

Powering the Future in a Sustainable way: Lithium Analysis and Forecasting

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Program

01

Lithium demand factors

Key aspects of the increasing demand of lithium worldwide

04

Forecast

Holt's Exponential smoothing, KNN regression, ARIMA

02

Lithium time series

Comparison of lithium exports in Chile and Australia

05

Conclusions

Summary of the main findings

03

Explainability analysis

Linear regression, Bass model, GBM, Competition model

06

Future of lithium

Expectations in the Lithium market for the future



01

Lithium demand factors

Key aspects for understanding the current demand for lithium



Net Zero by 2050

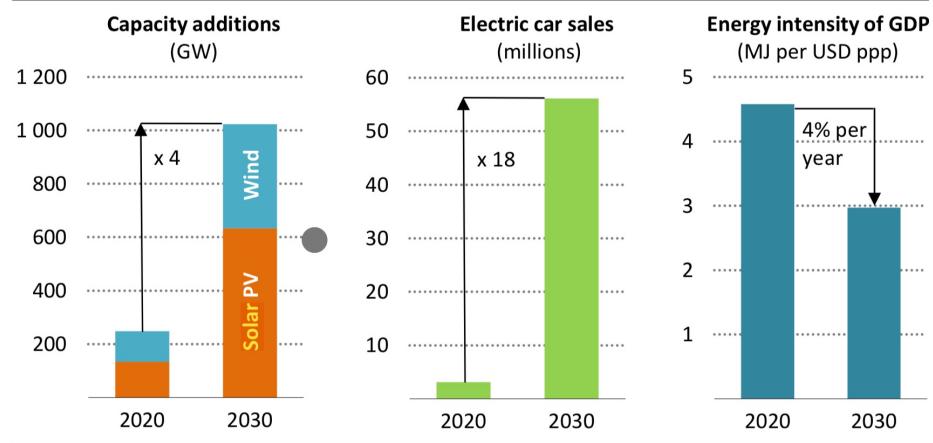
- To achieve global net-zero emissions by 2050 the conversion to electric transport is a cornerstone¹



Source: ¹[IEA \(A Roadmap for the Global Energy Sector\)](#)

A clean technology expansion by 2030 is needed

Key clean technologies ramp up by 2030 in the net zero pathway



Note: MJ = megajoules; GDP = gross domestic product in purchasing power parity.

Source: [IEA](#) (A Roadmap for the Global Energy Sector)



Sales of electric vehicles surge as fast-charging sites double across Australia in a year

EVs made up just 2% of new car sales in May 2022, but now 8.3% of new car sales in 2023 are battery powered



Source: [The Guardian](#)



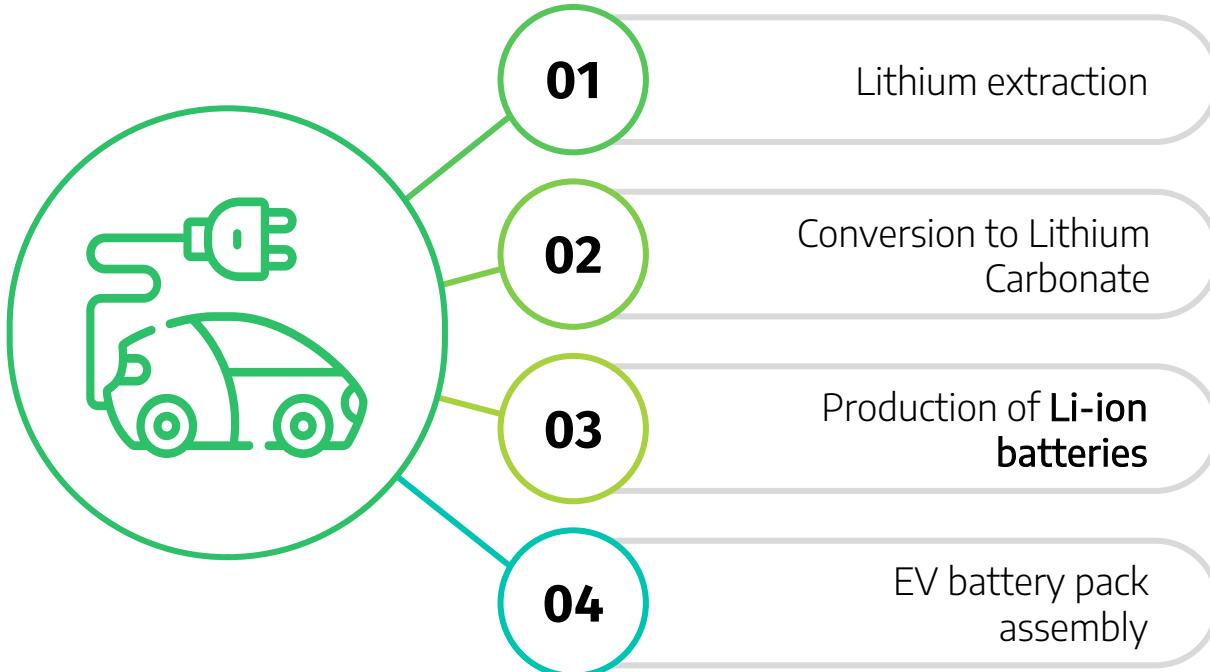
Sales of **electric vehicles** surge as fast-charging sites double across **Australia** in a year

EVs made up just 2% of new car sales in May 2022, but now 8.3% of new car sales in 2023 are **battery powered**



Source: [The Guardian](#)

Lithium: From the nature to an electric vehicle



What is Lithium?



Chemical Element



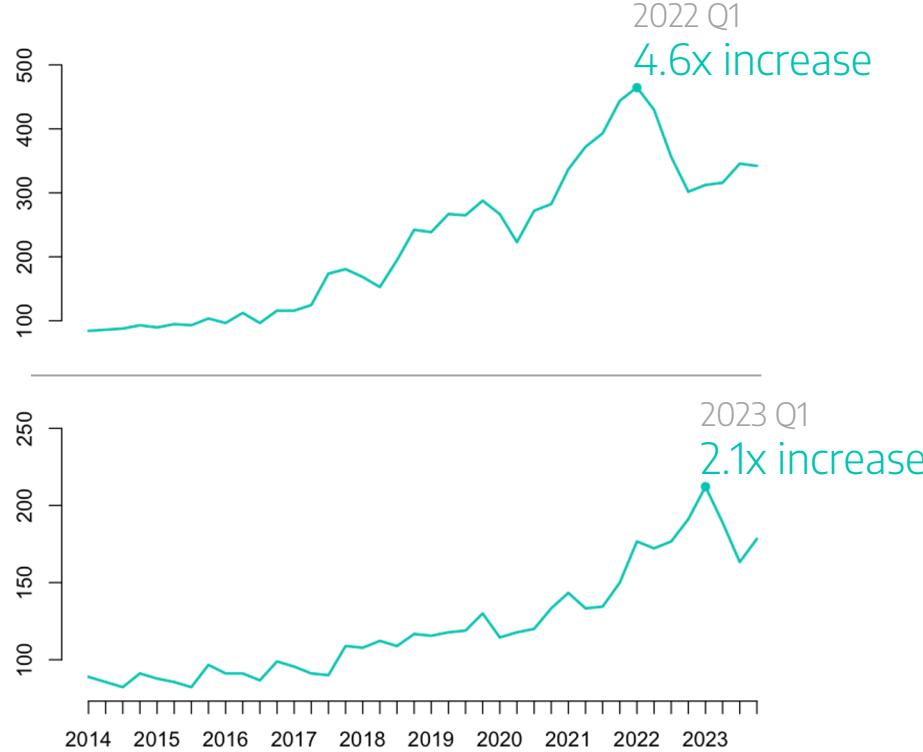
Abundant Element



Crucial for Batteries

Google Trends Interest over time

Electric vehicles



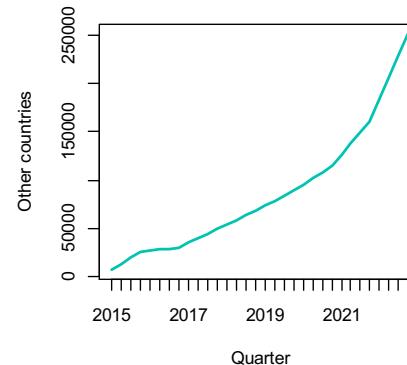
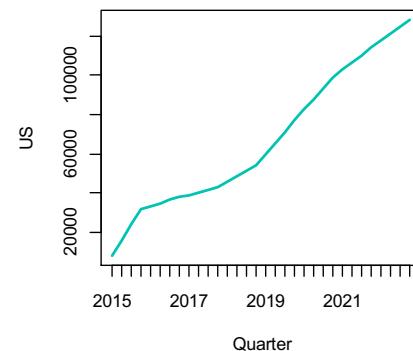
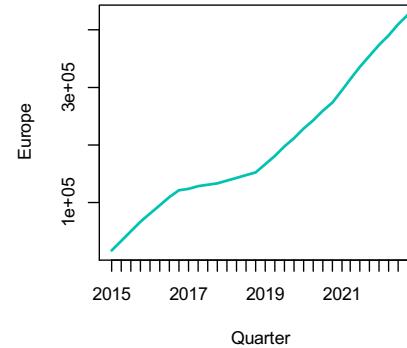
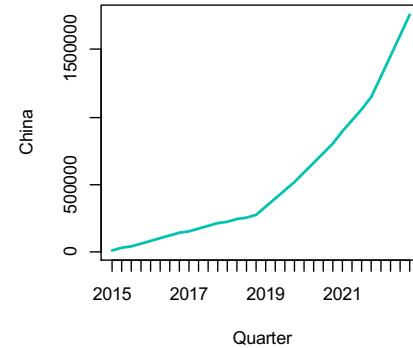
Lithium



Source: [Google Trends](#)

Electric chargers Number of fast and slow chargers

- Increasing trend in electric chargers
- China stands out as the country with the greatest number of fast¹ and slow² electric chargers available



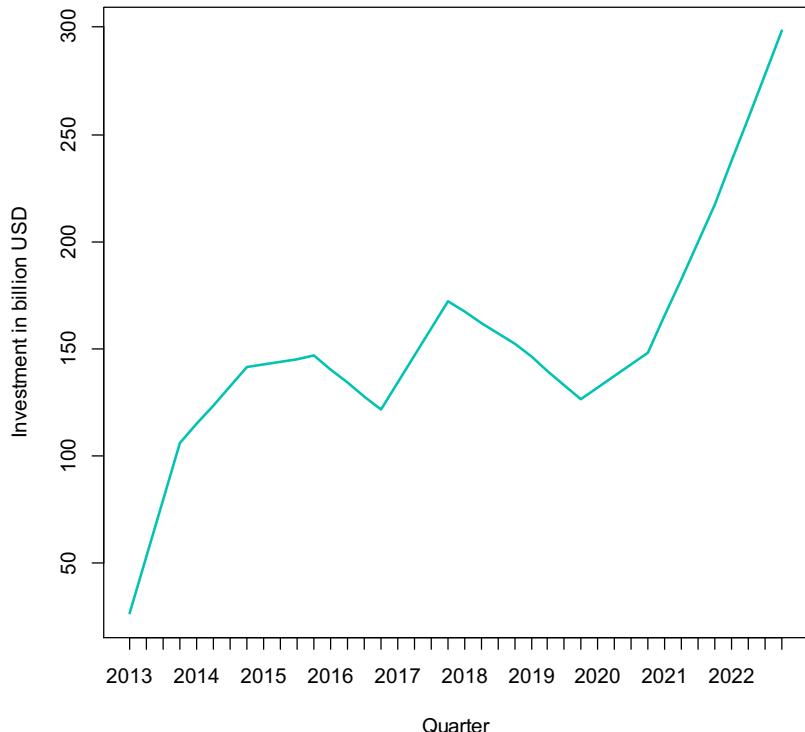
Source: ¹IEA (fast chargers),
²IEA (slow chargers)

Solar investment Billion USD

- It is related to energy storage
- It can also be seen as an indicator of interest in renewable energies



Source: [Statista](#)



02

Lithium Time Series

About the main time series

Lithium production Worldwide map (2022)

Australia and Chile account for the **77%** of the global production



Source: [United States Geological Survey](#)

Lithium production Australia and Chile

- Australia extracts lithium from hard rock mines
- Chile extracts the mineral from brines
- Common unit of measure: Lithium Carbonate Equivalent (LCE)
- We will focus on the exports of each country

Pilbara Minerals' Pilgangoora lithium tantalum mine, Australia



Brine pools and processing areas at SQM's lithium mine on the Atacama salt flat, Chile

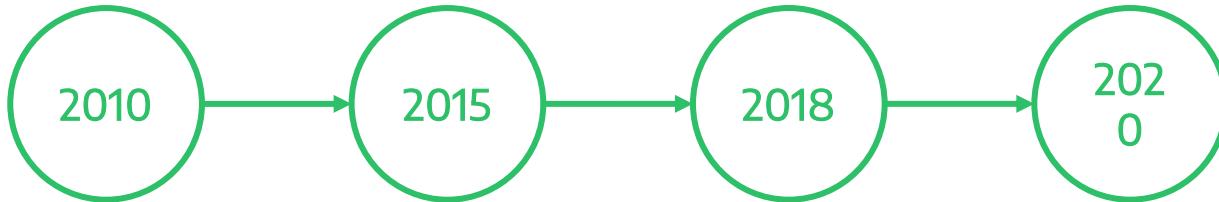
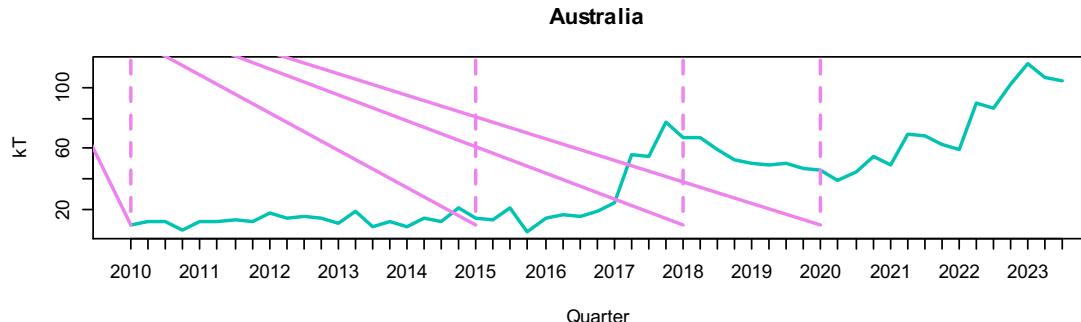
Lithium exports Australia

- Significant rise from 2016 to 2018
- Decline from 2018 to 2020
- Recovery from 2020 onwards
- Most of Australia's exports go to China



Source: Australian Government,
Department of Industry, Science and
Resources

Lithium Australia events



China began granting subsidies to EV buyers¹

China's government announced that it would phase out subsidies progressively from 2016 and by the end of 2020²

Excess of supply, slower demand growth for EV, criteria to qualify for subsidies became more stringent in China³

China's government extended the subsidies by two years to the end of 2022, due to the pandemic and the economic downturn⁴

Sources:

¹[China's National Development and Reform Commission](#)

²[International Council on Clean Transportation](#)

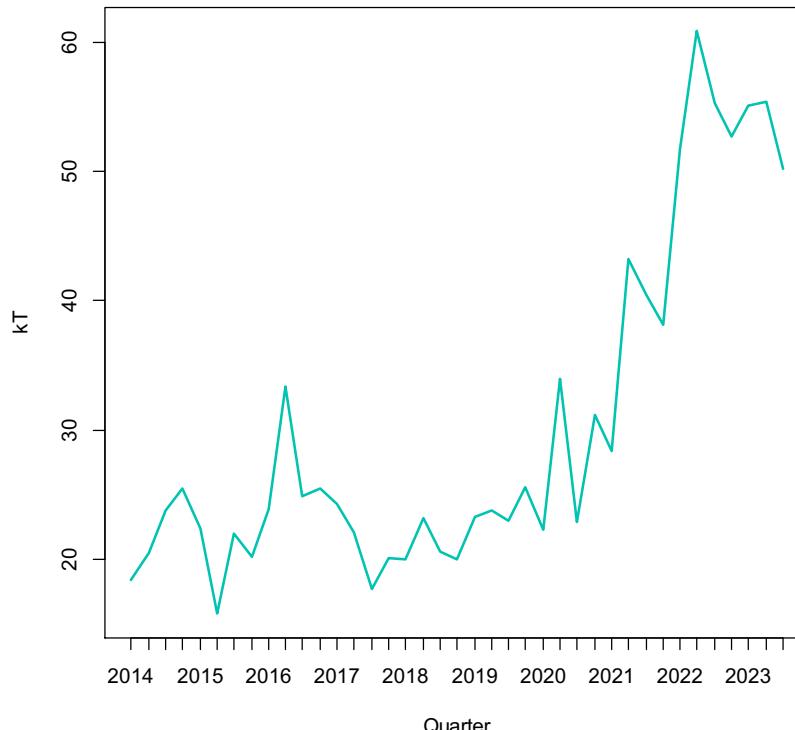
³[Reuters news agency](#)

⁴[Ministry of Finance of the People's Republic of China](#)



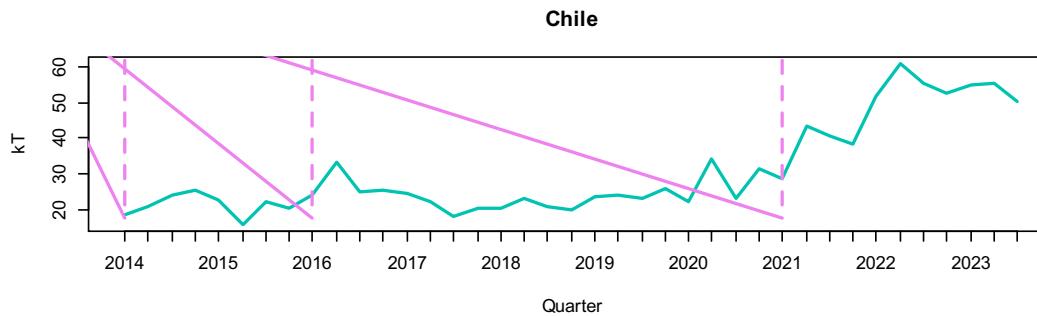
Lithium exports Chile

- Chile started a National Lithium Commission in 2014
- Rise from 2015 to mid-2016
- Decline from mid-2016 to 2018
- Significant rise from 2021 onwards, most of Chile's exports go to China, Japan and South Korea



Source: National Customs Service of Chile

Lithium Chile events



Decree: National
Lithium Commission
in 2014

Chile experienced a
decrease in its lithium
production and market
share¹

In the first five months of
2021, Chile increased
significantly the
exportation of lithium
carbonates (China, Japan,
South Korea)². Most of
Chile's exports go to
China³

Sources:

¹[Mining.com](#)

²[Reuters news agency](#)

³[World Integrated Trade Solution](#)

Lithium exports Australia and Chile

- Australia has a larger experience in Lithium extraction and exploration
- Both countries have experienced an increase in their exportations since 2020.
- Mainly explained by the growing interest in this product

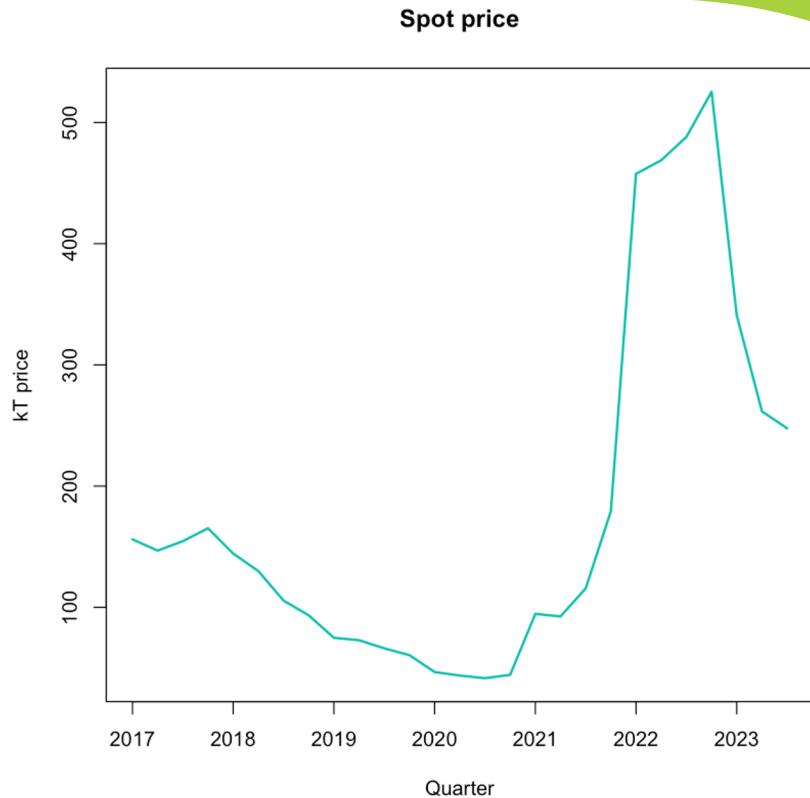


Lithium Carbonate Price per kilotonne (CNY)

- China spot price
- Exponential increase from 2020
- Slowing demand causing the price to fall.



Source: [Investing](#)



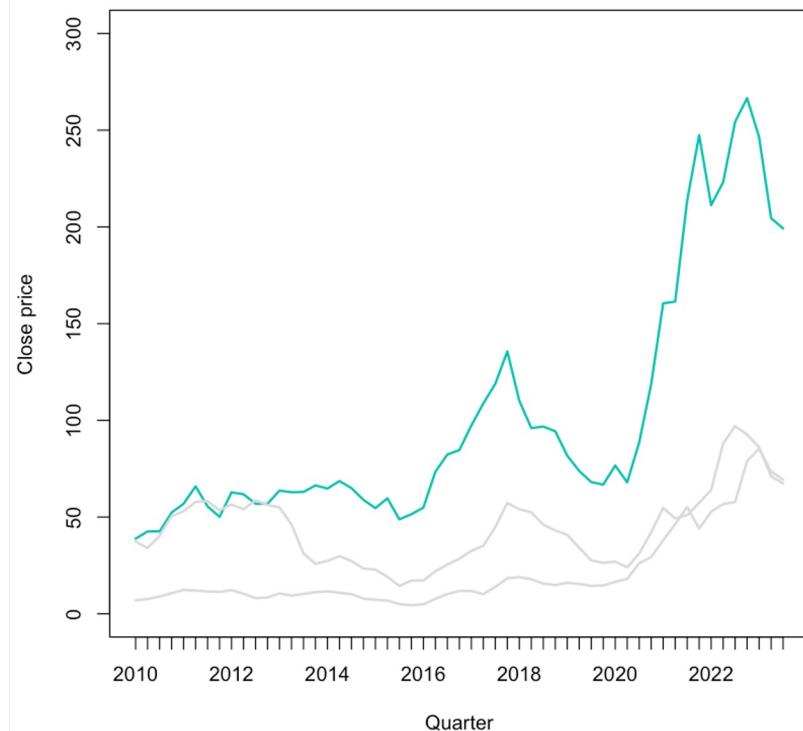
Stock Prices Lithium Companies

Albemarle Corporation

- US based company
- World's largest lithium producer
- Presence in Australia and Chile



Source: Yahoo! Finance



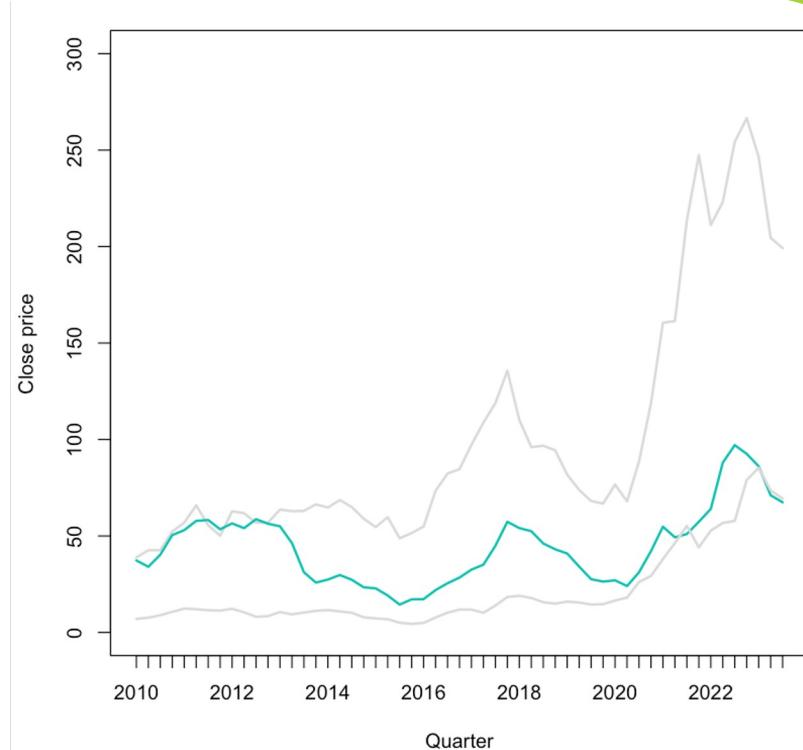
Stock Prices Lithium Companies

Sociedad Química y Minera de Chile S.A. (SQM)

- Chilean company
- World's second-largest lithium producer



Source: Yahoo! Finance



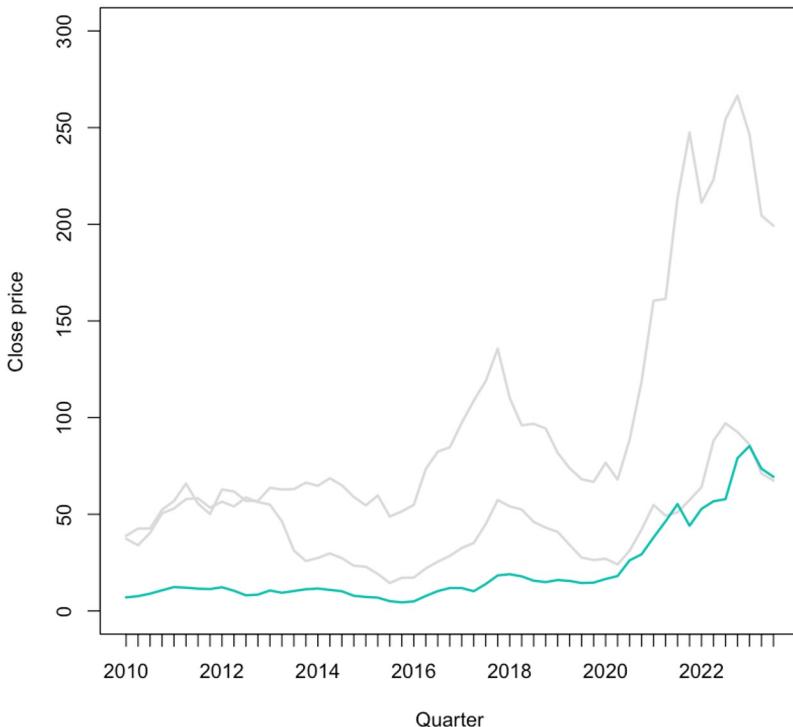
Stock Prices Lithium Companies

Mineral Resources Limited

- Australian company
- Operates hard rock lithium mines in Western Australia



Source: Yahoo! Finance



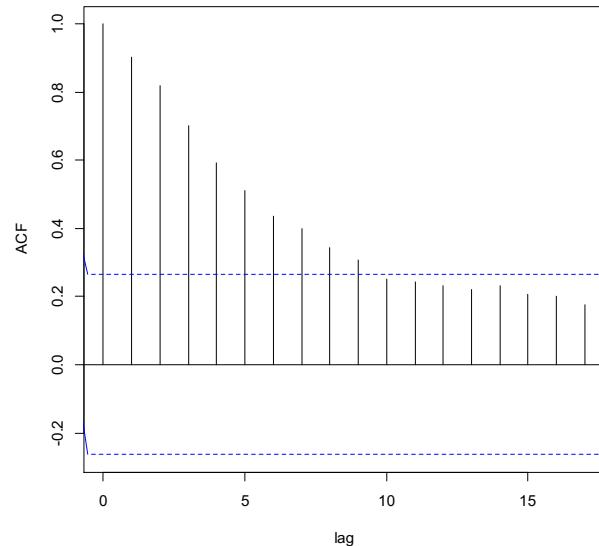
03

Explainability Analysis

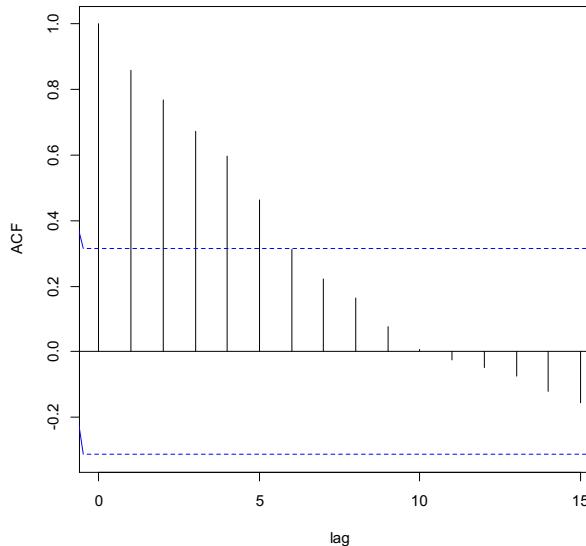
Linear Regression, Bass model, GBM, Competition model

Lithium time series Australia and Chile

Australia



Chile

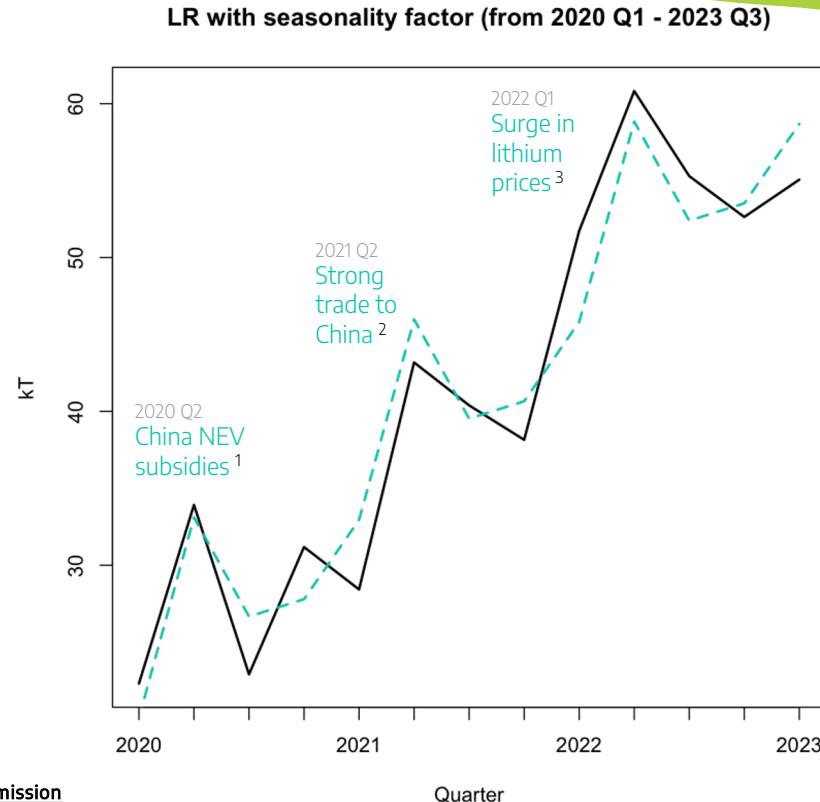


- Both cases: Trend is significant and Seasonality not significant (Linear Regression + season + trend) but...

Linear Regression Window from 2020 onwards

Chile

- Trend is significant
- Q2 is slightly significant, compared to Q1
- $R^2 = 0.94$



Sources:

¹China's National Development and Reform Commission

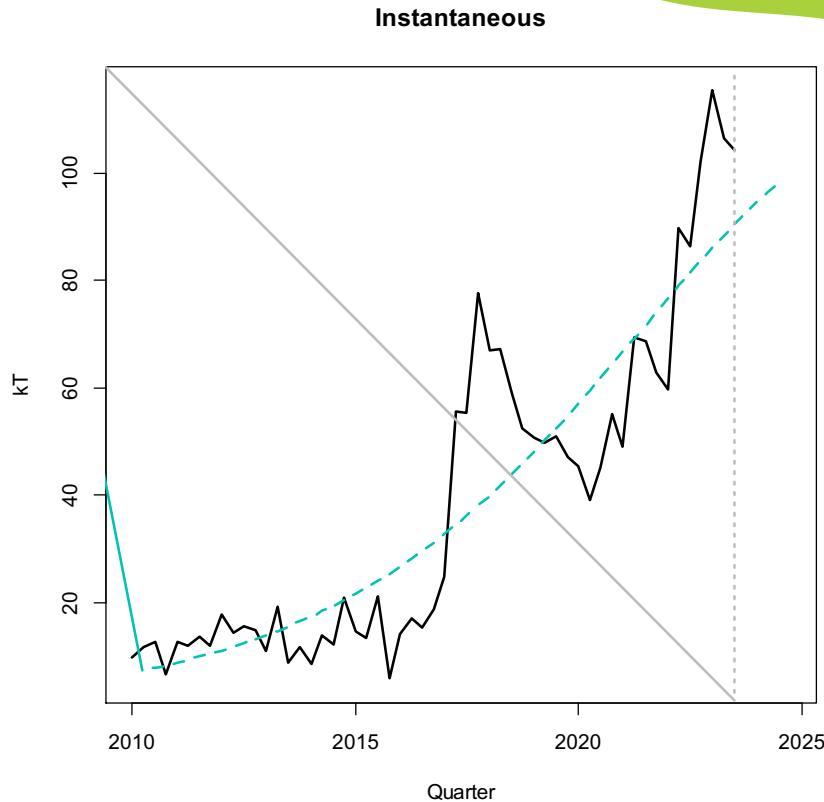
²S & P

³S & P

Bass model Australia

- Forecast for one year
- Slightly pessimistic
- Innovation and imitation are significant

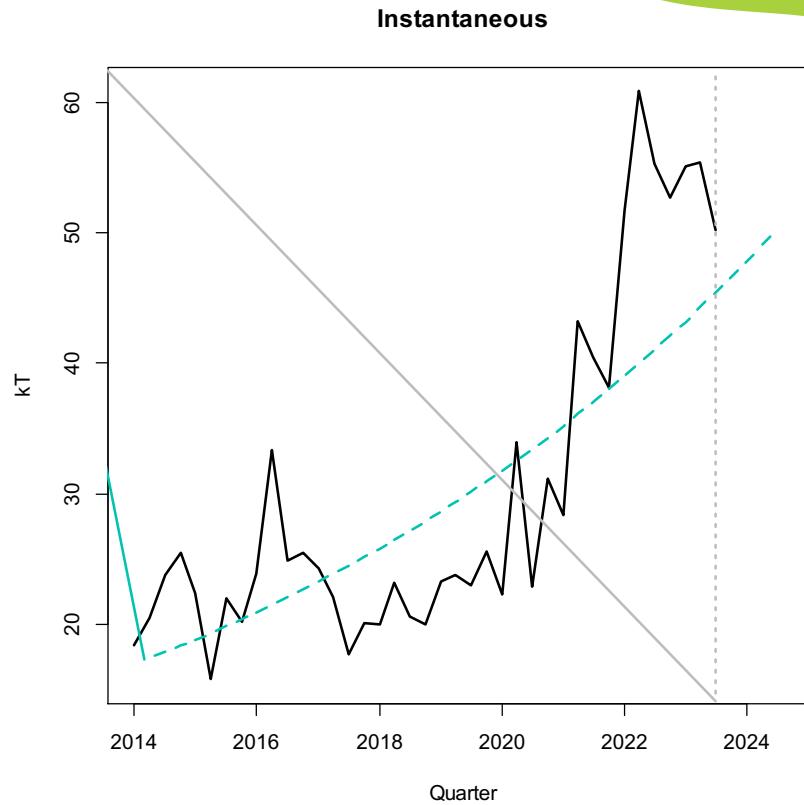
Market Potential	Innovation	Imitation	R ²
6.99e+03**	9.7e-04***	5.9e-02***	0.99



Bass model Chile

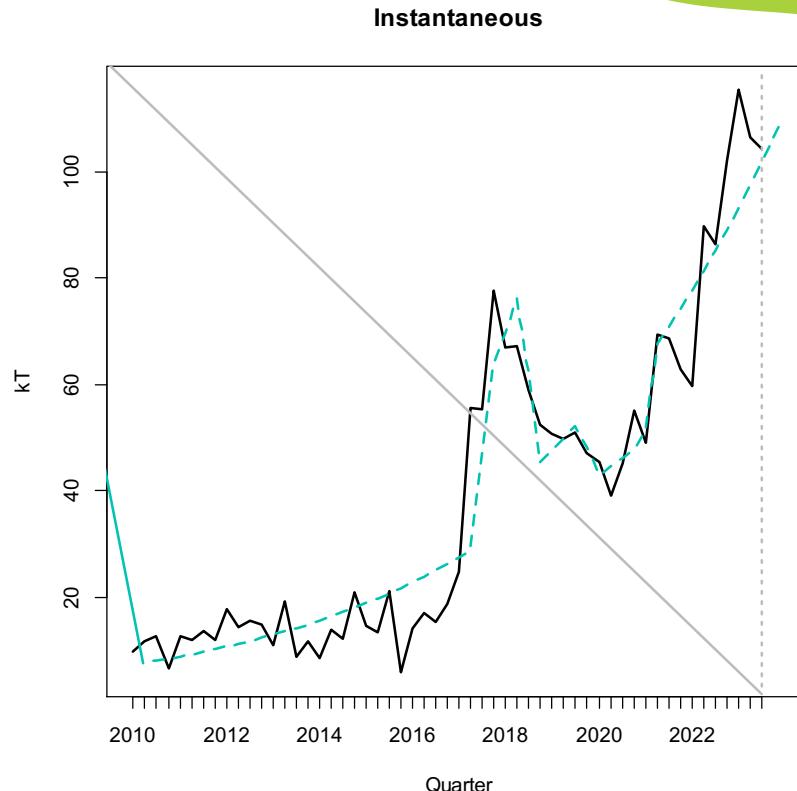
- No significant parameters
- Forecast for one year
- No indication that the peak has been reached

Market Potential	Innovation	Imitation	R ²
5.67+04	2.9-04	2.66-02	0.99



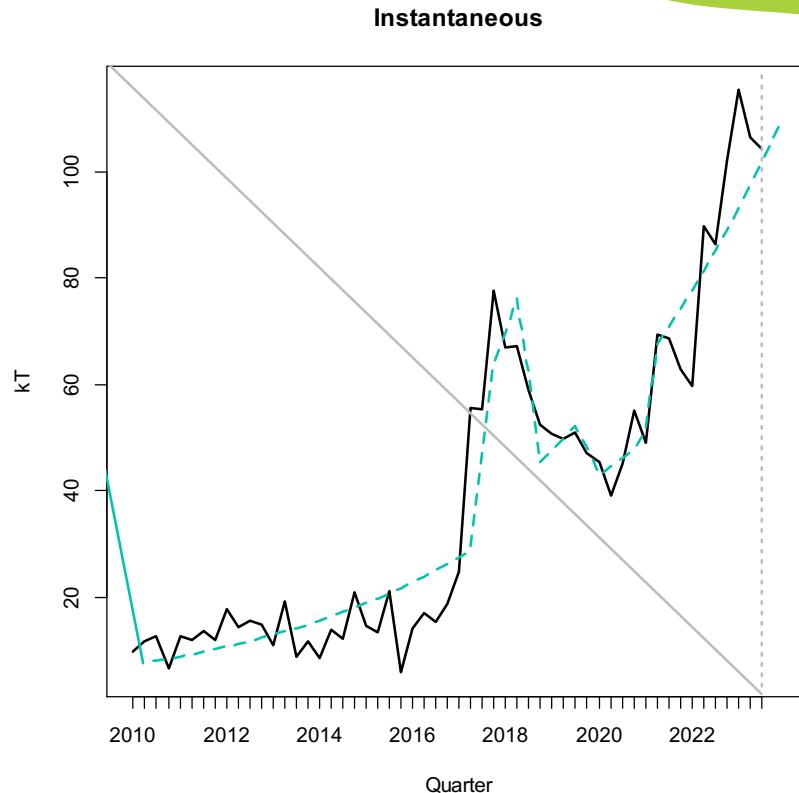
GBM Australia

- Double Rectangular shock between 2017.Q2 – 2018.Q2, and 2019.Q4 – 2021.Q1
- These periods could be approximately explained with the excess of supply of 2018, and the extension of the subsidies in 2020



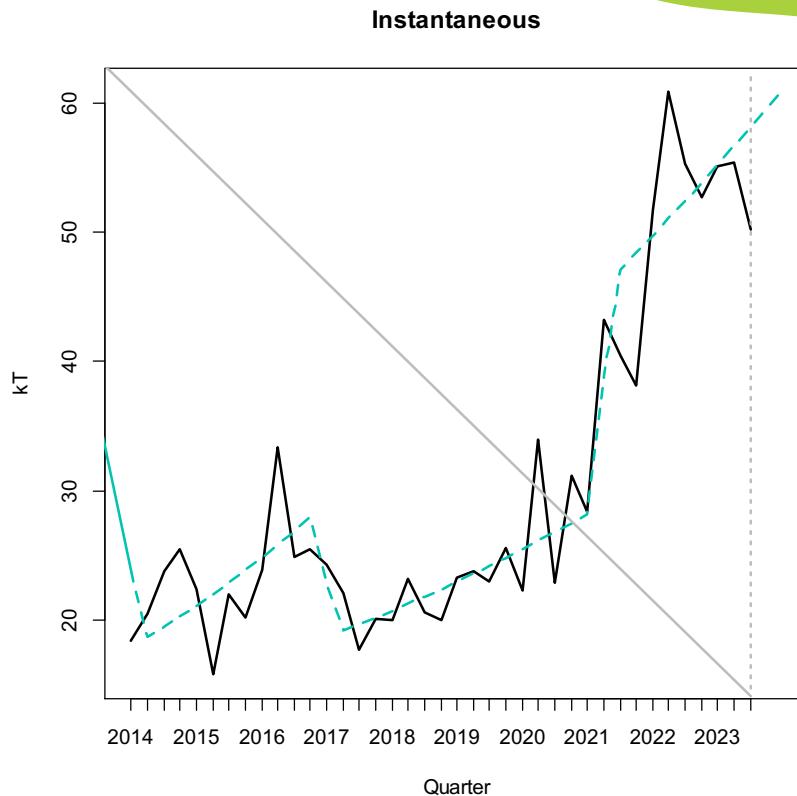
GBM Australia

- $R^2 = 0.998889$
- Significant auto-correlated residuals (better model found by using Double Exponential shock)
- Expected out-of-sample behaviour (four quarters)



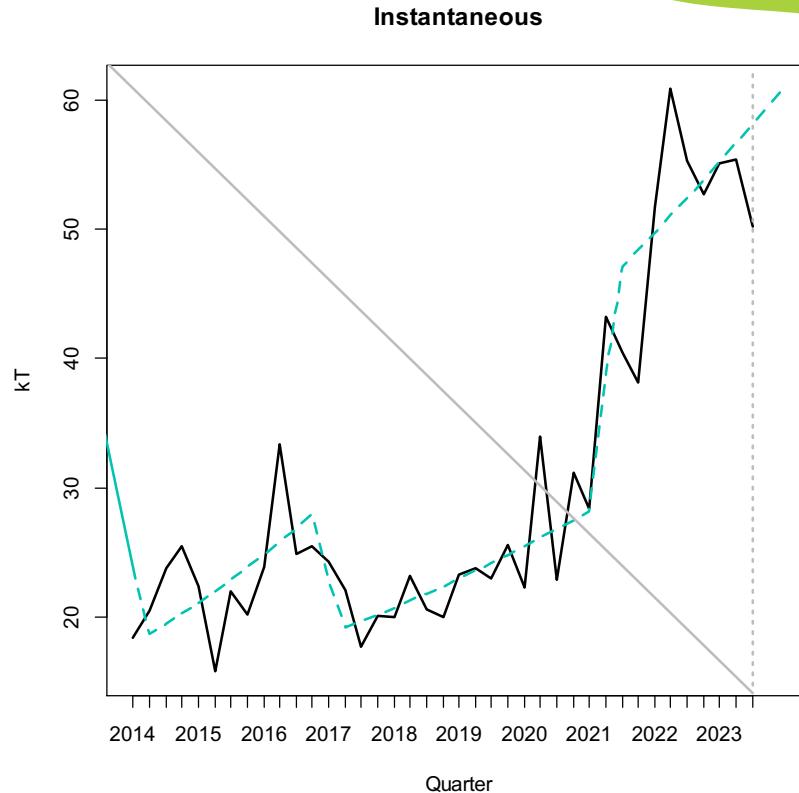
GBM Chile

- Mixed shock at 2016.Q4 (rectangular), and 2021.Q1 (exponential)
- These periods could be approximately explained with the decrease in lithium production and market share of 2016, and the increase in exportations to China, Japan, and South Korea from 2021 onwards



GBM Chile

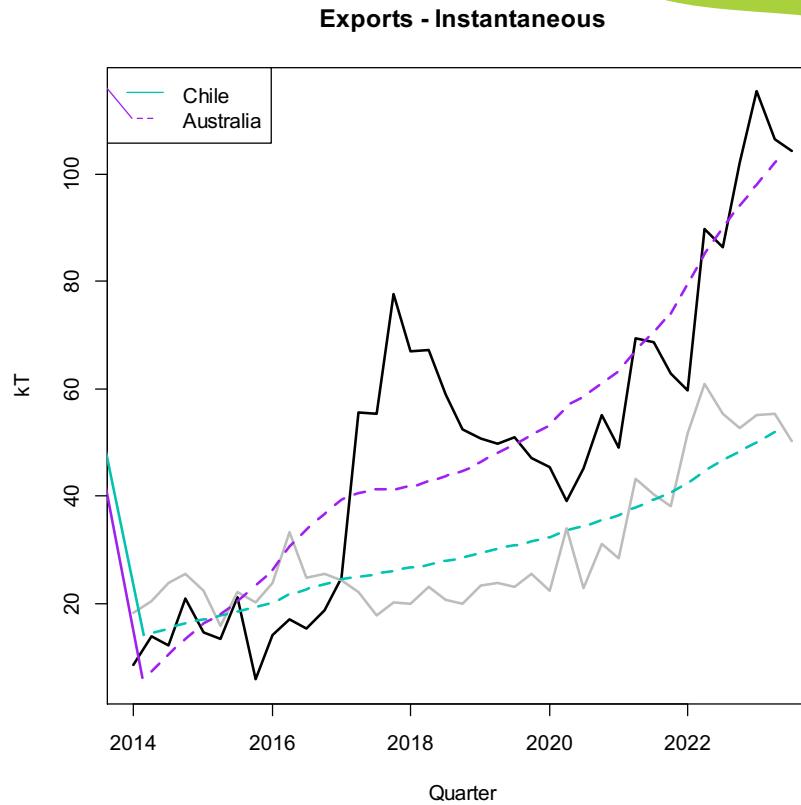
- Significant auto correlated residuals
- Expected out-of-sample behaviour (four quarters)



Competition model Australia vs. Chile

- Both series since 2014.Q1
- Chile collaborates with Australia, but Australia competes with Chile
- Both series show a better adjustment than Bass model

q1c	q2-gamma	R ²
1.47e-01**	-4.2e-03	0.83



04

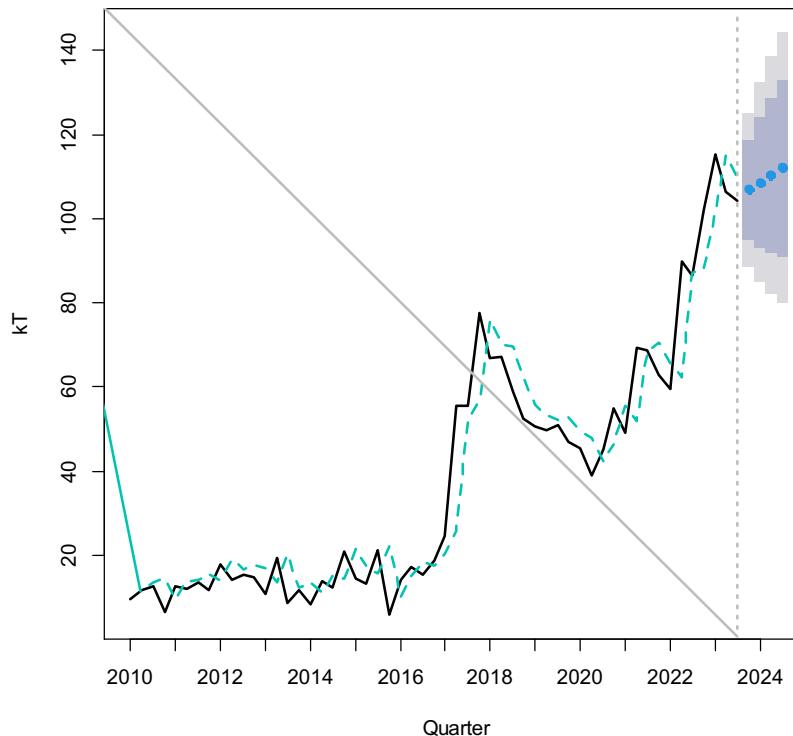
Forecast

Holt's exponential smoothing, KNN regression, ARIMA

Holt's exponential smoothing Australia

Comments:

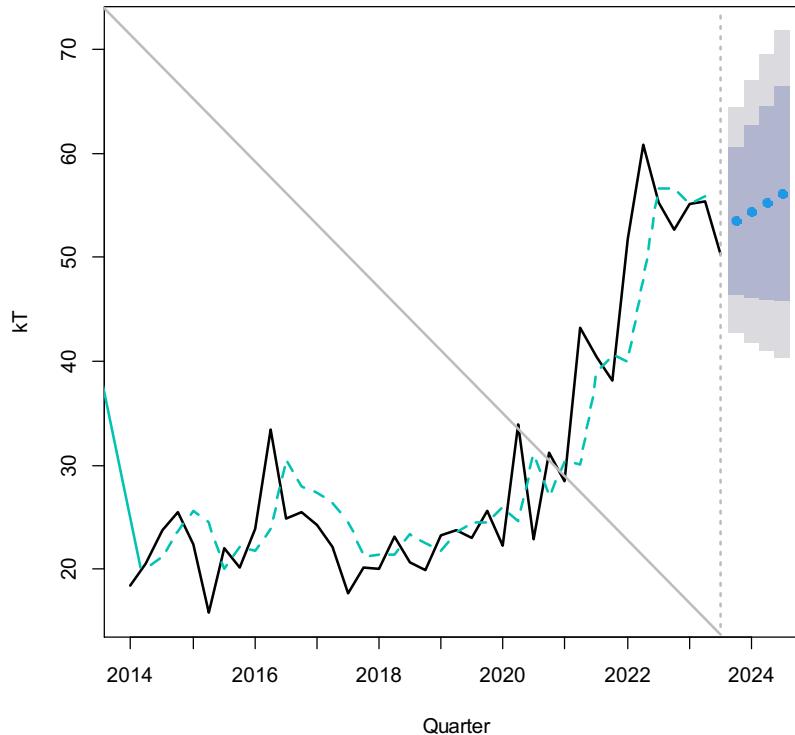
- MAPE 13.64
- Smoothing parameter chosen automatically (0.61)
- A time shift is observed
- Holt-Winters does not offer a better solution to this



Holt's exponential smoothing Chile

Comments:

- MAPE 26.68
- Smoothing parameter chosen automatically (0.84)
- A time shift is observed
- Holt-Winters does not offer a better solution to this



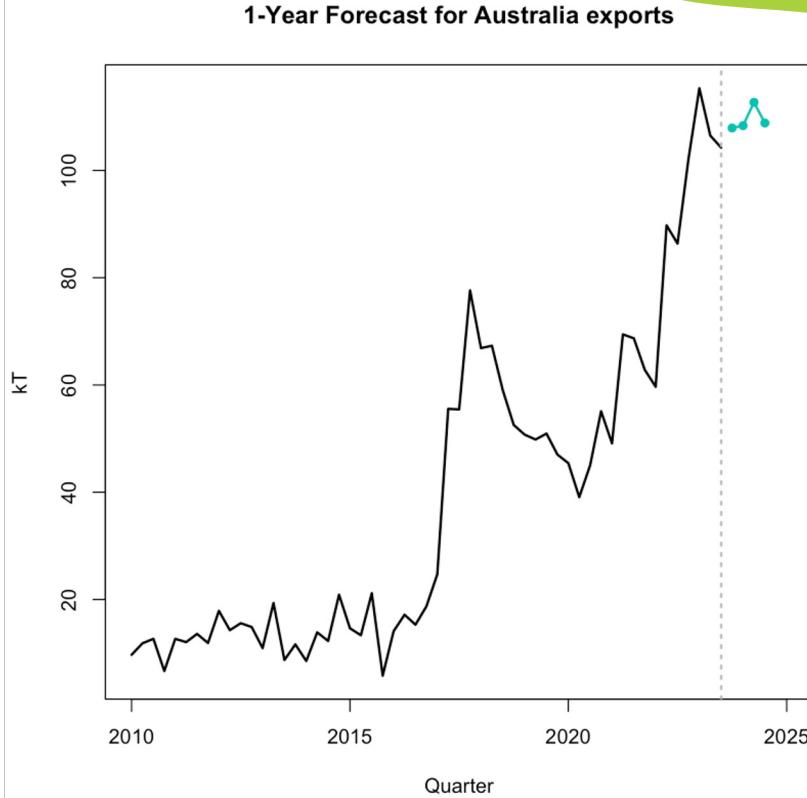
KNN Regression Australia

Comments:

- KNN adapted to time series using lagged values of the dependent variable¹.
- $k = 2$
- Recursive strategy for forecast
- Exports to be tightly balanced according to forecast



Source: [CRAN](#)



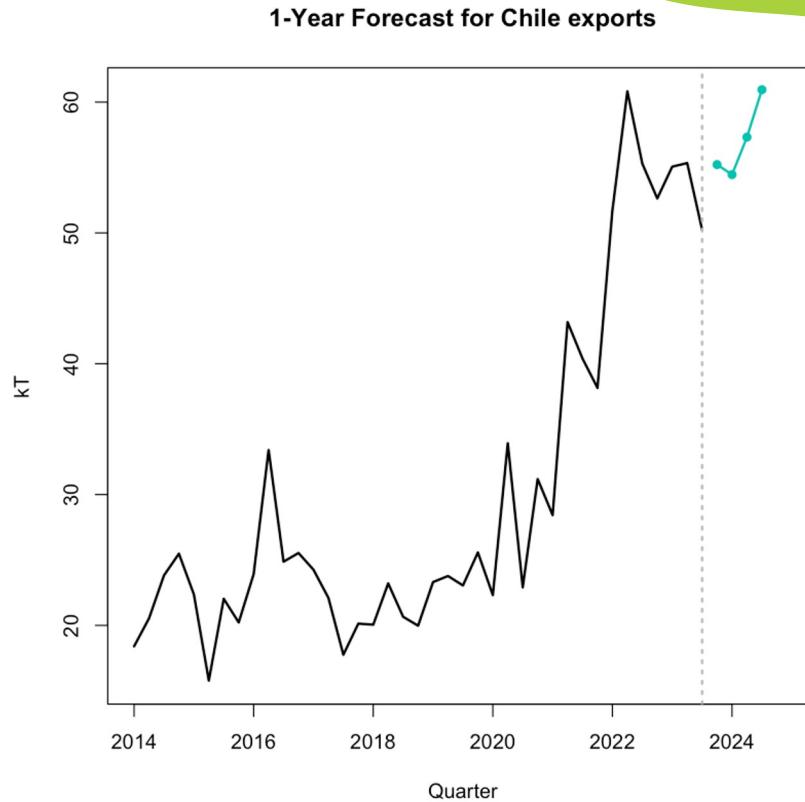
KNN Regression Chile

Comments:

- KNN adapted to time series using lagged values of the dependent variable¹.
- $k = 2$
- Recursive strategy for forecast
- Optimistic forecast

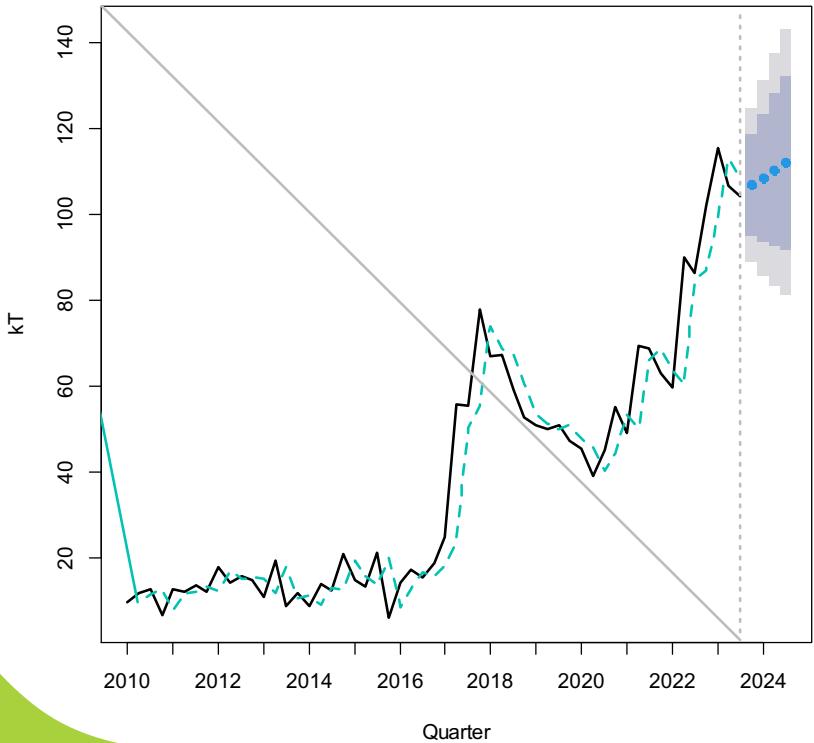


Source: [CRAN](#)

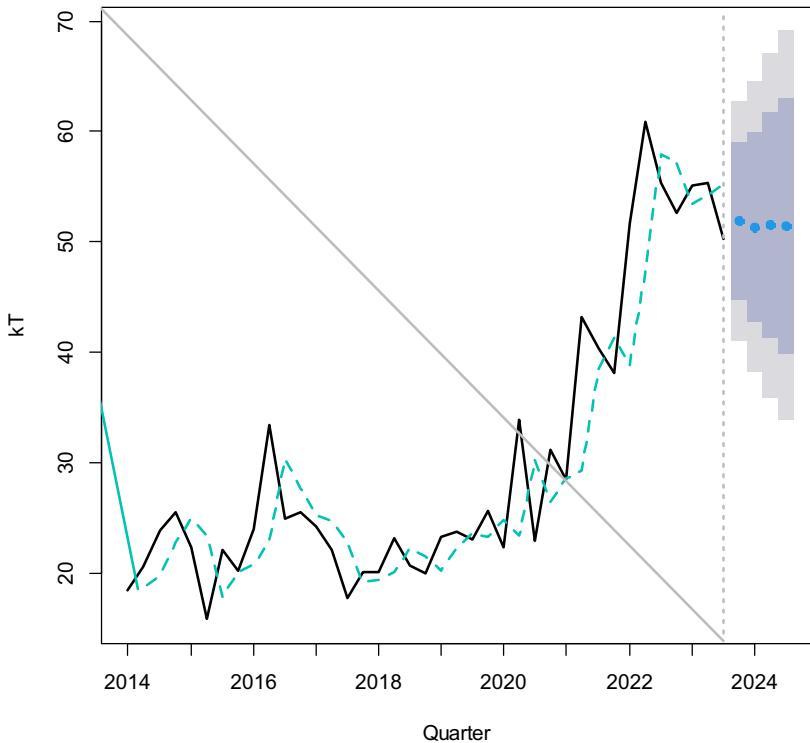


ARIMA Australia and Chile

Australia ARIMA (1,1,0) with drift



Chile ARIMA (1,1,0) with drift



ARMAX Explanatory Variables

Economic

- GDP
- GDP per capita (working population)
- Yearly variations

Energy related

- Electric Vehicles Stock (China, Europe, USA, Total)
- Fast and Slow chargers (China, Europe, USA, Total)
- Solar Investment

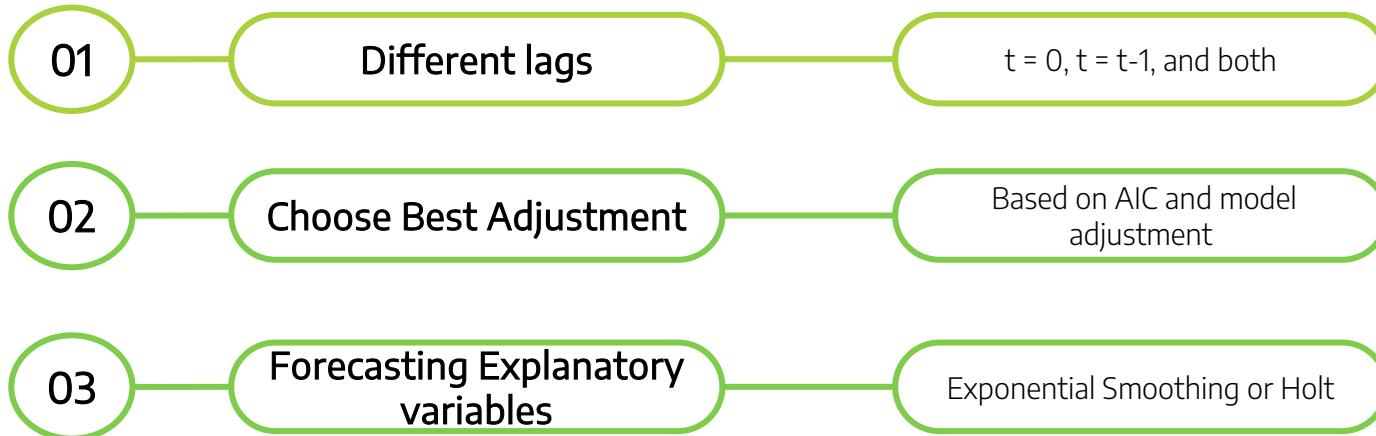
Google Trends

- E-cars (AUS, CHL, World)
- Lithium (AUS, CHL, World)
- Lithium Batteries (AUS, CHL, World)

Stock Market

- Albemarle
- Mineral Resources
- SQM

ARMAX Explanatory Variables

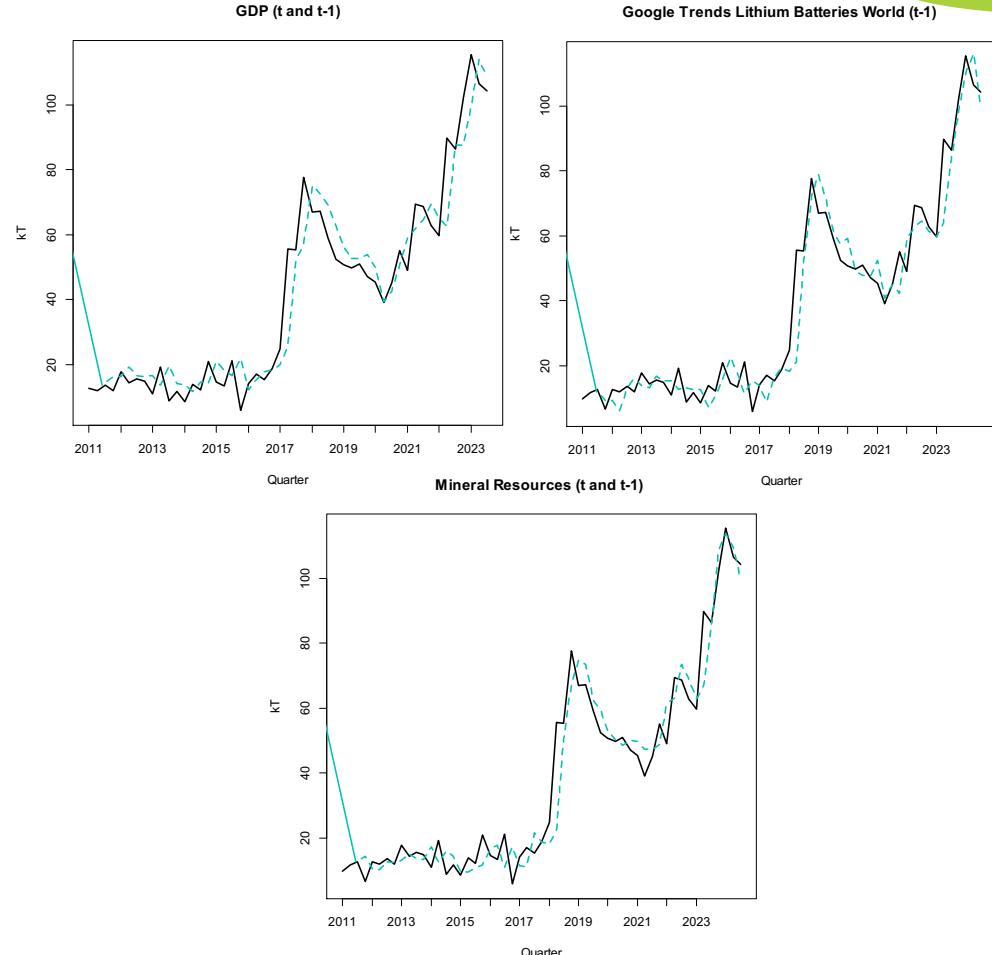


ARMAX Australia

Comments:

- Similar adjustments. Differences in certain periods.

Variable	Time
Mineral Resources	t, t-1
Lithium Batteries World Trends	t-1
GDP per capita Australia	t, t-1

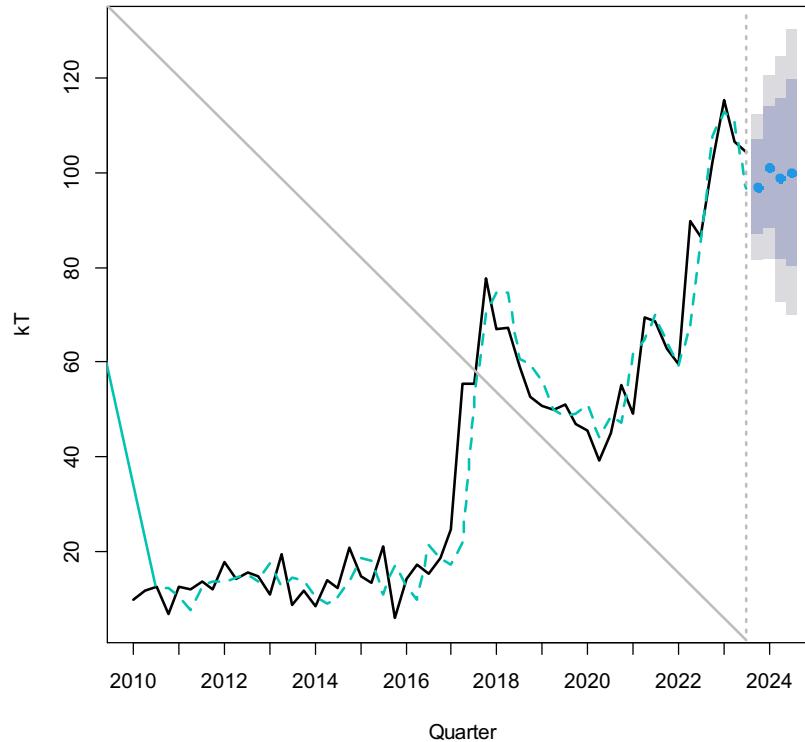


ARMAX Australia

Comments:

- ARIMA(2,1,0) with errors
- Slowly increasing forecasting

Variable	Time	Forecasting
Mineral Resources	t, t-1	Exponential Smoothing
Lithium Batteries World Trends	t-1	Exponential Smoothing

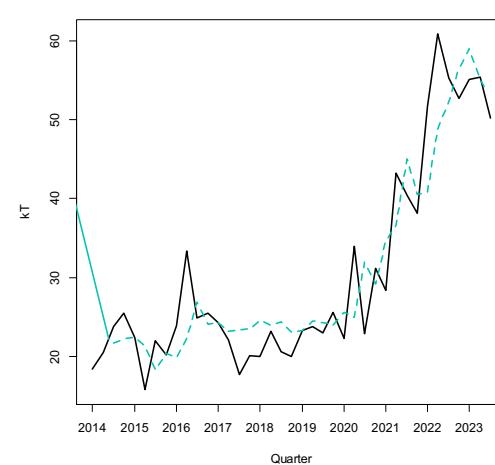
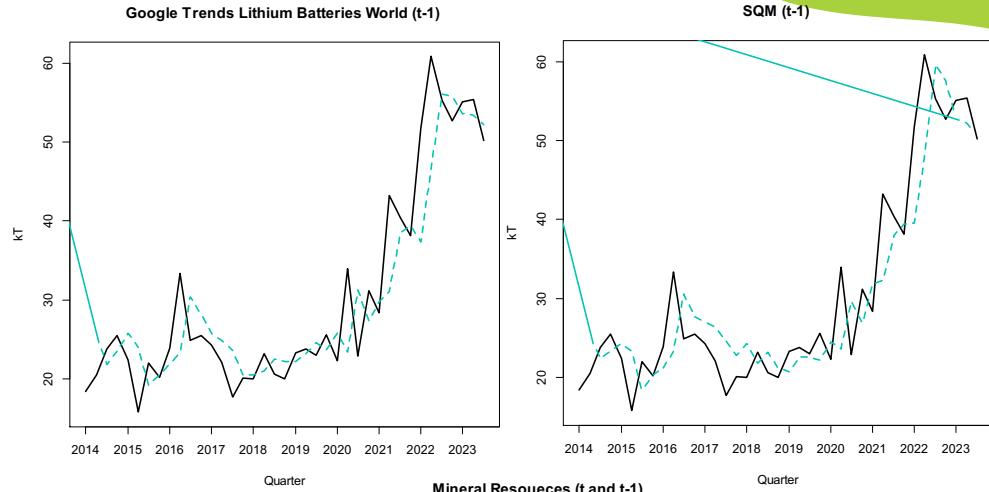


ARMAX Chile

Comments:

- Differences in the adjustment, by period and peaks

Variable	Time
Mineral Resources	t, t-1
Lithium Batteries World Trends	t-1
SQM stock	t-1

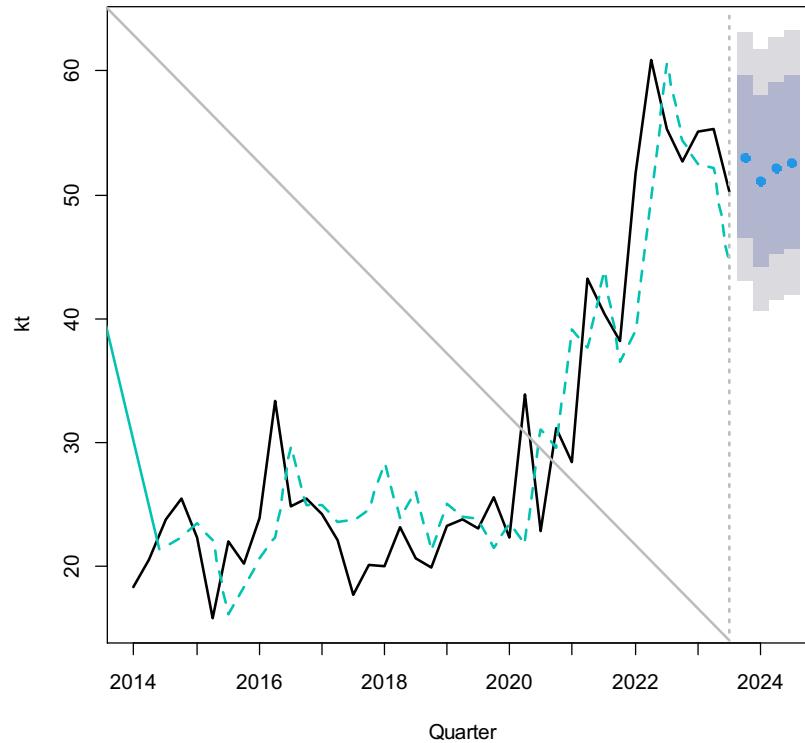


ARMAX Chile

Comments:

- ARIMA(1,1,0) with errors
- Moderate forecasting behaviour

Variable	Time	Forecasting
SQM stock	t-1	Exponential Smoothing
Lithium Batteries World Trends	t-1	Exponential Smoothing
Mineral Resources	t, t-1	Exponential Smoothing



05

Conclusions

Summary of the main findings

Product Growth Australia and Chile

Comments:

- The Bass Model and the Generalized Bass Model help to understand the general growth of Lithium up to now
- The market potential of Lithium has not been reached
- The Generalized Bass Model shows how the shocks impact the dynamics of the lithium export

Forecasting Australia

Model	MAPE
Holt's exponential smoothing	13.64
ARIMA(1,1,0)	26.25
ARMAX GDP	24.77
ARMAX Lithium Batteries World Trends	20.84
ARMAX Mineral Resources	22.48
ARMAX(2,1,0) with errors	21.87



Forecasting Chile

Model	MAPE
Holt's exponential smoothing	26.68
ARIMA(1,1,0)	13.17
ARMAX SQM	13.29
ARMAX Lithium Batteries World Trends	14.31
ARMAX Mineral Resources	12.77
ARMAX(1,1,0) with errors	12.48



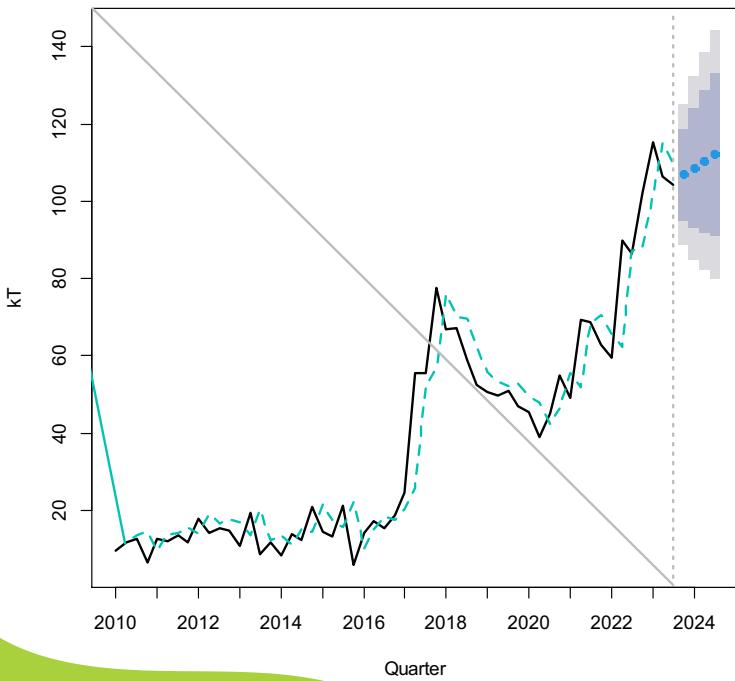
Forecasting Australia and Chile

Comments:

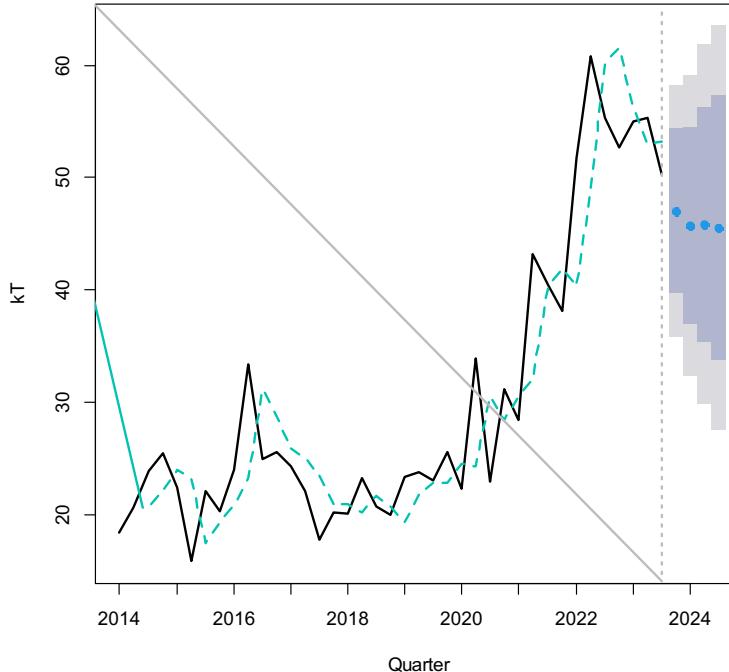
- In the case of Australia, Holt's Exponential Smoothing is the model with the best MAPE
- In the case of Chile, ARMAX with explanatory variables has the lowest MAPE
- The ARMAX model with multiple variables seems to have a better adjustment. However, forecasting will depend on the forecasting of explanatory variables
- It is expected that forecasting has a moderate or increasing behaviour
- Mineral Resources stock seems to be an important variable for **both countries**
- In general, **stock prices** of main companies and **trends of Google** can help forecast Lithium demands

Forecasting Australia and Chile

Holt's Exponential Smoothing Australia



ARIMA(1,1,0) with errors Chile
SQM stock, Lithium trends, Mineral Resources
stock



06

Future of Lithium

Expectations in the Lithium market for the future

Future of lithium

- Companies Albemarle and Tianqi, as well as the Australian Government through grants, are still investing in Western Australia for lithium extraction^{1,2}
- Australia is facing competition from the “lithium triangle” of Chile, Bolivia, and Argentina³
- Chile’s state-owned copper mining company Codelco reached a deal with miner SQM to take a majority stake in a new partnership for future lithium projects in the country until 2060⁴

Albemarle's lithium refinery plant, Australia



Latin America's Lithium Triangle

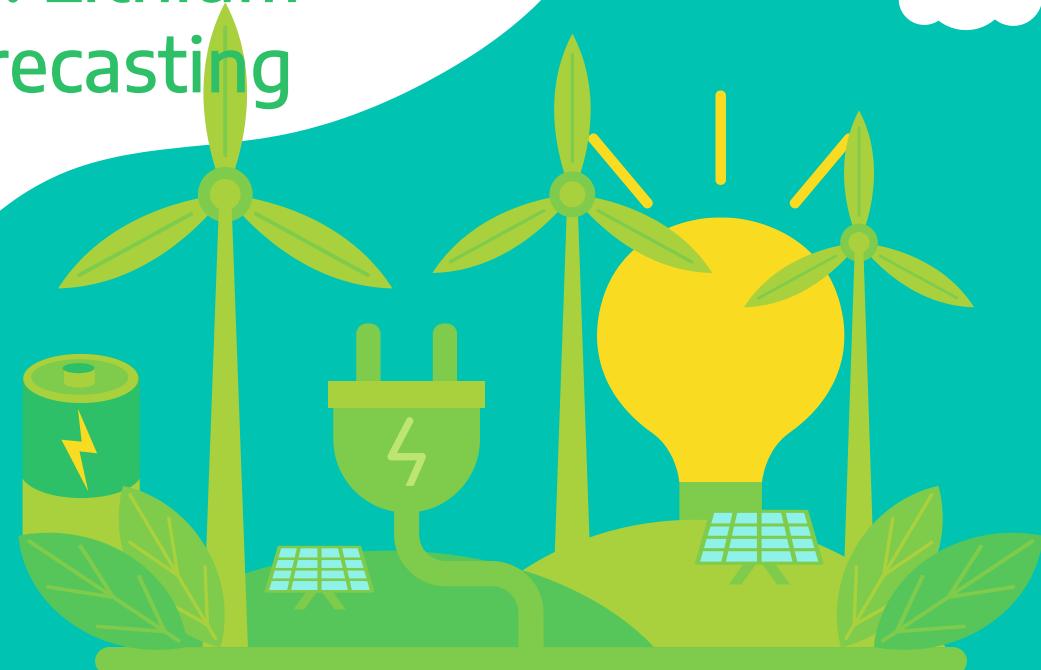
Source: ¹[ABC News Australia](#), ²[Australian Resources and Investment Mining Journal](#), ³[The Guardian](#), ⁴[Financial Times](#)

Padova, another player in the clean technology expansion race



Powering the Future in a Sustainable way: Lithium Analysis and Forecasting

José Chacón
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Mario Tapia



Annex

Models details

Bass model Australia

Residuals:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-92.441	-20.592	11.698	3.162	30.712	64.192

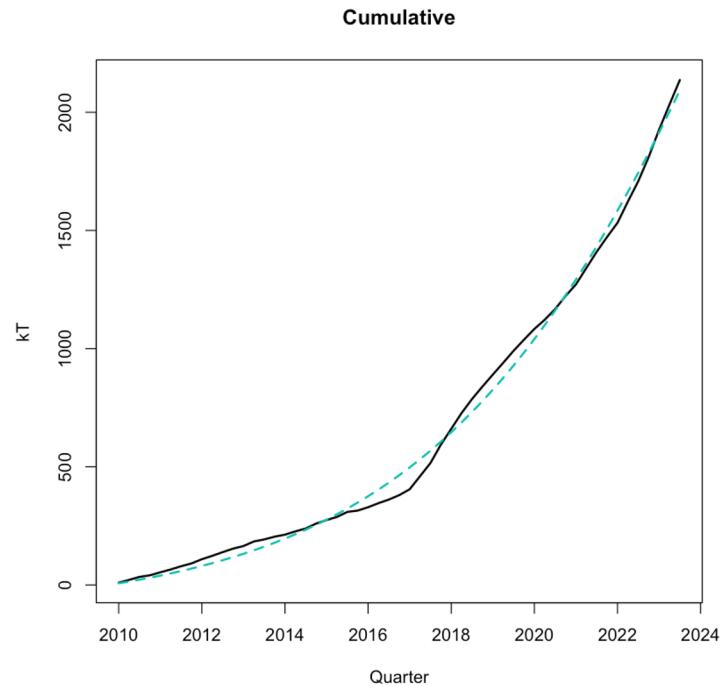
Coefficients:

	Estimate	Std. Error	Lower	Upper	p-value
m	6.996733e+03	2.023331e+03	3.031077e+03	1.096239e+04	1.09e-03 **
p	9.742995e-04	2.199984e-04	5.431106e-04	1.405488e-03	4.90e-05 ***
q	5.918484e-02	4.335764e-03	5.068690e-02	6.768278e-02	7.99e-19 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error 40.09025 on 52 degrees of freedom

Multiple R-squared: 0.998009 Residual sum of squares: 83575.88



Bass model Chile

Residuals:

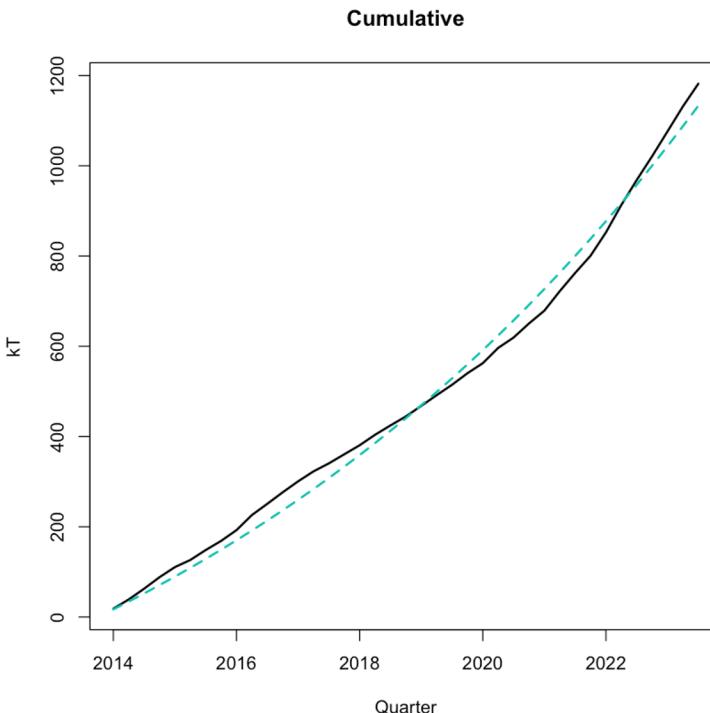
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-48.005	-16.801	11.857	5.834	24.918	48.728

Coefficients:

	Estimate	Std.Error	Lower	Upper	p-value
m	5.672974e+04	1.308179e+06	-2.507253e+06	2.620713e+06	0.966
p	2.953937e-04	6.789610e-03	-1.301200e-02	1.360278e-02	0.966
q	2.662497e-02	1.669377e-02	-6.094216e-03	5.934416e-02	0.119

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error 29.49248 on 36 degrees of freedom
Multiple R-squared: 0.997473 Residual sum of squares: 31313.04



GBM Australia

```

call: ( Generalized Bass model with 2 Rectangular shock )

GBM(series = lit.aus.exp.quarter.kton, shock = "rett", nshock = 2,
prelimestimates = c(5742.86, 0.001144887, 0.06183144, 29,
40, 0.1, 46, 48, -0.1), oos = 4)

Residuals:
    Min. 1st Qu. Median  Mean 3rd Qu. Max.
-41.558 -4.966  3.872  5.829 19.845 41.011

Coefficients:
            Estimate Std. Error      Lower      Upper p-value
m     8.416800e-04 4.715245e+05 -8.400030e-05 1.008339e+06 8.59e-01
p     8.566532e-05 4.920639e-04 -8.787622e-04 1.050093e-03 8.63e-01
q     4.724516e-02 6.224386e-03 3.504559e-02 5.944473e-02 1.20e-09 ***
a1    3.040689e+01 8.713057e-01 2.869916e+01 3.211462e+01 9.55e-35 ***
b1    3.448762e+01 6.650405e-01 3.318417e+01 3.579108e+01 1.84e-42 ***
c1    9.210240e-01 2.880427e-01 3.564706e-01 1.485577e+00 2.51e-03 **
a2    3.952839e+01 1.733262e+00 3.613126e+01 4.292553e+01 1.03e-26 ***
b2    4.486250e+01 1.652551e+00 4.162356e+01 4.810144e+01 5.86e-30 ***
c2   -2.382478e-01 1.196473e-01 -4.727523e-01 -3.743350e-03 5.24e-02
...
signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error 21.97029 on 46 degrees of freedom
Multiple R-squared:  0.998889 Residual sum of squares:  22203.92

```

Ljung-Box test

```

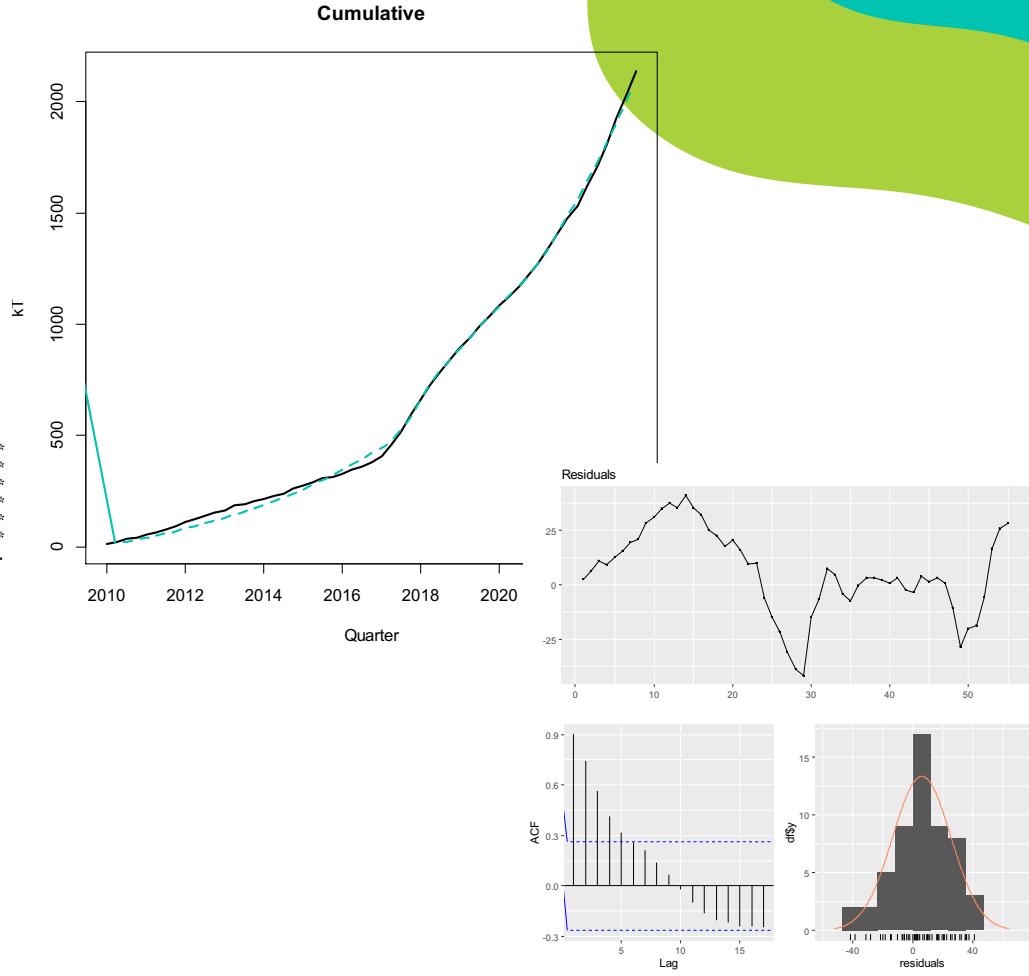
data: Residuals
Q* = 125.01, df = 10, p-value < 2.2e-16

```

```

Model df: 0.  Total lags used: 10

```



GBM Australia

Call: (Generalized Bass model with 2 Exponential shock)

```
GBM(series = lit.aus.exp.quarter.kton, shock = "exp", nshock = 2,
prelimestimates = c(5742.86, 0.001144887, 0.06183144, 27,
0.1, 0.1, 43, 0.1, 0.1), oos = 4)
```

Residuals:

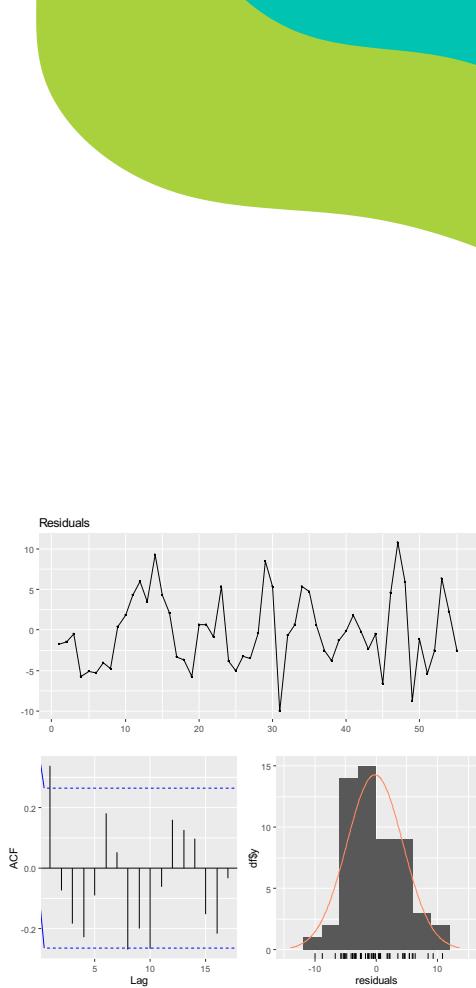
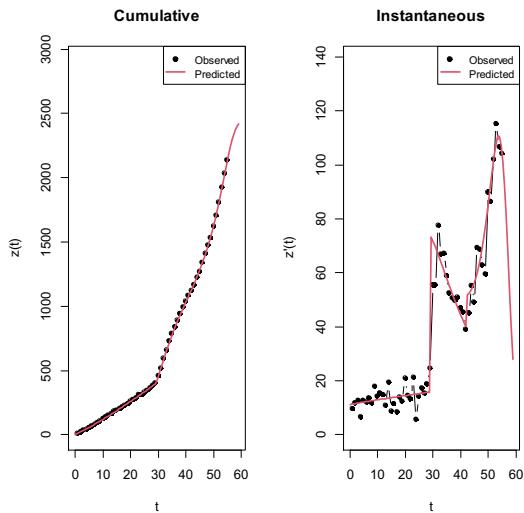
	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
	-10.0016	-3.5782	-0.5029	-0.2060	2.9150	10.8381

Coefficients:

	Estimate	Std. Error	Lower	Upper	p-value
m	2451.91235646	7.604703e-01	2.302863e+03	2.600962e+03	3.17e-33 ***
p	0.00461427	1.633521e-04	4.294106e-03	4.934434e-03	1.04e-30 ***
q	0.01920551	1.511792e-03	1.624245e-02	2.216857e-02	1.20e-16 ***
a1	29.25114472	9.603027e-02	2.906293e+01	2.943936e+01	1.13e-77 ***
b1	-0.08381038	7.952976e-03	9.939793e-02	-6.822284e-02	7.48e-14 ***
c1	3.63499244	1.657908e-01	3.310049e+00	3.959936e+00	5.43e-26 ***
a2	42.39489870	4.367424e-01	4.153890e+01	4.325090e+01	7.21e-55 ***
b2	0.23871050	1.630093e-02	2.067613e-01	2.706597e-01	6.17e-19 ***
c2	0.71115920	9.841783e-02	5.182638e-01	9.040546e-01	4.18e-09 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error 4.964882 on 46 degrees of freedom
Multiple R-squared: 0.999943 Residual sum of squares: 1133.903



GBM Chile

```

call: ( Generalized Bass model with 2 Mixed shock )

GBM(series = lit.chl.exp.quarter.kton, shock = "mixed", nshock = 2,
prelimestimates = c(56729.61, 0.0002953943, 0.02662497, 25,
0.1, 0.1, 7, 14, 0.1), oos = 4)

Residuals:
    Min. 1st Qu. Median Mean 3rd Qu. Max.
-9.835813 -1.744298 0.650401 0.005721 2.684263 7.171730

Coefficients:
            Estimate Std. Error Lower Upper p-value
m   6.034528e+04 1.409382e+06 -2.701993e+06 2.822684e+06 9.66e-01
p   1.850806e-04 4.336374e-03 -8.314055e-03 8.684216e-03 9.66e-01
q   2.623386e-02 1.183852e-02 3.030781e-03 4.943694e-02 3.44e-02 *
a1  2.941646e+01 3.407102e-01 2.874868e+01 3.008424e+01 1.60e-37 ***
b1  -3.484008e-02 5.799911e-02 -1.485162e-01 7.883609e-02 5.53e-01
c1  5.840652e-01 7.861523e-02 4.299822e-01 7.381483e-01 2.80e-08 ***
a2  -9.363961e-01 6.961145e-01 -2.300755e+00 4.279632e-01 1.89e-01
b2  1.240923e+01 4.155060e-01 1.159486e+01 1.322361e+01 7.12e-24 ***
c2  5.559315e-01 1.083900e-01 3.434910e-01 7.683721e-01 1.62e-05 ***

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error 4.283943 on 30 degrees of freedom
Multiple R-squared: 0.999864 Residual sum of squares: 550.5651

```

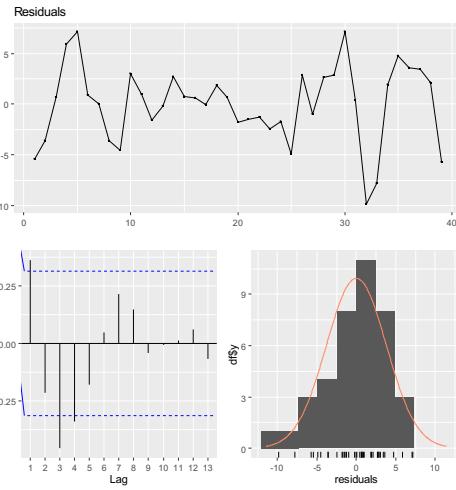
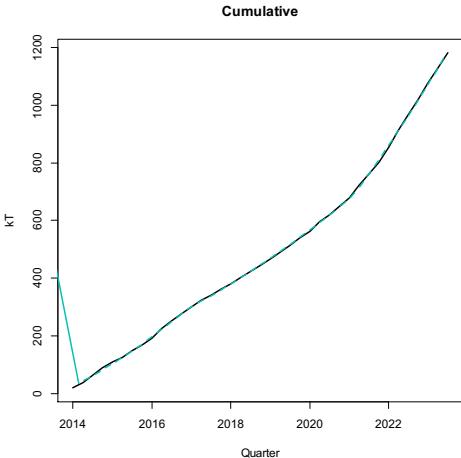
Ljung-Box test

```

data: Residuals
Q* = 26.922, df = 8, p-value = 0.0007294

Model df: 0. Total lags used: 8

```



Competition model Australia vs. Chile

Call: (UCRCD Model)

```
UCRCD(series1 = lit.aus.exp.quarter.kton2, series2 = lit.chl.exp.quarter.kton,
       display = T)
```

Residuals Series 1:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-19.71753	-11.67947	0.60355	0.00006	6.90625	36.25851

Residuals Series 2:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-11.480068	-6.052674	0.283861	0.000297	4.896704	16.228050

Coefficients:

	Estimate	Std.Error	Lower	Upper	p-value
mc	2.619344e+08	4.664402e+12	-9.141798e+12	9.142322e+12	1.00000
p1c	9.718295e-09	1.747179e-04	-3.424310e-04	3.424505e-04	1.00000
p2	4.927435e-08	8.859117e-04	-1.736306e-03	1.736404e-03	1.00000
q1c	1.472441e-01	4.531378e-02	5.843069e-02	2.360575e-01	0.00177 **
q2	4.123059e-02	4.470854e-02	-4.639653e-02	1.288577e-01	0.36000
delta	-1.839038e-01	6.964760e-02	-3.204106e-01	-4.739701e-02	0.01020 *
gamma	4.543773e-02	6.961459e-02	-9.100436e-02	1.818798e-01	0.51600

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

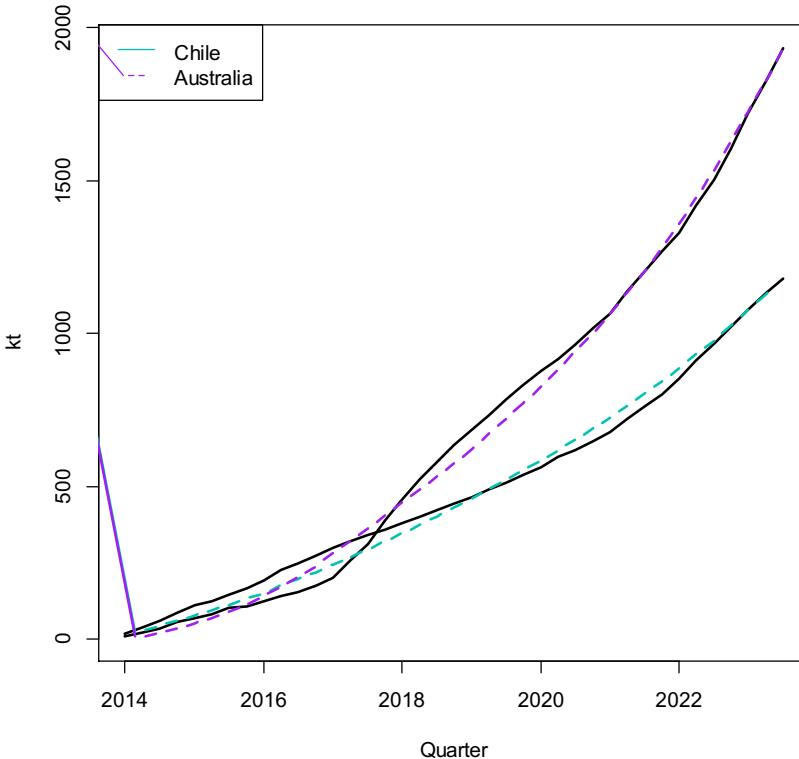
Residual standard error Series 1: 14.46583 on 32 degrees of freedom

Residual standard error Series 2: 7.359257 on 32 degrees of freedom

Multiple R-squared: 0.824858 Residual sum of squares: 8429.404

mc	p1c	q1c+delta	q1c	p2
2.619344e+08	9.718295e-09	-3.665973e-02	1.472441e-01	4.927435e-08
q2	q2-gamma			
4.123059e-02	-4.207138e-03			

Exports - Cumulative



Holt's exponential smoothing Australia

Forecast method: Holt's method

Model Information:
Holt's method

Call:
holt(y = lit.aus.exp.quarter.kton, h = 4)

Smoothing parameters:
alpha = 0.8402
beta = 1e-04

Initial states:
l = 8.4038
b = 1.7611

sigma: 9.273

AIC	AICC	BIC
471.2322	472.4567	481.2689

Error measures:

ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Training set	-0.0008214128	8.929441	6.418935	-11.96011	26.68084	0.9941299
	-0.0340883					

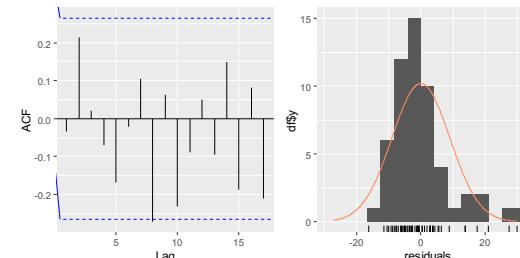
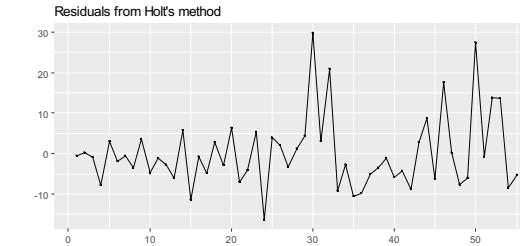
Forecasts:

Point	Forecast	Lo 80	Hi 80	Lo 95	Hi 95
56	106.8669	94.98309	118.7508	88.69216	125.0417
57	108.6281	93.10596	124.1502	84.88905	132.3671
58	110.3892	91.93197	128.8465	82.16129	138.6171
59	112.1504	91.16399	133.1367	80.05448	144.2462

Ljung-Box test

data: Residuals from Holt's method
Q* = 14.547, df = 10, p-value = 0.1495

Model df: 0. Total lags used: 10



Holt's exponential smoothing Chile

Forecast method: Holt's method

Model Information:
Holt's method

Call:
holt(y = lit.chl.exp.quarter.kton, h = 4)

Smoothing parameters:
alpha = 0.6064
beta = 1e-04

Initial states:
l = 19.6842
b = 0.8529

sigma: 5.5312

AIC	AICc	BIC
282.0701	283.8883	290.3880

Error measures:

ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Training set -0.009837826	5.239877	3.890208	-3.173123	13.64065	0.8893848	-0.01554943

Forecasts:

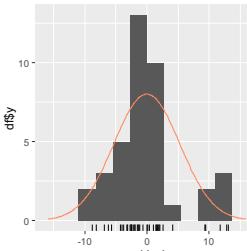
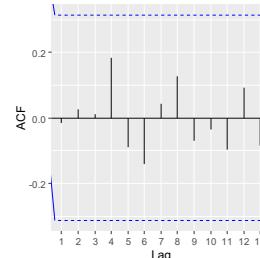
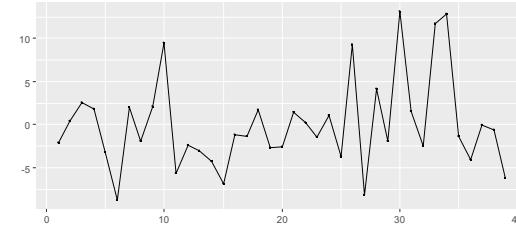
Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
40 53.52751	46.43900	60.61603	42.68656	64.36847
41 54.38041	46.09023	62.67058	41.70167	67.05914
42 55.23330	45.89450	64.57209	40.95084	69.51576
43 56.08619	45.80488	66.36750	40.36228	71.81009

Ljung-Box test

data: Residuals from Holt's method
Q* = 3.8423, df = 8, p-value = 0.8711

Model df: 0. Total lags used: 8

Residuals from Holt's method



KNN Regression Australia

```
Call: knn_forecasting(timeS = aus.exp.ts, h = 4, lags = 1:4, k = 2)
```

Multiple-Step Ahead Strategy: recursive

K (number of nearest neighbors): 2

Autoregressive lags: 1 2 3 4

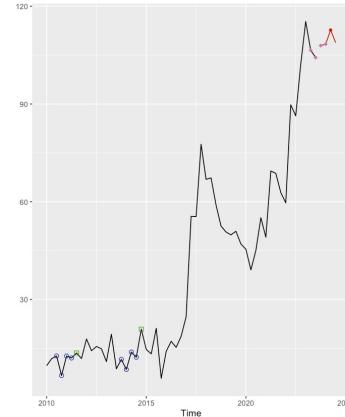
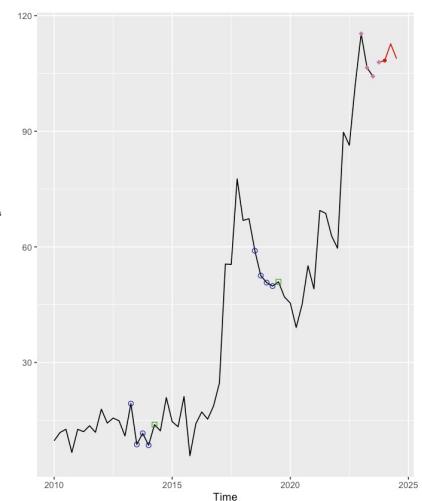
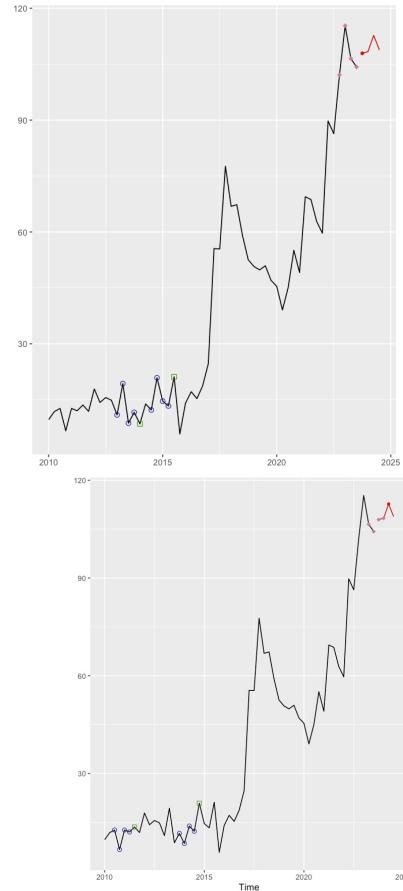
Number of examples: 51

Targets are combined using the mean function.

Forecasting horizon: 4

Forecast:

	Qtr1	Qtr2	Qtr3	Qtr4	
2023					107.9178
2024	108.3608	112.7049	108.8447		



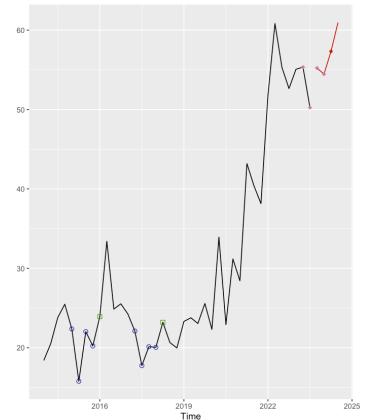
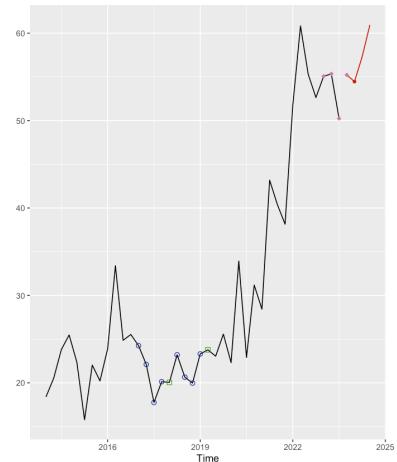
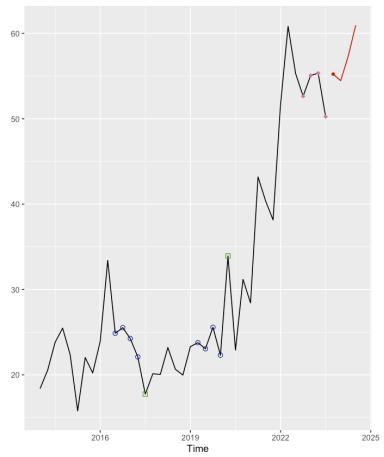
KNN Regression Chile

```
Call: knn_forecasting(timeS = ch.exp.ts, h = 4, lags = 1:4, k = 2,  
msas = "recursive", transform = "additive")
```

Multiple-Step Ahead Strategy: recursive
K (number of nearest neighbors): 2
Autoregressive lags: 1 2 3 4
Number of examples: 35
Targets are combined using the mean function.
Forecasting horizon: 4

Forecast:

	Qtr1	Qtr2	Qtr3	Qtr4
2023				55.22379
2024	54.45693	57.32102	60.95001	



ARMAX Australia: GDP Variation

Series: df\$lit_aus_exp_kton_met
Regression with ARIMA(1,1,0) errors

Coefficients:
ar1 drift var_0 var_1
-0.2407 1.9400 107.6573 21.3806
s.e. 0.1374 1.0453 63.6846 64.3608

sigma^2 = 87.05: log likelihood = -177.44
AIC=364.88 AICc=366.27 BIC=374.34

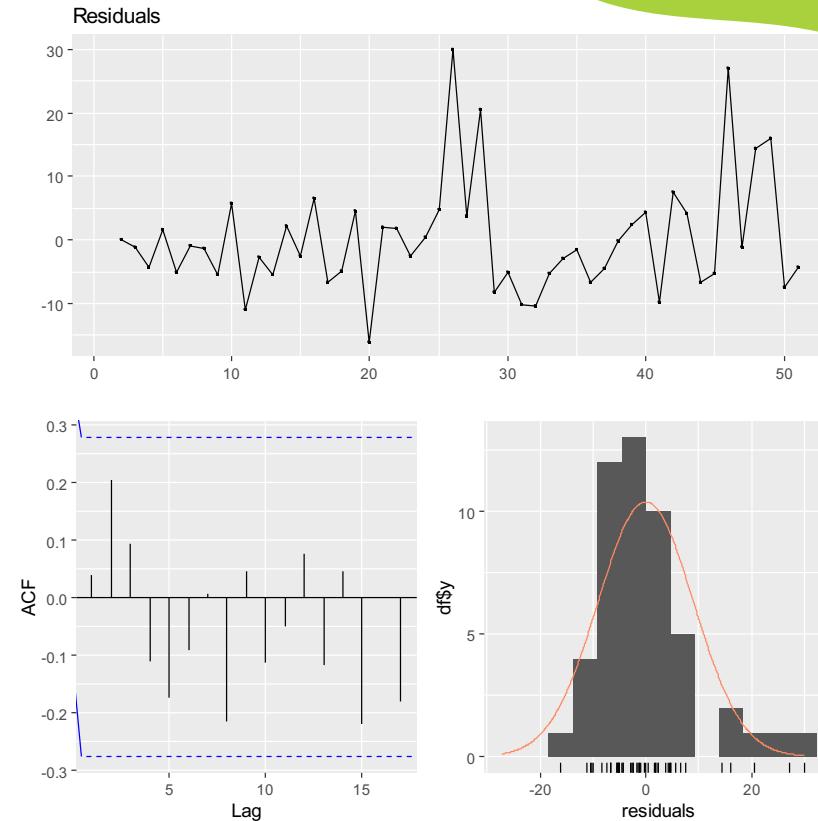
Training set error measures:

ME	RMSE	MAE	MPE	MAPE	MASE
----	------	-----	-----	------	------

Training set -0.004740116 8.949036 6.407176 -11.60387 24.77131 0.9600486

ACF1

Training set 0.03749308



ARMAX Australia: Lithium Batteries World Trends

Series: df\$lit_aus_exp_kton_met
Regression with ARIMA(0,1,2) errors

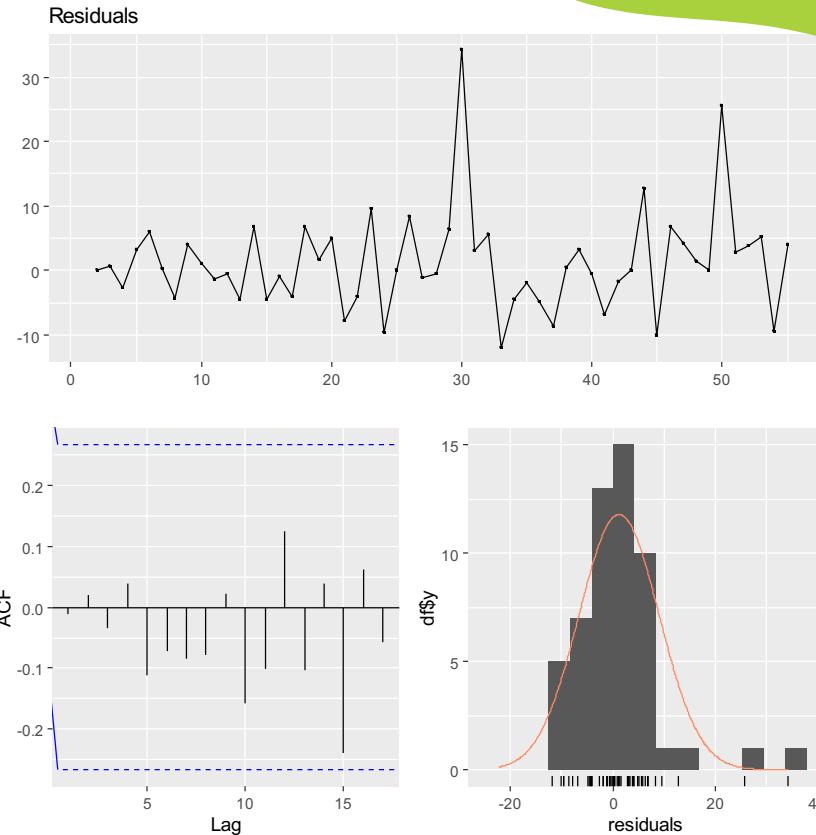
Coefficients:

ma1	ma2	xreg
-0.2043	0.4470	2.8502
s.e.	0.1314	0.1176
0.8257		

$\sigma^2 = 65.34$: log likelihood = -185.18
AIC=378.35 AICc=379.18 BIC=386.23

Training set error measures:

ME	RMSE	MAE	MPE	MAPE	MASE
Training set 1.230656	7.855367	5.192002	-1.49854	20.84089	0.8041091
ACF1					
Training set -0.01072171					



ARMAX Australia: Mineral Resources

Series: df\$lit_aus_exp_kton_met
Regression with ARIMA(2,1,0) errors

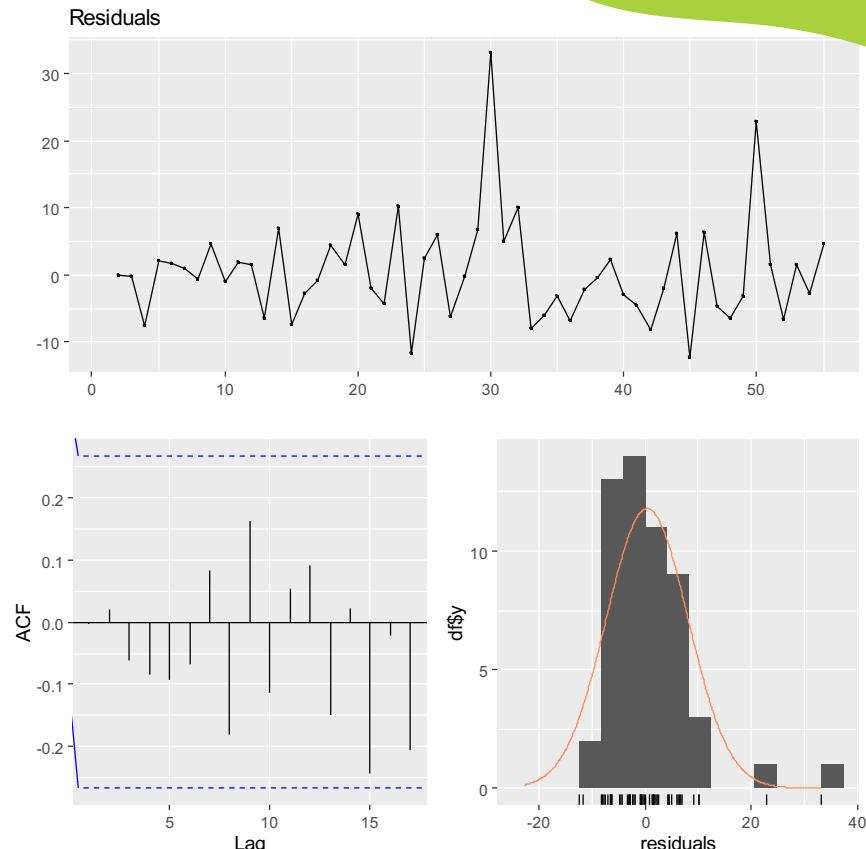
Coefficients:

	ar1	ar2	var_0	var_1
-	-0.2598	0.2144	0.7207	0.3937
s.e.	0.1343	0.1335	0.2070	0.2140

$\sigma^2 = 62.66$: log likelihood = -183.42
AIC=376.83 AICc=378.11 BIC=386.68

Training set error measures:

	ME	RMSE	MAE	MPE	MAPE
Training set	0.4077493	7.617112	5.283478	-4.858721	22.48089
	MASE	ACF1			
Training set	0.8182764	-0.002910985			



ARMAX Australia: Multivariable Model

Variable	Time	Forecasting
Mineral Resources	t, t-1	Exponential Smoothing
Lithium Bateries World Trends	t-1	Exponential Smoothing

Series: data_trends_aux\$lit_aus_exp_kton_met
 Regression with ARIMA(2,1,0) errors

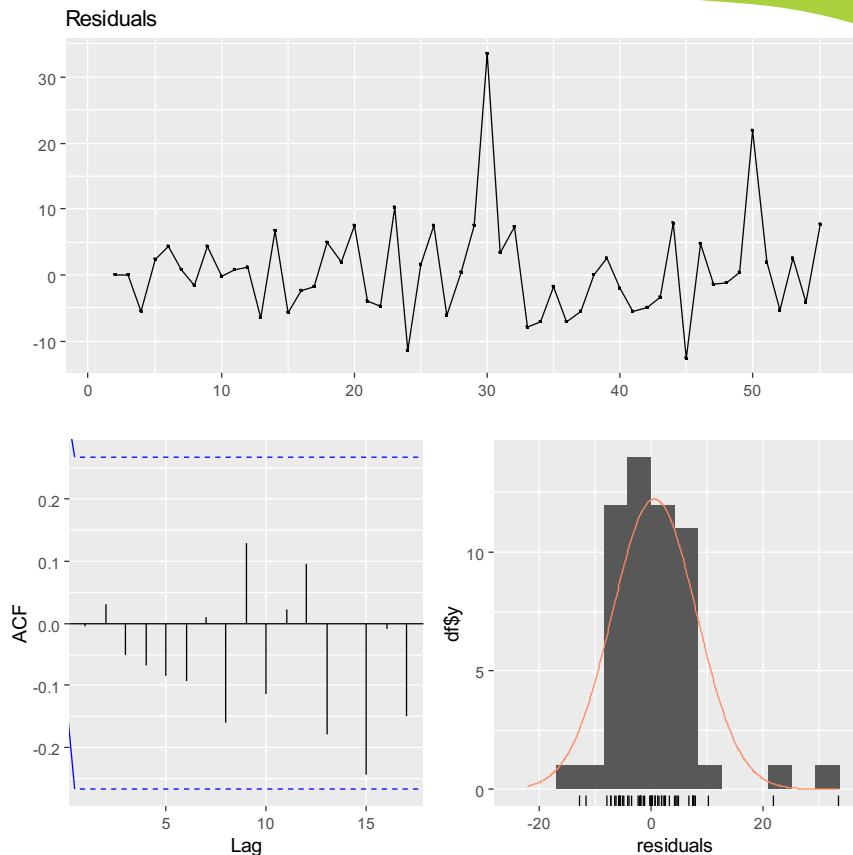
Coefficients:

	ar1	ar2	var0	var1	var2
-	-0.2280	0.2983	0.5803	0.2256	1.5752
s.e.	0.1329	0.1453	0.2268	0.2457	1.1635

$\sigma^2 = 61.7$: log likelihood = -182.51
 AIC=377.03 AICc=378.85 BIC=388.85

Training set error measures:

	ME	RMSE	MAE	MPE	MAPE
Training set	0.6490937	7.482392	5.097209	-3.704196	21.87899
	MASE	ACF1			
Training set	0.789428	-0.005767621			



ARMAX Chile: Lithium Batteries World Trends

Series: df\$lit_chl_exp_kton_met
Regression with ARIMA(2,0,0) errors

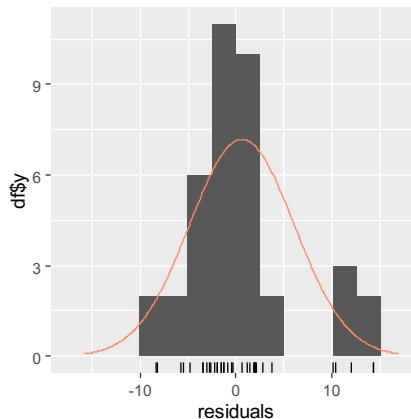
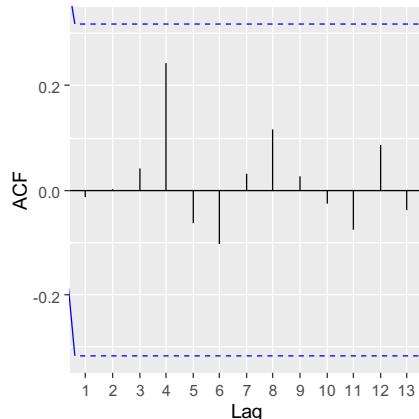
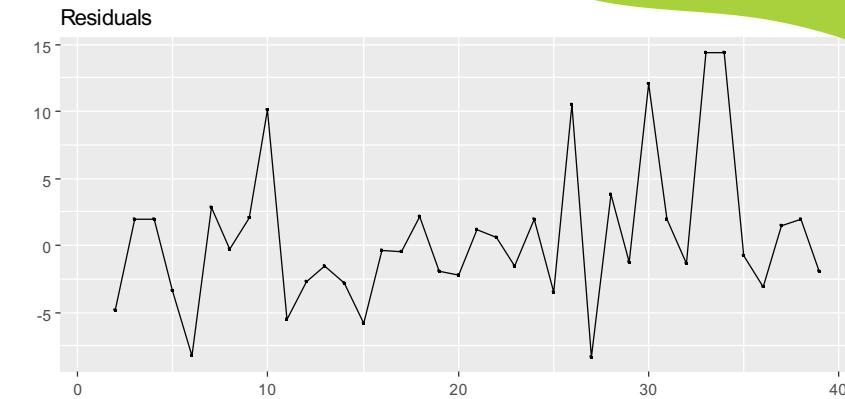
Coefficients:

ar1	ar2	intercept	xreg
0.6076	0.3227	27.2214	0.4743
s.e.	0.1600	0.1601	13.1100
			0.6923

$\sigma^2 = 32.01$: log likelihood = -119.14
AIC=248.28 AICc=250.15 BIC=256.47

Training set error measures:

ME	RMSE	MAE	MPE
0.6168364	5.429898	3.876251	-1.409716
MAPE	MASE	ACF1	
13.29638	0.886194	-0.01413027	



ARMAX Chile: SQM stock

Series: df\$lit_chl_exp_kton_met
Regression with ARIMA(1,0,1) errors

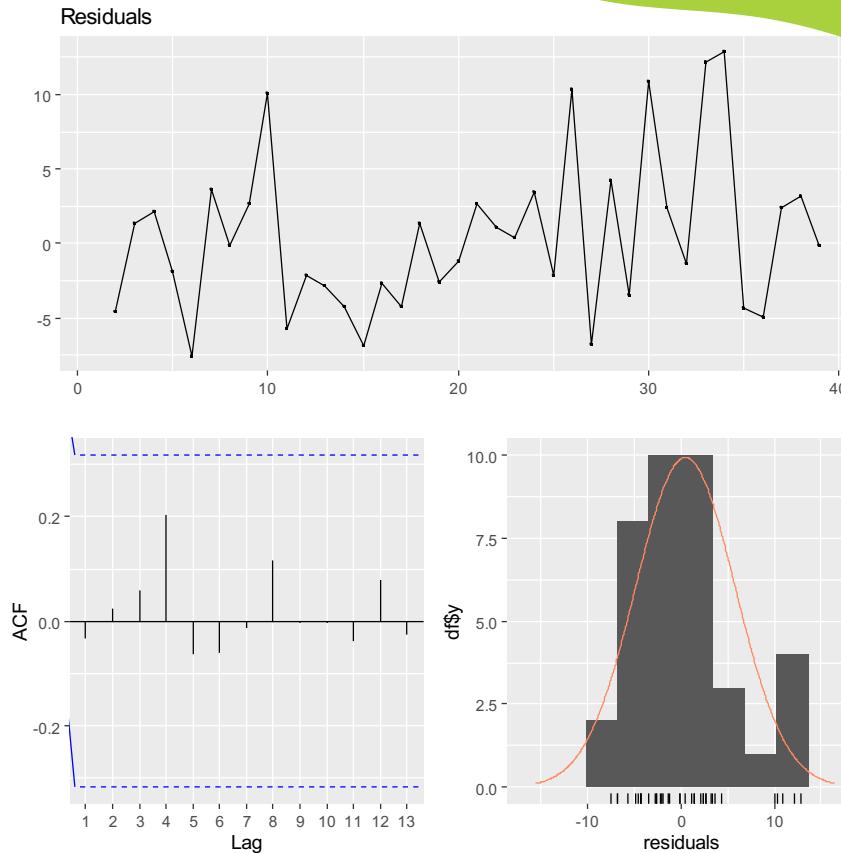
Coefficients:

	ar1	ma1	intercept	xreg
0.9313	-0.3262	24.0333	0.1960	
s.e.	0.0719	0.1609	8.6713	0.1105

$\sigma^2 = 30.32$: log likelihood = -117.88
AIC=245.77 AICc=247.64 BIC=253.95

Training set error measures:

ME	RMSE	MAE	MPE
Training set 0.4679963	5.284258	4.131787	-1.724194
MAPE	MASE	ACF1	
Training set 14.31525	0.9446149	-0.03275982	



ARMAX Chile: Mineral Resources

Series: df\$lit_chl_exp_kton_met
Regression with ARIMA(1,0,0) errors

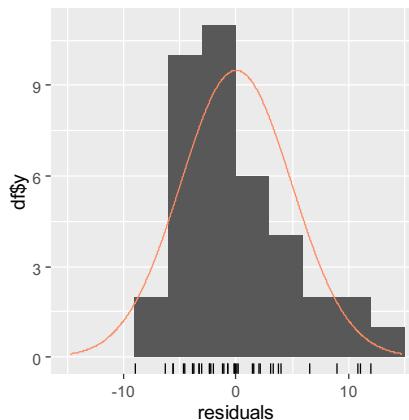
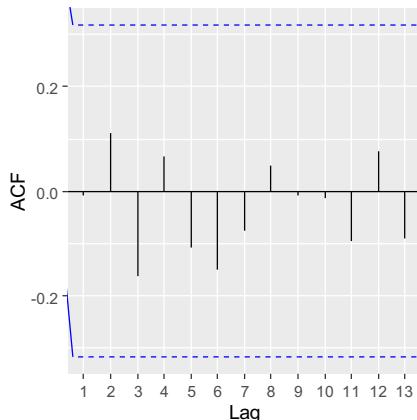
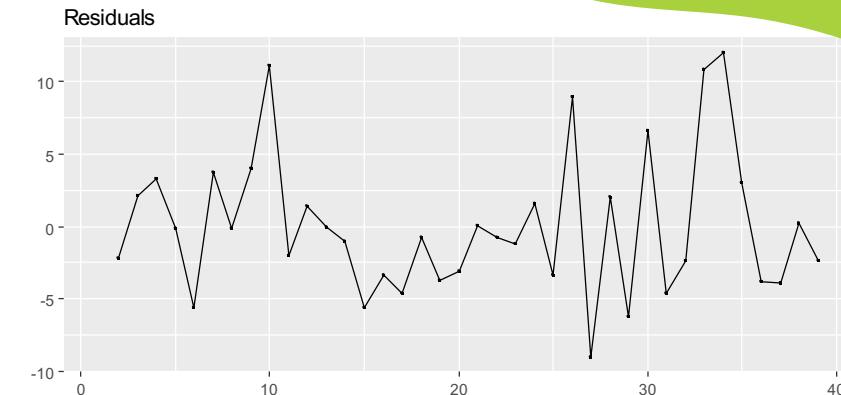
Coefficients:

ar1	intercept	var_0	var_1
0.3841	17.3434	0.3085	0.195
s.e.	0.1530	1.9247	0.1432
			0.146

$\sigma^2 = 25.62$: log likelihood = -114.06
AIC=238.12 AICc=239.99 BIC=246.31

Training set error measures:

ME	RMSE	MAE	MPE
Training set 0.02938578	4.857458	3.707437	-2.47511
MAPE	MASE	ACF1	
Training set 12.76616	0.8475995	-0.00855879	



ARMAX Chile: Multivariable Model

Variable	Time	Forecasting
SQM	t-1	Exponential Smoothing
Lithium Bateries World Trends	t-1	Exponential Smoothing
Mineral Resources	t, t-1	Exponential Smoothing

Series: data_trends_aux\$lit_ch1_exp_kton_met

Regression with ARIMA(1,1,0) errors

Coefficients:

ar1	var0	var1	var2	var3	
-0.3431	0.1435	0.0704	-0.2772	0.0666	
s.e.	0.1621	0.1927	0.1701	0.8166	0.1284

$\sigma^2 = 32.73$: log likelihood = -114.41

AIC=240.82 AICc=243.62 BIC=250.49

Training set error measures:

ME	RMSE	MAE	MPE	MAPE	MASE
Training set 0.5668778	5.2499	4.009516	0.06872461	13.39583	0.9166613
ACF1					
Training set -0.07035603					

