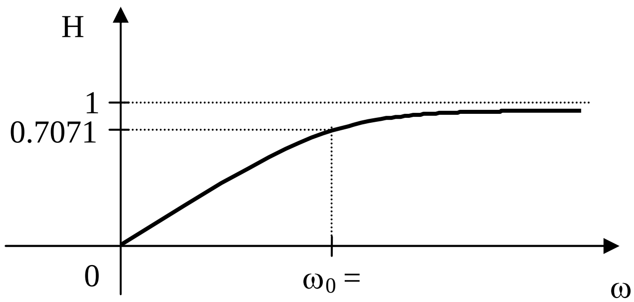
Chapter 14, Solution 1.

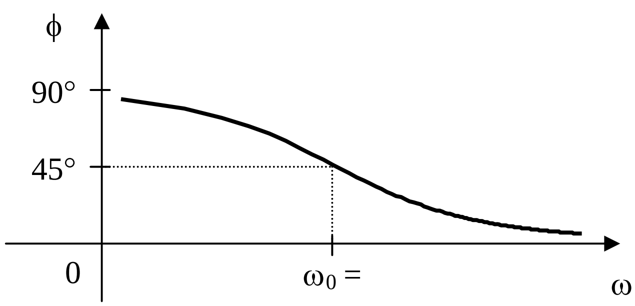
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{j\omega/\omega_0}}{\mathbf{1 + j\omega/\omega_0}}, \qquad \text{where } \boldsymbol{\omega_0} = \frac{\mathbf{1}}{\mathbf{RC}}$$

$$H = \left|\mathbf{H}(\omega)\right| = \frac{\omega/\omega_0}{\sqrt{1 + (\omega/\omega_0)^2}} \qquad \phi = \angle\mathbf{H}(\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

This is a highpass filter. The frequency response is the same as that for P.P.14.1 except that $\omega_0 = 1/RC$. Thus, the sketches of H and ϕ are shown below.





Chapter 14, Solution 2.

Using Fig. 14.69, design a problem to help other students to better understand how to determine transfer functions.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain the transfer function $\mathbf{V}_o/\mathbf{V}_i$ of the circuit in Fig. 14.66.

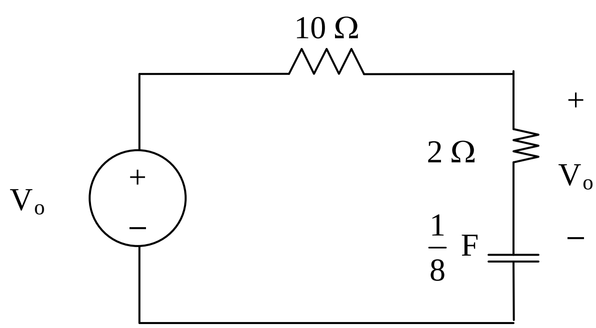


Figure 14.66 For Prob. 14.2.

Solution

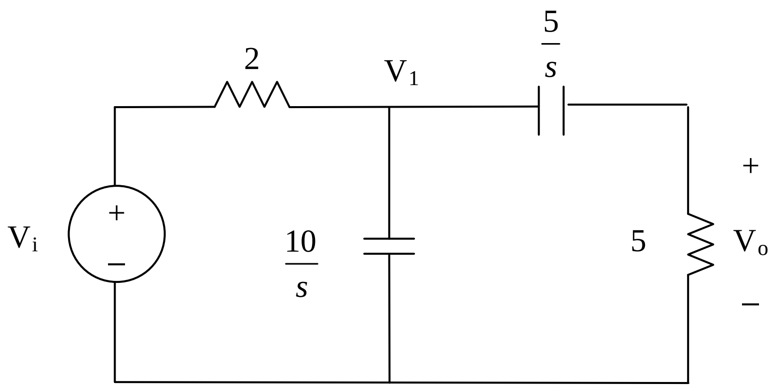
$$H(s) = \frac{V_o}{V_i} = \frac{2 + \frac{1}{s/8}}{10 + 20 + \frac{1}{s/8}} = \frac{2 + 8/s}{12 + 8/s} = \frac{1}{6}\,\frac{s+4}{s+0.6667}$$

Chapter 14, Solution 3.

$$0.2F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{s(0.2)} = \frac{5}{s}$$

$$0.1F \quad \longrightarrow \quad \frac{1}{s(0.1)} = \frac{10}{s}$$

The circuit becomes that shown below.



$$\text{Let } \; Z = \frac{10}{s} // (5 + \frac{5}{s}) = \frac{\frac{10}{s}(5+\frac{5}{s})}{5+\frac{15}{s}} = \frac{\frac{10}{s}5(\frac{1+s}{s})}{\frac{5}{s}(3+s)} = \frac{10(s+1)}{s(s+3)}$$

$$V_1 = \frac{Z}{Z+2} V_i$$

$$V_o = \frac{5}{5+5/s} V_1 = \frac{s}{s+1} V_1 = \frac{s}{s+1} \bullet \frac{Z}{Z+2} V_i$$

$$H(s) = V_o / V_i = \frac{s}{s+1} \bullet \frac{\frac{10(s+1)}{s(s+3)}}{2+\frac{10(s+1)}{s(s+3)}} = \frac{10s}{2s(s+3)+10(s+1)} = \underline{\frac{5s}{s^2+8s+5}}$$

H(s) = **5s/(s^2+8s+5)**

Chapter 14, Solution 4.

(a) $$R \parallel \frac{1}{j\omega C} = \frac{R}{1 + j\omega RC}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{R}{1 + j\omega RC}}{j\omega L + \dfrac{R}{1 + j\omega RC}} = \frac{R}{R + j\omega L\,(1 + j\omega RC)}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{R}}{\mathbf{-\,\omega^2 RLC + R + j\omega L}}$$

(b) $$\mathbf{H}(\omega) = \frac{R + j\omega L}{R + j\omega L + 1/j\omega C} = \frac{j\omega C\,(R + j\omega L)}{1 + j\omega C\,(R + j\omega L)}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{-\,\omega^2 LC + j\omega RC}}{\mathbf{1 - \omega^2 LC + j\omega RC}}$$

Chapter 14, Solution 5.

(a) Let $Z = R \,//\, sL = \dfrac{sRL}{R + sL}$

$$V_o = \frac{Z}{Z + R_s} V_s$$

$$H(s) = \frac{V_o}{V_s} = \frac{Z}{Z + R_s} = \frac{\dfrac{sRL}{R + sL}}{R_s + \dfrac{sRL}{R + sL}} = \underline{\frac{sRL}{RR_s + s(R + R_s)L}}$$

(b) Let $Z = R \,//\, \dfrac{1}{sC} = \dfrac{Rx\dfrac{1}{sC}}{R + \dfrac{1}{sC}} = \dfrac{R}{1 + sRC}$

$$V_o = \frac{Z}{Z + sL} V_s$$

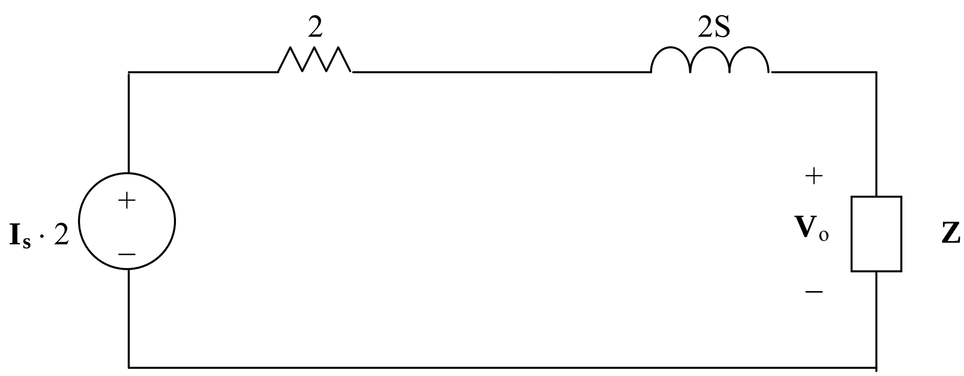
$$\mathrm{H(s)} = \frac{\mathrm{V_o}}{\mathrm{V_i}} = \frac{\mathrm{Z}}{\mathrm{Z + sL}} = \frac{\dfrac{\mathrm{R}}{1 + \mathrm{sRC}}}{\mathrm{sL} + \dfrac{\mathrm{R}}{1 + \mathrm{sRC}}} = \frac{\mathrm{R}}{\mathrm{s^2LRC + sL + R}}$$

Chapter 14, Solution 6.

The 2 H inductors become jω2 or 2s.

Let **Z** = 2s||2 = [(2s)(2)/(2s+2)] = 2s/(s+1)

We convert the current source to a voltage source as shown below.



$$\mathbf{V}_o = [(Z)/(Z+2s+2)](2I_s) = \frac{\frac{2s}{s+1}}{\frac{2s+2s^2+2s+2s+2}{s+1}}(2I_s) = \frac{2s}{s^2+3s+1}I_s$$ or

H(s) = I_o/I_s = **[2s/(s^2+3s+1)]**.

Chapter 14, Solution 7.

(a) $0.05 = 20\log_{10} H$

$2.5\times10^{-3} = \log_{10} H$

$H = 10^{2.5\times10^{-3}} = \mathbf{1.005773}$

(b) $-6.2 = 20\log_{10} H$

$-0.31 = \log_{10} H$

$H = 10^{-0.31} = \mathbf{0.4898}$

(c) $104.7 = 20\log_{10} H$

$5.235 = \log_{10} H$

$H = 10^{5.235} = \mathbf{1.718\times10^{5}}$

Chapter 14, Solution 8.

Design a problem to help other students to better calculate the magnitude in dB and phase in degrees of a variety of transfer functions at a single value of ω.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the magnitude (in dB) and the phase (in degrees) of $\mathbf{H}(\omega)$ at $\omega = 1$ if $\mathbf{H}(\omega)$ equals

(a) 0.05
(b) 125
(c) $\dfrac{10j\omega}{2+j\omega}$
(d) $\dfrac{3}{1+j\omega} + \dfrac{6}{2+j\omega}$

Solution

(a) $H = 0.05$
$H_{dB} = 20\log_{10} 0.05 = \mathbf{-26.02}$, $\quad \varphi = \mathbf{0°}$

(b) $H = 125$
$H_{dB} = 20\log_{10} 125 = \mathbf{41.94}$, $\quad \varphi = \mathbf{0°}$

(c) $H(1) = \dfrac{j10}{2+j} = 4.472\angle 63.43°$
$H_{dB} = 20\log_{10} 4.472 = \mathbf{13.01}$, $\quad \varphi = \mathbf{63.43°}$

(d) $H(1) = \dfrac{3}{1+j} + \dfrac{6}{2+j} = 3.9 - j2.7 = 4.743\angle -34.7°$
$H_{dB} = 20\log_{10} 4.743 = \mathbf{13.521}$, $\quad \varphi = \mathbf{-34.7°}$

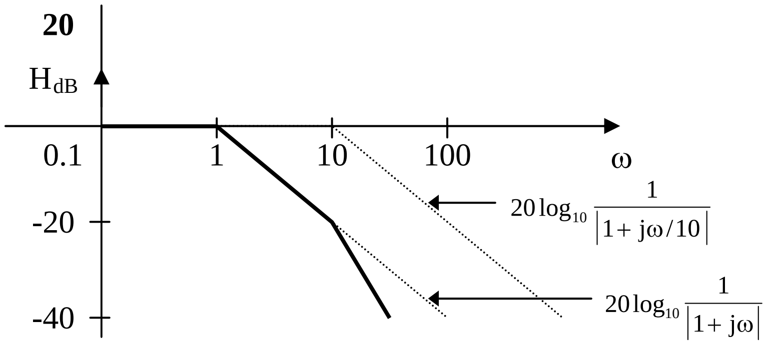
Chapter 14, Solution 9.

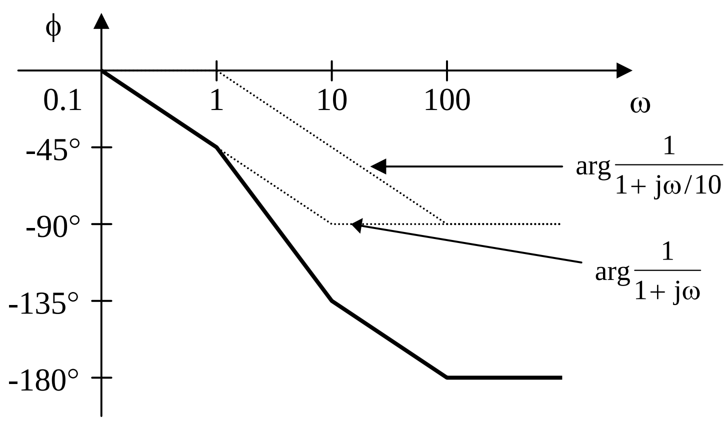
$$\mathbf{H}(\omega) = \frac{10}{10(1+j\omega)(1+j\omega/10)}$$

$$H_{dB} = 20\log_{10}|1| - 20\log_{10}|1+j\omega| - 20\log_{10}|1+j\omega/10|$$

$$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/10)$$

<u>The magnitude and phase plots are shown below</u>.





Chapter 14, Solution 10.

Design a problem to help other students to better understand how to determine the Bode magnitude and phase plots of a given transfer function in terms of jω.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

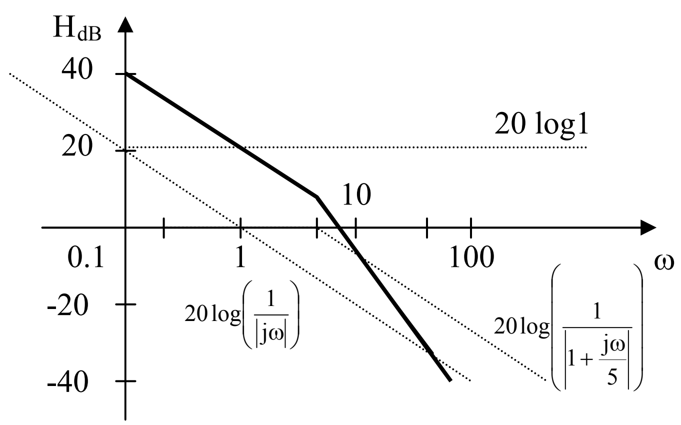
Problem

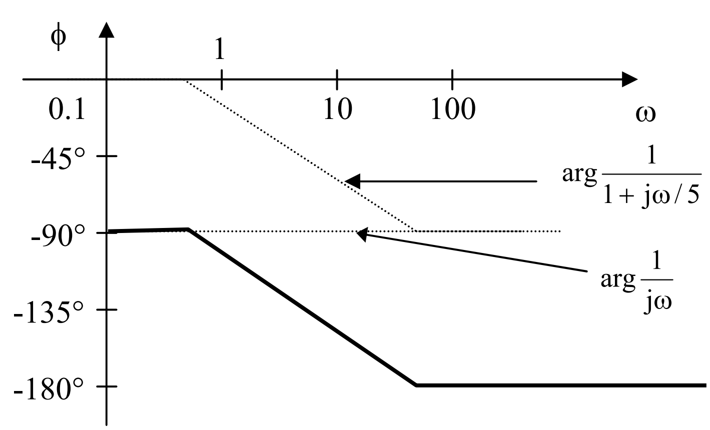
Sketch the Bode magnitude and phase plots of:

$$H(j\omega) = \frac{50}{j\omega(5 + j\omega)}$$

Solution

$$H(j\omega) = \frac{50}{j\omega(5 + j\omega)} = \frac{10}{1 j\omega\left(1 + \frac{j\omega}{5}\right)}$$





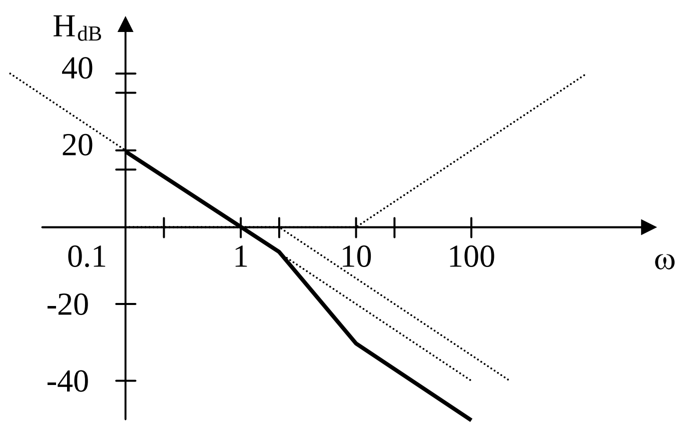
Chapter 14, Solution 11.

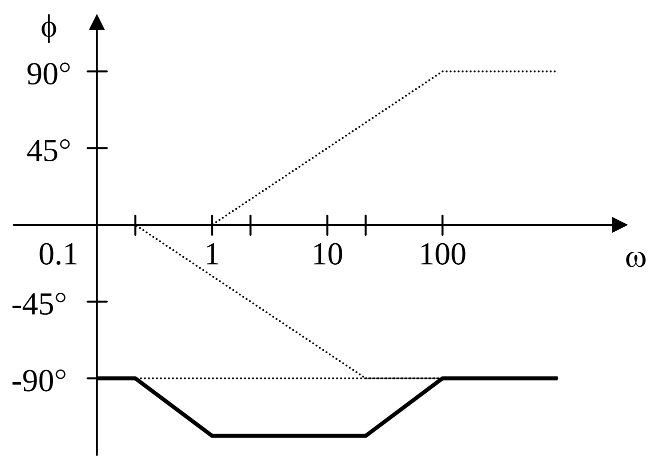
$$\mathbf{H}(\omega) = \frac{0.2x10(1 + j\omega/10)}{2[j\omega(1 + j\omega/2)]}$$

$$H_{dB} = 20\log_{10} 1 + 20\log_{10}\left|1 + j\omega/10\right| - 20\log_{10}\left|j\omega\right| - 20\log_{10}\left|1 + j\omega/2\right|$$

$$\phi = -90° + \tan^{-1}\omega/10 - \tan^{-1}\omega/2$$

The magnitude and phase plots are shown below.



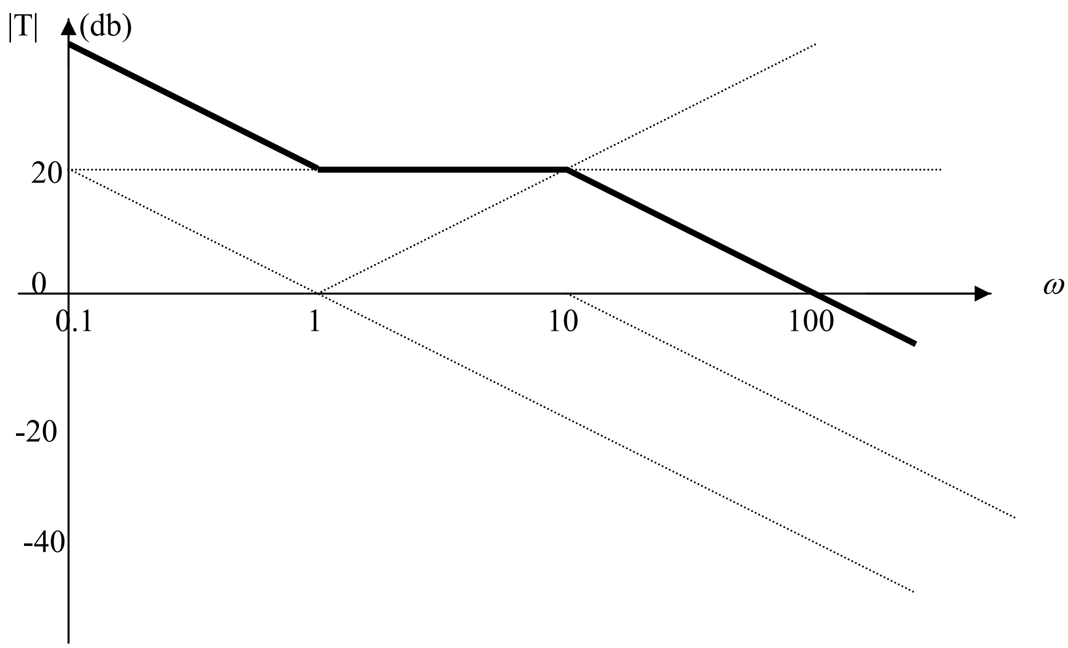


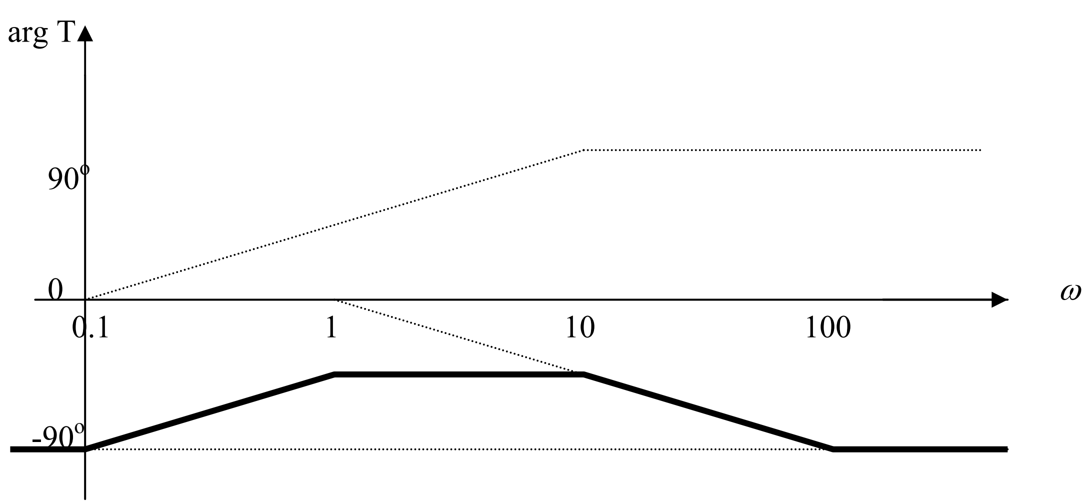
Chapter 14, Solution 12.

$$T(\omega) = \frac{10(1+j\omega)}{j\omega(1+j\omega/10)}$$

To sketch this we need $20\log_{10}|T(\omega)| = 20\log_{10}|10| + 20\log_{10}|1+j\omega| - 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega/10|$ and the phase is equal to $\tan^{-1}(\omega) - 90° - \tan^{-1}(\omega/10)$.

The plots are shown below.





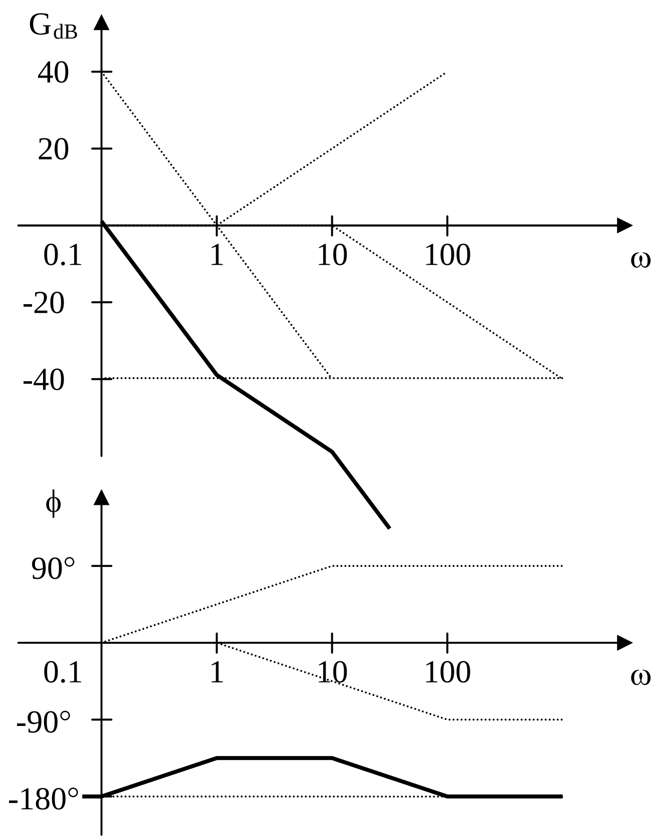
Chapter 14, Solution 13.

$$\mathbf{G}(\omega) = \frac{0.1(1 + j\omega)}{(j\omega)^2 (10 + j\omega)} = \frac{(1/100)(1 + j\omega)}{(j\omega)^2 (1 + j\omega/10)}$$

$$G_{dB} = -40 + 20\log_{10}\left|1 + j\omega\right| - 40\log_{10}\left|j\omega\right| - 20\log_{10}\left|1 + j\omega/10\right|$$

$$\phi = -180° + \tan^{-1}\omega - \tan^{-1}\omega/10$$

The magnitude and phase plots are shown below.



Chapter 14, Solution 14.

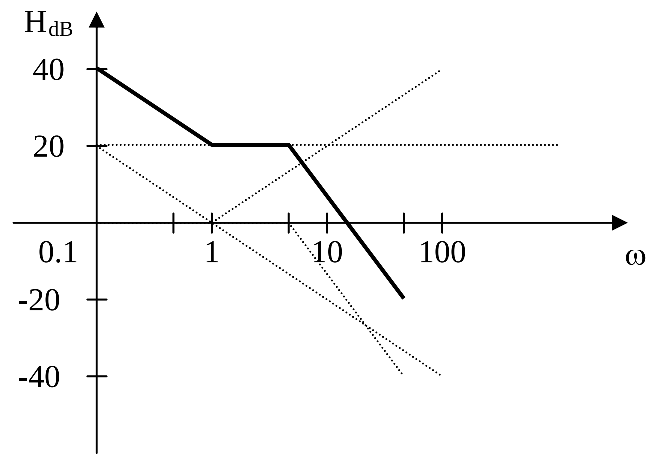
$$\mathbf{H}(\omega)=\frac{250}{25}\frac{1+j\omega}{j\omega\left(1+\frac{j\omega 10}{25}+\left(\frac{j\omega}{5}\right)^2\right)}$$

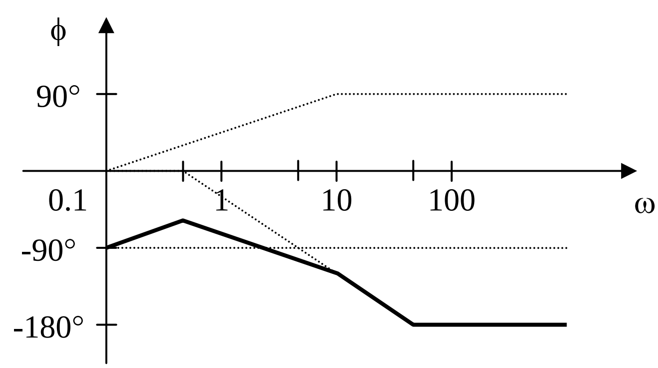
$$H_{dB}=20\log_{10}10+20\log_{10}|1+j\omega|-20\log_{10}|j\omega|$$

$$-20\log_{10}\left|1+\mathrm{j}\omega 2/5+(\mathrm{j}\omega/5)^2\right|$$

$$\phi=\text{-}90°+\tan^{-1}\omega-\tan^{-1}\left(\frac{\omega 10/25}{1-\omega^2/5}\right)$$

The magnitude and phase plots are shown below.



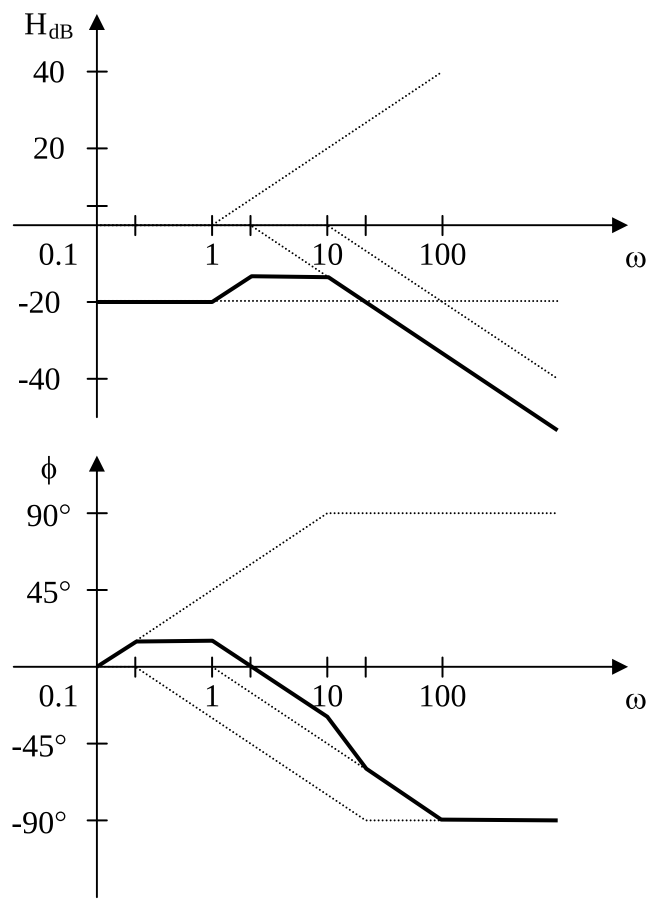


Chapter 14, Solution 15.

$$\mathbf{H}(\omega) = \frac{2(1 + j\omega)}{(2 + j\omega)(10 + j\omega)} = \frac{0.1(1 + j\omega)}{(1 + j\omega/2)(1 + j\omega/10)}$$

$$H_{dB} = 20\log_{10} 0.1 + 20\log_{10}|1 + j\omega| - 20\log_{10}|1 + j\omega/2| - 20\log_{10}|1 + j\omega/10|$$
$$\phi = \tan^{-1}\omega - \tan^{-1}\omega/2 - \tan^{-1}\omega/10$$

The magnitude and phase plots are shown below.

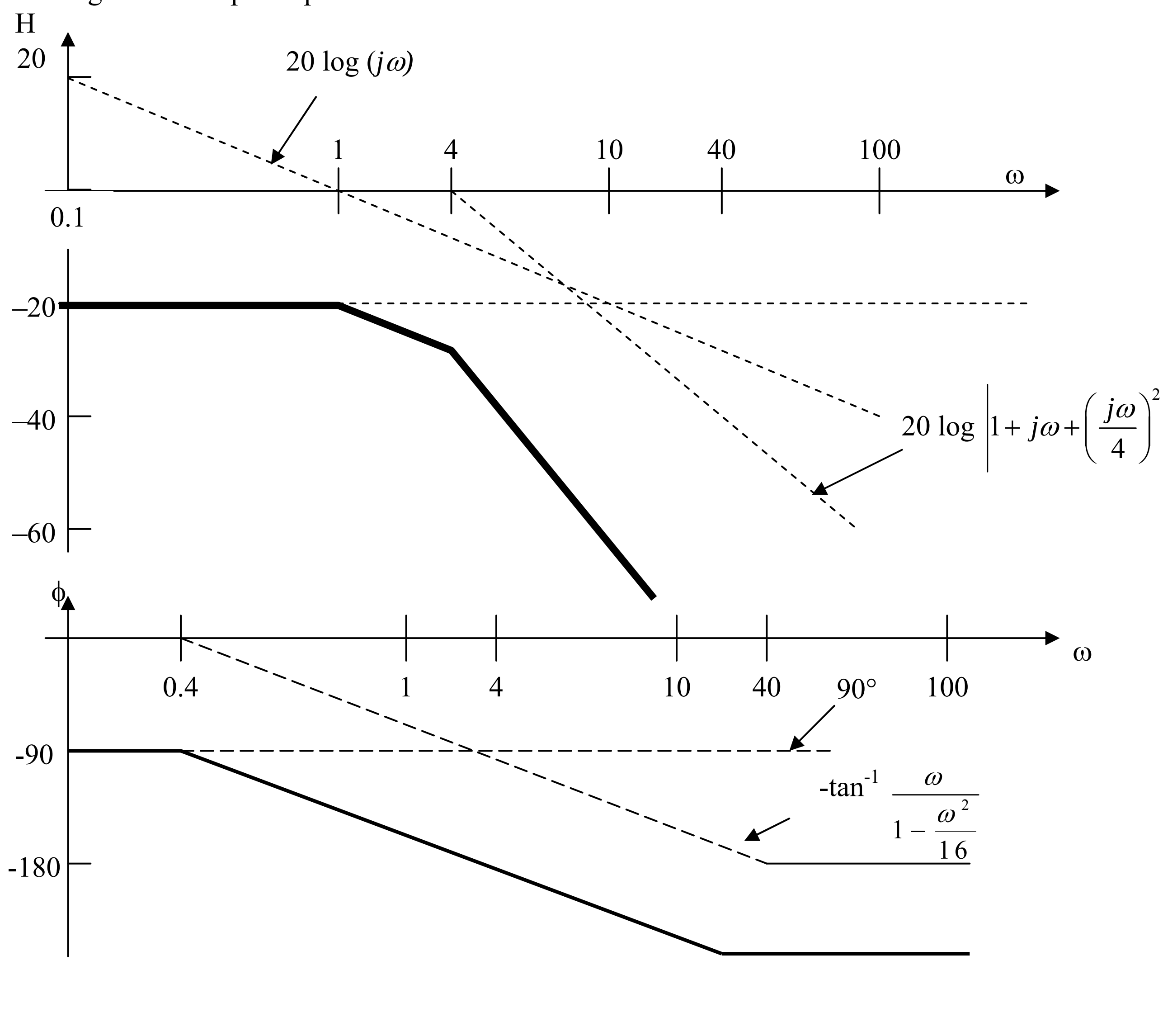


Chapter 14, Solution 16.

$$H(\omega) = \frac{\frac{1.6}{16}}{j\omega\left[1 + j\omega + \left(\frac{j\omega}{4}\right)^2\right]} = \frac{0.1}{j\omega\left[1 + j\omega + \left(\frac{j\omega}{4}\right)^2\right]}$$

$H_{db} = 20\log_{10}|0.1| - 20\text{olg}_{10}|j\omega| - 20\log_{10}|1+j\omega+(j\omega/4)^2|$

The magnitude and phase plots are shown below.



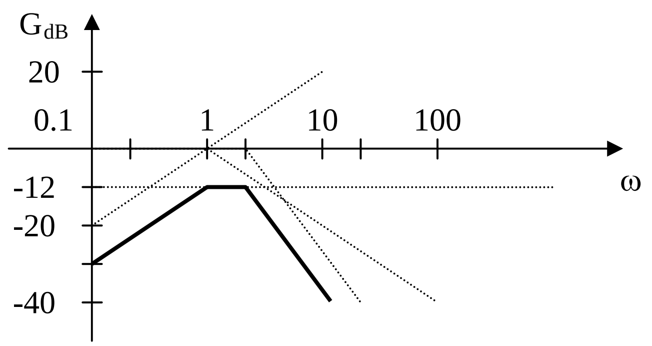
Chapter 14, Solution 17.

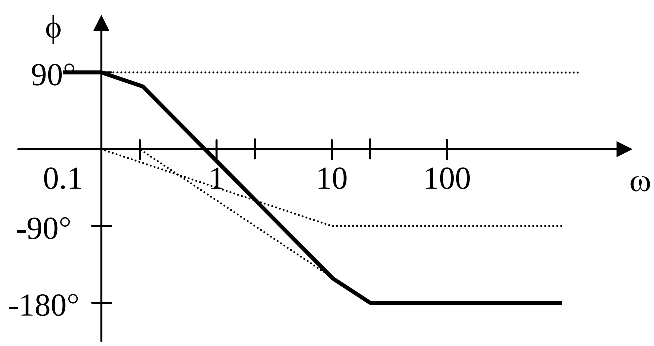
$$\mathbf{G}(\omega) = \frac{(1/4)\, j\omega}{(1+j\omega)(1+j\omega/2)^2}$$

$$G_{dB} = -20\log_{10} 4 + 20\log_{10}|\,j\omega| - 20\log_{10}|1+j\omega| - 40\log_{10}|1+j\omega/2|$$

$$\phi = -90° - \tan^{-1}\omega - 2\tan^{-1}\omega/2$$

The magnitude and phase plots are shown below.

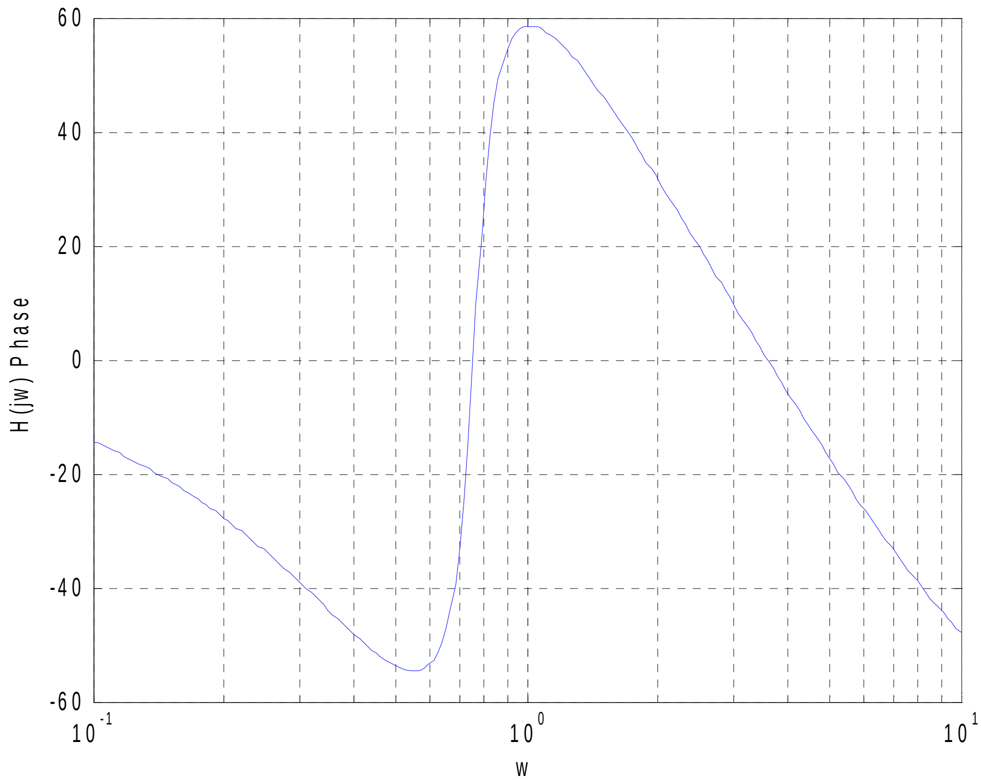




Chapter 14, Solution 18.

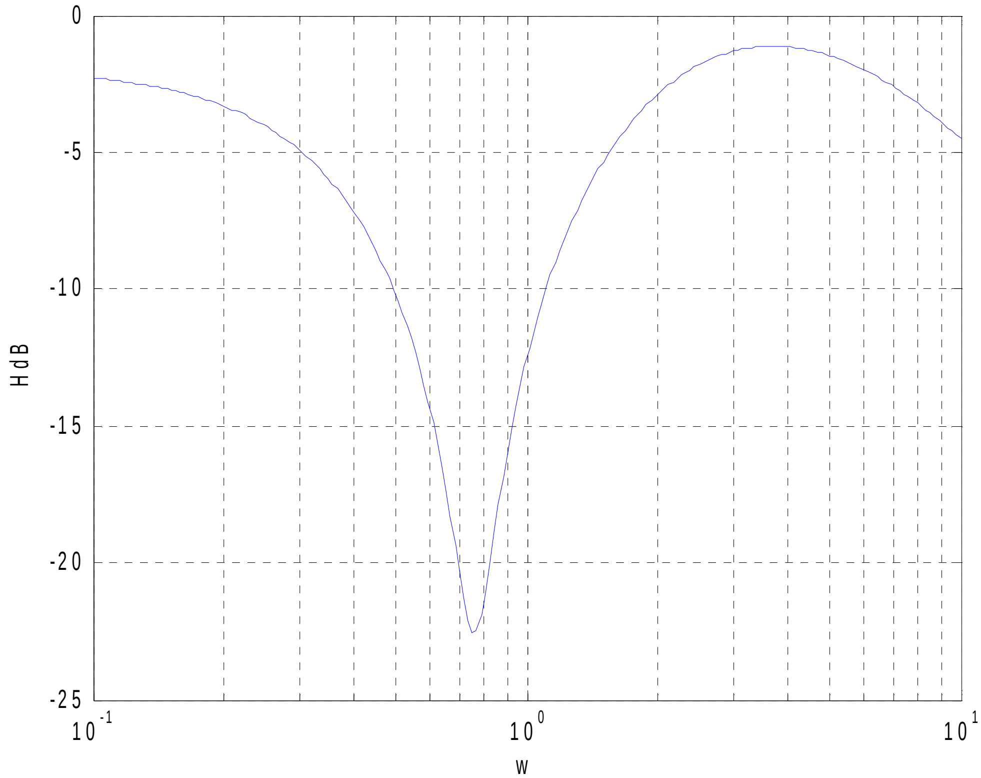
The MATLAB code is shown below.

```
>> w=logspace(-1,1,200);
>> s=i*w;
>> h=(7*s.^2+s+4)./(s.^3+8*s.^2+14*s+5);
>> Phase=unwrap(angle(h))*57.23;
>> semilogx(w,Phase)
>> grid on
```



Now for the magnitude, we need to add the following to the above,

```
>> H=abs(h);
>> HdB=20*log10(H);
>> semilogx(w,HdB);
>> grid on
```


0
-5
-10
-15
-20
-25
H dB
10^{-1}
10^{0}
10^{1}
w

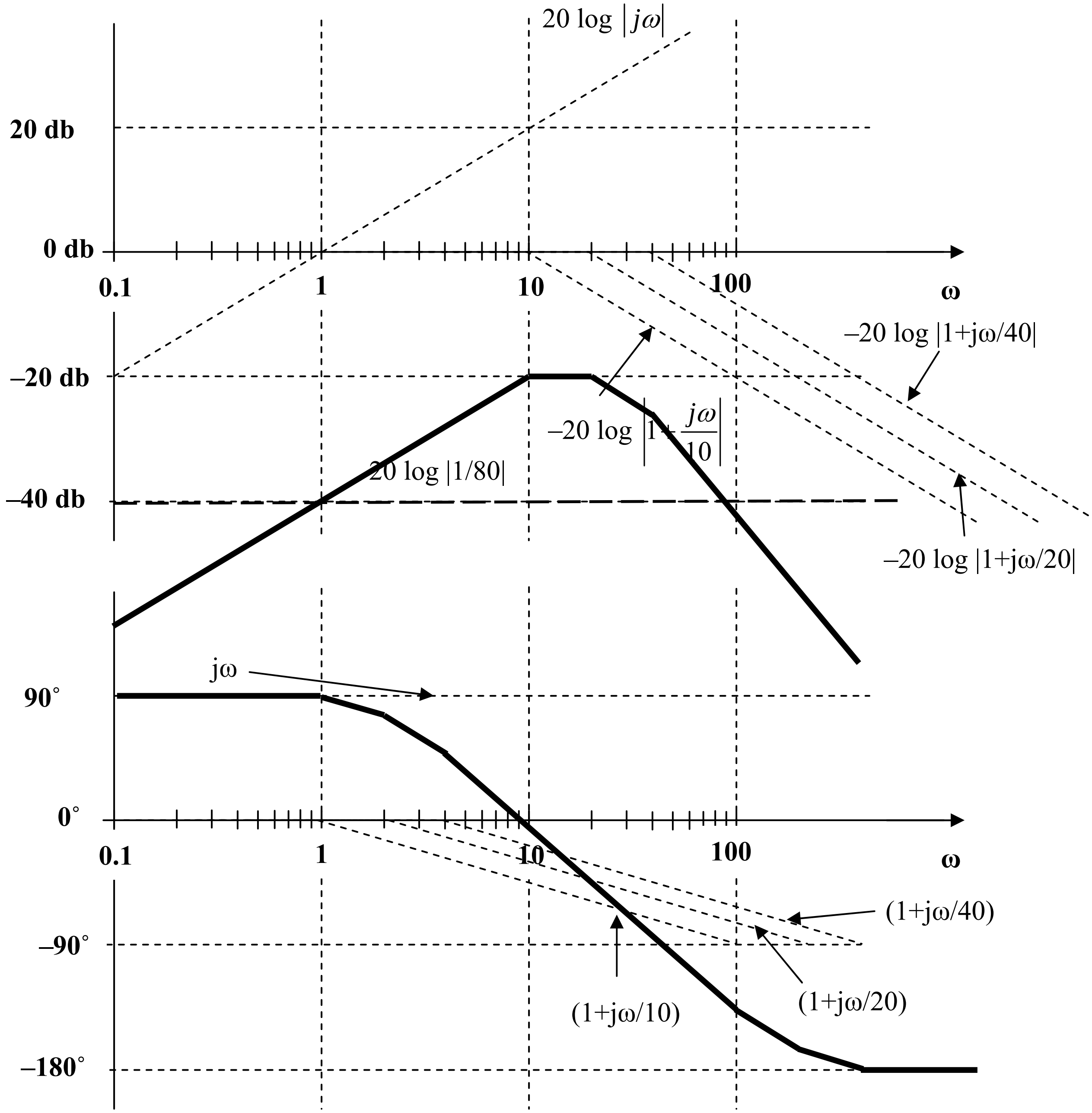
Chapter 14, Solution 19.

$H(\omega) = 80j\omega/[(10+j\omega)(20+j\omega)(40+j\omega)]$

$= [80/(10x20x40)](j\omega)/[(1+j\omega/10)(1+j\omega/20)(1+j\omega/40)]$

$H_{db} = 20\log_{10}|0.01| + 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega/10| - 20\log_{10}|1+j\omega/20| - 20\log_{10}|1+j\omega/40|$

The magnitude and phase plots are shown below.



Chapter 14, Solution 20.

Design a more complex problem than given in Prob. 14.10, to help other students to better understand how to determine the Bode magnitude and phase plots of a given transfer function in terms of jω. Include at least a second order repeated root.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Sketch the magnitude phase Bode plot for the transfer function

$$H(\omega) = \frac{25j\omega}{(j\omega+1)(j\omega+5)^2(j\omega+10)}$$

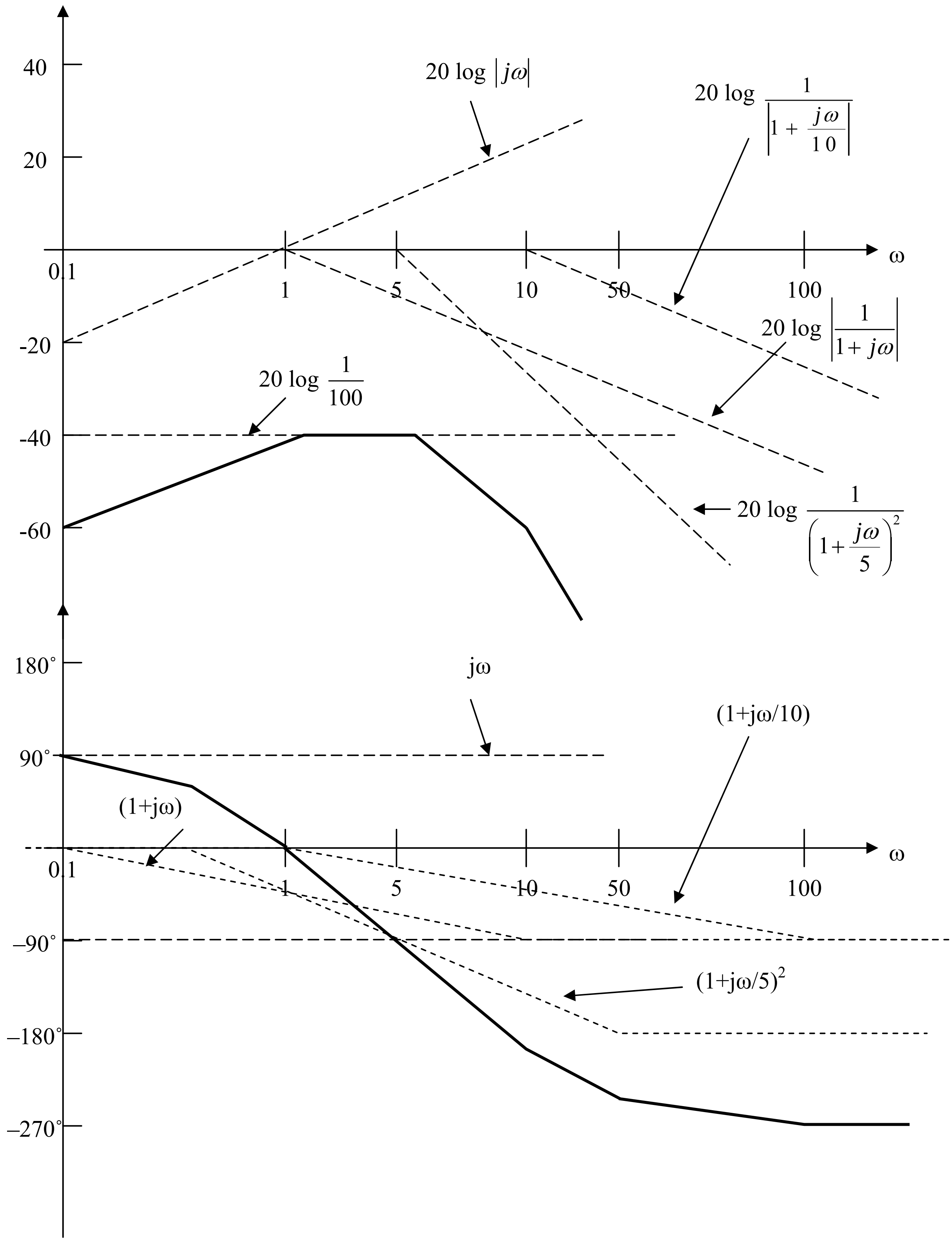
Solution

$$H(\omega) = \frac{\left(\frac{1}{100}\right)j\omega}{(1+j\omega)\left(1+\frac{j\omega}{5}\right)^2\left(1+\frac{j\omega}{10}\right)}$$

20log(1/100) = -40

For the plots, see the next page.

The magnitude and phase plots are shown below.



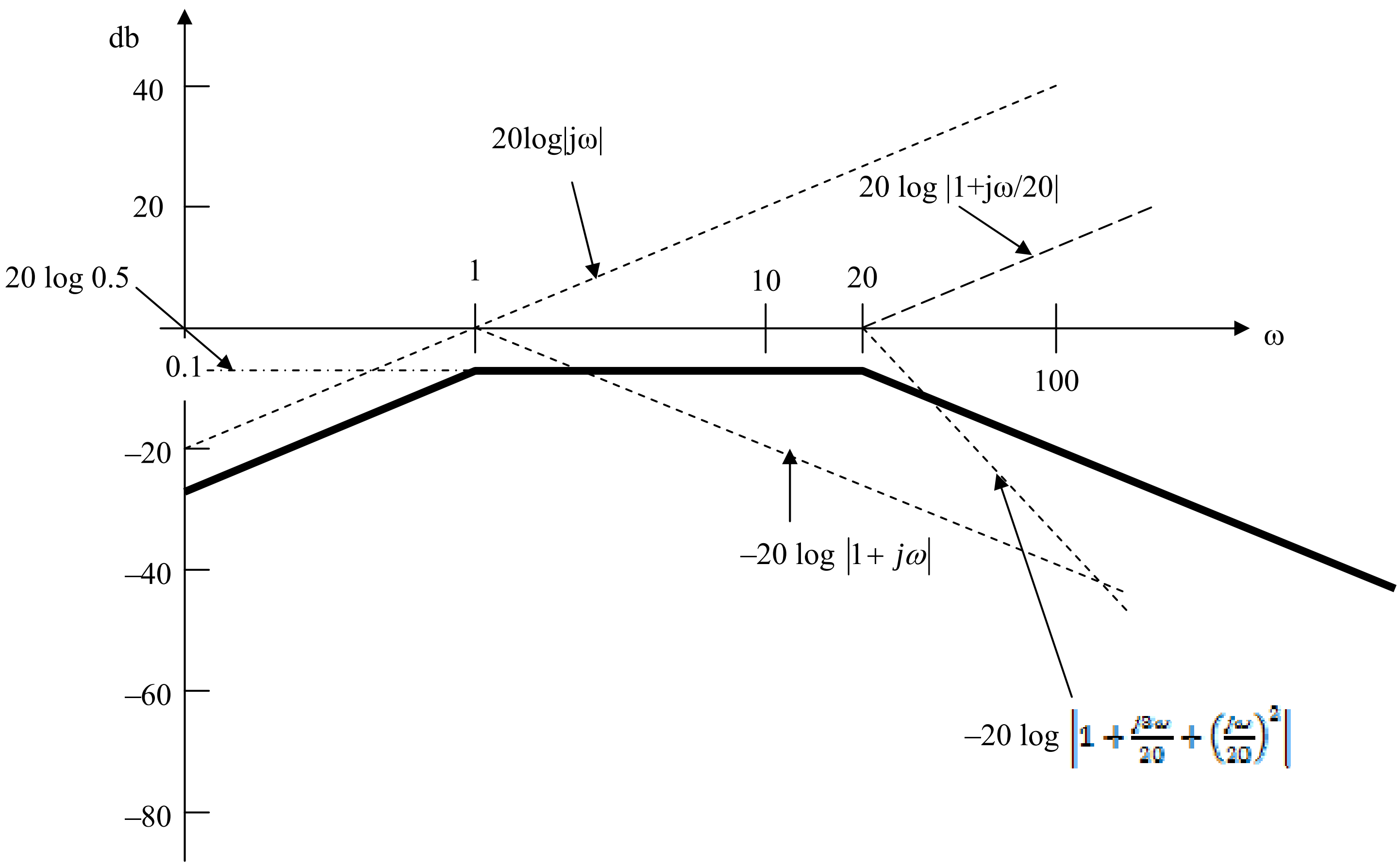
Chapter 14, Solution 21.

$H(\omega) = 10(j\omega)(20+j\omega)/[(1+j\omega)(400+60j\omega-\omega^2)]$

$= [10x20/400](j\omega)(1+j\omega/20)/[(1+j\omega)(1+(3j\omega/20)+(j\omega/20)^2)]$

$$H_{dB} = 20\log(0.5) + 20\log|j\omega| + 20\log\left|1+\frac{j\omega}{20}\right| - 20\log|1+j\omega| - 20\log\left|1+\frac{j3\omega}{20}+\left(\frac{j\omega}{20}\right)^2\right|$$

The magnitude plot is as sketched below. $20\log_{10}|0.5| = -6$ db



Chapter 14, Solution 22.

$$20 = 20\log_{10} k \quad \longrightarrow \quad k = 10$$

A zero of slope $+20\ \text{dB/dec}$ at $\omega = 2 \quad \longrightarrow \quad 1 + j\omega/2$

A pole of slope $-20\ \text{dB/dec}$ at $\omega = 20 \quad \longrightarrow \quad \dfrac{1}{1 + j\omega/20}$

A pole of slope $-20\ \text{dB/dec}$ at $\omega = 100 \quad \longrightarrow \quad \dfrac{1}{1 + j\omega/100}$

Hence,

$$\mathbf{H}(\omega) = \frac{10(1 + j\omega/2)}{(1 + j\omega/20)(1 + j\omega/100)}$$

$$\mathbf{H}(\omega) = \boldsymbol{\frac{10^4\,(2 + j\omega)}{(20 + j\omega)(100 + j\omega)}}$$

Chapter 14, Solution 23.

A zero of slope $+20\ \text{dB/dec}$ at the origin $\longrightarrow$ $j\omega$

A pole of slope $-20\ \text{dB/dec}$ at $\omega = 1$ $\longrightarrow$ $\dfrac{1}{1 + j\omega/1}$

A pole of slope $-40\ \text{dB/dec}$ at $\omega = 10$ $\longrightarrow$ $\dfrac{1}{(1 + j\omega/10)^2}$

Hence,

$$\mathbf{H}(\omega) = \frac{j\omega}{(1 + j\omega)(1 + j\omega/10)^2}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{100\ j\omega}}{\mathbf{(1 + j\omega)(10 + j\omega)^2}}$$

(It should be noted that this function could also have a minus sign out in front and still be correct. The magnitude plot does not contain this information. It can only be obtained from the phase plot.)

Chapter 14, Solution 24.

$$40 = 20\log_{10} K \quad \longrightarrow \quad K = 100$$

There is a pole at ω=50 giving 1/(1+jω/50)
There is a zero at ω=500 giving (1 + jω/500).
There is another pole at ω=2122 giving 1/(1 + jω/2122).
Thus,

$$H(j\omega) = 100(1+j\omega)/[(1+j\omega/50)(1+j\omega/2122)]$$

$$= [100(50\text{x}2122)/500](j\omega+500)/[(j\omega+50)(j\omega+2122)]$$

or

H(s) = **21220(s+500)/[(s+50)(s+2122)]**.

Chapter 14, Solution 25.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(40\times10^{-3})(1\times10^{-6})}} = 5\ \text{krad/s}$$

$$\mathbf{Z}(\omega_0) = R = \mathbf{2\ k\Omega}$$

$$\mathbf{Z}(\omega_0/4) = R + j\left(\frac{\omega_0}{4}L - \frac{4}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j\left(\frac{5\times10^3}{4}\cdot 40\times10^{-3} - \frac{4}{(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j(50 - 4000/5)$$

$$\mathbf{Z}(\omega_0/4) = \mathbf{2 - j0.75\ k\Omega}$$

$$\mathbf{Z}(\omega_0/2) = R + j\left(\frac{\omega_0}{2}L - \frac{2}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/2) = 2000 + j\left(\frac{(5\times10^3)}{2}(40\times10^{-3}) - \frac{2}{(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(\omega_0/2) = 200 + j(100 - 2000/5)$$

$$\mathbf{Z}(\omega_0/2) = \mathbf{2 - j0.3\ k\Omega}$$

$$\mathbf{Z}(2\omega_0) = R + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right)$$

$$\mathbf{Z}(2\omega_0) = 2000 + j\left((2)(5\times10^3)(40\times10^{-3}) - \frac{1}{(2)(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(2\omega_0) = \mathbf{2 + j0.3\ k\Omega}$$

$$\mathbf{Z}(4\omega_0) = R + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right)$$

$$\mathbf{Z}(4\omega_0) = 2000 + j\left((4)(5\times10^3)(40\times10^{-3}) - \frac{1}{(4)(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(4\omega_0) = \mathbf{2 + j0.75\ k\Omega}$$

Chapter 14, Solution 26.

Design a problem to help other students to better understand ω_o, Q, and B at resonance in series *RLC* circuits.

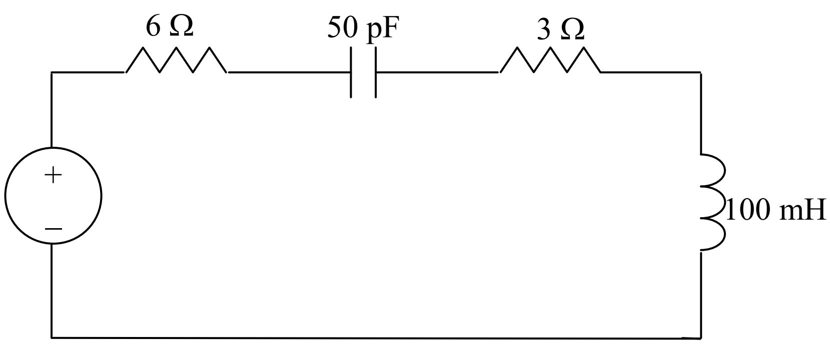
Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A coil with resistance 3 Ω and inductance 100 mH is connected in series with a capacitor of 50 pF, a resistor of 6 Ω, and a signal generator that gives 110V-rms at all frequencies. Calculate ω_o, Q, and B at resonance of the resultant series RLC circuit.

Solution

Consider the circuit as shown below. This is a series RLC resonant circuit.



R = 6 + 3 = 9 Ω

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-3}x50x10^{-12}}} = \underline{447.21 \text{ krad/s}}$$

$$Q = \frac{\omega_o L}{R} = \frac{447.21x10^3 x100x10^3}{9} = \underline{4969}$$

$$B = \frac{\omega_o}{Q} = \frac{447.21x10^3}{4969} = \underline{90 \text{ rad/s}}$$

Chapter 14, Solution 27.

$$\omega_o = \frac{1}{\sqrt{LC}} = 40 \quad \longrightarrow \quad LC = \frac{1}{40^2}$$

$$B = \frac{R}{L} = 10 \quad \longrightarrow \quad R = 10L$$

If we select R =**1 Ω**, then L = R/10 = **100 mH** and

$$C = \frac{1}{40^2 L} = \frac{1}{40^2 x 0.1} = \underline{6.25\ \text{mF}}$$

Chapter 14, Solution 28.

$R = 10\,\Omega$.

$$L = \frac{R}{B} = \frac{10}{20} = 0.5 \text{ H}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(1000)^2 (0.5)} = 2\ \mu\text{F}$$

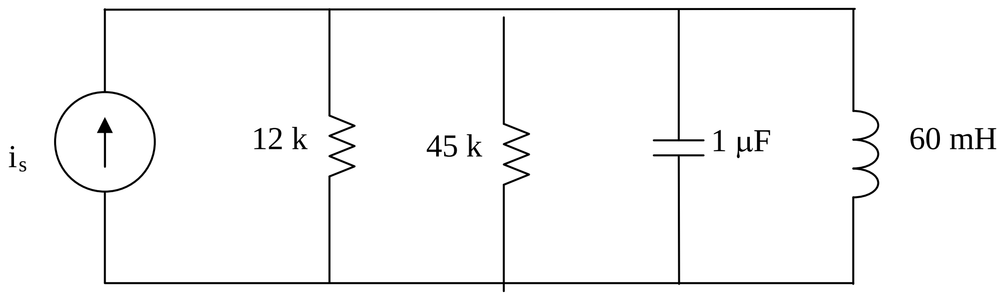
$$Q = \frac{\omega_0}{B} = \frac{1000}{20} = 50$$

Therefore, if $R = 10\,\Omega$ then

$L =$ **500 mH**, $C =$ **2 μF**, $Q =$ **50**

Chapter 14, Solution 29.

We convert the voltage source to a current source as shown below.



$i_s = \frac{20}{12}\cos\omega t$, R = 12//45= 12x45/57 = 9.4737 kΩ

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{60x10^{-3}x1x10^{-6}}} = \underline{4.082 \text{ krad/s}} = \mathbf{4.082\ krad/s}$$

$$B = \frac{1}{RC} = \frac{1}{9.4737x10^{3}x10^{-6}} = \underline{105.55 \text{ rad/s}} = \mathbf{105.55\ rad/s}$$

$$Q = \frac{\omega_o}{B} = \frac{4082}{105.55} = \underline{38.674} = \mathbf{38.67}$$

4.082 krads/s, 105.55 rad/s, 38.67

Chapter 14, Solution 30.

(a) $f_o = 15{,}000$ Hz leads to $\omega_o = 2\pi f_o = 94.25$ krad/s $= 1/(LC)^{0.5}$ or

$LC = 1/8.883\text{x}10^9$ or $C = 1/(8.883\text{x}10^9\text{x}10^{-2}) = 11.257\text{x}10^{-9}$ F = **11.257 pF**.

(b) since the capacitive reactance cancels out the inductive reactance at resonance, the current through the series circuit is given by

$$I = 120/20 = \mathbf{6\ A}.$$

(c) $Q = \omega_o L/R = 94.25\text{x}10^3(0.01)/20 =$ **47.12**.

Chapter 14, Solution 31.

$$R = 10\,\Omega.$$

$$L = \frac{R}{\omega_0 Q} = \frac{10}{(10)(20)} = 0.05 \text{ H} = 50 \text{ mH}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(100)(0.05)} = 0.2 \text{ F}$$

$$\mathbf{B} = \frac{1}{\mathbf{RC}} = \frac{1}{(10)(0.2)} = \mathbf{0.5 \text{ rad/s}}$$

Chapter 14, Solution 32.

Design a problem to help other students to better understand the quality factor, the resonant frequency, and bandwidth of a parallel *RLC* circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A parallel RLC circuit has the following values:

R = 60 Ω, L = 1 mH, and C = 50 μF

Find the quality factor, the resonant frequency, and the bandwidth of the RLC circuit.

Solution

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}x50x10^{-6}}} = \underline{4.472 \text{ krad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{60x50x10^{-6}} = \underline{333.33 \text{ rad/s}}$$

$$Q = \frac{\omega_o}{B} = \frac{4472}{333.33} = \underline{13.42}$$

Chapter 14, Solution 33.

$B = \omega_o/Q = 6\text{x}10^6/120 =$ **50 krad/s**.

$\omega_1 = \omega_o - B =$ **$5.95\text{x}10^6$ rad/s** and $\omega_2 = \omega_o + B =$ **$6.05\text{x}10^6$ rad/s**.

Chapter 14, Solution 34.

$$\mathbf{Q} = \omega_o \mathbf{RC} \quad \longrightarrow \quad \mathbf{C} = \frac{\mathbf{Q}}{2\pi \mathbf{f_o R}} = \frac{80}{2\pi \mathbf{x} 5.6 \mathbf{x} 10^6 \mathbf{x} 40 \mathbf{x} 10^3} = \mathbf{56.84\ pF}$$

$$Q = \frac{R}{\omega_o L} \quad \longrightarrow \quad L = \frac{R}{2\pi f_o Q} = \frac{40x10^3}{2\pi x 5.6x10^6 x80} = \mathbf{14.21\ \mu H}$$

Chapter 14, Solution 35.

(a) $$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8x10^{-3}x60x10^{-6}}} = \underline{1.443\,krad/s}$$

(b) $$B = \frac{1}{RC} = \frac{1}{5x10^{3}x60x10^{-6}} = \underline{3.33\,rad/s}$$

(c) $$Q = \omega_o RC = 1.443x10^{3}x5x10^{3}x60x10^{-6} = \underline{432.9}$$

Chapter 14, Solution 36.

At resonance,

$$\mathbf{Y} = \frac{1}{R} \longrightarrow R = \frac{1}{Y} = \frac{1}{25\times10^{-3}} = \mathbf{40\,\Omega}$$

$$Q = \omega_0 RC \longrightarrow C = \frac{Q}{\omega_0 R} = \frac{80}{(200\times10^{3})(40)} = \mathbf{10\ \mu F}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{(4\times10^{10})(10\times10^{-6})} = \mathbf{2.5\ \mu H}$$

$$B = \frac{\omega_0}{Q} = \frac{200\times10^3}{80} = \mathbf{2.5\ krad/s}$$

$$\omega_1 = \omega_0 - \frac{B}{2} = 200 - 1.25 = \mathbf{198.75\ krad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 200 + 1.25 = \mathbf{201.25\ krad/s}$$

Chapter 14, Solution 37.

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5000 \text{ rad/s}$$

$$\mathbf{Y}(\omega_0) = \frac{1}{R} \longrightarrow \mathbf{Z}(\omega_0) = R = \mathbf{2\ k\Omega}$$

$$\mathbf{Y}(\omega_0/4) = \frac{1}{R} + j\left(\frac{\omega_0}{4}C - \frac{4}{\omega_0 L}\right) = 0.5 - j18.75 \text{ mS}$$

$$\mathbf{Z}(\omega_0/4) = \frac{1}{0.0005 - j0.01875} = \mathbf{(1.4212 + j53.3)\ \Omega}$$

$$\mathbf{Y}(\omega_0/2) = \frac{1}{R} + j\left(\frac{\omega_0}{2}C - \frac{2}{\omega_0 L}\right) = 0.5 - j7.5 \text{ mS}$$

$$\mathbf{Z}(\omega_0/2) = \frac{1}{0.0005 - j0.0075} = \mathbf{(8.85 + j132.74)\ \Omega}$$

$$\mathbf{Y}(2\omega_0) = \frac{1}{R} + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right) = 0.5 + j7.5 \text{ mS}$$

$$\mathbf{Z}(2\omega_0) = \mathbf{(8.85 - j132.74)\ \Omega}$$

$$\mathbf{Y}(4\omega_0) = \frac{1}{R} + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right) = 0.5 + j18.75 \text{ mS}$$

$$\mathbf{Z}(4\omega_0) = \mathbf{(1.4212 - j53.3)\ \Omega}$$

Chapter 14, Solution 38.

$$Z = j\omega L \,//(R + \frac{1}{j\omega C}) = \frac{j\omega L(R + \frac{1}{j\omega C})}{R + \frac{1}{j\omega C} + j\omega L} = \frac{\left(\frac{L}{C} + j\omega LR\right)\left(R - j(\omega L - \frac{1}{\omega C})\right)}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$\mathrm{Im}(Z) = \frac{\omega LR^2 - \frac{L}{C}\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + (\omega L - \frac{1}{\omega C})^2} = 0 \qquad \longrightarrow \qquad \omega^2 (LC - R^2C^2) = 1$$

Thus,

$$\underline{\boldsymbol{\omega = \frac{1}{\sqrt{LC - R^2C^2}}}}$$

Chapter 14, Solution 39.

$$\mathbf{Y} = \frac{1}{R + j\omega L} + j\omega C = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

At resonance, $\text{Im}(\mathbf{Y}) = 0$, i.e.

$$\omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(40 \times 10^{-3})(1 \times 10^{-6})} - \left(\frac{50}{40 \times 10^{-3}}\right)^2}$$

$\omega_0 =$ **4.841 krad/s**

Chapter 14, Solution 40.

(a) $B = \omega_2 - \omega_1 = 2\pi(f_2 - f_1) = 2\pi(90 - 86)x10^3 = 8\pi krad/s$

$$\omega_o = \frac{1}{2}(\omega_1 + \omega_2) = 2\pi(88)x10^3 = 176\pi X10^3$$

$$B = \frac{1}{RC} \longrightarrow C = \frac{1}{BR} = \frac{1}{8\pi x10^3 x2x10^3} = \underline{19.89nF}$$

(b) $\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega^2{}_oC} = \frac{1}{(176\pi X10^3)^2 x19.89x10^{-9}}$ = **164.45 μH**

(c) $\omega_o = 176\pi = \underline{552.9krad/s}$

(d) $B = 8\pi = \underline{25.13krad/s}$

(e) $Q = \frac{\omega_o}{B} = \frac{176\pi}{8\pi} = \underline{22}$

Chapter 14, Solution 41.

Using Fig. 14.80, design a problem to help other students to better understand the quality factor, the resonant frequency, and bandwidth of an *RLC* circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in Example 14.9.

Problem

For the circuits in Fig. 14.80, find the resonant frequency ω_0, the quality factor Q, and the bandwidth B. Let C = 0.1 F, R_1 = 10 Ω, R_2 = 2 Ω, and L = 2 H.

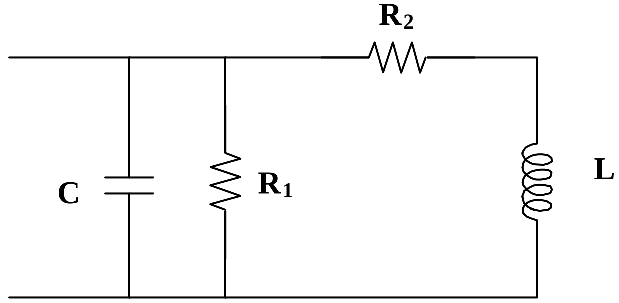


Figure 14.80
For Prob. 14.41.

Solution

To find ω_o, we need to find the input impedance or input admittance and set imaginary component equal to zero. Finding the input admittance seems to be the easiest approach.

$$\mathbf{Y} = j\omega 0.1 + 0.1 + 1/(2+j\omega 2) = j\omega 0.1 + 0.1 + [2/(4+4\omega^2)] - [j\omega 2/(4+4\omega^2)]$$

At resonance,

$$0.1\omega = 2\omega/(4+4\omega^2) \text{ or } 4\omega^2 + 4 = 20 \text{ or } \omega^2 = 4 \text{ or } \omega_o = \mathbf{2\ rad/s}$$

and,

$$\mathbf{Y} = 0.1 + 2/(4+16) = 0.1 + 0.1 = 0.2 \text{ S}$$

The bandwidth is define as the two values of ω such that $|\mathbf{Y}|$ = 1.4142(0.2) = 0.28284 S.

I do not know about you, but I sure would not want to solve this analytically. So how about using MATLAB or excel to solve for the two values of ω?

Using Excel, we get ω_1 = 1.414 rad/s and ω_2 = 3.741 rad/s or B = **2.327 rad/s**

We can now use the relationship between ω_o and the bandwidth.

$$Q = \omega_o/B = 2/2.327 = \mathbf{0.8595}$$

Chapter 14, Solution 42.

(a) This is a series RLC circuit.

$$R = 2 + 6 = 8\,\Omega, \qquad L = 1\,H, \qquad C = 0.4\,F$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4}} = \mathbf{1.5811\ rad/s}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1.5811}{8} = \mathbf{0.1976}$$

$$B = \frac{R}{L} = \mathbf{8\ rad/s}$$

(b) This is a parallel RLC circuit.

$$3\ \mu F \text{ and } 6\ \mu F \longrightarrow \frac{(3)(6)}{3+6} = 2\ \mu F$$

$$C = 2\ \mu F, \qquad R = 2\ k\Omega, \qquad L = 20\ mH$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times 10^{-6})(20\times 10^{-3})}} = \mathbf{5\ krad/s}$$

$$Q = \frac{R}{\omega_0 L} = \frac{2\times 10^{3}}{(5\times 10^{3})(20\times 10^{-3})} = \mathbf{20}$$

$$B = \frac{1}{RC} = \frac{1}{(2\times 10^{3})(2\times 10^{-6})} = \mathbf{250\ rad/s}$$

Chapter 14, Solution 43.

(a) $\mathbf{Z}_{in} = (1/j\omega C) \parallel (R + j\omega L)$

$$\mathbf{Z}_{in} = \frac{\dfrac{R + j\omega L}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{(R + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

$$0 = \omega_0 L(1 - \omega_0^2 LC) - \omega_0 R^2 C$$

$$\omega_0^2 L^2 C = L - R^2 C$$

$$\omega_0 = \sqrt{\frac{L - R^2 C}{L^2 C}} = \sqrt{\frac{\mathbf{1}}{\mathbf{LC}} - \frac{\mathbf{R}^2}{\mathbf{L}^2}}$$

(b) $\mathbf{Z}_{in} = R \parallel (j\omega L + 1/j\omega C)$

$$\mathbf{Z}_{in} = \frac{R\,(j\omega L + 1/j\omega C)}{R + j\omega L + 1/j\omega C} = \frac{R\,(1 - \omega^2 LC)}{(1 - \omega^2 LC) + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{R\,(1 - \omega^2 LC)[(1 - \omega^2 LC) - j\omega RC]}{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

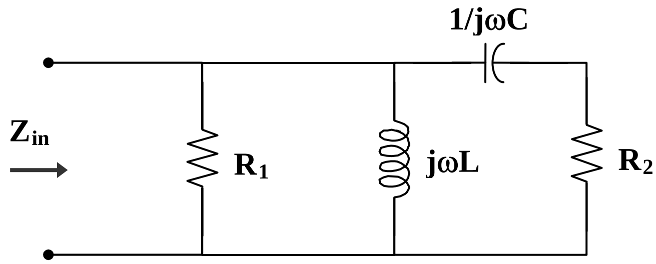
$$0 = R\,(1 - \omega^2 LC)\,\omega RC$$

$$1 - \omega^2 LC = 0$$

$$\omega_0 = \frac{\mathbf{1}}{\sqrt{\mathbf{LC}}}$$

Chapter 14, Solution 44.

Consider the circuit below.



(a) $$\mathbf{Z}_{in} = (R_1 \parallel j\omega L) \parallel (R_2 + 1/j\omega C)$$

$$\mathbf{Z}_{in} = \left(\frac{R_1 j\omega L}{R_1 + j\omega L}\right) \parallel \left(R_2 + \frac{1}{j\omega C}\right)$$

$$\mathbf{Z}_{in} = \frac{\dfrac{j\omega R_1 L}{R_1 + j\omega L} \cdot \left(R_2 + \dfrac{1}{j\omega C}\right)}{R_2 + \dfrac{1}{j\omega C} + \dfrac{jR_1\omega L}{R_1 + j\omega L}}$$

$$\mathbf{Z}_{in} = \frac{j\omega R_1 L(1 + j\omega R_2 C)}{(R_1 + j\omega L)(1 + j\omega R_2 C) - \omega^2 LCR_1}$$

$$\mathbf{Z}_{in} = \frac{-\omega^2 R_1 R_2 LC + j\omega R_1 L}{R_1 - \omega^2 LCR_1 - \omega^2 LCR_2 + j\omega(L + R_1 R_2 C)}$$

$$\mathbf{Z}_{in} = \frac{(-\omega^2 R_1 R_2 LC + j\omega R_1 L)[R_1 - \omega^2 LCR_1 - \omega^2 LCR_2 - j\omega(L + R_1 R_2 C)]}{(R_1 - \omega^2 LCR_1 - \omega^2 LCR_2)^2 + \omega^2 (L + R_1 R_2 C)^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

$$0 = \omega^3 R_1 R_2 LC(L + R_1 R_2 C) + \omega R_1 L(R_1 - \omega^2 LCR_1 - \omega^2 LCR_2)$$

$$0 = \omega^3 R_1^2 R_2^2 LC^2 + R_1^2 \omega L - \omega^3 R_1^2 L^2 C$$

$$0 = \omega^2 R_2^2 C^2 + 1 - \omega^2 LC$$

$$\omega^2 (LC - R_2^2 C^2) = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC - R_2^2 C^2}}$$

$$\omega_0 = \frac{1}{\sqrt{(0.02)(9\times 10^{-6}) - (0.1)^2 (9\times 10^{-6})^2}}$$

$$\omega_0 = \mathbf{2.357\ krad/s}$$

(b) At $\omega = \omega_0 = 2.357\ \text{krad/s}$,

$$j\omega L = j(2.357 \times 10^{3})(20 \times 10^{-3}) = j47.14$$

$$R_1 \parallel j\omega L = \frac{j47.14}{1 + j47.14} = 0.9996 + j0.0212$$

$$R_2 + \frac{1}{j\omega C} = 0.1 + \frac{1}{j(2.357 \times 10^{3})(9 \times 10^{-6})} = 0.1 - j47.14$$

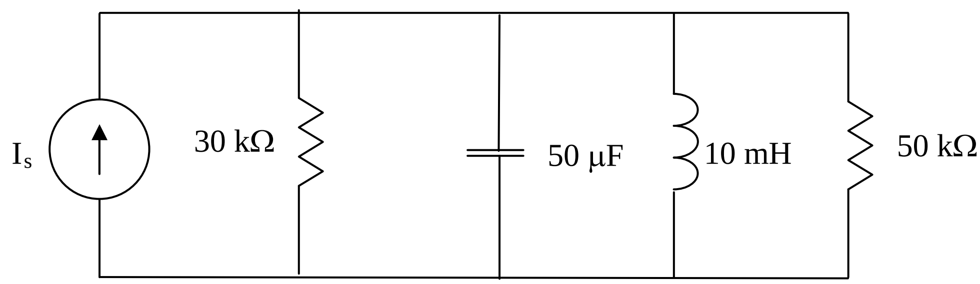
$$\mathbf{Z}_{in}(\omega_0) = (R_1 \parallel j\omega L) \parallel (R_2 + 1/j\omega C)$$

$$\mathbf{Z}_{in}(\omega_0) = \frac{(0.9996 + j0.0212)(0.1 - j47.14)}{(0.9996 + j0.0212) + (0.1 - j47.14)}$$

$$\mathbf{Z}_{in}(\omega_0) = \mathbf{1\,\Omega}$$

Chapter 14, Solution 45.

Convert the voltage source to a current source as shown below.



R = 30//50 = 30x50/80 = 18.75 kΩ

This is a parallel resonant circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10x10^{-3}\,x50x10^{-6}}} = \underline{447.21 \text{ rad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{18.75x10^{3}\,x50x10^{-6}} = \underline{1.067 \text{ rad/s}}$$

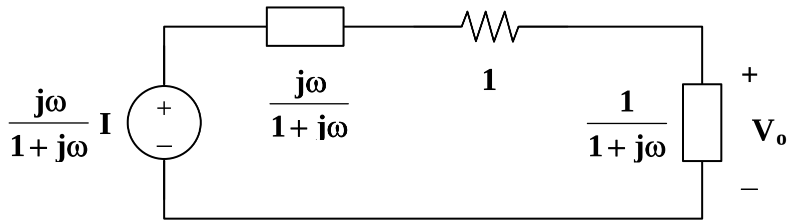
$$Q = \frac{\omega_o}{B} = \frac{447.21}{1.067} = \underline{419.13}$$

447.2 rad/s, 1.067 rad/s, 419.1

Chapter 14, Solution 46.

(a) $\quad 1 \,||\, j\omega = \dfrac{j\omega}{1+j\omega}, \qquad 1 \,||\, \dfrac{1}{j\omega} = \dfrac{1/j\omega}{1+1/j\omega} = \dfrac{1}{1+j\omega}$

Transform the current source gives the circuit below.



$$\mathbf{V}_o = \frac{\dfrac{1}{1+j\omega}}{1+\dfrac{1}{1+j\omega}+\dfrac{j\omega}{1+j\omega}} \cdot \frac{j\omega}{1+j\omega}\,\mathbf{I}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{I}} = \mathbf{\frac{j\omega}{2(1+j\omega)^2}}$$

(b) $\quad \mathbf{H}(1) = \dfrac{1}{2(1+j)^2}$

$$\left|\mathbf{H}(1)\right| = \frac{1}{2(\sqrt{2})^2} = \mathbf{0.25}$$

Chapter 14, Solution 47.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R}$$

$H(0) = 1$ and $H(\infty) = 0$ showing that this circuit is a lowpass filter.

At the corner frequency, $\left| H(\omega_c) \right| = \frac{1}{\sqrt{2}}$, i.e.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega_c L}{R}\right)^2}} \longrightarrow 1 = \frac{\omega_c L}{R} \qquad \text{or} \qquad \omega_c = \frac{R}{L}$$

Hence,

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$f_c = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{1}{2\pi} \cdot \frac{10 \times 10^3}{2 \times 10^{-3}} = \mathbf{796\ kHz}$$

Chapter 14, Solution 48.

$$\mathbf{H}(\omega) = \frac{R \parallel \dfrac{1}{j\omega C}}{j\omega L + R \parallel \dfrac{1}{j\omega C}}$$

$$\mathbf{H}(\omega) = \frac{\dfrac{R/j\omega C}{R + 1/j\omega C}}{j\omega L + \dfrac{R/j\omega C}{R + 1/j\omega C}}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{R}}{\mathbf{R + j\omega L - \omega^2 RLC}}$$

$H(0) = 1$ and $H(\infty) = 0$ showing that **this circuit is a lowpass filter**.

Chapter 14, Solution 49.

Design a problem to help other students to better understand lowpass filters described by transfer functions.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the cutoff frequency of the lowpass filter described by

$$H(\omega) = \frac{4}{2 + j\omega 10}$$

Find the gain in dB and phase of $\mathbf{H}(\omega)$ at $\omega = 2$ rad/s.

Solution

At dc, $H(0) = \frac{4}{2} = 2$.

Hence, $\left|H(\omega)\right| = \frac{1}{\sqrt{2}} H(0) = \frac{2}{\sqrt{2}}$

$$\frac{2}{\sqrt{2}} = \frac{4}{\sqrt{4 + 100\omega_c^2}}$$

$$4 + 100\omega_c^2 = 8 \longrightarrow \omega_c = 0.2$$

$$H(2) = \frac{4}{2 + j20} = \frac{2}{1 + j10}$$

$$\left|H(2)\right| = \frac{2}{\sqrt{101}} = 0.199$$

In dB, $20\log_{10}\left|H(2)\right| = \mathbf{-14.023}$

$\arg H(2) = -\tan^{-1} 10 = -84.3°$ or ω_c = **1.4713 rad/sec**.

Chapter 14, Solution 50.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L}$$

$H(0) = 0$ and $H(\infty) = 1$ showing that **this circuit is a highpass filter**.

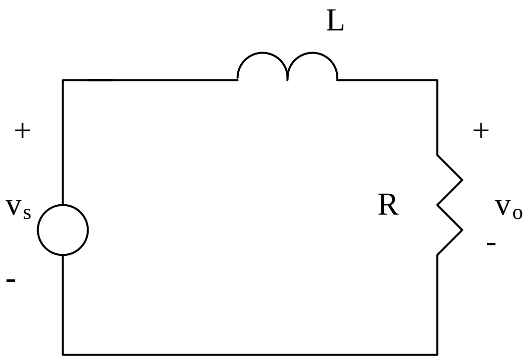
$$\mathbf{H}(\omega_c) = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\dfrac{R}{\omega_c L}\right)^2}} \longrightarrow 1 = \frac{R}{\omega_c L}$$

or $\qquad \omega_c = \dfrac{R}{L} = 2\pi f_c$

$$f_c = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{1}{2\pi} \cdot \frac{200}{0.1} = \mathbf{318.3\ Hz}$$

Chapter 14, Solution 51.

The lowpass RL filter is shown below.



$$H = \frac{V_o}{V_s} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L / R}$$

$$\boldsymbol{\omega_c = \frac{R}{L} = 2\pi f_c \quad \longrightarrow \quad R = 2\pi f_c L = 2\pi x 5x10^3 x 40x10^{-3} = \underline{1.256k\Omega}}$$

Chapter 14, Solution 52.

Design a problem to help other students to better understand passive highpass filters.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In a highpass *RL* filter with a cutoff frequency of 100 kHz, *L* = 40 mH. Find *R*.

Solution

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$R = 2\pi f_c L = (2\pi)(10^5)(40\times 10^{-3}) = \mathbf{25.13\ k\Omega}$$

Chapter 14, Solution 53.

$$\omega_1 = 2\pi f_1 = 20\pi \times 10^3$$
$$\omega_2 = 2\pi f_2 = 22\pi \times 10^3$$

$$B = \omega_2 - \omega_1 = 2\pi \times 10^3$$
$$\omega_0 = \frac{\omega_2 + \omega_1}{2} = 21\pi \times 10^3$$
$$Q = \frac{\omega_0}{B} = \frac{21\pi}{2\pi} = \mathbf{10.5}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega_0^2 C}$$
$$L = \frac{1}{(21\pi \times 10^3)^2 (80 \times 10^{-12})} = \mathbf{2.872\ H}$$

$$B = \frac{R}{L} \longrightarrow R = BL$$
$$R = (2\pi \times 10^3)(2.872) = \mathbf{18.045\ k\Omega}$$

Chapter 14, Solution 54.

We start with a series RLC circuit and the use the equations related to the circuit and the values for a bandstop filter.

$Q = \omega_o L/R = 1/(\omega_o CR) = 20$; $B = R/L = \omega_o/Q = 10/20 = 0.5$; $\omega_o = 1/(LC)^{0.5} = 10$

$(LC)^{0.5} = 0.1$ or $LC = 0.01$. Pick $L =$ **10 H** then $C =$ **1 mF**.

$Q = 20 = \omega_o L/R = 10x10/R$ or $R = 100/20 =$ **5 Ω**.

Chapter 14, Solution 55.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25\times10^{-3})(0.4\times10^{-6})}} = 10 \text{ krad/s}$$

$$B = \frac{R}{L} = \frac{10}{25\times10^{-3}} = 0.4 \text{ krad/s}$$

$$Q = \frac{10}{0.4} = \mathbf{25}$$

$$\omega_1 = \omega_o - B/2 = 10 - 0.2 = 9.8 \text{ krad/s} \qquad \text{or} \qquad f_1 = \frac{9.8}{2\pi} = 1.56 \text{ kHz}$$

$$\omega_2 = \omega_o + B/2 = 10 + 0.2 = 10.2 \text{ krad/s} \qquad \text{or} \qquad f_2 = \frac{10.2}{2\pi} = 1.62 \text{ kHz}$$

Therefore,

$$\mathbf{1.56 \text{ kHz} < f < 1.62 \text{ kHz}}$$

Chapter 14, Solution 56.

(a) From Eq 14.54,

$$\mathbf{H}(s) = \frac{R}{R + sL + \frac{1}{sC}} = \frac{sRC}{1 + sRC + s^2LC} = \frac{s\frac{R}{L}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

Since $B = \frac{R}{L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$,

$$\mathbf{H}(s) = \frac{\mathbf{sB}}{\mathbf{s^2 + sB + \omega_0^2}}$$

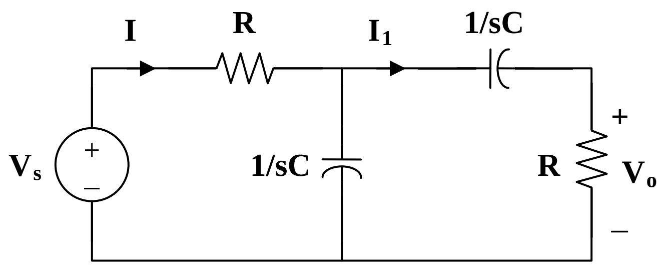
(b) From Eq. 14.56,

$$\mathbf{H}(s) = \frac{sL + \frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{s^2 + \frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

$$\mathbf{H}(s) = \frac{\mathbf{s^2 + \omega_0^2}}{\mathbf{s^2 + sB + \omega_0^2}}$$

Chapter 14, Solution 57.

(a) Consider the circuit below.



$$\mathbf{Z}(s) = R + \frac{1}{sC} \parallel \left(R + \frac{1}{sC}\right) = R + \frac{\frac{1}{sC}\left(R + \frac{1}{sC}\right)}{R + \frac{2}{sC}}$$

$$\mathbf{Z}(s) = R + \frac{1 + sRC}{sC\,(2 + sRC)}$$

$$\mathbf{Z}(s) = \frac{1 + 3sRC + s^2R^2C^2}{sC\,(2 + sRC)}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}$$

$$\mathbf{I}_1 = \frac{1/sC}{2/sC + R}\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}\,(2 + sRC)}$$

$$\mathbf{V}_o = \mathbf{I}_1 R = \frac{R\,\mathbf{V}_s}{2 + sRC} \cdot \frac{sC\,(2 + sRC)}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRC}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{1}{3}\left[\frac{\frac{3}{RC}s}{s^2 + \frac{3}{RC}s + \frac{1}{R^2C^2}}\right]$$

Thus, $\omega_0^2 = \dfrac{1}{R^2C^2}$ or $\omega_0 = \dfrac{1}{RC} = \mathbf{1\,rad/s}$

$B = \dfrac{3}{RC} = \mathbf{3\,rad/s}$

(b) Similarly,

$$\mathbf{Z}(s) = sL + R \parallel (R + sL) = sL + \frac{R\,(R + sL)}{2R + sL}$$

$$\mathbf{Z}(s) = \frac{R^2 + 3sRL + s^2L^2}{2R + sL}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}, \qquad \mathbf{I}_1 = \frac{R}{2R + sL}\mathbf{I} = \frac{R\,\mathbf{V}_s}{\mathbf{Z}\,(2R + sL)}$$

$$\mathbf{V}_o = \mathbf{I}_1 \cdot sL = \frac{sLR\,\mathbf{V}_s}{2R + sL} \cdot \frac{2R + sL}{R^2 + 3sRL + s^2L^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRL}{R^2 + 3sRL + s^2L^2} = \frac{\frac{1}{3}\left(\frac{3R}{L}s\right)}{s^2 + \frac{3R}{L}s + \frac{R^2}{L^2}}$$

Thus, $\omega_0 = \dfrac{R}{L} = \mathbf{1\,rad/s}$

$B = \dfrac{3R}{L} = \mathbf{3\,rad/s}$

Chapter 14, Solution 58.

(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.1)(40\times 10^{-12})}} = \mathbf{0.5\times 10^6\ rad/s}$$

(b) $$B = \frac{R}{L} = \frac{2\times 10^3}{0.1} = 2\times 10^4$$

$$Q = \frac{\omega_0}{B} = \frac{0.5\times 10^6}{2\times 10^4} = 25$$

As a high Q circuit,

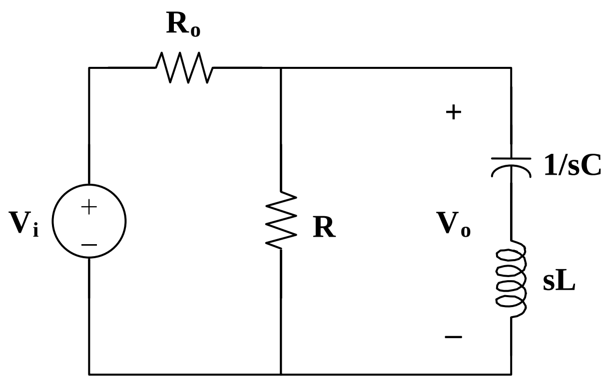
$$\omega_1 = \omega_0 - \frac{B}{2} = 10^4\,(50-1) = \mathbf{490\ krad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 10^4\,(50+1) = \mathbf{510\ krad/s}$$

(c) As seen in part (b), $Q = \mathbf{25}$

Chapter 14, Solution 59.

Consider the circuit below.



where L = 1 mH, C = 4 μF, R_o = 6 Ω, and R = 4 Ω.

$$\mathbf{Z}(s) = R \,||\left(sL + \frac{1}{sC} \right) = \frac{R\,(sL + 1/sC)}{R + sL + 1/sC}$$

$$\mathbf{Z}(s) = \frac{R\,(1 + s^2LC)}{1 + sRC + s^2LC}$$

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}}{\mathbf{Z} + R_o} = \frac{R\,(1 + s^2LC)}{R_o + sRR_oC + s^2LCR_o + R + s^2LCR}$$

$$\mathbf{Z}_{in} = R_o + \mathbf{Z} = R_o + \frac{R\,(1 + s^2LC)}{1 + sRC + s^2LC}$$

$$\mathbf{Z}_{in} = \frac{R_o + sRR_oC + s^2LCR_o + R + s^2LCR}{1 + sRC + s^2LC}$$

$s = j\omega$

$$\mathbf{Z}_{in} = \frac{R_o + j\omega RR_oC - \omega^2LCR_o + R - \omega^2LCR}{1 - \omega^2LC + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{(R_o + R - \omega^2LCR_o - \omega^2LCR + j\omega RR_oC)(1 - \omega^2LC - j\omega RC)}{(1 - \omega^2LC)^2 + (\omega RC)^2}$$

$\mathrm{Im}(\mathbf{Z}_{in}) = 0$ implies that

$$-\omega RC[R_o + R - \omega^2LCR_o - \omega^2LCR] + \omega RR_oC\,(1 - \omega^2LC) = 0$$

$$R_o + R - \omega^2 LCR_o - \omega^2 LCR - R_o + \omega^2 LCR_o = 0$$
$$\omega^2 LCR = R$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1\times 10^{-3})(4\times 10^{-6})}} = \mathbf{15.811\ krad/s}$$

$$\mathbf{H} = \frac{R\,(1-\omega^2 LC)}{R_o + j\omega RR_o C + R - \omega^2 LCR_o - \omega^2 LCR}$$

$$H_{max} = H(0) = \frac{R}{R_o + R}$$

or $$H_{max} = H(\infty) = \lim_{\omega\to\infty} \frac{R\left(\dfrac{1}{\omega^2} - LC\right)}{\dfrac{R_o + R}{\omega^2} + j\dfrac{RR_o C}{\omega} - LC(R + R_o)} = \frac{R}{R + R_o}$$

At ω_1 and ω_2, $\left|\mathbf{H}\right| = \dfrac{1}{\sqrt{2}} H_{mzx}$

$$\frac{R}{\sqrt{2}(R_o + R)} = \left| \frac{R\,(1-\omega^2 LC)}{R_o + R - \omega^2 LC(R_o + R) + j\omega RR_o C} \right|$$

$$\frac{1}{\sqrt{2}} = \frac{(R_o + R)(1-\omega^2 LC)}{\sqrt{(\omega RR_o C)^2 + (R_o + R - \omega^2 LC(R_o + R))^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{10\,(1-\omega^2\cdot 4\times 10^{-9})}{\sqrt{(96\times 10^{-6}\omega)^2 + (10-\omega^2\cdot 4\times 10^{-8})^2}}$$

$$0 = \frac{10\,(1-\omega^2\cdot 4\times 10^{-9})}{\sqrt{(96\times 10^{-6}\omega)^2 + (10-\omega^2\cdot 4\times 10^{-8})^2}} - \frac{1}{\sqrt{2}}$$

$$(10-\omega^2\cdot 4\times 10^{-8})(\sqrt{2}) - \sqrt{(96\times 10^{-6}\omega)^2 + (10-\omega^2\cdot 4\times 10^{-8})^2} = 0$$

$$(2)(10-\omega^2\cdot 4\times 10^{-8})^2 = (96\times 10^{-6}\omega)^2 + (10-\omega^2\cdot 4\times 10^{-8})^2$$

$$(96\times 10^{-6}\omega)^2 - (10-\omega^2\cdot 4\times 10^{-8})^2 = 0$$

$$1.6\times 10^{-15}\omega^4 - 8.092\times 10^{-7}\omega^2 + 100 = 0$$

$$\omega^4 - 5.058\times10^8 + 6.25\times10^{16} = 0$$

$$\omega^2 = \begin{cases} 2.9109\times10^8 \\ 2.1471\times10^8 \end{cases}$$

Hence,

$$\omega_1 = 14.653 \text{ krad/s}$$

$$\omega_2 = 17.061 \text{ krad/s}$$

$$B = \omega_2 - \omega_1 = 17.061 - 14.653 = \mathbf{2.408\ krad/s}$$

Chapter 14, Solution 60.

$$\mathbf{H}'(\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega}{j\omega + 1/RC} \qquad \text{(from Eq. 14.52)}$$

This has a unity passband gain, i.e. $H(\infty) = 1$.

$$\frac{1}{RC} = \omega_c = 50$$

$$\mathbf{H}^{\wedge}(\omega) = 10\,\mathbf{H}'(\omega) = \frac{j10\omega}{50 + j\omega}$$

$$\mathbf{H}(\omega) = \boldsymbol{\frac{j10\omega}{50 + j\omega}}$$

Chapter 14, Solution 61.

(a) $$\mathbf{V}_+ = \frac{1/j\omega C}{R + 1/j\omega C}\mathbf{V}_i, \qquad \mathbf{V}_- = \mathbf{V}_o$$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{1}{1 + j\omega RC}\mathbf{V}_i = \mathbf{V}_o$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{\frac{1}{1 + j\omega RC}}$$

(b) $$\mathbf{V}_+ = \frac{R}{R + 1/j\omega C}\mathbf{V}_i, \qquad \mathbf{V}_- = \mathbf{V}_o$$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i = \mathbf{V}_o$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{\frac{j\omega RC}{1 + j\omega RC}}$$

Chapter 14, Solution 62.

This is a highpass filter.

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{1}{1 - j/\omega RC}$$

$$\mathbf{H}(\omega) = \frac{1}{1 - j\omega_c/\omega}, \qquad \omega_c = \frac{1}{RC} = 2\pi(1000)$$

$$\mathbf{H}(\omega) = \frac{1}{1 - j f_c/f} = \frac{1}{1 - j1000/f}$$

(a) $$\mathbf{H}(f = 200\text{ Hz}) = \frac{1}{1 - j5} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$\left|\mathbf{V}_o\right| = \frac{120\text{ mV}}{\left|1 - j5\right|} = \mathbf{23.53\ mV}$$

(b) $$\mathbf{H}(f = 2\text{ kHz}) = \frac{1}{1 - j0.5} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$\left|\mathbf{V}_o\right| = \frac{120\text{ mV}}{\left|1 - j0.5\right|} = \mathbf{107.3\ mV}$$

(c) $$\mathbf{H}(f = 10\text{ kHz}) = \frac{1}{1 - j0.1} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$\left|\mathbf{V}_o\right| = \frac{120\text{ mV}}{\left|1 - j0.1\right|} = \mathbf{119.4\ mV}$$

Chapter 14, Solution 63.

For an active highpass filter,

$$H(s) = -\frac{sC_iR_f}{1 + sC_iR_i} \qquad (1)$$

But

$$H(s) = -\frac{10s}{1 + s/10} \qquad (2)$$

Comparing (1) and (2) leads to:

$$\mathbf{C_iR_f = 10 \quad \longrightarrow \quad R_f = \frac{10}{C_i} = \underline{10M\Omega}}$$

$$\mathbf{C_iR_i = 0.1 \quad \longrightarrow \quad R_i = \frac{0.1}{C_i} = \underline{100k\Omega}}$$

Chapter 14, Solution 64.

$$Z_f = R_f \parallel \frac{1}{j\omega C_f} = \frac{R_f}{1 + j\omega R_f C_f}$$

$$Z_i = R_i + \frac{1}{j\omega C_i} = \frac{1 + j\omega R_i C_i}{j\omega C_i}$$

Hence,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{\mathbf{-\,j\omega R_f C_i}}{\mathbf{(1 + j\omega R_f C_f)(1 + j\omega R_i C_i)}}$$

This is a bandpass filter. $\mathbf{H}(\omega)$ is similar to the product of the transfer function of a lowpass filter and a highpass filter.

Chapter 14, Solution 65.

$$\mathbf{V}_+ = \frac{R}{R + 1/j\omega C}\mathbf{V}_i = \frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i$$

$$\mathbf{V}_- = \frac{R_i}{R_i + R_f}\mathbf{V}_o$$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{R_i}{R_i + R_f}\mathbf{V}_o = \frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(\mathbf{1} + \frac{\mathbf{R_f}}{\mathbf{R_i}}\right)\left(\frac{\mathbf{j\omega RC}}{\mathbf{1 + j\omega RC}}\right)$$

It is evident that as $\omega \to \infty$, the gain is $\mathbf{1 + \frac{R_f}{R_i}}$ and that the corner frequency is $\mathbf{\frac{1}{RC}}$.

Chapter 14, Solution 66.

(a) **Proof**

(b) When $\mathbf{R_1 R_4 = R_2 R_3}$,

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \cdot \frac{s}{s + 1/R_2 C}$$

(c) When $\mathbf{R_3} \rightarrow \infty$,

$$\mathbf{H}(s) = \frac{-1/R_1 C}{s + 1/R_2 C}$$

Chapter 14, Solution 67.

$$\text{DC gain} = \frac{R_f}{R_i} = \frac{1}{4} \quad \longrightarrow \quad R_i = 4R_f$$

$$\text{Corner frequency} = \omega_c = \frac{1}{R_f C_f} = 2\pi(500) \text{ rad/s}$$

If we select $R_f = 20\text{ k}\Omega$, then $R_i = 80\text{ k}\Omega$ and

$$C = \frac{1}{(2\pi)(500)(20\times10^3)} = 15.915\text{ nF}$$

Therefore, if $R_f = \mathbf{20\ k\Omega}$, then $R_i = \mathbf{80\ k\Omega}$ and $C = \mathbf{15.915\ nF}$

Chapter 14, Solution 68.

Design a problem to help other students to better understand the design of active highpass filters when specifying a high-frequency gain and a corner frequency.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Design an active highpass filter with a high-frequency gain of 5 and a corner frequency of 200 Hz.

Solution

$$\text{High frequency gain} = 5 = \frac{R_f}{R_i} \longrightarrow R_f = 5R_i$$

$$\text{Corner frequency} = \omega_c = \frac{1}{R_i C_i} = 2\pi(200)\ \text{rad/s}$$

If we select $R_i = 20\ \text{k}\Omega$, then $R_f = 100\ \text{k}\Omega$ and

$$C = \frac{1}{(2\pi)(200)(20\times10^3)} = 39.8\ \text{nF}$$

Therefore, if $R_i = \mathbf{20\ k\Omega}$, then $R_f = \mathbf{100\ k\Omega}$ and $C = \mathbf{39.8\ nF}$

Chapter 14, Solution 69.

This is a highpass filter with $f_c = 2$ kHz.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$RC = \frac{1}{2\pi f_c} = \frac{1}{4\pi \times 10^3}$$

10^8 Hz may be regarded as high frequency. Hence the high-frequency gain is

$$\frac{-R_f}{R} = \frac{-10}{4} \qquad \text{or} \qquad R_f = 2.5R$$

If we let $R = \mathbf{10\ k\Omega}$, then $R_f = \mathbf{25\ k\Omega}$, and $C = \dfrac{1}{4000\pi \times 10^4} = \mathbf{7.96\ nF}$.

Chapter 14, Solution 70.

(a) $$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_4 (Y_1 + Y_2 + Y_3)}$$

where $Y_1 = \frac{1}{R_1} = G_1,\ Y_2 = \frac{1}{R_2} = G_2,\ Y_3 = sC_1,\ Y_4 = sC_2$.

$$\mathbf{H}(s) = \frac{\mathbf{G_1 G_2}}{\mathbf{G_1 G_2 + sC_2 (G_1 + G_2 + sC_1)}}$$

(b) $$H(0) = \frac{G_1 G_2}{G_1 G_2} = 1, \qquad H(\infty) = 0$$

showing that **this circuit is a lowpass filter**.

Chapter 14, Solution 71.

$R = 50\,\Omega,\ L = 40\text{ mH},\ C = 1\,\mu\text{F}$

$$L' = \frac{K_m}{K_f} L \longrightarrow 1 = \frac{K_m}{K_f} \cdot (40 \times 10^{-3})$$

$$25K_f = K_m \qquad (1)$$

$$C' = \frac{C}{K_m K_f} \longrightarrow 1 = \frac{10^{-6}}{K_m K_f}$$

$$10^6 K_f = \frac{1}{K_m} \qquad (2)$$

Substituting (1) into (2),

$$10^6 K_f = \frac{1}{25K_f}$$

$$K_f = \mathbf{2x10^{-4}}$$

$$K_m = 25K_f = \mathbf{5 \times 10^{-3}}$$

Chapter 14, Solution 72.

Design a problem to help other students to better understand magnitude and frequency scaling.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

What values of K_m and K_f will scale a 4-mH inductor and a 20-μF capacitor to 1 H and 2 F respectively?

Solution

$$L'C' = \frac{LC}{K_f^2} \longrightarrow K_f^2 = \frac{LC}{L'C'}$$

$$K_f^2 = \frac{(4\times 10^{-3})(20\times 10^{-6})}{(1)(2)} = 4\times 10^{-8}$$

$$K_f = \mathbf{2\times 10^{-4}}$$

$$\frac{L'}{C'} = \frac{L}{C}K_m^2 \longrightarrow K_m^2 = \frac{L'}{C'}\cdot\frac{C}{L}$$

$$K_m^2 = \frac{(1)(20\times 10^{-6})}{(2)(4\times 10^{-3})} = 2.5\times 10^{-3}$$

$$K_m = \mathbf{5\times 10^{-2}}$$

Chapter 14, Solution 73.

$$R' = K_m R = (12)(800\times10^{3}) = \mathbf{9.6\ M\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{800}{1000}(40\times10^{-6}) = \mathbf{32\ \mu F}$$

$$C' = \frac{C}{K_m K_f} = \frac{300\times10^{-9}}{(800)(1000)} = \mathbf{0.375\ pF}$$

Chapter 14, Solution 74.

$$R'_1 = K_m R_1 = 3x100 = \underline{300\Omega}$$

$$R'_2 = K_m R_2 = 10x100 = \underline{1\,k\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{10^2}{10^6}(2) = \underline{200\,\mu H}$$

$$C' = \frac{C}{K_m K_f} = \frac{\frac{1}{10}}{10^8} = \underline{1\,nF}$$

Chapter 14, Solution 75.

$$\mathbf{R' = K_m R = 20x10 = \underline{200\,\Omega}}$$

$$\mathbf{L' = \frac{K_m}{K_f} L = \frac{10}{10^5}(4) = \underline{400\,\mu H}}$$

$$\mathbf{C' = \frac{C}{K_m K_f} = \frac{1}{10x10^5} = \underline{1\,\mu F}}$$

Chapter 14, Solution 76.

$$R' = K_m R = 500x5x10^3 = \underline{25\ \text{M}\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{500}{10^5}(10mH) = \underline{50\ \mu\text{H}}$$

$$C' = \frac{C}{K_m K_f} = \frac{20x10^{-6}}{500x10^5} = \underline{0.4\ \text{pF}}$$

Chapter 14, Solution 77.

L and C are needed before scaling.

$$B = \frac{R}{L} \longrightarrow L = \frac{R}{B} = \frac{10}{5} = 2\ \text{H}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega_0^2 L} = \frac{1}{(1600)(2)} = 312.5\ \mu\text{F}$$

(a) $L' = K_m L = (600)(2) = \mathbf{1.200\ kH}$

$$C' = \frac{C}{K_m} = \frac{3.125\times10^{-4}}{600} = \mathbf{0.5208\ \mu F}$$

(b) $L' = \frac{L}{K_f} = \frac{2}{10^3} = \mathbf{2\ mH}$

$$C' = \frac{C}{K_f} = \frac{3.125\times10^{-4}}{10^3} = \mathbf{312.5\ nF}$$

(c) $L' = \frac{K_m}{K_f} L = \frac{(400)(2)}{10^5} = \mathbf{8\ mH}$

$$C' = \frac{C}{K_m K_f} = \frac{3.125\times10^{-4}}{(400)(10^5)} = \mathbf{7.81\ pF}$$

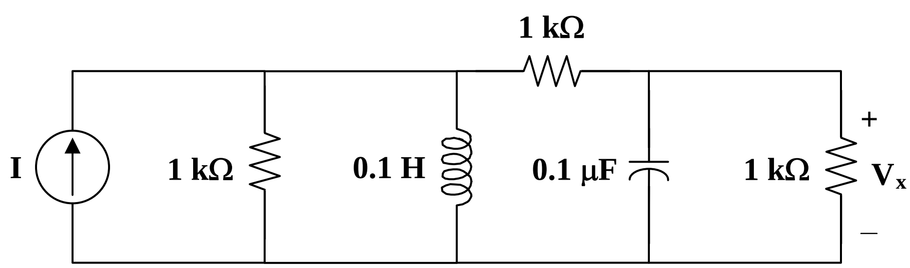
Chapter 14, Solution 78.

$$R' = K_m R = (1000)(1) = 1\text{ k}\Omega$$

$$L' = \frac{K_m}{K_f} L = \frac{10^3}{10^4}(1) = 0.1\text{ H}$$

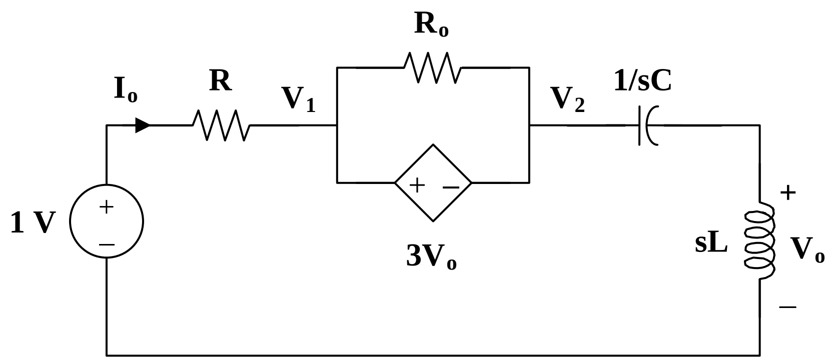
$$C' = \frac{C}{K_m K_f} = \frac{1}{(10^3)(10^4)} = 0.1\ \mu\text{F}$$

The new circuit is shown below.



Chapter 14, Solution 79.

(a) Insert a 1-V source at the input terminals.



There is a supernode.

$$\frac{1-\mathbf{V}_1}{R} = \frac{\mathbf{V}_2}{sL + 1/sC} \qquad (1)$$

But $\quad \mathbf{V}_1 = \mathbf{V}_2 + 3\mathbf{V}_o \quad \longrightarrow \quad \mathbf{V}_2 = \mathbf{V}_1 - 3\mathbf{V}_o \qquad (2)$

Also, $\quad \mathbf{V}_o = \dfrac{sL}{sL + 1/sC}\mathbf{V}_2 \quad \longrightarrow \quad \dfrac{\mathbf{V}_o}{sL} = \dfrac{\mathbf{V}_2}{sL + 1/sC} \qquad (3)$

Combining (2) and (3)

$$\mathbf{V}_2 = \mathbf{V}_1 - 3\mathbf{V}_o = \frac{sL + 1/sC}{sL}\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{s^2LC}{1 + 4s^2LC}\mathbf{V}_1 \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$\frac{1-\mathbf{V}_1}{R} = \frac{\mathbf{V}_o}{sL} = \frac{sC}{1 + 4s^2LC}\mathbf{V}_1$$

$$1 = \mathbf{V}_1 + \frac{sRC}{1 + 4s^2LC}\mathbf{V}_1 = \frac{1 + 4s^2LC + sRC}{1 + 4s^2LC}\mathbf{V}_1$$

$$\mathbf{V}_1 = \frac{1 + 4s^2LC}{1 + 4s^2LC + sRC}$$

$$\mathbf{I}_o = \frac{1-\mathbf{V}_1}{R} = \frac{sRC}{R\,(1 + 4s^2LC + sRC)}$$

$$\mathbf{Z}_{in} = \frac{1}{\mathbf{I}_o} = \frac{1 + sRC + 4s^2LC}{sC}$$

$$\mathbf{Z}_{in} = 4sL + R + \frac{1}{sC} \qquad (5)$$

When $R = 5$, $L = 2$, $C = 0.1$,

$$\mathbf{Z}_{in}(s) = \mathbf{8s + 5 + \frac{10}{s}}$$

At resonance,

$$\mathrm{Im}(\mathbf{Z}_{in}) = 0 = 4\omega L - \frac{1}{\omega C}$$

or $$\omega_0 = \frac{1}{2\sqrt{LC}} = \frac{1}{2\sqrt{(0.1)(2)}} = \mathbf{1.118\ rad/s}$$

(b) After scaling,

$$R' \longrightarrow K_m R$$

$$4\,\Omega \longrightarrow 40\,\Omega$$

$$5\,\Omega \longrightarrow 50\,\Omega$$

$$L' = \frac{K_m}{K_f} L = \frac{10}{100}(2) = 0.2 \text{ H}$$

$$C' = \frac{C}{K_m K_f} = \frac{0.1}{(10)(100)} = 10^{-4}$$

From (5),

$$\mathbf{Z}_{in}(s) = \mathbf{0.8s + 50 + \frac{10^4}{s}}$$

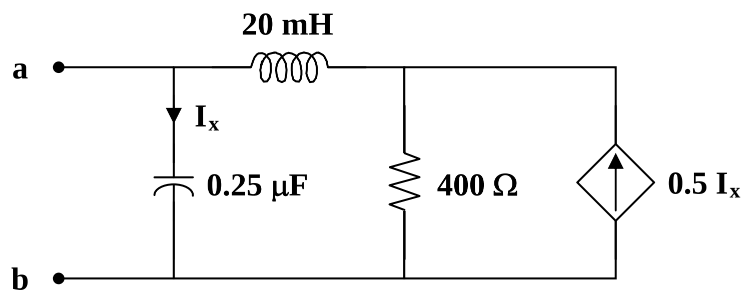
$$\omega_0 = \frac{1}{2\sqrt{LC}} = \frac{1}{2\sqrt{(0.2)(10^{-4})}} = \mathbf{111.8\ rad/s}$$

Chapter 14, Solution 80.

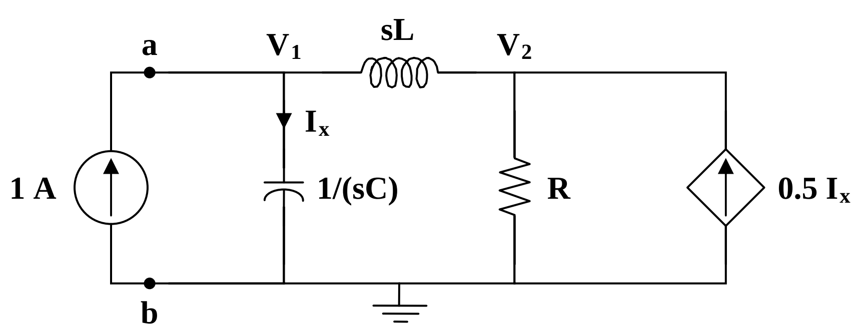
(a) $R' = K_m R = (200)(2) = 400\ \Omega$

$$L' = \frac{K_m L}{K_f} = \frac{(200)(1)}{10^4} = 20\text{ mH}$$
$$C' = \frac{C}{K_m K_f} = \frac{0.5}{(200)(10^4)} = 0.25\ \mu\text{F}$$

The new circuit is shown below.



(b) Insert a 1-A source at the terminals a-b.



At node 1,

$$1 = sC\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} + 0.5\mathbf{I}_x = \frac{\mathbf{V}_2}{R}$$

But, $\mathbf{I}_x = sC\,\mathbf{V}_1$.

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} + 0.5sC\,\mathbf{V}_1 = \frac{\mathbf{V}_2}{R} \qquad (2)$$

Solving (1) and (2),

$$\mathbf{V}_1 = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_1}{1} = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

At $\omega = 10^4$,

$$\mathbf{Z}_{Th} = \frac{(j10^4)(20\times10^{-3}) + 400}{(j10^4)^2(20\times10^{-3})(0.25\times10^{-6}) + 0.5(j10^4)(0.25\times10^{-6})(400) + 1}$$

$$\mathbf{Z}_{Th} = \frac{400 + j200}{0.5 + j0.5} = 600 - j200$$

$$\mathbf{Z}_{Th} = \mathbf{632.5\angle -18.435° \text{ ohms}}$$

Chapter 14, Solution 81.

(a)

$$\frac{1}{Z} = G + j\omega C + \frac{1}{R + j\omega L} = \frac{(G + j\omega C)(R + j\omega L) + 1}{R + j\omega L}$$

which leads to $$Z = \frac{j\omega L + R}{-\omega^2 LC + j\omega(RC + LG) + GR + 1}$$

$$Z(\omega) = \frac{j\dfrac{\omega}{C} + \dfrac{R}{LC}}{-\omega^2 + j\omega\left(\dfrac{R}{L} + \dfrac{G}{C}\right) + \dfrac{GR + 1}{LC}} \qquad (1)$$

We compare this with the given impedance:

$$Z(\omega) = \frac{1000(j\omega + 1)}{-\omega^2 + 2j\omega + 1 + 2500} \qquad (2)$$

Comparing (1) and (2) shows that

$$\frac{1}{C} = 1000 \quad \longrightarrow \quad C = 1\,\text{mF}, \quad R/L = 1 \quad \longrightarrow \quad R = L$$

$$\frac{R}{L} + \frac{G}{C} = 2 \quad \longrightarrow \quad G = C = 1 \text{ mS}$$

$$2501 = \frac{GR + 1}{LC} = \frac{10^{-3}R + 1}{10^{-3}R} \quad \longrightarrow \quad R = 0.4 = L$$

Thus,

R = **0.4Ω**, L = **0.4 H**, C = **1 mF**, G = **1 mS**

(b) By frequency-scaling, K_f =1000.

R’ = **0.4 Ω**, G’ = **1 mS**

$$\mathbf{L' = \frac{L}{K_f} = \frac{0.4}{10^3} = \underline{0.4mH}}, \qquad \mathbf{C' = \frac{C}{K_f} = \frac{10^{-3}}{10^{-3}} = \underline{1\mu F}}$$

Chapter 14, Solution 82.

$$C' = \frac{C}{K_m K_f}$$

$$K_f = \frac{\omega'_c}{\omega} = \frac{200}{1} = 200$$

$$K_m = \frac{C}{C'} \cdot \frac{1}{K_f} = \frac{1}{10^{-6}} \cdot \frac{1}{200} = 5000$$

$R' = K_m R =$ **5 kΩ**, thus, $R'_f = 2R_i =$ **10 kΩ**

Chapter 14, Solution 83.

$$1\mu F \longrightarrow C' = \frac{1}{K_m K_f} C = \frac{10^{-6}}{100x10^5} = \underline{0.1\,pF}$$

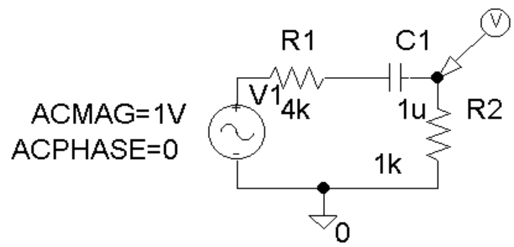
$$5\mu F \longrightarrow C' = \underline{0.5\,pF}$$

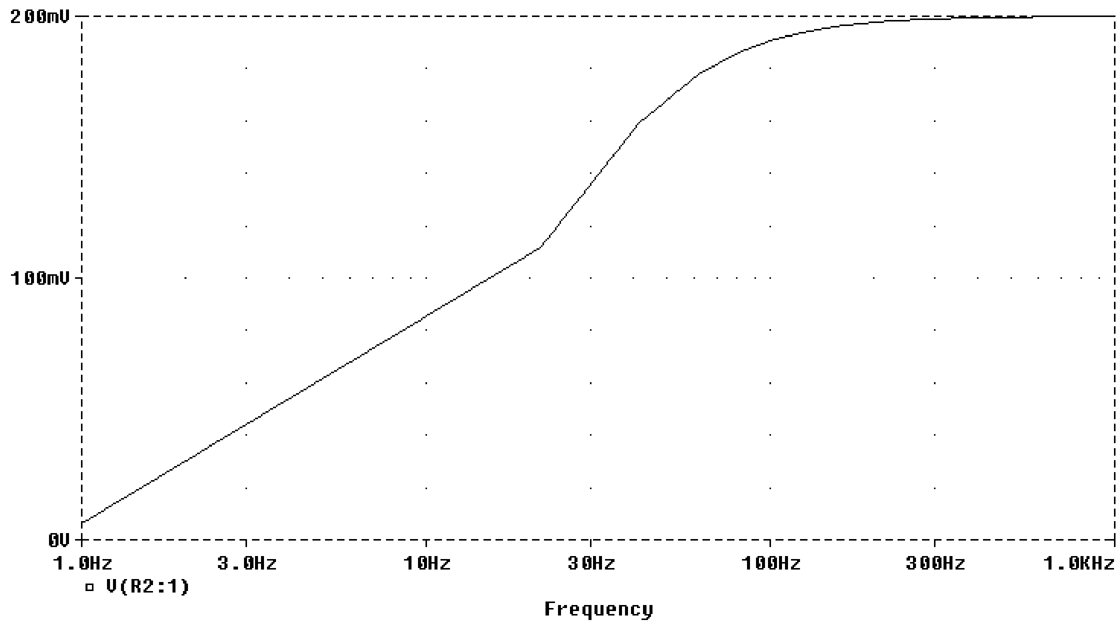
$$10\,k\Omega \longrightarrow R' = K_m R = 100x10\,k\Omega = \underline{1\,M\Omega}$$

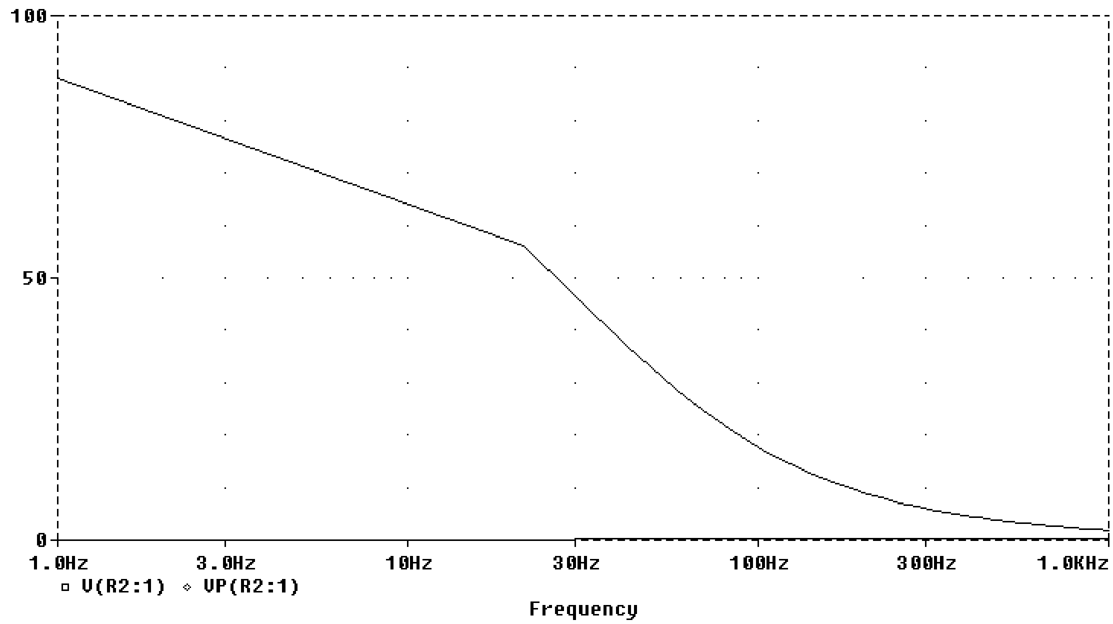
$$20\,k\Omega \longrightarrow R' = \underline{2\,M\Omega}$$

Chapter 14, Solution 84.

The schematic is shown below. A voltage marker is inserted to measure v_o. In the AC sweep box, we select Total Points = 50, Start Frequency = 1, and End Frequency = 1000. After saving and simulation, we obtain the magnitude and phase plots in the probe menu as shown below.



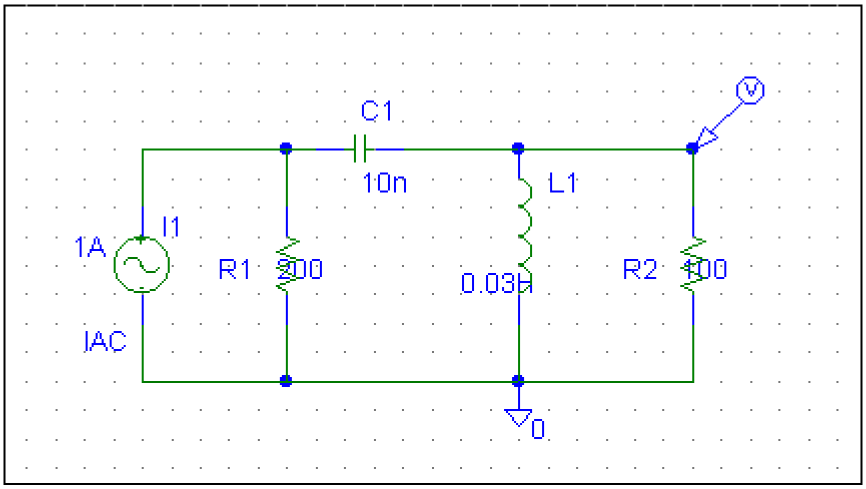


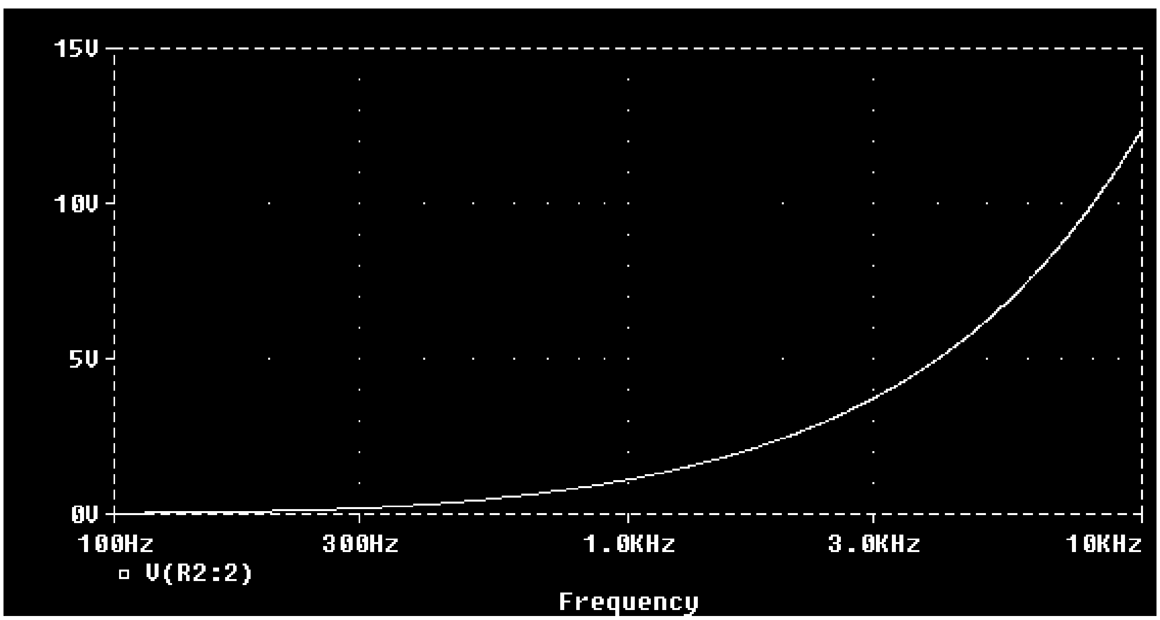


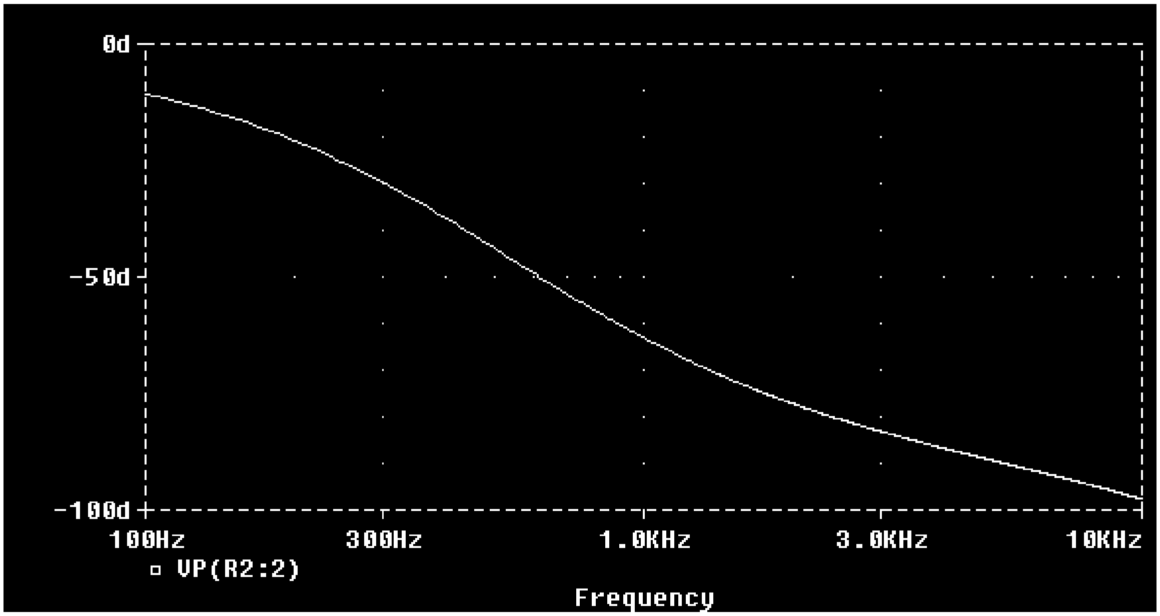
100
50
0
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
V(R2:1)
VP(R2:1)
Frequency

Chapter 14, Solution 85.

We let $I_s = 1\angle 0^o$ A so that $V_o / I_s = V_o$. The schematic is shown below. The circuit is simulated for 100 < f < 10 kHz.






0d
-50d
-100d
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
VP(R2:2)
Frequency

Chapter 14, Solution 86.

Using Fig. 14.103, design a problem to help other students to better understand how to use PSpice to obtain the frequency response (magnitude and phase of I) in electrical circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use *PSpice* to provide the frequency response (magnitude and phase of *i*) of the circuit in Fig. 14.103. Use linear frequency sweep from 1 to 10,000 Hz.

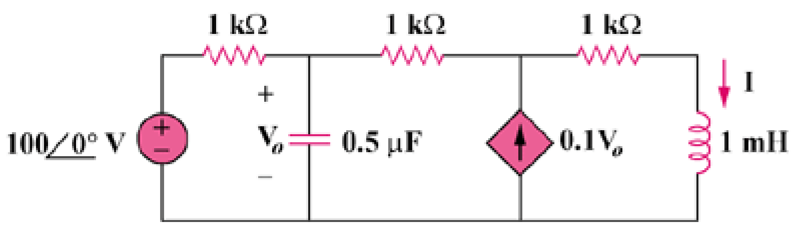
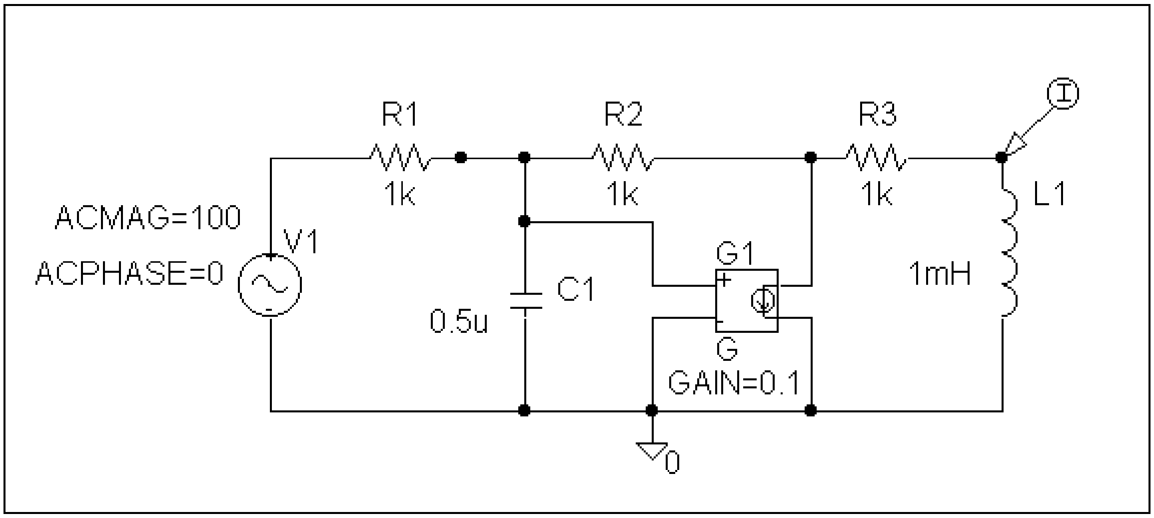
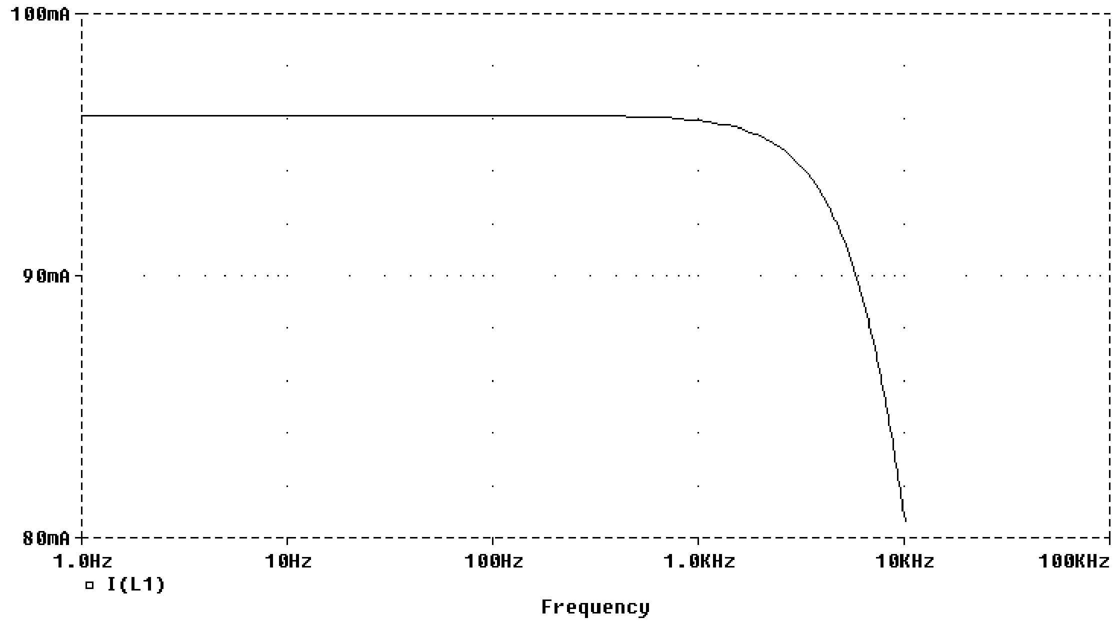


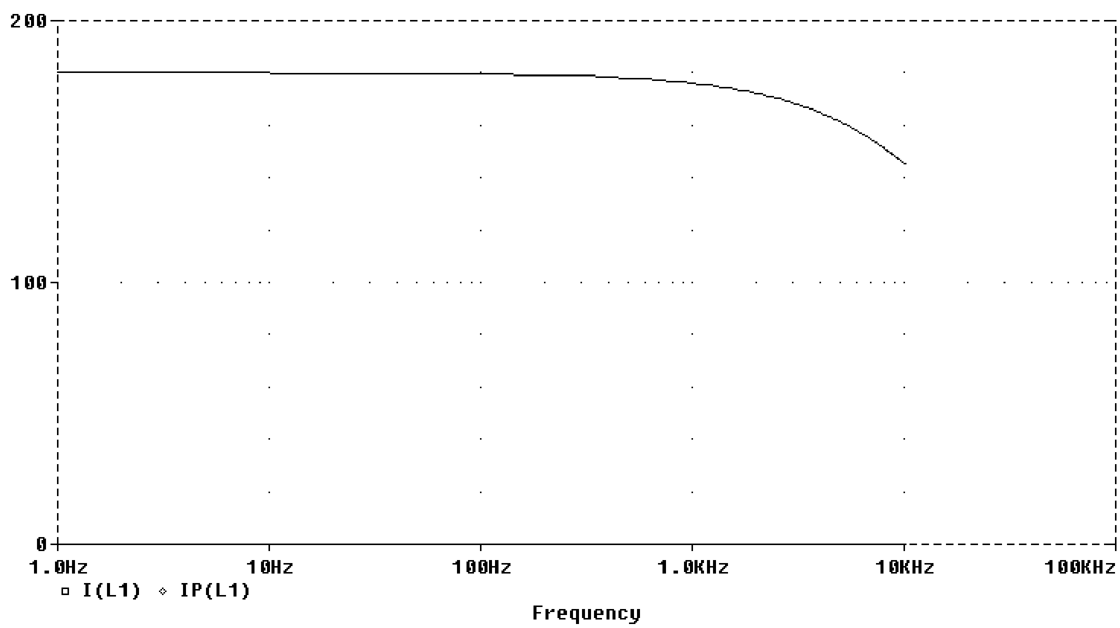
Figure 14.103

Solution

The schematic is shown below. A current marker is inserted to measure **I**. We set Total Points = 101, start Frequency = 1, and End Frequency = 10 kHz in the AC sweep box. After simulation, the magnitude and phase plots are obtained in the Probe menu as shown below.

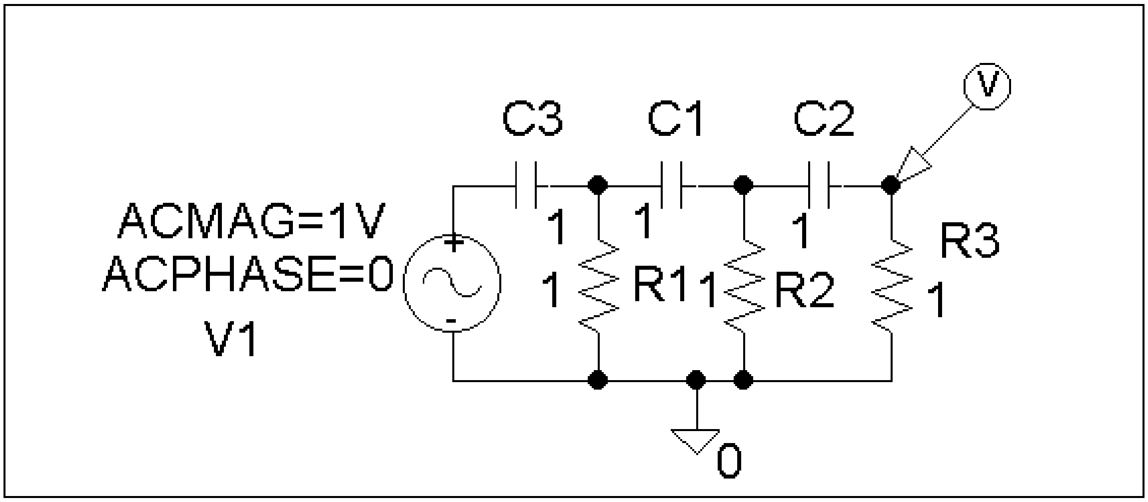


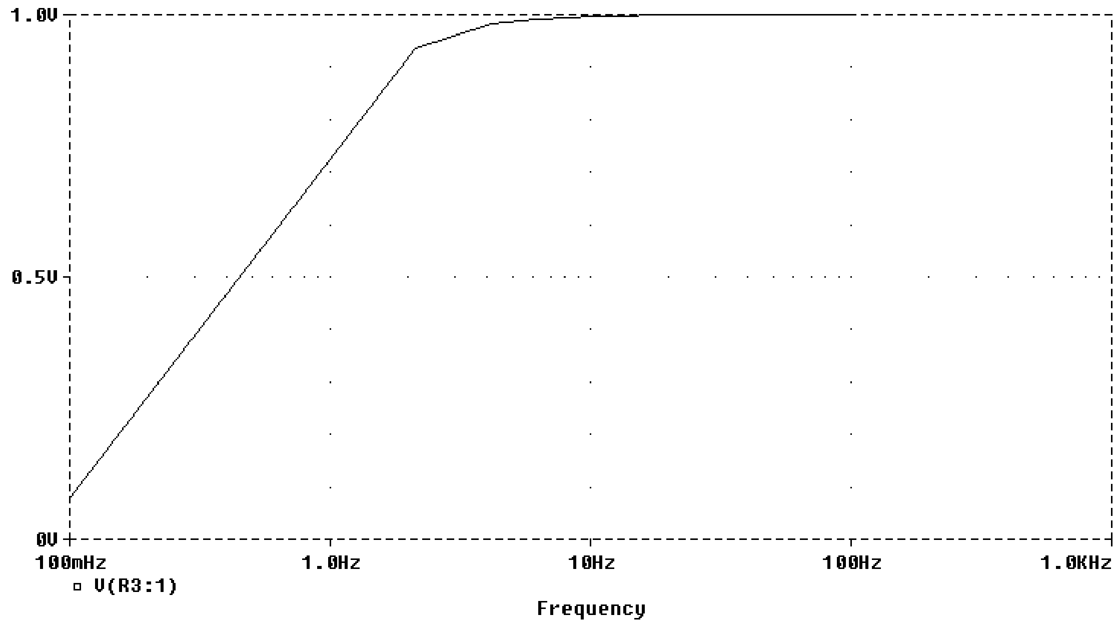

100mA
90mA
80mA
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
100KHz
I(L1)
Frequency


200
100
0
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
100KHz
I(L1) IP(L1)
Frequency

Chapter 14, Solution 87.

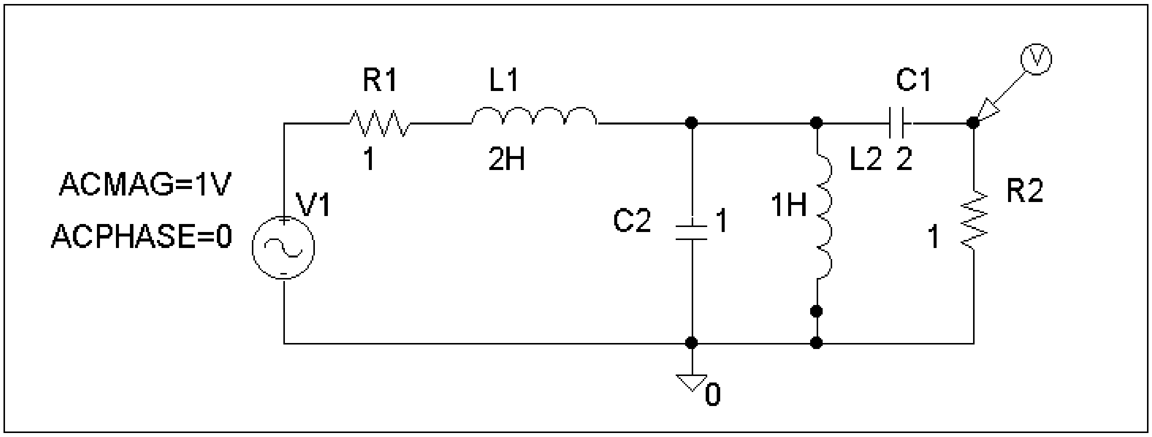
The schematic is shown below. I_n the AC Sweep box, we set Total Points = 50, Start Frequency = 1, and End Frequency = 100. After simulation, we obtain the magnitude response as shown below. It is evident from the response that the circuit represents a high-pass filter.

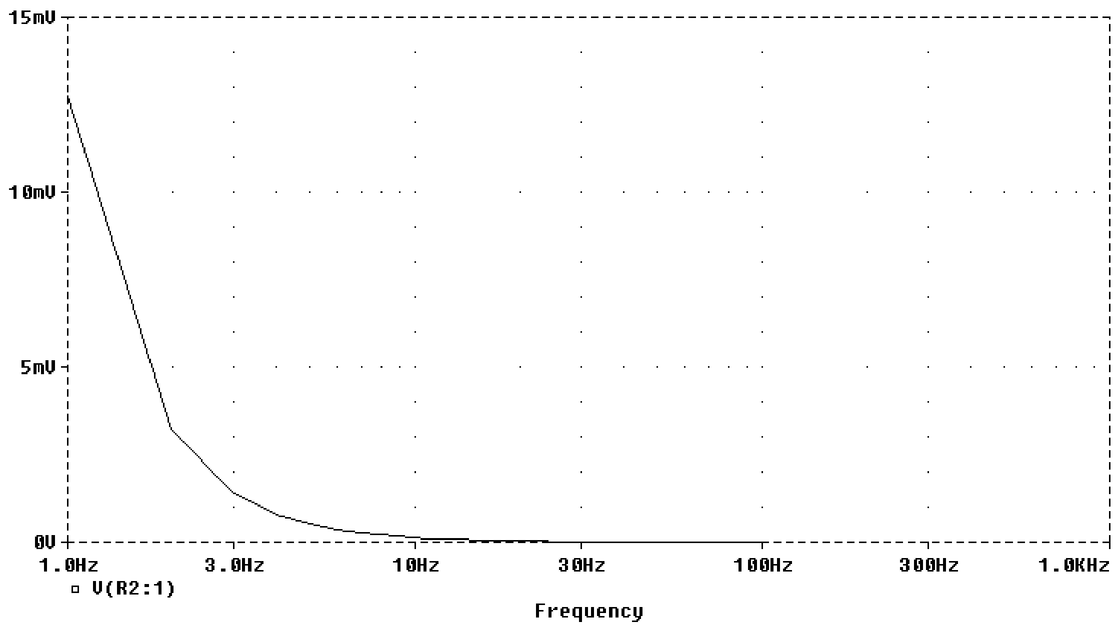


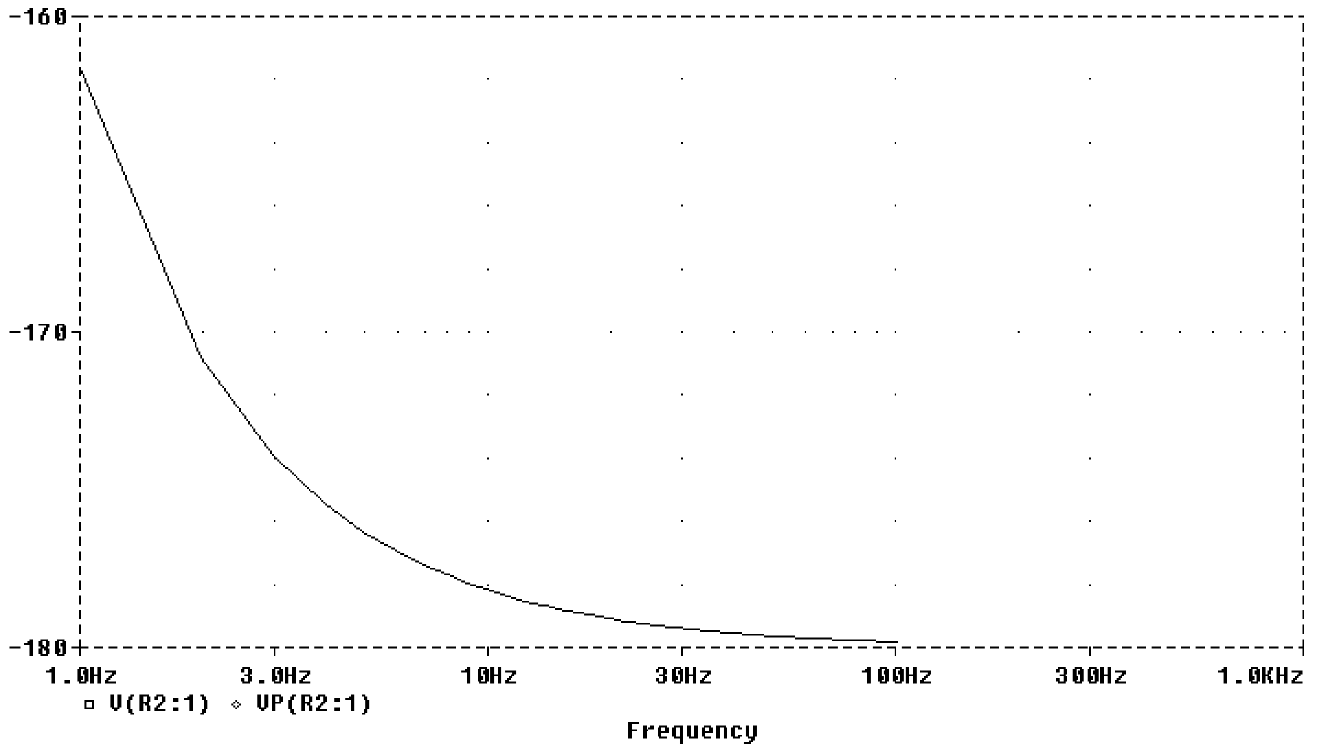


Chapter 14, Solution 88.

The schematic is shown below. We insert a voltage marker to measure V_o. In the AC Sweep box, we set Total Points = 101, Start Frequency = 1, and End Frequency = 100. After simulation, we obtain the magnitude and phase plots of V_o as shown below.

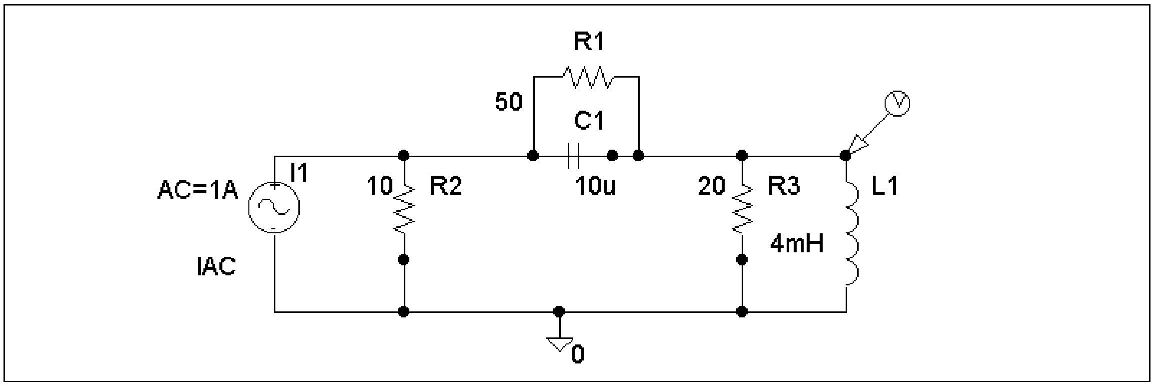


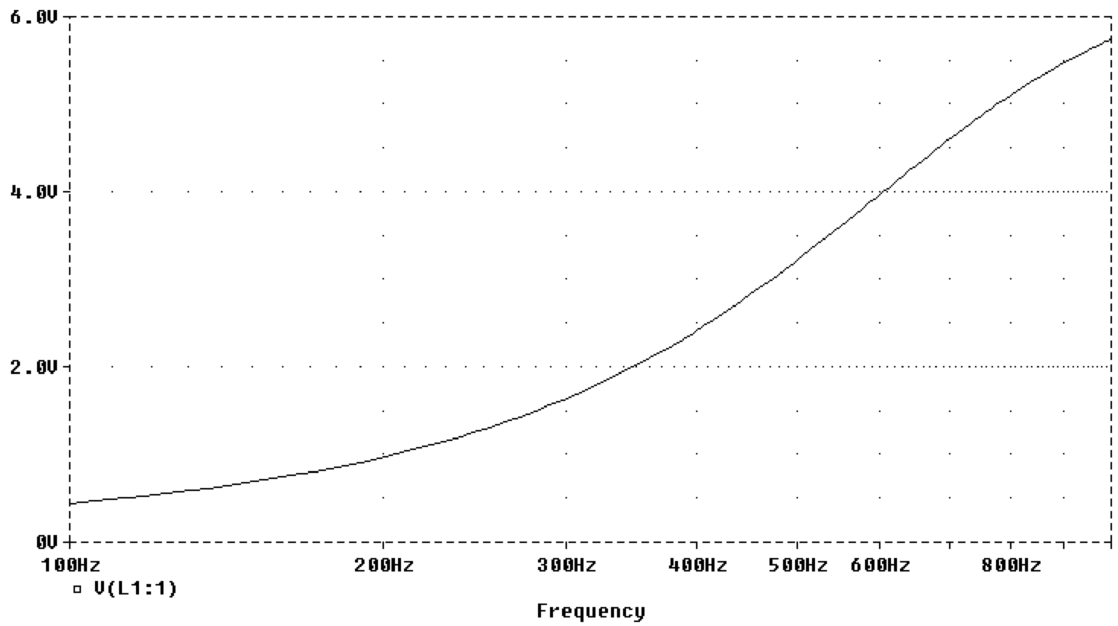



-160
-170
-180
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
V(R2:1)
VP(R2:1)
Frequency

Chapter 14, Solution 89.

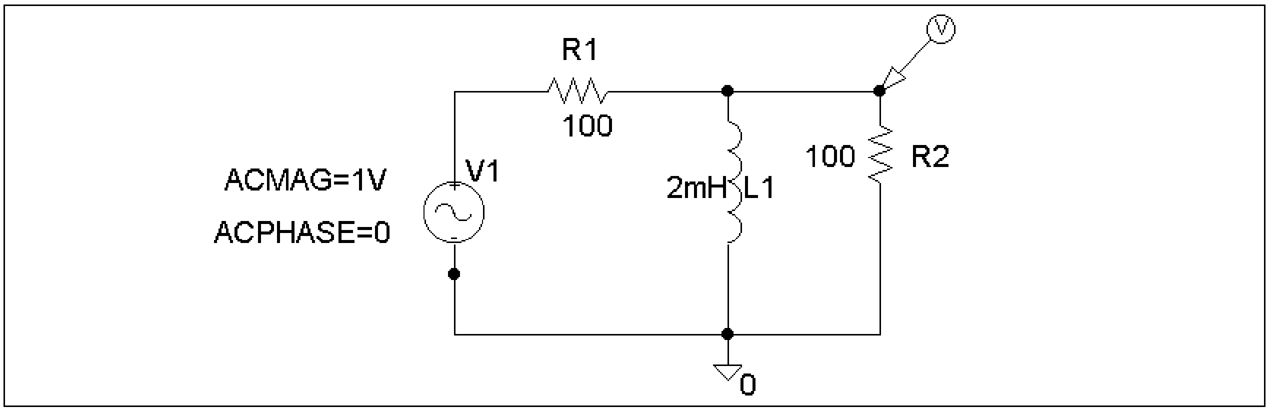
The schematic is shown below. In the AC Sweep box, we type Total Points = 101, Start Frequency = 100, and End Frequency = 1 k. After simulation, the magnitude plot of the response V_o is obtained as shown below.

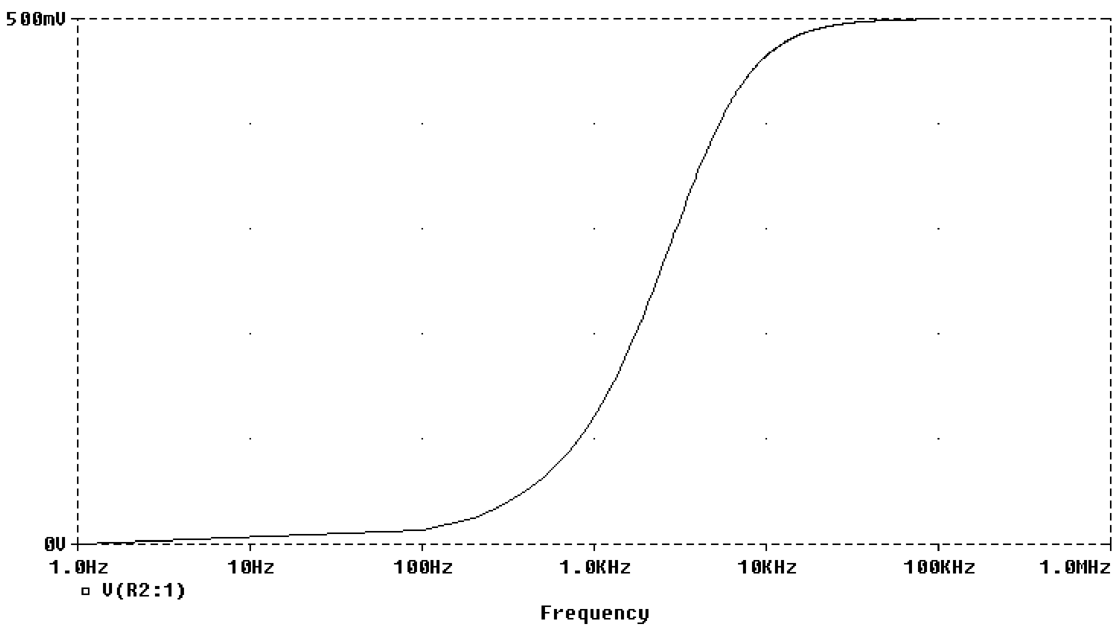




Chapter 14, Solution 90.

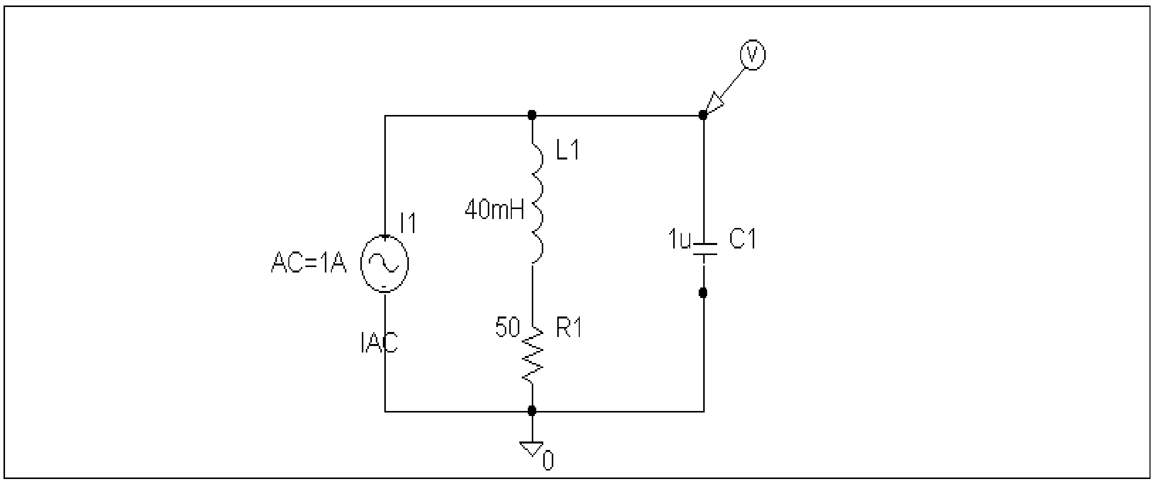
The schematic is shown below. In the AC Sweep box, we set Total Points = 1001, Start Frequency = 1, and End Frequency = 100k. After simulation, we obtain the magnitude plot of the response as shown below. The response shows that the circuit is a high-pass filter.

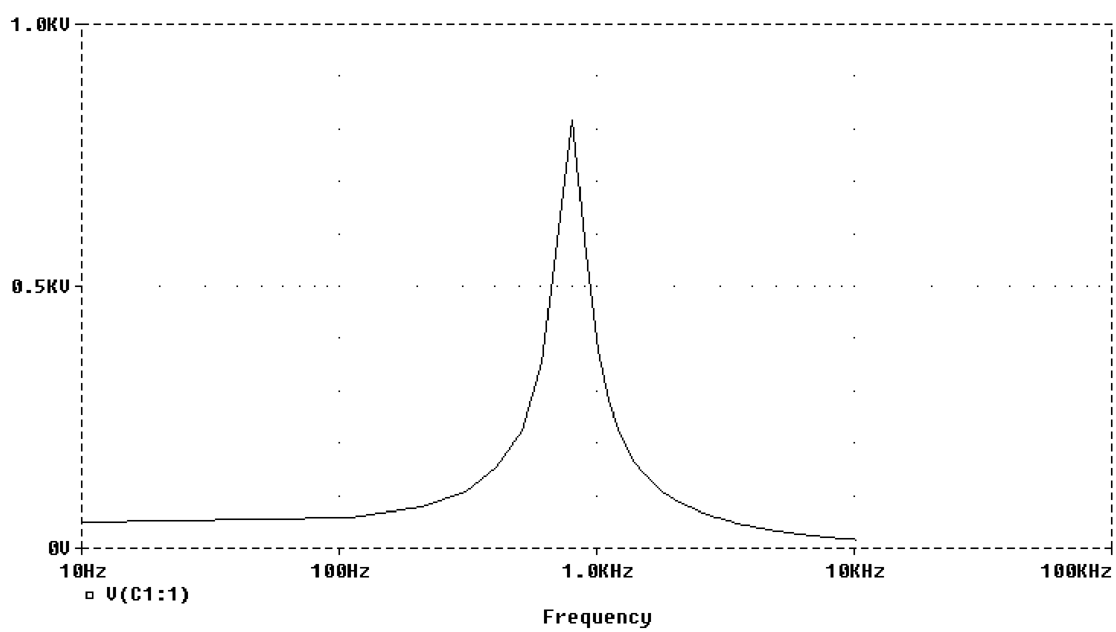




Chapter 14, Solution 91.

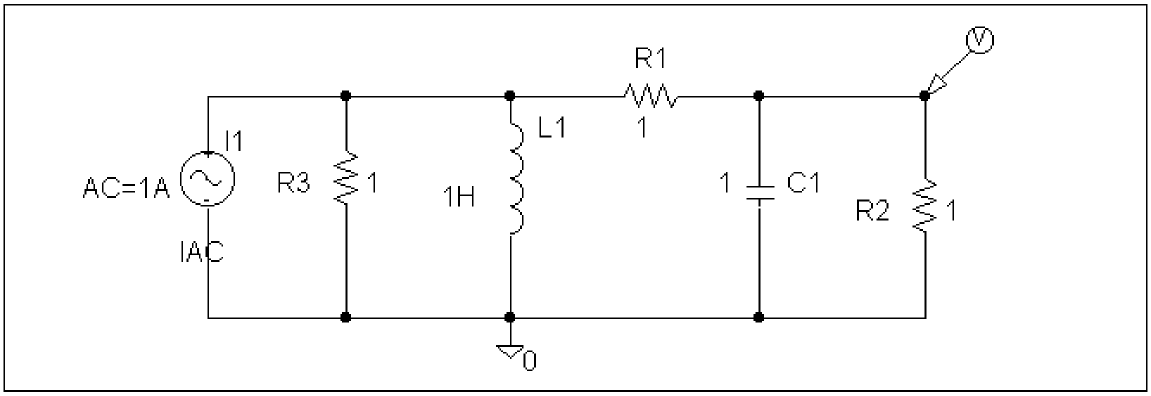
The schematic is shown below. In the AC Sweep box, we select Total Points = 101, Start Frequency = 10, and End Frequency = 10 k. After simulation, the magnitude plot of the frequency response is obtained. From the plot, we obtain the resonant frequency f_o is approximately equal to **800 Hz** so that $\omega_o = 2\pi f_o = 5026$ rad/s.

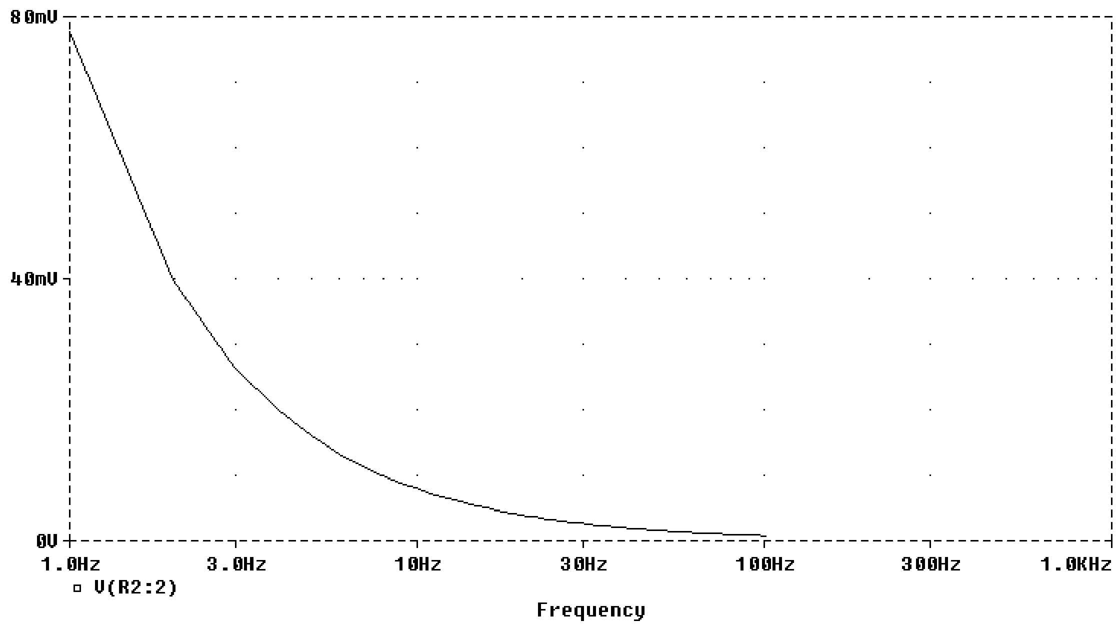




Chapter 14, Solution 92.

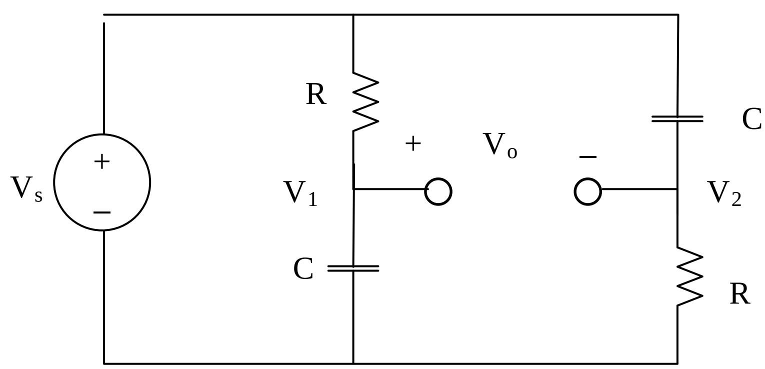
The schematic is shown below. We type Total Points = 101, Start Frequency = 1, and End Frequency = 100 in the AC Sweep box. After simulating the circuit, the magnitude plot of the frequency response is shown below.





Chapter 14, Solution 93.

Consider the circuit as shown below.



$$V_1 = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_s = \frac{V}{1 + sRC}$$

$$V_2 = \frac{R}{R + sC} V_s = \frac{sRC}{1 + sRC} V_s$$

$$V_o = V_1 - V_2 = \frac{1 - sRC}{1 + sRC} V_s$$

Hence,

$$H(s) = \frac{V_o}{V_s} = \frac{1 - sRC}{1 + sRC}$$

Chapter 14, Solution 94.

$$\omega_c = \frac{1}{RC}$$

We make R and C as small as possible. To achieve this, we connect 1.8 kΩ and 3.3 kΩ in parallel so that

$$R = \frac{1.8x3.3}{1.8 + 3.3} = 1.164\ k\Omega$$

We place the 10-pF and 30-pF capacitors in series so that

C = (10x30)/40 = 7.5 pF

Hence,

$$\boldsymbol{\omega_c = \frac{1}{RC} = \frac{1}{1.164x10^3\, x7.5x10^{-12}} = \underline{114.55x10^6\ rad/s}}$$

Chapter 14, Solution 95.

(a) $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

When $C = 360\text{ pF}$,

$$f_0 = \frac{1}{2\pi\sqrt{(240\times10^{-6})(360\times10^{-12})}} = 0.541\text{ MHz}$$

When $C = 40\text{ pF}$,

$$f_0 = \frac{1}{2\pi\sqrt{(240\times10^{-6})(40\times10^{-12})}} = 1.624\text{ MHz}$$

Therefore, the frequency range is

$$\mathbf{0.541\ MHz < f_0 < 1.624\ MHz}$$

(b) $$Q = \frac{2\pi fL}{R}$$

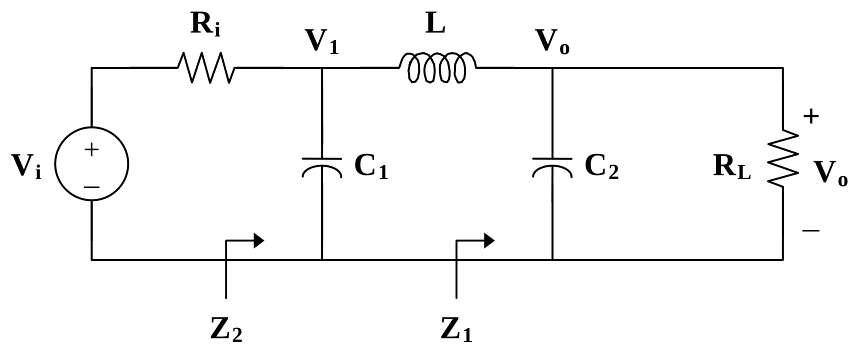
At $f_0 = 0.541\text{ MHz}$,

$$Q = \frac{(2\pi)(0.541\times10^{6})(240\times10^{-6})}{12} = \mathbf{67.98}$$

At $f_0 = 1.624\text{ MHz}$,

$$Q = \frac{(2\pi)(1.624\times10^{6})(240\times10^{-6})}{12} = \mathbf{204.1}$$

Chapter 14, Solution 96.



$$\mathbf{Z}_1 = R_L \parallel \frac{1}{sC_2} = \frac{R_L}{1 + sR_2C_2}$$

$$\mathbf{Z}_2 = \frac{1}{sC_1} \parallel (sL + \mathbf{Z}_1) = \frac{1}{sC_1} \parallel \left(\frac{sL + R_L + s^2R_LC_2L}{1 + sR_LC_2} \right)$$

$$\mathbf{Z}_2 = \frac{\dfrac{1}{sC_1} \cdot \dfrac{sL + R_L + s^2R_LC_2L}{1 + sR_LC_2}}{\dfrac{1}{sC_1} + \dfrac{sL + R_L + s^2R_LC_2L}{1 + sR_LC_2}}$$

$$\mathbf{Z}_2 = \frac{sL + R_L + s^2R_LLC_2}{1 + sR_LC_2 + s^2LC_1 + sR_LC_1 + s^3R_LLC_1C_2}$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_i}\mathbf{V}_i$$

$$\mathbf{V}_o = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL}\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_2} \cdot \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL}\mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_2} \cdot \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL}$$

where

$$\frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_2} =$$

$$\frac{sL + R_L + s^2R_LLC_2}{sL + R_L + s^2R_LLC_2 + R_i + sR_iR_LC_2 + s^2R_iLC_1 + sR_iR_LC_1 + s^3R_iR_LLC_1C_2}$$

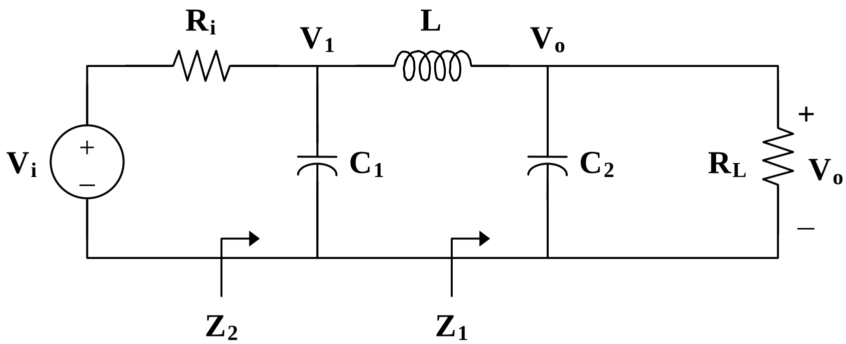
and $$\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL} = \frac{R_L}{R_L + sL + s^2R_LLC_2}$$

Therefore,

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{R_L(sL + R_L + s^2 R_L L C_2)}}{\begin{array}{l}\mathbf{(sL + R_L + s^2 R_L L C_2 + R_i + sR_i R_L C_2 + s^2 R_i L C_1 + sR_i R_L C_1} \\ \quad \mathbf{+ s^3 R_i R_L L C_1 C_2)(R_L + sL + s^2 R_L L C_2)}\end{array}}$$

where $s = j\omega$.

Chapter 14, Solution 97.



$$\mathbf{Z} = sL \,\|\, \left(R_L + \frac{1}{sC_2} \right) = \frac{sL\,(R_L + 1/sC_2)}{R_L + sL + 1/sC_2}, \qquad s = j\omega$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}}{\mathbf{Z} + R_i + 1/sC_1}\mathbf{V}_i$$

$$\mathbf{V}_o = \frac{R_L}{R_L + 1/sC_2}\mathbf{V}_1 = \frac{R_L}{R_L + 1/sC_2} \cdot \frac{\mathbf{Z}}{\mathbf{Z} + R_i + 1/sC_1}\mathbf{V}_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_L}{R_L + 1/sC_2} \cdot \frac{sL\,(R_L + 1/sC_2)}{sL\,(R_L + 1/sC_2) + (R_i + 1/sC_1)(R_L + sL + 1/sC_2)}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{s^3 L R_L C_1 C_2}}{\mathbf{(sR_i C_1 + 1)(s^2 L C_2 + sR_L C_2 + 1) + s^2 L C_1 (sR_L C_2 + 1)}}$$

where $s = j\omega$.

Chapter 14, Solution 98.

$$B = \omega_2 - \omega_1 = 2\pi(f_2 - f_1) = 2\pi(454 - 432) = 44\pi$$

$$\omega_0 = 2\pi f_0 = QB = (20)(44\pi)$$

$$f_0 = \frac{(20)(44\pi)}{2\pi} = (20)(22) = \mathbf{440\ Hz}$$

Chapter 14, Solution 99.

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f\, C}$$

$$C = \frac{1}{2\pi f\, X_c} = \frac{1}{(2\pi)(2\times 10^6)(5\times 10^3)} = \frac{10^{-9}}{20\pi}$$

$$X_L = \omega L = 2\pi f\, L$$

$$L = \frac{X_L}{2\pi f} = \frac{300}{(2\pi)(2\times 10^6)} = \frac{3\times 10^{-4}}{4\pi}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{3\times 10^{-4}}{4\pi}\cdot\frac{10^{-9}}{20\pi}}} = \mathbf{8.165\ MHz}$$

$$B = \frac{R}{L} = (100)\left(\frac{4\pi}{3\times 10^{-4}}\right) = \mathbf{4.188\times 10^6\ rad/s}$$

Chapter 14, Solution 100.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$R = \frac{1}{2\pi f_c C} = \frac{1}{(2\pi)(20\times10^{3})(0.5\times10^{-6})} = \mathbf{15.91\,\Omega}$$

Chapter 14, Solution 101.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$R = \frac{1}{2\pi f_c C} = \frac{1}{(2\pi)(15)(10 \times 10^{-6})} = \mathbf{1.061\ k\Omega}$$

Chapter 14, Solution 102.

(a) When $R_s = 0$ and $R_L = \infty$, we have a low-pass filter.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(4\times 10^{3})(40\times 10^{-9})} = \mathbf{994.7\ Hz}$$

(b) We obtain R_{Th} across the capacitor.

$$R_{Th} = R_L \parallel (R + R_s)$$

$$R_{Th} = 5 \parallel (4+1) = 2.5\ \text{k}\Omega$$

$$f_c = \frac{1}{2\pi R_{Th} C} = \frac{1}{(2\pi)(2.5\times 10^{3})(40\times 10^{-9})}$$

$$f_c = \mathbf{1.59\ kHz}$$

Chapter 14, Solution 103.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2}{R_2 + R_1 \parallel 1/j\omega C}, \qquad s = j\omega$$

$$\mathbf{H}(s) = \frac{R_2}{R_2 + \dfrac{R_1(1/sC)}{R_1 + 1/sC}} = \frac{R_2(R_1 + 1/sC)}{R_1 R_2 + (R_1 + R_2)(1/sC)}$$

$$\mathbf{H}(s) = \frac{\mathbf{R_2(1 + sCR_1)}}{\mathbf{R_1 + R_2 + sCR_1R_2}}$$

Chapter 14, Solution 104.

The schematic is shown below. We click Analysis/Setup/AC Sweep and enter Total Points = 1001, Start Frequency = 100, and End Frequency = 100 k. After simulation, we obtain the magnitude plot of the response as shown.