Section 4/7: Midterm Review Template

Tuesday, April 7, 2020 11:53 AM

<u>Announcements:</u>

Office Hours: I will hold office hours **tomorrow** 12

Midterm is **Tomorrow** Weds at 2:00 pm PST.
- Get by email (or on Bcourses if email doesn't come)
- 50 minutes, **due at 3:05 pm PST.**
- 2 cheat sheets allowed.
- Your answers should be submitted on **4 sheets of paper**, one for each question.
- Write **name, SID, start and end time, and oath to not cheat** on first page.

Second chance midterm for Pass/Fail students.
- **If** you are taking the class Pass/Fail **then** you have until **2 pm** tomorrow to submit a **second version** of your midterm 2.
- For this version, you may use your book.
- Your second version will replace your first version if it positively effects your grade, i.e. makes your Fail into a Pass.
- Your second version will **not** be used if you are taking this class for a letter grade.

A/B/C students will be required to learn some extra material from Sections 12.5 and 12.6. Pass/Fail students do not need to learn this and will not need to answer these questions on the final.

<u>Midterm Review</u>

(9.1, 9.2,9.6) Multivariable Derivatives/Integrals. Need to know how to

- Take partial derivatives, partial integrals and double integrals (**useful to have a table of integrals and derivatives in cheat sheet!).**

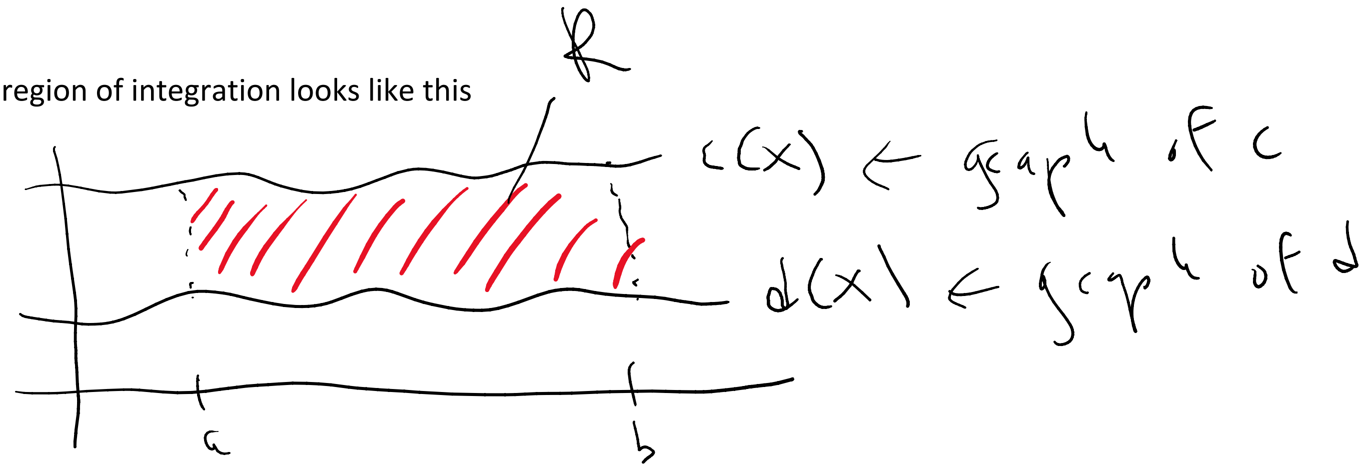
<u>Fubini:</u> You can switch the bounds of integration **when the bounds are not dependent on x and y.**

$$\int_b^a \int_d^c f(x,y)dxdy = \int_d^c \int_b^a f(x,y)dydx$$

<u>Drawing Domain Of Integration:</u> If you have an integral of the form

$$\int_b^a \int_{d(x)}^{c(x)} f(x,y)dydx$$

then the region of integration looks like this



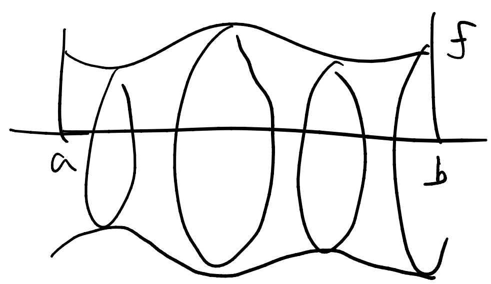
(8.2) Volumes and averages.

<u>Average:</u> The average of a function $f(x)$ from a to b is

$$(\int_a^b f(x)dx)/(b-a)$$

<u>Volume Of Solids Of Rotation:</u> The volume of a solid of rotation a function $f(x)$ from a to b is

$$\int_a^b \pi\, f(x)^2 dx$$



(10.1-10.4) ODEs. You need to know how to

- Use integration factors, separation of variables and Euler's method.
- Turn a word problem into an ODE.

<u>Equilibrium Solutions:</u> Suppose you have an ODE of the form

$$y'(t) = F(y(t))$$

An **equilibrium solution** is a solution that is constant, i.e. a solution

$$y(t) = c$$

The solution is **stable** if $F'(c) < 0$ and **unstable** if $F'(c) > 0$. If you draw solutions y(t) in the (t,y)-plane, then equilibrium solutions look like horizontal lines.

Solutions near a stable solution slope towards the stable solution at t gets big. Solutions near an unstable solution slope away from the unstable solution as t gets big.

Integration Factors: Given the ODE

$y'(t) + p(t)y(t) = f(t)$ (and $y(a) = b$ for some a) then use

$y(t) = e^{-I(t)} \int e^{I(t)} f(t)dt$ where $I(t) = \int p(t)dt$

Separation Of Variables: Given the ODE

$y'(t) = \frac{p(t)}{q(y)}$ (and maybe $y(a) = b$ for some a) then use

Solve the equation Q(y) = $\int q(y)dy = \int p(t)dt = P(t) + C$
for y in terms of t.

Euler's Method: Given the ODE

$y'(t) = F(y(t))$ and $y(a) = A$

You can **approximate** the solution y(b) by doing k steps of Euler's method as so.

1) First calculate the step size $s = \frac{b-a}{k}$.
1. Then do k steps of Euler's method, as so.

$$(1)\ y(a + 1 * s) \approx y(a) + s * y'(a) = y(a) + s * F(a)$$
$$(2) y(a + 2 * s) \approx y(a + 1 * s) + s * F(a + 1 * s)$$
$$(3) y(a + 3 * s) \approx y(a + 2 * s) + s * F(a + 2 * s)$$
.....
$$(k)\ y(b) = y(a + k * s) \approx y(a + (k - 1) * s) + s * F(a + (k - 1) * s)$$

(6.6,9.5) Differentials/Total Differentials: You need to know how to:

- Compute differnetials and total differentials
- Approximate functions using their differentials.
- Read a word problem about approximating and set it up using differentials.

Total Differentials: If $f(x)$ and $g(x, y)$ are functions, then the total differentials are

$$df = f'(x)dx \qquad dg = g_x dx + g_y dy$$

Approximation: For small dx and dy we have

$$f(x + dx) = f(x) + df = f(x) + f'(x) * dx$$

$$g(x+dx, y+dy) = g(x,y) + dg = g(x,y) + \; g_x(x,y)dx + \; g_y(x,y)dy$$

(12.3) Taylor Polynomials: You need to know how to

- Compute Taylor polynomials

Taylor Polynomial: The kth order taylor polynomial of $f(x)$ is given by

$$P_k(x) = \sum_{i=1}^{k} \frac{f^{(i)}(0)}{i!} x^i = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(k)}(0)}{k!}x^k$$

If $P_k(x)$ is the kth Taylor polynomial of $f(x)$ and $Q_k(x)$ is the kth Taylor polynomial of $g(x)$ then

- The kth Taylor polynomial of $f(x) + g(x)$ is $P_k(x) + Q_k(x)$.
- The kth Taylor polynomial of $k * f(x)$ is $k * P_k(x)$
- The kth Taylor polynomial of $f(x) * g(x)$ is the polynomial $P_k(x) * Q_k(x)$ with the terms of order more than k removed.

Approximation: For small dx we have $P_k(dx) \; \approx f(dx)$ and this approximation gets better as k gets bigger.

(12.1) Sequences: A sequence is just a sequence of numbers a_i and the partial sums are given by

$$\sum_{n=1}^{N} a_n$$

Geometric Sequences: A geometric sequence is given by the formula

$a_i = ar^{i-1}$ for some a, r

The partial sums of a geometric series are given by the formula.

$$\sum_{n=1}^{N} a_n = \sum_{n=1}^{N} ar^{n-1} = a\frac{r^N - 1}{r - 1}$$

(12.4) Series: A series is an infinite sum of a sequence. It either **converges** (meaning the partial sums converge) or **diverges** (meaning the partial sums diverge). If it converges, then the

Geometric Series: If $a_n = ar^{n-1}$ is a geometric series, then

$$\sum_{n=1}^{\infty} a_n = \frac{a}{1-r} \quad \text{if} \quad |r| < 1$$

If $|r| \geq 1$ then the series diverges.

(12.2) Annuities: An annuity is a regularly paying source of income into an account earning interest. The amount of annuity is the amount you have at the end of the payment period (accounting for interest). The present value is the lump sum needed to earn the same amount using interest.

<u>Regular Payment Annuities:</u> An annuity consisting of annual payments of $\$R$ per period, for n consecutive interest periods with interest i (decimal interest, not percent). Then the amount of annuity S is given by

$$S = R\frac{(1+i)^n - 1}{i}$$

The present value P of an annuity is given by

$$P = (1+i)^{-n}S = R\,\frac{1-(1+i)^{-n}}{i}$$

The present value P of an annuity is given by

$$P = (1+i)^n S$$