Math 535A: Smooth Manifolds

Course Description. This course is a graduate introduction to smooth manifolds. Topics will include smooth manifolds, smooth functions, vector bundles, tensors, special maps (immersions, submersions, embeddings), sub-manifolds, Lie groups and de Rham cohomology.

Beware! This course is primarily aimed at doctoral students preparing for their screening exams. It will cover a lot of material, very quickly, and the problem sets will be very demanding.

Instructor. Julian Chaidez (`julian.chaidez@usc.edu`). OH: F 1-3 in KAP 245.

Assistant. Wenhan Jiang (`wenhanji@usc.edu`).

Lecture Information. MWF 11-12 in KAP 134

Textbook. J. M. Lee. *Introduction To Smooth Manifolds.* Springer Graduate Series (2013).

Course Grades. At the end of the semester, grades will be computed by the following formula.

$$\text{Course Grade} \;=\; .6 \times \text{Problem Sets} \;+\; .2 \times \text{Midterm} \;+\; .2 \times \text{Final Exam}$$

Problem Sets. Weekly problem sets consisting of textbook exercises will be assigned as follows.

- **Due Day.** Solutions are due on Monday in class (see schedule below). Printed or written solutions must be provided in person, at the start of class (11 am).
- **Formatting.** Students are encouraged (but not required) to write solutions in LaTeX.
- **Collaboration Policy.** Students may collaborate on their solutions. However, all students in a collaborative group must write the names of their partners.
- **Late/Drop Policy.** No late problem sets will be accepted. The bottom two problem sets will be dropped from the students overall problem set grade. This allows any student to skip one or two problem sets if needed.
- **Grading Policy.** Solutions will be graded **very carefully** for clarity, rigor and correctness. Grading will be taken very seriously!

Exams. The midterm and final will each consists of two equally weighted components.

- **Take-Home Component.** This part of the exam will be a long assignment consisting of original (non-book) problems. This will replace the homework for that week.
- **In-Class Component.** This part of the exam will be a 45 minute exam administered during class. It will consist of textbook problems.
- **Collaboration Policy.** Collaboration is permitted on the take-home components, with the inclusion of the names of partners, but every student must write original solutions.

  If a group of solutions is found to be written in an identical manner, every person in that group will receive a 20% deduction to their score on that problem.

Lecture Schedule (First Half)

Date	Topics	Reading
	Basics (Ch 1,2)	
M 1/8	introduction, topological manifolds (with boundary)	1-10, 25-27
W 1/10	smooth manifolds (with boundary)	11-24
F 1/12	smooth functions and maps, partitions of unity	32-48
W 1/17	examples: implicit function theorem, products, quotients	1-24
	Vector Bundle (Ch 10, 3, 11)	
F 1/19	vector bundles, sections	249-261
M 1/22	bundle homomorphisms, pullback, sub-bundles	261-268
W 1/24	tangent vectors, tangent bundle, tangent map	50-60
F 1/26	cotangent vectors, cotangent bundle, differentials	272-287
	Immersions And Submersions (Ch 4,5)	
M 1/29	maps of constant rank, immersions, submersions	77-91
W 1/31	embeddings, sub-manifolds, normal bundles	98-108
	Lie Groups And Flows (Ch 7,9,8)	
F 2/2	Lie groups, examples, homomorphisms	150-156
M 2/5	Lie subgroups, Lie group actions, equivariance	156-171
W 2/7	flows, vector-fields, integral curves	205-217
F 2/9	Lie bracket, Lie algebras	186-199
	Tensors And Tensor-fields (Ch 12)	
M 2/12	categories, functors, universal properties, Vect	73-75
W 2/14	multilinear algebra, tensors, tensor-fields	304-313
F 2/16	symmetric and antisymmetric tensor(-fields)	313-316
	Riemannian manifolds (Ch 13)	
W 2/21	inner products, Riemannian metrics	327-337
F 2/23	length, distance, musical isomorphism	337-343
M 2/26	geodesics, geodesic flow, examples	
W 2/28	**Midterm (In Class)**	

Lecture Schedule (Second Half)

Date	Topics	Reading
	Differential Forms (Ch 14,15)	
F 3/1	exterior algebra	349-359
M 3/4	differential forms, exterior derivative	359-367
W 3/6	Cartan's magic formula	369-373
F 3/8	orientations, volume forms	377-388
	Integration (Ch 16)	
M 3/18	geometry of volume, integration of differential forms	400-411
W 3/20	Stokes' theorem	411-415
F 3/22	divergence theorem	415-426
	de Rham Cohomology (Ch 17)	
M 3/25	(co)chain complexes, chain maps, chain homotopies, examples	
W 3/27	de Rham cohomology, basic properties	440-443
F 3/29	homotopy invariance of de Rham cohomology	443-448
M 4/1	Mayer-Vietoris: statement and applications	448-457
W 4/3	Mayer-Vietoris: proof	460-464
	Other Homologies	
F 4/5	overview of simplicial (co)homology	
M 4/8	overview of singular (co)homology	467-473
	de Rham Theorem (Ch 18)	
W 4/10	smoothing chains	473-477
F 4/12	smooth singular homology	477-480
M 4/15	de Rham map and naturality	480-484
W 4/17	de Rham map is an isomorphism	484-487
	Intersection Theory	
F 4/19	Sard's theorem	125-131
M 4/22	transversality	
W 4/24	intersection numbers and homology	
F 4/26	**Final (In Class)**	

Homework Schedule

HW	Problems	Due Date
1	Ch 1: 1,2,9. Ch 2: 3a, 9, 10, 14	1/22
2	Ch 10: 1, 12. Ch 3: 1, 3, 4	1/29
3	Ch 11: 5, 6, 7. Ch 4: 2, 4. Ch 5: 1, 2, 6	2/5
4	Ch 7: 1, 2, 5, 14. Ch 8: 10, 19, 22, 23, 29.	2/12
5	Ch 9: 4, 5. Ch 12: 1, 2, 4, 5, 11, 12.	2/21
	Midterm	
6	Ch 14: 1, 5, 6, 7. Ch 15: 1, 3, 5	3/18
7	Ch 16: 1, 2, 3, 5, 6, 9	3/25
8	Ch 16: 18, 21, 22. Ch 17: 1, 2	4/1
9	Ch 17: 4, 6, 7, 11, 12	4/8
10	Ch 18: 1, 2, 6, 7, 8, 9	4/22