

FINAL FOLLOWAGE

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Alpha Numeric

Repository from where we will work:

Page from where we will work:

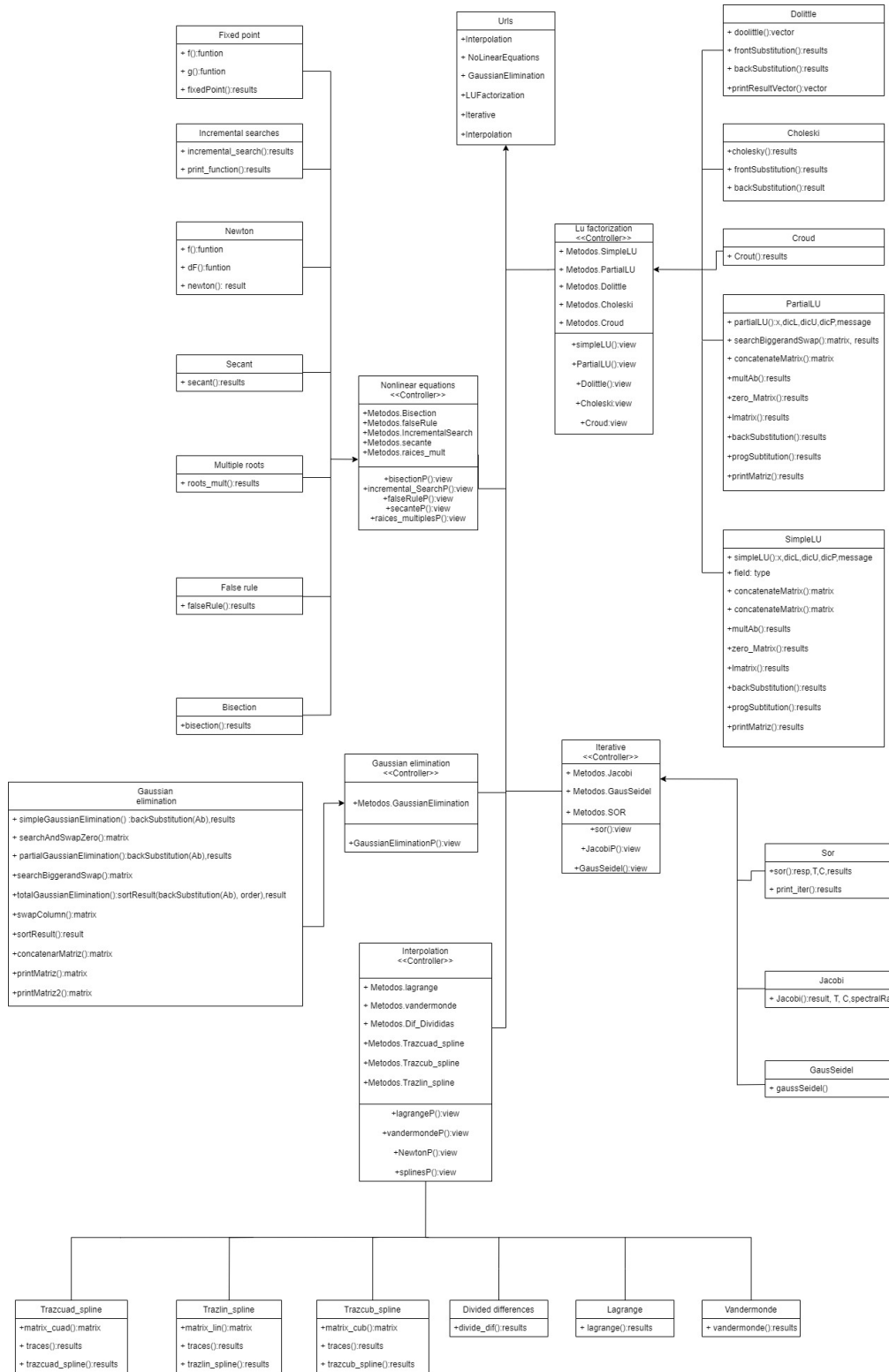
<https://dry-beyond-30251.herokuapp.com/>

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Class diagram



Use case diagram

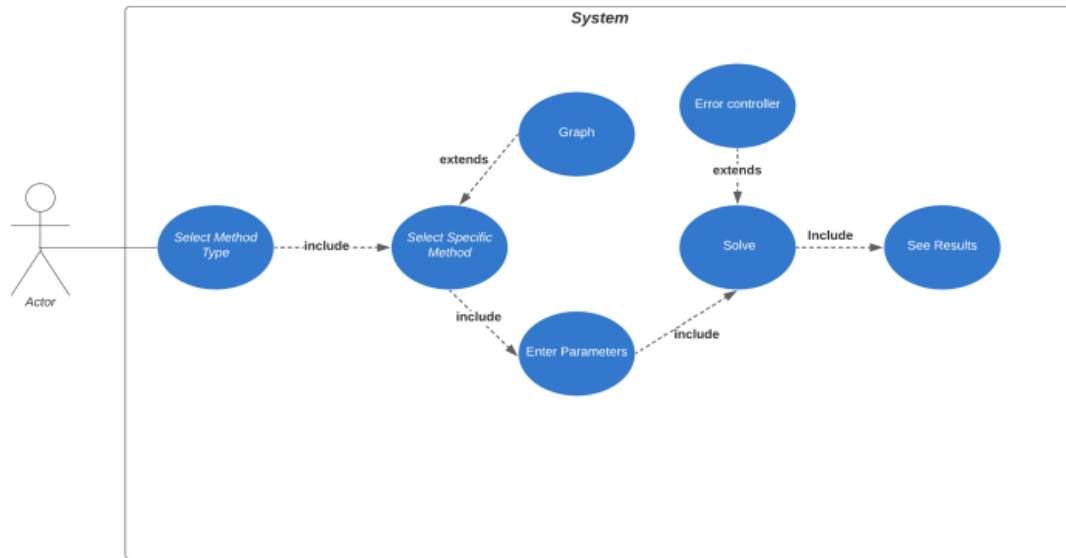
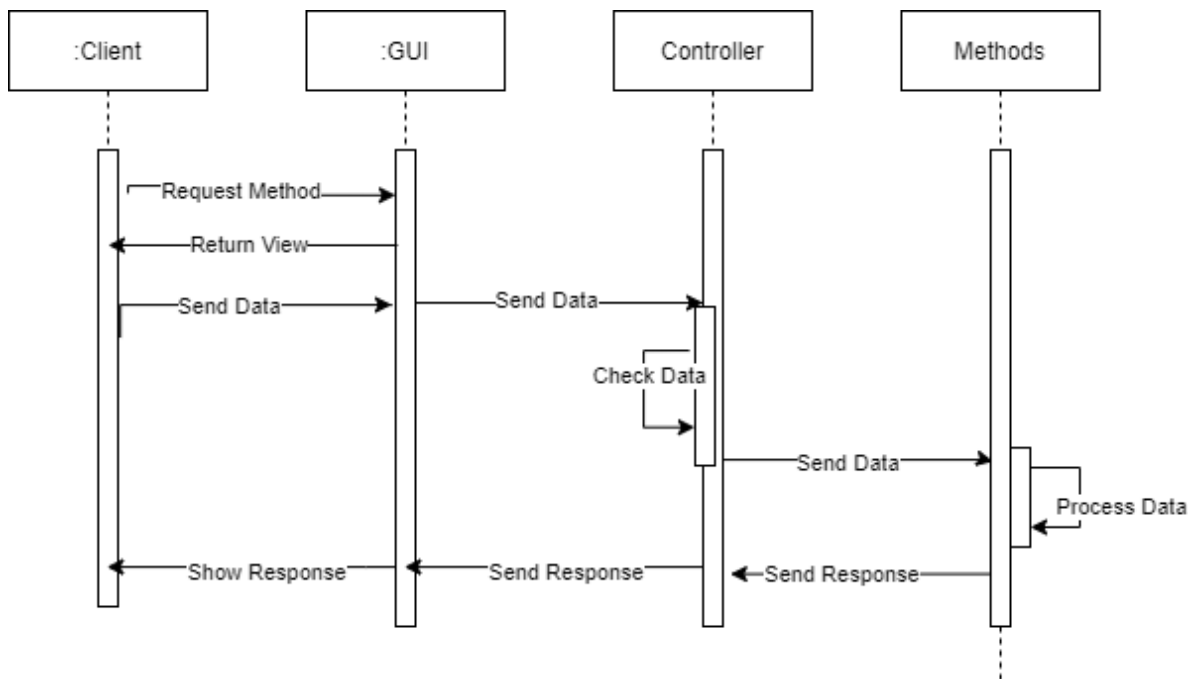


Diagram sequence



Pseudocodes

Bisection:

read xi, xs, tolerancia, niter, funcion

```

    fxi = f(xi)
    fxs = f(xs)

    if (fxi == 0)
        write "xi es raíz"
    else if (fxs == 0)
        write "xs es raíz"
    else if (fxi * fxs < 0 then)

        xm = (xi + xs)/2
        fxm = f(xm)
        error = tolerancia + 1
        count = 1

    while ( fxm != 0 and error > tolerancia and contador < niter)
        if (fxi * fxm < 0 )
            xs = xm
        else
            xi = xm
            fxi = fxm
        end if

        xaux = xm
        xm = (xi + xs) / 2
        fxm = f(xm)
        error = abs(xm - xaux)
        count = count + 1
    end while

    else
        write "el intervalo es inadecuado"
    end if
end if

```

```
end if  
end
```

Cholesky:

```
read A, b  
determinante=det(A);  
  
if determinante==0 then  
return "El sistema no tiene una única solución"  
end if  
  
n = lenght A  
L,U = Iniciar LU(n)  
for k=1 hasta n  
    suma1=0;  
    for p=1:k-1  
suma1=suma1+L(k,p)* U(p,k);  
  
    end for  
L(k,k)* U(k,k)=A(k,k) - suma1;
```



```

    for i=k+1 hasta n
        suma2= 0;
        for p = 1 hasta k-1
            suma2= suma2+ L(i,p)* U(p,k);
        end for
        L(i,k) = (A(i,k)-suma2)/U(k,k);
    end for

    for j=k+1 hasta n
        suma3= 0;
        for p = 1 hasta k-1
            suma3=suma3+ L(k,p)* U(p,j);
        end for
        U(k,j) = (A(k,j)- suma3)/L(k,k);
    end for
end for
return L,U
end

```

Crout:

```
read A, b
n = lenght A
L,U = Iniciar LU(n)
for k=0 hasta n
    U(k,k)=1;
    for i=k hasta n
        suma1 = 0
        for p=0 hasta k
            suma1=suma1+L(k,p)* U(p,k);

        end for
        U(k,j)=A(k,j) - suma1;

        for j=k+1 hasta n
            suma2= 0;
            for p = 0 hasta k
                suma2= suma2+ L(k,p)* U(p,j);
            end for
            U(k,j) = (A(k,j)-suma2)/L(k,k);
        end for
    end for

    detA = 1;
    for i=1 hasta n
        detA = detA* L(i,i)
    end for

    if (detA  $\neq$  0 then)
```

```

    z = sustituir(L,b)
    x = sustituir(U,z)
    else
return "Hay infinitas soluciones o no tiene solución"
    end if
return L,U

```

Divide diferencias:

```

    leer x,y:

    n = longitud(x)
    D = matriz de 0

    D[:,0] = y->traspuesta

    para i hasta n:
    aux0 = D[i-1:n,i-1]

```

```

    aux1 = diferenciaAdyacente(aux0)
    aux2 = restaVectorial(x[i:n],x[0:n-1-i+1])
    D[i:n,i] = DivisionVectorial(aux1,traspuesta(aux2))
    fin

    res = diagonal(D)

    r = res[0]
    m = '(x' + (-x[0]) + ')'
    para i hasta n:
        r += res[i] + m
        m += '(x' + -x[i] + ')'
    fin

    escribir('Matrix D: \ n',D)
    escribir('Coef: ',res)
    escribir('Newton Polinom : ', r)
    fin

```

Doolittle:

```

    read A, b
    L,U = Iniciar LU(n)
    for k=0 hasta n
        L(k,k)=1;
        for j=k hasta n
            suma1 = 0
            for p=0 hasta k
                suma1=suma1+L(k,p)* U(p,k);
            end for
            U(k,j)=A(k,j) - suma1;
        end for
    end for

```

```

    for i=k+1 hasta n
        suma2= 0;
        for p = 0 hasta k
            suma2= suma2+ L(i,p)* U(p,k);
        end for
        L(i,k) = (A(i,k)-suma2)/U(k,k);
    end for
end for

detA = 1;
for i=1 hasta n
    detA = detA* U(i,i)
end for

if (detA  $\neq$  0 then)
    z = substituir(L,b)
    x = substituir(U,z)
else
return "Hay infinitas soluciones o no tiene solución"
end if

return L,U

```

Fixed point:

```
read xa, tol, iter

y = f(xa)
cont = 0
error = tol + 1

while(y != 0 & error > tol & cont < iter)
    xn = g(xa)
    y = f(xn)
    error = abs(xn - xa)
    xa = xn
    cont += 1
end while
if (y == 0)
    write "xa is root"
else if (error < tol)
write "xa approximate root with tolerance: tol)"
    else
write "Fail in iteration: iter"
    end if
```

Gaussian Elimination:

```
simpleGaussianElimination
read a, b
```

```

AB = concatenar(a, b)
n = len(AB)
while k < n do
    write step k
    write AB
    if AB[k][k] == 0
AB = searchAndSwapZero(Ab, n, k)
    while i = k+1 < n do
        mult = Ab[i][k]/Ab[k][k]
        while j = k < n+1 do
            Ab[i][j] = Ab[i][j] - mult* Ab[k][j]

```

partialGaussianElimination

```

    read a, b
    AB = concatenar(a, b)
    n = len(AB)
    while k < n do
        write step k
        write AB
AB = searchBiggerandSwap(Ab, n, k)
    while i = k+1 < n do
        mult = Ab[i][k]/Ab[k][k]
        while j = k < n+1 do
            Ab[i][j] = Ab[i][j] - mult* Ab[k][j]

```

totalGaussianElimination

```

    read a, b
    AB = concatenar(a, b)

```

```

    n = len(AB)
    while k < n do
        write step k
        write AB
    Ab, order = searchTheBiggestandSwap(Ab, n, k, order)
    while i = k+1 < n do
        mult = Ab[i][k]/Ab[k][k]
        while j = k < n+1 do
            Ab[i][j] = Ab[i][j] - mult* Ab[k][j]

```

Gauss Seidel:

```

read A,b,x0,tol,iter
n = lenght A
cont = 0
error = tol + 1
while(error > tol & cont < iter)
    for i=0 hasta n do
        sum=0
        for j=0 hasta i do

```



```

        sum = sum + A(i,j)* x(j)
    end for

    for j=i+1 hasta n do
        sum = sum + A(i,j)* x(j)
    end for
    x(i) = ((b(i)-sum)/A(i,i))
end for

error = errorAbsolute(x0,x)
cont++

for i=0 hasta n do
    x0(i)= x(i)
end for

mostrar cont vector x error
end while

if(error < tol)
return "La solución del sistema es: "
    mostrar vector x
else
return "Fracaso en " cont
    mostrar vector x
end

```

IncrementalSearch:

read x_inicial, delta, limite_Iteraciones, funcion:

if delta <= 0:

write "El delta debe ser positivo"

sys.exit(1)

elif limite_Iteraciones > 0:

x_anterior = x_inicial

x_actual = x_anterior+delta

f_anterior = f(x_anterior)

f_actual = F(x_actual)

contador = 0

while (contador < limite_Iteraciones):

if f_actual*f_anterior<0:

resultados[contador] <- [x_anterior,x_actual]

x_anterior = x_actual

x_actual = x_actual + delta

f_anterior = f_actual

f_actual = f(x_actual)

contador = contador + 1

endwhile:

devolver resultados

write(aux)

endif:

else:

```

write "Las iteraciones deben ser un numero positivo"
    sys.exit(1)
end else:
    end

```

Jacobi:

```

Jacobi ():
    read A
    read b
    read t
    read iter
    read x0
    n = length of A
    l = length of A [0]
    if (n! = l):

```

```
write ("A is not a square matrix please check and run again.")
```

```
if not:
```

```
x = [with the size of nxn]
```

```
aux = 0
```

```
cont = 0
```

```
error = t + 1
```

```
iteration = 1
```

```
T = [with the size of nxn]
```

```
C = [with the size of n]
```

```
while (error > t and cont < o = iter):
```

```
write ("iteration: # " + str (iteration))
```

```
error = 0
```

```
for i from 0 to n:
```

```
sum = 0
```

```
for j from 0 to n:
```

```
if (i != j):
```

```
sum = sum + A [i] [j] * x0 [j]
```

```
T [i] [j] = -A [i] [j] / A [i] [i]
```

```
C [i] = b [i] / A [i] [i]
```

```
x [i] = (b [i] - sum) / A [i] [i]
```

```
aux = x [i] - x0 [i]
```

```
error = error + math.pow (aux, 2)
```

```
error = error raised to 0.5
```

```
write (error)
```

```

    for i from 0 to n:
        x0 [i] = x [i]
write ("x" + i + 1 + ":" + x0 [i])

```

LagrangeP:

```

read x, y

n = len(x)
l = []
ld = []

for i in x do
    l1 = ""
    l2 = 1
    for j in x do
        if i!=j then
            if j<0 then
                l1+= '(x + '+str(abs(j))+')+' '
            else:
                l1+= '(x - '+str(abs(j))+')+' '
            l2 = l2* (i-j)
        l.append(l1)

```

```

ld.append(l2)

i = 0
lf = []
l1 = 1
while i<len(y) do
    l1 = y[i]/ld[i]
    lf.append(l1)
    i+=1

i = 0
polinomio = ''

while i<len(l) do
    if(lf[i]>=0):
polinomio+= ' + '+str(lf[i])+l[i]
    else:
polinomio+= ' '+str(lf[i])+l[i]
    i+=1
print(polinomio)

```

Newton:

```
read x0, iter, tol
  fx = f(x0)
  dFx = dF(x0)
  cont = 1
  error = tol + 1

while fx != 0 and error > tol and dFx != 0 and cont < iter
  x1 = x0 - fx/dFx
  fx = f(x1)
  dFx = df(x1)
  error = abs(x1 - x0)
  x0 = x1
  cont += 1
end while

if fx = 0
  write "X0 is root"
else if error < tolerancia
  write "X0 approximate root with tolerance: tol"
else if dfx = 0
  write "X0 is probably a multiple root"
else
  write "Fail in iteration: iter"
end if
```

Partial LU:

```
leer A, b :
```

```

    n = longitud(A)
    u = MatrizdeCeros(n)
    l = MatrizDiagonal(n)
    p = MatrizDiagonal(n)

    para k hasta (n):
A, p = IntercambiarFilas(A, n, k, p)

    para i hasta(k + 1, n):
        mult = A[i][k] / A[k][k]
        l[i][k] = mult
        para j hasta (k, n):
            A[i][j] = A[i][j] - mult * A[k][j]

    escribir('u step', k)
    para i hasta (n):
        u[k][i] = A[k][i]

Pb = Producto_Punto(p, b)
lpb = concatenarMatriz(l, Pb)
z = SustitucionProgresiva(lpb)
uz = concatenarMatriz(u, z)
x = SustitucionRegresiva(uz)
    escribir('z', z)
    escribir('x', x)

```


Multiple roots:

```
read tol, x0, nIteration, function1, function2, function3

fx = function1 (x0)
dfx = function2 (x0)
d2fx = function3 (x0)

counter = 0
error = tol + 1
den = (dfx ^ 2) - (fx * d2fx)
while (error > tol and fx <> 0 and den <> 0 counter < nIteration)
  x1 = x0 - ((fx * dfx) / den)
  fx = function1 (x1)
  dfx = function2 (x1)
  d2fx = function3 (x1)
  den = (dfx ^ 2) - (fx * d2fx)
  error = abs (x1-x0)
  x0 = x1
  counter = counter + 1
end while
if (fx == 0)
  x0 "is a root"
else if (error < tol)
  x1 "was found as an approximation to a root with a tolerance of =" tol
  else if (den == 0)
    "it is an indeterminacy"
  Else
    "method failed in" nIteration "iterations"
  end if
```

Secant:

read x1, x0, tol, nIteration

fun0 = f (x0)

if (fun0 = 0)

write "x0 is root"

else

fun1 = f (x1)

cont = 0

error = tol + 1

while (fun1 <> 0 and error > tol and den <> 0 and counter <nIteration)

x2 = x1 - ((fun1 * (x1 - x0)) / den)

error = absolute_value ((x2 - x1) / x2)

x0 = x1

fun0 = fun1

x1 = x2

fun1 = f (x1)

den = fun1 - fun0

cont = cont + 1

```

end while

if (fun1 = 0)

write "x1 was found as root"

else if (error < tol)

write x1 + "Found as an approximation with a tolerance of" + tol

else if (den = 0)

write "There is a possible multiple root"

else

write "Failure in" + nIteration + " iterations "

end if

end if

```

Simple LU:

```

leer A, b:

n = longitud(A)
u = MatrizCeros(n)
l = MatrizDiagonal(n)

para k hasta (n):

if (A[k][k] == 0):
A = IntercambiarFilas(A, n, k)

```

```

para i hasta(k + 1, n):
    mult = A[i][k] / A[k][k]
    l[i][k] = mult
    para j hasta (k, n):
        A[i][j] = A[i][j] - mult * A[k][j]

escribir('u step', k)
para i hasta(n):
    u[k][i] = A[k][i]
    escribir(u)
escribir('u', u)
lb = concatenarMatriz(l, b)
z = SustitucionProgresiva(lb)
uz = concatenarMatriz(u, z)
x = SustitucionRegresiva(uz)
escribir('z ', z)
escribir('x ', x)

```

Sor:

```

leer sor A, b, x0, w, tol, Nmax:

results = { }
D = Diagonal(A)
L = -TraingularIferior(A) + D
U = -TraingularSuperiror(A) + D
T = Inversa(D - (w * L)) * ((1 - w) * D + (w * U))
C = w * Inversa(D - (w * L))* (b)
xant = x0
E = 1000

```

```

        cont = 0
        val = ValoresPropios(T)
        resp = max(abs(val))
        Mientras E > tol and cont < Nmax:
            xact = T* xant + C
            E = Normal(xant - xact)
            xant = xact
            cont = cont + 1
        results[cont] = [float(E), xact]
        fin

        x = xact
        escribir('Radio espectral', resp)
        escribir('X', x)
        escribir('T', T)
        escribir('C', C)
        escribir(results)
        fin

```

Spline Quadratic:

```

matrix_cuad ():
    read x
    read b

a = [[0 for i from 0 to ((length (x) -1) * 3)] for j from 0 to ((length (x) -1) * 3)]
a [0] [0] = x [0] raised to the 2
a [0] [1] = x [0]
a [0] [2] = 1
a [1] [0] = x [1] to the power of 2
a [1] [1] = x [1]
a [1] [2] = 1

```

$$j = 3$$

for i from 2 to length (x):

$$a[i][j] = x[i] \text{ raised to the } 2$$

$$a[i][j + 1] = x[i]$$

$$a[i][j + 2] = 1$$

$$j + = 3$$

$$i = 1$$

$$j = 0$$

for k of length (x) up to ((length (x) raised to the 2) -2)):

$$b + = [0]$$

$$a[k][j] = x[i] \text{ raised to the } 2$$

$$a[k][j + 1] = x[i]$$

$$a[k][j + 2] = 1$$

$$a[k][j + 3] = - (x[i] \text{ to the power of } 2)$$

$$a[k][j + 4] = -x[i]$$

$$a[k][j + 5] = -1$$

$$i + = 1$$

$$j + = 3$$

$$i = 1$$

$$j = 0$$

for k from (length (x) raised to the 2) -2) to length (a) -1):

$$b + = [0]$$

$$a[k][j] = 2 * x[i]$$

$$a[k][j + 1] = 1$$

$$a[k][j + 2] = 0$$

$$a[k][j + 3] = -2 * x[i]$$

$$a[k][j + 4] = -1$$

```

a [k] [j + 5] = 0
i += 1
j += 3

b += [0]
a [len (a) -1] [0] = 2
return a, b

traces ():
    read x
    result = empty
    j = 0
for i in from 0 to length (x):
    if j == 0:
        if x [i]>= 0.0:
            result += x [i]) + "x * * 2"
        else:
            result += x [i] + "x * * 2"
    elif j == 1:
        if x [i]>= 0.0:
            result += "+" + x [i] + "x"
        else:
            result += x [i] + "x"
        else:
            if x [i]>= 0.0:
                result += "+" + x [i] + ""
            else:
                result += x [i] + ""
    j = -1

```

```

j += 1
write ("Traces:")
for i in result.split (""):
    write (i)

```

Spline cubic:

```

def matrix_cub ():
    read x
    read b
a = [[0 for i from 0 to ((length (x) -1) * 4)] for j from 0 to ((length (x) -1) * 4)]
a [0] [0] = x [0] raised to the 3
a [0] [1] = x [0] raised to the 2
a [0] [2] = x [0]
a [0] [3] = 1
a [1] [0] = x [1] raised to the 3
a [1] [1] = x [1] to the power of 2
a [1] [2] = x [1]
a [1] [3] = 1

j = 4

for i from 2 to length (x):

```


$$a[i][j] = x[i] \text{ raised to the } 3$$

$$a[i][j + 1] = x[i] \text{ raised to the } 2$$

$$a[i][j + 2] = x[i]$$

$$a[i][j + 3] = 1$$

$$j += 4$$

$$i = 1$$

$$j = 0$$

for k of length (x) up to ((length (x) raised to the 2) -2)):

$$b += [0]$$

$$a[k][j] = x[i] \text{ raised to the } 3$$

$$a[k][j + 1] = x[i] \text{ raised to the } 2$$

$$a[k][j + 2] = x[i]$$

$$a[k][j + 3] = 1$$

$$a[k][j + 4] = - (x[i] \text{ to the power of } 3)$$

$$a[k][j + 5] = - (x[i] \text{ to the power of } 2)$$

$$a[k][j + 6] = -x[i]$$

$$a[k][j + 7] = -1$$

$$i += 1$$

$$j += 4$$

$$i = 1$$

$$j = 0$$

for k from (length (x) raised to the 2) -2) to length (x) * 3 -4):

$$b += [0]$$

$$a[k][j] = 3 * (x[i] \text{ to the power of } 2)$$

$$a[k][j + 1] = 2 * x[i]$$

$$a[k][j + 2] = 1$$

$$a[k][j+3] = 0$$

$$a[k][j+4] = -(3 * (x[i] \text{ to the power of } 2))$$

$$a[k][j+5] = -(2 * x[i])$$

$$a[k][j+6] = -1$$

$$a[k][j+7] = 0$$

$$i += 1$$

$$j += 4$$

$$i = 1$$

$$j = 0$$

for k from (length (x) raised to the 3) -4 to length ((x) * 4) -6):

$$b += [0]$$

$$a[k][j] = 6 * x[i]$$

$$a[k][j+1] = 2$$

$$a[k][j+2] = 0$$

$$a[k][j+3] = 0$$

$$a[k][j+4] = -6 * x[i]$$

$$a[k][j+5] = -2$$

$$a[k][j+6] = 0$$

$$a[k][j+7] = 0$$

$$i += 1$$

$$j += 4$$

$$b += [0] * 2$$

$$a[\text{len}(a)-2][0] = 6 * x[0]$$

$$a[\text{len}(a)-2][1] = 2$$

$$a[\text{len}(a)-1][\text{len}(a)-4] = 6 * x[\text{len}(x)-1]$$

$$a[\text{len}(a)-1][\text{len}(a)-3] = 2$$

```
return a, b
```

```
def traces (x):  
    result = empty  
    j = 0  
    for i from 0 to length (x):  
        if j == 0:  
            if x [i]>= 0.0:  
result += "+" + x [i] + "x raised to 3"  
                else:  
result += x [i] + "x raised to 3"  
            elif j == 1:  
                if x [i]>= 0.0:  
result += "+" + x [i] + "x raised to 2"  
                    else:  
result += x [i] + "x raised to 2"  
            elif j == 2:  
                if x [i]>= 0.0:  
result += "+" + x [i] + "x"  
                    else:  
result += x [i] + "x"  
            else:  
                if x [i]>= 0.0:  
result += "+" + x [i] + ""  
                    else:  
result += x [i] + ""  
        j = -1
```

```

j += 1
write ("Traces: \ n")
for i in result.split (""):
    write (i)

```

Spline linear:

```

matrix_lin ():
    read x
    read b

a = [[0 for i from 0 to ((length (x) -1) * 2)] for j from 0 to ((length (x) -1) * 2)]
a [0] [0] = x [0]
a [0] [1] = 1
a [1] [0] = x [1]
a [1] [1] = 1

j = 2
for i from 2 to length (x):
    a [i] [j] = x [i]
    a [i] [j + 1] = 1
    j += 2

i = 1
j = 0

for k of length (x) up to ((length (x) raised to the 2) -2)):
    b += [0]
    a [k] [j] = x [i]
    a [k] [j + 1] = 1

```

```

a [k] [j + 2] = -x [i]
a [k] [j + 3] = -1
    i += 1
    j += 2

return a, b

traces ():
    read x

result = empty

for i from 0 to length x:
    if i% 2 == 0:
        if x [i]>= 0.0:
            result += "+" + x [i] + "x"
        else:
            result += x [i] + "x"
        else:
            if x [i]>= 0.0:
                result += "+" + x [i] + ""
            else:
                result += x [i] + ""

write ("Traces:")
for i in result.split (""):
    write (i)

```

```

Vandermonde:
    read x, y

    matriz = []

    while i < len(x) do
        matriz.append([])
        i++

    fila = 0

    for i in x do
        j = len(x)-1
        while j >= 0 do
            matriz[fila].append(i* * j)
            fila++
        totalGaussianElimination(matriz,y)

```

Codes

```

Bisection:

import sympy as sm

def bisection(funcion, xi, xs, nIter, iter):

```

```

        results = { }

        if nIter > 0:

            x = sm.symbols('x')

            fxi = sm.sympify(funcion).subs(x, xi)
            fxs = sm.sympify(funcion).subs(x, xs)

            sm.plot(funcion)

            if (fxi == 0):

                print(fxi)

            elif (fxs == 0):

                print(fxs)

            elif (fxs * fxi < 0):

                xm = (xi + xs) / 2

                fxm = sm.sympify(funcion).subs(x, xm)

                count = 1

                error = iter + 1

results[count] = [float(xi), float(xm), float(xs), float(fxm), float(error)]

            while ((error > iter) and (count < nIter)):

                if (fxi * fxm < 0):

                    xs = xm

                    else:

                        xi = xm

                   iaux = xm

                    xm = (xi + xs) / 2

                    fxm = sm.sympify(funcion).subs(x, xm)

                    error = abs(xm -iaux)

                    count += 1

results[count] = [float(xi), float(xm), float(xs), float(fxm), float(error)]

            print(results)

            return results

```

```

else:
results['message'] = 'Error'
print('el intervalo no sirve')
return results

```

```

Cholesky:
import numpy as np
import math

def cholesky(A, b):
    # Inicialización
    n = len(A)
    L = np.eye(n)
    U = np.eye(n)

    # factorization
    for i in range(n-1):
        suma = 0
        for j in range(i):
            suma += (L[i][j] * U[j][i])

```



```

L[i][i] = math.sqrt(A[i][i] - suma)
U[i][i] = L[i][i]

for k in range(i+1,n):
    suma = 0
    for j in range(i):
        suma += (L[k][j] * U[j][i])
    L[k][i] = (A[k][i] - suma) / U[i][i]

for k in range(i+1,n):
    suma = 0
    for j in range(i):
        suma += (L[i][j] * U[j][k])
    U[i][k] = (A[i][k] - suma) / L[i][i]

    suma = 0
    for j in range(n-1):
        suma += (L[n-1][j] * U[j][n-1])
    L[n-1][n-1] = math.sqrt(A[n-1][n-1] - suma)
    U[n-1][n-1] = L[n-1][n-1]

print("Matriz L")
print(L)
print("Matriz U")
print(U)
z = frontSubstitution(L, b)
x = backSubstitution(U, z)
print(x)

def frontSubstitution(A, b):
    n = len(A)

```

```

x = np.zeros((n))
for i in range(n):
    sum = 0
    for j in range(i):
        sum += A[i][j] * x[j]
    x[i] = (b[i] - sum) / A[i][i]
return x

def backSubstitution(A, b):
    n = len(A)
    x = np.zeros((n))
    for i in range(n-1, -1, -1):
        sum = 0.0
        for j in range (i+1, n):
            sum += A[i][j] * x[j]
        x[i] = (b[i] - sum) / A[i][i]
    return x

```

```

    Crout:
import numpy as np

def Crout(a, b):
    cout = 0

    m, n = a.shape
    if (m !=n ):
print("Crout cannot be used.")
        else:
            l = np.zeros((n,n))
            u = np.zeros((n,n))

            s1 = 0
            s2 = 0

            for m in range(1,n+1):
print("Stage " + str(m) + ": ")
                for i in range(n):
                    l[i][0] = a[i][0]
                    u[i][i] = 1

                    for j in range(1, n):
                        u[0][j] = a[0][j] / l[0][0]

                        for k in range(1, n):
                            for i in range(k, n):
                                for r in range(k): s1 += l[i][r] * u[r][k]
                                    l[i][k] = a[i][k] - s1
                                        s1 = 0

                                    for j in range(k+1, n):
                                        for r in range(k): s2 += l[k][r] * u[r][j]
                                            u[k][j] = (a[k][j] - s2) / l[k][k]
                                                s2 = 0

```

```

print("U: ")
print(u)
print("L: ")
print(l)

```

```

y = np.zeros(n)
s3 = 0
y[0] = b[0] / l[0][0]
for k in range(1, n):
    for r in range(k):
        s3 += l[k][r] * y[r]
    y[k] = (b[k]-s3) / l[k][k]
    s3 = 0

```

```

x = np.zeros(n)
s4 = 0
x[n-1] = y[n-1]
for k in range(n-2, -1, -1):
    for r in range(k+1, n):
        s4 += u[k][r] * x[r]
    x[k] = y[k] - s4
    s4 = 0

```

```

for i in range(n):
    print("x" + str(i + 1) + " = ", x[i])

```

Divide differences:

```

import numpy as np

```

```

def divide_dif(x,y):
    n = len(x)
    D = np.zeros((n,n))

    D[:,0] = np.conjugate(y)

    for i in range(1,n):
        aux0 = D[i-1:n,i-1]
        aux1 = np.diff(aux0)
        aux2 = np.subtract(x[i:n],x[0:n-1-i+1])
        D[i:n,i] = np.divide(aux1,np.transpose(aux2))

    res = np.diag(D)

    r = '' + '{ 0:+} '.format(res[0])
    m = '(x' + '{ 0:+} '.format(-x[0]) + ')'
    for i in range(1,n):
        r += '{ 0:+} '.format(res[i]) + m
        m += '(x' + '{ 0:+} '.format(-x[i]) + ')'
    r = r.replace('x+0','x')
    print('Matrix D: \ n',D)
    print('Coef: ',res)
    print('Newton Polinom : ', r)

    return (r,D)

```

Doolittle:

```

import sympy as sm
import math
import sys
import json

```

```

import base64

import numpy as np

def doolittle(A,b,size):
    A = np.array(A)
    b = np.array(b)
    L = np.eye(size)
    U = np.eye(size)
    print("Etapla 0:")
    print("Matriz L: ")
        print(L)
    print("Matriz U: ")
        print(U)
    for i in range(size):
        print("Etapla " + str(i+1))
        for k in range(i, size):
            suma = 0;
            for j in range(i):
                suma += (L[i][j] * U[j][k]);
            U[i][k] = A[i][k] - suma;
        for k in range(i, size):
            if (i == k):
                L[i][i] = 1;
            else:
                suma = 0;
                for j in range(i):
                    suma += (L[k][j] * U[j][i]);
                L[k][i] = ((A[k][i] - suma)/U[i][i]);

        print("Matriz L: ")

```

```

        print(L)
        print("Matriz U: ")
        print(U)
    z = frontSubstitution(L, b)
    x = backSubstitution(U, z)
    printResultVector(x)

```

```

def frontSubstitution(A, b):
    n = len(A)
    x = np.zeros((n))
    for i in range(n):
        sum = 0
        for j in range(i):
            sum += A[i][j] * x[j]
        x[i] = (b[i] - sum) / A[i][i]
    return x

```

```

def backSubstitution(A, b):
    n = len(A)
    x = np.zeros((n))
    for i in range(n-1, -1, -1):
        sum = 0.0
        for j in range (i+1, n):
            sum += A[i][j] * x[j]
        x[i] = (b[i] - sum) / A[i][i]
    return x

```

```

def printResultVector(vector):
    n = len(vector)
    for i in range(n):

```

```
print('x' + str(i + 1) + ': ' + str(vector[i]))
```

```
FalseRule:
import sympy as sm
import math
import sys

def falseRule(a, b, funcion, limite_iteraciones, tolerancia):
    results = { }
    x = sm.symbols('x')
    funcion = sm.sympify(funcion)
    fa = funcion.subs(x, a)
    fb = funcion.subs(x, b)
    if(fa == 0):
        print('a es raiz')
    elif(fb == 0):
        print('b es raiz')
```



```

        elif(fa * fb < 0):
            error = 1
            cont = 1
            c = (fb* a - fa* b)/(fb - fa)
            fc = funcion.subs(x, c)
            print('iter|   a   |   c   |   b   |   fc   |   error')
while(fc != 0 and cont < limite_iteraciones and error > tolerancia):
    results[cont]=[float(a),float(c),float(b),float(fc),float(error)]
    print(cont,a,c,b,fc,error)
    if (fa * fc < 0):
        b = c
        fb = funcion.subs(x, c)
    else:
        a = c
        fa = funcion.subs(x, c)
        caux = c
        c = (fb* a - fa* b)/(fb - fa)
        fc = funcion.subs(x, c)
        error = abs(caux - c)
        cont+=1
    if (fc == 0):
        print('c is a root '+ str(c))
    results['message']='c is a root '+ str(c)
    elif (error < tolerancia):
print('c is an approximation of the root c: '+ str(c) +' error: '+ str(error)+' in the it-
    eration '+str(cont))

    results['message']='c is an approximation of the root c: '+ str(c) +' er-
        ror: '+ str(error)+' in the iteration '+str(cont)

        else:

print('number of maximum iterations reached, convergence was not reached')

```

```

    results['message']='number of maximum iterations reached, conver-
        gence was not reached'

    else:

    print('inadequate interval, does not satisfy the theorem fa * fb < 0')
    results['message']='inadequate interval, does not satisfy the theorem fa *
        fb < 0'

    return results

```

Fixed Point:

```

import math

def f(x):
    return x* * 3 + 4 * x* * 2 - 10

def g(x):
    return math.sqrt(10/(x+4))

def fixedPoint(xa, iter, tol):
    fx = f(xa)
    cont = 0
    error = tol + 1
    xn = 0

    while((fx != 0) and error > tol and cont < iter):
        xn = g(xa)
        fx = f(xn)
        error = abs(xn - xa)
        xa = xn

```

```

        cont += 1

    if fx == 0:
        print("Xa: ", xa, " is a root")
        elif error < tol :
print("Xa: ", xa, " approximate root with tolerance: ", tol)
        else:
print("Fail in iteration: ", iter)

```

Gaussian Elimination:

```

import numpy as np
import math
import copy

def simpleGaussianElimination(A, b):
    Ab = concatenarMatriz(A, b)
    n = len(Ab)
    result = { }
    for k in range(n):
        print('step ',k)
        print(Ab)
        result[k]=copy.deepcopy(Ab)
        if(Ab[k][k]==0):
            Ab = searchAndSwapZero(Ab, n, k)
        for i in range(k+1, n):
            mult = Ab[i][k]/Ab[k][k]
            for j in range(k, n+1):

```

```

        Ab[i][j] = Ab[i][j] - mult* Ab[k][j]

    print('x ',backSubstitution(Ab))
    return(backSubstitution(Ab),result)

def searchAndSwapZero(Ab, n, i):
    for j in range(i+1,n):
        if(Ab[j][i]!=0):
            temp = Ab[i]
            Ab[i] = Ab[j]
            Ab[j] = temp
            break
    return Ab

def partialGaussianElimination(A, b):
    Ab = concatenarMatriz(A, b)

    order = []
    result = { }
    n = len(Ab)
    for k in range(n):
        print('step ',k)
        printMatriz(Ab)
        result[k]=copy.deepcopy(Ab)
    Ab = searchBiggerandSwap(Ab, n, k)
    for i in range(k+1, n):
        mult = Ab[i][k]/Ab[k][k]
        for j in range(k, n+1):
            Ab[i][j] = Ab[i][j] - mult* Ab[k][j]
    print('X ',backSubstitution(Ab))
    return(backSubstitution(Ab),result)

```

```

def searchBiggerandSwap(Ab, n, i):
    row = i
    for j in range(i+1,n):
        if(abs(Ab[row][i]) < abs(Ab[j][i])):
            row = j
            temp = Ab[i]
            Ab[i] = Ab[row]
            Ab[row] = temp
    return Ab

def totalGaussianElimination(A, b):
    order = []
    result = { }
    Ab = concatenarMatriz(A, b)
    n = len(Ab)
    for k in range(n):
        print('step ',k)
        printMatriz(Ab)
        result[k]=copy.deepcopy(Ab)
    Ab, order = searchTheBiggestandSwap(Ab, n, k, order)
    for i in range(k+1, n):
        mult = Ab[i][k]/Ab[k][k]
        for j in range(k, n+1):
            Ab[i][j] = Ab[i][j] - mult* Ab[k][j]
        # retorna las x
    return(sortResult(backSubstitution(Ab), order),result)

def searchTheBiggestandSwap(Ab, n, k, order):
    row = k
    column = k

```

```

    for i in range(k,n):
        for j in range(k,n):
            if(abs(Ab[row][column]) < abs(Ab[i][j])):
                row = i
                column = j
                temp = Ab[k]
                Ab[k] = Ab[row]
                Ab[row] = temp
    Ab = swapColumn(Ab, k, column)
    order.append((k, column))
    return (Ab, order)

```

```

def swapColumn(Ab, c1, c2):
    for i in range(len(Ab)):
        temp = Ab[i][c1]
        Ab[i][c1] = Ab[i][c2]
        Ab[i][c2] = temp
    return Ab

```

```

def sortResult(x, order):
    for i in range(len(order)-1, -1, -1):
        temp = x[order[i][0]]
        x[order[i][0]] = x[order[i][1]]
        x[order[i][1]] = temp
    return x

```

```

def concatenarMatriz(A, b):
    n = len(A)
    for i in range(n):
        A[i].append(b[i])

```

```

        return A

# b = [[4],[5],[6]] this is the format
def concatenar(a,b):
    a = np.array(a)
    b = np.array(b)
matriz = np.concatenate((a, b), axis=1)
    return matriz

def backSubstitution(Ab):
    n= len(Ab)
    x = []
    for i in range(n):
        x.append(0)

    for i in range(n-1, -1, -1):
        sum = 0
        for j in range(i+1, n):
            sum+= Ab[i][j]* x[j]
        x[i] = (Ab[i][n]-sum)/Ab[i][i]
    return x

def printMatriz(M):
    result=""
    for i in range(len(M)):
        print(M[i])

def printMatriz2(M,k,result):
    for i in range(len(M)):
        print(M[i])
    result[k]=M

```

```
return result
```

Gauss Seidel:

```
import math
import numpy as np

def gaussSeidel(A, b, t, iter, x0) :
    n = len(A)
    l = len(A)
    if (n != l) :
        print("A is nor a square matrix.")
        return 0
    else:
        x = [None]* n
        aux = 0
        cont = 0
        E = t + 1
        iteration = 1
        while (E > t and cont <= iter):
            print("iter: " , iteration)
            E = 0
            for i in range(0,n):
                suma = 0
                for j in range(0,n):
                    if (i != j):
                        suma = suma + A[i][j] * x0[j]
                x[i] = ( ((b[i] - suma) / A[i][i]))
```



```

        aux = x[i] - x0[i]
        E = E + math.pow(aux, 2)
        x0[i] = x[i]
    print("x" , (i + 1) , ": " , x0[i])

    E = math.pow(E, 0.5)
    print("E = " , E)
    print("")
    iteration = iteration + 1
    cont = cont+1

    if (E < t):
        return x
    else:
print("Can not find a solution in " , iter , " iterations")
        return 0

```

```

A = [[4, -1, 0, 3],
      [1, 15.5, 3, 8],
      [0, -1.3, -4, 1.1],
      [14, 5, -2, 30]]
b = [1, 1, 1, 1]
x0 = [0, 0, 0, 0]
t = math.pow(10, -7)
iter = 100
gaussSeidel(A, b, t, iter, x0)

```

```

D = np.diag(np.diag(A))

```

```

U = -np.triu(A,1)
L = -np.tril(A,-1)
T = (np.dot((np.linalg.inv(D-L)), U))
C = (np.dot((np.linalg.inv(D-L)), b))
    print("T: ")
        print(T)
            print("C:")
                print(C)
values, normalized_eigenvectors = np.linalg.eig(T) # T es la matriz
    spectral_radius = max(abs(values))
print("\ nSpectral Radius: ", spectral_radius)

```

Incremental search:

```

import sympy as sm
import sys
import pandas as pd

```

```

def incremental_search(funcion,xi, delta, nIter):
    results = { }
    if delta <= 0:
        print("El delta debe ser positivo")
        sys.exit(1)
    elif nIter > 0:

        x = sm.symbols('x')
        x_a = xi
        current_X = x_a+delta
        f_a = sm.sympify(funcion).subs(x,x_a)
        currentF = sm.sympify(funcion).subs(x,current_X)
        contador = 0
        while (contador < nIter):
            if currentF*f_a<0:
                results[contador] = [float(x_a),float(current_X)]
                x_a = current_X
                current_X = current_X + delta
                f_a = currentF
                currentF = sm.sympify(funcion).subs(x,current_X)
                contador = contador + 1

            return results

        else:
            print("Las iteraciones deben ser un numero positivo")
            results['message'] = 'Error'
            print('el intervalo no sirve')
            return results

```

```

def print_function(results):
    index = []
    x1 = []
    x2 = []
    for i in results:
        index.append(i)
        x1.append(results[i][0])
        x2.append(results[i][1])

    data = { 'xi': x1,
             'xs': x2,
             }
    df = pd.DataFrame(data, index=index)
    print(df)

```

Jacobi:

```

import math
import numpy as np
def Jacobi(A, b, t, iter, x0):
    n = len(A)
    l = len(A[0])
    result = { }
    if (n!=l):

```

```
return("A is not a square matrix please check and run again.")
```

```
else:
```

```
x=[None]*n
```

```
aux=0
```

```
cont = 0
```

```
error = t + 1
```

```
iteration = 1
```

```
T = np.zeros((n, n))
```

```
C = np.zeros(n)
```

```
while(error > t and cont <= iter):
```

```
    error = 0
```

```
    for i in range(0,n):
```

```
        sum = 0
```

```
        for j in range(0,n):
```

```
            if (i != j):
```

```
                sum = sum + A[i][j] * x0[j]
```

```
                T[i][j] = -A[i][j] / A[i][i]
```

```
                C[i] = b[i] / A[i][i]
```

```
            x[i] = (b[i] - sum) / A[i][i]
```

```
            aux = x[i] - x0[i]
```

```
        error = error + math.pow(aux, 2)
```

```
    error = math.pow(error, 0.5)
```

```
    for i in range(0,n):
```

```
        x0[i] = x[i]
```

```
print("x"+str(i+1)+": "+str(round(x0[i],4)))
```

```
result[iteration]=(float(error),x)
```

```

        iteration=iteration+1

        cont = cont+1

        print(result)
        print("")
        print("T: \ n"+str(T))
        print("")
        print("C: \ n"+str(C))
        print("")
        spectralRadius = np.amax(abs(T))
        print("Spectral radius: \ n"+str(spectralRadius))


        if (error < t):
            return(result, T, C,spectralRadius )
        else:
            print ("no solution reached in " + str (iter) + " iterations")
            return

```

Lagrange:

```
import math

def lagrange(x, y):
    n = len(x)
    l = []
    ld = []
    for i in x:
        l1 = ''
        l2 = 1
        for j in x:
            if(i!=j):
                if(j<0):
                    l1+= '(x + '+str(abs(j))+')+'
                else:
                    l1+= '(x - '+str(abs(j))+')+'
                l2 = l2* (i-j)
        l.append(l1)
        ld.append(l2)

    i = 0
    lf = []
    l1 = 1
    while(i<len(y)):
        l1 = y[i]/ld[i]
        lf.append(l1)
        i+=1

    i = 0
```

```

    polinomio = ''
    while(i<len(l)):
        if(lf[i]>=0):
            polinomio+= ' + '+str(lf[i])+l[i]
        else:
            polinomio+= ' '+str(lf[i])+l[i]
        i+=1
    return(polinomio)

```

Newton:

```

import numpy as np
import matplotlib.pyplot as plt

def f(x):
    return x* * 3 - np.cos(x)

def dF(x):

```



```

return 3 * x * x * 2 + np.sin(x)

def newton(x0, iter, tol):
    fx = f(x0)
    dFx = dF(x0)
    cont = 1
    error = tol + 1

    while( (fx != 0) and error > tol and (dFx != 0) and cont < iter):
        x1 = x0 - fx/dFx
        fx = f(x1)
        dFx = dF(x1)
        error = abs(x1-x0)
        x0 = x1
        cont += 1

        if fx == 0:
            print("X0: ", x0, " is root")
            elif error < tol:
                print("X0: ", x0, " approximate root with tolerance: ", tol)
                elif dFx == 0:
                    print("X0: ", x0, " is probably a multiple root")
                else:
                    print("Fail in iteration: ", iter)

def draw():
    x = np.linspace(-2, 2, 100)
    plt.plot(x, x * x * 3 - np.cos(x))
    plt.grid()
    plt.show()

```

```
draw()
```

Partial LU:

```
import copy
import numpy as np

def partialLU(A, b):
    n = len(A)
    message = ''
    det = np.linalg.det(A)
    if(det != 0):
        u = zero_Matrix(n)
        l = lmatrix(n)
        p = lmatrix(n)
        dicL = { }
        dicU = { }
        dicP = { }
    for k in range(n):
```

```

        print('step ', k)
        printMatriz(A)
        print('L step', k)
        printMatriz(l)
        dicL[k] = copy.deepcopy(l)
        dicU[k] = copy.deepcopy(u)
A, p = searchBiggerandSwap(A, n, k, p)
        for i in range(k + 1, n):
            mult = A[i][k] / A[k][k]
            l[i][k] = mult
            for j in range(k, n):
                A[i][j] = A[i][j] - mult * A[k][j]

        print('u step', k)
        for i in range(n):
            u[k][i] = A[k][i]
        printMatriz(u)
        dicU[k] = copy.deepcopy(u)
        print('P step', k)
        printMatriz(p)
        dicP[k] = copy.deepcopy(p)

        Pb = multAb(p, b)
        lpb = concatenateMatrix(l, Pb)
        z = progSubstitution(lpb)
        uz = concatenateMatrix(u, z)
        x = backSubstitution(uz)
        print('det', det)
    return (x,dicL,dicU,dicP,message)
else:

```

```

        message = 'Error'
        return (",",",",message)

def searchBiggerandSwap(Ab, n, i, p):
    row = i

    for j in range(i + 1, n):
        if (abs(Ab[row][i]) < abs(Ab[j][i])):
            row = j
            temp = Ab[i]
            aux = p[i]
            Ab[i] = Ab[row]
            p[i] = p[row]
            Ab[row] = temp
            p[row] = aux
    return Ab, p

def concatenateMatrix(A, b):
    n = len(A)
    for i in range(n):
        A[i].append(b[i], )
    return A

def multAb(A, b):
    n = len(A)
    mult = []
    for i in range(n):
        suma = 0
        for j in range(n):
            suma += b[j] * A[i][j]

```

```

        mult.append(suma)

    return mult

def zero_Matrix(n):
    u = []
    for i in range(n):
        u.append([0] * n)
    return u

def lmatrix(n):
    l = zero_Matrix(n)
    for i in range(n):
        l[i][i] = 1
    return l

def backSubstitution(Ab):
    n = len(Ab)
    x = []
    for i in range(n):
        x.append(0)

    for i in range(n - 1, -1, -1):
        sum = 0
        for j in range(i + 1, n):
            sum += Ab[i][j] * x[j]
        x[i] = (Ab[i][n] - sum) / Ab[i][i]
    return x

def progSubtitution(Ab):
    n = len(Ab)
    x = []

```

```

    for i in range(n):
        x.append(0)

    for i in range(n):
        sum = 0
        for j in range(i):
            sum += Ab[i][j] * x[j]
        x[i] = ((Ab[i][n] - sum) / Ab[i][i])

    return x

def printMatriz(M):
    for i in range(len(M)):
        print(M[i])

```

Multiple roots:

```
import sympy as sm
import math

def roots_mult(x0, nIterations, tol, function, function2, function3):

    results = { }
    add = { }

    x = sm.symbols ('x')

    fx0 = sm.sympify (function) .subs (x, x0)
    dfx0 = sm.sympify (function2) .subs (x, x0)
    d2fx0 = sm.sympify (function3) .subs (x, x0)

    cont = 0
    error = tol + 1
    xn = 0

    det = (math.pow (dfx0,2)) - (fx0 * d2fx0)
    results [cont] = [float (x0), float (fx0), float (0)]
    while (fx0!= 0 and error> tol and det != 0 and cont <nIterations):

        xn = x0 - ((fx0 * dfx0) / det)

        fx0 = sm.sympify (function) .subs (x, xn)
        dfx0 = sm.sympify (function2) .subs (x, xn)
        d2fx0 = sm.sympify (function3) .subs (x, xn)
        det = (math.pow (dfx0,2)) - (fx0 * d2fx0)

        error = abs (xn-x0)

        x0 = xn

        cont = cont + 1

    results [cont] =([round(float (x0),18), round(float (fx0),18), round(float (error),18)])]
```

```

        if (fx0 == 0):
            add =(str (x0) + "is a root")

        elif (error<tol):
            add =(str (x0) + "was found as an approximation with a tolerance of =" + str (tol))

        elif (det == 0):
            add =( "is an indeterminacy")

        else:
            add =( "The method failed in" + str (nIterations) + "iterations")

    return results,add

```

Secant:

```

import abc

import sympy as sm

def secant(x0, x1, nIterations, tol, function):
    results = { }

```



```

add = { }

x = sm.symbols('x')
fx0 = sm.sympify(function).subs(x, x0)

if (fx0 == 0):
    print(x0 + "is root")
else:
    fx1 = sm.sympify(function).subs(x, x1)
    cont = 1
    error = tol + 1
    det = fx1 - fx0
    xi = 0
    results [cont-1] = [float (x0), float (fx0), float (0)]
    results [cont] = [float (x1), float (fx1), float (0)]
while (fx1 != 0 and error > tol and det != 0 and cont < nIterations):

    xi = x1 - ((fx1 * (x1-x0)) / det)
    error = abs(xi-x1)
    x0 = x1
    fx0 = fx1
    x1 = xi
    fx1 = sm.sympify(function).subs(x, x1)
    det = fx1 - fx0
    cont = cont + 1
results [cont] = ([round(float (xi),10), round(float (fx1),10), round(float (error),10)])

```

```

        if (fx1 == 0):

            add = (str(x0)+" is a root")

        elif (error <tol):

            add= (str(x1)+" was found as an approximation with a tolerance of = "+str(tol))

        elif (det == 0):

            add= (str(x1)+"is a possible multiple root")

        else:

            add=("The method failed in "+str(nIterations)+ " iterations")

    return results,add

```

Simple LU:

```

import numpy as np
import math as math
import copy

def simpleLU(A, b):

    n = len(A)

    det = np.linalg.det(A)

    message = "
    if(det != 0):

```

```

u = zero_Matrix(n)
l = lmatrix(n)
dicL = { }
dicU = { }

for k in range(n):
    print('step ', k)
    printMatriz(A)
    print('L step', k)
    printMatriz(l)
    dicL[k] = copy.deepcopy(l)
    dicU[k] = copy.deepcopy(u)
    if (A[k][k] == 0):
A = searchAndSwapZero(A, n, k)
    for i in range(k + 1, n):
        mult = A[i][k] / A[k][k]

        l[i][k] = mult
        for j in range(k, n):
            A[i][j] = A[i][j] - mult * A[k][j]

    print('u step', k)
    for i in range(n):
        u[k][i] = A[k][i]
    printMatriz(u)
    dicU[k] = copy.deepcopy(u)

    print('u', dicU)
lb = concatenateMatrix(l, b)
z = progSubstitution(lb)

```

```

        uz = concatenateMatrix(u, z)
        x = backSubstitution(uz)
        print('x ', x)

    return (x,dicL,dicU,message)
    else:
        message = 'Error'
        return (" ", " ", " ", message)

def searchAndSwapZero(Ab, n, i):
    for j in range(i + 1, n):
        if (Ab[j][i] != 0):
            temp = Ab[i]
            Ab[i] = Ab[j]
            Ab[j] = temp
            break
    return Ab

def concatenateMatrix(A, b):
    n = len(A)
    for i in range(n):
        A[i].append(b[i], )
    return A

def multAb(A, b):
    n = len(A)
    mult = []
    for i in range(n):
        suma = 0
        for j in range(n):
            suma += b[j] * A[i][j]

```

```

        mult.append(suma)

    return mult

def zero_Matrix(n):
    u = []
    for i in range(n):
        u.append([0] * n)
    return u

def lmatrix(n):
    l = zero_Matrix(n)
    for i in range(n):
        l[i][i] = 1
    return l

def backSubstitution(Ab):
    n = len(Ab)
    x = []
    for i in range(n):
        x.append(0)

    for i in range(n - 1, -1, -1):
        sum = 0
        for j in range(i + 1, n):
            sum += Ab[i][j] * x[j]
        x[i] = (Ab[i][n] - sum) / Ab[i][i]
    return x

def progSubtitution(Ab):
    n = len(Ab)
    x = []

```

```

    for i in range(n):
        x.append(0)

    for i in range(n):
        sum = 0
        for j in range(i):
            sum += Ab[i][j] * x[j]
        x[i] = ((Ab[i][n] - sum) / Ab[i][i])

    return x

def printMatriz(M):
    for i in range(len(M)):
        print(M[i])

```

Sor:

```

import numpy as np
from scipy import linalg as LA
import pandas as pd
import copy

def sor(A, b, x0, w, tol, Nmax):
    results = { }
    det = np.linalg.det(A)
    if(det != 0):
        d = np.diag(A)
        D = np.diagflat(d)
        L = -np.tril(A) + D
        U = -np.triu(A) + D
        T = np.linalg.inv(D - (w * L)).dot((1 - w) * D + (w * U))
        C = w * np.linalg.inv(D - (w * L)).dot(b)

```

```

        xant = x0
        E = 1000
        cont = 0
        val, evec = np.linalg.eig(T)
        resp = max(abs(val))
        while E > tol and cont < Nmax:
            xact = np.dot(T, xant) + C
            short = ['{ :.6f} '.format(elem) for elem in xact]
            E = np.linalg.norm(xant - xact)
            xant = xact
            cont = cont + 1
            results[cont] = [float(E), short]

        print_iter(results)
        print('Spectral Radius ', resp)
        print('T', T)
        print('C', C)

        return (resp,T,C,results)
    else:
        results['message'] = 'Error'
        return ("","",results)

def print_iter(results):
    index = []
    x = []
    error = []
    for i in results:
        index.append(i)
        error.append(results[i][0])

```

```

x.append(results[i][1])

data = { 'Error': error,
         'X': x
       }

df = pd.DataFrame(data, index=index)

print(df)

```

Spline Quadratic

```

from Metodos.GaussianElimination_splines import simpleGaussianElimination, partialGaussianElimination, totalGaussianElimination, backSubstitution, sortResult

```

```

def matrix_cuad(x, b):
    a = [[0 for i in range((len(x)-1)* 3)] for j in range((len(x)-1)* 3)]

    a[0][0] = x[0]* * 2
    a[0][1] = x[0]
    a[0][2] = 1
    a[1][0] = x[1]* * 2
    a[1][1] = x[1]
    a[1][2] = 1

    j = 3
    for i in range(2,len(x)):
        a[i][j] = x[i]* * 2
        a[i][j+1] = x[i]
        a[i][j+2] = 1

```



```

    j += 3

    i = 1
    j = 0
    for k in range(len(x),((len(x)* 2)-2)):
        b += [0]
        a[k][j] = x[i]* * 2
        a[k][j+1] = x[i]
        a[k][j+2] = 1
        a[k][j+3] = -(x[i]* * 2)
        a[k][j+4] = -x[i]
        a[k][j+5] = -1
        i += 1
        j += 3

    i = 1
    j = 0
    for k in range(((len(x)* 2)-2),len(a)-1):
        b += [0]
        a[k][j] = 2* x[i]
        a[k][j+1] = 1
        a[k][j+2] = 0
        a[k][j+3] = -2* x[i]
        a[k][j+4] = -1
        a[k][j+5] = 0
        i += 1
        j += 3

    b += [0]

```

```

a[len(a)-1][0] = 2
return a,b

def traces(x):
    result = ""
    j = 0
    for i in range(len(x)):
        if j == 0:
            # print(x[i])
            if x[i] >= 0.0:
result += "+" + str(x[i]) + "x* * 2"
                else:
result += str(x[i]) + "x* * 2"
            elif j == 1:
                if x[i] >= 0.0:
result += "+" + str(x[i]) + "x"
                    else:
result += str(x[i]) + "x"
                else:
                    if x[i] >= 0.0:
result += "+" + str(x[i]) + " "
                        else:
result += str(x[i]) + " "
                    j = -1
                    j += 1

    return(result)

```

```

def trazcuad_spline(x,y,d):

```

```

''' x = [-1,0,3,4]
y = [15.5,3,8,1]'''

b = y
A, b = matrix_cuad(x,b)

if(d=="T"):
t1=totalGaussianElimination(A, b)
return traces(t1[0]),A,b,t1[1],t1[0]

elif(d=="P"):
t2=partialGaussianElimination(A, b)
return traces(t2[0]),A,b,t2[1],t2[0]

elif(d=="S"):
t3=simpleGaussianElimination(A, b)
return traces(t3[0]),A,b,t3[1],t3[0]

```

Spline Linear:

```
from Metodos.GaussianElimination_splines import simpleGaussianElimination,partialGaussianElimination,totalGaussianElimination
```

```
def matrix_lin(x, b):  
    a = [[0 for i in range((len(x)-1)* 2)] for j in range((len(x)-1)* 2)]  
    a[0][0] = x[0]  
    a[0][1] = 1  
    a[1][0] = x[1]  
    a[1][1] = 1  
  
    j = 2  
    for i in range(2,len(x)):  
        a[i][j] = x[i]  
        a[i][j+1] = 1  
        j += 2  
  
    i = 1  
    j = 0  
    for k in range(len(x),((len(x)* 2)-2)):  
        b += [0]  
        a[k][j] = x[i]  
        a[k][j+1] = 1  
        a[k][j+2] = -x[i]  
        a[k][j+3] = -1  
        i += 1  
        j += 2  
  
    return a,b
```

```
def traces(x):
```

```
    result = ""
```

```
    for i in range(len(x)):
```

```
        if i % 2 == 0:
```

```
            if x[i] >= 0.0:
```

```
                result += "+" + str(x[i]) + "x"
```

```
            else:
```

```
                result += str(x[i]) + "x"
```

```
            else:
```

```
                if x[i] >= 0.0:
```

```
                    result += "+" + str(x[i]) + " "
```

```
                else:
```

```
                    result += str(x[i]) + " "
```

```
    return(result)
```

```
def trazlin_spline(x,y,d):
```

```
    b = y
```

```
    A, b = matrix_lin(x,b)
```

```
    if(d=="T"):
```

```
        t1=totalGaussianElimination(A, b)
```

```
        return traces(t1[0]),A,b,t1[1],t1[0]
```

```
    elif(d=="P"):
```

```

t2=partialGaussianElimination(A, b)
    return traces(t2[0]),A,b,t2[1],t2[0]
        elif(d=="S"):
t3=simpleGaussianElimination(A, b)
    return traces(t3[0]),A,b,t3[1],t3[0]

```

Spline Cubic

```

from Metodos.GaussianElimination_splines import simpleGaussianElimination,partialGaussianElimination,totalGaussianElimination, backSubstitution, sortResult

```

```

def matrix_cub(x, b):
a = [[0 for i in range((len(x)-1)* 4)] for j in range((len(x)-1)* 4)]
    a[0][0] = x[0]* * 3
    a[0][1] = x[0]* * 2
    a[0][2] = x[0]

```

$$a[0][3] = 1$$

$$a[1][0] = x[1] * * 3$$

$$a[1][1] = x[1] * * 2$$

$$a[1][2] = x[1]$$

$$a[1][3] = 1$$

$$j = 4$$

for i in range(2,len(x)):

$$a[i][j] = x[i] * * 3$$

$$a[i][j+1] = x[i] * * 2$$

$$a[i][j+2] = x[i]$$

$$a[i][j+3] = 1$$

$$j += 4$$

$$i = 1$$

$$j = 0$$

for k in range(len(x),((len(x)* 2)-2)):

$$b += [0]$$

$$a[k][j] = x[i] * * 3$$

$$a[k][j+1] = x[i] * * 2$$

$$a[k][j+2] = x[i]$$

$$a[k][j+3] = 1$$

$$a[k][j+4] = -(x[i] * * 3)$$

$$a[k][j+5] = -(x[i] * * 2)$$

$$a[k][j+6] = -x[i]$$

$$a[k][j+7] = -1$$

$$i += 1$$

$$j += 4$$

$$i = 1$$

```

j = 0
for k in range(((len(x)* 2)-2),((len(x)* 3)-4)):
    b += [0]
    a[k][j] = 3* (x[i]* * 2)
    a[k][j+1] = 2* x[i]
    a[k][j+2] = 1
    a[k][j+3] = 0
    a[k][j+4] = -(3* (x[i]* * 2))
    a[k][j+5] = -(2* x[i])
    a[k][j+6] = -1
    a[k][j+7] = 0
    i += 1
    j += 4

```

```

i = 1
j = 0
for k in range(((len(x)* 3)-4),((len(x)* 4)-6)):
    b += [0]
    a[k][j] = 6* x[i]
    a[k][j+1] = 2
    a[k][j+2] = 0
    a[k][j+3] = 0
    a[k][j+4] = -6* x[i]
    a[k][j+5] = -2
    a[k][j+6] = 0
    a[k][j+7] = 0
    i += 1
    j += 4

```



```

        else:
            result += str(x[i])+" "
            j = -1
            j += 1

    return(result)

def trazcub_spline(x,y,d):

    b = y
    A, b = matrix_cub(x,b)

    if(d=="T"):
        t1=totalGaussianElimination(A, b)
        return traces(t1[0]),A,b,t1[1],t1[0]

    elif(d=="P"):
        t2=partialGaussianElimination(A, b)
        return traces(t2[0]),A,b,t2[1],t2[0]

    elif(d=="S"):
        t3=simpleGaussianElimination(A, b)
        return traces(t3[0]),A,b,t3[1],t3[0]

```

Vandermonde:

```
from Metodos.GaussianElimination import *

def vandermonde(x, y):
    matriz = []
    for i in range(len(x)):
        matriz.append([])
        fila = 0

        for i in x:
            for j in range(len(x)-1,-1,-1):
                matriz[fila].append(i* * j)
            fila+=1
        print('A:')
        printMatriz(matriz)
    return totalGaussianElimination(matriz, y)[0]
```