Math 641: Floer Theory (3 Units)

Course Description. This is a graduate course on Floer homology in symplectic geometry. In the first part, we will cover the construction of Hamiltonian-Floer homology and the Floer moduli space in detail, including Fredholm theory, index calculations, asymptotics, compactness, transversality and gluing. In the second part, we will survey the most prominent flavors of Floer theory and their applications: symplectic homology, Lagrangian-Floer theory, Fukaya categories, Heegaard-Floer homology and symplectic field theory (SFT).

Prerequisites. Students should have a strong background in real analysis (525a), differential topology (535a) and algebraic topology (540). The basics of PDE (555a), functional analysis and Riemannian/symplectic geometry (535b) will also be helpful.

Course Goals. This course aims to provide sufficient mastery of important research tools in the symplectic geometry. By the end of the the course, the student should obtain

- good understanding of the basic ingredients and proof methods of Floer theory: basic Fredholm theory, index calculations, compactness, gluing, transversality.
- good understanding of the contemporary landscape of Floer theory, including open questions and current research.

Instructor. Julian Chaidez (`julian.chaidez@usc.edu`).

Lecture Information. WF 1:30-3 in KAP 427.

Office Hours Information. WF 4-5 in KAP Math Center.

Course Website. The course website is `julianchaidez.net/2025s_math641.html`. Thanks to the small class size, we will not need Brightspace for this class.

Textbook. For Part I of the course, we will use the following two texts.

- **[S]** D. Salamon. *Lectures On Floer Homology.* IAS Park City Graduate Summer School Notes. Available at `people.math.ethz.ch/~salamon/PREPRINTS/floer.pdf`
- **[AD]** Michele Audin, Mihai Damian. *Morse Theory And Floer Homology.* Springer, 2014.

The second part of the course will use various survey articles listed below.

Course Grades. At the end of the semester, grades will be computed by the following formula.

$$\text{Grade} \;=\; .5 \times \text{Problem Sets} \;+\; .3 \times \text{Final Paper} + \;.2 \times \text{Final Presentation}$$

Students who are taking the course under a Pass-Fail grade option are permitted to submit the final project as the sole basis of their grade.

Readings. Readings are assigned for every class meeting following the schedule below. Any student who wants to really learn the material must do the readings weekly!

Problem Sets. Problem sets consisting of textbook exercises will be assigned as follows.

- **Due Day.** Solutions are due on Wednesdays in class, following the schedule below.

- **Formatting.** Students are encouraged (but not required) to write solutions in LaTeX.
- **Collaboration Policy.** Students may collaborate on their solutions.
- **Late Policy.** Every student will be allowed two problem set extensions by one week. Further extensions will be considered on a case-by-case basis and the DGS (Prof. Sami Assaf) will be alerted about the case.
- **Grading Policy.** Solutions will be graded carefully for clarity, rigor and correctness.

Final Project. Students will submit a final 10-15 page expository paper on an application of Floer theory. Each paper should include

- A summary of the construction of the Floer homology of interest, including a comparison of the construction to standard Floer theory.
- A summary of the main application discussed in the paper and why it is interesting to the community.
- A short discussion of open problems relating to the main applicatyion, it any.

Each student will also prepare a short 15 minute presentation about their topic. Final papers are due on **Friday 5/2.** On that day, final presentations will take place in class.

Statement For Students With Disabilities. Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me (or to TA) as early in the semester as possible. DSP is located in GFS 120 and is open 8:30 a.m.-5:00 p.m., Monday through Friday. Website for DSP (`https://dsp.usc.edu/`) and contact information: (213) 740-0776 (Phone), (213) 740-6948 (TDD only), (213) 740-8216 (FAX) `dspfrontdesk@usc.edu`.

Statement On Academic Integrity. USC seeks to maintain an optimal learning environment. General principles of academic honesty include the concept of respect for the intellectual property of others, the expectation that individual work will be submitted unless otherwise allowed by an instructor, and the obligations both to protect one's own academic work from misuse by others as well as to avoid using another's work as one's own. All students are expected to understand and abide by these principles. SCampus, the Student Guidebook, contains the University Student Conduct Code (see University Governance, Section 11.00), while the recommended sanctions are located in Appendix A.

Emergency Preparedness/Course Continuity In A Crisis. In case of a declared emergency if travel to campus is not feasible, USC executive leadership will announce an electronic way for instructors to teach students in their residence halls or homes using a combination of Blackboard, teleconferencing, and other technologies. See the university's site on Campus Safety and Emergency Preparedness.

Lecture Schedule (Part I)

Date	Topics	Read [S]	Read [AD]
	Introduction		
W 1/15	Morse theory, Morse homology, moduli of flowlines	1.3	1.1-3.5 $\star$
F 1/17	symplectic action, Floer equation, moduli of Floer solutions	1.4-1.6	6.1-6.4
	Background		
W 1/22	Geometry: Hamiltonians, complex structures		5.1-5.6 $\star$
F 1/24	Analysis: Banach manifolds, Fredholm maps/index	2.1	C.1-C.5
W 1/29	Analysis: Properties of Fredholm operators/index	2.1	C.1-C.5
	The Floer Map And Flowlines		
F 1/31	Floer solutions, energy, elliptic regularity, C^k-topology	1.4-1.6, 2.7	6.5-6.6
W 2/5	asymptotics, compactness of parametrized Floer trajectories	1.4-1.6, 2.7	6.5-6.6
	Linearization And Index		
F 2/7	linearization, exponential decay	2.2-2.3	8.2, 8.4,8.9
W 2/12	Fredholm property	2.2-2.3	8.7
F 2/14	rotation number, Conley-Zehnder index, index of orbit	2.4	7.1-7.2
W 2/19	Fredholm index formula	2.5	8.8
	Generic Transversality		
F 2/21	perturbation space, somewhere injectivity		8.3,8.6
W 2/26	generic tranversality of Floer map		8.5
	Broken Trajectories And Gluing		
F 2/28	space of trajectories, gluing statement, pre-gluing map	3.1	9.1-9.3
W 3/5	construction of gluing map, sketch of immersion/submersion	3.3	9.4-9.6
	Floer To Morse		
F 3/7	review of Floer construction, linearized flow operator	3.2, 3.5	10.1-10.2
W 3/12	generic regularity, trajectory bijection	3.5	10.3-10.4
	Invariance		
F 3/14	sketch, continuation map, isotopy/composition homotopy	3.4	11.1-11.6 $\star$

Reading Notes. Note that two reading assignments are provided for most weeks above. The Salamon [S] readings are much shorter, but also easier to follow and more big-picture. The Audin-Damian [AD] readings are more detailed but harder to follow. Note that there are small differences in notation and treatment. The readings marked with the symbol $\star$ are just for reference. It is not necessary to read them in detail. However if might be good to skim them to check if there's anything unfamiliar.

Lecture Schedule (Part II)

Date	Topics	Reading
	Lagrangian Floer Homology And Fukaya Categories	
W 3/26	Lagrangians, Hamiltonian chords, Lagrangian-Floer homology	Au §1
F 3/28	A_∞ algebras/categories/functors, weak equivalence	Au §2
W 4/2	Disk tree compactness, Fukaya category A_∞-operations	Au §2-4
	Quantitative Floer Theory	
F 4/4	filtered complexes/chain maps/homotopies, examples	
W 4/9	spectral invariants in Morse/Floer homology, applications	
	Symplectic Homology	
F 4/11	Liouville domains, maximum principle, SH construction, properties	We §2-3
W 4/16	S^1-equivariant version, applications (dynamics, capacities, etc)	We §4
	Symplectic Field Theory (SFT)	
F 4/18	Hamiltonian manifolds, symplectic cobordisms, SFT ACS	EGH, BE+ §1-3
W 4/23	Deligne-Mumford space, energy, holomorphic buildings	BE+ §4-8
F 4/25	SFT index, SFT compactness, applications and special cases	BE+ §10-12
W 4/30	flavors of SFT (contact dga, cylindrical CH, ECH etc), applications	
F 5/2	**Final Presentations**	

Reading List

- **[Au]** D. Auroux. *A beginner's introduction to Fukaya categories.* `arXiv:1301.7056`
- **[BE+]** F. Bourgeois, Y. Eliashberg, H. Hofer, K. Wysocki, E. Zehnder. *Compactness results in symplectic field theory.* `arXiv:math/0308183`
- **[EGH]** Y. Eliashberg, A. Givental, H. Hofer. *Introduction to symplectic field theory.* `arXiv:math/0010059`
- **[We]** C. Wendl. *A beginner's overview of symplectic homology.* `www.mathematik.hu-berlin.de/~wendl/pub/SH.pdf`

Homework Schedule

HW	Problems	Due Date
1	[S] 1.2, 1.7, 1.8, 1.9, 1.10, 1.11	W 1/29
2	[S] 1.22, 1.23, 2.1, [AD] 6.28, 6.30	W 2/5
3	[AD] 7.38, 7.39, 7.40, 7.41, 7.43	W 2/26
4	[S] 2.8, 2.9, 2.10 [AD] 10.46, 6.25, 11.47	W 3/14
5	Project proposal (1 paragraph by email)	W 4/2
6	Project draft (5 pages, pdf by email)	W 4/23
7	Final project (10-15 pages, pdf by email)	F 5/2

Homework Notes

The problems below are from Audin-Damian [AD], Oszvath-Szabo [OZ] and Salamon [S]. In Audin-Damian, the questions are denoted by X.Y if the assignment corresponds to Chapter X and is Exercise Y in the chapter containing exercises (Chapter 14). Sorry about the numbering.