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- Solar-terrestrial relationships
- Formation of the ionosphere
- Morphology
- Probe the ionosphere
- Space science and engineering

Advanced Space Science- upper atmosphere and ionosphere Syllabus

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Chapter 1

Solar-terrestrial relationships

1.1 The solar-terrestrial system

- The Earth's upper atmosphere is ionized by solar radiations

(a) electromagnetic: radio - - X-ray; speed 300000 km/s, traveling time 8.3 min.

(b) particle (corpuscular): solar wind (H^+ and e^- mainly); speed 300-1000 km/s

- Solar terrestrial system: The Sun, the interplanetary medium, and the Earth's magnetosphere, Ionosphere and neutral atmosphere.

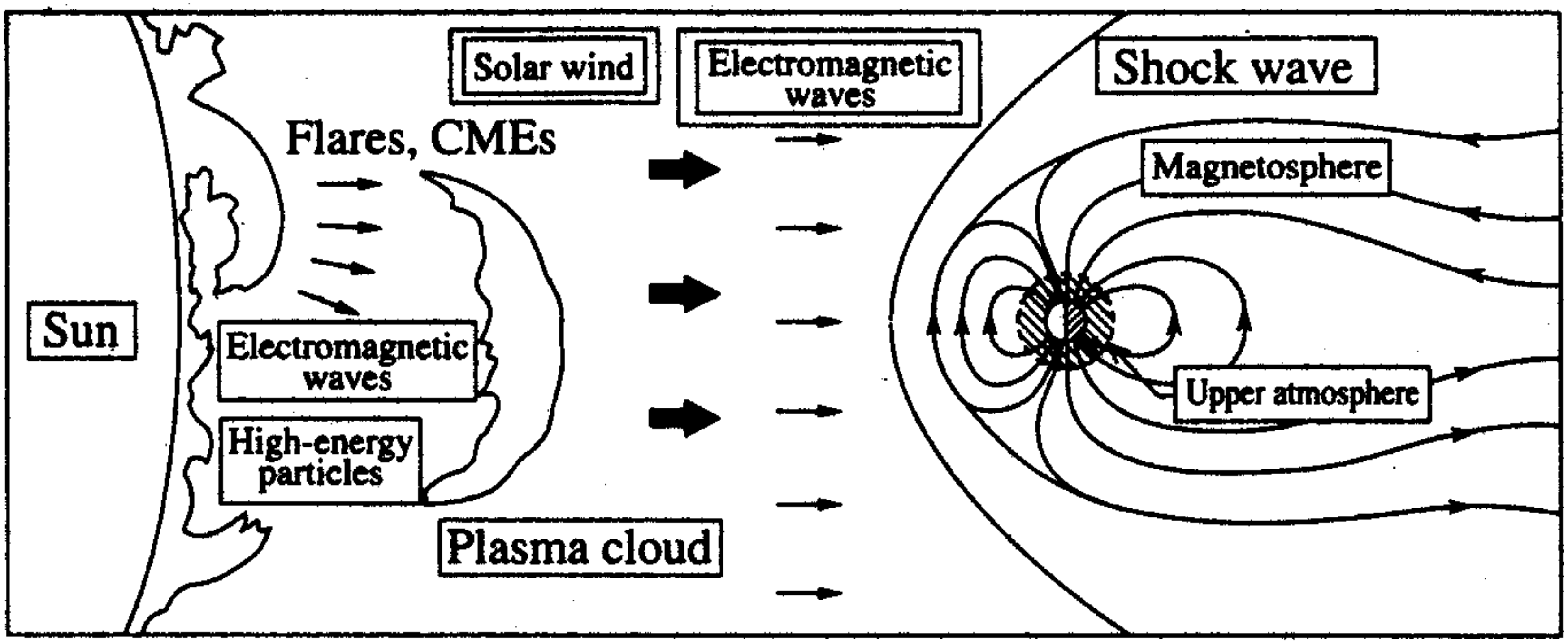


Fig. 1.1 The various forms of solar energy and the space environment

1.2 The Sun

- The quiet Sun
- The Sun: radius Rs=696000 km (~110Re); Ms=330000 Me; photospheres temp=5800 K.
- Solar structure: core 15M K, irradiative interior, convection zone, photosphere, chromosphere, and corona 1M K.
- The Sun’s composition: hydrogen 92.1%; helium 7.8%; oxygen 0.06%; carbon 0.03%; the rest<0.01%.
- The sharp edge results from the photospheric density change.
- Limb darkening
- The Sun radiates as a black body at about 6000 K.
- The flux of visible solar energy varies very little and, however, the emissions at shorter wavelengths, the UV, EUV and X-ray, vary by orders of magnitude depending on sunspot number and solar activity.

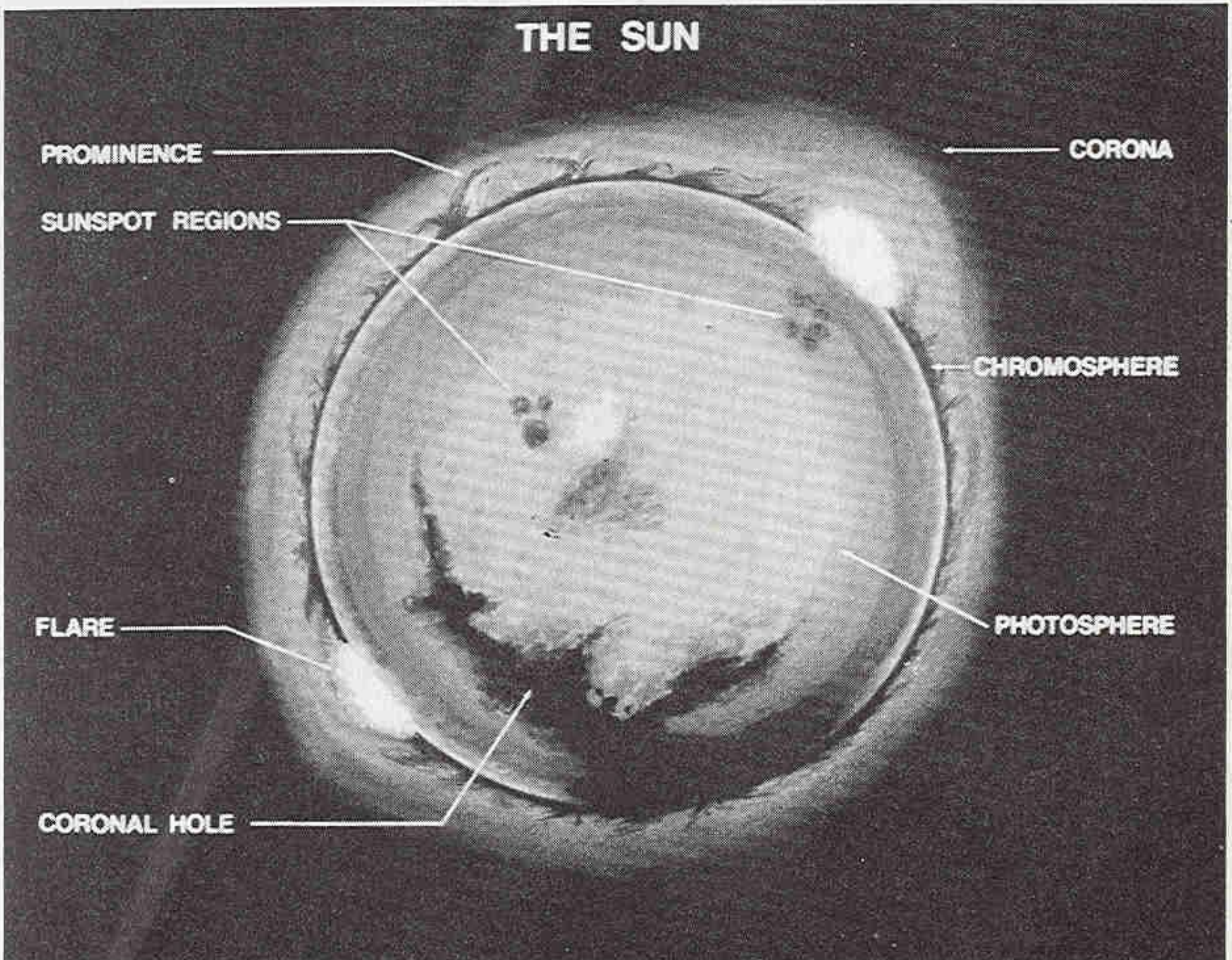
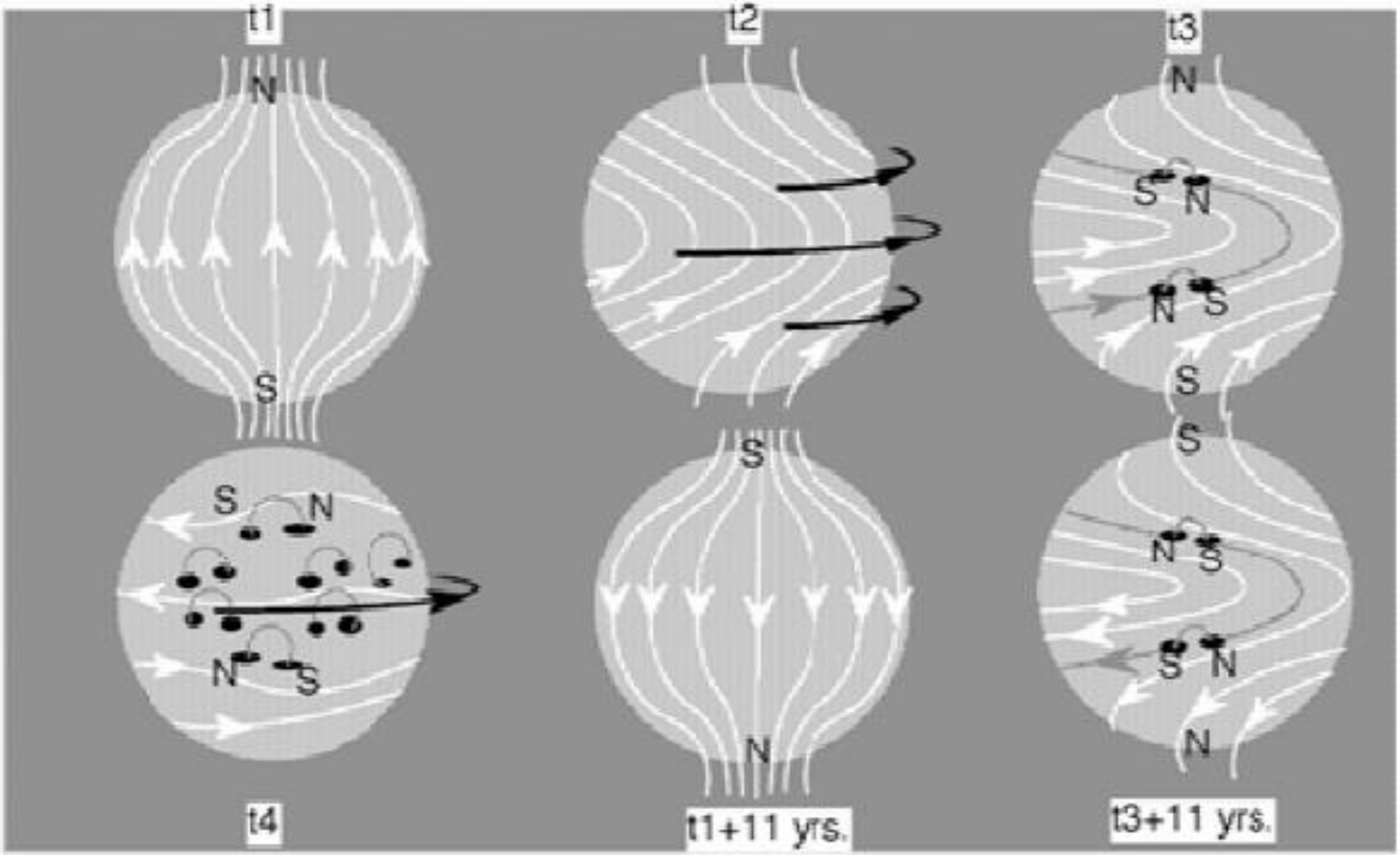
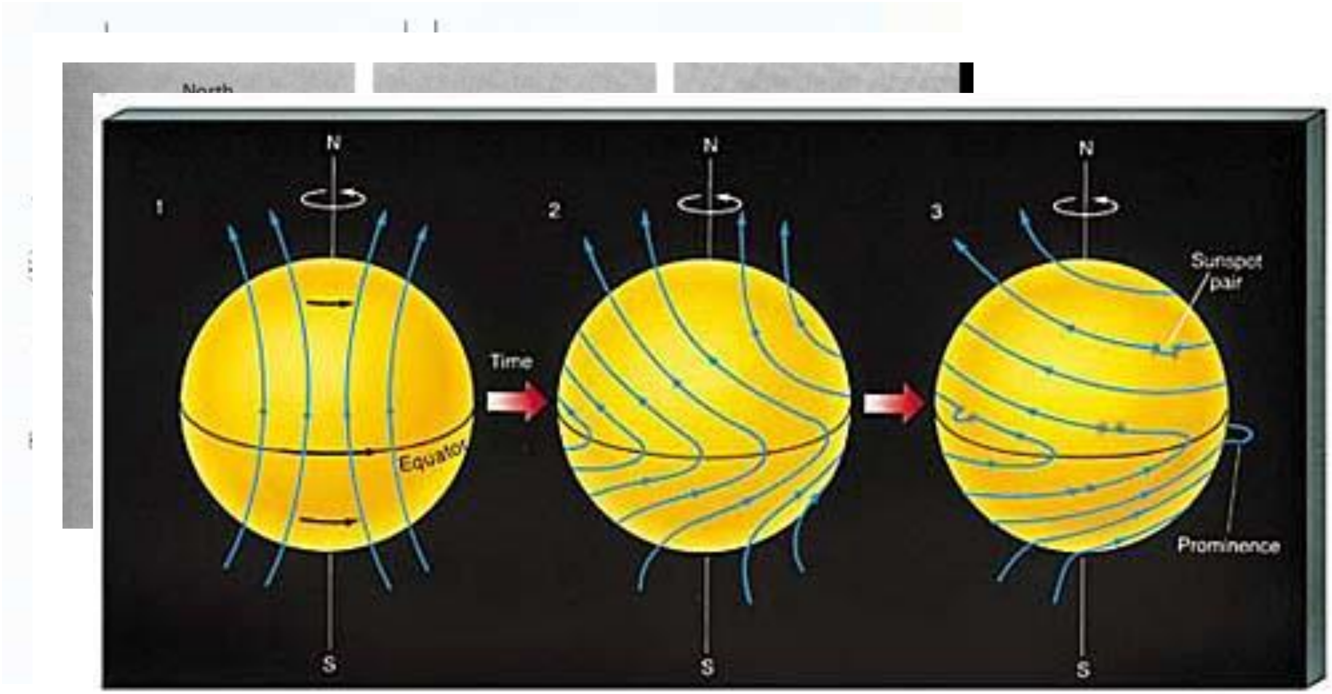
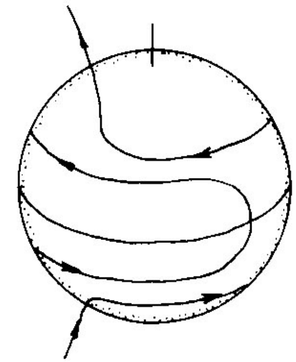


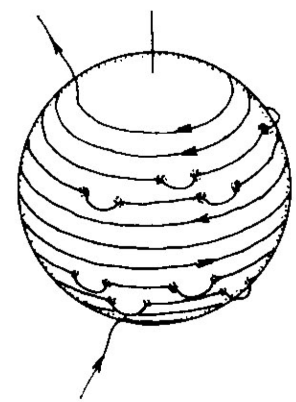
Fig. 2.2 *Solar features (observed at various times on different wavelengths) that can have important consequences on the ionosphere (NASA photograph MSFC–5/80–STE 3826)*


t1
N
S
t2
t3
N
S
N
N
S
S
S
S
N
S
N
N
S
t4
S
N
N
S
N
t1+11 yrs.
t3+11 yrs.

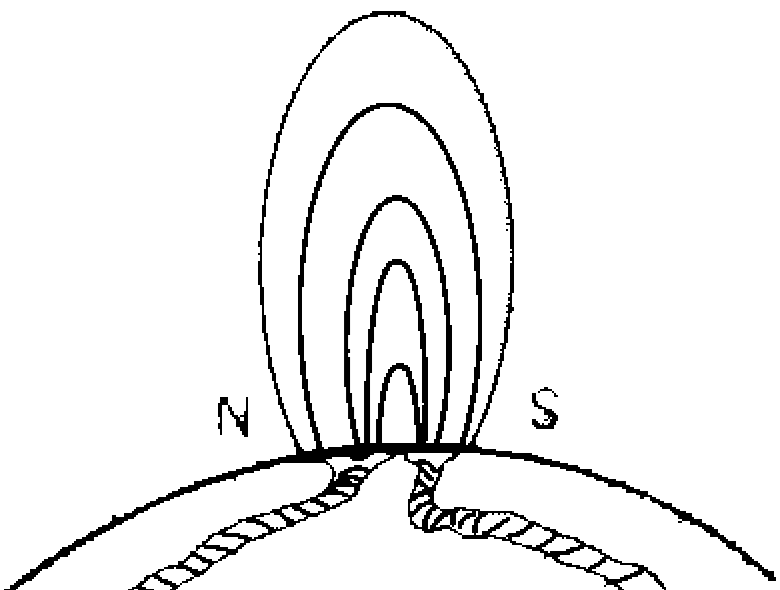
sunspot







(b)



(a)

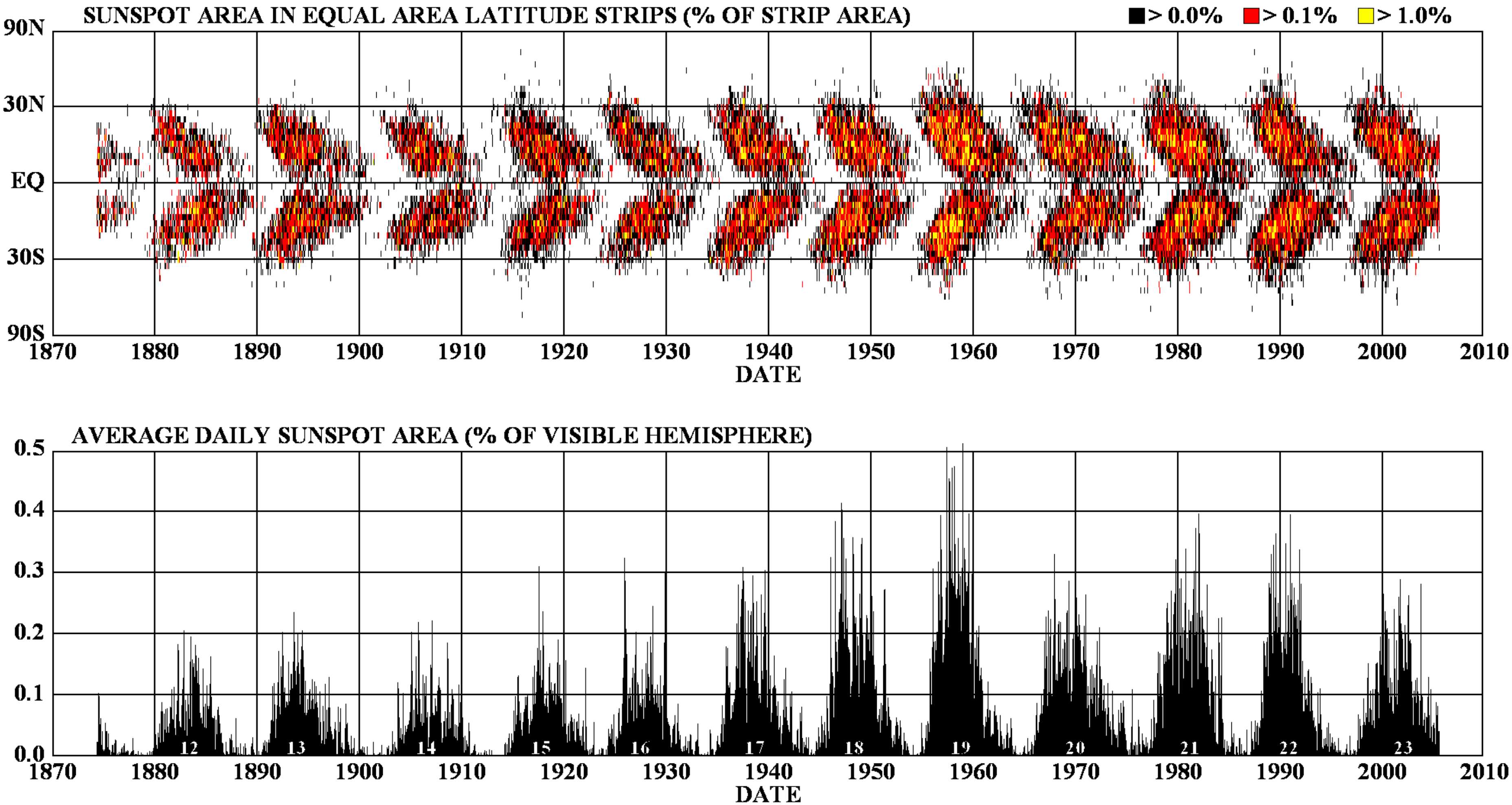
cle.

- Sunspots: low temp ~3000 K, strong magnetic field 0.4 T (4000 G), cycle ~ 11 yrs.
- The period between about 1640 and 1710 is called the Maunder minimum. Solar activity vs. climate.
- The Wolf (or Zurich) sunspot number $R=k(10g+s)$, where g and s are the group and individual spot number, and k denotes the correction factor.
- The spots, after a sunspot min., occur at the latitudes of 20° to 30° north and south whereas at sunspot max. spots occur at ±15° and, as the sunspot number declines, spots occur in the latitudes of 5° and 10°.
- The 12-month smoothed relative sunspot number

$$R_{12} = (0.5R_{-6} + R_{-5} + - - - + R_5 + 0.5\ R_6)/12$$

- Solar activity index: sunspots, F10.7

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS



http://science.msfc.nasa.gov/ssl/pad/solar/images/bfly.gif

NASA/NSSTC/HATHAWAY 2005/09

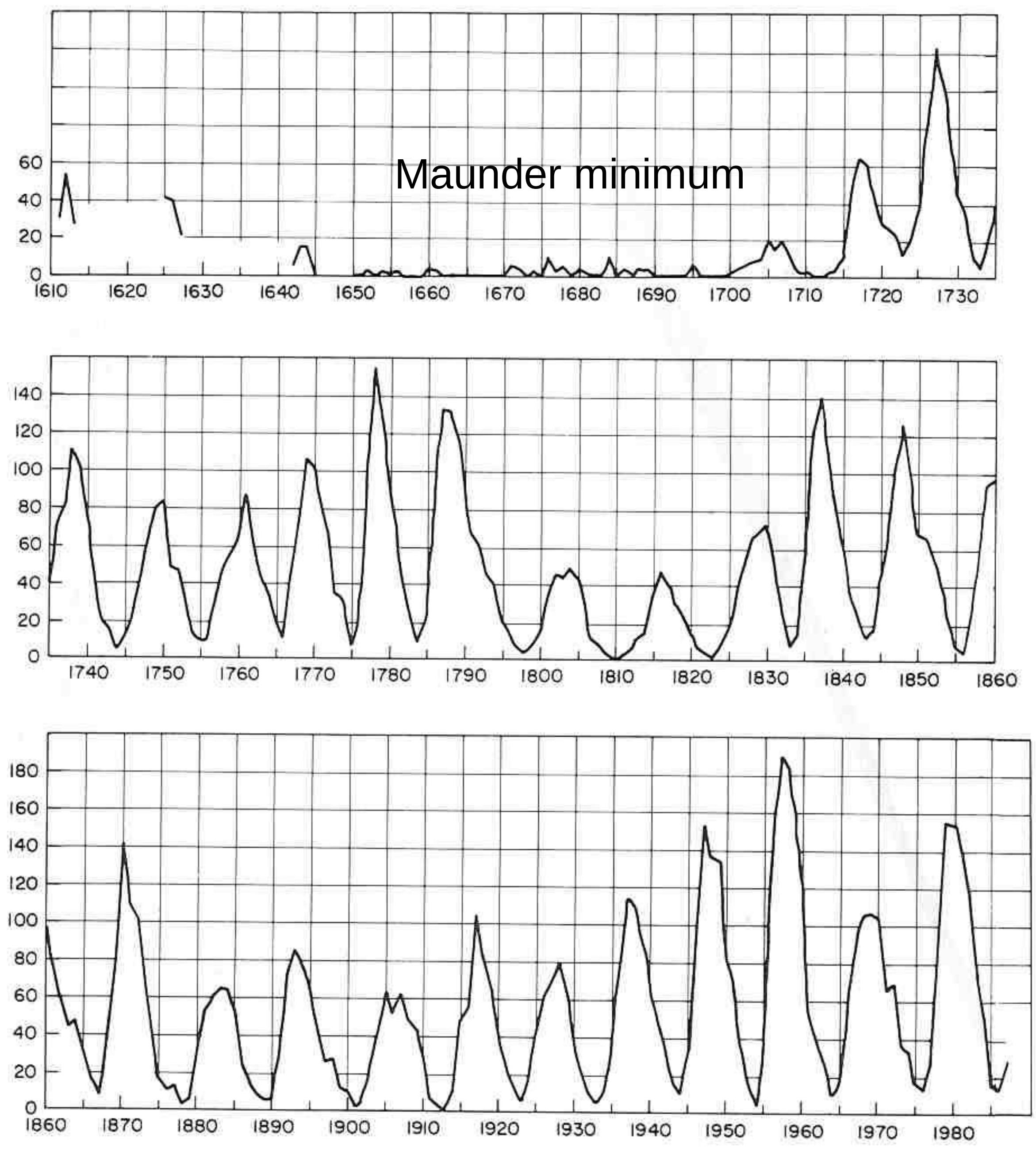


Fig. 2.3 *Yearly sunspot numbers from 1600 to 1980 (Courtesy of J. A. Eddy)*
The period between about 1640 and 1710 is called the Maunder minimum

- plages, spicules, filaments (prominences), corona hole.
- The ionospheric D-region is produced by Lyman α (1216A) and X-rays (<10A). The E-region is formed by X-rays (30-100A), Lyman β (1025.7A) and EUV(910-980A). The F-region is ionized Lyman continuum and band 350-200A.
- Solar X-ray fluxes are measured in W/m^2 and denoted by A, B, C (10^{-6} W/m^2), M (10^{-5}), and X (10^{-4}). X17.2=17.2×10^{-4} W/m^2.
- Solar radio noise is produced by plasma oscillations, cyclotron emissions, and random oscillations of electrons and heavy particles.
- The minimum base level of random oscillations is the thermal emission from the quiet Sun.
- F10.7: the general level of solar activity; the daily full-disk radio noise flux on a wave length of 10.7 cm (2800 MHz).

- The active Sun
- Flare
- A flare is a burst of “light” (easily observed at red Hα 6563A) occurring in the chromosphere near a sunspot. Life time 3 min up to 2 hr.
- Hα-flares are ranked in the size (importance, 1-4) and brilliance (f, n, b).
- X-ray flares are classified by power flux level of 1-8A (C, M, X).
- Radio burst
- Radio bursts are generally associated with flares and originate from all levels of the solar atmosphere.

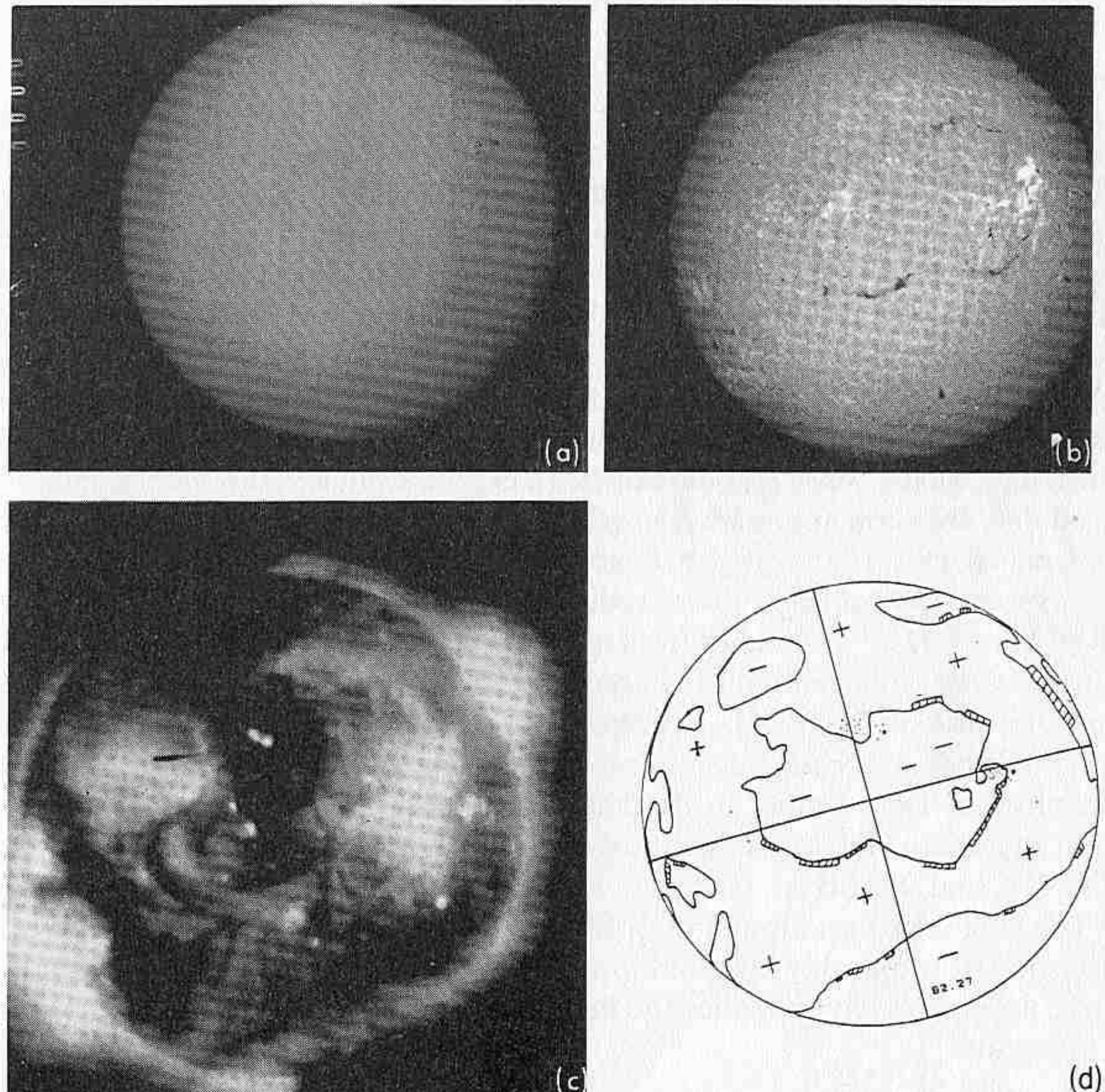
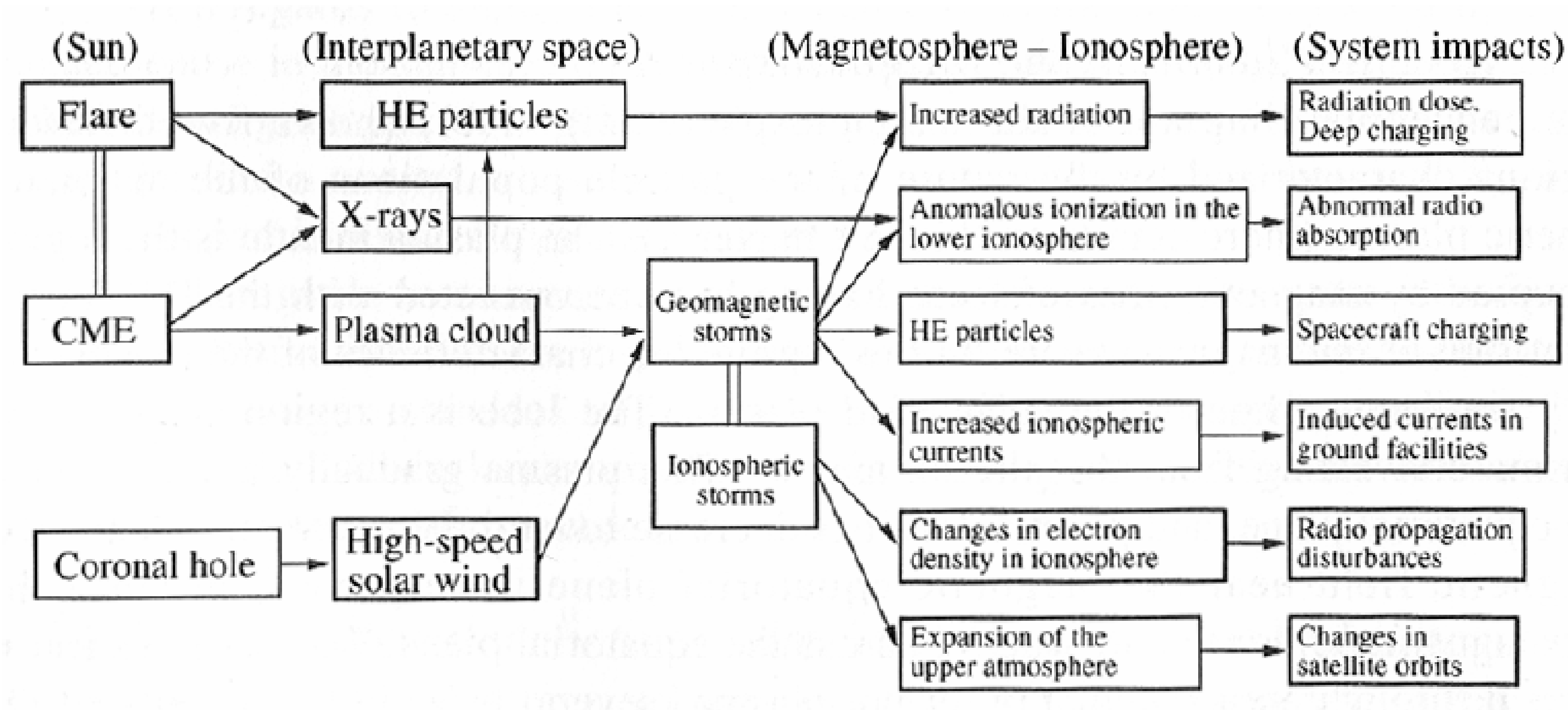


Fig. 2.5 *The Sun seen (a) in white light, (b) in 6563 Å* Hα *light, and (c) in soft X-rays; (d) the Sun's magnetic field (Courtesy of W. Wagner)*
The + sign indicates an outward-pointing field and the − sign indicates an inward-pointing field

Solar energy	Effects on space environment
Steady energy radiation	Control of the structure and dynamics of the atmosphere.
Electromagnetic waves	Formation of the ionosphere, production of ionospheric currents.
Particle radiation (solar wind)	Formation of the magnetosphere, drives magnetospheric convection.
Transient energy radiation	
Electromagnetic waves (flares)	Anomalous ionization of the ionosphere, radio noise.
	Magnetocspheric and ionospheric storms.
Plasma cloud (CME)	Production of high-energy particles.
High-energy particles	Radiation, anomalous ionization in the polar ionosphere.


(Sun)
(Interplanetary space)
(Magnetosphere – Ionosphere)
(System impacts)
Flare
HE particles
Increased radiation
Radiation dose. Deep charging
X-rays
Anomalous ionization in the lower ionosphere
Abnormal radio absorption
CME
Plasma cloud
Geomagnetic storms
HE particles
Spacecraft charging
Increased ionospheric currents
Induced currents in ground facilities
Ionospheric storms
Coronal hole
High-speed solar wind
Changes in electron density in ionosphere
Radio propagation disturbances
Expansion of the upper atmosphere
Changes in satellite orbits

Type	Time to earth	Consequence
Solar flare	8.5 minute	Sudden ionospheric disturbance Short wave (1-30MHz) fadeout
CME	1-2 day	Magnetic storm Ionospheric storm (1-2 day later)
High-energy particle	1-2 hour	Electronic equipment damage

	Type	Rate
Energy radiation from sun	Electromagnetic wave	3.8×10^{26} W/sec
	Solar wind	4.1×10^{20} W/sec
	Explosive event	7×10^{18} W/sec
Rate of mass-lose from sun	Electromagnetic wave	4.2×10^{9} kg/sec
	Solar wind	1.4×10^{9} kg/sec

1.3 The interplanetary medium

- The solar wind
- Solar wind: from corona hole with speed 400-1000 km/s; density near the Earth: 5 pair/cm^3 (5x10^6 proton/m^3); irregularity size: 105 km; Energy: proton 0.5 keV + electron 0.25eV; travel time from Sun to Earth 4.5 days.
- The interplanetary magnetic filed
- The IMF has a spiral form. Toward and away sectors as well as northward and southward directions.

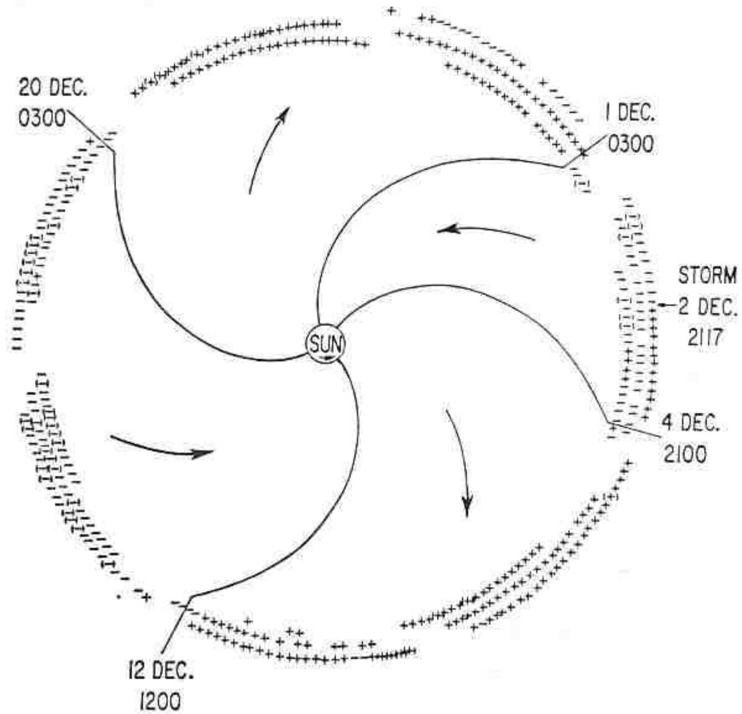


Fig. 2.8 *Spiral structure of the interplanetary magnetic field, showing sectors in which the interplanetary magnetic field points inward and outward (From Wilcox 1968 © Reidel Publishing Co.; reprinted with permission)*

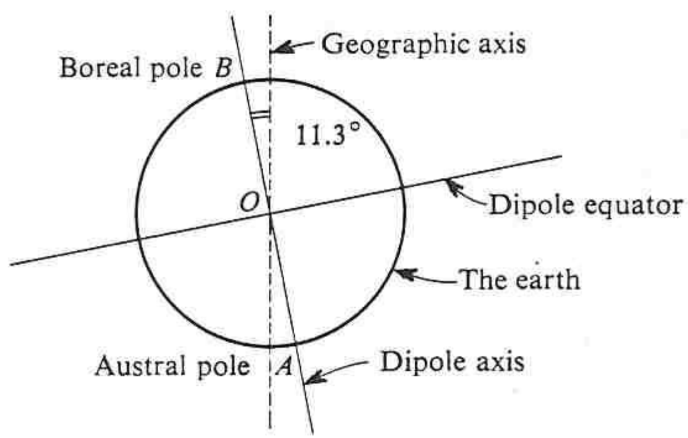


Fig. 2.9 *Earth's dipole field showing the north (B) and south (A) dipole poles and the dipole equator*

- <u>The magnetosphere</u>
- Bow shock, Magnetosheath, magnetopause.
- The magnetosphere: charged particle motion is dominated by forces due to the Earth's magnetic and electric fields.
- Polar cusps or clefts, plasmasphere, magnetospheric tail
- The plasmasphere: the lower part of the magnetosphere, which rotates with the Earth and which contains cold plasma.
- The magnetospheric tail consists of oppositely directly field lines separately by a neutral sheet of nearly zero magnetic filed. Surrounding the neutral sheet is a plasma of hot particles.
- Throughout the inner magnetosphere are trapped particles (radiation belts and ring current). Some of these particles originate in the ionosphere and some come from the solar wind.
- The charged particle motions in the magnetosphere: (1) gyration, (2) bouncing motion, and (3) zonal draft.

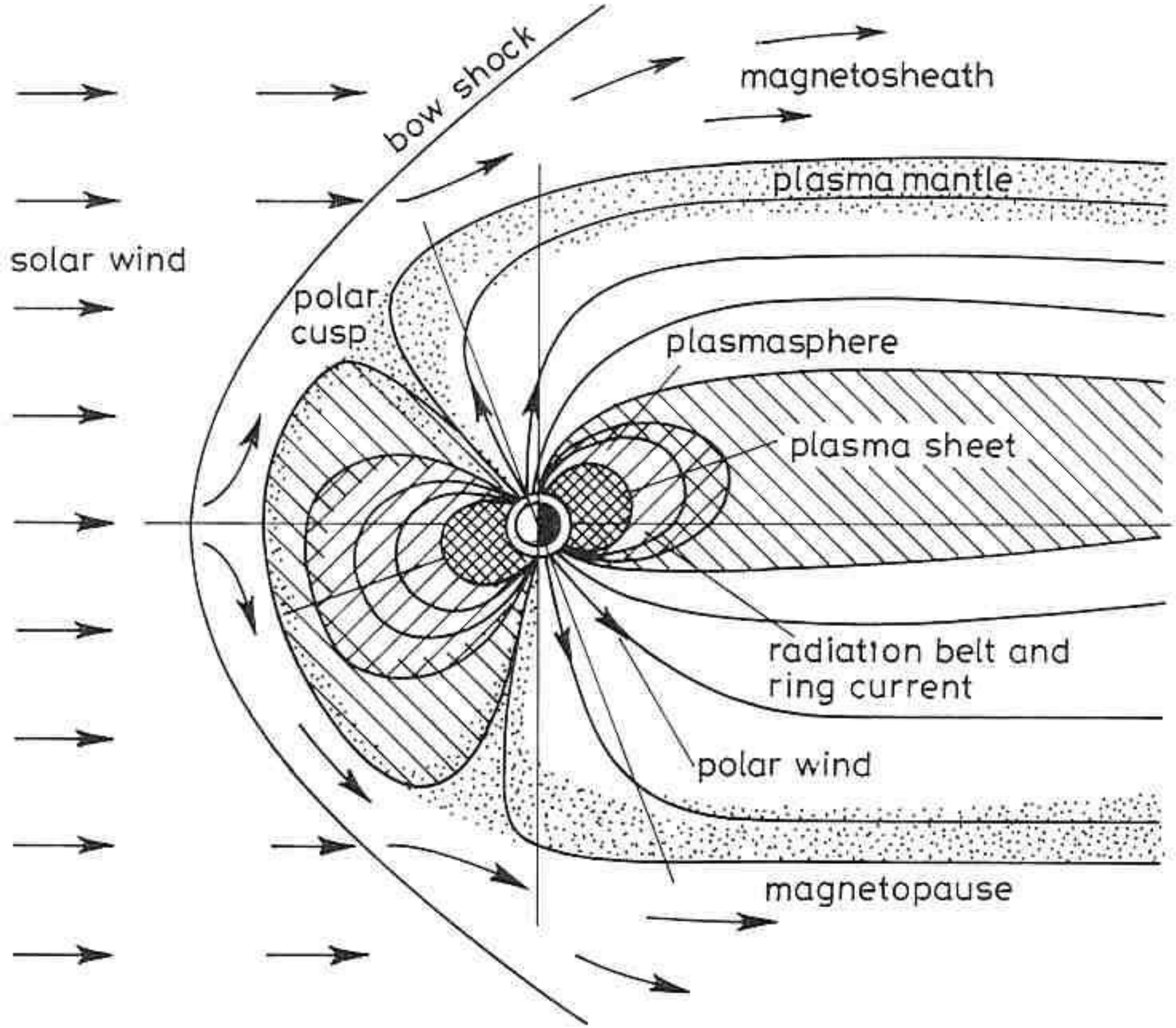


Fig. 2.12 *Magnetosphere formed by the interaction of the solar wind with the Earth's magnetic field*

The magnetic field is compressed on the dayside and drawn out on the nightside

- The pitch angle
- The particle mirror

$$\frac{\sin^2 \alpha}{B} = \text{ a constant} \qquad B_m = \frac{B_{00}}{\sin^2 \alpha_0}$$

- Magnetic variations
- The Earth's magnetic field: (1) diurnal, (2) seasonal, and (3) solar activity variations.
- The net variations at the Earth's surface is the sum of the two parts: (1) that caused by external currents and (2) that produced by induced Earth currents.
- Quiet (q) days and disturbed (d) days
- q day magnetic variations result from the solar daily (*Sq*) variations (mainly) and the lunar daily (*L*) variations.
- The variations are caused by (external) currents flowing above the Earth's surface, mostly in the E-region where the electrical conductivity maximizes.
- The *Sq* currents are stronger by day than by night, stronger in summer than in winter, and about 50% stronger at sunspot maximum than at the sunspot minimum.

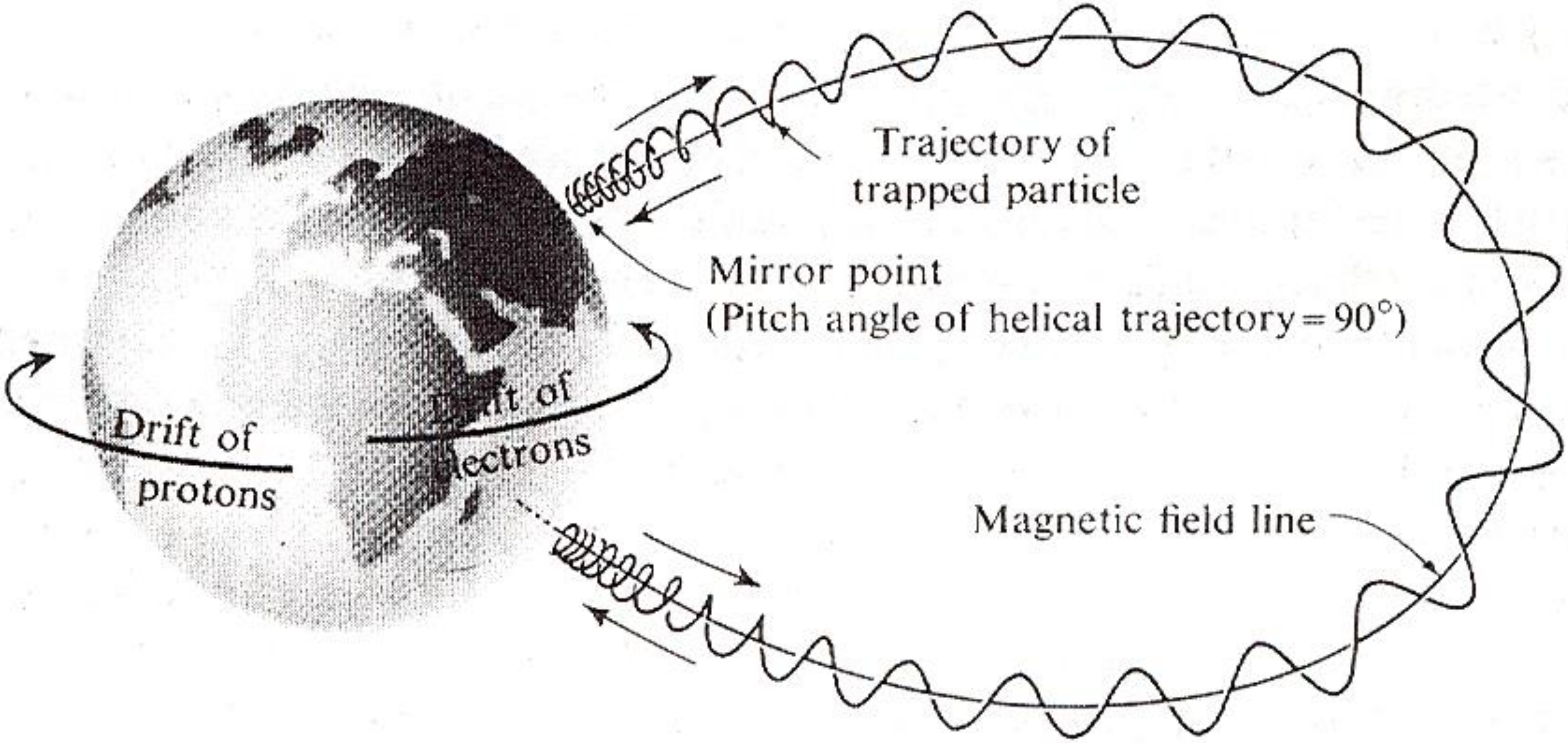


Fig. 2.13 *Motion of a charged particle in a dipole field (From Hess, 1968 © Blaisdell Publishing Co.; reprinted with permission)*
For low energy particles the motion can be divided into three components: (1) spiraling along field lines, (2) bouncing between hemispheres, and (3) drifting in longitude around the Earth, with electrons drifting east and protons west

- Equatorial electrojet: the concentration of enhanced currents in a narrow strip along the magnetic equator.
- The currents during a magnetic disturbance are strong in high latitudes and are often stronger over the night than over the day hemisphere.
- DP (disturbance polar) currents are especially concentrated along the auroral zones (auroral electrojet).
- The storm time is subdivided into an initial positive phase, a main negative phase, and a recovery phase.
- The storm has Universal Time (UT) and Local Time (LT) component. The UT part is called the storm time variation D_{st}, and LT part is named the disturbance daily variation.
- A storm begin either gradually or with an abrupt change call a storm sudden commencement (SSC) caused by a solar wind shock hitting the magnetopause and compressing the magnetic filed.
- Sudden impulses are short-lived disturbances.
- Quasi-sinusoidal oscillations of the magnetic field in the range of 0.2 sec-10 min, are called pulsations
- A short –lived disturbance in the magnetic field is called a substorm or magnetic bay.

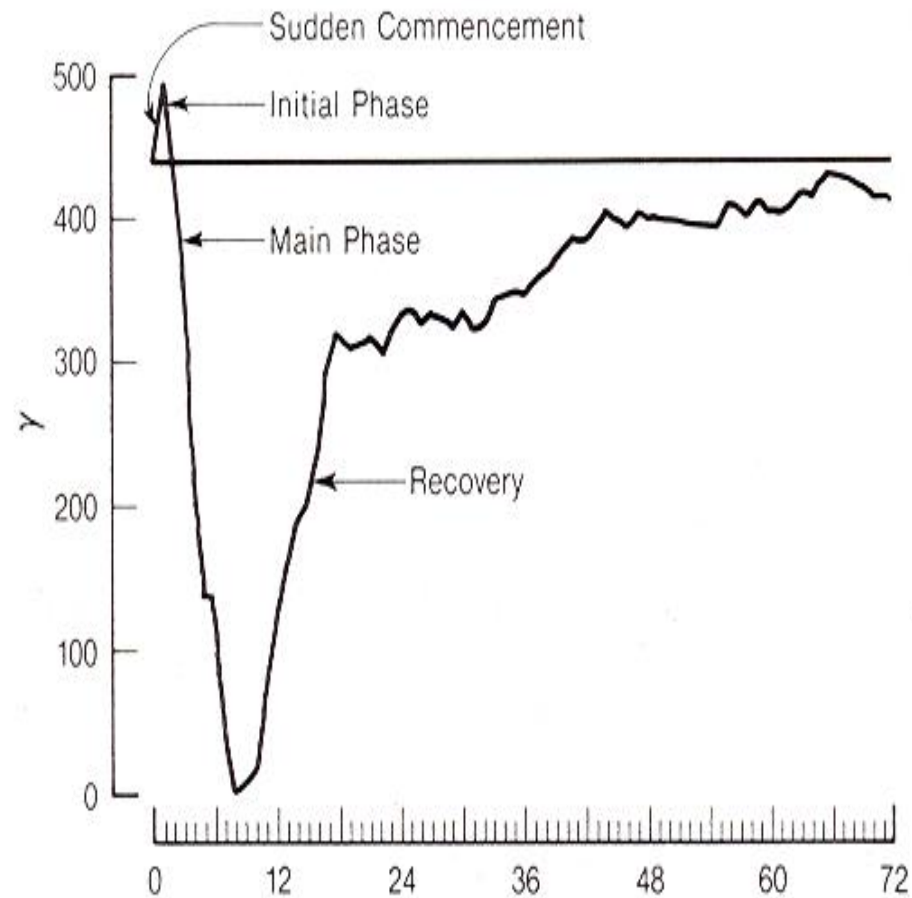
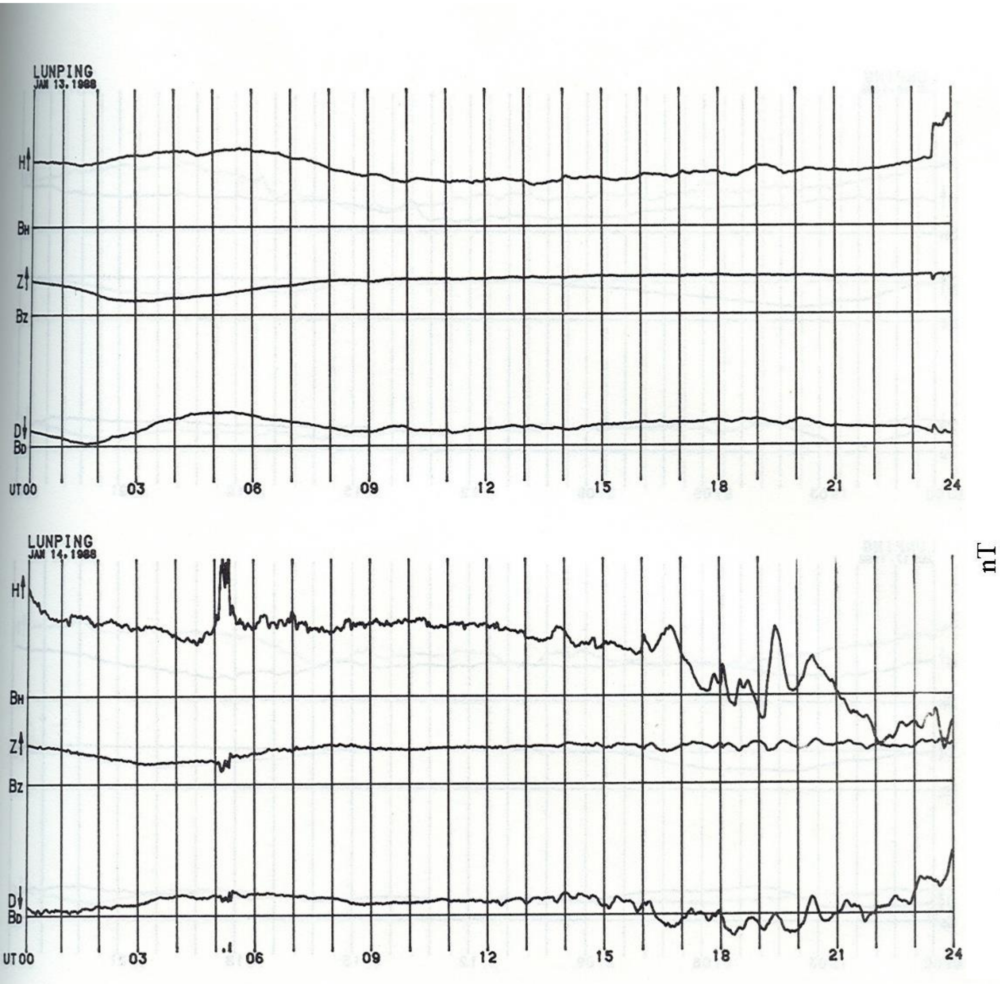
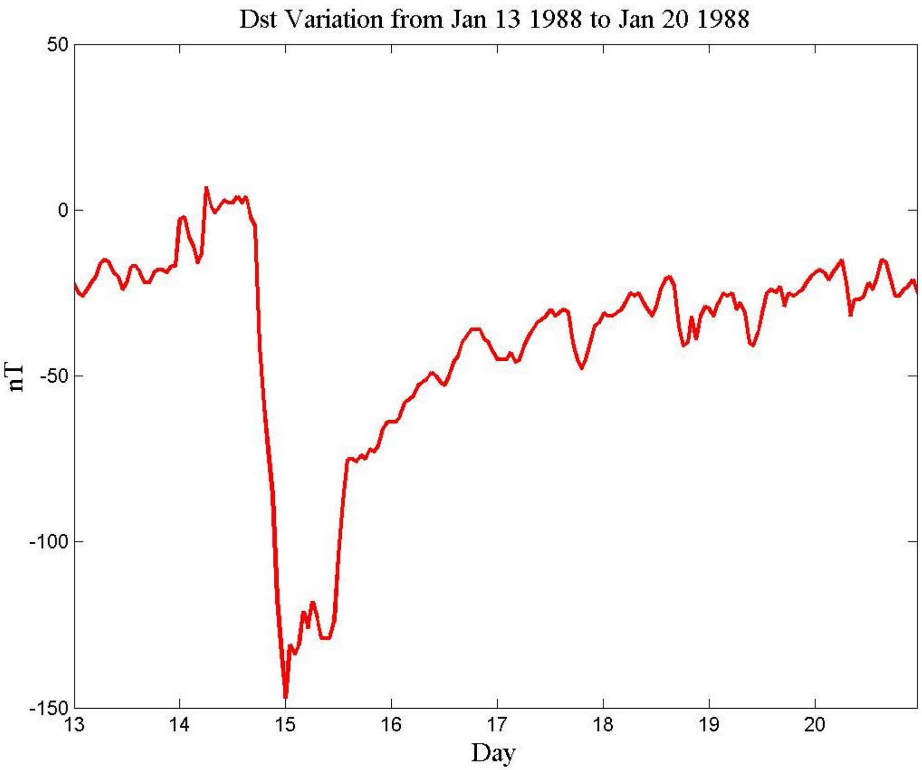


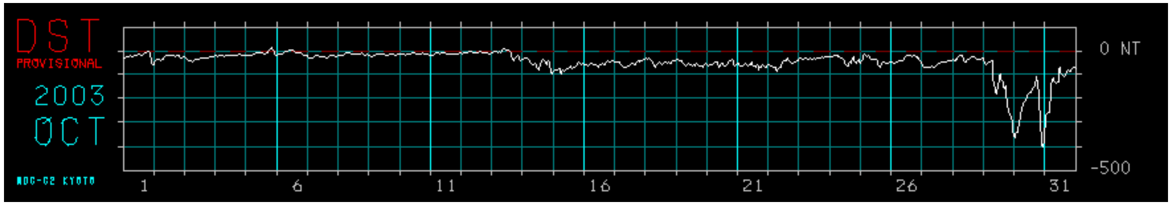
Fig. 2.14 *Magnetic storm variation consisting of a sudden commencement, an initial (positive) phase, a main (negative) phase, and a recovery phase*

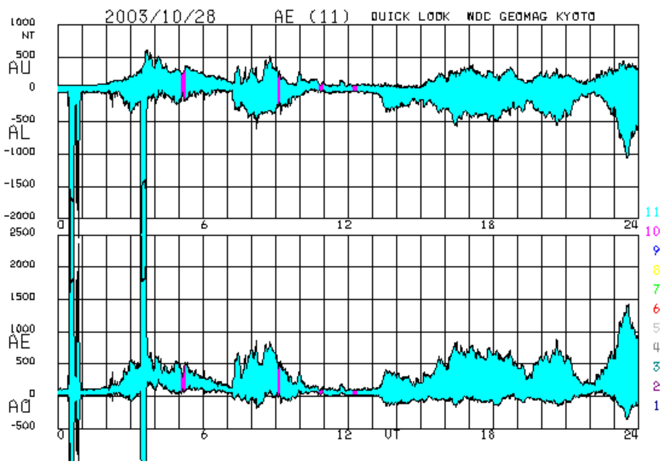
Geomagnetic Index

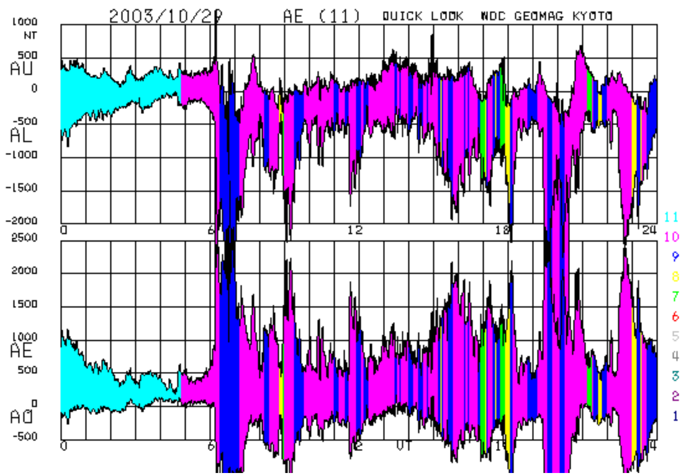


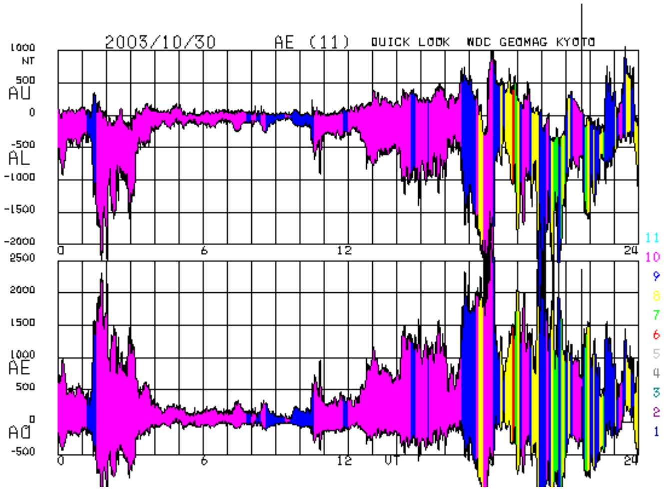


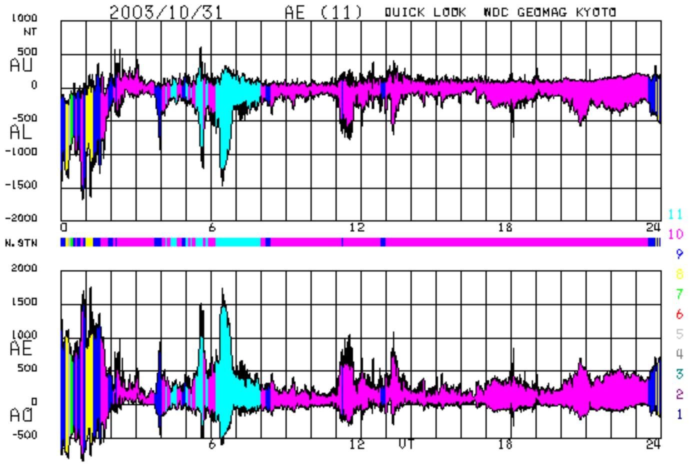
Halloween Storm











0.4 Magnetic indices

- Magnetic indices: local indices and world (global) indices.
- The *K* indices
- The *K* index is a 3-hr range designed to measure the irregular variations associated with magnetic disturbances. Each observatory assigns an integer from 0 to 9 to each of eight 3-hr UT interval (0000-0300, 0300-0600, - - - 2100-2400).
- In assessing *K,* the *Sq, L,* and solar flare effects as well as the after-effect of a disturbance are eliminated.
- Observatories have their own *R* range but have essentially the same *K*.
- The K scale is quasi-logarithmic.
- The planetary 3-hr index K_p is designed to give a global measure of geomagnetic activity. K_p indices are obtained from local variations for 12 stations in geomagnetic latitudes of 63 to 48 in both the Southern and Northern Hemispheres. K_p is an index in 28 grades from 0_0, 0_+, 1_-, 1_0, 1_+, - - - 9_0.

- The *A* index
- The *A* index is a daily index derived from K_p-a_p table. The daily A_p index is the average of the eight a_p values.
- The auroral electrojet index AE
- The index is a measure of the currents flowing in the auroral zone; it is derived from data obtained at stations distributed in latitudinal near the auroral zone. Daily H magnetograms from the stations are superimposed, and upper and lower envelops defined AU and AL respectively. AE = AU –AL; AO=(AU+AL)/2
- The *Dst* index
- The hourly index measures the strength of the ring current, using magnetic field variations near the dipole equator, averaged over local time.
- Quiet and disturbed days

- Selected quiet and disturbed days
- (1) the sum of the eight K_p values, (2) the sum of the squares of the eight K_p values, and (3) the greatest K_p value. The average of the three numbers is taken for each day. The averages are used to find the quiet and disturbed days.

Kp	0o	0+	1-	1o	1+	2-	2o	2+	3-	3o	3+	4-	4o	4+
ap	0	2	3	4	5	6	7	9	12	15	18	22	27	32

Kp	5-	5o	5+	6-	6o	6+	7-	7o	7+	8-	8o	8+	9-	9o
ap	39	48	56	67	80	94	111	132	154	179	207	236	300	400

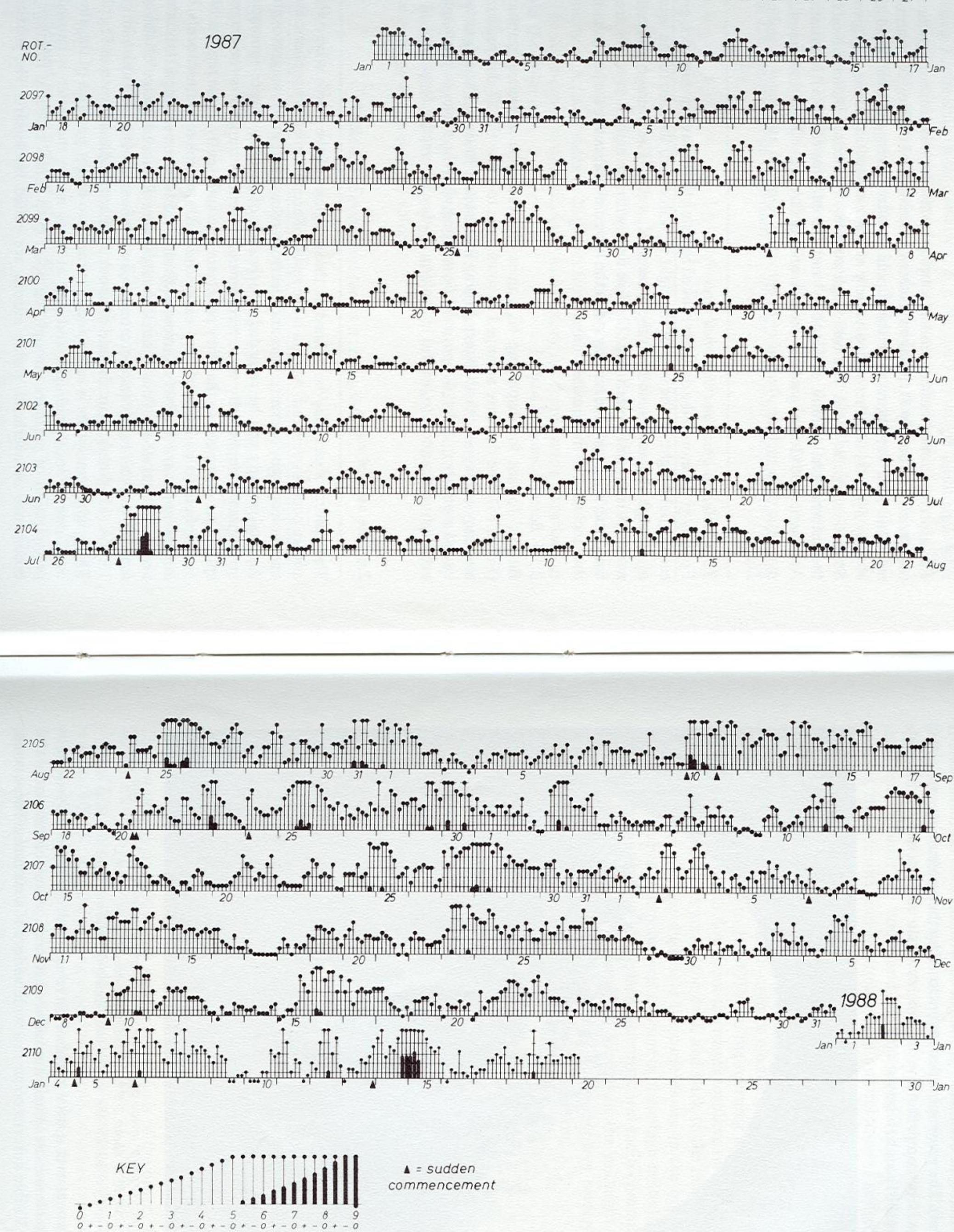


Fig. 2.15 *Bartels' 'musical scales' (By permission of the Institut für Geophysik, University of Göttingen)*
Planetary 3-h-range indices (K_p) are arranged in 27-day rows to show recurring pattern of magnetic disturbance (data for Jan. 1, 1987, to Jan. 20, 1988)

0.5 The auroral zones

- Aurora is the result of excitation of atmospheric atoms and molecules by energetic ions and electrons released from the magnetosphere (magnetosphere-ionosphere-atmosphere coupling).
- The auroral zones: those region where visible aurora occurs overhead, 64°-70°, geomagnetic latitude. The maximum isochasm is located at about 67°.
- The aurora emits over a wide range of wavelengths from radio to X-ray. In general, the intensity and position of the auroral zone is related to geomagnetic disturbance, the latitude of the zone decreasing with increasing magnetic disturbance.
- The more important lines in auroral spectra include oxygen 5577A (green) and 6300A (red), and molecular bands from singly ionized N_2 molecules.
- The most active and brilliant auroras are located in the mid-night sector of the auroral zone.

Fig. 2.16 *Dynamics Explorer image of the quiescent auroral oval taken at 1304 Å and 1356 Å at an altitude of about 3·5 Earth radii (Courtesy of L. A. Frank)*
The bright portions of the auroral oval appear at local midnight and local noon

0.6 The plasmasphere

- The positive ions are predominately protons (H^+) and, hence, the plasmasphere (1000 km altitude to L=6) is practically synonymous with the protonosphere.
- The protons are produced by charge exchange between oxygen ions and hydrogen.

  $O^+ + H \leftrightarrow H^+ + O$
- There is no ion production in the plasmasphere. During daytime, plasma diffuses upward from the ionosphere into the plasmasphere. During nighttime, the diffusion are in the opposite direction.
- Within the plasmasphere the ion temperature is about 1000 K, and the plasma is said to be cold, where outside the plasmapause, the energy is of the order of 1000 eV (10^7 K).

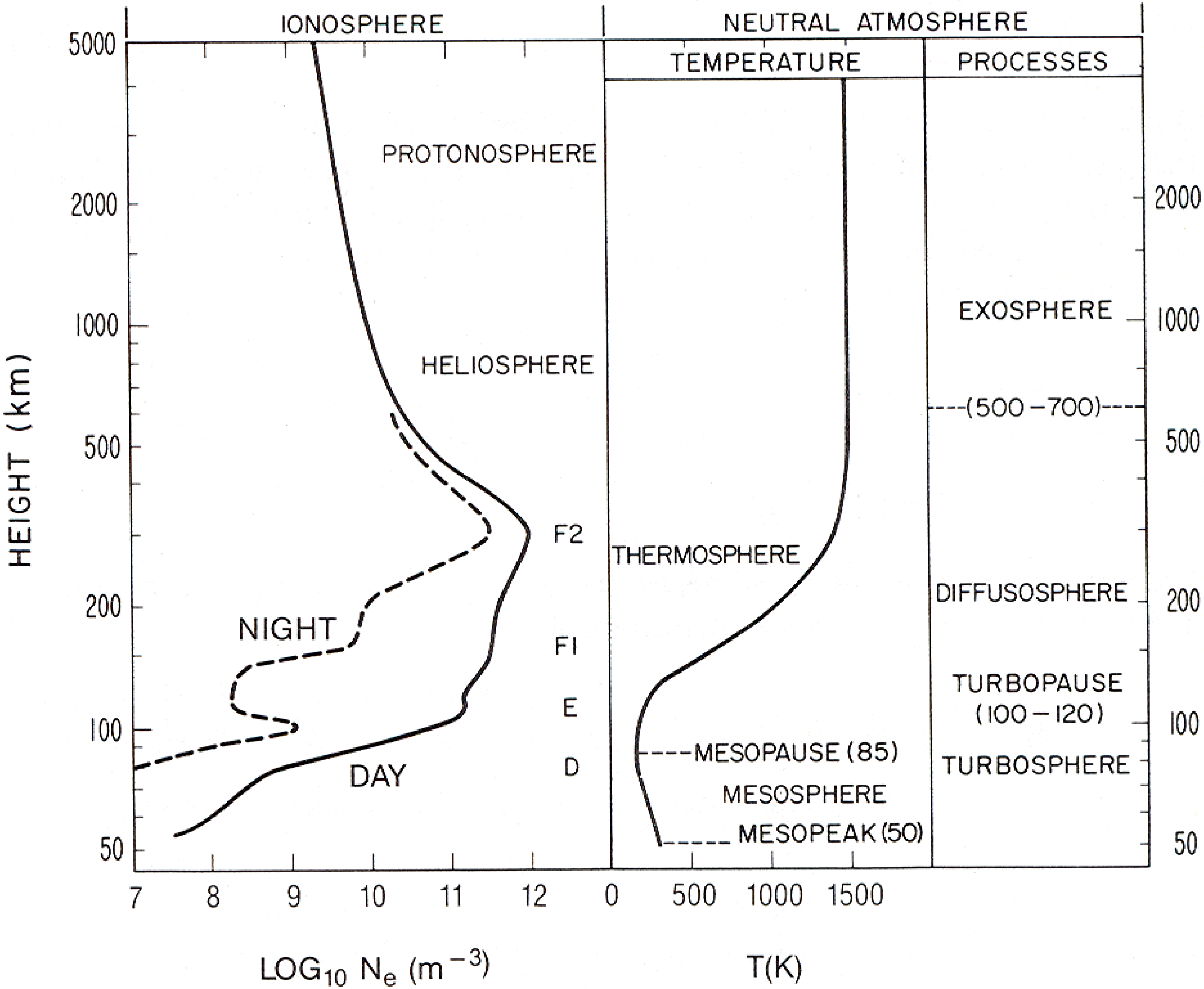


Fig. 2.18 *Atmosphere division by temperature and dynamics*

reaching a mesopause minimum near 85 km above which the temperature steadily increases up through the thermosphere.

2.8.2 Pressure, density, and temperature variations

The relationship between pressure p, mass density ρ, and temperature T, at any height h, is given by the barometric equation

$$p = p_0 \exp\left(-\int_{h_0}^{h} \frac{dh}{H_p}\right) \tag{2.23}$$

and

$$\rho T = \rho_0 T_0 \exp\left(-\int_{h_0}^{h} \frac{dh}{H_p}\right) \tag{2.24}$$

where p_0, ρ_0, and T_0 are the values of p, ρ, and T at the reference height h_0 and

$$H_p = \frac{KT}{\bar{m}g} \tag{2.25}$$

is the pressure scale height, which is a function of the mean molecular mass $\bar{m}$, the acceleration due to gravity g, and temperature T, all of which are height dependent. The scale height varies from about 8 km at the Earth's surface to several hundreds of kilometers in the upper ionosphere.

In an isothermal atmosphere

$$p = p_0 \exp\left(-\frac{h - h_0}{H}\right) \tag{2.26}$$

$$\rho = \rho_0 \exp\left(-\frac{h - h_0}{H}\right) \tag{2.27}$$

From the perfect gas law ($p = nKT$) and $\rho = \bar{m}n$ we have, in a temperature-, density-, and composition-varying atmosphere,

$$-\frac{1}{H_p} = \frac{1}{p}\frac{dp}{dh} = \frac{1}{T}\frac{dT}{dh} + \frac{1}{n}\frac{dn}{dh} = \frac{1}{T}\frac{dT}{dh} + \frac{1}{\rho}\frac{d\rho}{dh} - \frac{1}{\bar{m}}\frac{d\bar{m}}{dh} \tag{2.28}$$

The following formulas are useful in estimating the scale height H in kilometers at any altitude h:

$$H = 0{\cdot}85\left(1 + \frac{h}{a}\right)^2 \frac{T}{M} \tag{2.29}$$

where M is in gram mols, and a (≈ 6370 km) is the mean Earth radius. At 550 km, $H = T/M$, which is correct to within 10% between 200 km and 900 km.

It is often convenient to use a reduced height z defined by

$$z = \int_{h_0}^{h} \frac{dh}{H_p} \tag{2.30}$$

which, for an isothermal atmosphere, becomes

$$z = \frac{h - h_0}{H_p} \tag{2.31}$$

Above 120 km the temperature increases asymptotically to a value T_∞ called the exospheric temperature.

The temperature at a height h can be approximated by curves of the form

$$T = T_\infty - (T_\infty - T_{120}) \exp\left[-s(h - 120)\right] \tag{2.32}$$

where T_{120} is the temperature at 120 km and s is a parameter that depends on T_∞ (CIRA 1972). The exospheric temperatures can vary with time, especially over the sunspot cycle, and with geomagnetic activity. The nighttime minimum of the global exospheric temperature T_c (when $K_p = 0$) is related to the 10·7 cm radio noise flux $F_{10{\cdot}7}$ by

$$T_c = 379 + 3{\cdot}24\bar{F}_{10{\cdot}7} + 1{\cdot}3(F_{10{\cdot}7} - \bar{F}_{10{\cdot}7}) \tag{2.33}$$

where $\bar{F}$ is a long-term average (CIRA 1972). The dependence on geomagnetic activity is

$$\Delta T_\infty = 28K_p + 0{\cdot}03 \exp(K_p) \tag{2.34}$$

where K_p is the 3 hour index. These equations express the fact that the exospheric temperature increases with solar flux (sunspot number) and with magnetic disturbance.

2.8.3 Chemical composition

Below the mesopause (≈ 85 km) turbulent mixing keeps the composition of the atmosphere essentially homogeneous. The percentages of the number densities of the major species (molecular nitrogen, molecular oxygen, and atomic argon) at sea level are as follows:

N_2	O_2	Ar	Sum	$\bar{M}$
78·08	20·95	0·93	99·96	28·96

where $\bar{M}$ is the mean molecular weight.

Above the mesopause, molecular oxygen is dissociated by solar UV with wavelengths shorter than 1759 Å (175·9 nm). This process is so rapid that turbulence cannot keep O_2 and O mixed and, therefore, the number density of O increases rapidly with height to a peak near 100 km. The fraction of O also increases until, above about 150 km, the atmosphere becomes mostly atomic oxygen. Above about 100 km (the turbopause) each constituent obeys a separate barometric equation with its own 'partial scale height'

$$H_i = KT/m_i g \tag{2.35}$$

where i denotes the ith constituent (VanZandt 1967). The structure of the diffusosphere is sensitive to the height of the turbopause, which probably varies diurnally, seasonally, and with solar activity. The height variations of the

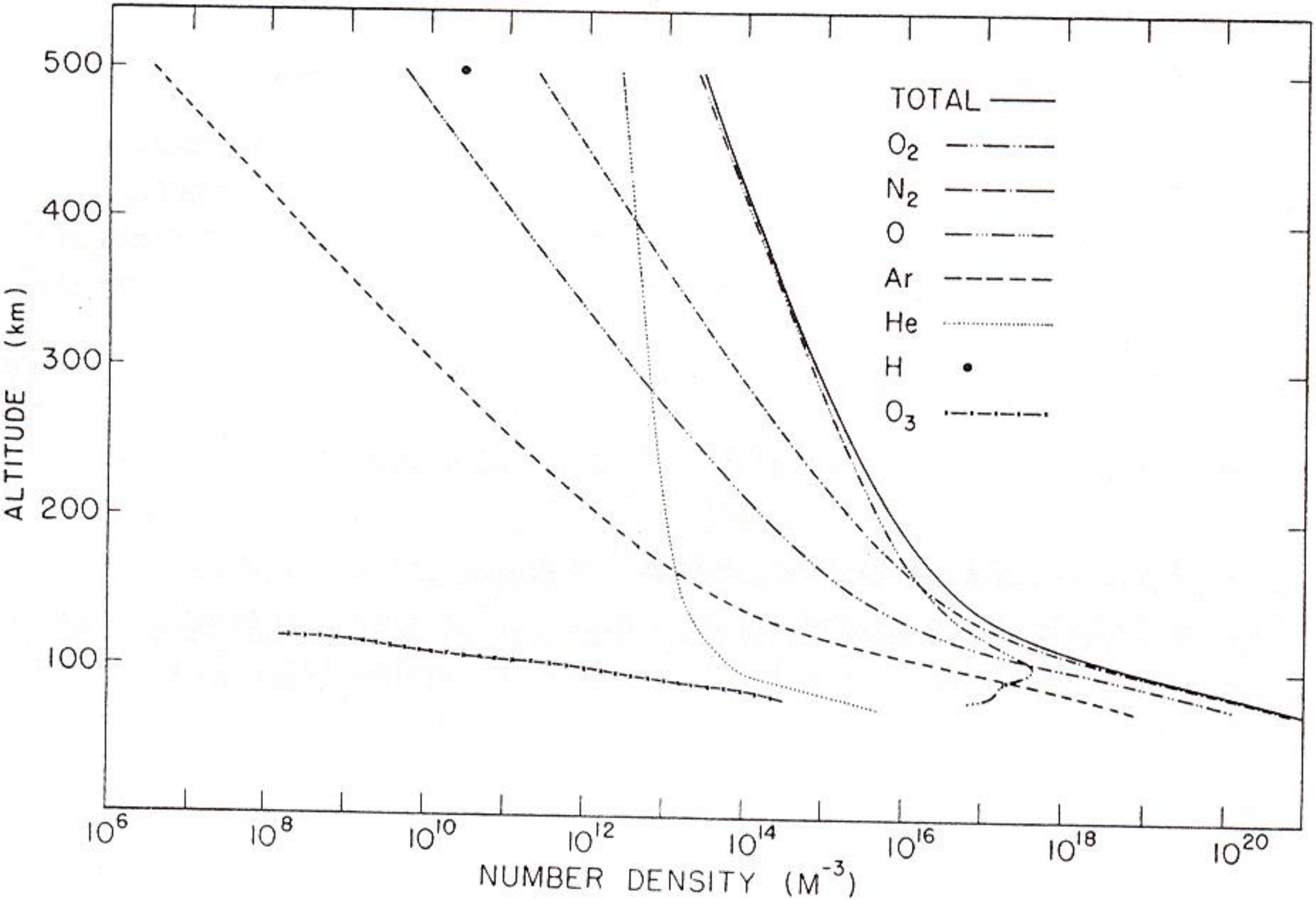


Fig. 2.19 *Height variations of the main atmospheric constituents according to the mean COSPAR International Reference Atmosphere*

2. Physics of Space Plasma

2.2 SINGLE-PARTICLE MOTION

$$\mathbf{F}_L = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \qquad \text{(Lorentz-force law)} \tag{2.1}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} + \mathbf{F}_g \tag{2.2}$$

Let Uniform B, E=0, B in the z direction

$$m\dot{v}_x = qv_yB; \qquad m\dot{v}_y = -qv_xB \tag{2.3}$$

By substitution,

$$\ddot{v}_j = -(qB/m)^2 v_j = -\Omega_c^2 v_j \quad \text{or} \quad \ddot{x}_j = -\Omega_c^2 x_j \tag{2.4}$$

TABLE 2.1. Maxwell's Equations in Different Systems of Units

SI Units	Gaussian Units	
$\nabla \cdot \mathbf{D} = \rho_q$	$\nabla \cdot \mathbf{D} = 4\pi\rho_q$	Poisson's equation
$\dfrac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$	$\dfrac{1}{c}\dfrac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{j} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \dfrac{4\pi}{c}\mathbf{j} + \dfrac{1}{c}\dfrac{\partial \mathbf{D}}{\partial t}$	Ampère's law
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$	Divergenceless **B**
$\mathbf{F} = q[\mathbf{E} + \mathbf{u} \times \mathbf{B}]$	$\mathbf{F} = q[\mathbf{E} + \dfrac{1}{c}\mathbf{u} \times \mathbf{B}]$	Lorentz-force law

Gyro frequency, cyclotron frequency

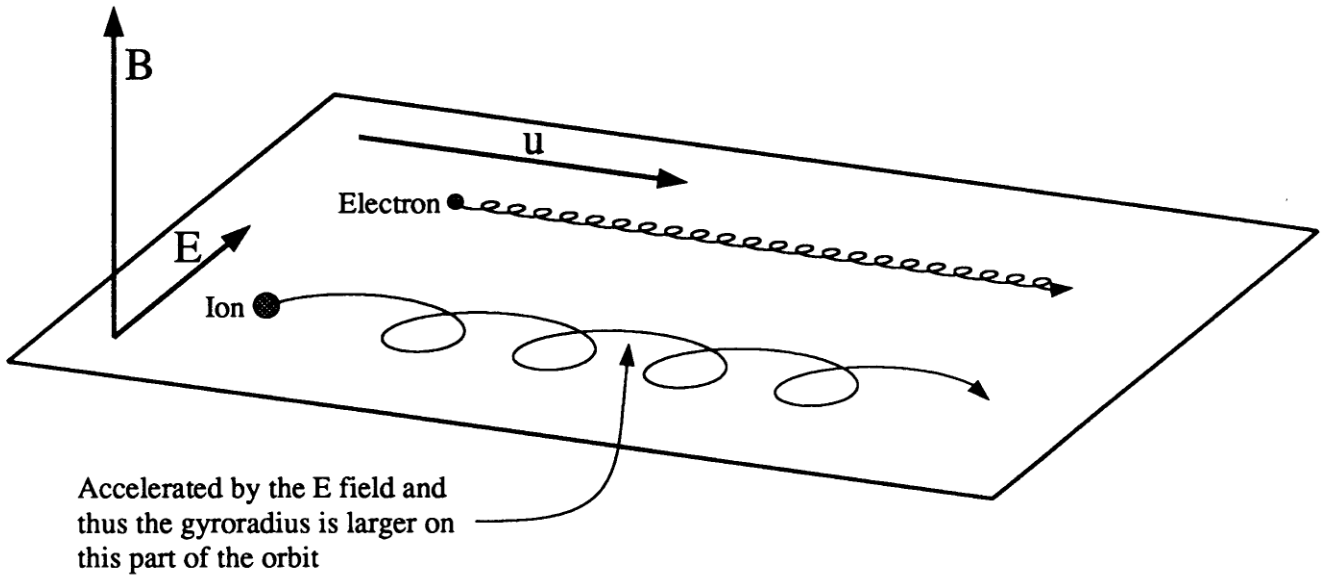
$$\Omega_c = |q|B/m \tag{2.5}$$

Gyro radius, cyclotron radius, Larmor

$$\rho_c = \frac{v_\perp}{\Omega_c} = \frac{mv_\perp}{qB} \tag{2.6}$$

$$m\,\frac{d\mathbf{v}}{dt}\cdot\mathbf{v} = \frac{d(\frac{1}{2}mv^2)}{dt} = q\mathbf{v}\cdot(\mathbf{v}\times\mathbf{B}) = 0 \tag{2.7}$$

FIG. 2.1. Schematic showing the motions of ions (charge e) and electrons (charge $-e$) in a uniform magnetic field **B** in the presence of an electric field **E** perpendicular to **B**. The diagram represents motion in a plane perpendicular to the magnetic field. For both signs of the charge, the motion along the magnetic field is at constant velocity, unaffected by the presence of the fields.



ExB drift No current

$$\mathbf{u}_E = \mathbf{E} \times \mathbf{B}/B^2 \tag{2.8}$$

External force drift Current

$$\mathbf{u}_F = \mathbf{F} \times \mathbf{B}/qB^2 \tag{2.9}$$

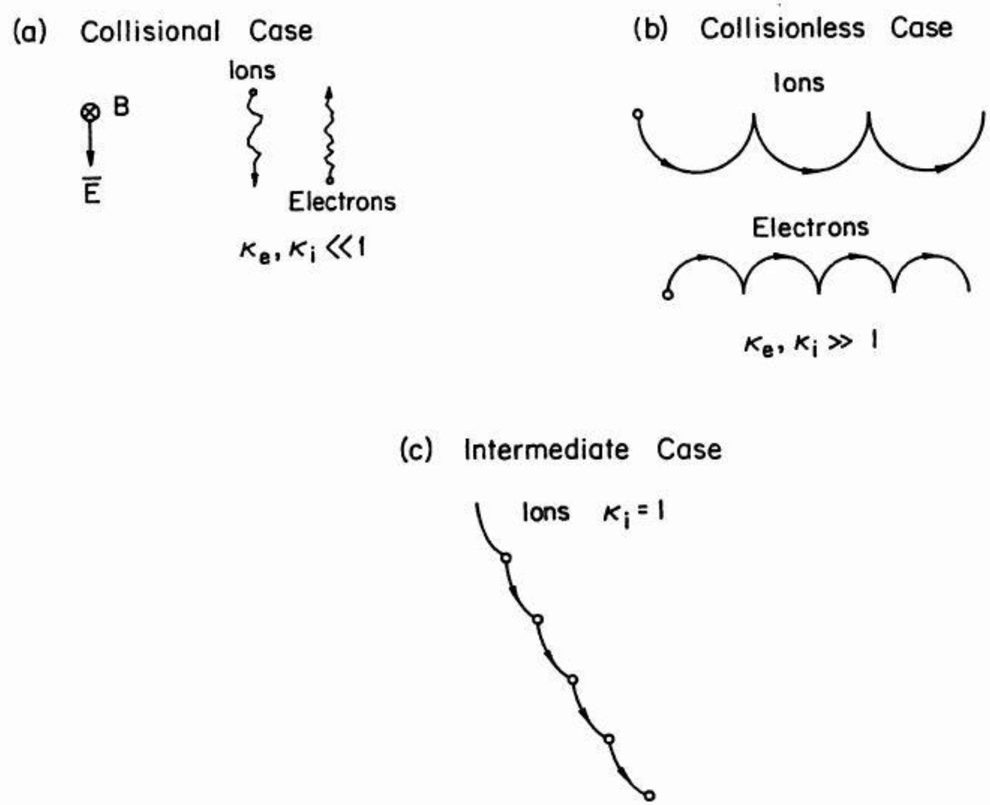


Fig. 2.4. Ion and electron trajectories for various values of κ.

Gradient B drift

$$\mathbf{u}_g = \tfrac{1}{2} m v_\perp^2 \mathbf{B} \times \nabla \mathbf{B} / q B^3 \tag{2.10}$$

$$\frac{\hat{\mathbf{n}}}{R_c} = -(\hat{\mathbf{b}} \cdot \nabla)\hat{\mathbf{b}} \tag{2.11}$$

where $\hat{\mathbf{b}} = \mathbf{B}/B$, and $\hat{\mathbf{n}}$ is a unit vector perpendicular to $\mathbf{B}$ that points away from the center of curvature, one can express the curvature drift velocity $\mathbf{u}_c$ as

$$\mathbf{u}_c = \frac{m v_\parallel^2 \mathbf{B} \times (\hat{\mathbf{b}} \cdot \nabla)\hat{\mathbf{b}}}{q B^2} = -\frac{m v_\parallel^2 \mathbf{B} \times \hat{\mathbf{n}}}{R_c \, q B^2} \tag{2.12}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \text{(Faraday's law)} \tag{2.13}$$

The first adiabatic invariant

μ is called a magnetic moment, $\boldsymbol{\mu} = \frac{1}{2}\mathbf{r} \times \mathbf{j}$

$$\mu = \frac{\frac{1}{2}mv_\perp^2}{B} \tag{2.14}$$

$$\mathbf{F} = \boldsymbol{\mu} \cdot \nabla B = -\mu \frac{dB}{dz} \tag{2.15}$$

The mirror point

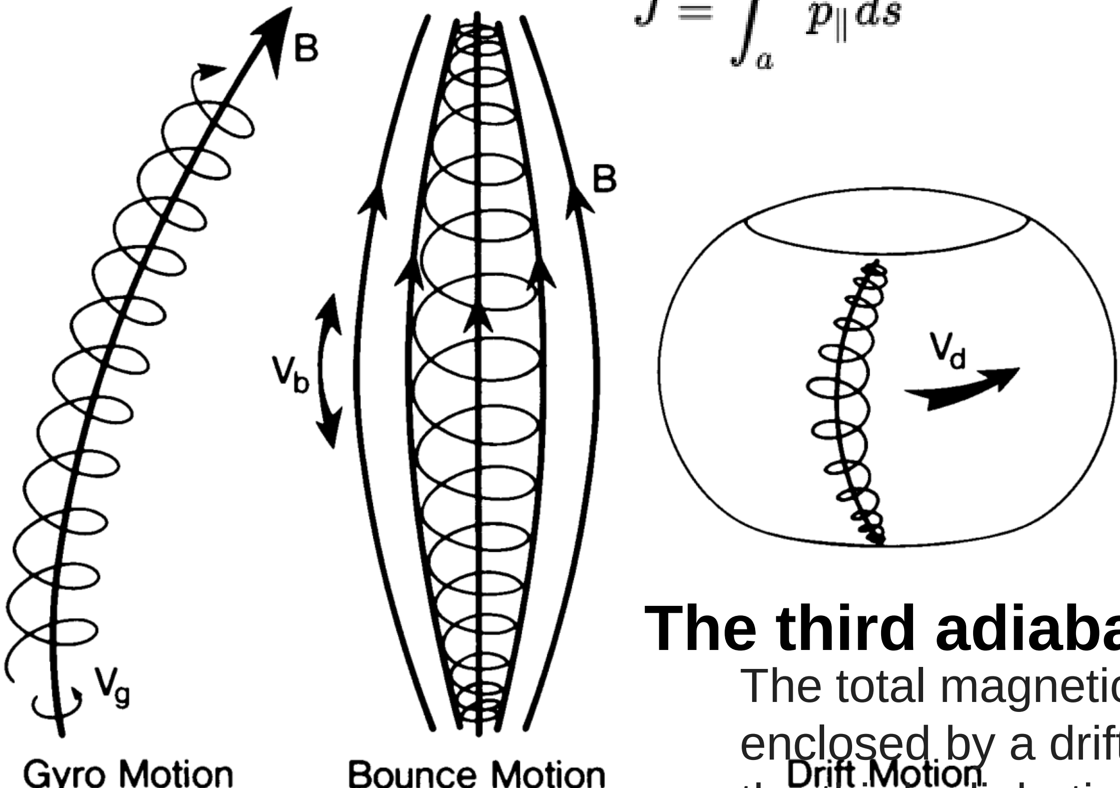
$$B = \tfrac{1}{2}mv^2\mu$$

The first adiabatic invariant

$$\mu = \frac{\frac{1}{2}mv_\perp^2}{B}$$

The second adiabatic invariant

$$J = \int_a^b p_\parallel \, ds$$



The third adiabatic invarian

The total magnetic flux Φ enclosed by a drift surface is the third adiabatic invariant