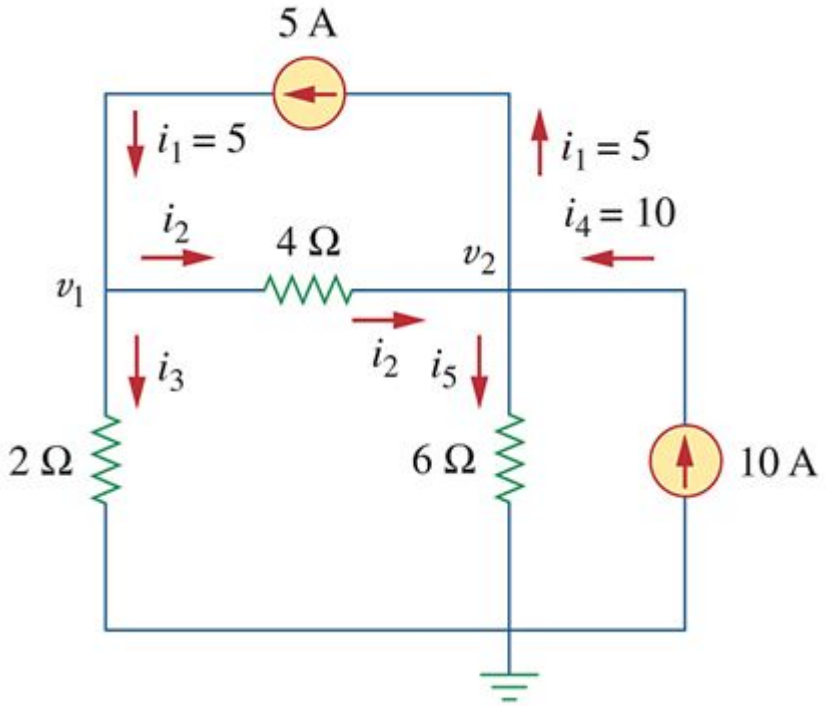
Problem 3.3

- 다음에서 전류 I_1, I_2, I_3, I_4 와 v_0를 구하라



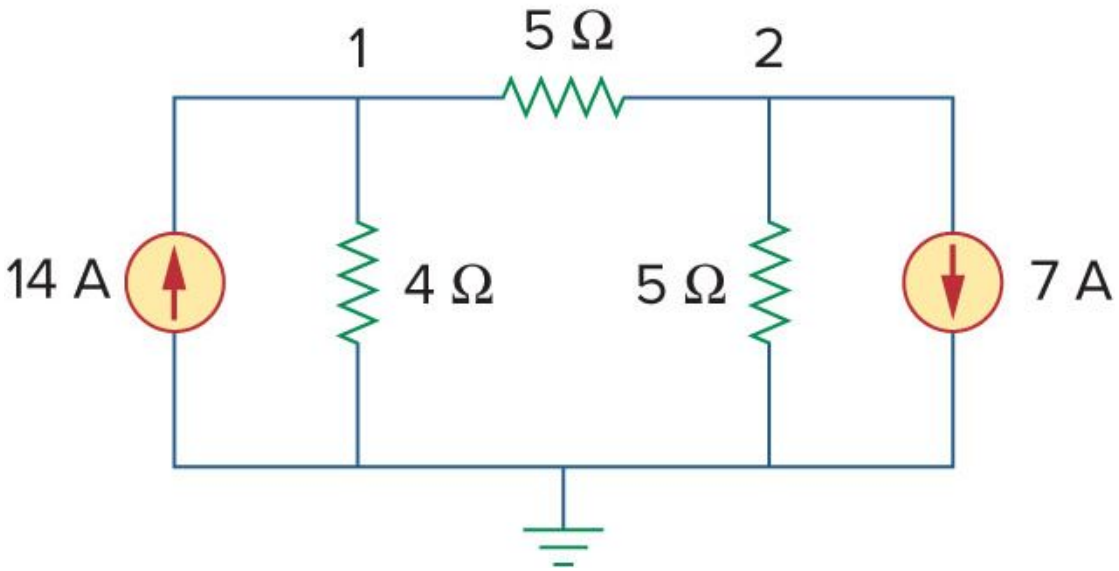
Example 3.1





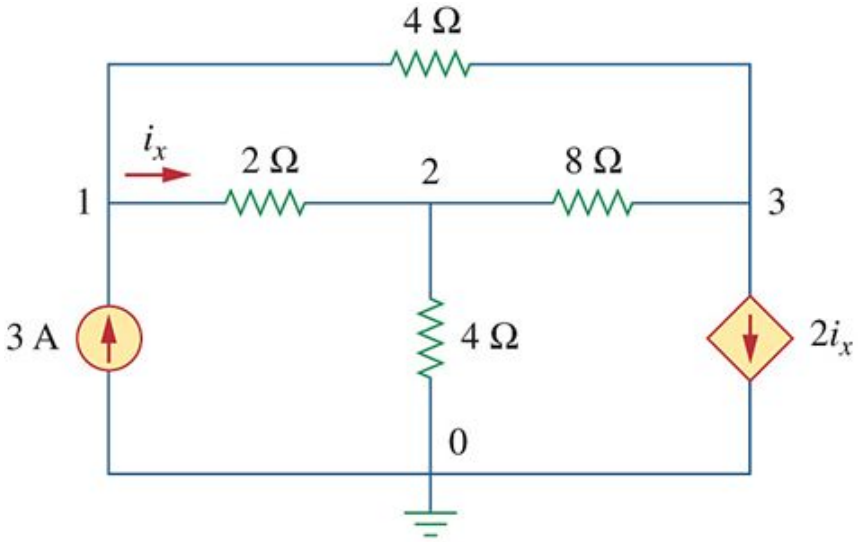
Practice 3.1

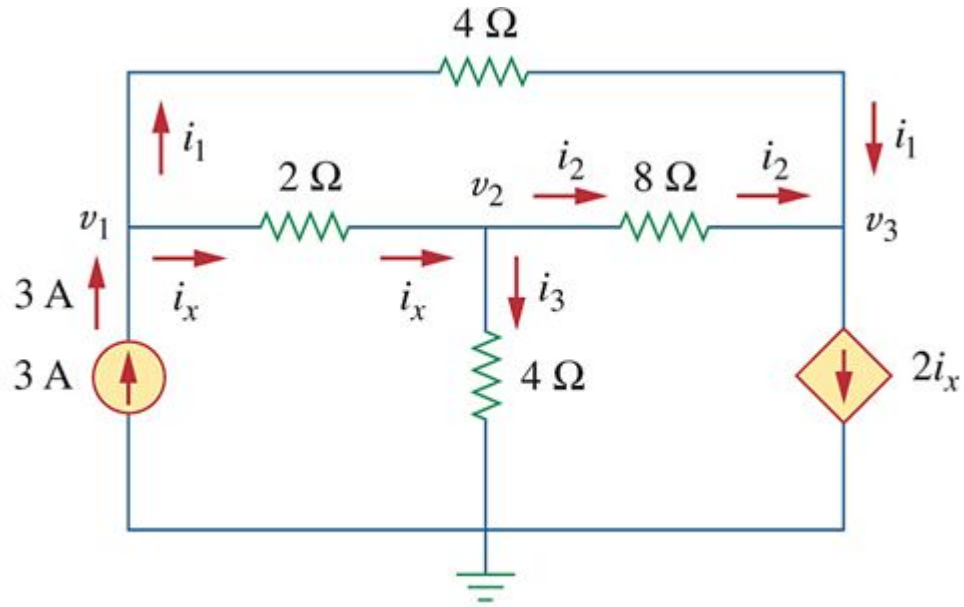
- 다음에서 노드 전압을 구하라



Example 3.2

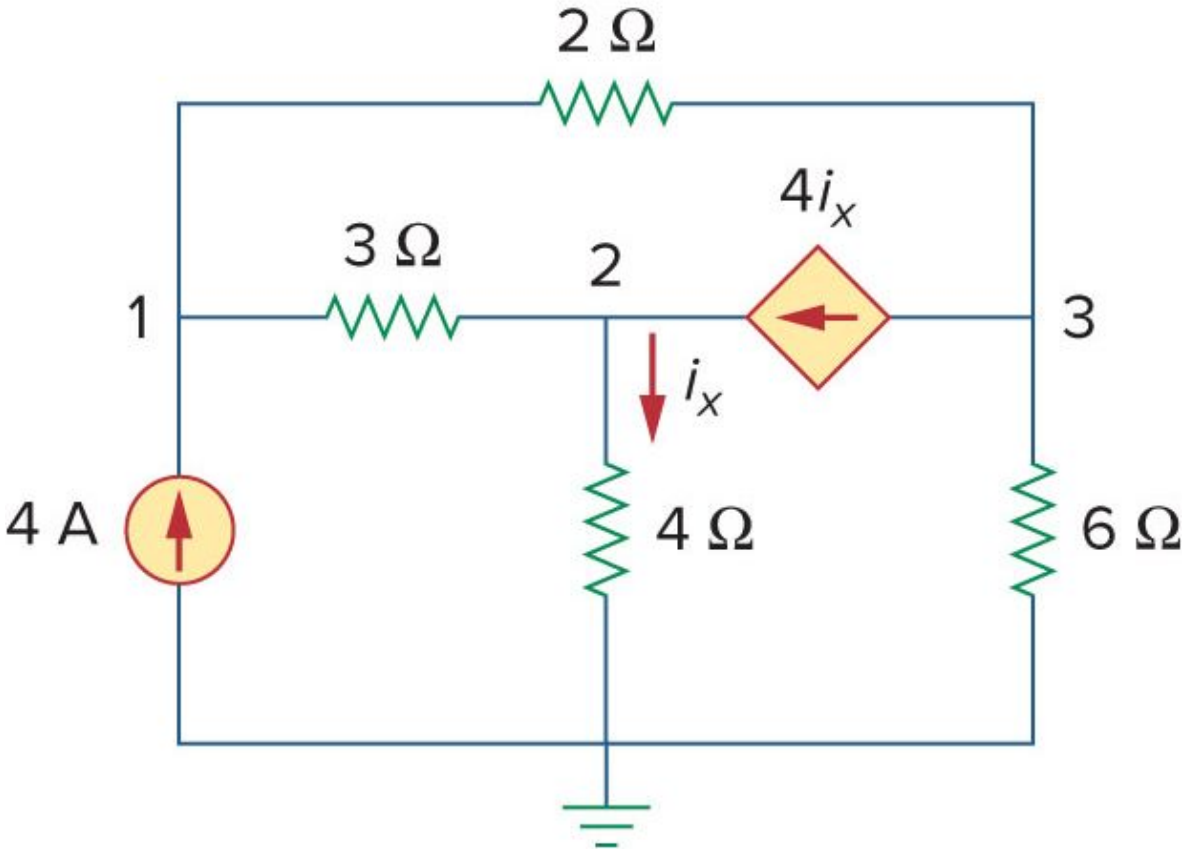
- 다음에서 노드 전압을 구하라



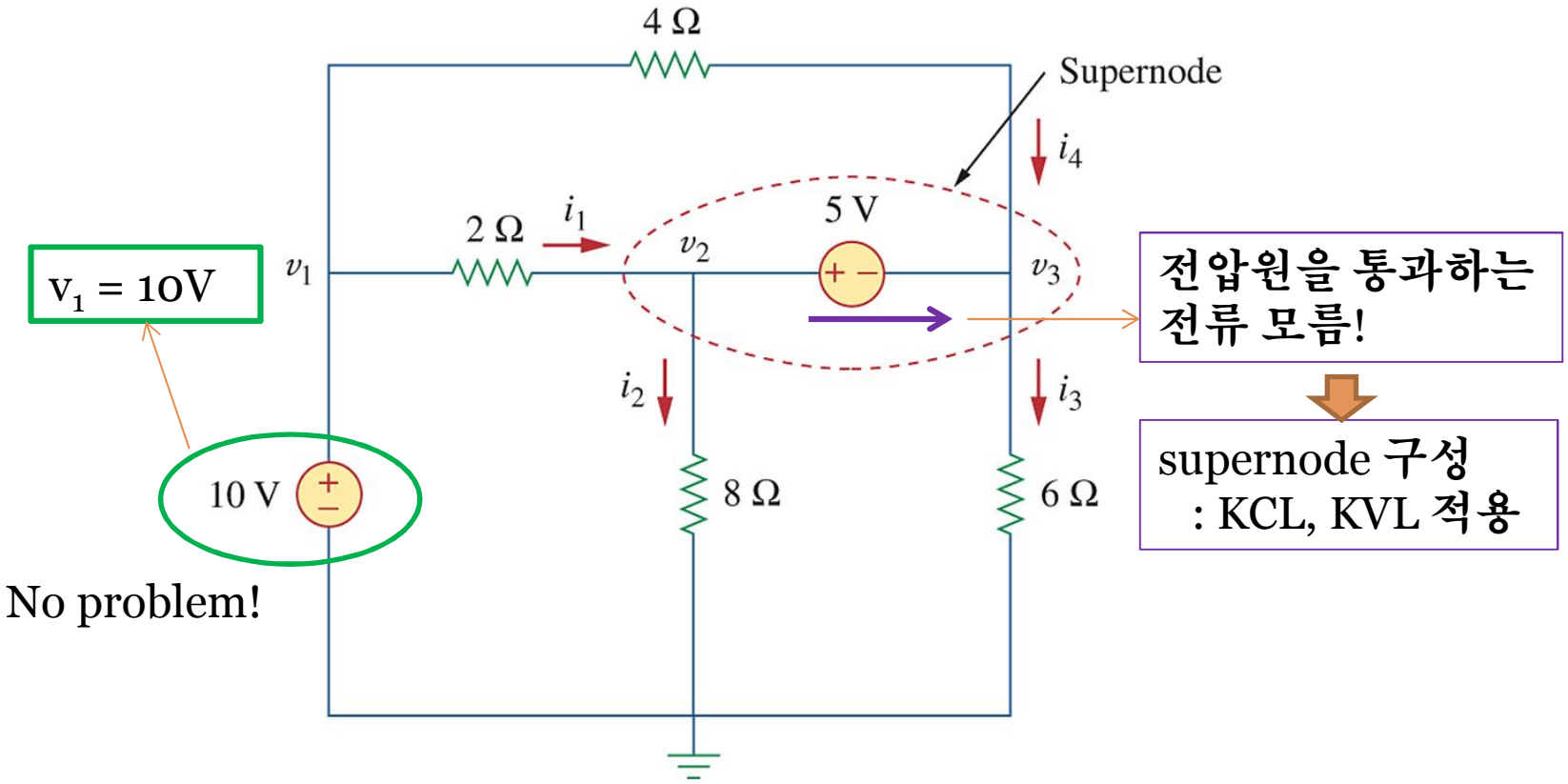


Practice 3.2

- 다음에서 노드 전압을 구하라



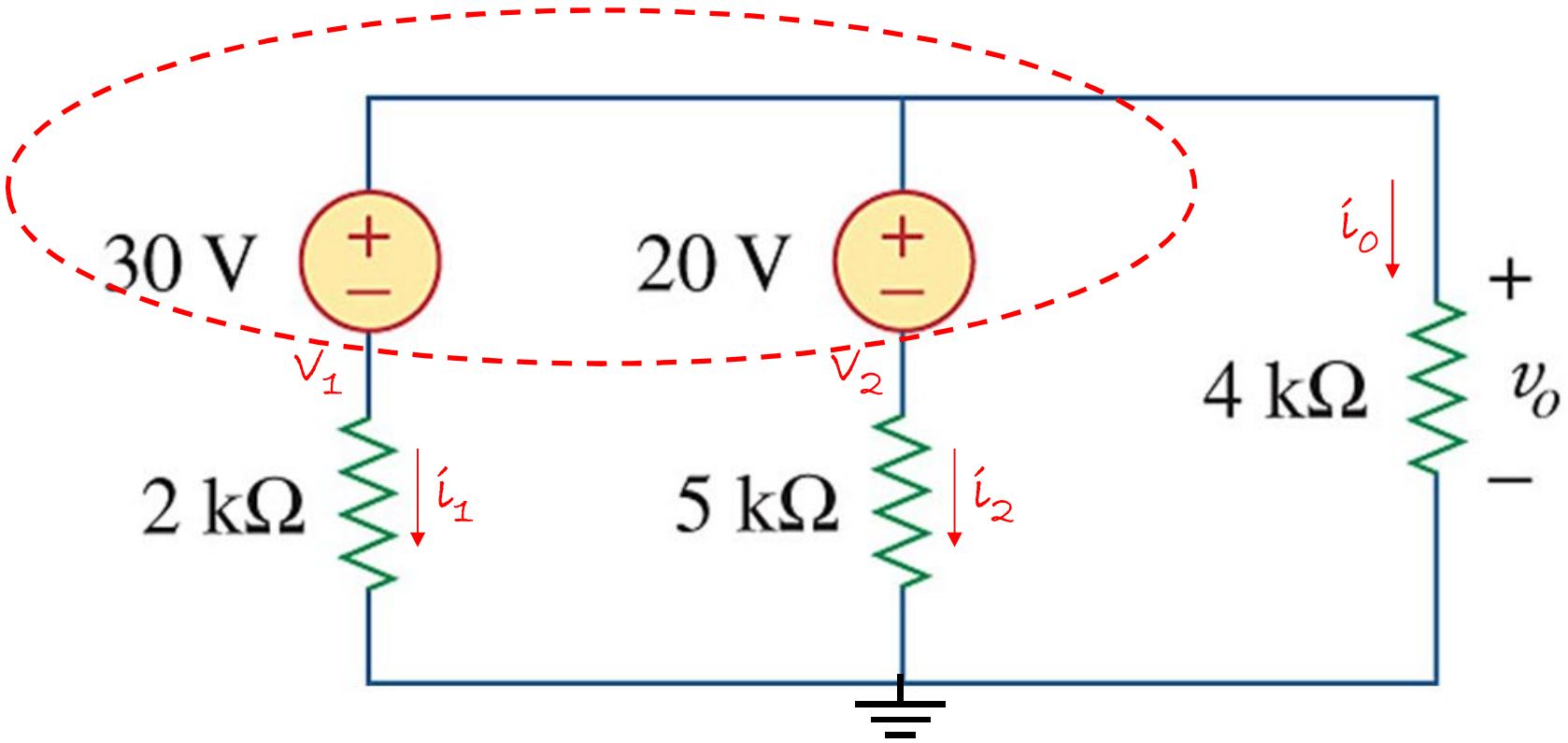
3.3 Nodal Analysis with voltage sources



Case 1 : V source is between reference node & nonreference node

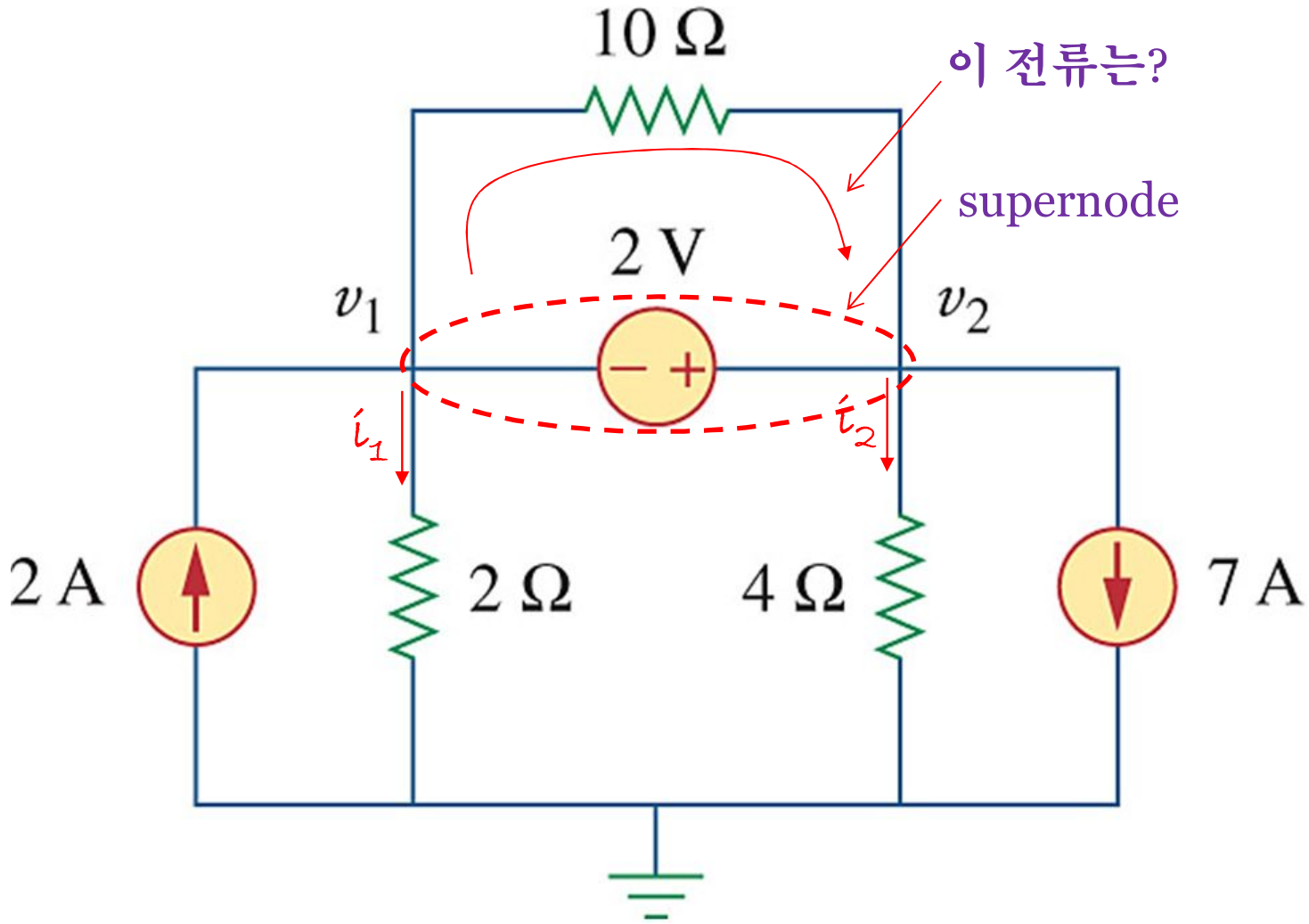
Case 2 : V source is between 2 nonreference nodes

Problem 3.x

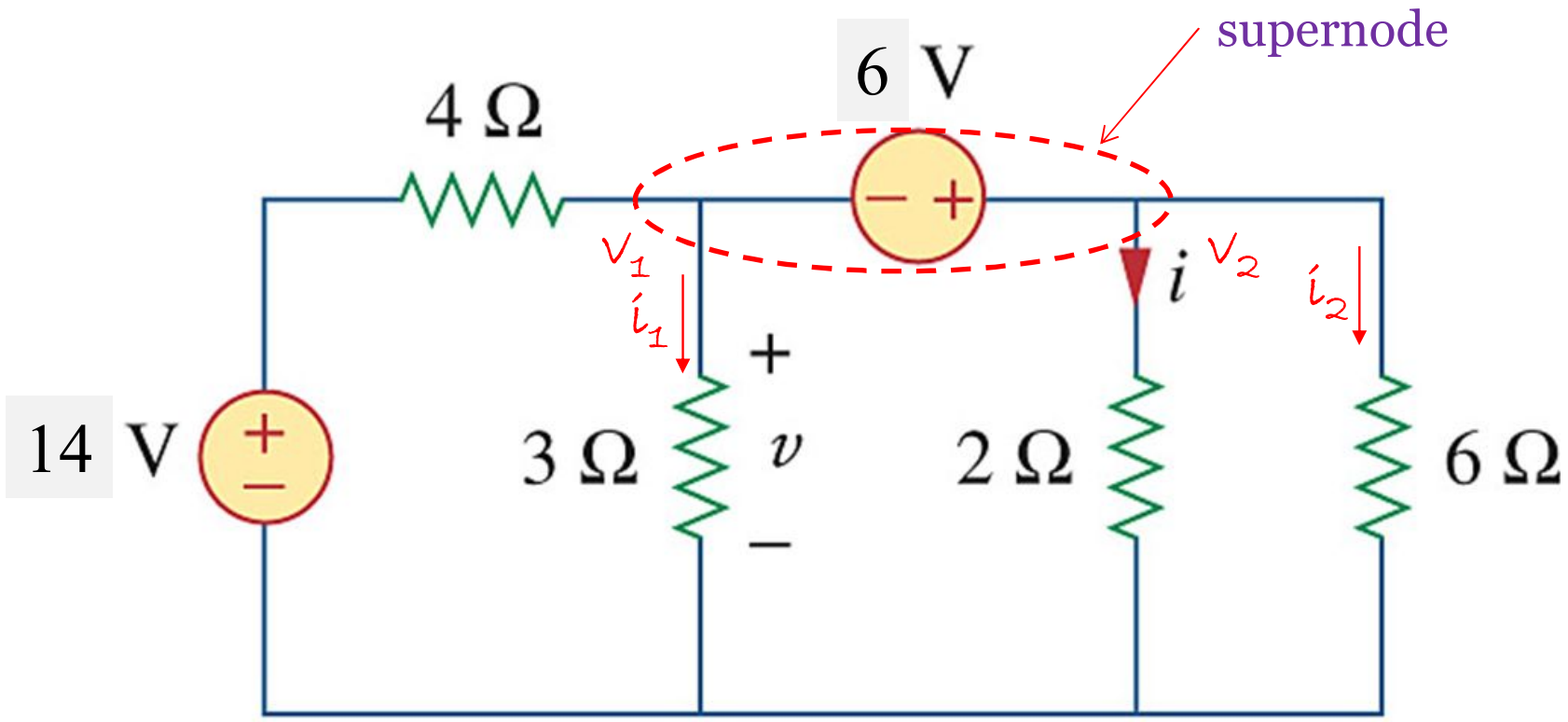


Case 3 : V source가 저항과 직렬 연결을 구성할 때

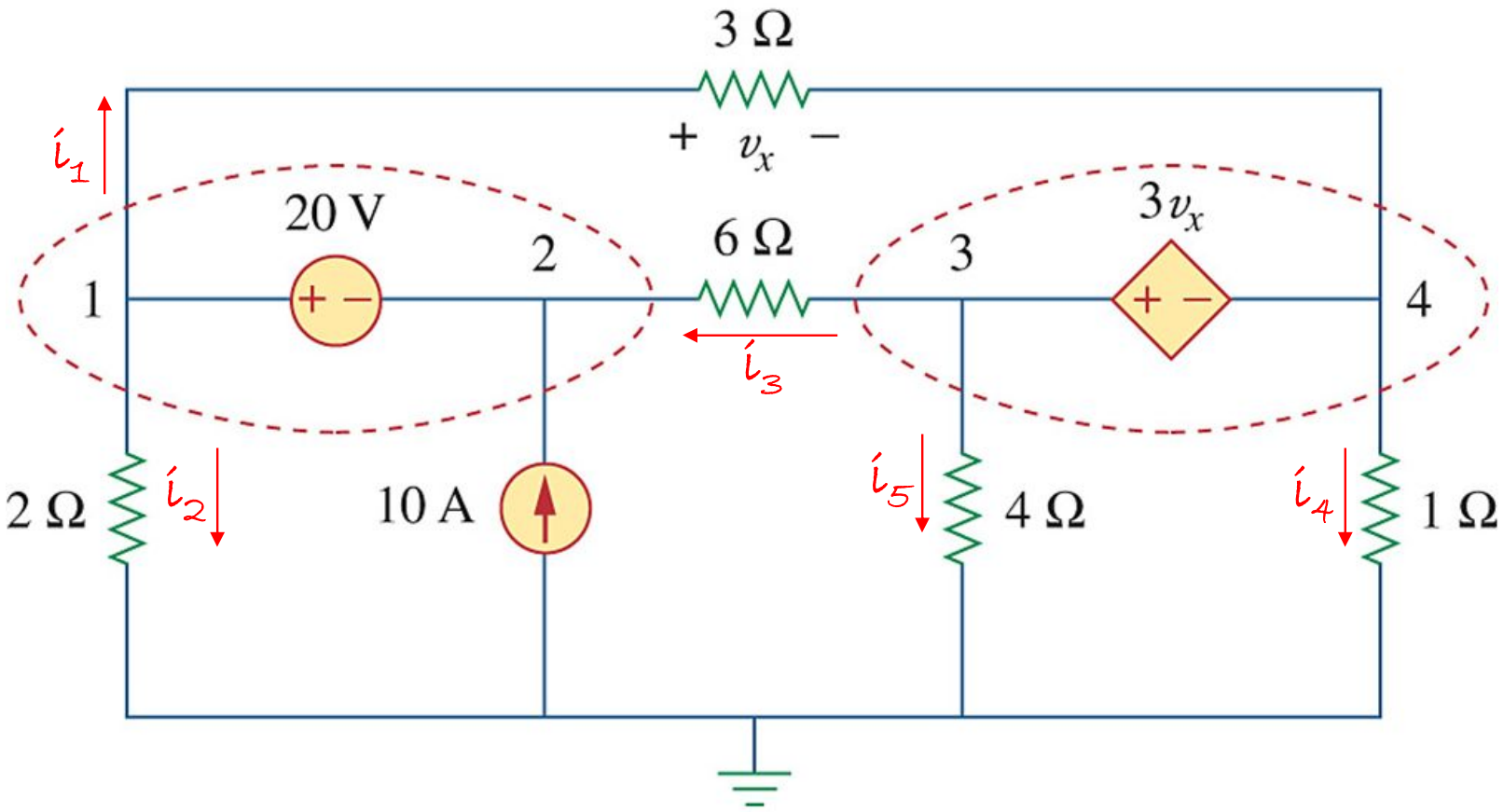
Example 3.3



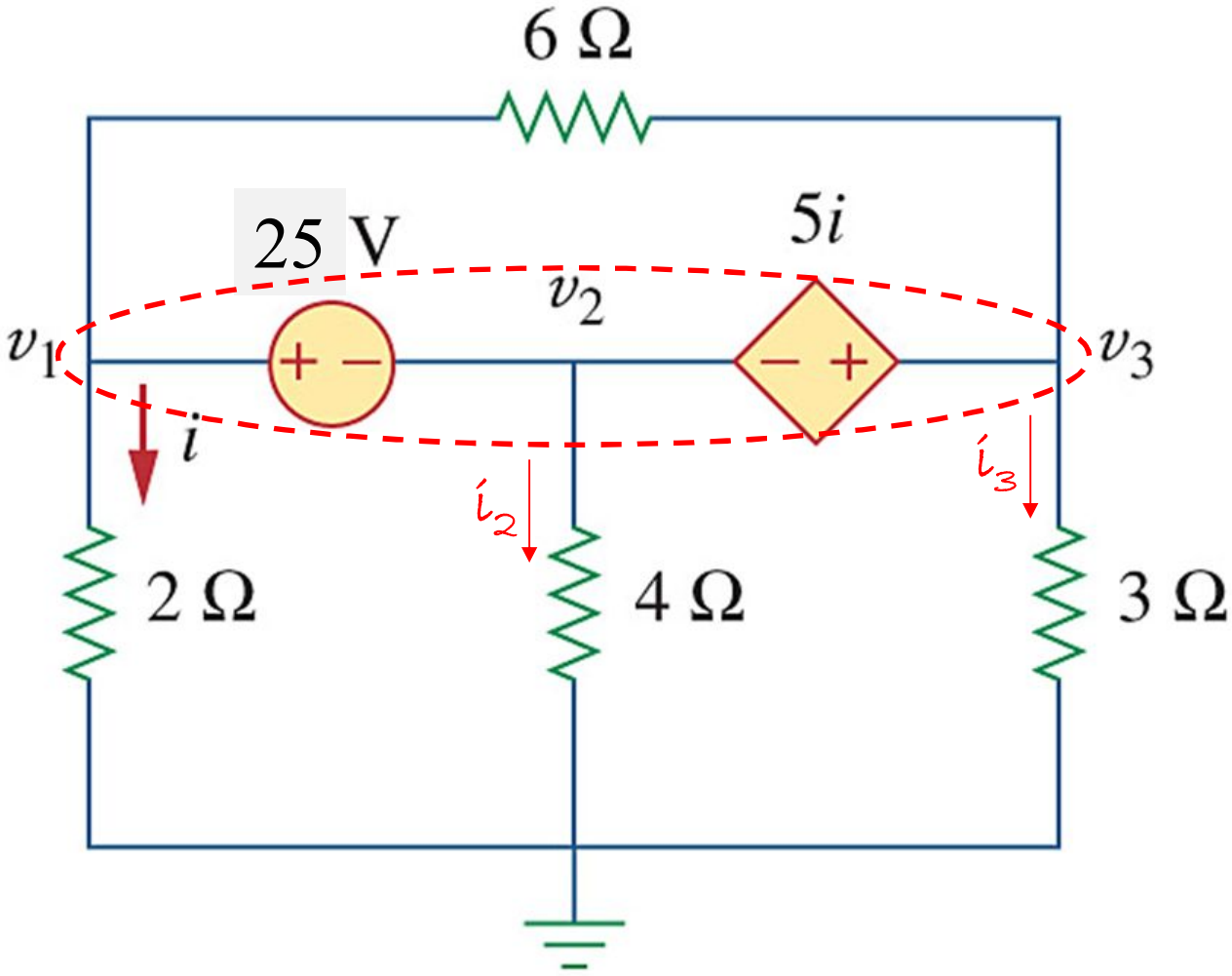
Practice 3.3



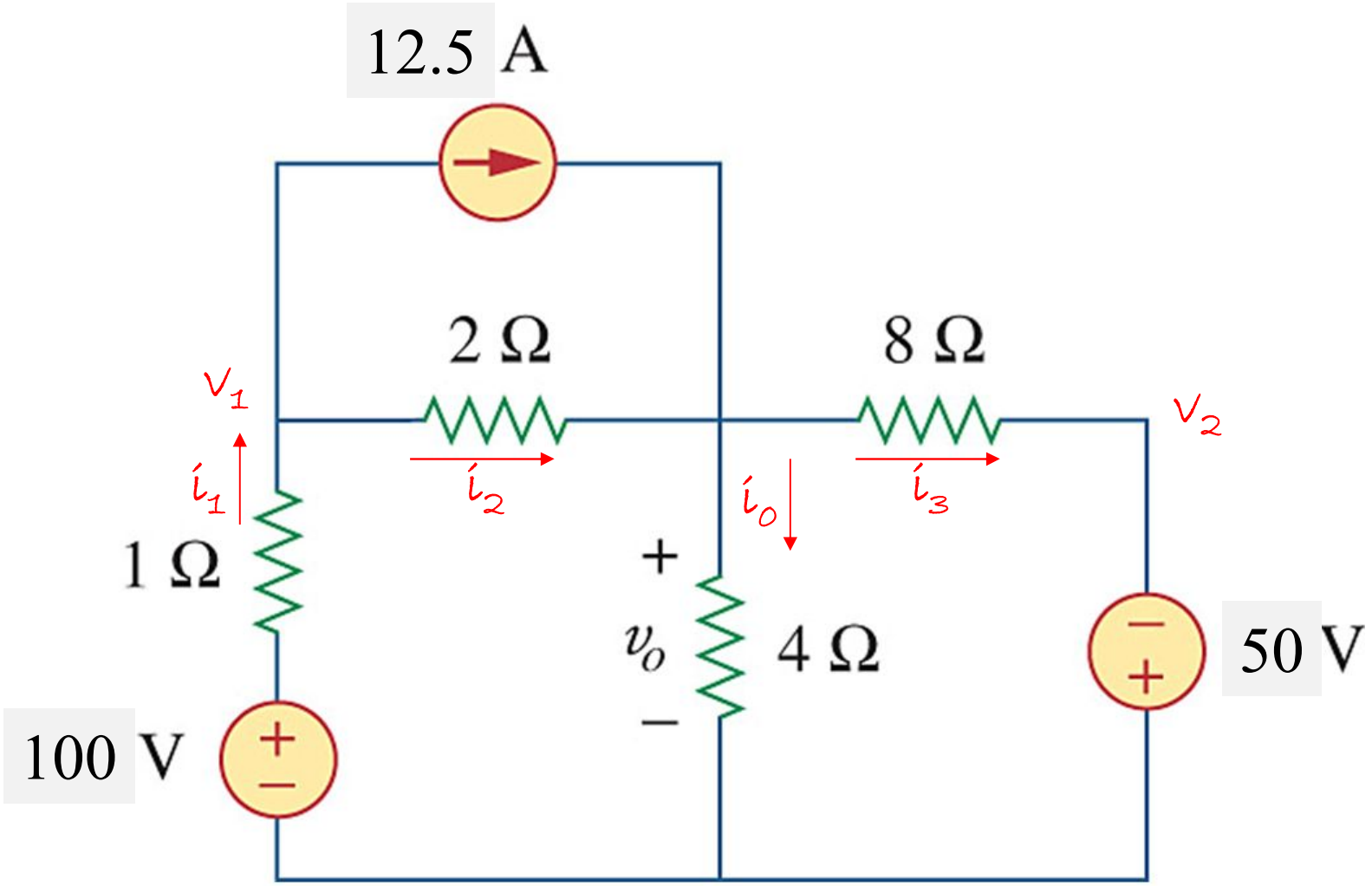
Example 3.4



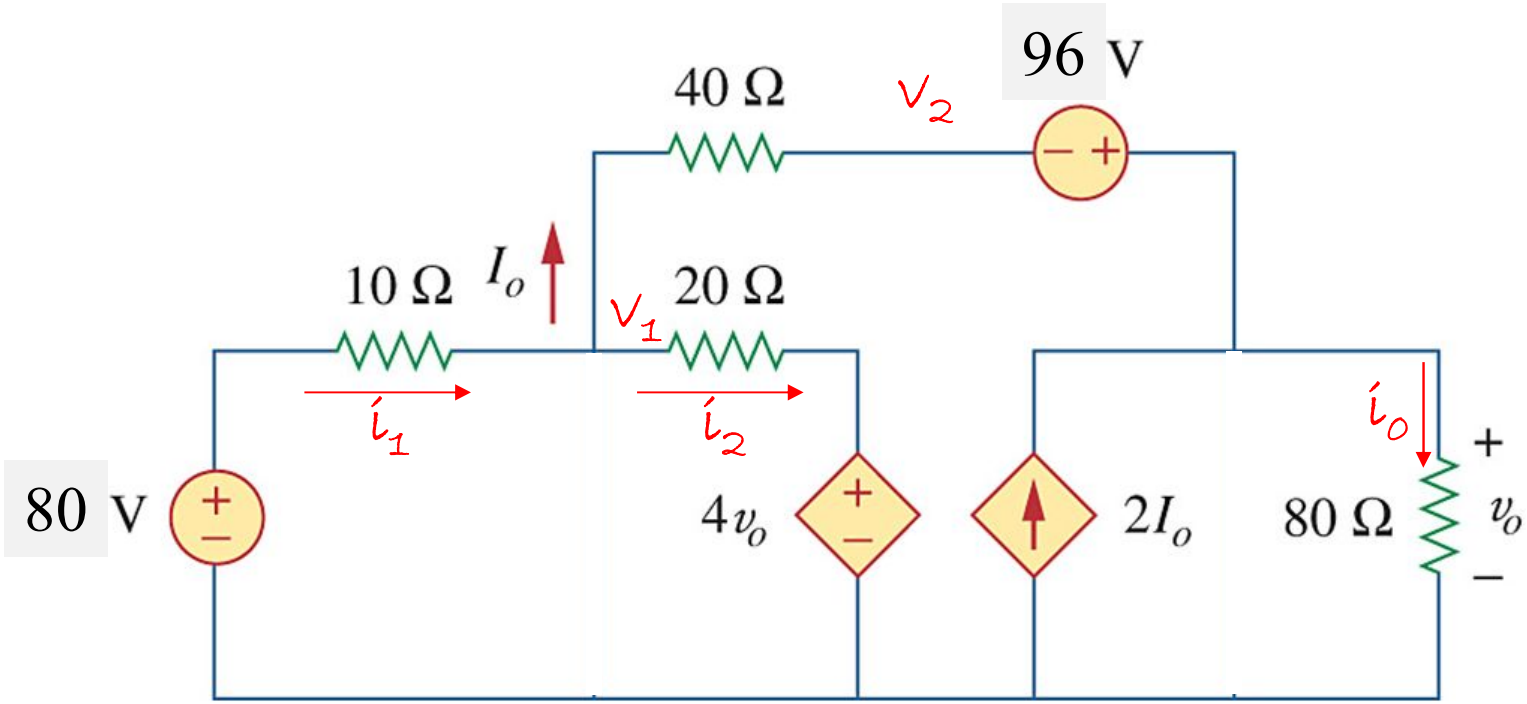
Practice 3.4



Problem 3.14

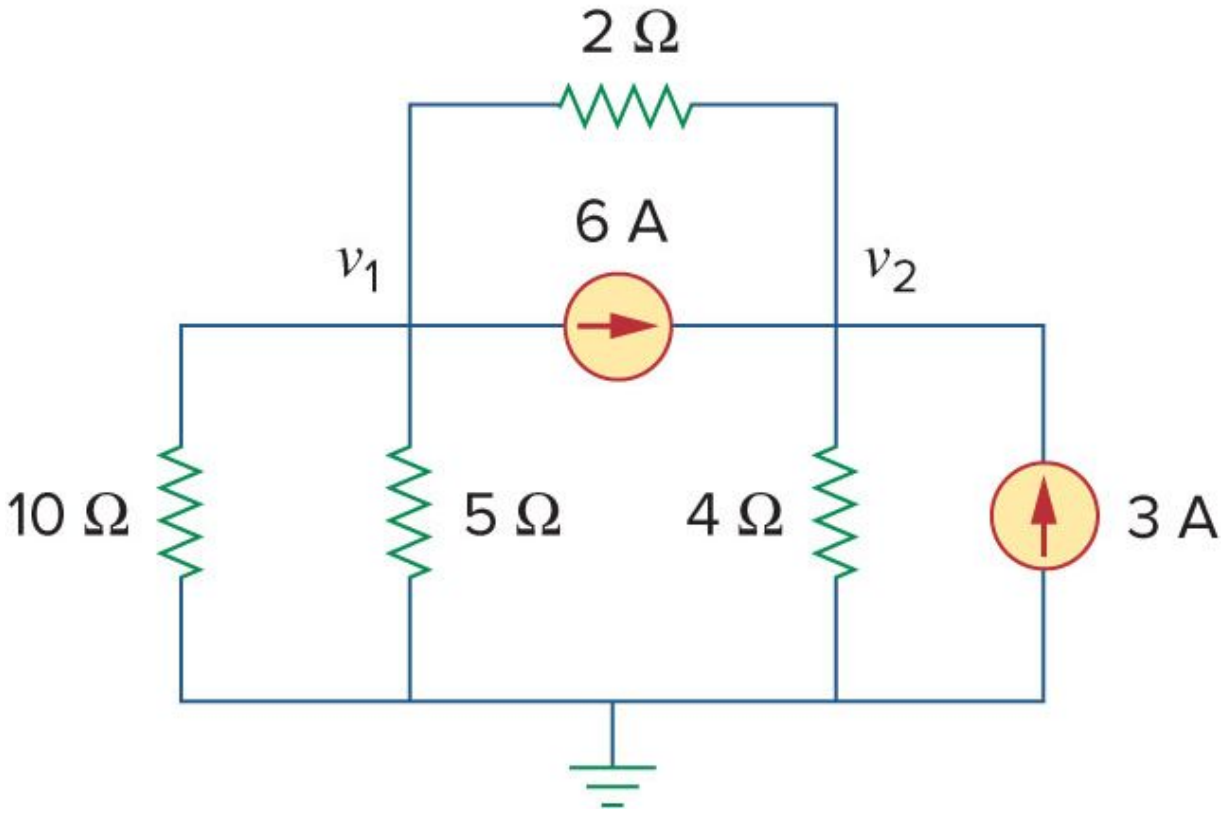


Problem 3.30



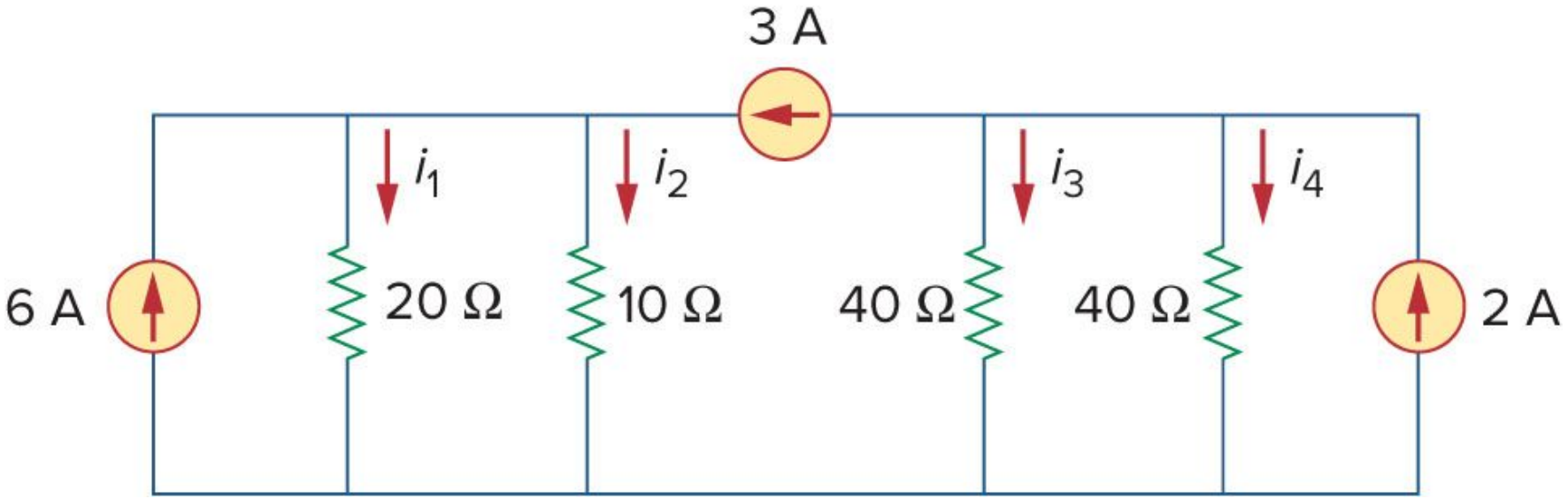
Homework #4

- 다음 수업 시간까지 (5문제)
- #1. Problem 3.2, 다음 회로에서 v_1, v_2를 구하라



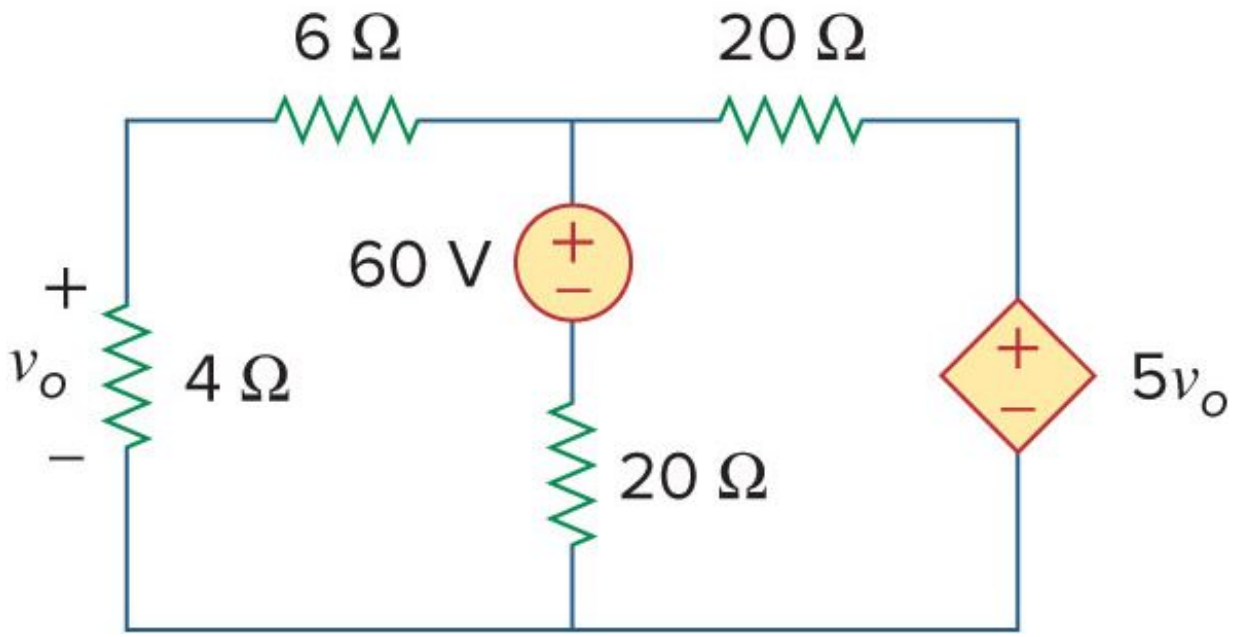
Homework #4

- #2. Problem 3.4, 다음 회로에서 $i_1 \sim i_4$를 구하라



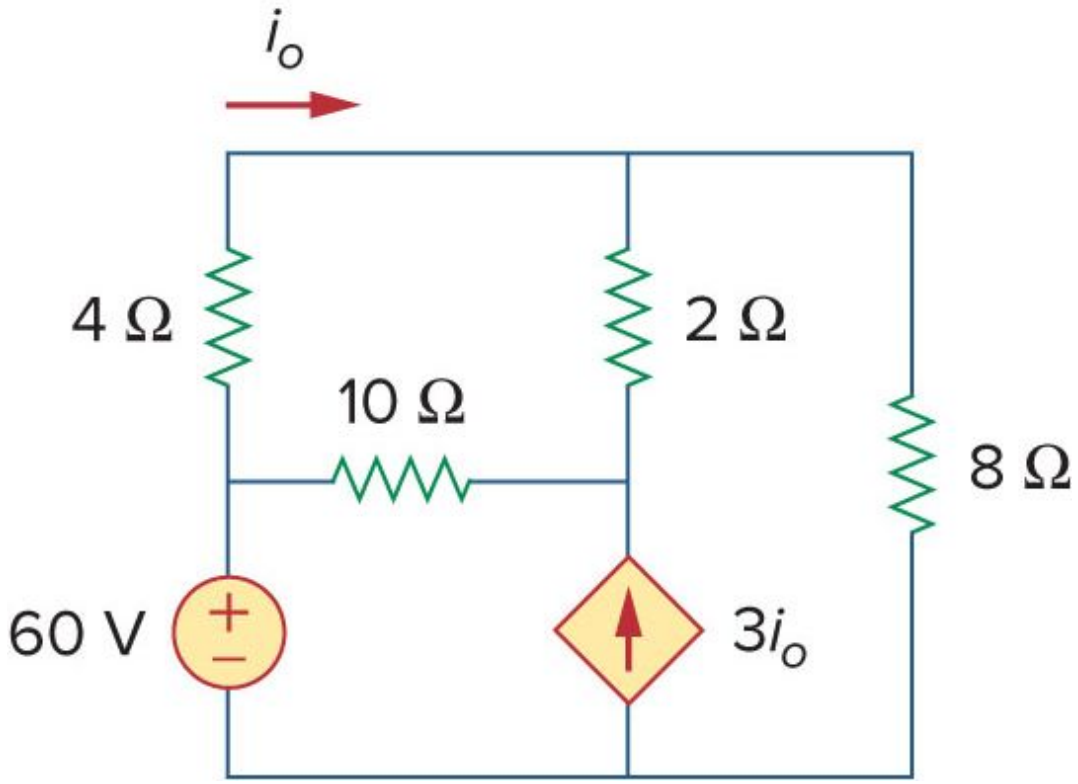
Homework #4

- #3, Problem 3.8 노드 해석을 이용하여 v_0를 구하라



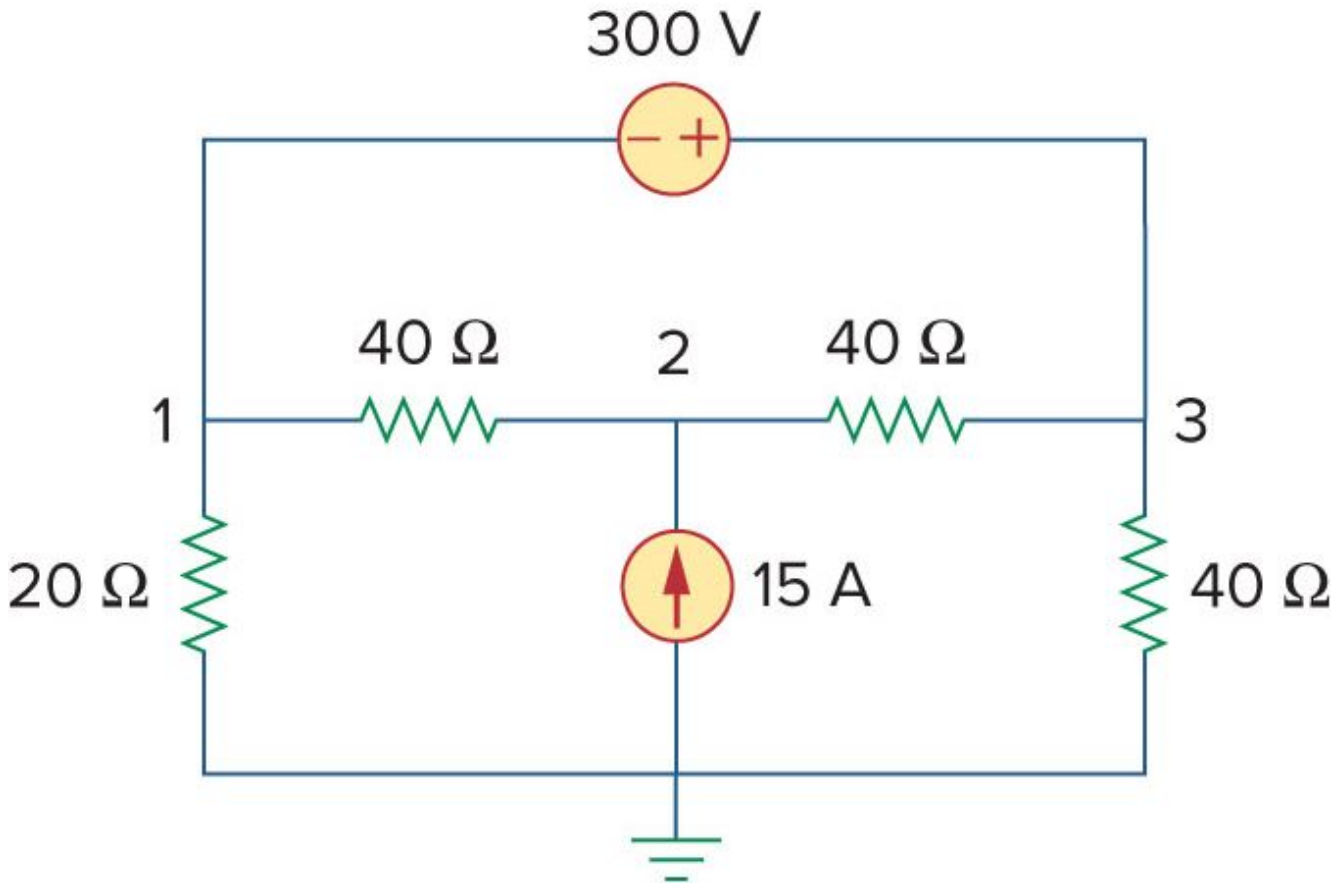
Homework #4

- #4, Problem 3.17 노드 해석을 이용하여 i_0를 구하라



Homework #4

- #5, Problem 3.18 노드 해석을 이용하여 노드 전압을 구하라



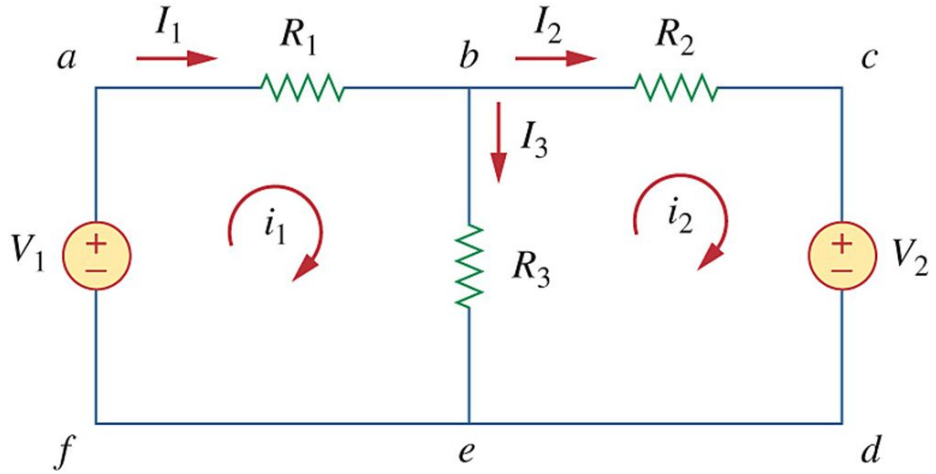
3.4 Mesh Analysis

Mesh Analysis

- Mesh (망로) : 내부에 다른 루프를 포함하지 않는 루프
- 방법 : KVL 사용 ($\sum v_k = 0$)
- 변수 : mesh current

  (옴의 법칙에 의해

  $v_k = iR$ 이므로)



Step 1 :

n 개의 Mesh 에 대해 Mesh Current $i_1, i_2, \ldots i_n$을 할당

Step 2 :

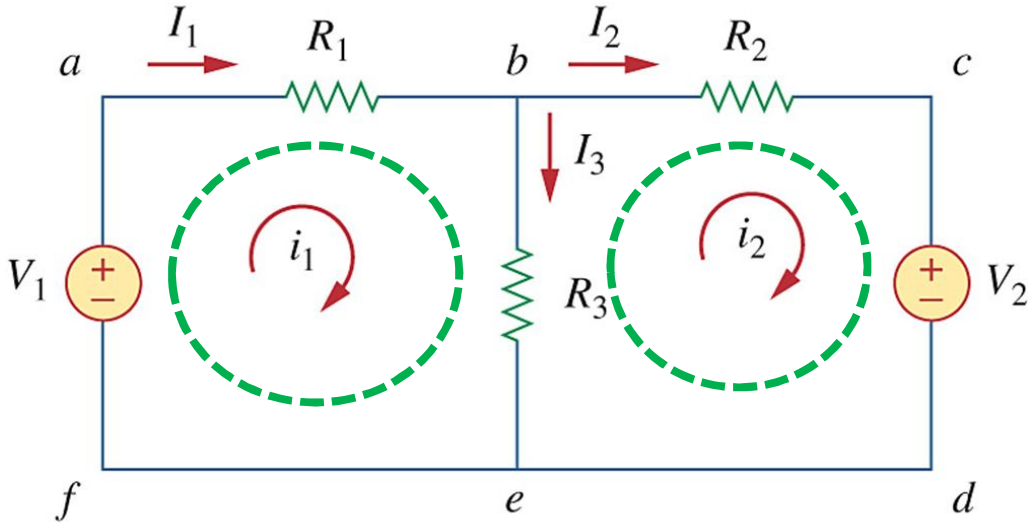
n개의 Mesh 각각에 KVL을 적용 ← ohm의 법칙 이용

Step 3 :

연립 방정식 계산

3.4 Mesh Analysis without current sources

- Step 1
  - 각 mesh에 mesh current $i_1, i_2, \ldots, i_n$ 할당

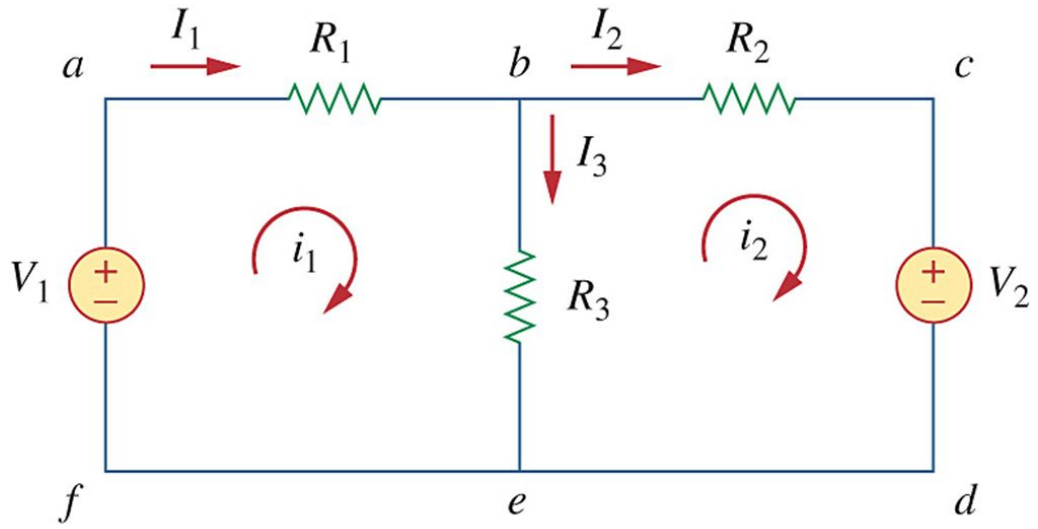


Note :

- i_1, i_2 : 실제 회로에 흐르는 전류와 다를 수 있음.
  (예) $I_3 = i_1 - i_2$

3.4 Mesh Analysis without current sources

- Step 2
  - 각 mesh에 KVL 적용



$$-V_1 + i_1R_1 + (i_1 - i_2)R_3 = 0 \qquad i_2R_2 + V_2 + (i_2 - i_1)R_3 = 0$$

$$i_1(R_1 + R_3) - i_2R_3 = V_1 \qquad -i_1R_3 + i_2(R_2 + R_3) = -V_2$$

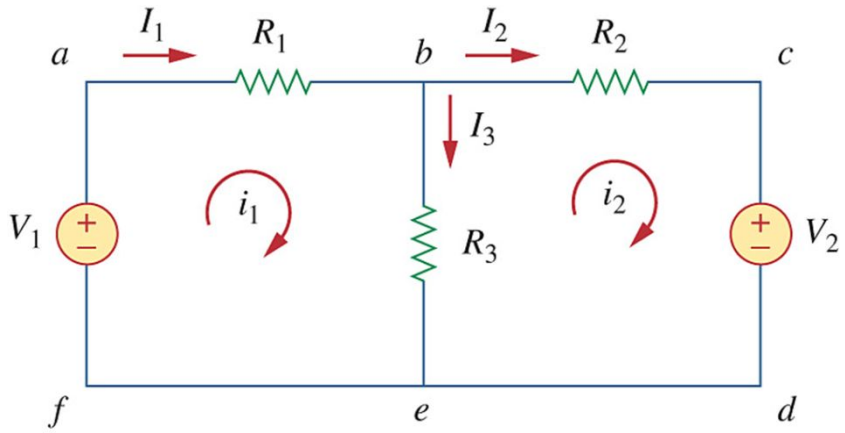
3.4 Mesh Analysis without current sources

- Step 3
  - **연립방정식 풀이**

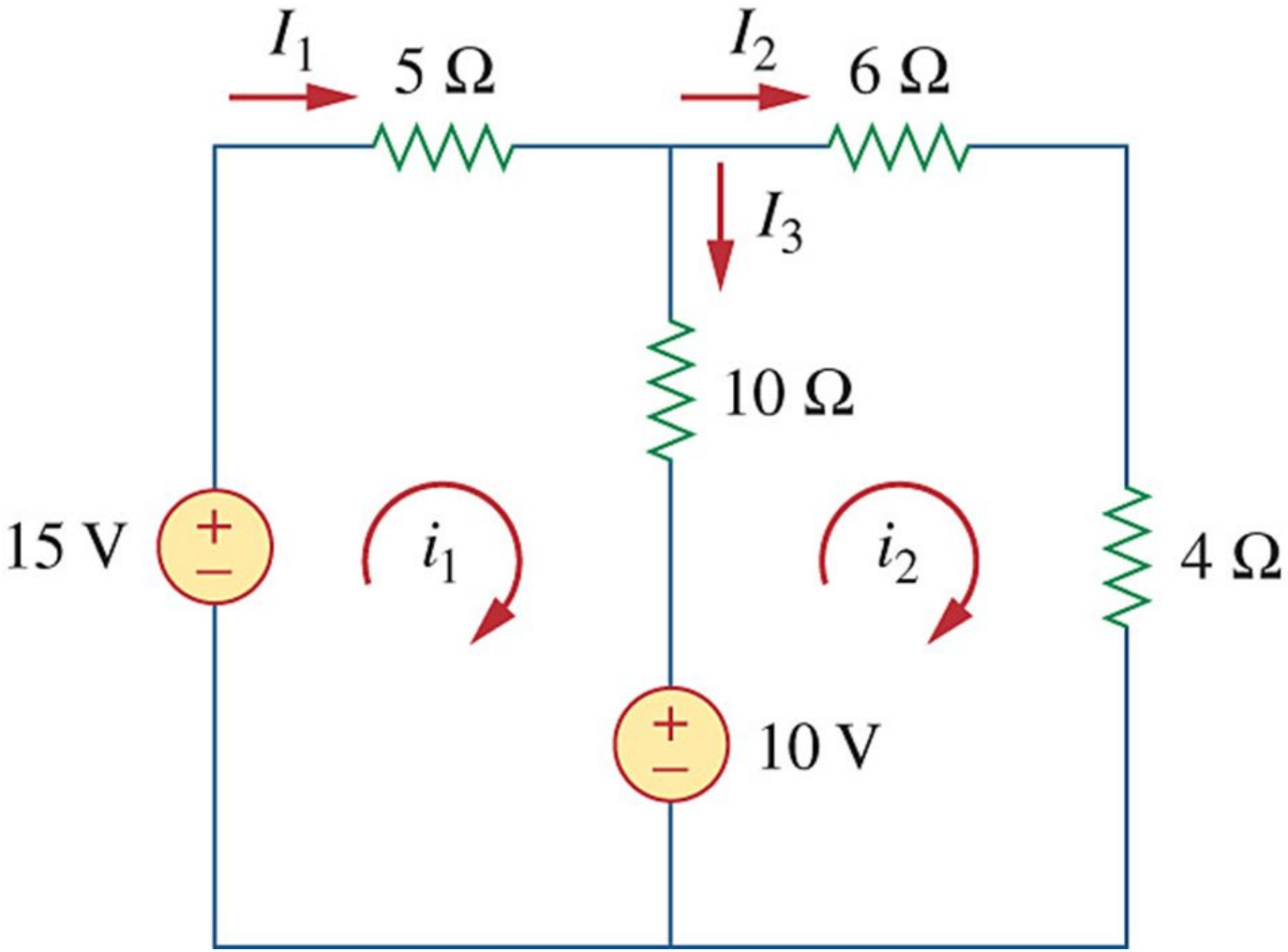
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

  - **실제 전류로 변환**

$$I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2$$

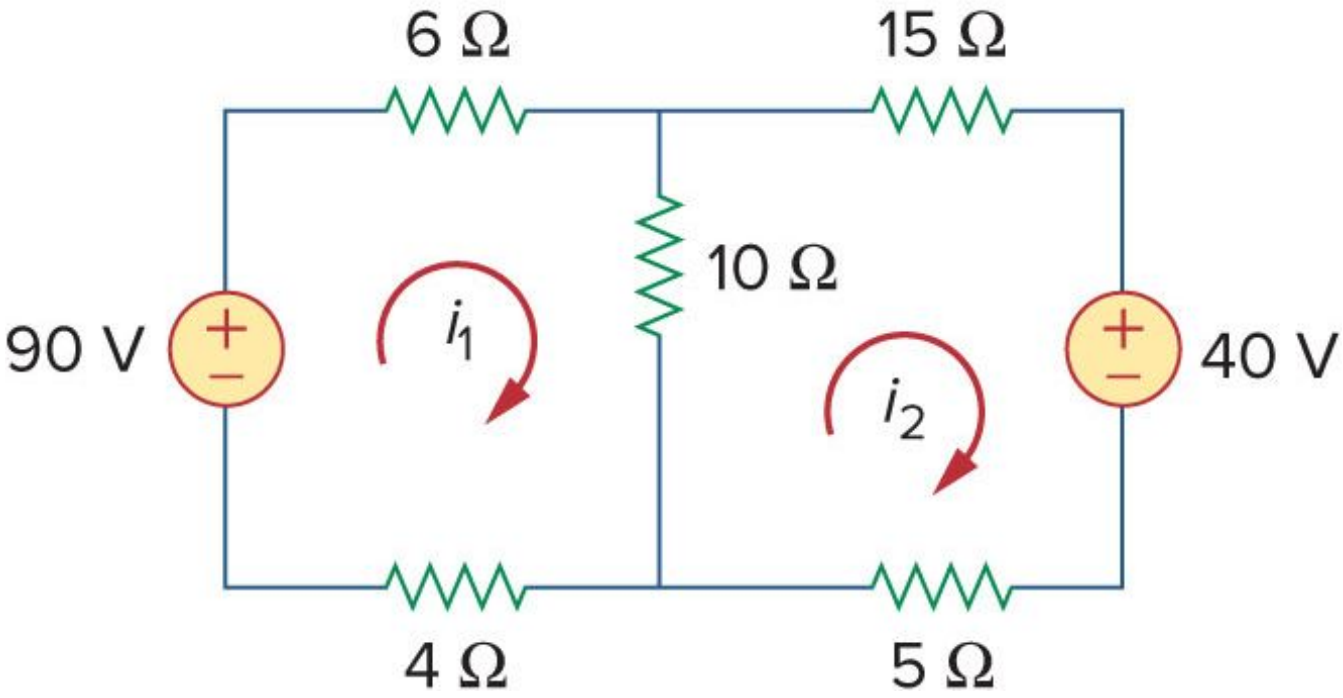


Example 3.5

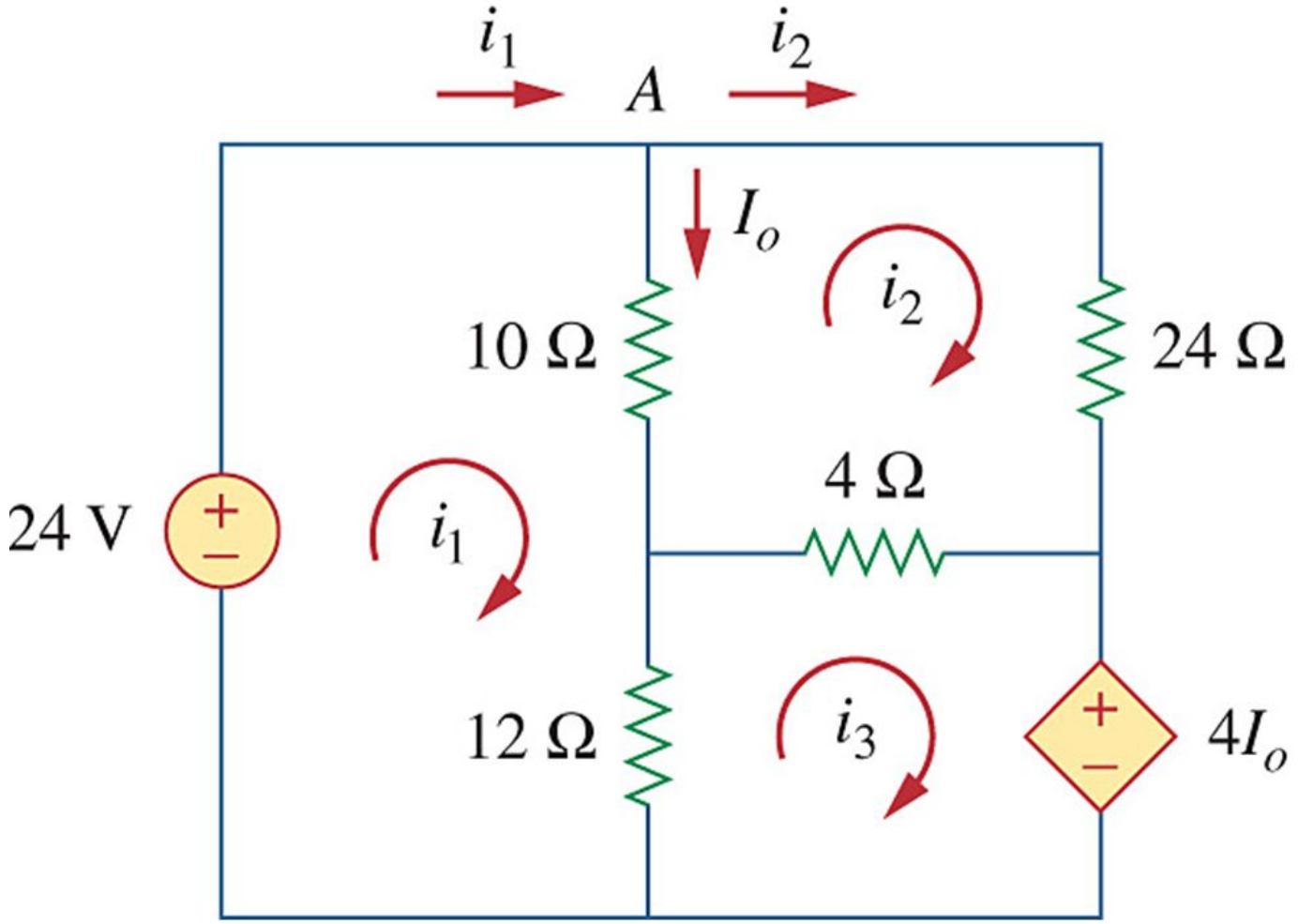


Practice 3.5

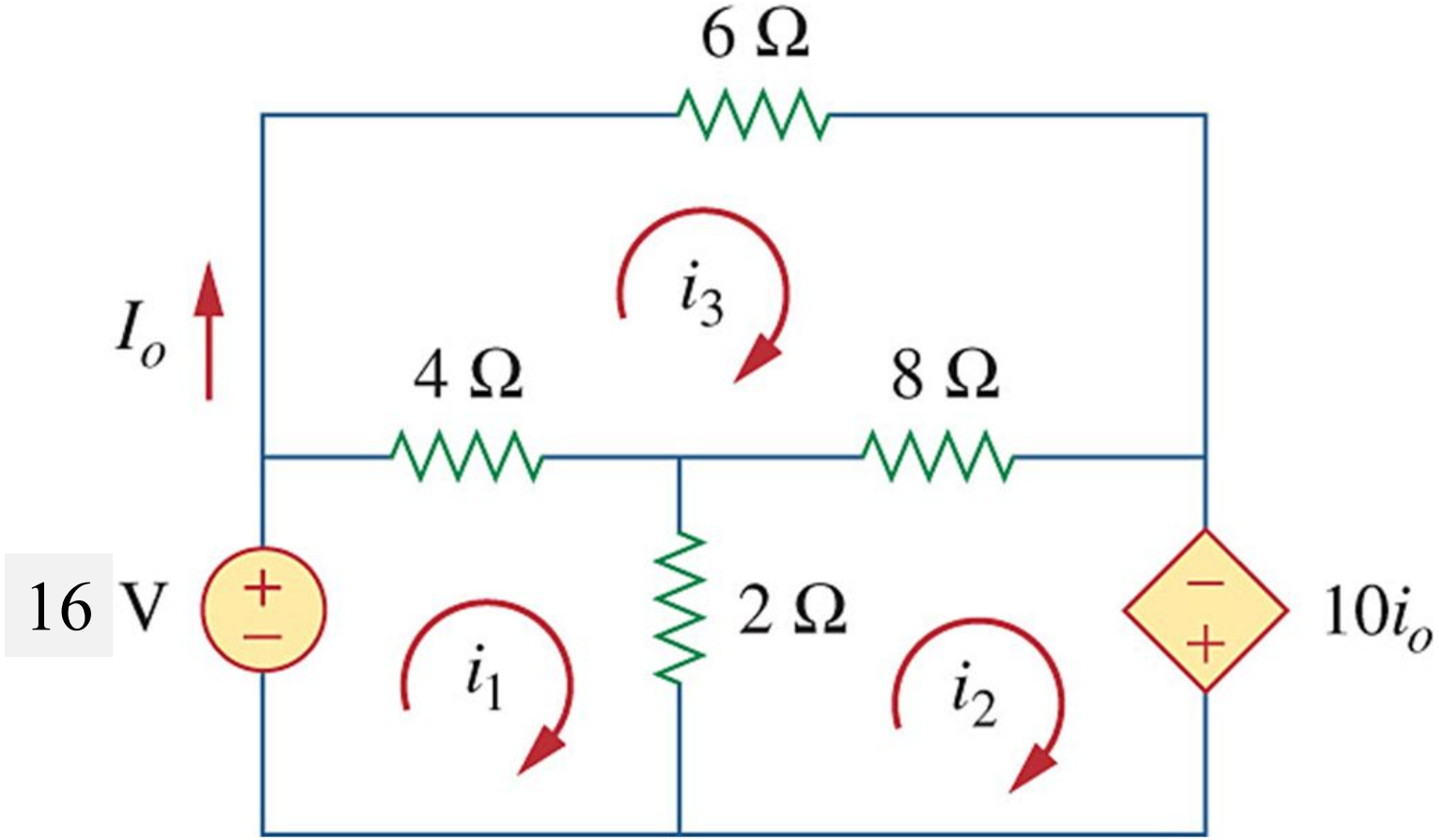
- 메시 전류 i_1, i_2를 구하라



Example 3.6

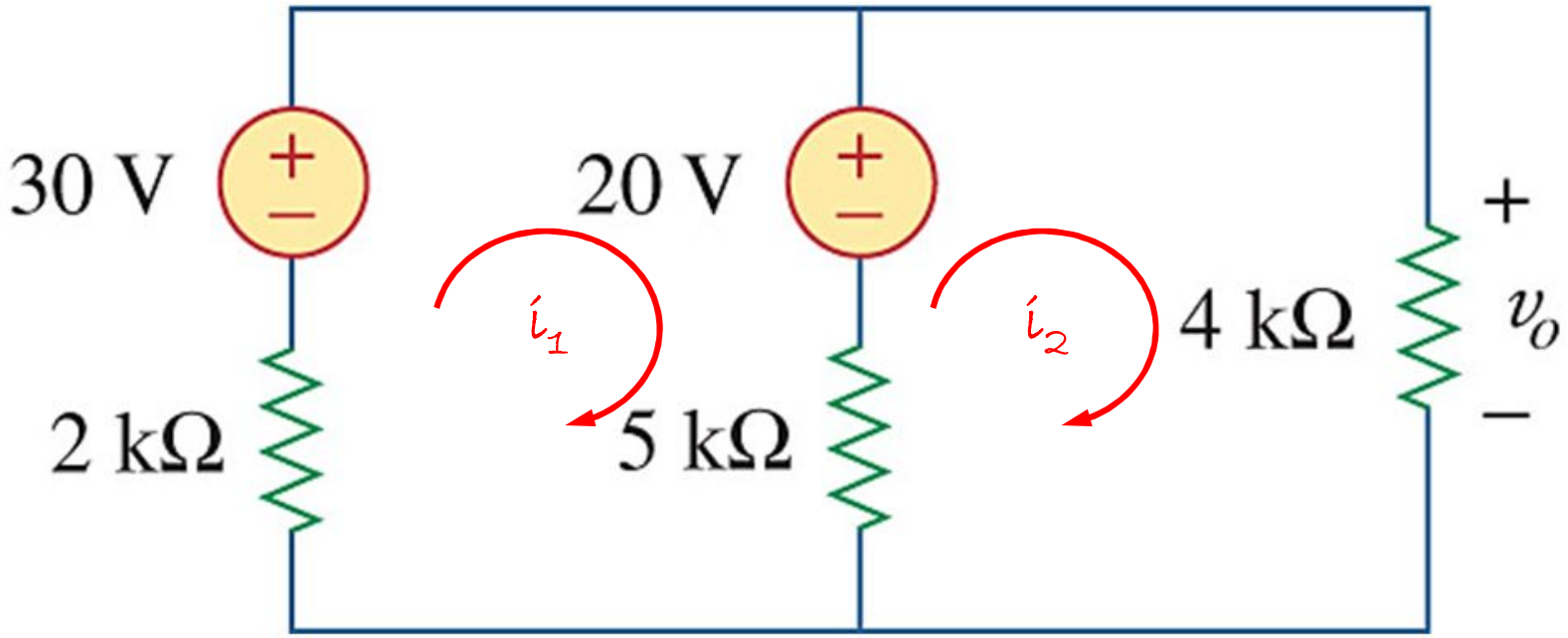


Practice 3.6



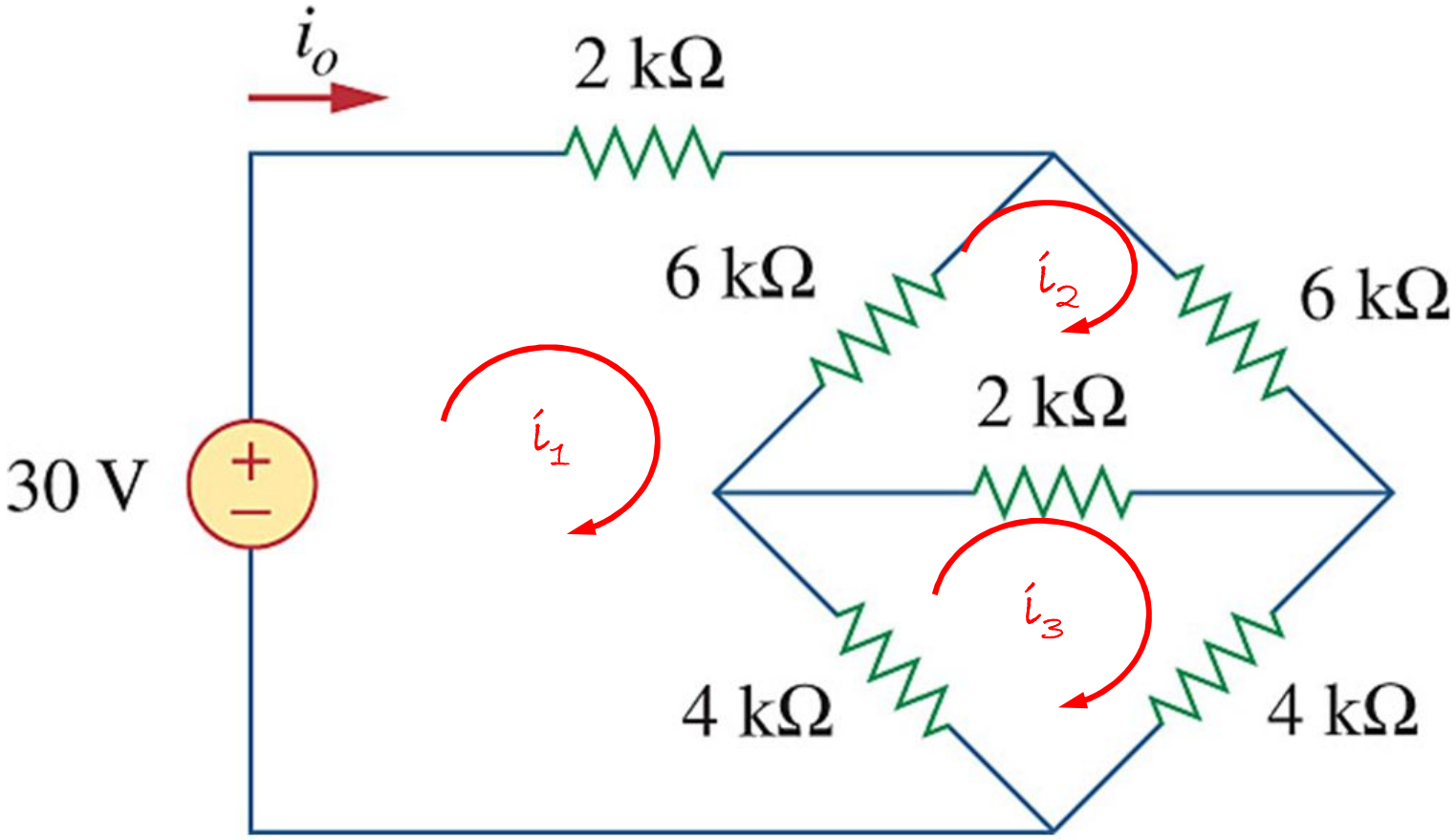
Problem 3.x

- Using mesh analysis



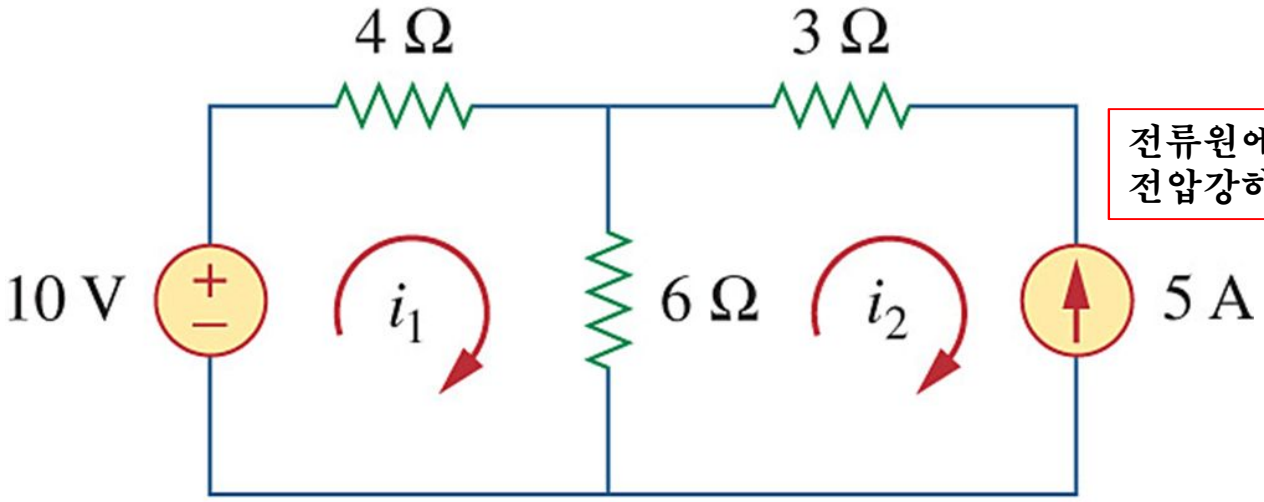
Problem 3.40

- Using mesh analysis



3.5 Mesh Analysis with current sources

- Case 1
  - 전류원이 하나의 mesh에 존재할 때



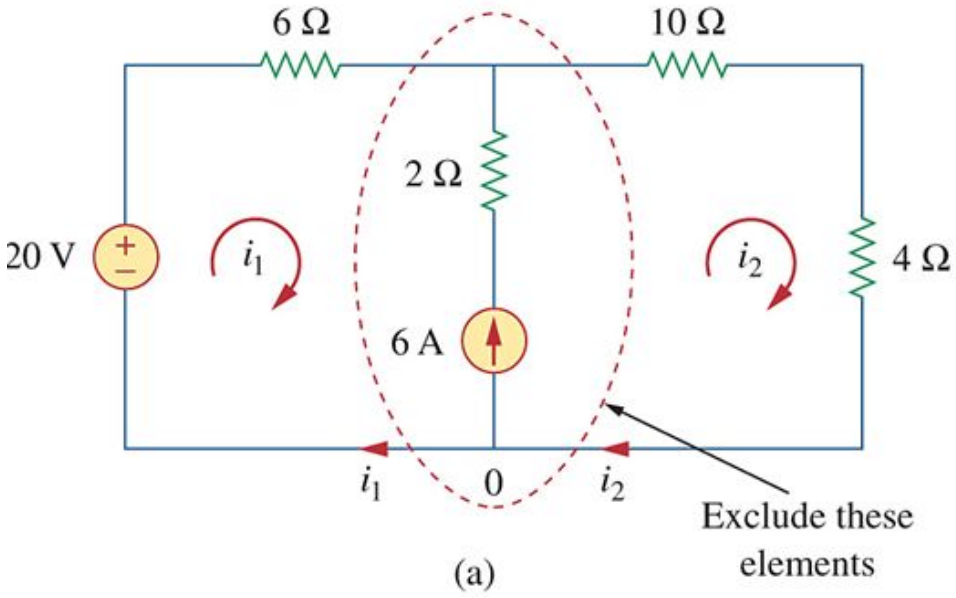
전류원에 대해서는
전압강하를 정할 수 없다.

이 mesh에만 KVL 적용!

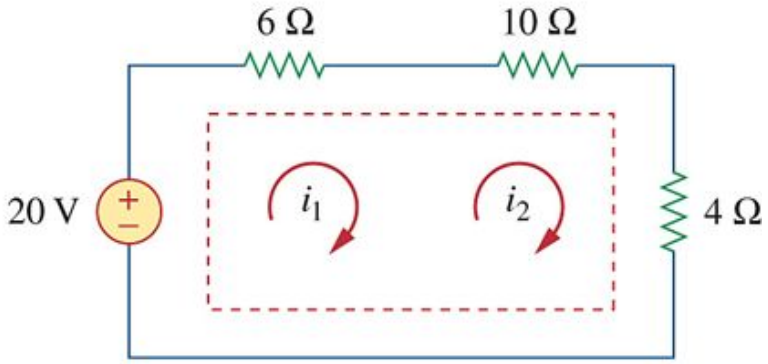
Just use i_2 = -5!
Do not apply KVL!

3.5 Mesh Analysis with current sources

- Case 2
  - 전류원이 2개의 mesh 사이에 존재할 때
  - supermesh 형성 (∵ 전류원에 대해서는 KVL 적용 어려움)
  - supermesh에 대하여 KVL 적용



(a)

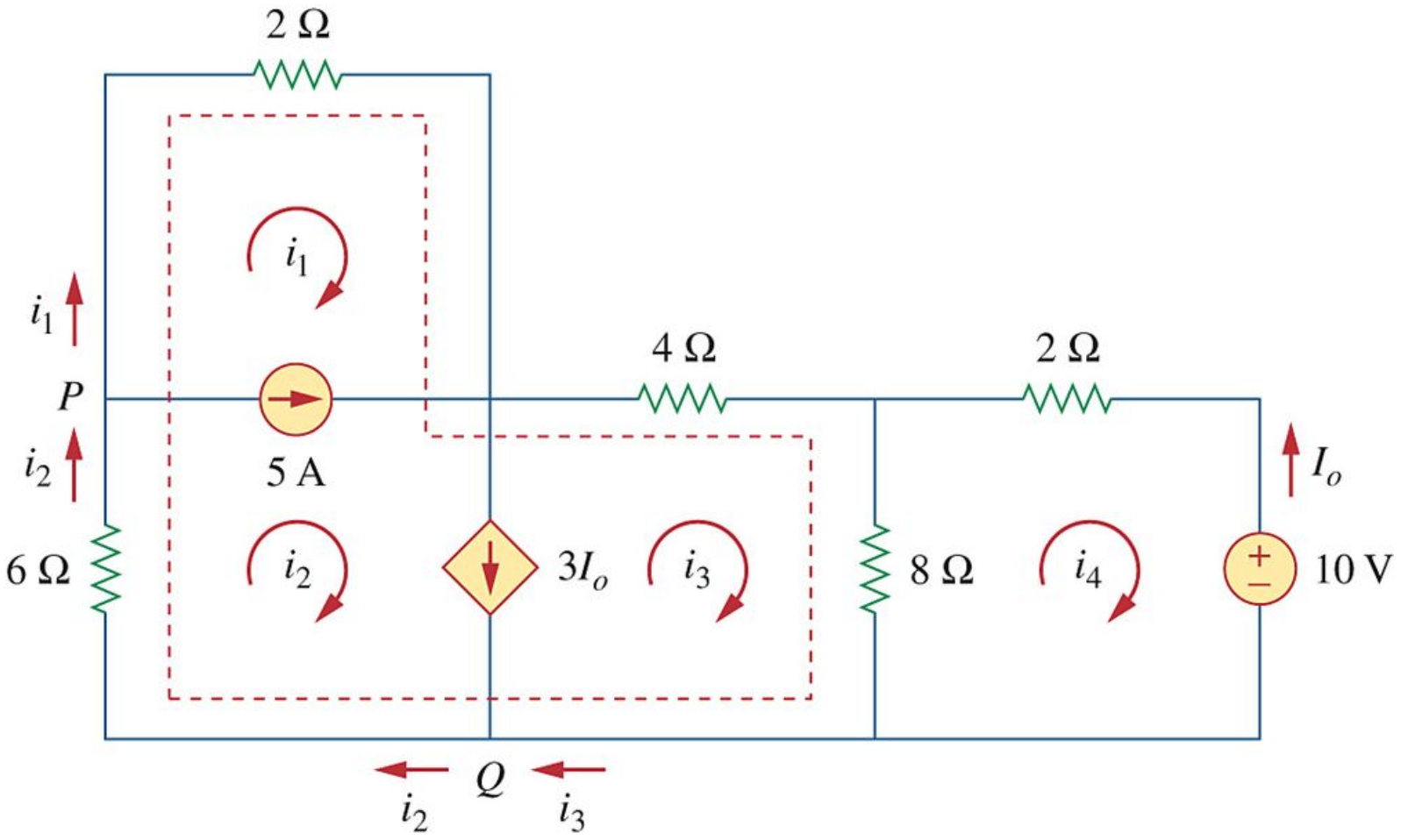


(b)

KVL : $6i_1 + 14i_2 = 20$

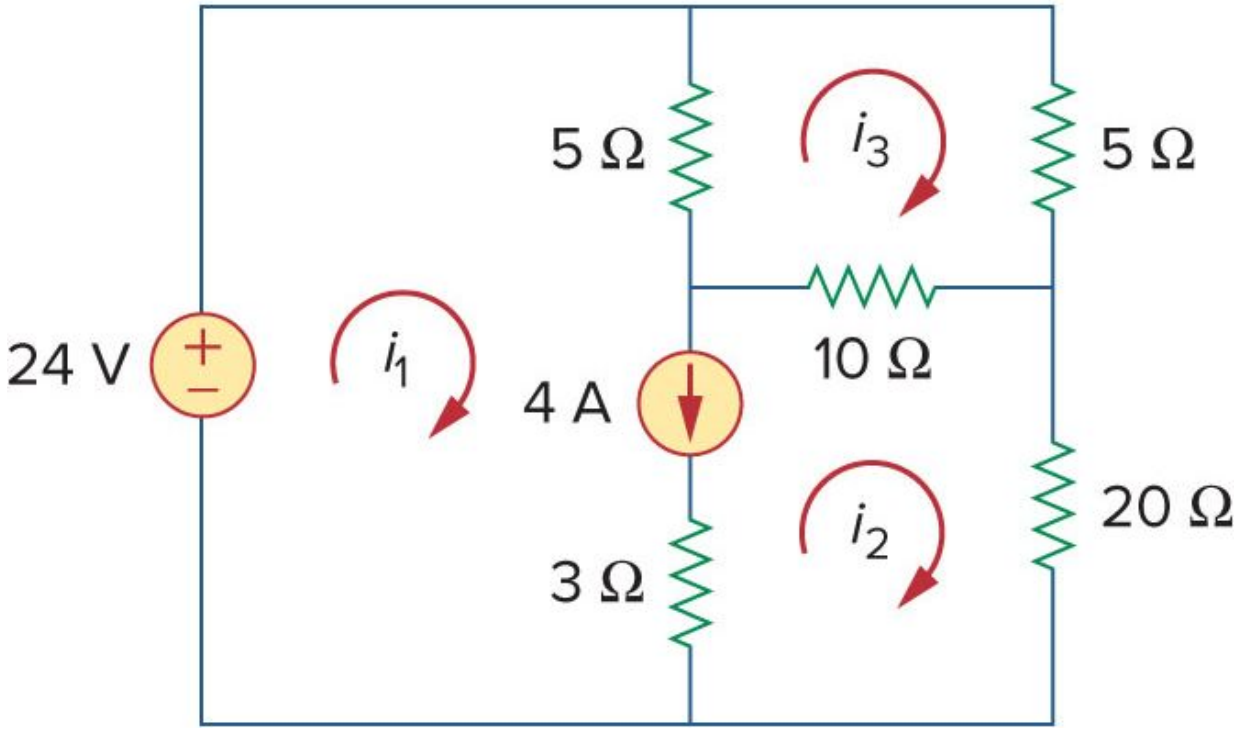
KCL : $i_2 = i_1 + 6$

Example 3.7

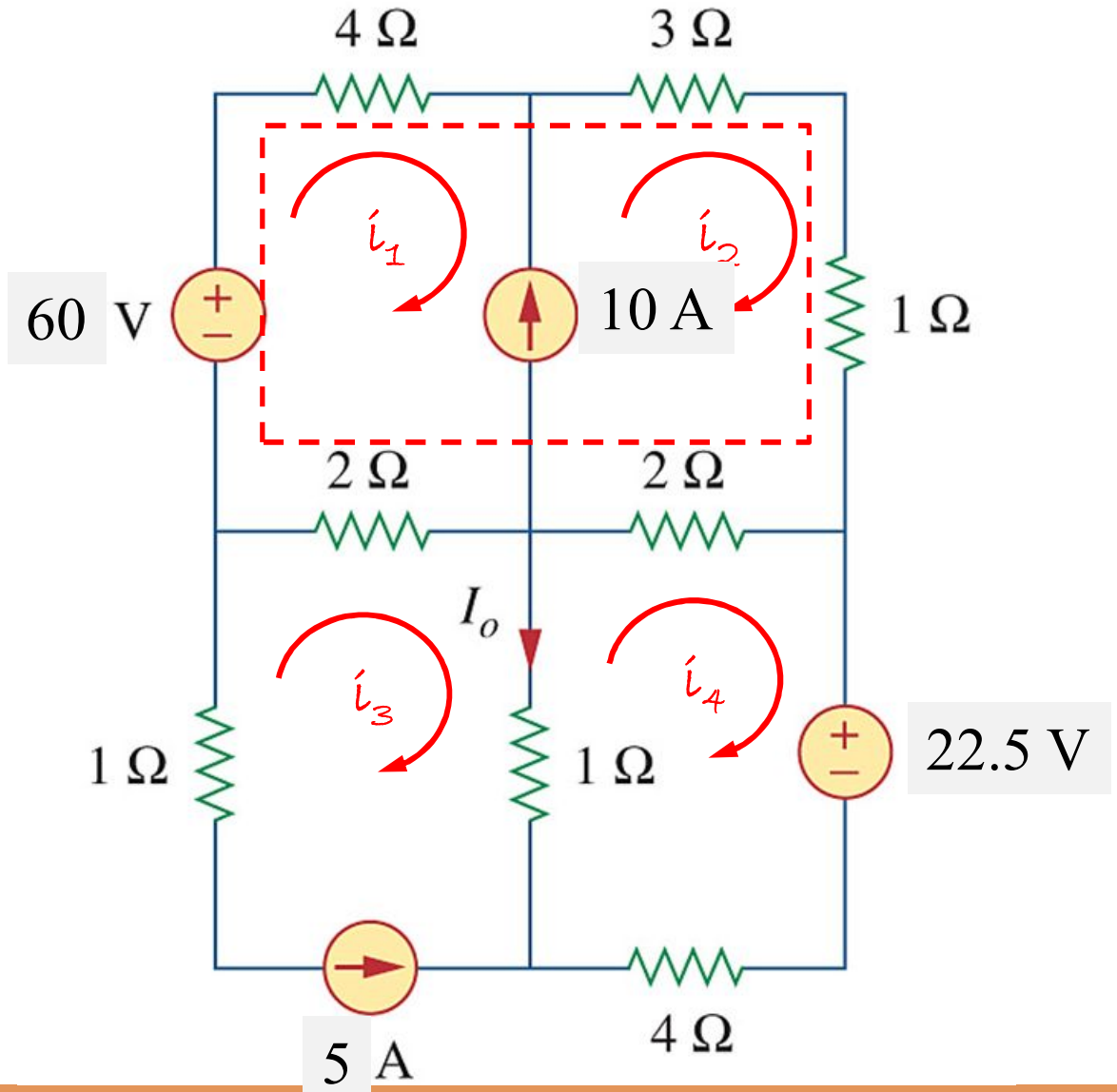


Practice 3.7

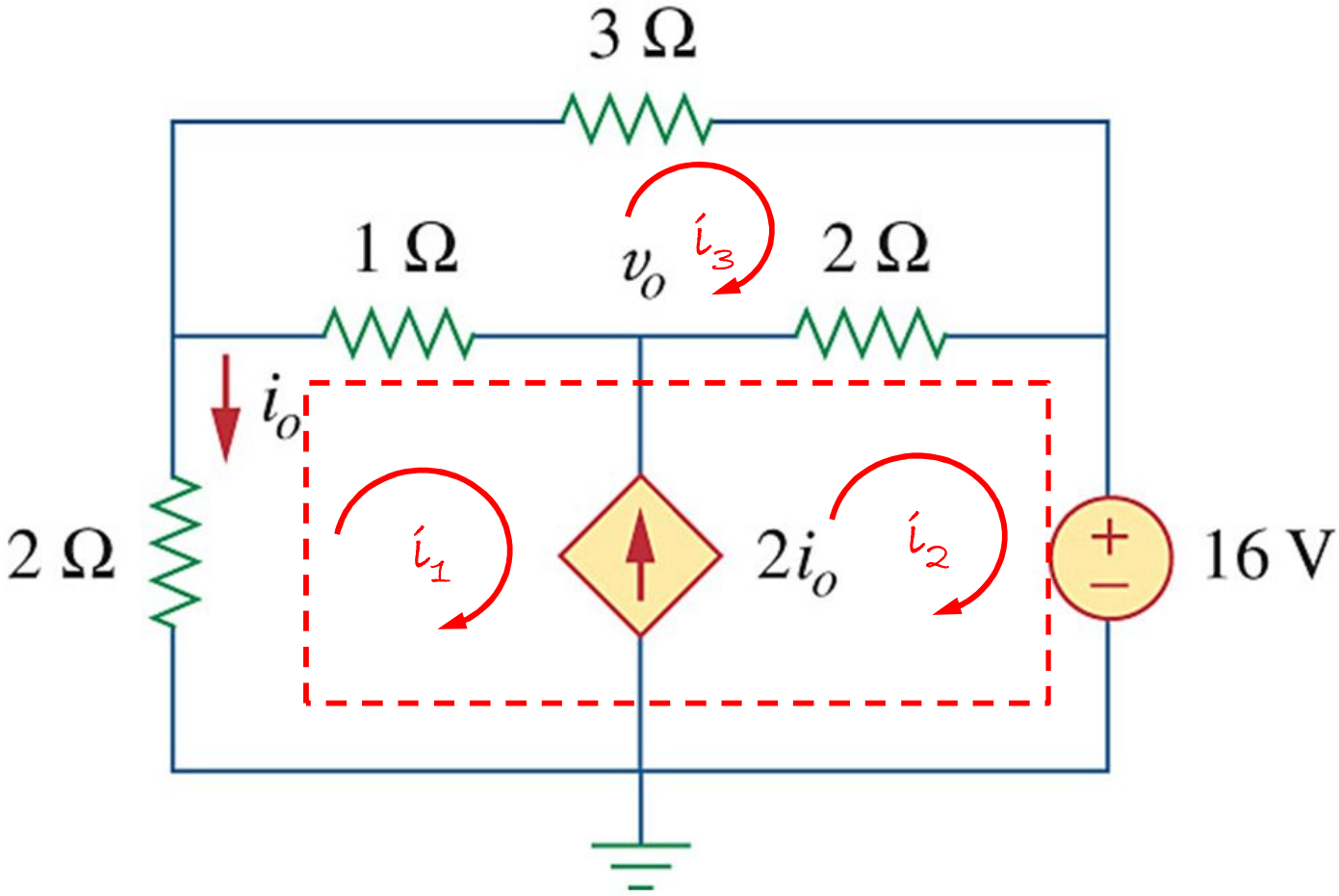
- 메시 전류 i_1, i_2, i_3를 구하라



Problem 3.38

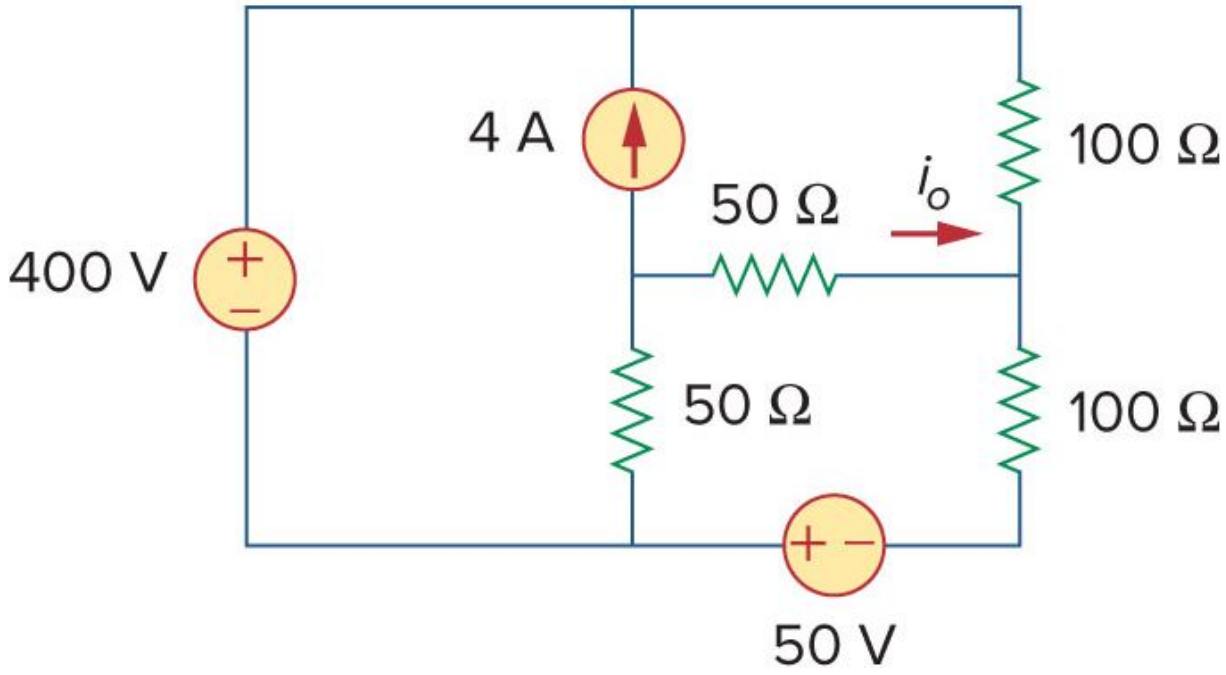


Problem 3.49



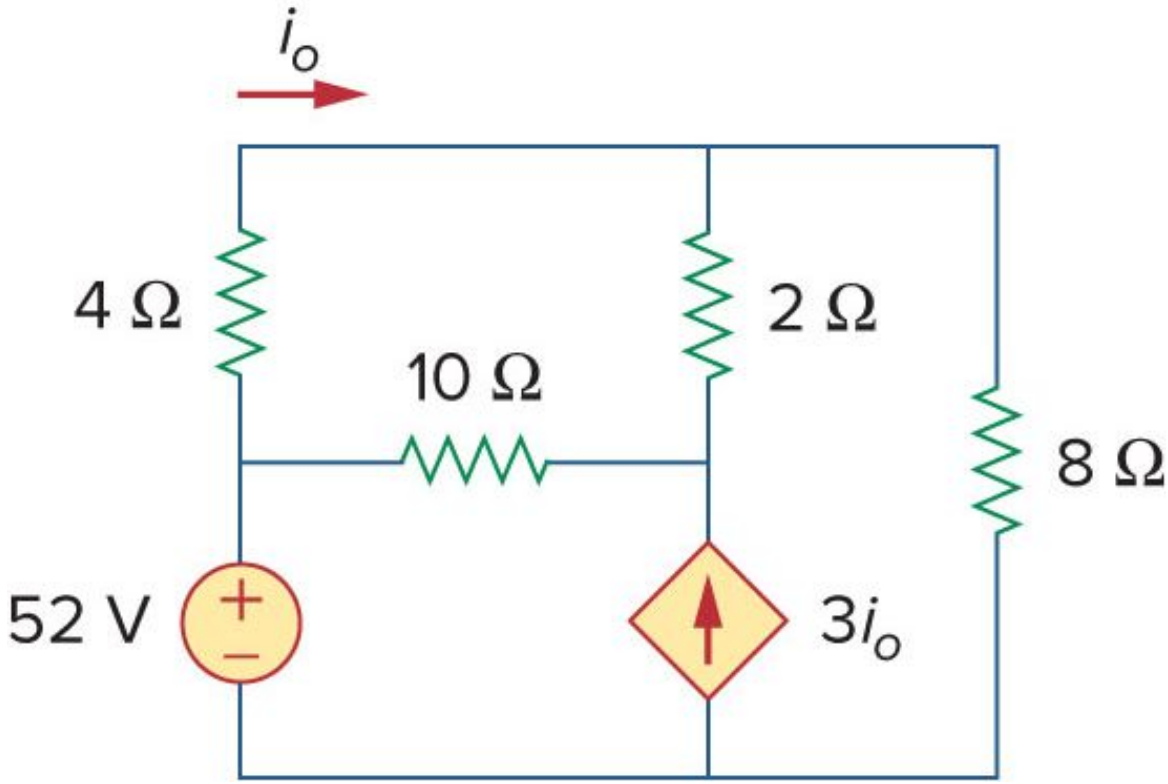
Homework #5

- 다음 수업시간까지 (5문제) : mesh analysis 사용할 것!!!
- #1. Problem 3.44 다음에서 i_0를 구하라



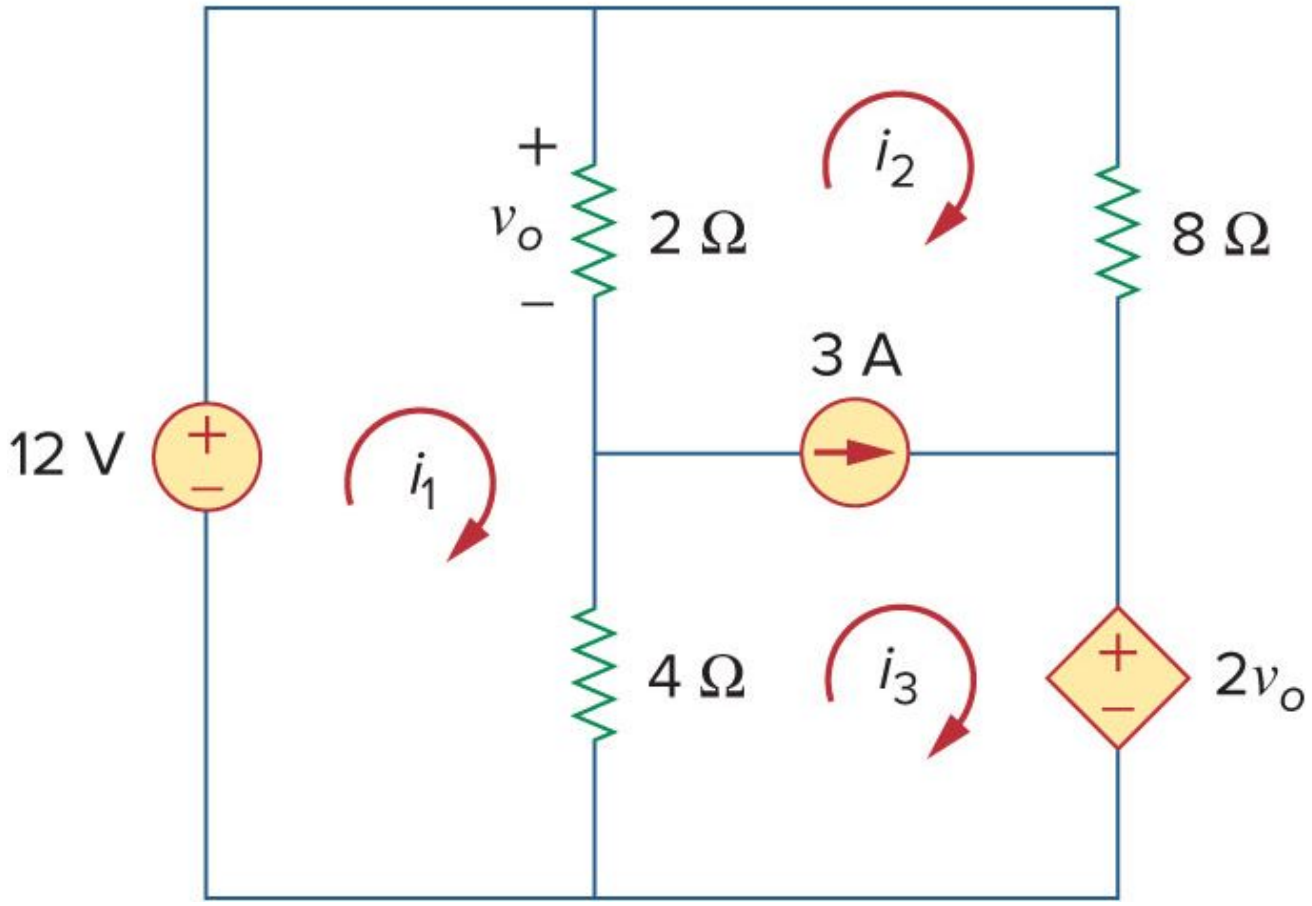
Homework #5

- #2. Problem 3.50 다음에서 i_0를 구하라



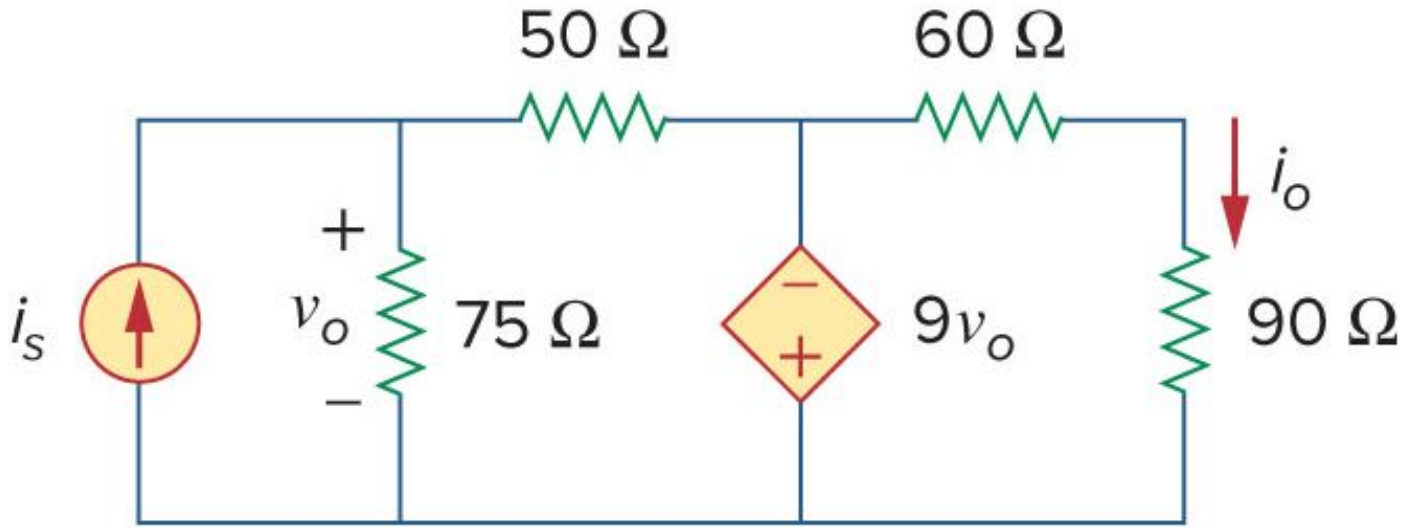
Homework #5

- #3. Problem 3.52 다음에서 i_1, i_2, i_3를 구하라



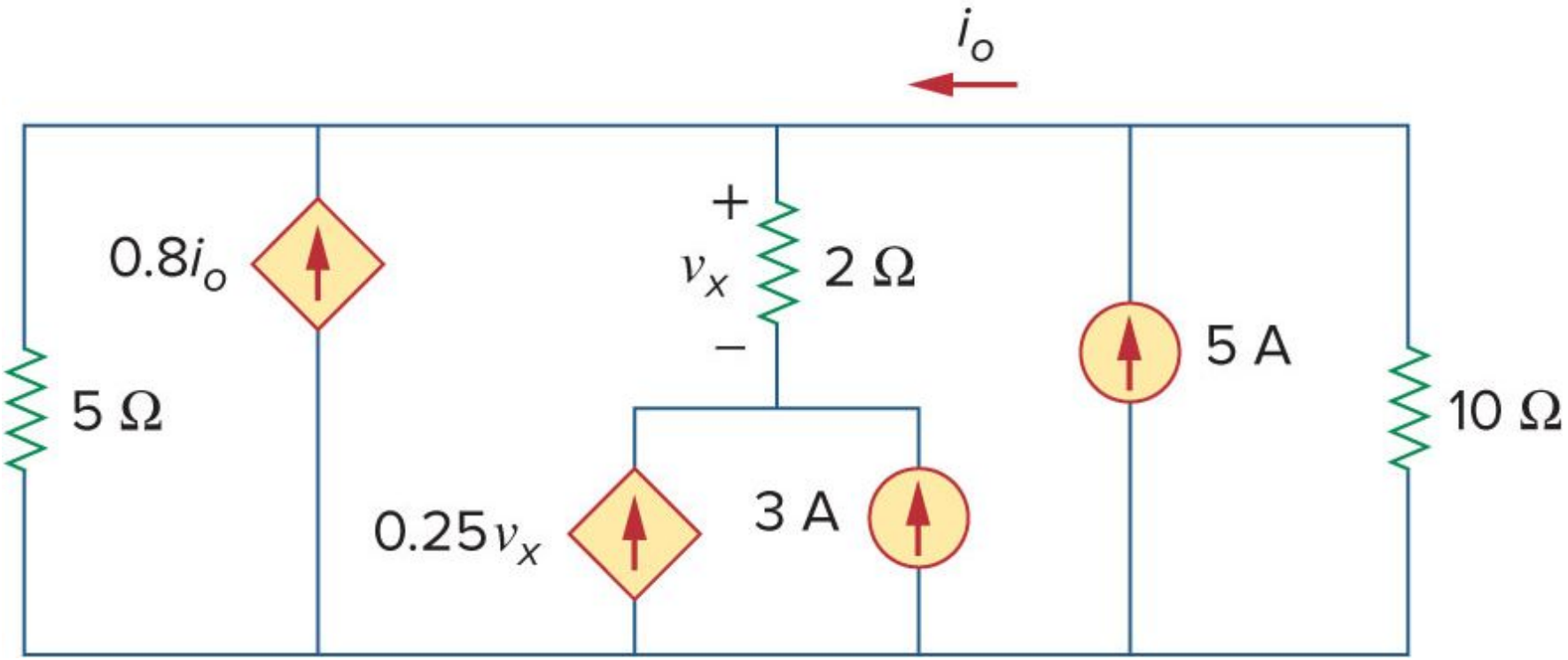
Homework #5

- #4. Problem 3.61 다음 회로에서 전류이득 i_o/i_s를 구하라



Homework #5

- #4. Problem 3.63 다음 회로에서 i_o, v_x를 구하라



3.7 Nodal vs. Mesh Analysis

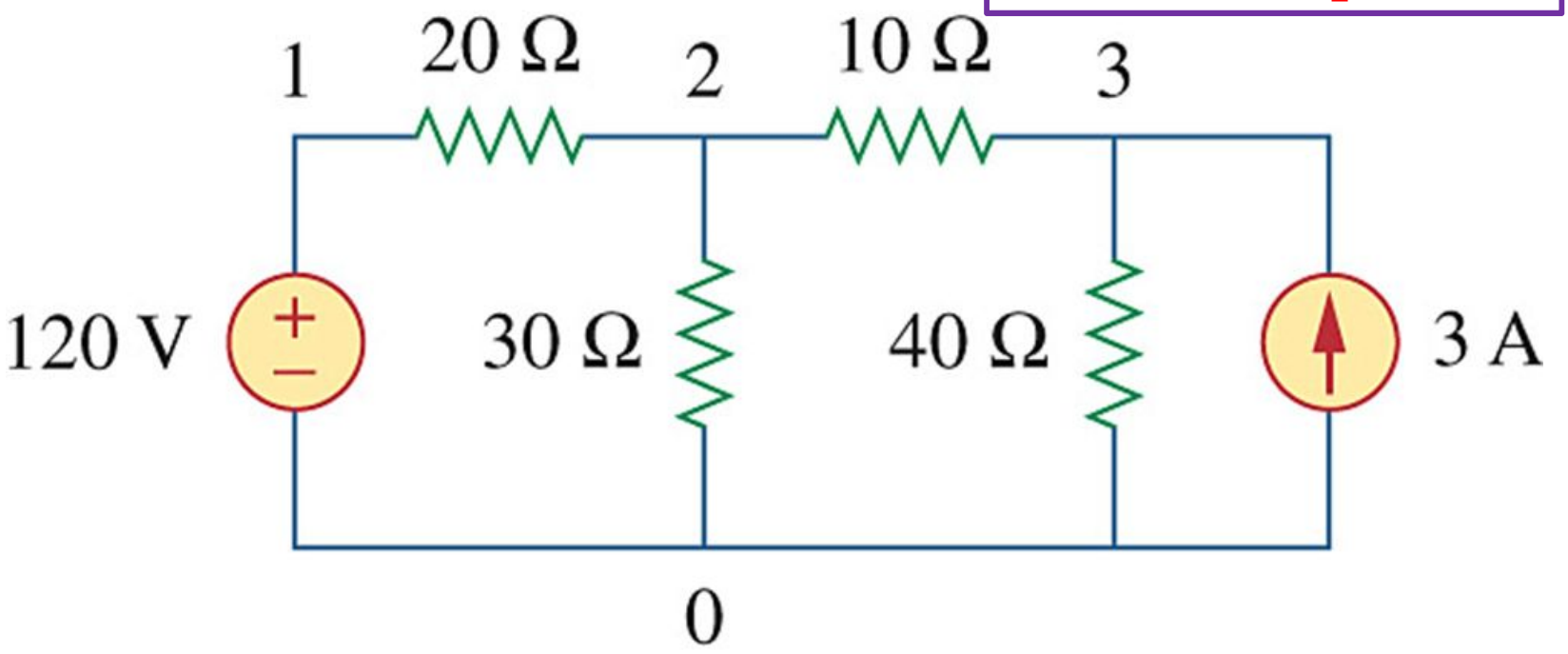
- 어떤 방법을 선택할 것인가?
  - 회로의 형태
    - 직렬 vs. 병렬 ➔
    - 전압원 vs. 전류원 ➔
    - 변수 (또는 식)이 적은 방법 선택
  - 구하자고 하는 정보
    - node 전압 vs. mesh 전류 ➔

- 한 가지 방법만 적용되는 예
  - Transistor 회로 ➔ mesh analysis만 가능
  - Op amp 회로 ➔ nodal analysis만 가능
  - 비평면 회로 ➔ nodal analysis만 가능

3.8 Circuit Analysis with *PSpice*

- pspice student version install!!

전압원 : **VDC, VAC, VSRC**
전류원 : **IDC, IAC, ISRC**
접지 : **AGND**
전류관찰 : **iprobe**
전압관찰 : **viewpoint**



3.8 Circuit Analysis with *PSpice*

VCVS : **E**
CCCS : **F**
VCCS : **G**
CCVS : **H**