Math 54 Section 4: Quiz 1

Problem 1 Compute the following determinant with cofactor expansion.

$$\begin{vmatrix} 3 & 2 & 4 \\ -1 & 2 & 1 \\ 0 & 3 & 2 \end{vmatrix}$$

Problem 2 Find the area of the following parallelogram $(-2,3),(4,2),(1,5),(7,4)$.

Problem 3 Compute the following determinants by examination (cofactor expansion will take forever in some cases!). Write the answer to the right of the matrix.

$$\text{(a)} \begin{vmatrix} 20 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{vmatrix} \qquad \text{(b)} \begin{vmatrix} 1 & 2 & 1 & 0 & -1 \\ -2 & -4 & -2 & 0 & 2 \\ 3 & -4 & 30 & 1 & -1 \\ 1 & 5 & 2 & -3 & 3 \\ 0 & 2 & 1 & -2 & 0 \end{vmatrix}$$

$$\text{(c)} \begin{vmatrix} -1 & 2 & 4 & 3 & 3 \\ 0 & 5 & 1 & 7 & 2 \\ 0 & 0 & 2 & 2 & -1 \\ 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix} \qquad \text{(d)} \begin{vmatrix} 0 & 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix}$$

Problem 4 True or False? No need to give counter-examples or show work.

(a) If $A^3 = 0$ then $\det(A) = 0$.

(b) $\det(A+B) = \det(A) + \det(B)$

(c) $\det(A-B) = 0$ then $A = B$.

(d) $\det((A^{-1})^T) = \det(A)$

(e) $\det(A^T A) \geq 0$.

(f) If A, B, C are $n \times n$ matrices and D is the block matrix:

$$D = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

Then $\det(D) = \det(A) + \det(B) + \det(C)$

Problem 5 Let J be the $2n \times 2n$ block matrix:

$$J = \begin{bmatrix} J_2 & 0 & 0 & \dots & 0 \\ 0 & J_2 & 0 & \dots & 0 \\ 0 & 0 & J_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & J_2 \end{bmatrix}$$

where J_2 is the 2×2 matrix:

$$J_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Show that if $M^T J M = J$ then $\det(M) = \pm 1$.