

# Extracting information from FX Options

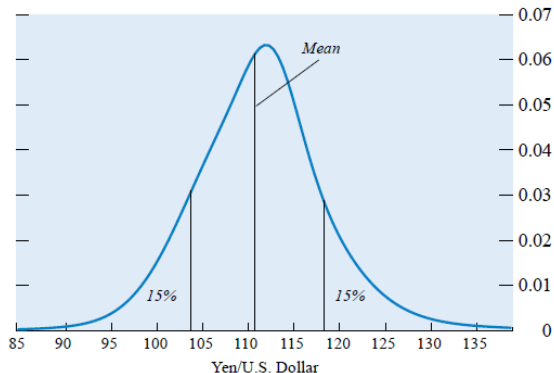
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# Objectives

- We want to build this

**Figure 50. Distribution for Yen-Dollar Exchange Rate in Early September 1997 Implied by Options Prices on May 20, 1997**



## What we will cover

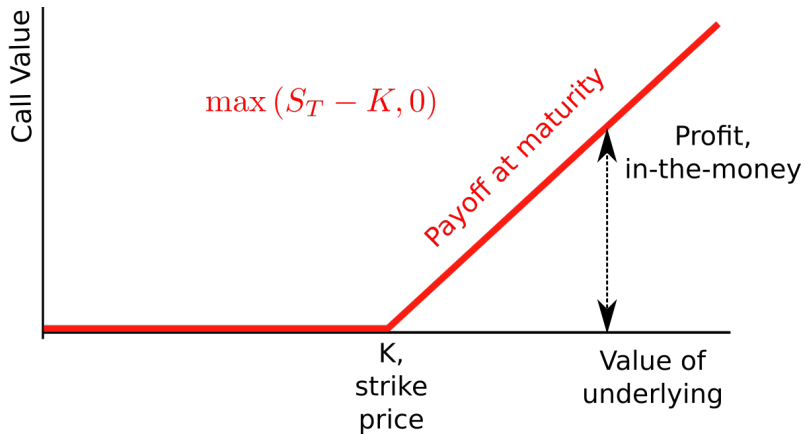
- ▶ Learn about basic FX structures
- ▶ Develop some intuition about FX option prices
- ▶ Get comfortable with a variety of methods
- ▶ Side benefit: learn R and RStudio!
- ▶ Lecture notes available at

[https://jchanlauimf.github.io/IMF\\_FXOptions](https://jchanlauimf.github.io/IMF_FXOptions)

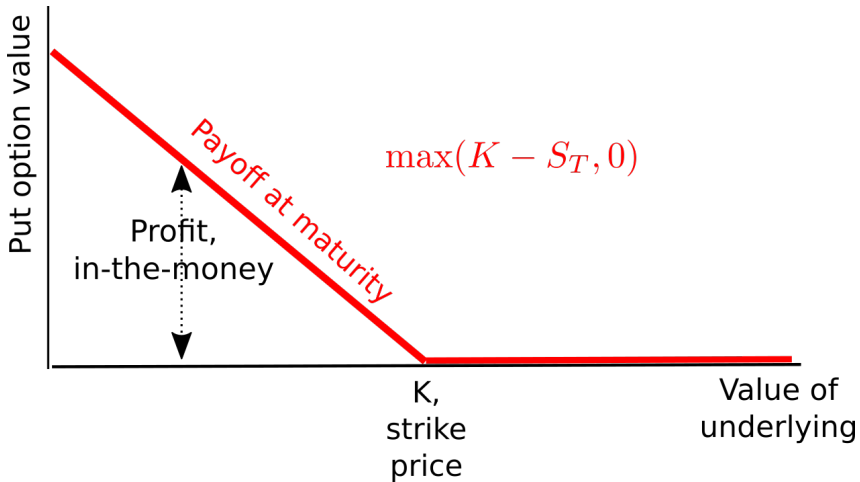
## Part A:

### The Very Basics

## Basics: a plain vanilla call option



## Basics: a plain vanilla put option



# Market reality

- ▶ Price quote available for
  - ▶ ATM plain options
- ▶ Price quotes available for **FX structures**
  - ▶ risk reversals
  - ▶ butterfly spreads
- ▶ Main data sources
  - ▶ Bloomberg
  - ▶ Reuters
  - ▶ Investment banks' data portals

## Part B:

### Eyeballing the Data



## Getting the data in (1)

1. Start RStudio
2. Set the working directory to where we placed the data file
3. Issue this command in the console (replace with your directory)

```
my_wdir = "D:/IMF_FXCourse"  # replace with your directory  
setwd(my_wdir)
```

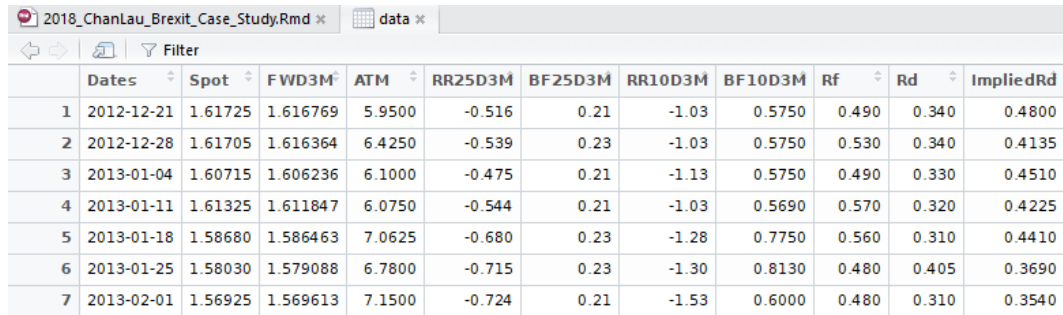
## Getting the data in (2)

Clean memory, set up the needed libraries

```
rm(list=ls())           # Clean up memory
library(ggplot2)        # Graphic library
library(lubridate)       # Date manipulation library
library(dplyr)           # Data manipulation library
source("auxFunctions.R") # Auxiliary functions
```

## Getting the data in (3)

```
filename = "2018_IET_Options_data.csv"
data = read.csv(filename, header=TRUE)
data$Dates = mdy_hm(as.character(data$Dates))
```



	Dates	Spot	FWD3M	ATM	RR25D3M	BF25D3M	RR10D3M	BF10D3M	Rf	Rd	ImpliedRd
1	2012-12-21	1.61725	1.616769	5.9500	-0.516	0.21	-1.03	0.5750	0.490	0.340	0.4800
2	2012-12-28	1.61705	1.616364	6.4250	-0.539	0.23	-1.03	0.5750	0.530	0.340	0.4135
3	2013-01-04	1.60715	1.606236	6.1000	-0.475	0.21	-1.13	0.5750	0.490	0.330	0.4510
4	2013-01-11	1.61325	1.611847	6.0750	-0.544	0.21	-1.03	0.5690	0.570	0.320	0.4225
5	2013-01-18	1.58680	1.586463	7.0625	-0.680	0.23	-1.28	0.7750	0.560	0.310	0.4410
6	2013-01-25	1.58030	1.579088	6.7800	-0.715	0.23	-1.30	0.8130	0.480	0.405	0.3690
7	2013-02-01	1.56925	1.569613	7.1500	-0.724	0.21	-1.53	0.6000	0.480	0.310	0.3540

## The data (1)

### FX related

- ▶ Spot: the GBPUSD spot exchange rate, i.e. USD per GBP
- ▶ FWD3M: the 3-month GBPUSD forward exchange rate
- ▶  $R_f$ : the 3-month GBP money market deposit rate, annualized (in percent)
- ▶  $R_d$ : the 3-month USD money market deposit rate, annualized (in percent)

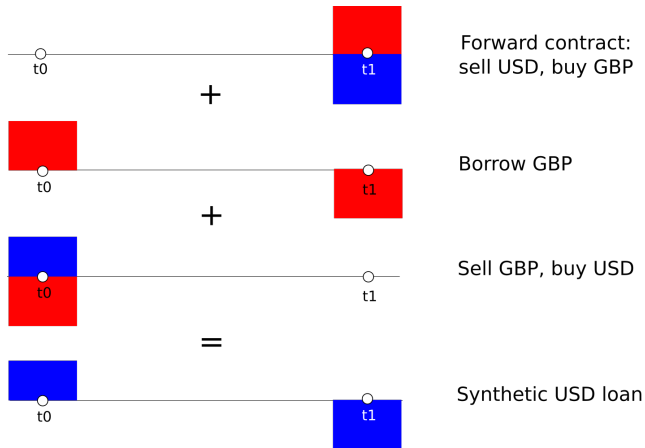
## The data (2)

### FX option related

- ▶ ATM: the at-the-money implied volatility of a GBPUSD option with strike price equal to ATM
- ▶ RR25D3M: the price of a  $25\Delta$  risk reversal, in annualized volatility units (in percent)
- ▶ BF25D3M: the price of a  $25\Delta$  butterfly spread, in annualized volatility units (in percent)
- ▶ RR10D3M: the price of a  $10\Delta$  risk reversal, in annualized volatility units (in percent)
- ▶ BF10D3M: the price of a  $10\Delta$  butterfly spread, in annualized volatility units (in percent)

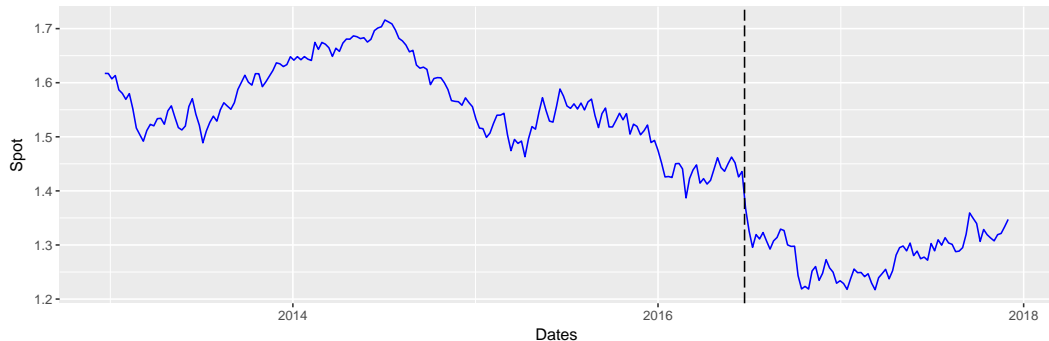
## The data (3)

Implied domestic rate from covered interest parity:  $F = S \frac{\exp(\text{Implied } R_d \times T)}{\exp(R_f \times T)}$



## What the data tell us: spot and forward FX rates (1)

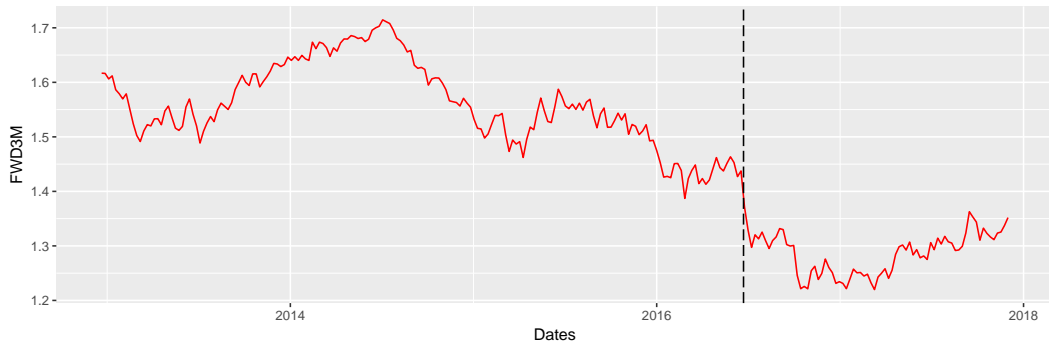
```
# Plot of the spot exchange rate with vertical line at Brexit vote  
p1 = ggplot(data, aes(Dates, Spot)) + geom_line(colour="blue")  
p1 = p1 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),  
                      linetype="longdash")  
p1
```



## What the data tell us: spot and forward FX rates (2)

```
p2 = ggplot(data, aes(Dates, FWD3M)) + geom_line(colour="red")  
p2 = p2 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),  
                      linetype="longdash")
```

p2

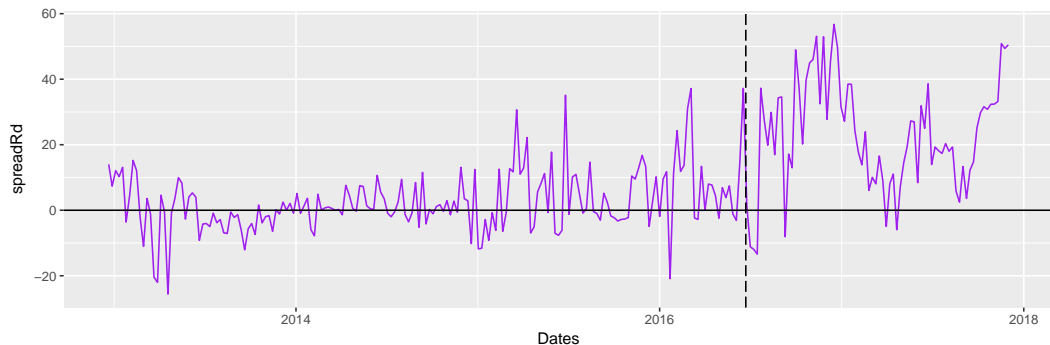




## What the data tell us: domestic rate differentials

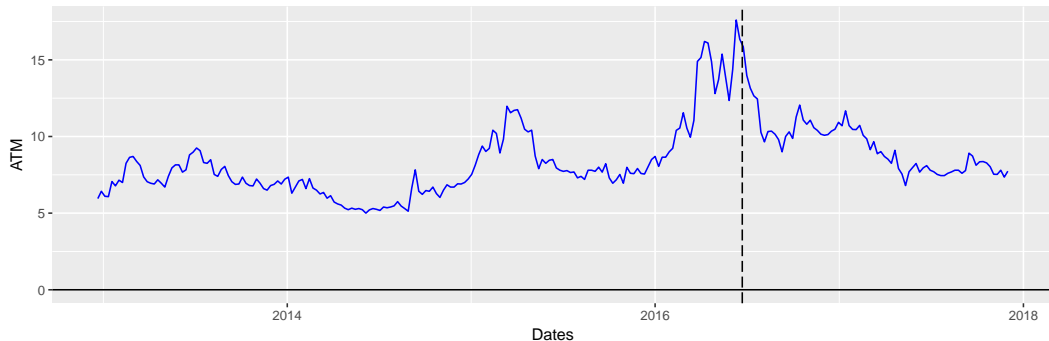
```
data$spreadRd = (data$ImpliedRd - data$Rd)*100  
p3 = ggplot(data, aes(Dates, spreadRd)) + geom_line(colour="purple") +  
      geom_hline(yintercept=0)  
p3 = p3 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),  
                     linetype="longdash")
```

p3



## What the data tell us: ATM volatility

```
p4 = ggplot(data, aes(Dates, ATM)) + geom_line(colour="blue") +  
  geom_hline(yintercept=0)  
p4 = p4 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),  
  linetype="longdash")  
p4
```



# What the data tell us: risk reversals (1)

## Risk reversal (measure of vol-skew) [\[ edit \]](#)

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Risk reversal can refer to the manner in which similar out-of-the-money call and put options, usually [foreign exchange options](#), are quoted by [finance](#) dealers. Instead of quoting these options' prices, dealers quote their [volatility](#).

$$R_{25} = \sigma_{call,25} - \sigma_{put,25}$$

In other words, for a given maturity, the 25 risk reversal is the vol of the *25 delta call* **less** the vol of the *25 delta put*. The *25 delta put* is the put whose strike has been chosen such that the delta is -25%.

Figure 3: Risk reversal definition (Wikipedia)

## What the data tell us: risk reversals (2)

- ▶ Full understanding of the risk reversal requires knowing what  $\Delta$  is
- ▶ But without knowing, we still use risk reversal quotes to assess the prices market participants place on potential exchange rate movements.

$$RR_{25\Delta} = \sigma_{25\Delta C} - \sigma_{25\Delta P}$$

- ▶ Pay  $\sigma_{25\Delta C}$  for owning the call
- ▶ Offset cost somewhat by selling the put at  $\sigma_{25\Delta P}$

## What the data tell us: risk reversals (3)

- Risk reversal payoff, long a call and short a put, both of them OTM:

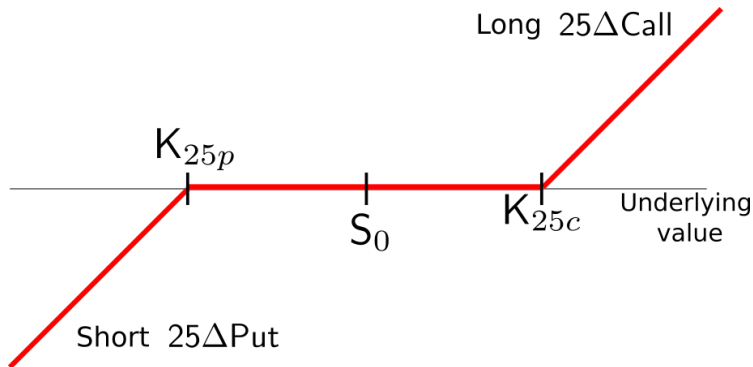


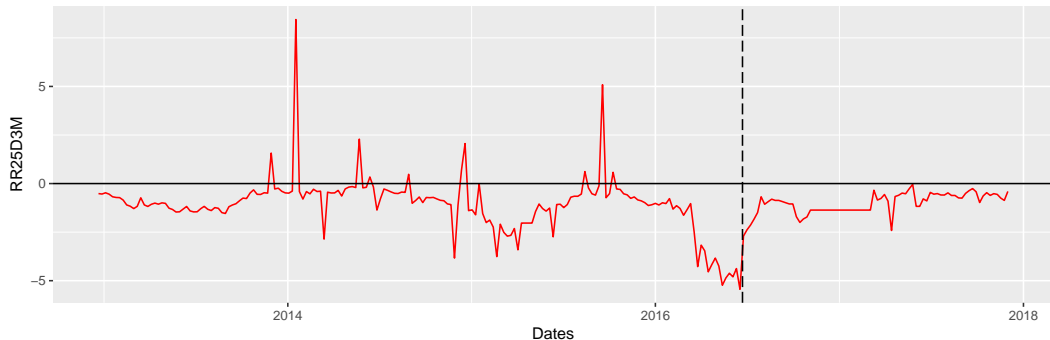
Figure 4: Risk reversal payoff at maturity

- Explain why the price could be positive (or negative)?

## What the data tell us: risk reversals (4)

```
p5 = ggplot(data, aes(Dates, RR25D3M)) + geom_line(colour="red") +  
  geom_hline(yintercept=0)  
p5 = p5 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),  
  linetype="longdash")
```

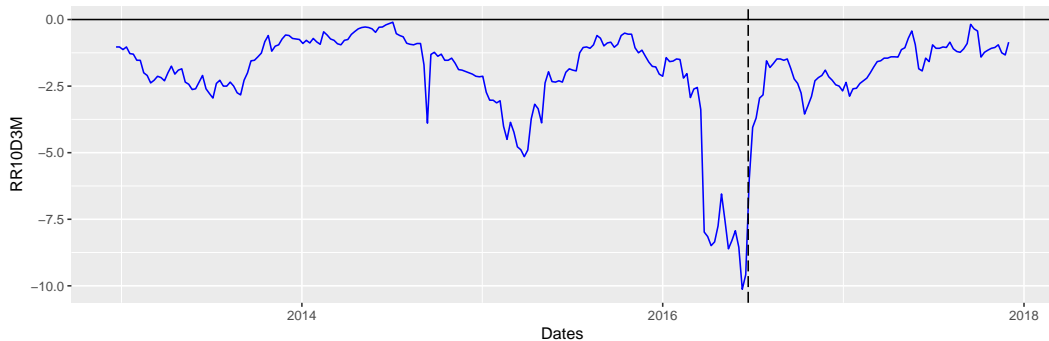
p5



## What the data tell us: risk reversals (5)

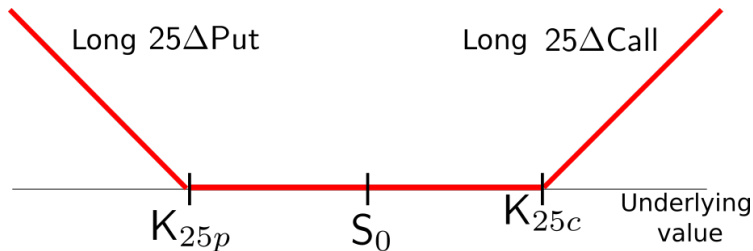
```
p6 = ggplot(data, aes(Dates, RR10D3M)) + geom_line(colour="blue") +  
  geom_hline(yintercept=0)  
p6 = p6 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),  
  linetype="longdash")
```

p6



## What the data tells us: butterfly spreads (1)

- ▶ Suppose we want to profit from large movements of the exchange rate
- ▶ The strangle would make our day !!





## What the data tells us: butterfly spreads (2)

- ▶ The cost of the strangle is:

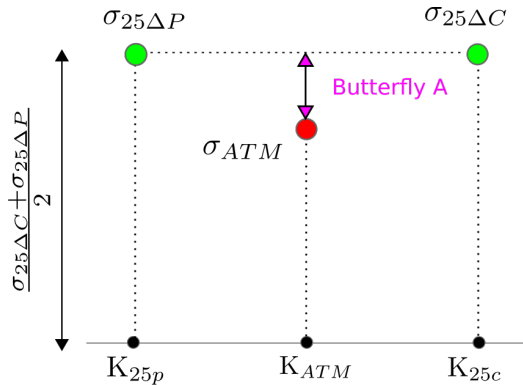
$$S_{25\Delta} = \sigma_{25\Delta C} + \sigma_{25\Delta P}$$

- ▶ But markets don't quote strangles
- ▶ They quote **Butterfly Spreads**

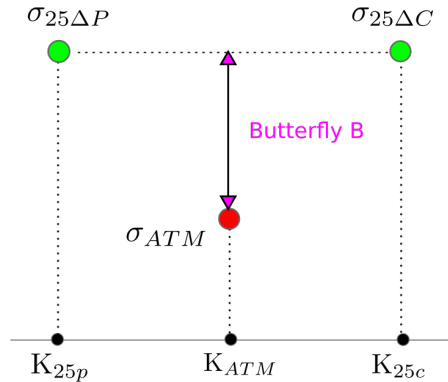
$$BF_{25\Delta} = \frac{\sigma_{25\Delta C} + \sigma_{25\Delta P}}{2} - \sigma_{ATM}$$

## What the data tells us: butterfly spreads (3)

- ▶ Butterfly spreads convey more information



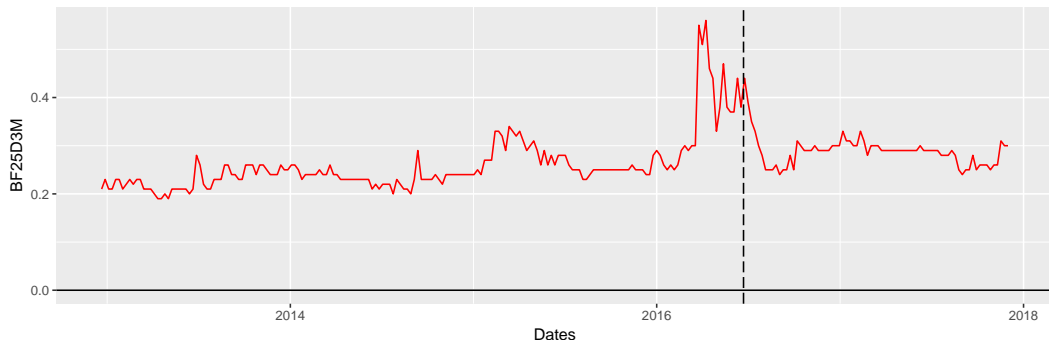
## Case A



## Case B

## What the data tells us: butterfly spreads (4)

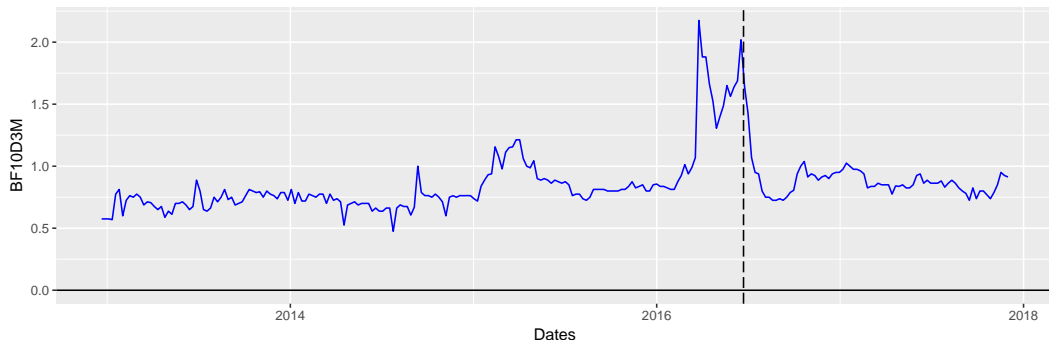
```
p7 = ggplot(data, aes(Dates, BF25D3M)) + geom_line(colour="red") +  
  geom_hline(yintercept=0)  
p7 = p7 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),  
  linetype="longdash")  
p7
```



## What the data tells us: butterfly spreads (5)

```
p8 = ggplot(data, aes(Dates, BF10D3M)) + geom_line(colour="blue") +  
  geom_hline(yintercept=0)  
p8 = p8 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),  
  linetype="longdash")
```

p8



Part C:

A Little Bit of Option Pricing

## What is $\Delta$ (1)

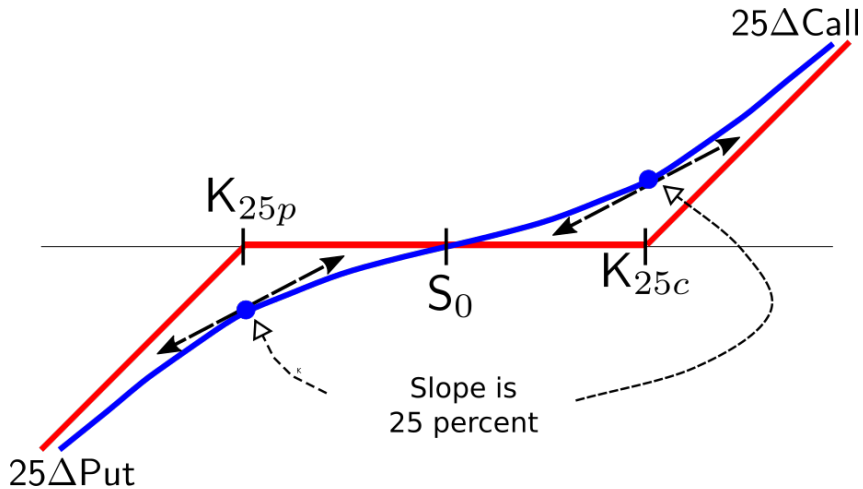
- ▶ The  $\Delta$  of a call

$$\Delta_C = \frac{\partial C}{\partial S} \geq 0$$

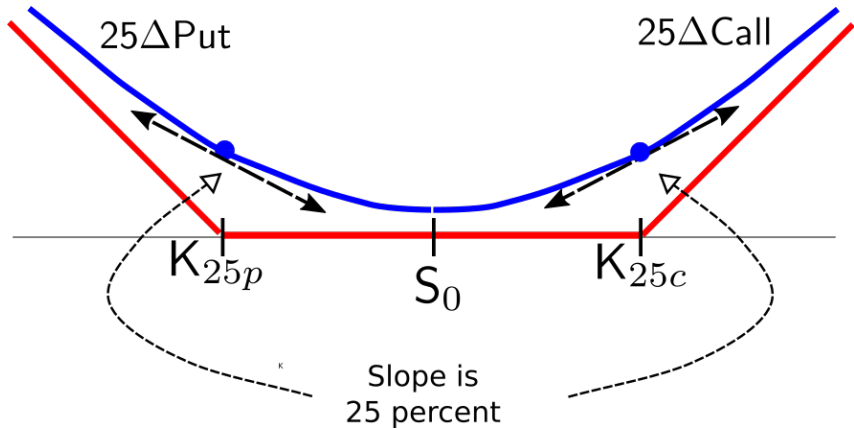
- ▶ The  $\Delta$  of a put

$$\Delta_P = \frac{\partial P}{\partial S} \leq 0$$

What is  $\Delta$  (2)

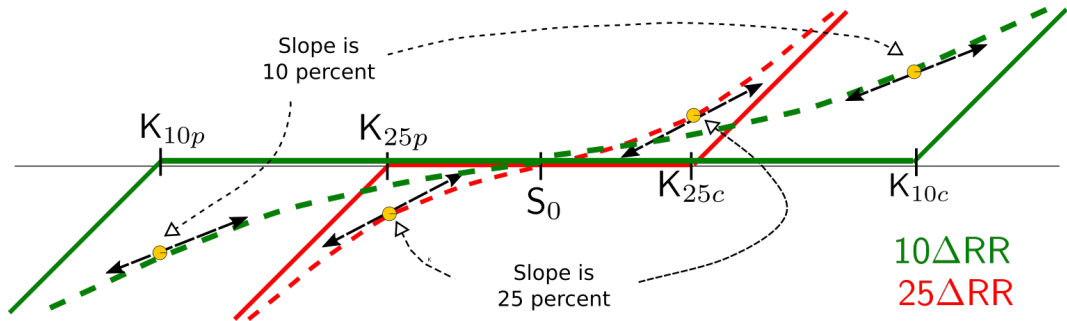


What is  $\Delta$  (3)





What is  $\Delta$  (4)



# Option prices = replication cost (1)

Dealer costs when buying call option from client

- ▶ Funding cost: borrow  $C$  at  $R_d$ :  $R_d C_t \delta t$
- ▶ Hedging cost (using delta-hedging)
  - ▶ Borrow and sell  $\Delta$  units of foreign currency
  - ▶ Receive  $\Delta S$  and reinvest at  $R_d$
  - ▶ Pay accrued interest on borrowed amount of currency
  - ▶ Net gain:  $(R_d - R_f) \Delta S_t \delta t$

## Option price = replication cost (2)

- ▶ Time decay of option: loses value as maturity approaches

$$\frac{\partial C_t}{\partial t} \delta t$$

- ▶ Convexity gains since options are non-linear

$$\frac{\partial^2 C_t}{\partial S^2} (\delta S_t)^2$$

## Option price = replication cost (3)

- Gains must offset costs, yielding the pricing partial differential equation:

$$\frac{\partial C_t}{\partial t} \delta t + (R_d - R_f) \Delta S_t \delta t + \frac{\partial^2 C}{\partial S_t^2} (\delta S_t)^2 = R_d C_t \delta t$$

## Garman-Kohlhagen formula (1)

Assume FX follows a geometric brownian motion, the call option price is:

$$C(K, S_t, R_d, R_f, T, \sigma) = S_t \exp(-R_f \times (T - t))N(d_1) - K \exp(-R_d \times (T - t))N(d_2)$$

and the put option price is:

$$P(K, S_t, R_d, R_f, T, \sigma) = K \exp(-R_d \times (T - t))N(-d_2) - S_t \exp(-R_f \times (T - t))N(-d_1)$$

## Garman-Kohlhagen formula (2)

- ▶  $K$  is the strike price,
- ▶  $S_t$  is the current spot exchange rate,
- ▶  $R_d$  is the domestic interest rate,
- ▶  $R_f$  is the foreign interest rate,
- ▶  $T - t$  is the remaining life of an option maturing at time  $T$ ,
- ▶  $\sigma$  is the implied volatility of the exchange rate used to price the option,

▶

$$d_1 = \frac{\ln(S_t/K) + (R_d - R_f + \sigma^2/2)(T - t)}{\sigma \times (T - t)}$$

$$d_2 = d_1 - \sigma \times (T - t)$$

## Implied volatility $\sigma$ (1)

- ▶ All GK formula inputs are observable except implied volatility  $\sigma$
- ▶ Option prices are quoted as **vols**, i.e. volatility
- ▶ To obtain price
  - ▶ Take quoted vol
  - ▶ Use reference values for all other variables
  - ▶ Plug into GK formula

## Implied volatility $\sigma$ (2)

### Implied volatility

- ▶ is different from historical or realized volatility
- ▶ is not a forecast of future volatility
- ▶ yields an GK option premium reflecting
  - ▶ profit margin
  - ▶ hedging costs
  - ▶ demand and supply in the FX market



## Part D: The Volatility Smile

## Finding vols for different $\Delta$ s

- ▶ For a given  $\Delta$  we have prices for:
  - ▶ Risk reversal (RR)
  - ▶ Butterfly spread (BF)
- ▶ The ATM vol,  $\sigma_{ATM}$  is also given
- ▶ From the definitions of the RR and the BF

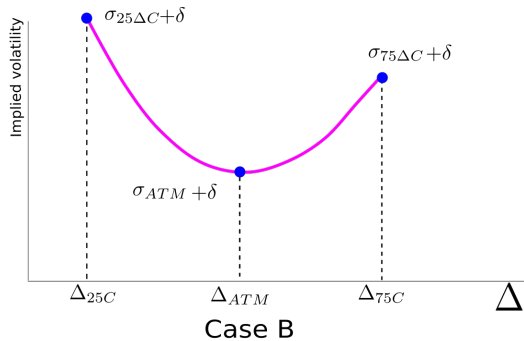
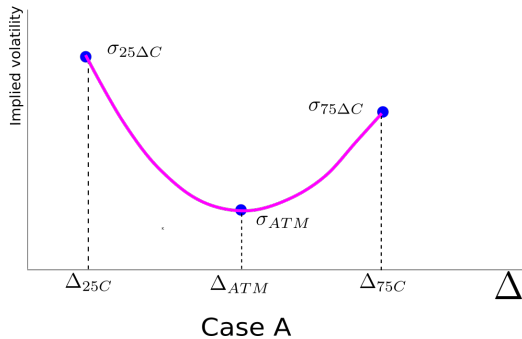
$$\sigma_{25\Delta C} = \sigma_{ATM} + BF_{25\Delta} + \frac{1}{2}RR_{25\Delta}$$

$$\sigma_{25\Delta P} = \sigma_{ATM} + BF_{25\Delta} - \frac{1}{2}RR_{25\Delta}$$

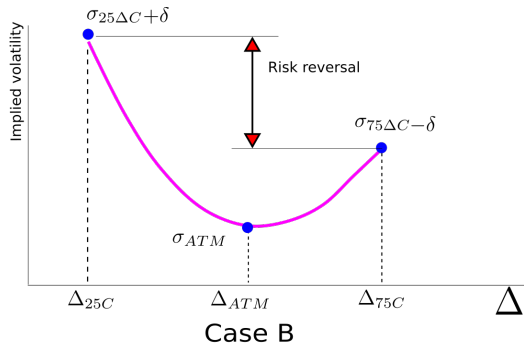
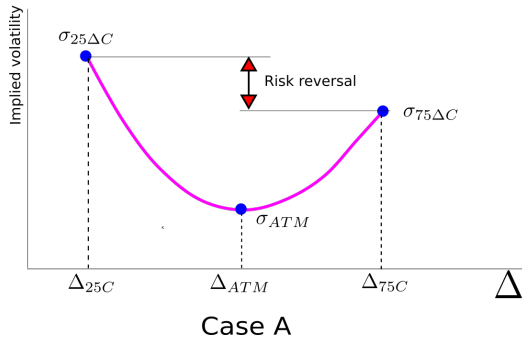
$$\sigma_{75\Delta C} = \sigma_{25\Delta P} \text{ by put-call parity}$$

- ▶ Plotting  $\sigma$  against  $\Delta$  yields the volatility smile

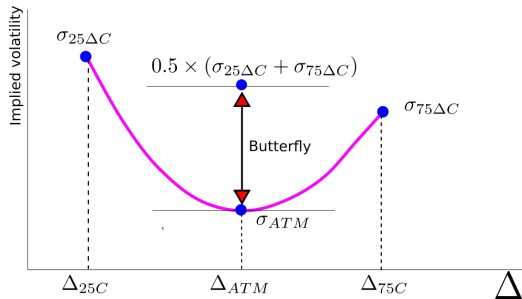
## The level of the smile: $\sigma_{ATM}$



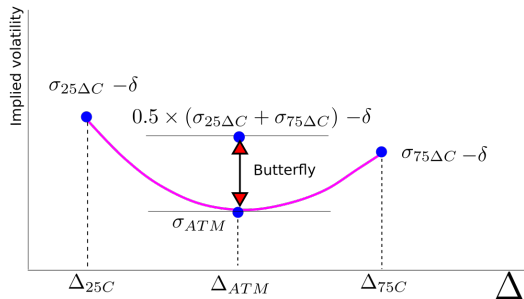
# The slope of the smile: the risk reversal



# The curvature of the smile: the butterfly spread



Case A



Case B

Part E:

Constructing the Volatility Smile

A Brexit case study

## Selecting the dates

We are interested in the behavior of the GBPUSD for three dates:

- ▶ January 8, 2016 (Pre-Brexit)
- ▶ June 24, 2016 (Brexit)
- ▶ December 30, 2016 (Post-Brexit)

## Getting the data (1)

```
rm(list=ls())  
filename = "2018_IET_Options_data.csv"  
data = read.csv(filename, header=TRUE)  
data$Dates = mdy_hm(as.character(data$Dates))  
rownames(data)=NULL
```

*# Specify dates for analysis*

```
date01 = as.Date("2016-01-08 UTC")  
date02 = as.Date("2016-06-24 UTC")  
date03 = as.Date("2016-12-30 UTC")
```

```
# Clean up memory  
# Name of CSV data file  
# Load datafile  
# convert dates to Date  
# remove row names
```



## Getting the data (2)

```
# Create data frame this.data
```

```
this.data = rbind(
  data[which(data$Dates==date01),],
  data[which(data$Dates==date02),],
  data[which(data$Dates==date03),])
```

```
# Delete row names and change the names of the columns
```

[illegible]

## Get additional vols

In addition to the  $25\Delta$  and  $75\Delta$  vols, obtain the  $10\Delta$  and  $90\Delta$  vols:

$$\sigma_{10\Delta C} = \sigma_{ATM} + BF_{10\Delta} + \frac{1}{2}RR_{10\Delta}$$

$$\sigma_{10\Delta P} = \sigma_{ATM} + BF_{10\Delta} - \frac{1}{2}RR_{10\Delta}$$

$$\sigma_{90\Delta C} = \sigma_{10\Delta P}$$

## Calculating the implied vols (1)

*# Vols are in percent, expressed them as simple numbers*

```
this.data$atm  = this.data$atm/100  
this.data$rr25 = this.data$rr25/100  
this.data$bf25 = this.data$bf25/100  
this.data$rr10 = this.data$rr10/100  
this.data$bf10 = this.data$bf10/100  
this.data$rf    = this.data$rf/100  
this.data$rd    = this.data$rd/100  
this.data$imp_rd=this.data$imp_rd/100
```

*# We will use this.data repeatedly*

*# Attach it to access its elements*

```
attach(this.data)
```

## Calculating the implied vols (2)

*# Recover vols for different deltas and put them in the data frame*

```
this.data$sigma10c = atm + bf10 + 0.5*rr10  
this.data$sigma25c = atm + bf25 + 0.5*rr25  
this.data$sigma75c = this.data$sigma25c - rr25  
this.data$sigma90c = this.data$sigma10c - rr10  
this.data$sigmaatm = atm
```

Tenor = 3/12    *# Maturity of options, 3 months, in years*

## What is the proper *ATM* strike?

- ▶ Retail products

$$K_{ATM} = S$$

$$\Delta_{ATM} = N\left(\frac{\log(F/S) + \frac{1}{2}\sigma_{ATM}^2 T}{\sigma_{ATM}\sqrt{T}}\right)$$

- ▶ EM currencies, maturities more than one year

$$K_{ATM} = F$$

$$\Delta_{ATM} = \exp(-Rf \times T) N\left(\frac{1}{2}\sigma_{ATM}\sqrt{T}\right)$$

- ▶ Major currencies, maturities of one year or less

$$K_{ATM} = F \times \exp\left(0.5\sigma_{ATM}^2 \times T\right)$$

$$\Delta_{ATM} = 0.5 \times \exp(-Rf \times T) \simeq 0.5$$

Calculate the  $\Delta_{ATM}$

```
# Calculate the strike of the ATM option  
K_atm = forward*exp((0.5*(this.data$sigmaatm)^2)*Tenor)  
deltaATM = 0.5*exp(-rf*Tenor)  
deltaATM  
  
## [1] 0.4989636 0.4993504 0.4994441
```

## Rough volatility smile (1)

```
# Select only the vols for each delta
```

```
list_variables = c("sigma10c", "sigma25c", "sigmaatm",  
                  "sigma75c", "sigma90c")
```

```
# Read the data as a matrix
```

```
vol_data = t(as.matrix(subset(this.data, select=list_variables)))
```

```
# Group the deltas in a vector, to be used in the x-axis
```

```
delta_vector = c(0.10, 0.25, 0.5, 0.75, 0.9)
```

```
# Create the data frame for the chart
```

```
vol.smile = data.frame(delta_vector, vol_data)
```

```
rownames(vol.smile) = NULL
```

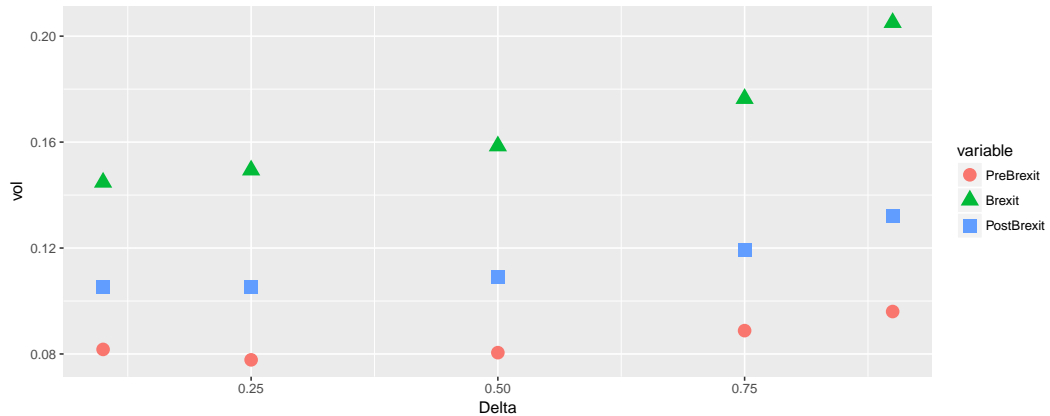
```
colnames(vol.smile) = c("Delta", "PreBrexit", "Brexit", "PostBrexit")
```

## Rough volatility smile (2)

```
library(reshape2)
vol.data = melt(vol.smile, id="Delta")
ggplot(data=vol.data, aes(x=Delta, y=value, shape=variable)) +
  geom_point(aes(colour=variable), size=4) +
  labs(y="vol")
```



## Rough volatility smile (3)



## A more refined volatility smile (1)

```
# Fit second degree polynomial to smile
fit.Vol = function(data.vol, data.delta,delta.range)
{
  poly.fit = lm(data.vol ~ poly(data.delta, 2, raw=TRUE))

  # Use fitted polynomial to interpolate Delta-Vol Curve
  delta.square = delta.range*delta.range
  delta.interc = rep(1,length(delta.range))

  X = cbind(delta.interc, delta.range, delta.square)
  iVolInterpol = t(t(X)*poly.fit$coefficients)
  iVolInterpol = rowSums(iVolInterpol)

  return(iVolInterpol)
}
```

## A more refined volatility smile (2)

```
# Get the data points from the vol.smile data frame
```

```
data.delta = vol.smile$Delta
```

```
data.vol   = vol.smile$PreBrexit
```

```
# Interpolation and extrapolation range
```

```
delta.range = seq(from=0.01, to = 0.99, by=0.005)
```

```
# Obtain the volatility smiles
```

```
smile.preBrexit = fit.Vol(vol.smile$PreBrexit, data.delta, delta.range)
```

```
smile.Brexit    = fit.Vol(vol.smile$Brexit, data.delta, delta.range)
```

```
smile.postBrexit= fit.Vol(vol.smile$PostBrexit, data.delta, delta.range)
```

## A more refined volatility smile (3)

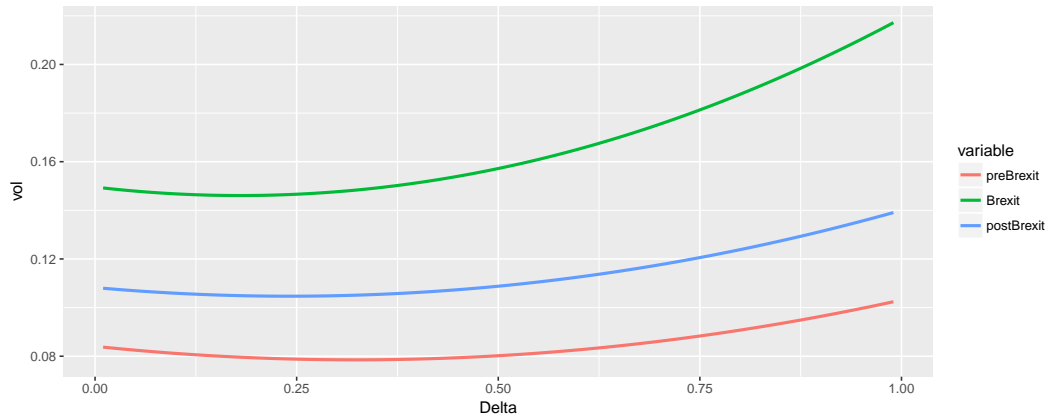
*# Group the extended volatility smiles in a data frame*

```
smile.df = data.frame(delta.range, smile.preBrexit,  
                      smile.Brexit, smile.postBrexit)  
rownames(smile.df) = NULL  
colnames(smile.df) = c("Delta", "preBrexit", "Brexit", "postBrexit")
```

*# Create the chart*

```
library(reshape2)  
smile.data = melt(smile.df, id="Delta")  
ggplot(data = smile.data, aes(x=Delta, y=value, colour=variable)) +  
  geom_line(size=1) + labs(y="vol")
```

## A more refined volatility smile (4)



## Part F: Extracting Risk-Neutral Densities

# Market views are not forecasts

## Market views

- ▶ They are not forecast of the real-world FX distribution
- ▶ Market prices weigh risk aversion of
  - ▶ hedgers
  - ▶ speculators
  - ▶ market makers
- ▶ “Technical” factors may affect weight of events
- ▶ Risk-neutral, weighting expectations and risk aversion
  - ▶ Not easy to disentangle them

# Extraction methods

Methods fall under one of three categories:

- ▶ Parametric methods
- ▶ Semiparametric methods
- ▶ Non-parametric methods



## Parametric methods

- ▶ Specify a parametric distribution function  $F$  for the exchange rate
- ▶ For the distribution parameters  $\Lambda$ , the price of a call option is

$$C^F(K) = \exp(-R_d \times T) \int_K^\infty (S_T - K) dF(S_T; \Lambda),$$

- ▶ The parameters that fit the market-implied distribution better solve

$$\arg \min_{\Lambda} \sum_{j=1}^N |C^F(K_j) - C^{\text{Market}}(K_j)|^2$$

## Semi-parametric methods

- ▶ Start with a particular density
- ▶ Add expansion terms to distribution
- ▶ Each term helps to approximate market-implied distribution

## Non-parametric methods

- ▶ Plot value of calls against different strike prices, i.e. **market call function**
- ▶ Breeden and Litzenberger: risk-neutral distribution proportional to second derivative of the call function:

$$\left. \frac{\partial^2 C}{\partial K^2} \right|_{K=S} = \exp(-R_d \times T) q(S).$$

- ▶ Use numerical methods to calculate derivative and find risk neutral distribution

## Numerical estimation exercise

- ▶ Risk neutral density extraction
  - ▶ Generalized beta density (parametric)
  - ▶ Edgeworth expansion (semi-parametric)
  - ▶ Shimko (non-parametric)
- ▶ All methods require constructing call and/or put functions
  - ▶ Option premia vs. strike price

Part F:

Extracting Risk-Neutral Densities

Constructing option premium functions

## Constructing the market call function (1)

- ▶ We need to obtain the market call function
- ▶ In other words, move from  $\Delta - \sigma$  space to *Option premium - Strike* space
- ▶ Two useful functions in `auxFunctions.R`
  - ▶ `get.strike()`
  - ▶ `GKoption.premium()`

## Constructing the market call function (2)

```
get.strike = function(vol,delta,S0,fwd,rf,Tenor)
{
  aux = qnorm(delta*exp(rf*Tenor))
  aux = aux*vol*sqrt(Tenor)
  aux = aux - 0.5*vol*vol*Tenor
  K = exp(-aux)*fwd
  return(K)
}
```

## Constructing the market call function (3)

```
GKoption.premium = function(K,sigma,S,Tenor,fwd,rf,option_type)
{
  if (option_type == "c") {w=1}
  if (option_type == "p") {w=-1}
  d1 = log(fwd/K)+0.5*sigma*sigma*Tenor
  d1 = d1/(sigma*sqrt(Tenor))
  d2 = d1 - sigma*sqrt(Tenor)
  rd = log(fwd/S)/Tenor + rf
  premium = exp(-rd*Tenor)*(w*fwd*pnorm(w*d1) - w*K*pnorm(w*d2))
  return(premium)
}
```



## Constructing the market call function (3)

Strike ranges for the selected dates:

```
K_preBrexit = mapply(get.strike,smile.df$preBrexit, smile.df$Delta,  
                     S0=spot[1], fwd=forward[1], rf=rf[1], Tenor=0.25)  
K_Brexit     = mapply(get.strike,smile.df$Brexit, smile.df$Delta,  
                     S0=spot[2], fwd=forward[2], rf=rf[2], Tenor=0.25)  
K_postBrexit= mapply(get.strike,smile.df$postBrexit, smile.df$Delta,  
                     S0=spot[3], fwd=forward[3], rf=rf[3], Tenor=0.25)
```

## Constructing the market call function (4)

Generate the call premia:

```
Call_preBrexite = mapply(GKoption.premium, K_preBrexite,  
                          smile.df$preBrexite, S=spot[1], Tenor=Tenor,  
                          fwd=forward[1], rf=rf[1], option_type="c")
```

```
Call_Brexite = mapply(GKoption.premium, K_Brexite,  
                      smile.df$Brexite, S=spot[2], Tenor=Tenor,  
                      fwd=forward[2], rf=rf[2], option_type="c")
```

```
Call_postBrexite = mapply(GKoption.premium, K_postBrexite,  
                          smile.df$postBrexite, S=spot[3], Tenor=Tenor,  
                          fwd=forward[3], rf=rf[3], option_type="c")
```

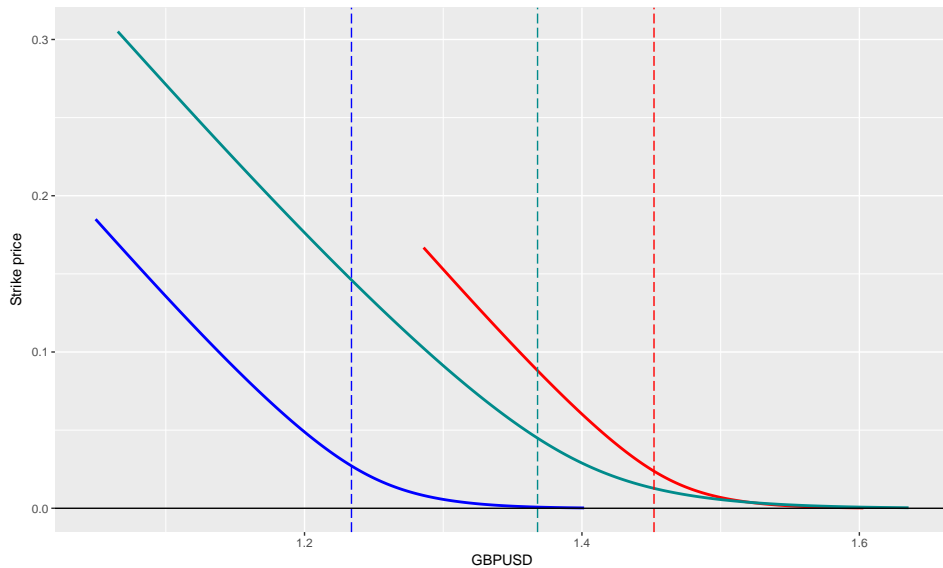


## Constructing the market call function (6)

Plot the market call function

```
ggplot(callstrike.df) +  
  geom_line(aes(x=K_preBrexite,y=Call_preBrexite), col="red", size=1) +  
  geom_line(aes(x=K_Brexite, y=Call_Brexite), col="darkcyan", size=1) +  
  geom_line(aes(x=K_postBrexite, y=Call_postBrexite), col="blue", size=1) +  
  labs(x="GBPUSD", y="Strike price") +  
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +  
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +  
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +  
  geom_hline(yintercept = 0, col="black")
```

## Constructing the market call function (7)



## Constructing the market call function (8)

Question: the call premium-strike functions are downward sloping, i.e. the call premium is higher for lower strike prices. Does this make sense? Explain why.

## Constructing the market put function (1)

```
Put_preBrexid = mapply(GKoption.premium, K_preBrexid,  
                        smile.df$preBrexid, S=spot[1], Tenor=Tenor,  
                        fwd=forward[1], rf=rf[1], option_type="p")
```

```
Put_Brexid = mapply(GKoption.premium, K_Brexid, smile.df$Brexid,  
                    S=spot[2], Tenor=Tenor, fwd=forward[2],  
                    rf=rf[2], option_type="p")
```

```
Put_postBrexid = mapply(GKoption.premium, K_postBrexid,  
                        smile.df$postBrexid, S=spot[3], Tenor=Tenor,  
                        fwd=forward[3], rf=rf[3], option_type="p")
```

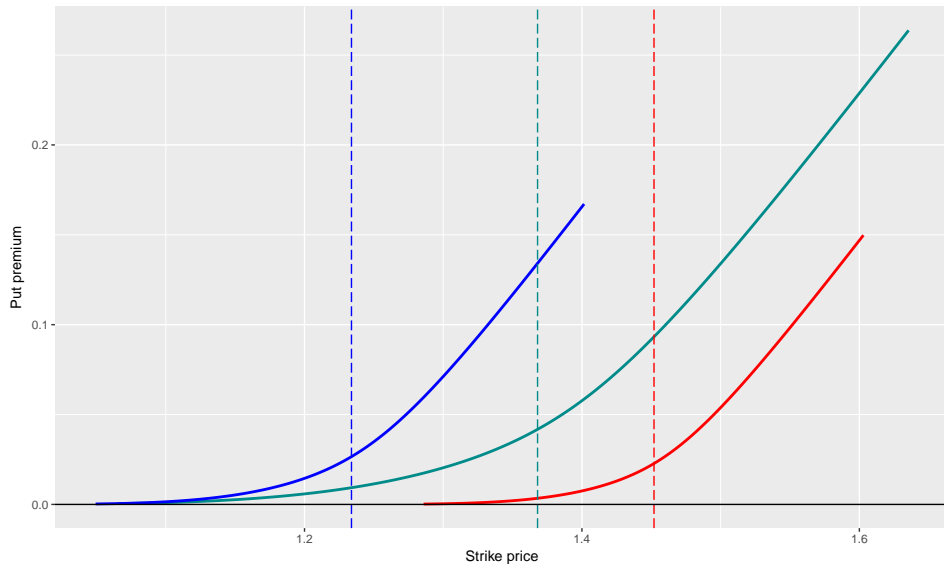




## Constructing the market put function (3)

```
ggplot(putstrike.df) +  
  geom_line(aes(x=K_preBrexit,y=Put_preBrexit), col="red", size=1) +  
  geom_line(aes(x=K_Brexit, y=Put_Brexit), col="darkcyan", size=1) +  
  geom_line(aes(x=K_postBrexit, y=Put_postBrexit), col="blue", size=1) +  
  labs(x="Strike price", y="Put premium") +  
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +  
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +  
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +  
  geom_hline(yintercept = 0, col="black")
```

## Constructing the market put function (4)



Part F:

Extracting Risk-Neutral Densities

Generalized Beta Density

# Generalized Beta Density (1)

- ▶ Parametric method
- ▶ Implemented in package RND
- ▶ Requires as inputs
  - ▶ call premia and strike range
  - ▶ put premia and strike range
  - ▶ spot rate
  - ▶ implied domestic interest rate
  - ▶ option tenor

## Generalized Beta Density (2)

Let's start with the pre-Brexit period

```
r = imp_rd[1]           # domestic interest rate
te= Tenor               # tenor of the option
y = rf[1]              # foreign interest rate
s0= spot[1]            # spot exchange rate
call.premium = Call_preBrexit # vector of call premium values
call.strikes = K_preBrexit   # vector of corresponding call strikes
put.premium = Put_preBrexit  # vector of put premium values
put.strikes = K_preBrexit    # vector of corresponding put strikes
```



## Generalized Beta Density (4)

Obtain the risk neutral distribution by passing a range of strike prices

```
Krange = seq(0.9*min(K_preBrexite, K_Brexite, K_postBrexite),  
             1.1*max(K_preBrexite, K_Brexite, K_postBrexite),  
             0.01)
```

to the function `dgb()`:

```
gb = gb.preBrexite  
gb.rnd.preBrexite = dgb(Krange,gb$a, gb$b, gb$v, gb$w)
```

## Generalized Beta Density (5)

Do the same for the Brexit ...

```
r = imp_rd[2]
y = rf[2]
s0= spot[2]
call.premium = Call_Brexit
call.strikes = K_Brexit
put.premium = Put_Brexit
put.strikes = K_Brexit
```



## Generalized Beta Density (6)

```
gb.Brexit = extract.gb.density(initial.values=c(NA,NA,NA,NA),  
                               r=r, te=te, y=y, s0=s0,  
                               market.calls=call.premium,  
                               call.strikes = call.strikes,  
                               call.weights =1,  
                               market.puts = put.premium,  
                               put.strikes = put.strikes,  
                               put.weights = 1,  
                               lambda=1, hessian.flag=F)  
  
gb = gb.Brexit  
gb.rnd.Brexit = dgb(Krange,gb$a, gb$b, gb$v, gb$w)
```

## Generalized Beta Density (7)

And more of the same for post-Brexit:

```
r = imp_rd[3]
y = rf[3]
s0= spot[3]
call.premium = Call_postBrexit
call.strikes = K_postBrexit
put.premium = Put_postBrexit
put.strikes = K_postBrexit
```

## Generalized Beta Density (8)

```
gb.postBrexit = extract.gb.density(initial.values=c(NA,NA,NA,NA),  
                                   r=r, te=te, y=y, s0=s0,  
                                   market.calls=call.premium,  
                                   call.strikes = call.strikes,  
                                   call.weights =1,  
                                   market.puts = put.premium,  
                                   put.strikes = put.strikes,  
                                   put.weights = 1,  
                                   lambda=1, hessian.flag=F)  
gb = gb.postBrexit  
gb.rnd.postBrexit = dgb(Krange,gb$a, gb$b, gb$v, gb$w)
```

## Generalized Beta Density (9)

```
gb.rnd.df = data.frame(Krange,gb.rnd.preBrexit,  
                        gb.rnd.Brexit, gb.rnd.postBrexit)
```

```
ggplot(data=gb.rnd.df, aes(x=Krange)) +  
  geom_line(aes(y=gb.rnd.preBrexit), col="red", size=1.25) +  
  geom_line(aes(y=gb.rnd.Brexit), col="darkcyan", size=1.25) +  
  geom_line(aes(y=gb.rnd.postBrexit), col="blue", size=1.25) +  
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +  
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +  
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +  
  geom_hline(yintercept=0, col="black", size=0.5) +  
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

## Generalized Beta Density (10)

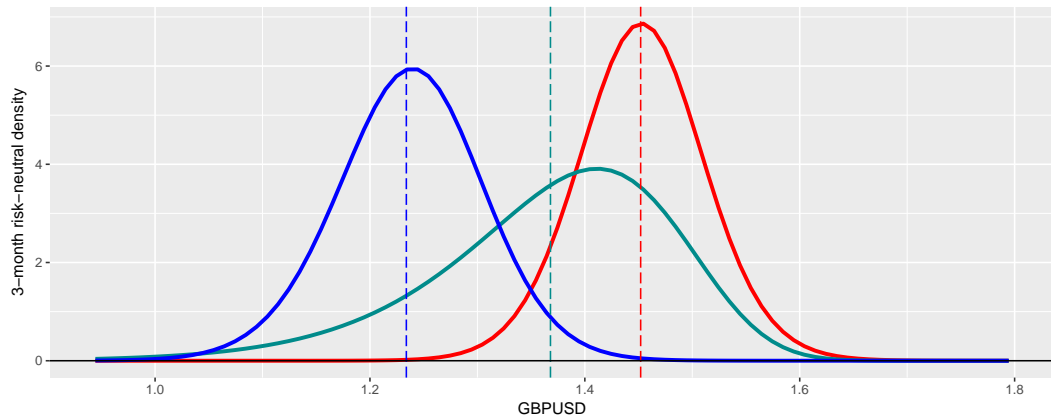


Figure 5: Generalized beta risk neutral distributions. Pre-Brexit: red; Brexit: cyan; post-Brexit: blue

Part F:

Extracting Risk-Neutral Densities

Edgeworth expansion

# Edgeworth expansion (1)

- ▶ Semi-parametric method
- ▶ Implemented in package RND
- ▶ Requires as inputs
  - ▶ call premia and strike range
  - ▶ spot rate
  - ▶ implied domestic interest rate
  - ▶ option tenor

## Edgeworth expansion (2)

Calculations in Brexit period

```
r = imp_rd[1]           # domestic interest rate
te= Tenor               # tenor of the option
y = rf[1]              # foreign interest rate
s0= spot[1]            # spot exchange rate
call.premium = Call_preBrexit # vector of call premium values
call.strikes = K_preBrexit  # vector of corresponding call strikes
```



## Edgeworth expansion (3)

The function `extract.ew.density()` calculates the parameters of the Edgeworth expansion:

```
ew.preBrexite = extract.ew.density(initial.values = rep(NA,2),  
                                   r=r, y=y, te=te, s0=s0,  
                                   market.calls=call.premium,  
                                   call.strikes = call.strikes,  
                                   call.weights =1, lambda=1, hessian.flag=F,  
                                   cl = list(maxit=10000))
```

which we then input into the function `dew()` to obtain the distribution.

```
ew = ew.preBrexite  
ew.rnd.preBrexite = dew(Krange,r,y,te,s0,  
                        ew$sigma, ew$skew, ew$kurt)
```

## Edgeworth expansion (4)

Repeat the recipe for Brexit ...

```
r = imp_rd[2]           # domestic interest rate
te= Tenor               # tenor of the option
y = rf[2]              # foreign interest rate
s0= spot[2]            # spot exchange rate
call.premium = Call_Brexit # vector of call premium values
call.strikes = K_Brexit  # vector of corresponding call strikes
```

## Edgeworth expansion (5)

```
ew.Brexit = extract.ew.density(initial.values = rep(NA,2),  
                               r=r, y=y, te=te, s0=s0,  
                               market.calls=call.premium,  
                               call.strikes = call.strikes,  
                               call.weights =1, lambda=1, hessian.flag=F,  
                               cl = list(maxit=10000))  
  
ew = ew.Brexit  
ew.rnd.Brexit  = dew(Krange,r,y,te,s0,ew$sigma, ew$skew, ew$kurt)
```

## Edgeworth expansion (6)

And for post-Brexit

```
r = imp_rd[3]           # domestic interest rate
te= Tenor                # tenor of the option
y = rf[3]                # foreign interest rate
s0= spot[3]              # spot exchange rate
call.premium = Call_postBrexit # vector of call premium values
call.strikes = K_postBrexit  # vector of corresponding call strikes
```

## Edgeworth expansion (7)

```
ew.postBrexIt = extract.ew.density(initial.values = rep(NA,2),  
                                   r=r, y=y, te=te, s0=s0,  
                                   market.calls=call.premium,  
                                   call.strikes = call.strikes,  
                                   call.weights =1, lambda=1, hessian.flag=F,  
                                   cl = list(maxit=10000))  
  
ew = ew.postBrexIt  
ew.rnd.postBrexIt = dew(Krange,r,y,te,s0,ew$sigma, ew$skew, ew$kurt)
```

## Edgeworth expansion (8)

Produce a pretty chart

```
ew.rnd.df = data.frame(Krange, ew.rnd.preBrexit,  
                        ew.rnd.Brexit, ew.rnd.postBrexit)  
  
ggplot(data=ew.rnd.df, aes(x=Krange)) +  
  geom_line(aes(y=ew.rnd.preBrexit), col="red", size=1.25) +  
  geom_line(aes(y=ew.rnd.Brexit), col="darkcyan", size=1.25) +  
  geom_line(aes(y=ew.rnd.postBrexit), col="blue", size=1.25) +  
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +  
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +  
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +  
  geom_hline(yintercept=0, col="black", size=0.5) +  
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

## Edgeworth expansion (9)

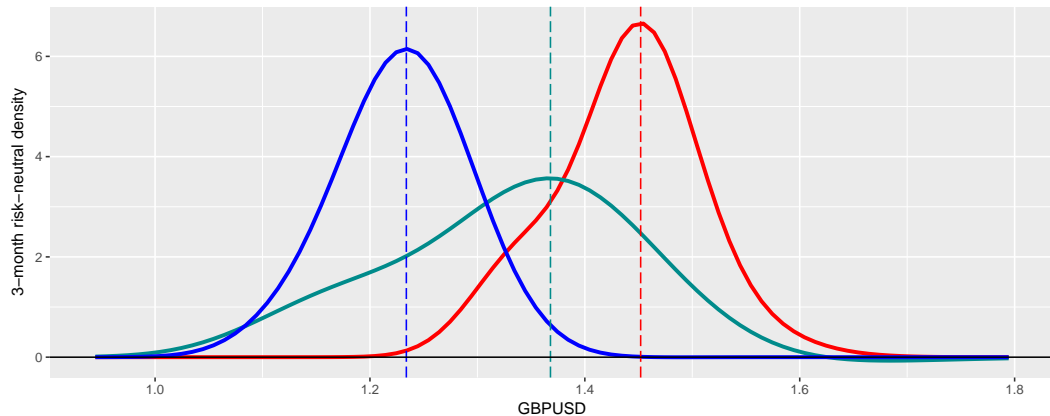


Figure 6: Edgeworth expansion risk neutral distributions. Pre-Brexit: red; Brexit: cyan; post-Brexit: blue

Part F:

Extracting Risk-Neutral Densities

Shimko method



## Shimko method (1)

- ▶ Nonparametric method
- ▶ Implemented in package RND
- ▶ Method requires a volatility smile in the  $\sigma - K$  space

## Shimko method (2)

Construct the volatility smile in the  $\sigma - K$  in the Brexit period

```
vol.preBrexit = vol.data[1:5,3]          # obtain implied volatility
delta.shimko = vol.data[1:5,1]          # obtain the deltas
```

```
# Calculate the strike prices corresponding to the observed deltas
```

```
Kshimko_preBrexit = mapply(get.strike, vol.preBrexit,
                           delta.shimko, S0=spot[1],
                           fwd=forward[1], rf=rf[1], Tenor=0.25)
```

```
# Calculate the call premium
```

```
shimko_preBrexit = mapply(GKoption.premium, Kshimko_preBrexit,
                           vol.preBrexit, S=spot[1], Tenor=Tenor,
                           fwd=forward[1], rf=rf[1], option_type="c")
```

## Shimko method (3)

The function in the RND package that calculates the parameters of the Shimko's quadratic approximation to the volatility smile is `extract.shimko.density()`:

```
# Inputs for extract.shimko.density()
```

[illegible]

## Shimko method (4)

The parameters are then fed to the function `dshimko()` to obtain the risk neutral distribution for a wider range of strikes, `Krange`:

```
shimko = shimko.preBrexite
```

```
shimko.rnd.preBrexite = dshimko(r=r, te=Tenor,  
                                s0=s0, k=Krange, y=y,  
                                a0=shimko[[1]]$a0, a1=shimko[[1]]$a1,  
                                a2=shimko[[1]]$a2)
```

## Shimko method (5)

Repeat for Brexit

```
vol.Brexit = vol.data[6:10,3]           # obtain implied volatility
```

```
Kshimko_Brexit = mapply(get.strike, vol.Brexit,  
                        delta.shimko, S0=spot[2],  
                        fwd=forward[2], rf=rf[2], Tenor=0.25)
```

```
shimko_Brexit = mapply(GKoption.premium, Kshimko_Brexit,  
                      vol.Brexit, S=spot[2], Tenor=Tenor,  
                      fwd=forward[2], rf=rf[2], option_type="c")
```

## Shimko method (6)

```
r = imp_rd[2]
y = rf[2]
s0= spot[2]
call.premium = shimko_Brexit
call.strikes = Kshimko_Brexit
b=r-y
```

```
shimko.Brexit = extract.shimko.density(market.calls=call.premium,
                                       call.strikes = call.strikes,
                                       r=r, y=b, t=te, s0=s0,
                                       lower=0, upper= 30)
```

```
shimko = shimko.Brexit
shimko.rnd.Brexit = dshimko(r=r, te=Tenor, s0=s0, k=Krange, y=y,
                           a0=shimko[[1]]$a0, a1=shimko[[1]]$a1,
                           a2=shimko[[1]]$a2)
```

## Shimko method (7)

Once more for post-Brexit

```
vol.postBrexit = vol.data[11:15,3]           # obtain implied volatility
```

```
Kshimko_postBrexit = mapply(get.strike, vol.postBrexit,  
                             delta.shimko, S0=spot[3],  
                             fwd=forward[3], rf=rf[3], Tenor=0.25)
```

```
shimko_postBrexit = mapply(GKoption.premium, Kshimko_postBrexit,  
                           vol.postBrexit, S=spot[3], Tenor=Tenor,  
                           fwd=forward[3], rf=rf[3], option_type="c")
```

## Shimko method (8)

```
r = imp_rd[3]
y = rf[3]
s0= spot[3]
call.premium = shimko_postBrexit
call.strikes = Kshimko_postBrexit
b=r-y
```

```
shimko.postBrexit = extract.shimko.density(market.calls=call.premium,
                                           call.strikes = call.strikes,
                                           r=r, y=b, t=te, s0=s0,
                                           lower=0, upper= 30)
```

```
shimko = shimko.postBrexit
shimko.rnd.postBrexit = dshimko(r=r, te=Tenor, s0=s0, k=Krange, y=y,
                                a0=shimko[[1]]$a0, a1=shimko[[1]]$a1,
                                a2=shimko[[1]]$a2)
```



## Shimko method (9)

Fun part: plots

```
shimko.rnd.df = data.frame(Krange,shimko.rnd.preBrexit,  
                           shimko.rnd.Brexit, shimko.rnd.postBrexit)  
  
ggplot(data=shimko.rnd.df, aes(x=Krange)) +  
  geom_line(aes(y=shimko.rnd.preBrexit), col="red", size=1.25) +  
  geom_line(aes(y=shimko.rnd.Brexit), col="darkcyan", size=1.5) +  
  geom_line(aes(y=shimko.rnd.postBrexit), col="blue", size=1.25) +  
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +  
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +  
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +  
  geom_hline(yintercept=0, col="black", size=0.5) +  
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

## Shimko method (10)

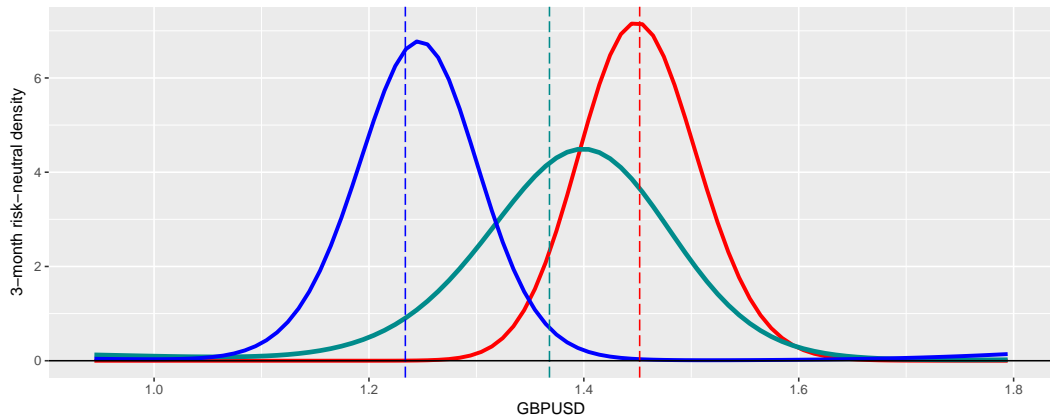


Figure 7: Shimko risk neutral distributions. Pre-Brexit: red; Brexit: cyan; post-Brexit: blue

Part G:

Comparing distributions

## Pre-Brexit (1)

```
ggplot(data=shimko.rnd.df, aes(x=Krange)) +  
  geom_line(aes(y=shimko.rnd.preBrexit), col="red", size=1.25) +  
  geom_line(data=gb.rnd.df, aes(x=Krange, y=gb.rnd.preBrexit),  
            col="darkcyan", size=1.25) +  
  geom_line(data=ew.rnd.df, aes(x=Krange, y=ew.rnd.preBrexit),  
            col="blue", size=1.25) +  
  geom_hline(yintercept=0, col="black", size=0.5) +  
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

## Pre-Brexit (2)

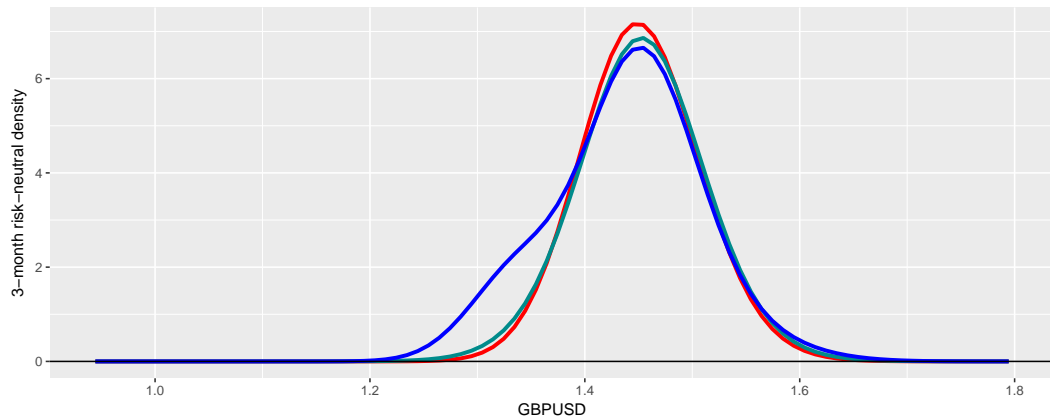


Figure 8: Pre-Brexit risk neutral distributions. Generalized beta: cyan; Edgeworth expansion: blue; Shimko: red.

## Brexit (1)

```
ggplot(data=shimko.rnd.df, aes(x=Krange)) +  
  geom_line(aes(y=shimko.rnd.Brexit), col="red", size=1.25) +  
  geom_line(data=gb.rnd.df, aes(x=Krange, y=gb.rnd.Brexit),  
            col="darkcyan", size=1.25) +  
  geom_line(data=ew.rnd.df, aes(x=Krange, y=ew.rnd.Brexit),  
            col="blue", size=1.25) +  
  geom_hline(yintercept=0, col="black", size=0.5) +  
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

## Brexit (2)

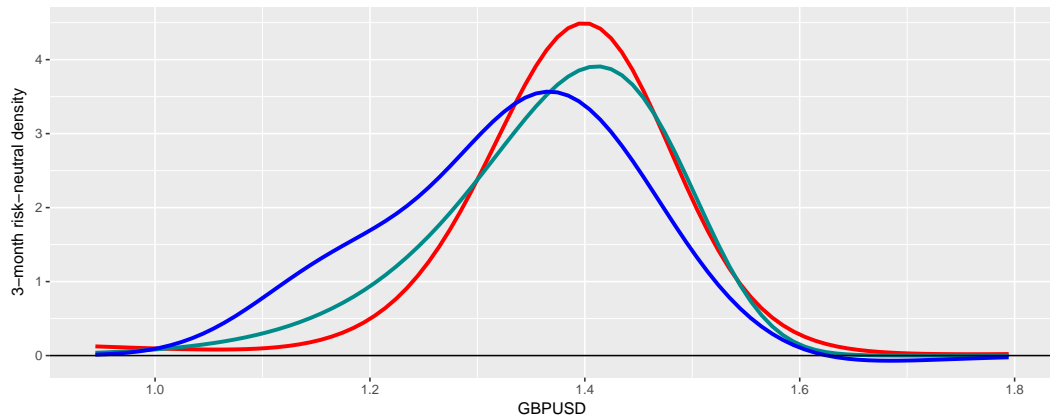


Figure 9: Brexit risk neutral distributions. Generalized beta: cyan; Edgeworth expansion: blue; Shimko: red.

## Post-Brexit (1)

```
ggplot(data=shimko.rnd.df, aes(x=Krange)) +  
  geom_line(aes(y=shimko.rnd.postBrexit), col="red", size=1.25) +  
  geom_line(data=gb.rnd.df, aes(x=Krange, y=gb.rnd.postBrexit),  
            col="darkcyan", size=1.25) +  
  geom_line(data=ew.rnd.df, aes(x=Krange, y=ew.rnd.postBrexit),  
            col="blue", size=1.25) +  
  geom_hline(yintercept=0, col="black", size=0.5) +  
  labs(x="GBPUSD", y="3-month risk-neutral density")
```



## Post-Brexit(2)

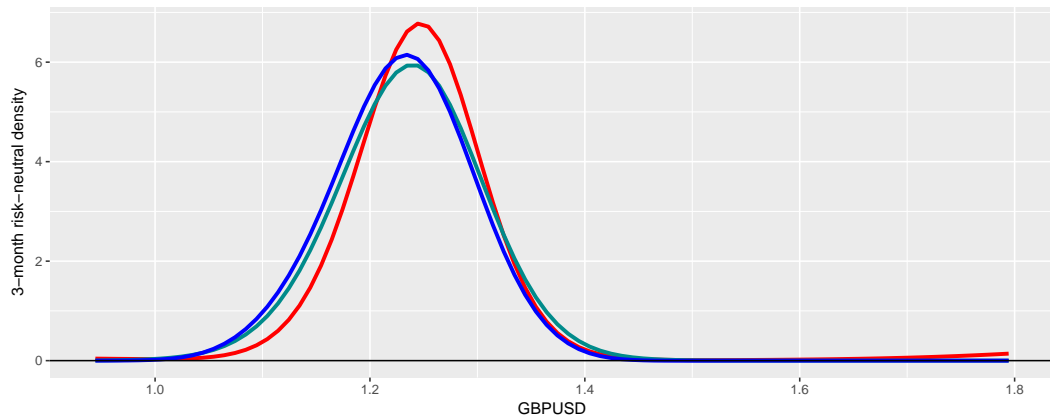


Figure 10: Post-Brexit risk neutral distributions. Generalized beta: cyan; Edgeworth expansion: blue.

Thank You

And Have Fun!