Math 55 Section 101 Quiz 9

Problem 1 (9.1 # 7d,e) Determine whether the relation on R on the set of all integers is reflexive, symmetric, anti-symmetric and/or transitive, when $(x, y) \in R$ if and only if:

1.A (2 pt) $x \equiv y \mod 7$

Solution: This is equivalent to the relation $x - y = 0 \mod 7$, or $x - y$ divisible by 7. Since $x - x = 0$ (which is 0 mod 7), this relation is reflexive. If $x - y$ is divisible by 7, then so is $y - x = -(x - y)$, so the relation is symmetric. It therefore cannot be anti-symmetric. If $x - y = 0 \mod 7$ and $y - z = 0 \mod 7$ then $x - z = x - y + y - z = 0 \mod 7$. Thus it is transitive (and an equivalence relation).

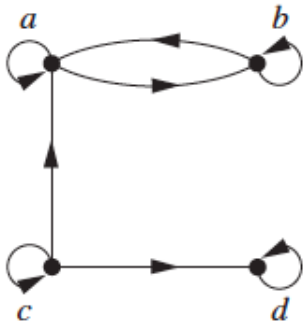
1.B (2 pt) x is an integer multiple of y.

Solution: x is a multiple of itself (by 1) so the relation is reflexive. It is not symmetric: 6 is a multiple of 2 but not the other way around, so that is a counter-example. It is *not* anti-symmetric: if x is a multipe of y and y is a multiple of x then $x = \pm y$, but it does not mean they are equal. This would be true if we were using the set of positive integers. Finally, if $x = ay$ is a multiple of $y = bz$ and y is a multiple of z, then $x = abz$ so x is a multiple of z.

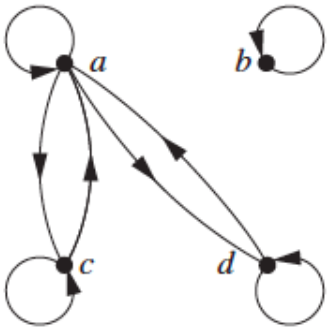
Problem 2 (2 pt) (Like 9.3 # 21) Draw the directed graph corresponding to the following matrix (where the rows and columns correspond to the integers in increasing order). The set is $\{a, b, c, d\}$ with column indices increasing left to right and row indices increasing top to bottom.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Solution:



Problem 3 (4 pt) Determine whether or not this graph determines an equivalence relation.



Solution: No. It is not transitive, because dRa and aRc but we do not have dRc (d and c are not connected).