Worksheet 10-27

Problem 1 Construct the matrix with rank 1 that has $Av = 12u$ where $v = \frac{1}{2}(1,1,1,1)$ and $u = \frac{1}{3}(2,2,1)$.

Problem 2 If A has SVD $U\Sigma V^T$, what are the SVDs of $4A$, A^T and A^{-1}?

Problem 3 Any symmetric matrix A has an orthonormal eigenbasis u_i. This can be used to write an SVD decomposition $U\Lambda U^T$ where the U and V matrices are equal. That is, U is just the matrix of eigenvectors and Λ is the matrix of eigenvalues.

Use this to show that any positive semi-definite symmetric matrix A has an nth root, i.e. a matrix A' such that $(A')^n = A$.