Finite Element Method

Student #

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1 Introduction

Here comes the introduction.

General model, equation

Present the model, matrices Displacement Convergence

The linear elasticity equation:

$$\nabla \boldsymbol{\sigma}(\boldsymbol{u}) = -\boldsymbol{f}. \tag{1}$$

1.1 Convergence analysis

We test our code on the problem

$$f_x = \frac{E}{1-\nu^2}(-2y^2 - x^2 + \nu x^2 - 2\nu xy - 2xy + 3 - \nu) \tag{2}$$

$$f_y = \frac{E}{1-\nu^2}(-2x^2 - y^2 + \nu y^2 - 2\nu xy - 2xy + 3 - \nu) \tag{3}$$

with homogeneous Dirichlet boundary conditions on all boundaries. Compared to the analytical solution

$$\boldsymbol{u} = \begin{bmatrix} (x^2-1)(y^2-1) \\ (x^2-1)(y^2-1) \end{bmatrix} \tag{4}$$

we get the error plot shown in Figure 1. Also, increasing the grid size in each spatial direction by a factor 2, we obtain an accuracy convergence of order 2, as shown in Figure 2. This implies that our code is correct.

Same shit as 2D

Present the model, matrices Convergence

Stress analysis

2 Stress

The code has so far only calculated the displacement in the geometry. We will now look at the stress $\boldsymbol{\sigma}$ which measures how much forces per area are acting on a particular spatial point.

To calculate the stress in 3 dimensions, we used the stress matrices from [1].

$\boldsymbol{C}$ gives us the relation between shear strain $\boldsymbol{\epsilon}$ and shear stress $\boldsymbol{\sigma}$.

$$\boldsymbol{\sigma} = \boldsymbol{C}\,\boldsymbol{\epsilon}$$

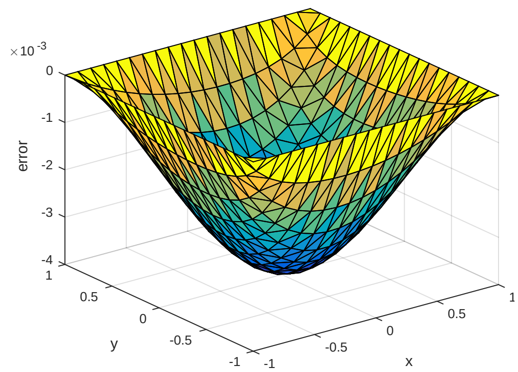

Figure 1: Error between the numerical solution to the linear elasticity problem and the analytical solution with $N = 20$ grid points in each spatial direction. The numerical solution is smaller than the analytical solution.

$$\boldsymbol{C} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & 0 & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$\boldsymbol{\epsilon} = \boldsymbol{B}\boldsymbol{u}^e$$

where $\boldsymbol{u}^e$ is the displacement field, $\boldsymbol{B}$ is the strain-displacement matrix

$$\boldsymbol{B} = \begin{bmatrix} \bar{\boldsymbol{\partial}}\boldsymbol{\phi_1} & \bar{\boldsymbol{\partial}}\boldsymbol{\phi_2} & \bar{\boldsymbol{\partial}}\boldsymbol{\phi_3} & \bar{\boldsymbol{\partial}}\boldsymbol{\phi_4} \end{bmatrix}, \qquad \bar{\boldsymbol{\partial}} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix}$$

and $\boldsymbol{\phi_i}$ is the shape function for the i-th node in the current element $\boldsymbol{u}^e$.
The resulting system for the stress is

$$\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{B}\boldsymbol{u}^e$$

where
Our geometry (bridge) Figures
Problems along the way
Conclusion, discussion

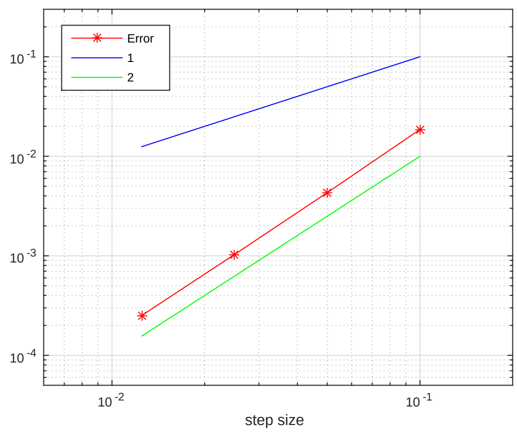

Figure 2: Error convergence plot of the linear elasticity problem with $N = 10$, 20, 40 and 80 grid points in each spatial direction.

References

[1] Colorado University
http://www.colorado.edu/engineering/cas/courses.d/IFEM.d/IFEM.Ch14.d/IFEM.Ch14.pdf

[2] Colorado University
http://www.colorado.edu/engineering/cas/courses.d/IFEM.d/IFEM.Ch28.d/IFEM.Ch28.pdf