# hw5

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```
library(tidyverse)
library(patchwork)
library(modelr)
library(leaps)
library(purrr)
library(glmnet)
library(knitr)
library(caret)
```

```
state= state.x77 %>%
  as.tibble()%>%
  janitor::clean_names()%>%
  select(life_exp,everything())
```

## a)

```
sum = function(variable){
  tibble(
    mean = mean(variable),
    sd = sd(variable),
    median = median(variable),
    maximum = max(variable),
    minimum = min(variable),
    IQR = IQR(variable)
  )
}

map(state, sum) %>%
  bind_rows() %>%
  mutate(variable = names(state)) %>%
  select(variable,everything()) %>%
  knitr::kable(digits = 2,
               caption = "Descriptive statistics of continuous variables")
```

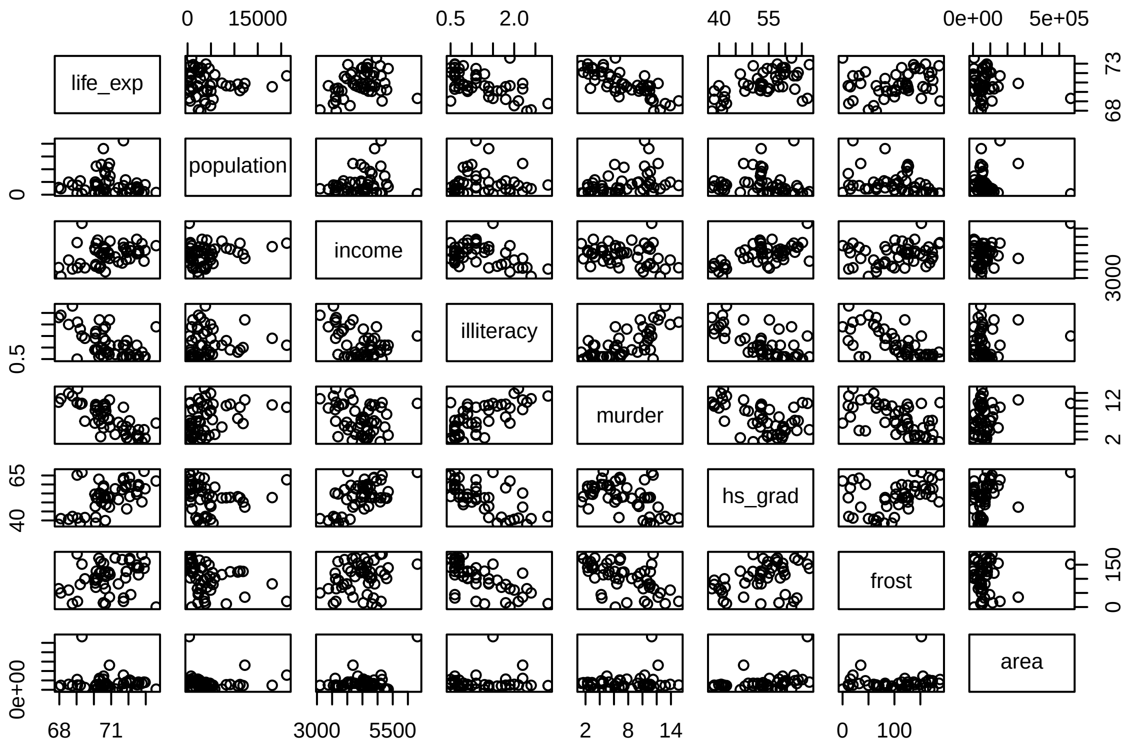
Table 1: Descriptive statistics of continuous variables

| variable | mean | sd | median | maximum | minimum | IQR |
|---|---|---|---|---|---|---|
| life_exp | 70.88 | 1.34 | 70.67 | 73.6 | 67.96 | 1.78 |
| population | 4246.42 | 4464.49 | 2838.50 | 21198.0 | 365.00 | 3889.00 |
| income | 4435.80 | 614.47 | 4519.00 | 6315.0 | 3098.00 | 820.75 |

| variable | mean | sd | median | maximum | minimum | IQR |
|---|---|---|---|---|---|---|
| illiteracy | 1.17 | 0.61 | 0.95 | 2.8 | 0.50 | 0.95 |
| murder | 7.38 | 3.69 | 6.85 | 15.1 | 1.40 | 6.32 |
| hs_grad | 53.11 | 8.08 | 53.25 | 67.3 | 37.80 | 11.10 |
| frost | 104.46 | 51.98 | 114.50 | 188.0 | 0.00 | 73.50 |
| area | 70735.88 | 85327.30 | 54277.00 | 566432.0 | 1049.00 | 44177.25 |

## b)

```
plot(state)
```



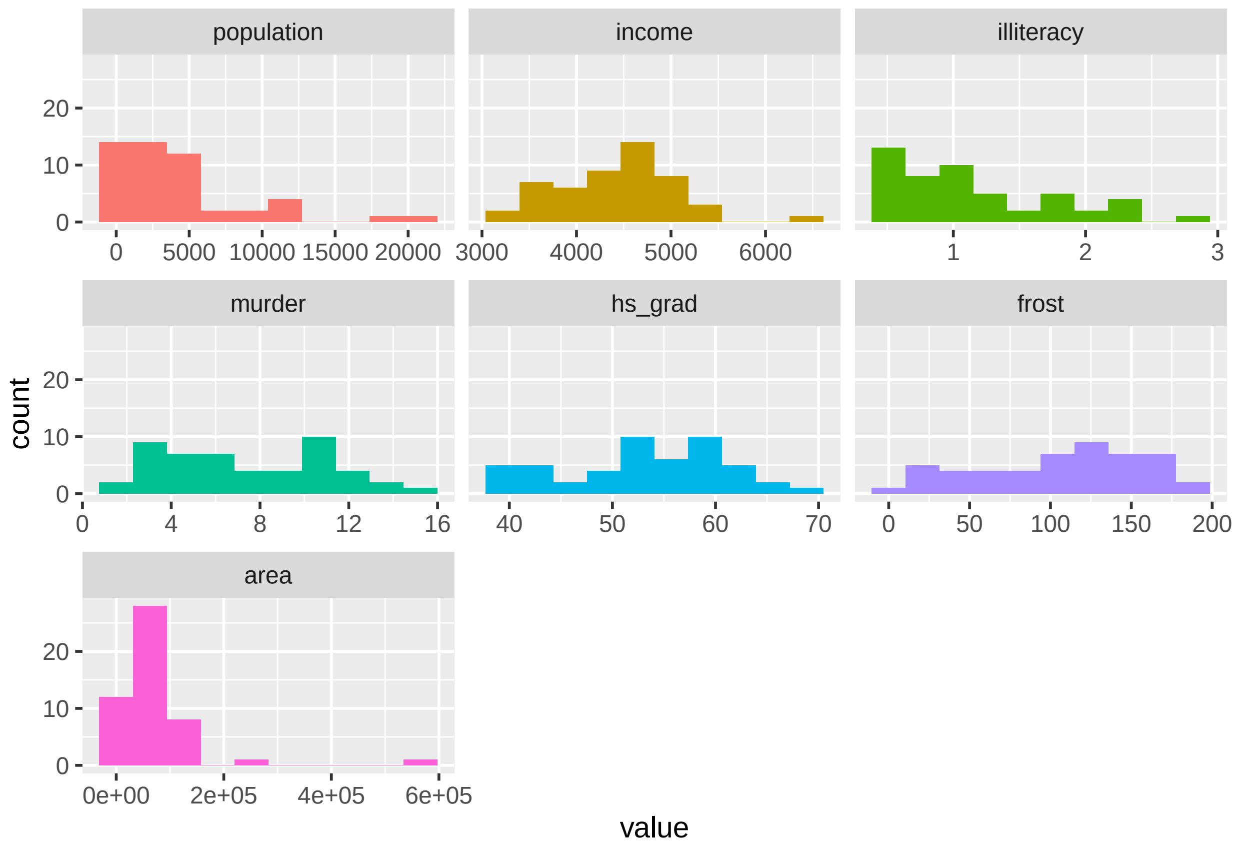

```
cor(state) %>%
  knitr::kable(digits=2,caption="Correlation for all variables")
```

Table 2: Correlation for all variables

| | life_exp | population | income | illiteracy | murder | hs_grad | frost | area |
|---|---|---|---|---|---|---|---|---|
| life_exp | 1.00 | -0.07 | 0.34 | -0.59 | -0.78 | 0.58 | 0.26 | -0.11 |
| population | -0.07 | 1.00 | 0.21 | 0.11 | 0.34 | -0.10 | -0.33 | 0.02 |
| income | 0.34 | 0.21 | 1.00 | -0.44 | -0.23 | 0.62 | 0.23 | 0.36 |
| illiteracy | -0.59 | 0.11 | -0.44 | 1.00 | 0.70 | -0.66 | -0.67 | 0.08 |
| murder | -0.78 | 0.34 | -0.23 | 0.70 | 1.00 | -0.49 | -0.54 | 0.23 |
| hs_grad | 0.58 | -0.10 | 0.62 | -0.66 | -0.49 | 1.00 | 0.37 | 0.33 |

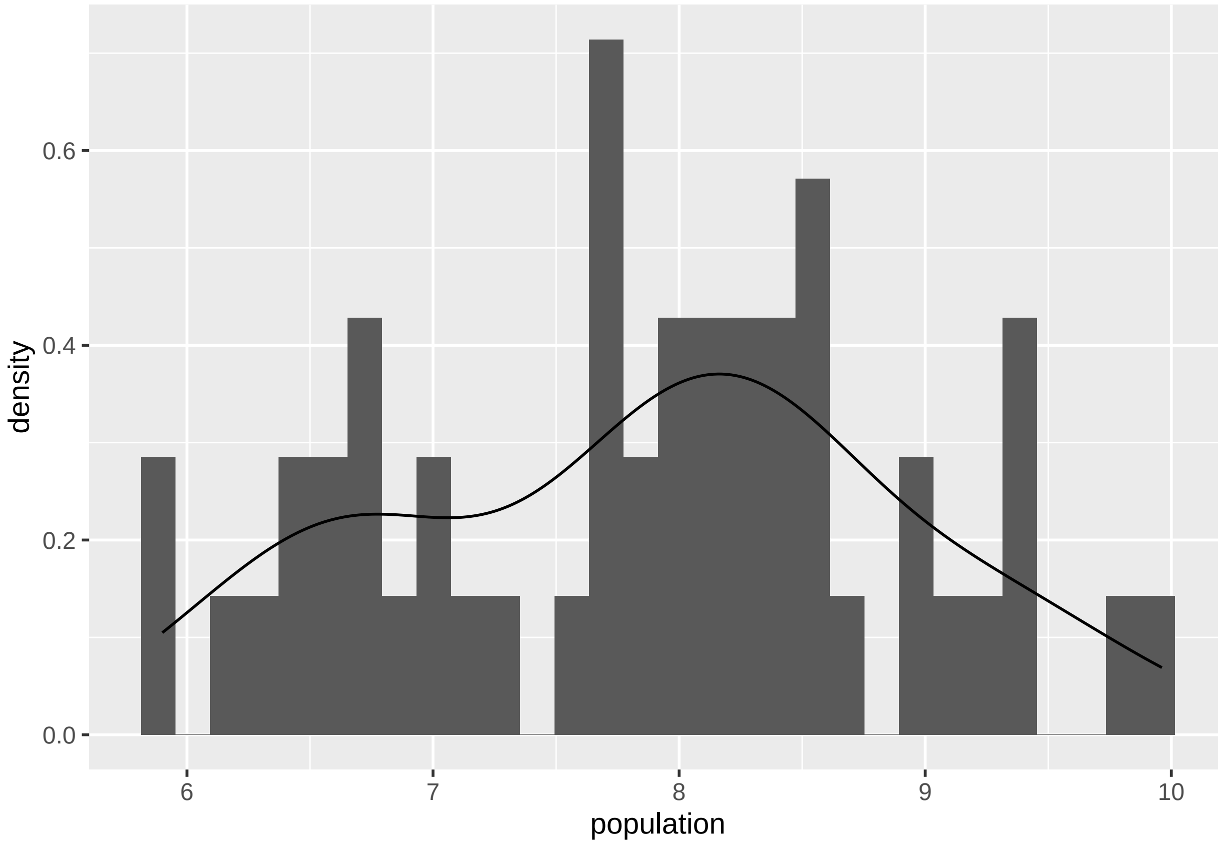
| | life_exp | population | income | illiteracy | murder | hs_grad | frost | area |
|---|---|---|---|---|---|---|---|---|
| frost | 0.26 | -0.33 | 0.23 | -0.67 | -0.54 | 0.37 | 1.00 | 0.06 |
| area | -0.11 | 0.02 | 0.36 | 0.08 | 0.23 | 0.33 | 0.06 | 1.00 |

```
state%>% select(-life_exp)%>%
  funModeling::plot_num()
```

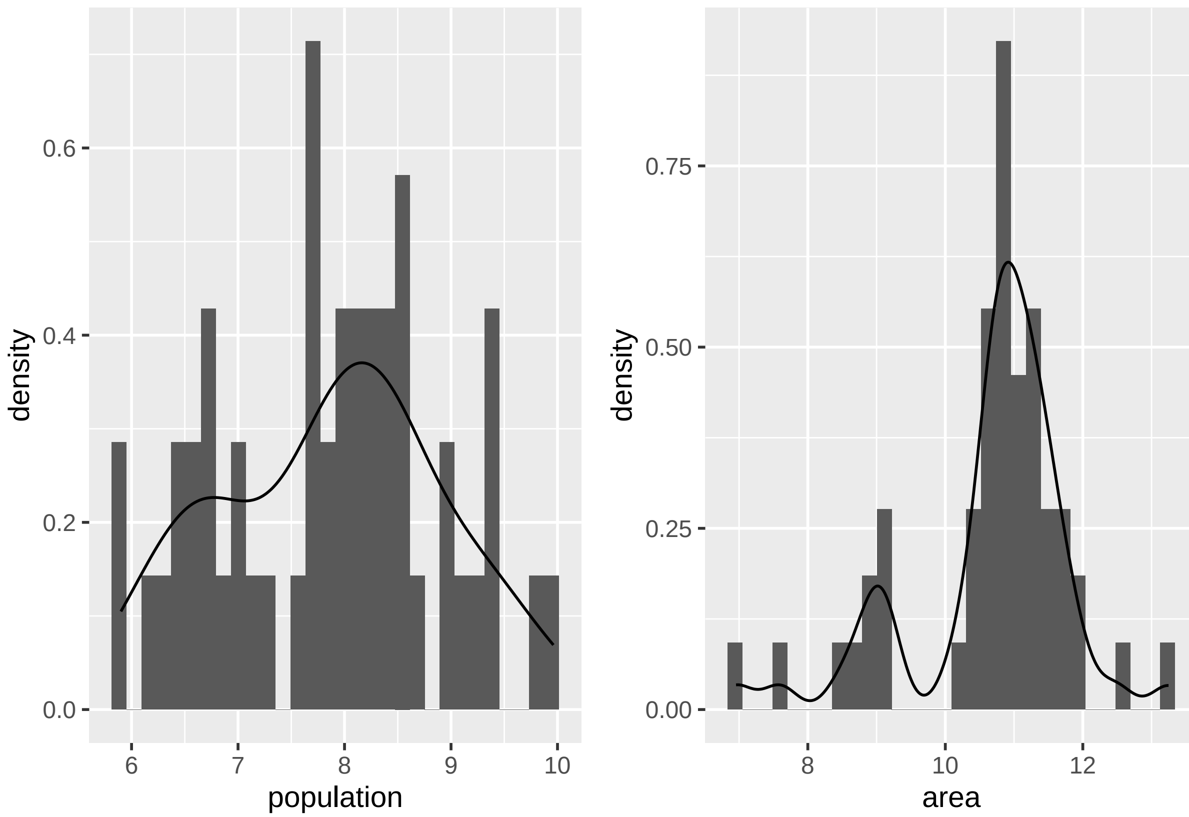



From the above plot, we can see that population and area are skewed, while all other variables are pretty normal distributed. We would want to try to make transformations on population and area.

```
ggl_p =
state %>%
  mutate(population = log(population)) %>%
 ggplot(aes(x=population,y=..density..))+
 geom_histogram()+
 geom_line(stat = 'density')+
    labs(x = "population")
ggl_p
```




```
ggl_a=state %>%
  mutate(area = log(area)) %>%
 ggplot(aes(x=area,..density..))+
 geom_histogram()+geom_line(stat = 'density')+
    labs(x = "area")
ggl_p+ggl_a
```




### c)

```
multi.fit=lm(life_exp ~ ., data = state)
summary(multi.fit)
```

```
##
## Call:
## lm(formula = life_exp ~ ., data = state)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.48895 -0.51232 -0.02747  0.57002  1.49447
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.094e+01  1.748e+00  40.586  < 2e-16 ***
## population   5.180e-05  2.919e-05   1.775   0.0832 .
## income      -2.180e-05  2.444e-04  -0.089   0.9293
## illiteracy   3.382e-02  3.663e-01   0.092   0.9269
## murder      -3.011e-01  4.662e-02  -6.459 8.68e-08 ***
## hs_grad      4.893e-02  2.332e-02   2.098   0.0420 *
## frost       -5.735e-03  3.143e-03  -1.825   0.0752 .
## area        -7.383e-08  1.668e-06  -0.044   0.9649
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.7448 on 42 degrees of freedom
## Multiple R-squared:  0.7362, Adjusted R-squared:  0.6922
## F-statistic: 16.74 on 7 and 42 DF,  p-value: 2.534e-10
```

1) **Method I: Backward elimination**

By looking at the summary of full model regression, backward elimination starts eliminating the one with largest p value, we stop remove variables when their p-value are all less than 0.05. so we **remove area** first

```
step1 <- update(multi.fit, . ~ . -area)
summary(step1)
```

```
##
## Call:
## lm(formula = life_exp ~ population + income + illiteracy + murder +
##     hs_grad + frost, data = state)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.49047 -0.52533 -0.02546  0.57160  1.50374
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.099e+01  1.387e+00  51.165  < 2e-16 ***
## population   5.188e-05  2.879e-05   1.802   0.0785 .
## income      -2.444e-05  2.343e-04  -0.104   0.9174
## illiteracy   2.846e-02  3.416e-01   0.083   0.9340
## murder      -3.018e-01  4.334e-02  -6.963 1.45e-08 ***
## hs_grad      4.847e-02  2.067e-02   2.345   0.0237 *
## frost       -5.776e-03  2.970e-03  -1.945   0.0584 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7361 on 43 degrees of freedom
## Multiple R-squared:  0.7361, Adjusted R-squared:  0.6993
## F-statistic: 19.99 on 6 and 43 DF,  p-value: 5.362e-11
```

Then we **remove illiteracy**

```
step2 <- update(step1, . ~ . -illiteracy)
summary(step2)
```

```
##
## Call:
## lm(formula = life_exp ~ population + income + murder + hs_grad +
##     frost, data = state)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.4892 -0.5122 -0.0329  0.5645  1.5166
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.107e+01  1.029e+00  69.067  < 2e-16 ***
```

```
## population   5.115e-05  2.709e-05   1.888   0.0657 .
## income      -2.477e-05  2.316e-04  -0.107   0.9153
## murder      -3.000e-01  3.704e-02  -8.099 2.91e-10 ***
## hs_grad      4.776e-02  1.859e-02   2.569   0.0137 *
## frost       -5.910e-03  2.468e-03  -2.395   0.0210 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7277 on 44 degrees of freedom
## Multiple R-squared:  0.7361, Adjusted R-squared:  0.7061
## F-statistic: 24.55 on 5 and 44 DF,  p-value: 1.019e-11
```

Then we **remove income**

```
step3 <- update(step2, . ~ . -income)
summary(step3)
```

```
##
## Call:
## lm(formula = life_exp ~ population + murder + hs_grad + frost,
##     data = state)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.47095 -0.53464 -0.03701  0.57621  1.50683
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.103e+01  9.529e-01  74.542  < 2e-16 ***
## population   5.014e-05  2.512e-05   1.996  0.05201 .
## murder      -3.001e-01  3.661e-02  -8.199 1.77e-10 ***
## hs_grad      4.658e-02  1.483e-02   3.142  0.00297 **
## frost       -5.943e-03  2.421e-03  -2.455  0.01802 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7197 on 45 degrees of freedom
## Multiple R-squared:  0.736,  Adjusted R-squared:  0.7126
## F-statistic: 31.37 on 4 and 45 DF,  p-value: 1.696e-12
```

Then we **remove population**

```
step4 <- update(step3, . ~ . -population)
summary(step4)
```

```
##
## Call:
## lm(formula = life_exp ~ murder + hs_grad + frost, data = state)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.5015 -0.5391  0.1014  0.5921  1.2268
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept) 71.036379   0.983262  72.246  < 2e-16 ***
## murder      -0.283065   0.036731  -7.706 8.04e-10 ***
## hs_grad      0.049949   0.015201   3.286  0.00195 **
## frost       -0.006912   0.002447  -2.824  0.00699 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7427 on 46 degrees of freedom
## Multiple R-squared:  0.7127, Adjusted R-squared:  0.6939
## F-statistic: 38.03 on 3 and 46 DF,  p-value: 1.634e-12
```

**Result**: backward selection model is

life expectancy = 71 - 0.3Murder + 0.047hs_grad - 0.006frost

2) **Method II: Forward elimination**

```
variable=names(state)

map(.x=variable,~lm(substitute(life_exp ~ i, list(i = as.name(.x))), data = state)) %>%
  map_df(.,broom::tidy)%>%
  filter(term!="(Intercept)") %>%
  select(term,p.value)%>%
  arrange(p.value)
```

```
## # A tibble: 7 x 2
##   term        p.value
##   <chr>         <dbl>
## 1 murder     2.26e-11
## 2 illiteracy 6.97e- 6
## 3 hs_grad    9.20e- 6
## 4 income     1.56e- 2
## 5 frost      6.60e- 2
## 6 area       4.58e- 1
## 7 population 6.39e- 1
```

So we first enter the one with the lowest p-value 2.26e-11 < 0.05: `murder`.

```
forward1 = lm(life_exp ~ murder, data = state)
summary(forward1)
```

```
##
## Call:
## lm(formula = life_exp ~ murder, data = state)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.81690 -0.48139  0.09591  0.39769  2.38691
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 72.97356    0.26997  270.30  < 2e-16 ***
## murder      -0.28395    0.03279   -8.66 2.26e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.8473 on 48 degrees of freedom
## Multiple R-squared:  0.6097, Adjusted R-squared:  0.6016
## F-statistic: 74.99 on 1 and 48 DF,  p-value: 2.26e-11
```

```
variable=names(state)
map(.x = variable, ~update(forward1, substitute(.~. + i, list(i = as.name(.x))))) %>%
  map_df(., broom::tidy) %>%
  filter(term != "(Intercept)", term != "murder") %>%
  select(term,p.value) %>%
  arrange(p.value)
```

```
## # A tibble: 6 x 2
##   term       p.value
##   <chr>        <dbl>
## 1 hs_grad    0.00909
## 2 population 0.0164
## 3 frost      0.0352
## 4 income     0.0666
## 5 area       0.424
## 6 illiteracy 0.543
```

Enter the one with the lowest p-value 0.00909: `hs_grad`.

```
forward2 <- update(forward1, . ~ . + hs_grad)
summary(forward2)
```

```
##
## Call:
## lm(formula = life_exp ~ murder + hs_grad, data = state)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.66758 -0.41801  0.05602  0.55913  2.05625
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 70.29708    1.01567  69.213  < 2e-16 ***
## murder      -0.23709    0.03529  -6.719 2.18e-08 ***
## hs_grad      0.04389    0.01613   2.721  0.00909 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7959 on 47 degrees of freedom
## Multiple R-squared:  0.6628, Adjusted R-squared:  0.6485
## F-statistic:  46.2 on 2 and 47 DF,  p-value: 8.016e-12
```

```
variable=names(state)
map(.x = variable, ~update(forward2, substitute(.~. + i, list(i = as.name(.x))))) %>%
  map_df(., broom::tidy) %>%
  filter(term != "(Intercept)", term != "murder",term!="hs_grad") %>%
  arrange(p.value)
```

```
## # A tibble: 5 x 5
```

```
##   term          estimate  std.error statistic p.value
##   <chr>            <dbl>      <dbl>     <dbl>   <dbl>
## 1 frost      -0.00691    0.00245       -2.82  0.00699
## 2 population  0.0000625  0.0000259      2.41  0.0199
## 3 illiteracy  0.254      0.305          0.833 0.409
## 4 area       -0.00000106 0.00000162    -0.658 0.514
## 5 income      0.0000953  0.000239       0.398 0.692
```

Enter the one with the lowest p-value 0.00699: `frost`.

```
forward3 <- update(forward2, . ~ . + frost)
summary(forward3)
```

```
##
## Call:
## lm(formula = life_exp ~ murder + hs_grad + frost, data = state)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.5015 -0.5391  0.1014  0.5921  1.2268
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 71.036379   0.983262  72.246  < 2e-16 ***
## murder      -0.283065   0.036731  -7.706 8.04e-10 ***
## hs_grad      0.049949   0.015201   3.286  0.00195 **
## frost       -0.006912   0.002447  -2.824  0.00699 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7427 on 46 degrees of freedom
## Multiple R-squared:  0.7127, Adjusted R-squared:  0.6939
## F-statistic: 38.03 on 3 and 46 DF,  p-value: 1.634e-12
```

```
variable=names(state)
map(.x = variable, ~update(forward3, substitute(.~. + i, list(i = as.name(.x))))) %>%
  map_df(., broom::tidy) %>%
  filter(term != "(Intercept)", term != "murder",term!="hs_grad",term!="frost") %>%
  arrange(p.value)
```

```
## # A tibble: 4 x 5
##   term           estimate  std.error statistic p.value
##   <chr>             <dbl>      <dbl>     <dbl>   <dbl>
## 1 population  0.0000501   0.0000251      2.00   0.0520
## 2 income      0.000127    0.000223       0.571  0.571
## 3 illiteracy -0.182       0.328         -0.554  0.582
## 4 area       -0.000000329 0.00000154    -0.214  0.832
```

P-value of all new added variables are larger than 0.05, which means that they are not significant predictor, so we stop here.

```
forward_fit = lm(life_exp ~ murder + hs_grad + frost, data = state) %>%
summary() %>% broom::tidy()
```

The model we obtained by forward elimination is life_exp ~ murder + hs_grad + frost.

**Method III: stepwise regression**

```
step.fit <- lm(life_exp ~ ., data = state)
step(step.fit, direction = 'both') # select by AIC
```

```
## Start:  AIC=-22.18
## life_exp ~ population + income + illiteracy + murder + hs_grad +
##     frost + area
##
##              Df Sum of Sq    RSS     AIC
## - area        1    0.0011 23.298 -24.182
## - income      1    0.0044 23.302 -24.175
## - illiteracy  1    0.0047 23.302 -24.174
## <none>                    23.297 -22.185
## - population  1    1.7472 25.044 -20.569
## - frost       1    1.8466 25.144 -20.371
## - hs_grad     1    2.4413 25.738 -19.202
## - murder      1   23.1411 46.438  10.305
##
## Step:  AIC=-24.18
## life_exp ~ population + income + illiteracy + murder + hs_grad +
##     frost
##
##              Df Sum of Sq    RSS     AIC
## - illiteracy  1    0.0038 23.302 -26.174
## - income      1    0.0059 23.304 -26.170
## <none>                    23.298 -24.182
## - population  1    1.7599 25.058 -22.541
## + area        1    0.0011 23.297 -22.185
## - frost       1    2.0488 25.347 -21.968
## - hs_grad     1    2.9804 26.279 -20.163
## - murder      1   26.2721 49.570  11.569
##
## Step:  AIC=-26.17
## life_exp ~ population + income + murder + hs_grad + frost
##
##              Df Sum of Sq    RSS     AIC
## - income      1     0.006 23.308 -28.161
## <none>                    23.302 -26.174
## - population  1     1.887 25.189 -24.280
## + illiteracy  1     0.004 23.298 -24.182
## + area        1     0.000 23.302 -24.174
## - frost       1     3.037 26.339 -22.048
## - hs_grad     1     3.495 26.797 -21.187
## - murder      1    34.739 58.041  17.456
##
## Step:  AIC=-28.16
## life_exp ~ population + murder + hs_grad + frost
##
##              Df Sum of Sq    RSS     AIC
## <none>                    23.308 -28.161
## + income      1     0.006 23.302 -26.174
## + illiteracy  1     0.004 23.304 -26.170
## + area        1     0.001 23.307 -26.163
## - population  1     2.064 25.372 -25.920
```

```
## - frost       1     3.122 26.430 -23.877
## - hs_grad     1     5.112 28.420 -20.246
## - murder      1    34.816 58.124  15.528
```

```
##
## Call:
## lm(formula = life_exp ~ population + murder + hs_grad + frost,
##     data = state)
##
## Coefficients:
## (Intercept)   population       murder      hs_grad        frost
##   7.103e+01    5.014e-05   -3.001e-01    4.658e-02   -5.943e-03
```

We choose the one with smallest AIC, hence the model selected by stepwise regression procedure is:

life_exp = 71 + 0.00005population - 0.3murder + 0.047hs_grad - 0.006frost

- Do the procedures generate the same model?

Backward elimination and forward elimination generated the same model: `life_exp ~ murder + hs_grad + frost`. However, stepwise regression generated a larger model with add a pridictor `population`.

- Is there any variable a close call? What was your decision: keep or discard? Provide arguments for your choice.

The variable `population` is a close call, with p-value = 0.052. I would keep it, because its p-value is close to 0.05. This model has a better AIC than a smaller model. And adding 'population' contributes to the goodness of fit by increasing the adjusted R2 from 0.6939 to 0.7126.

- Is there any association between 'Illiteracy' and 'HS graduation rate'? Does your 'subset' contain both?

The correlation coefficient between 'Illiteracy' and 'HS graduation rate' is -0.66, indicating a moderate association. My subset only contains 'HS graduate rate". 'Illiteracy' is not included.

## d)

```
leaps(x = state %>% select(-life_exp), y = state[[1]], nbest = 1, method = "Cp")
```

```
## $which
##       1     2     3    4     5     6     7
## 1 FALSE FALSE FALSE TRUE FALSE FALSE FALSE
## 2 FALSE FALSE FALSE TRUE  TRUE FALSE FALSE
## 3 FALSE FALSE FALSE TRUE  TRUE  TRUE FALSE
## 4  TRUE FALSE FALSE TRUE  TRUE  TRUE FALSE
## 5  TRUE  TRUE FALSE TRUE  TRUE  TRUE FALSE
## 6  TRUE  TRUE  TRUE TRUE  TRUE  TRUE FALSE
## 7  TRUE  TRUE  TRUE TRUE  TRUE  TRUE  TRUE
##
## $label
## [1] "(Intercept)" "1"           "2"           "3"           "4"
## [6] "5"           "6"           "7"
##
```

```
## $size
## [1] 2 3 4 5 6 7 8
##
## $Cp
## [1] 16.126760  9.669894  3.739878  2.019659  4.008737  6.001959  8.000000
```

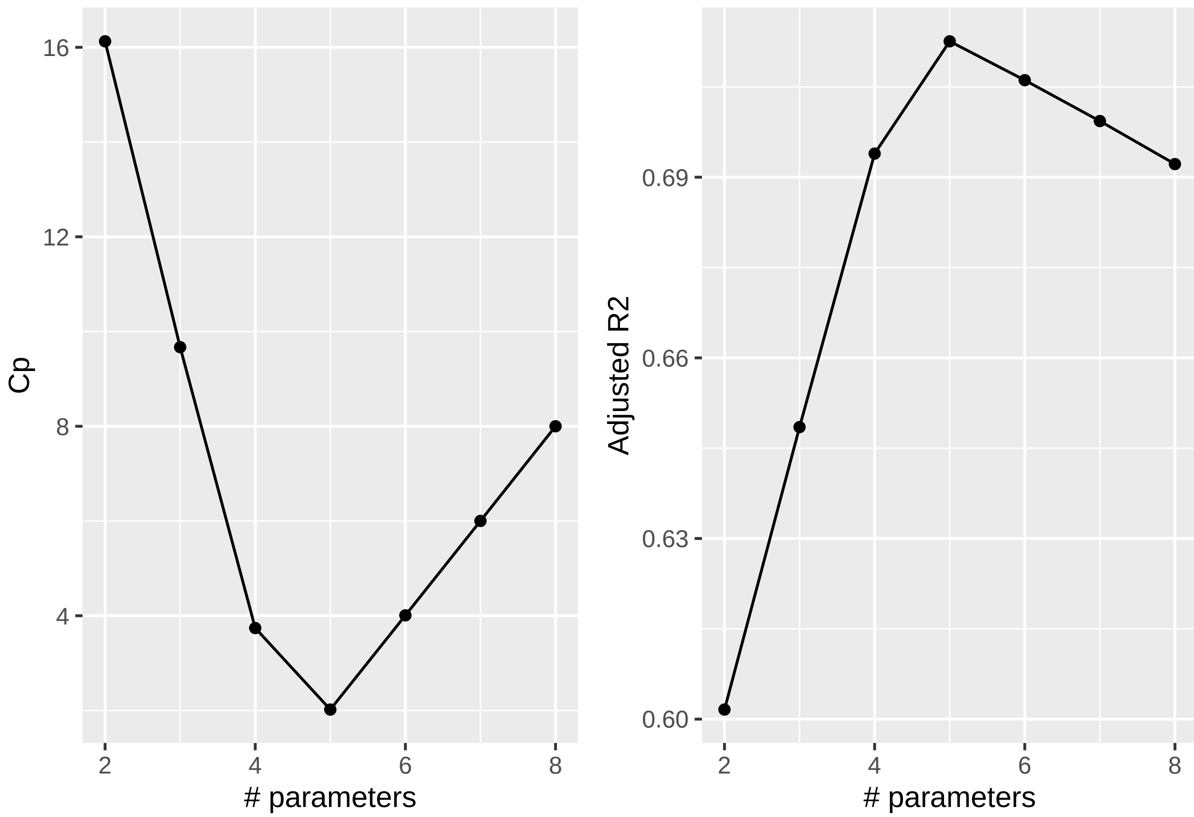
```
leaps(x = state %>% select(-life_exp), y = state[[1]], nbest = 1, method = "adjr2")
```

```
## $which
##       1     2     3    4     5     6     7
## 1 FALSE FALSE FALSE TRUE FALSE FALSE FALSE
## 2 FALSE FALSE FALSE TRUE  TRUE FALSE FALSE
## 3 FALSE FALSE FALSE TRUE  TRUE  TRUE FALSE
## 4  TRUE FALSE FALSE TRUE  TRUE  TRUE FALSE
## 5  TRUE  TRUE FALSE TRUE  TRUE  TRUE FALSE
## 6  TRUE  TRUE  TRUE TRUE  TRUE  TRUE FALSE
## 7  TRUE  TRUE  TRUE TRUE  TRUE  TRUE  TRUE
##
## $label
## [1] "(Intercept)" "1"           "2"           "3"           "4"
## [6] "5"           "6"           "7"
##
## $size
## [1] 2 3 4 5 6 7 8
##
## $adjr2
## [1] 0.6015893 0.6484991 0.6939230 0.7125690 0.7061129 0.6993268 0.6921823
```

```
sub = regsubsets(life_exp ~ ., data = state)
summ=summary(sub)
```

```
plot_cp =
  tibble(x = 2:8, y = summ$cp) %>%
  ggplot(aes(x = x, y = y)) +
    geom_point() + geom_line()+
    labs(x = "# parameters", y = "Cp")

plot_adjr2 =
  tibble(x = 2:8, y = summ$adjr2) %>%
  ggplot(aes(x = x, y = y)) +
    geom_point() + geom_line()+
    labs(x = "# parameters", y = "Adjusted R2")
plot_cp + plot_adjr2
```




Based on the Cp and adjusted R2 criterion, I would choose the 4-predictors (5 parameters) model. The best 4-predictors model is life_exp ~ population + murder + hs_grad + frost. It has the highest adjusted R2 and the lowest Cp value.

### e)

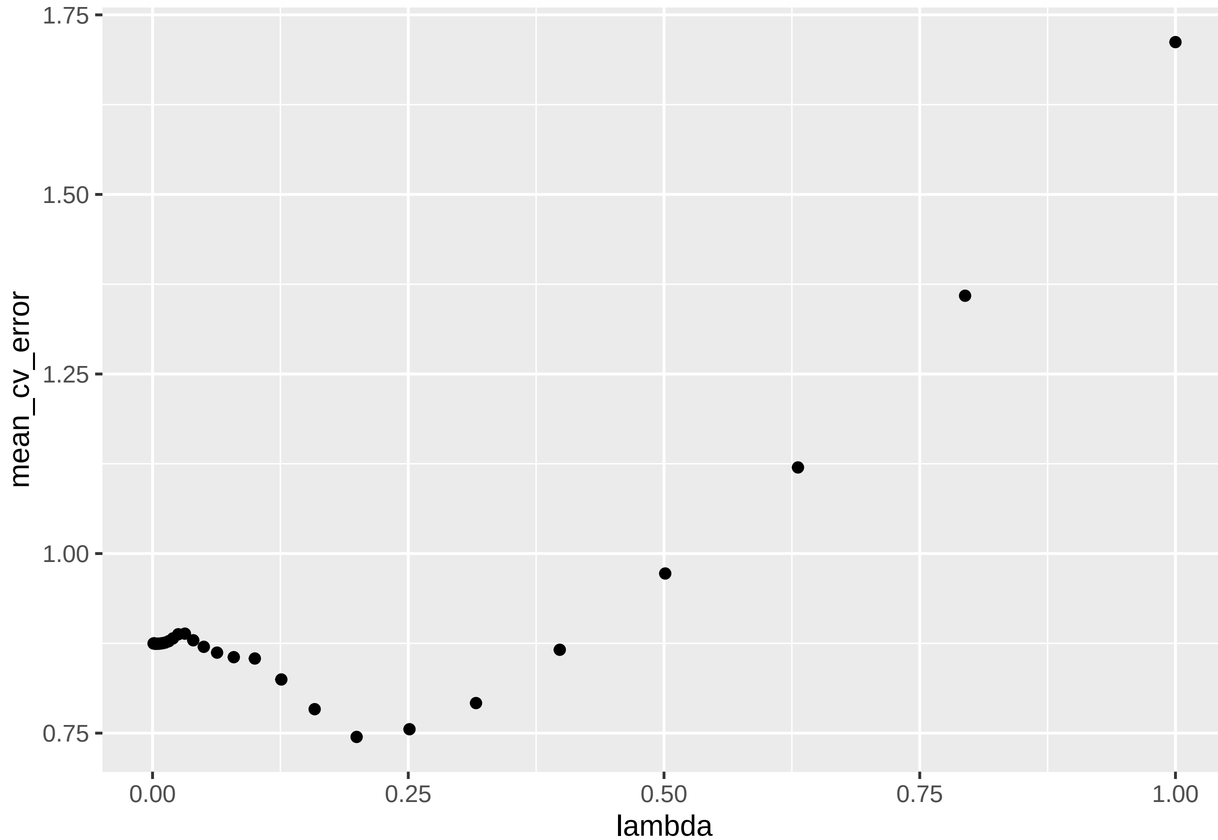
```
lambda_seq <- 10^seq(-3, 0, by = .1)
set.seed(2022)
cv_object <- cv.glmnet(as.matrix(state[2:8]), state$life_exp,
lambda = lambda_seq,
nfolds = 5)
cv_object
```

```
##
## Call:  cv.glmnet(x = as.matrix(state[2:8]), y = state$life_exp, lambda = lambda_seq,      nfolds = 5)
##
## Measure: Mean-Squared Error
##
##     Lambda Index Measure     SE Nonzero
## min 0.1995     8  0.7447 0.1706       2
## 1se 0.3981     5  0.8661 0.1603       2
```

```
cv_object$lambda.min
```

```
## [1] 0.1995262
```

```r
tibble(lambda = cv_object$lambda,
mean_cv_error = cv_object$cvm) %>%
ggplot(aes(x = lambda, y = mean_cv_error)) +
geom_point()
```




When lambda is 0.1995, it will generate the lowest cv error.

Now I will try to refit the model with the best lambda.

```r
fit_bestcv <- glmnet(as.matrix(state[2:8]), state$life_exp, lambda = cv_object$lambda.min)
coef(fit_bestcv)
```

```
## 8 x 1 sparse Matrix of class "dgCMatrix"
##                      s0
## (Intercept) 70.91716697
## population   .
## income       .
## illiteracy   .
## murder      -0.20040550
## hs_grad      0.02711503
## frost        .
## area         .
```

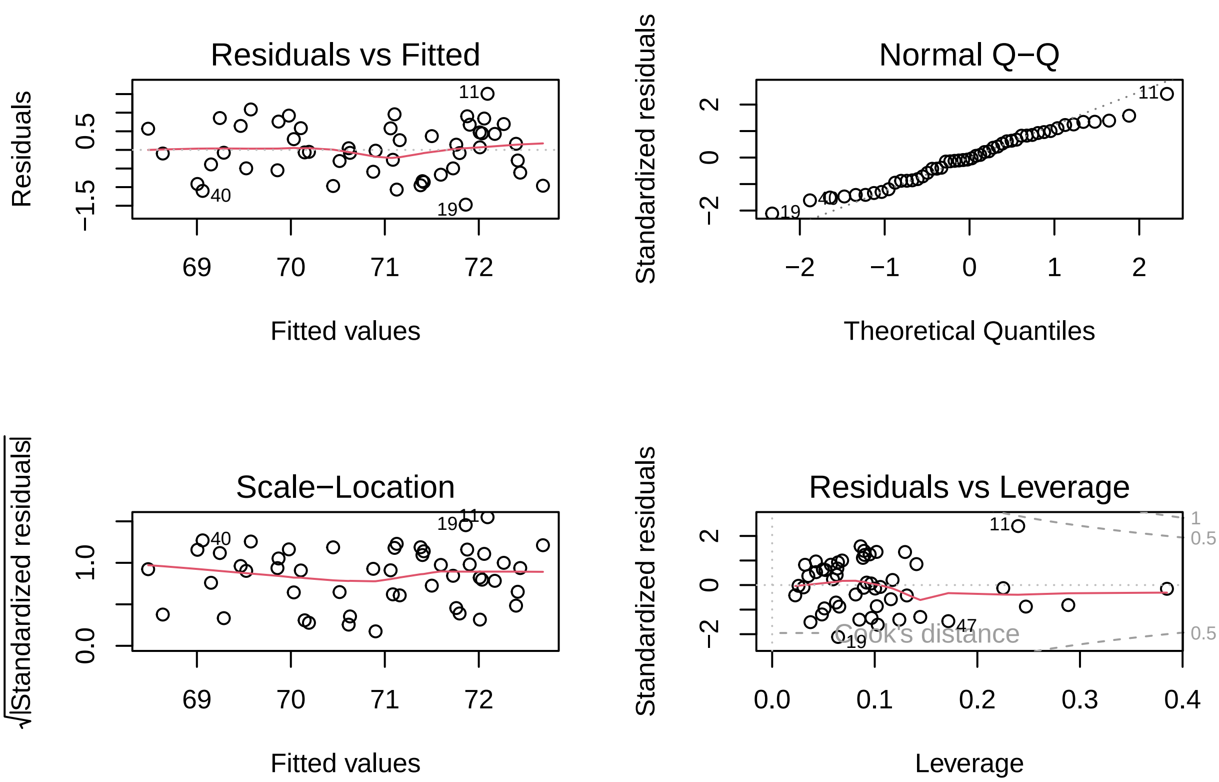
The results shows that `murder` and `hs_grad` should be included in the model.

## f)

### Diagnostics

Based on the models chosen from c), d), and e), forward elimination, stepwise regression, and the criterion-based procedures, they all ended up with the model: life_exp~ population + murder+ hs_grad + frost. I will use this as my final model.

```
final_model=lm(life_exp~population+murder+hs_grad+frost,data=state)
par(mfrow=c(2,2))
plot(final_model)
```




The residuals scattered evenly along the fitted values. We can assume that residuals have a mean of 0, and a constant variance, and independent of each other. The QQ plot shows that the tail slightly deviates from the straight line, this may indicates that there exists outliers. Additionally, there is no influential observations according to the residual vs leverage plot.

### 10-fold Cross Validation

```
train_control = trainControl(method ="cv", number = 10, savePredictions = TRUE)

model = train(life_exp ~ murder + hs_grad + frost + population,
              data = state,
              trControl = train_control,
              method = 'lm')
model
```

```
## Linear Regression
##
## 50 samples
##  4 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 46, 43, 45, 45, 44, 44, ...
## Resampling results:
##
##   RMSE       Rsquared   MAE
##   0.7853639  0.7254467  0.6724146
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```r
predictions = model$resample
```

For 10-fold cross-validation, the `RMSE` is about 0.74, `Rsquare` is about 0.73, `MAE` is about 0.62. There are about 73% of variation explained by our model.

## g)

The final model that I chose is life_exp ~ population + murder + hs_grad + frost. Some predictors are known as junk predictors, which will not provide help for us to predict life expectancy. The four predictors that was included, shows statistically significant when predicting life expectancy. After choosing the model, I tried to validate it. The R square shows to be 0.73, which means the model explained pretty well on the observed data. Overall, the model that was chosen provided a quite well prediction on the life expectancy.