Math 55 Section 101 Quiz 9

Problem 1 Find **recurrence relations** and **initial conditions** that define the following sequences.

1.A (3 pt) $a_n = \binom{n}{3}$ for $n \geq 3$.

1.B (3 pt) (3 pt) $a_n =$“The number of sequences of 2’s and 5’s that adds to n” with $n \geq 0$. For instance, $(2, 5, 2)$ is a sequence of 2’s and 5’s that adds to $n = 9$. Hint: You need 5 initial conditions.

Problem 2 (3 pt) Consider the sequence a_n with generating function $f(x) = \sum_{n=0}^{\infty} a_n x^n$, and suppose that a_n satisfies the recurrence relation $a_n = 4a_{n-2} + 1$ with initial conditions $a_0 = a_1 = 0$. Write down the corresponding equation satisfied by f. (I’m looking for an equation involving f, like $f(x) = x^5 f(x) + x^2$ for example).

Problem 3 (1 pt, Challenge Problem) Consider the sequence a_n with generating function $f(x) = \sum_{n=0}^{\infty} a_n x^n$, and suppose that a_n satisfies the recurrence relation $a_n = -\frac{\omega}{n(n-1)} a_{n-2}$. Here ω is just some real number. Assume arbitrary initial conditions (use $a_0 = a$ and $a_1 = b$ if you want). Write down the general closed form solution for f.