Extracting information from FX Options

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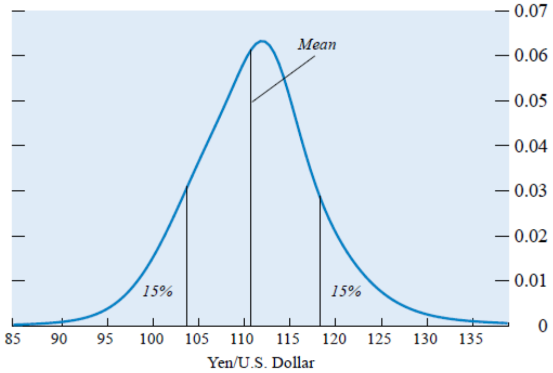
Objectives

- We want to build this

Figure 50. Distribution for Yen-Dollar Exchange Rate in Early September 1997 Implied by Options Prices on May 20, 1997

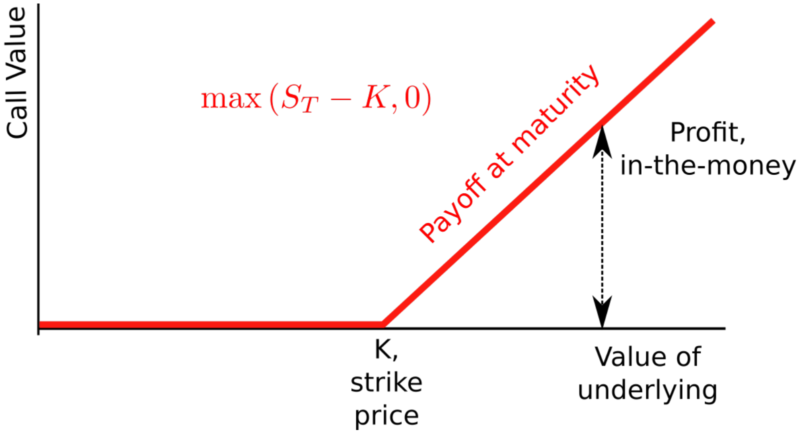


What we will cover

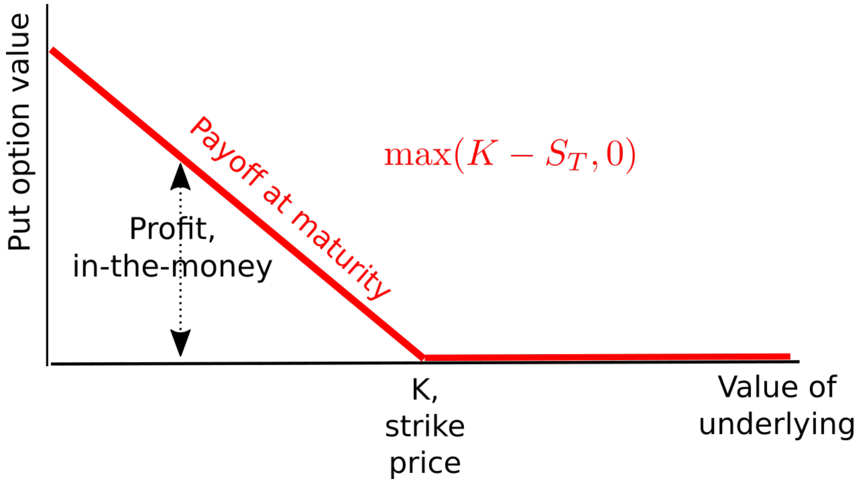
- Learn about basic FX structures
- Develop some intuition about FX option prices
- Get comfortable with a variety of methods
- Side benefit: learn `R` and `RStudio`!
- Lecture notes available at https://jchanlauimf.github.io/IMF_FXOptions

Part A:
The Very Basics

Basics: a plain vanilla call option



Basics: a plain vanilla put option



Market reality

- Price quote available for
  - ATM plain options
- Price quotes available for **FX structures**
  - risk reversals
  - butterfly spreads
- Main data sources
  - Bloomberg
  - Reuters
  - Investment banks' data portals

Part B:
Eyeballing the Data

Getting the data in (1)

1. Start `RStudio`
2. Set the working directory to where we placed the data file
3. Issue this command in the console (replace with your directory)

```r
my_wdir = "D:/IMF_FXCourse"  # replace with your directory
setwd(my_wdir)
```

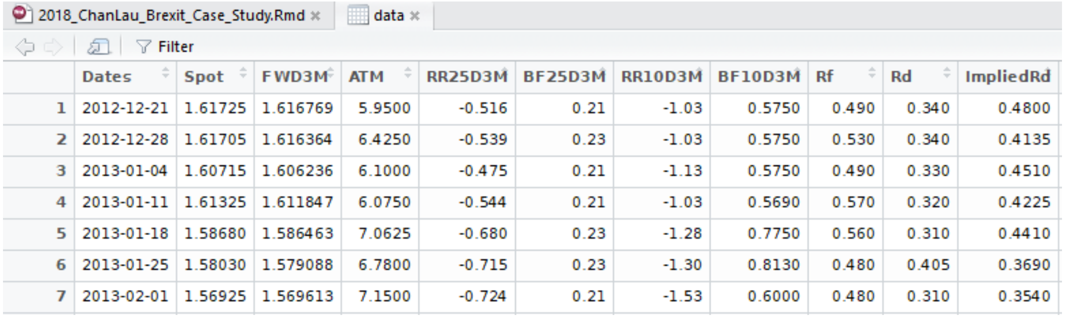
Getting the data in (2)

Clean memory, set up the needed libraries

```r
rm(list=ls())                # Clean up memory
library(ggplot2)             # Graphic library
library(lubridate)           # Date manipulation library
library(dplyr)               # Data manipulation library
source("auxFunctions.R")     # Auxiliary functions
```

Getting the data in (3)

```
filename = "2018_IET_Options_data.csv"
data = read.csv(filename, header=TRUE)
data$Dates = mdy_hm(as.character(data$Dates))
```



	Dates	Spot	FWD3M	ATM	RR25D3M	BF25D3M	RR10D3M	BF10D3M	Rf	Rd	ImpliedRd
1	2012-12-21	1.61725	1.616769	5.9500	-0.516	0.21	-1.03	0.5750	0.490	0.340	0.4800
2	2012-12-28	1.61705	1.616364	6.4250	-0.539	0.23	-1.03	0.5750	0.530	0.340	0.4135
3	2013-01-04	1.60715	1.606236	6.1000	-0.475	0.21	-1.13	0.5750	0.490	0.330	0.4510
4	2013-01-11	1.61325	1.611847	6.0750	-0.544	0.21	-1.03	0.5690	0.570	0.320	0.4225
5	2013-01-18	1.58680	1.586463	7.0625	-0.680	0.23	-1.28	0.7750	0.560	0.310	0.4410
6	2013-01-25	1.58030	1.579088	6.7800	-0.715	0.23	-1.30	0.8130	0.480	0.405	0.3690
7	2013-02-01	1.56925	1.569613	7.1500	-0.724	0.21	-1.53	0.6000	0.480	0.310	0.3540

The data (1)

FX related

- Spot: the GBPUSD spot exchange rate, i.e. USD per GBP
- FWD3M: the 3-month GBPUSD forward exchange rate
- Rf: the 3-month GBP money market deposit rate, annualized (in percent)
- Rd: the 3-month USD money market deposit rate, annualized (in percent)

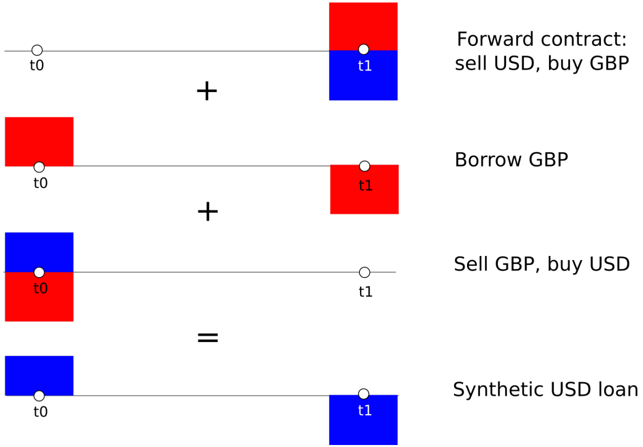
The data (2)

FX option related

- ATM: the at-the-money implied volatility of a GBPUSD option with strike price equal to ATM
- RR25D3M: the price of a 25Δ risk reversal, in annualized volatility units (in percent)
- BF25D3M: the price of a 25Δ butterfly spread, in annualized volatility units (in percent)
- RR10D3M: the price of a 10Δ risk reversal, in annualized volatility units (in percent)
- BF10D3M: the price of a 10Δ butterfly spread, in annualized volatility units (in percent)

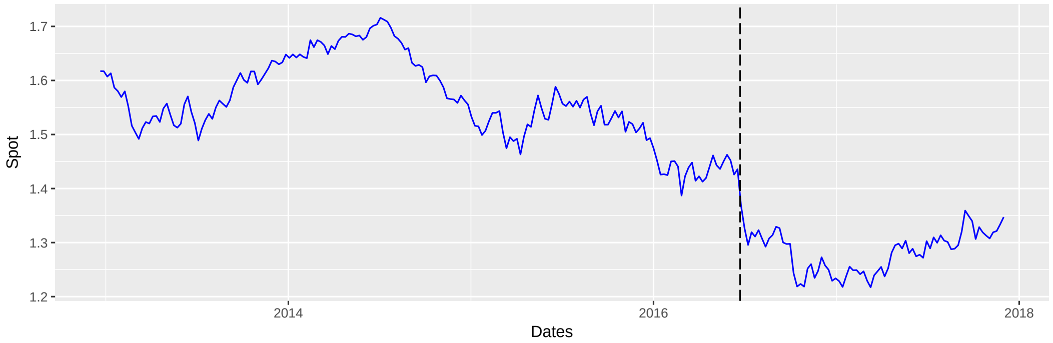
The data (3)

Implied domestic rate from covered interest parity: $F = S\frac{\exp\left(\text{Implied } R_d \times T\right)}{\exp\left(R_f \times T\right)}$



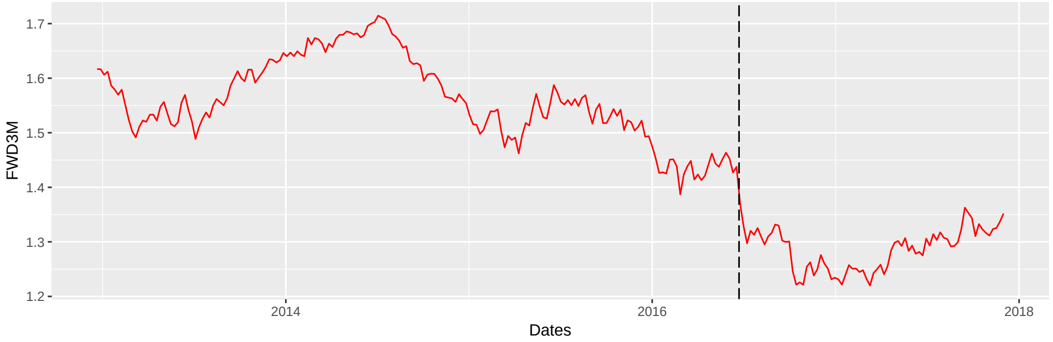
What the data tell us: spot and forward FX rates (1)

```
# Plot of the spot exchange rate with vertical line at Brexit vote
p1 = ggplot(data, aes(Dates, Spot)) + geom_line(colour="blue")
p1 = p1 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),
                     linetype="longdash")
p1
```



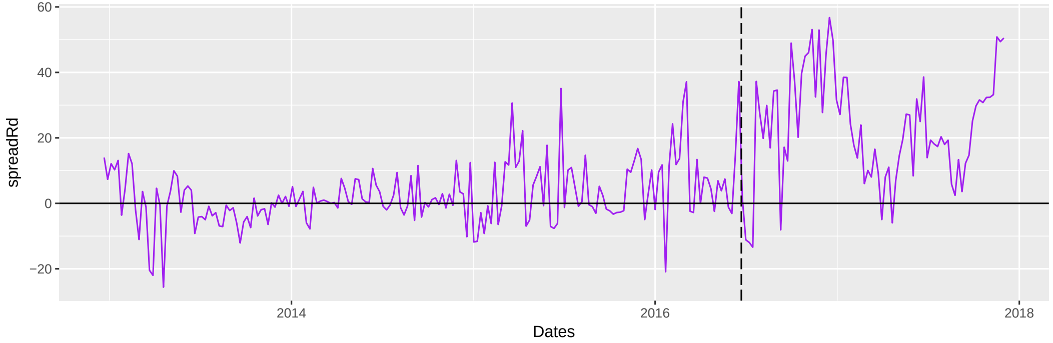
What the data tell us: spot and forward FX rates (2)

```
p2 = ggplot(data, aes(Dates, FWD3M)) + geom_line(colour="red")
p2 = p2 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),
                     linetype="longdash")
p2
```



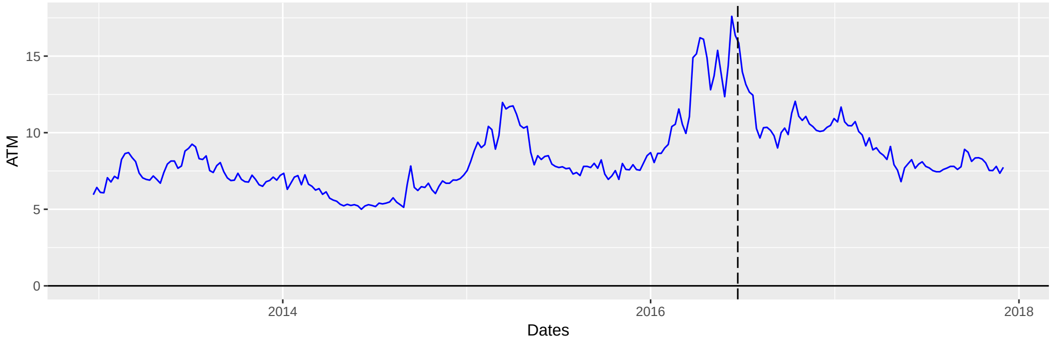
What the data tell us: domestic rate differentials

```r
data$spreadRd = (data$ImpliedRd - data$Rd)*100
p3 = ggplot(data, aes(Dates, spreadRd)) + geom_line(colour="purple") +
        geom_hline(yintercept=0)
p3 = p3 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),
                     linetype="longdash")
p3
```

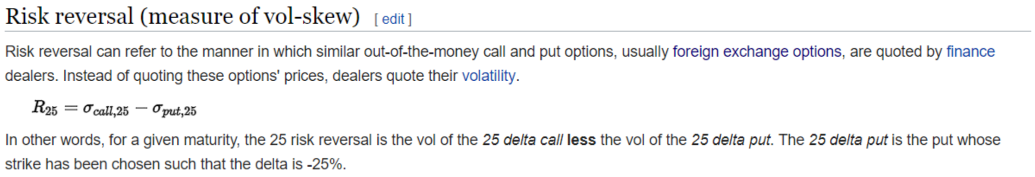


What the data tell us: ATM volatility

```
p4 = ggplot(data, aes(Dates, ATM)) + geom_line(colour="blue") +
        geom_hline(yintercept=0)
p4 = p4 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),
                     linetype="longdash")
p4
```



What the data tell us: risk reversals (1)



Risk reversal (measure of vol-skew) [edit]

Risk reversal can refer to the manner in which similar out-of-the-money call and put options, usually foreign exchange options, are quoted by finance dealers. Instead of quoting these options' prices, dealers quote their volatility.

$$R_{25} = \sigma_{call,25} - \sigma_{put,25}$$

In other words, for a given maturity, the 25 risk reversal is the vol of the *25 delta call* **less** the vol of the *25 delta put*. The *25 delta put* is the put whose strike has been chosen such that the delta is -25%.

Figure 3: Risk reversal definition (Wikipedia)

What the data tell us: risk reversals (2)

- Full understanding of the risk reversal requires knowing what Δ is
- But without knowing, we still use risk reversal quotes to assess the prices market participants place on potential exchange rate movements.

$$RR_{25\Delta} = \sigma_{25\Delta C} - \sigma_{25\Delta P}$$

- Pay $\sigma_{25\Delta C}$ for owning the call
- Offset cost somewhat by selling the put at $\sigma_{25\Delta P}$

What the data tell us: risk reversals (3)

- Risk reversal payoff, long a call and short a put, both of them OTM:

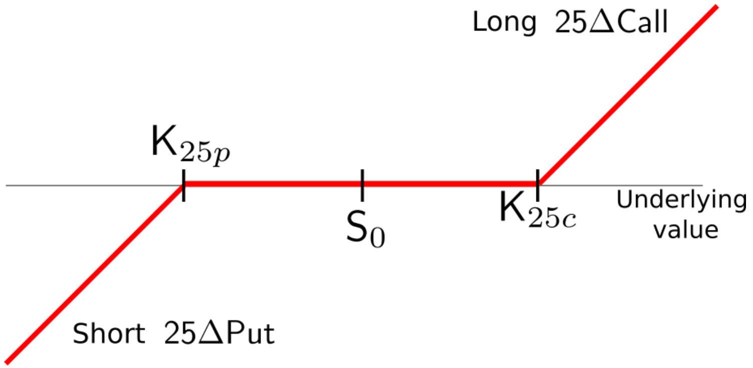
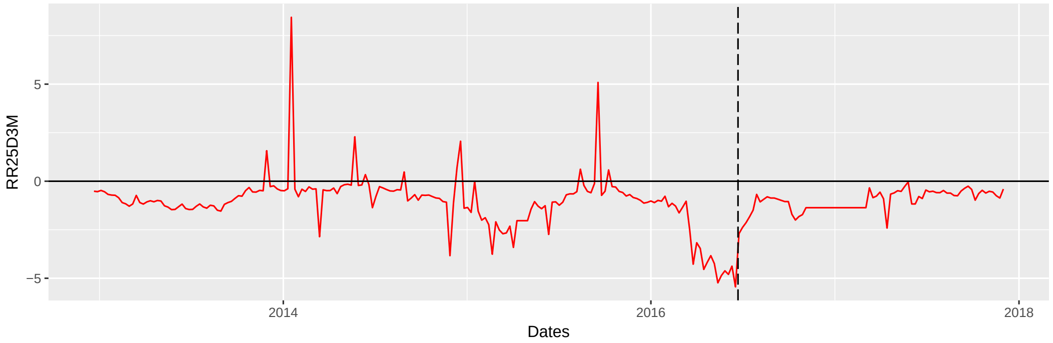


Figure 4: Risk reversal payoff at maturity

- Explain why the price could be positive (or negative)?

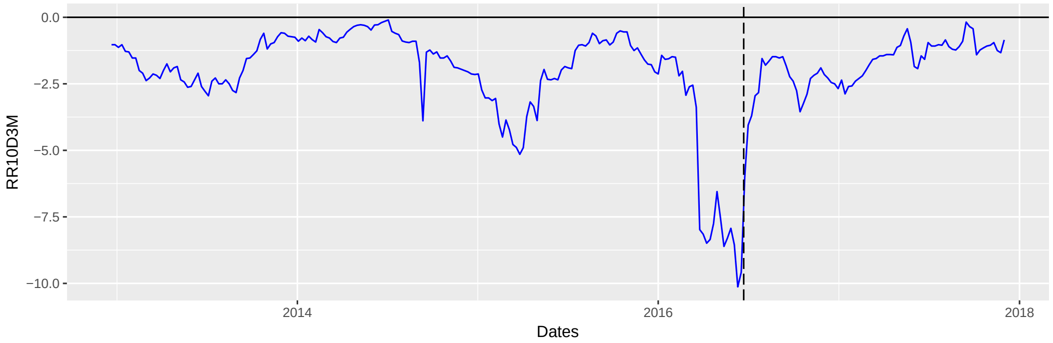
What the data tell us: risk reversals (4)

```
p5 = ggplot(data, aes(Dates, RR25D3M)) + geom_line(colour="red") +
        geom_hline(yintercept=0)
p5 = p5 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),
                     linetype="longdash")
p5
```



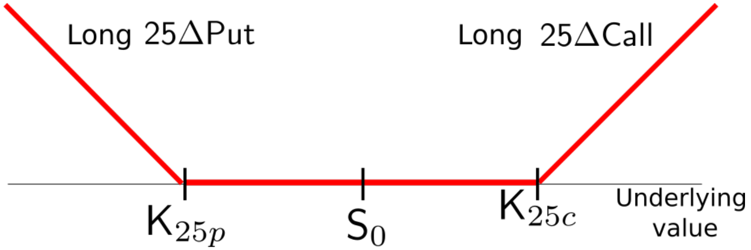
What the data tell us: risk reversals (5)

```
p6 = ggplot(data, aes(Dates, RR10D3M)) + geom_line(colour="blue") +
        geom_hline(yintercept=0)
p6 = p6 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),
                     linetype="longdash")
p6
```



What the data tells us: butterfly spreads (1)

- Suppose we want to profit from large movements of the exchange rate
- The strangle would make our day !!



What the data tells us: butterfly spreads (2)

- The cost of the strangle is:

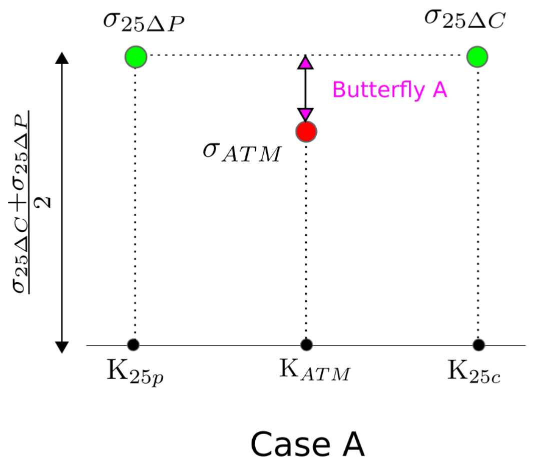
$$S_{25\Delta} = \sigma_{25\Delta C} + \sigma_{25\Delta P}$$

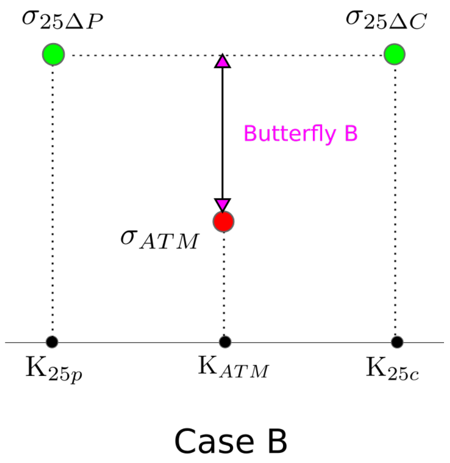
- But markets don't quote strangles
- They quote **Butterfly Spreads**

$$BF_{25\Delta} = \frac{\sigma_{25\Delta C} + \sigma_{25\Delta P}}{2} - \sigma_{ATM}$$

What the data tells us: butterfly spreads (3)

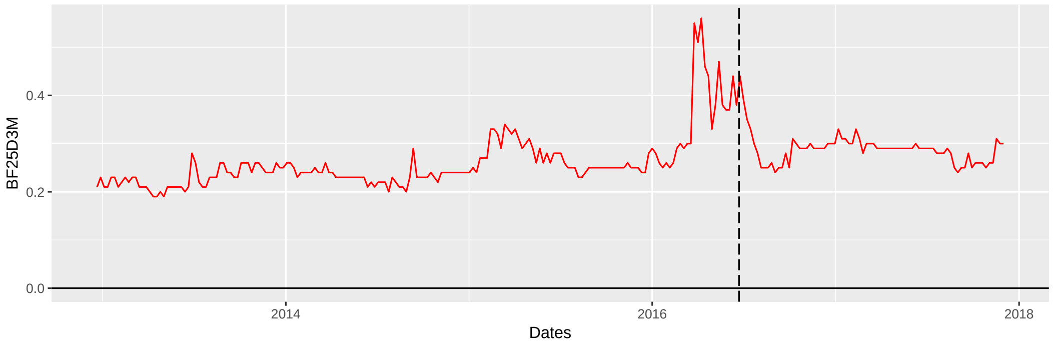
- Butterfly spreads convey more information





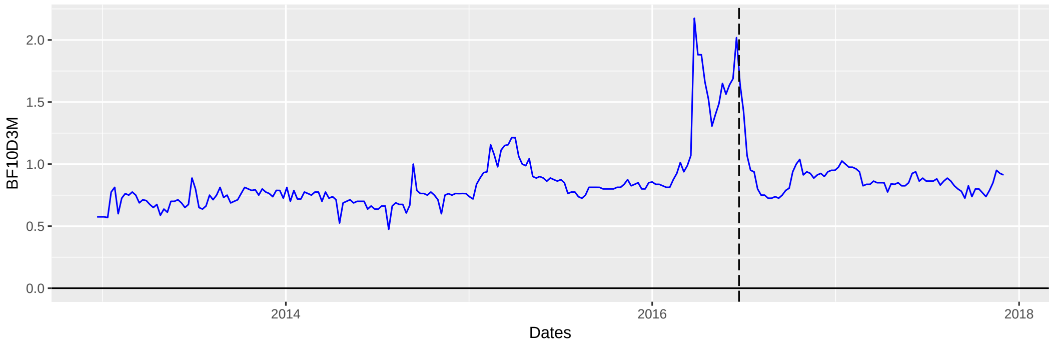
What the data tells us: butterfly spreads (4)

```
p7 = ggplot(data, aes(Dates, BF25D3M)) + geom_line(colour="red") +
        geom_hline(yintercept=0)
p7 = p7 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),
                     linetype="longdash")
p7
```



What the data tells us: butterfly spreads (5)

```
p8 = ggplot(data, aes(Dates, BF10D3M)) + geom_line(colour="blue") +
        geom_hline(yintercept=0)
p8 = p8 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),
                     linetype="longdash")
p8
```



Part C:
A Little Bit of Option Pricing

What is Δ (1)

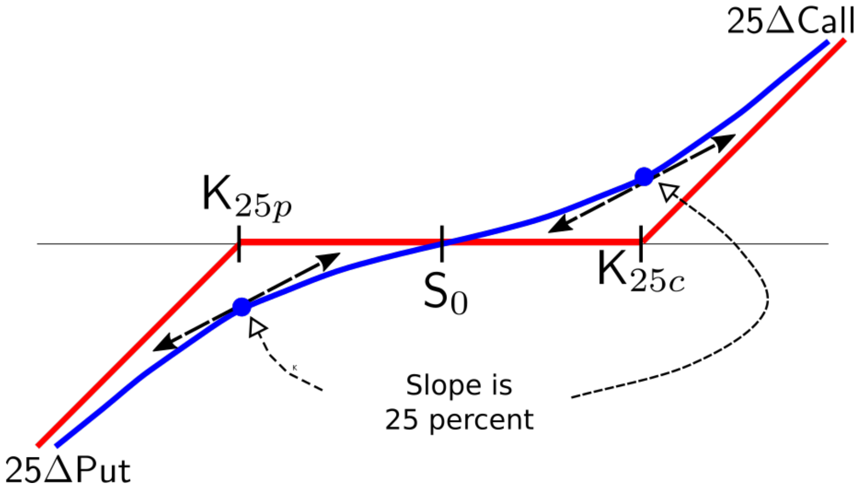
- The Δ of a call

$$\Delta_C = \frac{\partial C}{\partial S} \geq 0$$

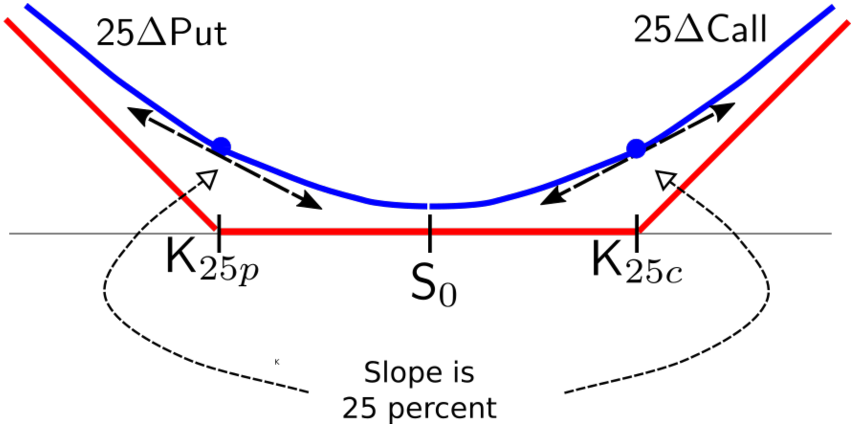
- The Δ of a put

$$\Delta_P = \frac{\partial P}{\partial S} \leq 0$$

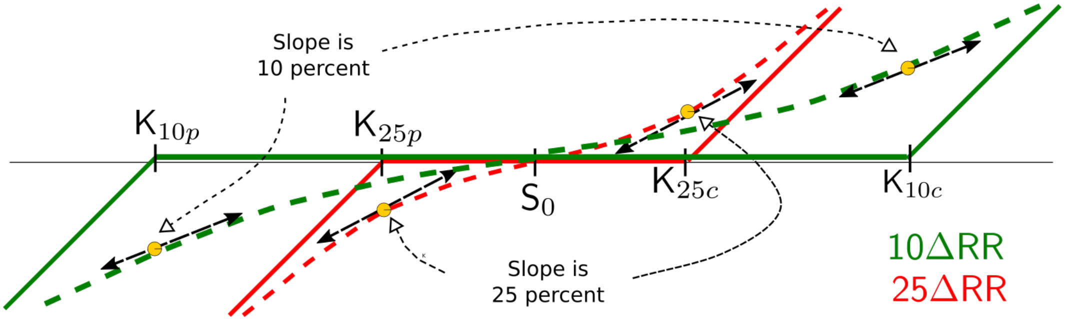
What is Δ (2)



What is Δ (3)



What is Δ (4)



Slope is 10 percent
Δ
K_{10p}
K_{25p}
S_0
K_{25c}
K_{10c}
Slope is 25 percent
10ΔRR
25ΔRR

Option prices = replication cost (1)

Dealer costs when buying call option from client

- Funding cost: borrow C at R_d: $R_d C_t \delta t$
- Hedging cost (using delta-hedging)
  - Borrow and sell Δ units of foreign currency
  - Receive ΔS and reinvest at R_d
  - Pay accrued interest on borrowed amount of currency
  - Net gain: $(R_d - R_f)\Delta S_t \delta t$

Option price = replication cost (2)

- Time decay of option: loses value as maturity approaches

$$\frac{\partial C_t}{\partial t}\delta t$$

- Convexity gains since options are non-linear

$$\frac{\partial^2 C_t}{\partial S^2}(\delta S_t)^2$$

Option price = replication cost (3)

- Gains must offset costs, yielding the pricing partial differential equation:

$$\frac{\partial C_t}{\partial t}\delta t + (R_d - R_f)\Delta S_t \delta t + \frac{\partial^2 C}{\partial S_t^2}(\delta S_t)^2 = R_d C_t \delta t$$

Garman-Kohlhagen formula (1)

Assume FX follows a geometric brownian motion, the call option price is:

$$\boxed{C(K, S_t, R_d, R_f, T, \sigma) = S_t \exp(-R_f \times (T - t))N(d_1) - K \exp(-R_d \times (T - t))N(d_2)}$$

and the put option price is:

$$\boxed{P(K, S_t, R_d, R_f, T, \sigma) = K \exp(-R_d \times (T - t))N(-d_2) - S_t \exp(-R_f \times (T - t))N(-d_1)}$$

Garman-Kohlhagen formula (2)

- K is the strike price,
- S_t is the current spot exchange rate,
- R_d is the domestic interest rate,
- R_f is the foreign interest rate,
- $T - t$ is the remaining life of an option maturing at time T,
- σ is the implied volatility of the exchange rate used to price the option,
- $$d_1 = \frac{\ln(S_t/K) + (R_d - R_f + \sigma^2/2)(T-t)}{\sigma \times (T-t)}$$
  $$d_2 = d_1 - \sigma \times (T-t)$$

Implied volatility σ (1)

- All GK formula inputs are observable except implied volatility σ
- Option prices are quoted as **vols**, i.e. volatility
- To obtain price
  - Take quoted vol
  - Use reference values for all other variables
  - Plug into GK formula

Implied volatility σ (2)

Implied volatility

- is different from historical or realized volatility

- is not a forecast of future volatility

- yields an GK option premium reflecting
  - profit margin
  - hedging costs
  - demand and supply in the FX market

Part D:
The Volatility Smile

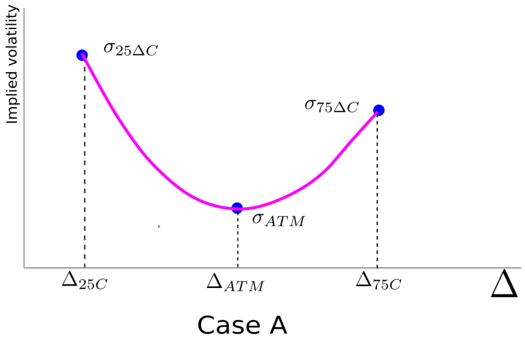
Finding vols for different Δs

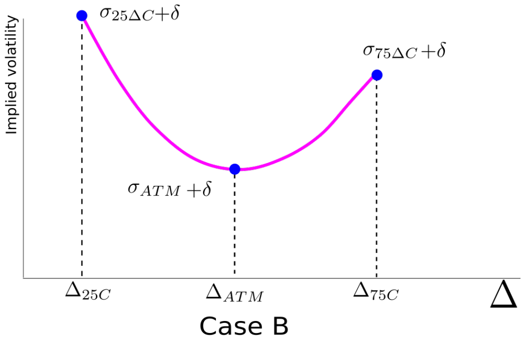
- For a given Δ we have prices for:
  - Risk reversal (RR)
  - Butterfly spread (BF)
- The ATM vol, σ_{ATM} is also given
- From the definitions of the RR and the BF

$$\sigma_{25\Delta C} = \sigma_{ATM} + BF_{25\Delta} + \frac{1}{2} RR_{25\Delta}$$
$$\sigma_{25\Delta P} = \sigma_{ATM} + BF_{25\Delta} - \frac{1}{2} RR_{25\Delta}$$
$$\sigma_{75\Delta C} = \sigma_{25\Delta P} \text{ by put-call parity}$$

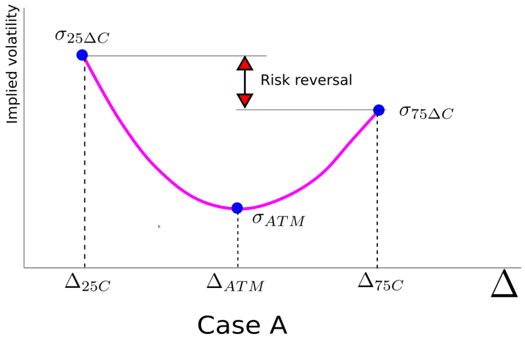
- Plotting σ against Δ yields the volatility smile

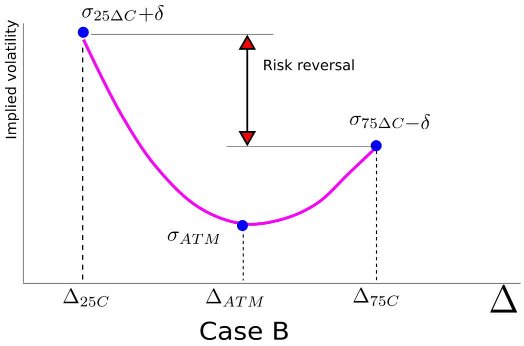
The level of the smile: σ_{ATM}



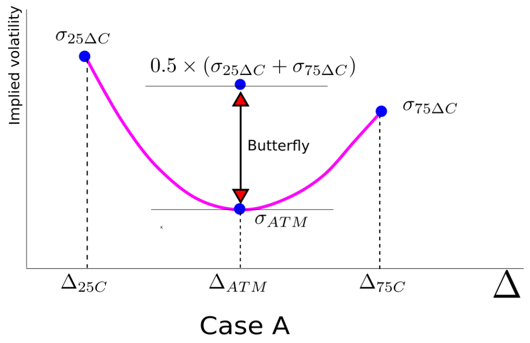


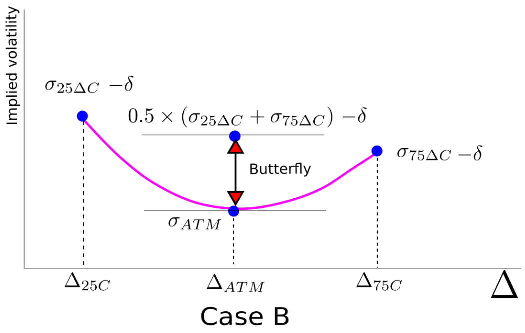
The slope of the smile: the risk reversal





The curvature of the smile: the butterfly spread





Implied volatility
$\sigma_{25\Delta C}\ -\delta$
$0.5 \times (\sigma_{25\Delta C} + \sigma_{75\Delta C})\ -\delta$
$\sigma_{75\Delta C}\ -\delta$
Butterfly
σ_{ATM}
Δ_{25C}
Δ_{ATM}
Δ_{75C}
Δ
Case B

Part E:
Constructing the Volatility Smile A Brexit case study

Selecting the dates

We are interested in the behavior of the GBPUSD for three dates:

- January 8, 2016 (Pre-Brexit)
- June 24, 2016 (Brexit)
- December 30, 2016 (Post-Brexit)

Getting the data (1)

```
rm(list=ls())                                               # Clean up memory
filename = "2018_IET_Options_data.csv"                      # Name of CSV data file
data = read.csv(filename, header=TRUE)                      # Load datafile
data$Dates = mdy_hm(as.character(data$Dates))               # convert dates to Date
rownames(data)=NULL                                         # remove row names

# Specify dates for analysis

date01 = as.Date("2016-01-08 UTC")
date02 = as.Date("2016-06-24 UTC")
date03 = as.Date("2016-12-30 UTC")
```

Getting the data (2)

```
# Create data frame this.data

this.data = rbind(
  data[which(data$Dates==date01),],
  data[which(data$Dates==date02),],
  data[which(data$Dates==date03),])

# Delete row names and change the names of the columns

rownames(this.data) = NULL
colnames(this.data) = c("Date","spot","forward","atm", "rr25","bf25",
                        "rr10","bf10","rf","rd","imp_rd")
```

Get additional vols

In addition to the 25Δ and 75Δ vols, obtain the 10Δ and 90Δ vols:

$$
\begin{aligned}
\sigma_{10\Delta C} &= \sigma_{ATM} + BF_{10\Delta} + \frac{1}{2} RR_{10\Delta} \\
\sigma_{10\Delta P} &= \sigma_{ATM} + BF_{10\Delta} - \frac{1}{2} RR_{10\Delta} \\
\sigma_{90\Delta C} &= \sigma_{10\Delta P}
\end{aligned}
$$

Calculating the implied vols (1)

```
# Vols are in percent, expressed them as simple numbers
this.data$atm  = this.data$atm/100
this.data$rr25 = this.data$rr25/100
this.data$bf25 = this.data$bf25/100
this.data$rr10 = this.data$rr10/100
this.data$bf10 = this.data$bf10/100
this.data$rf   = this.data$rf/100
this.data$rd   = this.data$rd/100
this.data$imp_rd=this.data$imp_rd/100

# We will use this.data repeatedly
# Attach it to access its elements

attach(this.data)
```

Calculating the implied vols (2)

```
# Recover vols for different deltas and put them in the data frame

this.data$sigma10c = atm + bf10 + 0.5*rr10
this.data$sigma25c = atm + bf25 + 0.5*rr25
this.data$sigma75c = this.data$sigma25c - rr25
this.data$sigma90c = this.data$sigma10c - rr10
this.data$sigmaatm = atm

Tenor = 3/12    # Maturity of options, 3 months, in years
```

What is the proper *ATM* strike?

- Retail products

$$K_{ATM} = S$$
$$\Delta_{ATM} = N\left(\frac{\log(F/S) + \frac{1}{2}\sigma_{ATM}^2 T}{\sigma_{ATM}\sqrt{T}}\right)$$

- EM currencies, maturities more than one year

$$K_{ATM} = F$$
$$\Delta_{ATM} = \exp(-Rf \times T)N(\frac{1}{2}\sigma_{ATM}\sqrt{T})$$

- Major currencies, maturities of one year or less

$$K_{ATM} = F \times \exp\left(0.5\sigma_{ATM}^2 \times T\right)$$
$$\Delta_{ATM} = 0.5 \times exp(-Rf \times T) \simeq 0.5$$

Calculate the Δ_{ATM}

```
# Calculate the strike of the ATM option
K_atm = forward*exp((0.5*(this.data$sigmaatm)^2)*Tenor)
deltaATM = 0.5*exp(-rf*Tenor)
deltaATM
```

```
## [1] 0.4989636 0.4993504 0.4994441
```

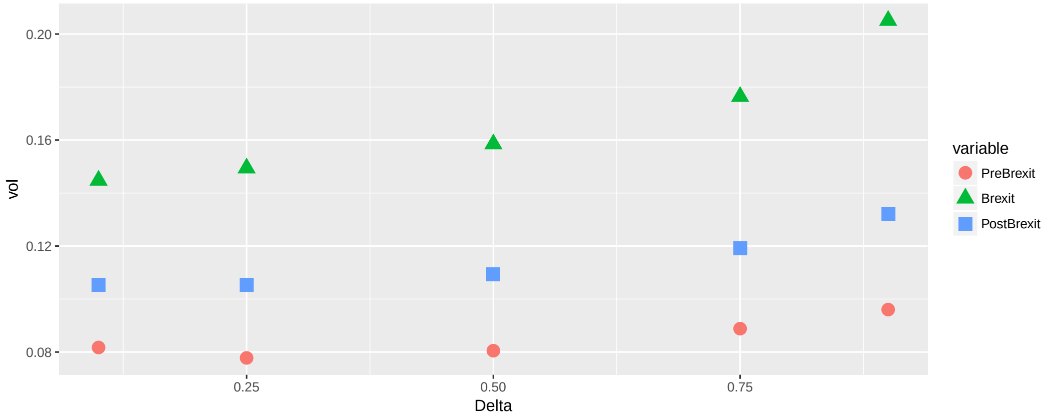
Rough volatility smile (1)

```r
# Select only the vols for each delta
list_variables = c("sigma10c", "sigma25c", "sigmaatm",
                   "sigma75c", "sigma90c")

# Read the data as a matrix
vol_data = t(as.matrix(subset(this.data, select=list_variables)))

# Group the deltas in a vector, to be used in the x-axis
delta_vector = c(0.10, 0.25, 0.5, 0.75, 0.9)

# Create the data frame for the chart
vol.smile  = data.frame(delta_vector, vol_data)
rownames(vol.smile) = NULL
colnames(vol.smile) = c("Delta","PreBrexit","Brexit","PostBrexit")
```

Rough volatility smile (2)

```
library(reshape2)
vol.data = melt(vol.smile, id="Delta")
ggplot(data=vol.data, aes(x=Delta, y=value, shape=variable)) +
   geom_point(aes(colour=variable), size=4) +
   labs(y="vol")
```

Rough volatility smile (3)



A more refined volatility smile (1)

```r
# Fit second degree polynomial to smile
fit.Vol = function(data.vol, data.delta,delta.range)
{
  poly.fit = lm(data.vol ~ poly(data.delta, 2, raw=TRUE))

  # Use fitted polynomial to interpolate Delta-Vol Curve
  delta.square = delta.range*delta.range
  delta.interc = rep(1,length(delta.range))

  X = cbind(delta.interc, delta.range, delta.square)
  iVolInterpol = t(t(X)*poly.fit$coefficients)
  iVolInterpol = rowSums(iVolInterpol)

  return(iVolInterpol)
}
```

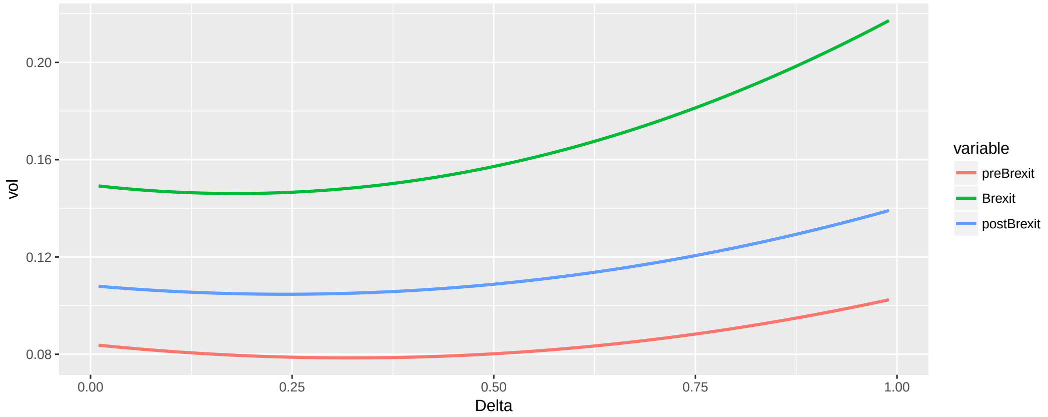
A more refined volatility smile (2)

```
# Get the data points from the vol.smile data frame
data.delta = vol.smile$Delta
data.vol  = vol.smile$PreBrexit

# Interpolation and extrapolation range
delta.range = seq(from=0.01, to = 0.99, by=0.005)

# Obtain the volatility smiles
smile.preBrexit =fit.Vol(vol.smile$PreBrexit, data.delta, delta.range)
smile.Brexit    =fit.Vol(vol.smile$Brexit, data.delta, delta.range)
smile.postBrexit=fit.Vol(vol.smile$PostBrexit, data.delta, delta.range)
```

A more refined volatility smile (3)

```r
# Group the extended volatility smiles in a data frame
smile.df = data.frame(delta.range, smile.preBrexit,
                      smile.Brexit, smile.postBrexit)
rownames(smile.df) = NULL
colnames(smile.df) = c("Delta","preBrexit","Brexit", "postBrexit")

# Create the chart
library(reshape2)
smile.data = melt(smile.df, id="Delta")
ggplot(data = smile.data, aes(x=Delta, y=value, colour=variable)) +
  geom_line(size=1) + labs(y="vol")
```

A more refined volatility smile (4)



Part F:
Extracting Risk-Neutral Densities

Market views are not forecasts

Market views

- They are not forecast of the real-world FX distribution
- Market prices weigh risk aversion of
  - hedgers
  - speculators
  - market makers
- “Technical” factors may affect weight of events
- Risk-neutral, weighting expectations and risk aversion
  - Not easy to disentangle them

Extraction methods

Methods fall under one of three categories:

- Parametric methods
- Semiparametric methods
- Non-parametric methods

Parametric methods

- Specify a parametric distribution function F for the exchange rate
- For the distribution parameters Λ, the price of a call option is

$$C^F(K) = \exp(-R_d \times T) \int_K^\infty (S_T - K)\, dF(S_T; \Lambda),$$

- The parameters that fit the market-implied distribution better solve

$$\arg\min_\Lambda \sum_{j=1}^{N} |C^F(K_j) - C^{\text{Market}}(K_j)|^2$$

Semi-parametric methods

- Start with a particular density
- Add expansion terms to distribution
- Each term helps to approximate market-implied distribution

Non-parametric methods

- Plot value of calls against different strike prices, i.e. **market call function**
- Breeden and Litzenberger: risk-neutral distribution proportional to second derivative of the call function:

$$\left.\frac{\partial^2 C}{\partial K^2}\right|_{K=S} = \exp(-R_d \times T)q(S).$$

- Use numerical methods to calculate derivative and find risk neutral distribution

Numerical estimation exercise

- Risk neutral density extraction
  - Generalized beta density (parametric)
  - Edgeworth expansion (semi-parametric)
  - Shimko (non-parametric)
- All methods require constructing call and/or put functions
  - Option premia vs. strike price

Part F:
Extracting Risk-Neutral Densities
Constructing option premium functions

Constructing the market call function (1)

- We need to obtain the market call function
- In other words, move from Δ - σ space to *Option premium* - *Strike* space
- Two useful functions in `auxFunctions.R`
  - `get.strike()`
  - `GKoption.premium()`

Constructing the market call function (2)

```r
get.strike = function(vol,delta,S0,fwd,rf,Tenor)
{
  aux = qnorm(delta*exp(rf*Tenor))
  aux = aux*vol*sqrt(Tenor)
  aux = aux - 0.5*vol*vol*Tenor
  K = exp(-aux)*fwd
  return(K)
}
```

Constructing the market call function (3)

```
GKoption.premium = function(K,sigma,S,Tenor,fwd,rf,option_type)
{
  if (option_type =="c") {w=1}
  if (option_type =="p") {w=-1}
  d1 = log(fwd/K)+0.5*sigma*sigma*Tenor
  d1 = d1/(sigma*sqrt(Tenor))
  d2 = d1 - sigma*sqrt(Tenor)
  rd = log(fwd/S)/Tenor + rf
  premium = exp(-rd*Tenor)*(w*fwd*pnorm(w*d1) - w*K*pnorm(w*d2))
  return(premium)
}
```

Constructing the market call function (3)

Strike ranges for the selected dates:

```
K_preBrexit = mapply(get.strike,smile.df$preBrexit, smile.df$Delta,
                     S0=spot[1], fwd=forward[1], rf=rf[1], Tenor=0.25)
K_Brexit    = mapply(get.strike,smile.df$Brexit, smile.df$Delta,
                     S0=spot[2], fwd=forward[2], rf=rf[2], Tenor=0.25)
K_postBrexit= mapply(get.strike,smile.df$postBrexit, smile.df$Delta,
                     S0=spot[3], fwd=forward[3], rf=rf[3], Tenor=0.25)
```

Constructing the market call function (4)

Generate the call premia:

```
Call_preBrexit = mapply(GKoption.premium, K_preBrexit,
                        smile.df$preBrexit, S=spot[1],Tenor=Tenor,
                        fwd=forward[1],rf=rf[1], option_type="c")

Call_Brexit = mapply(GKoption.premium, K_Brexit,
                     smile.df$Brexit, S=spot[2],Tenor=Tenor,
                     fwd=forward[2],rf=rf[2],option_type="c")

Call_postBrexit = mapply(GKoption.premium, K_postBrexit,
                         smile.df$postBrexit,S=spot[3],Tenor=Tenor,
                         fwd=forward[3],rf=rf[3],option_type="c")
```

Constructing the market call function (5)

The data frame `callstrike.df` collects the call premium-strike functions for the selected dates:

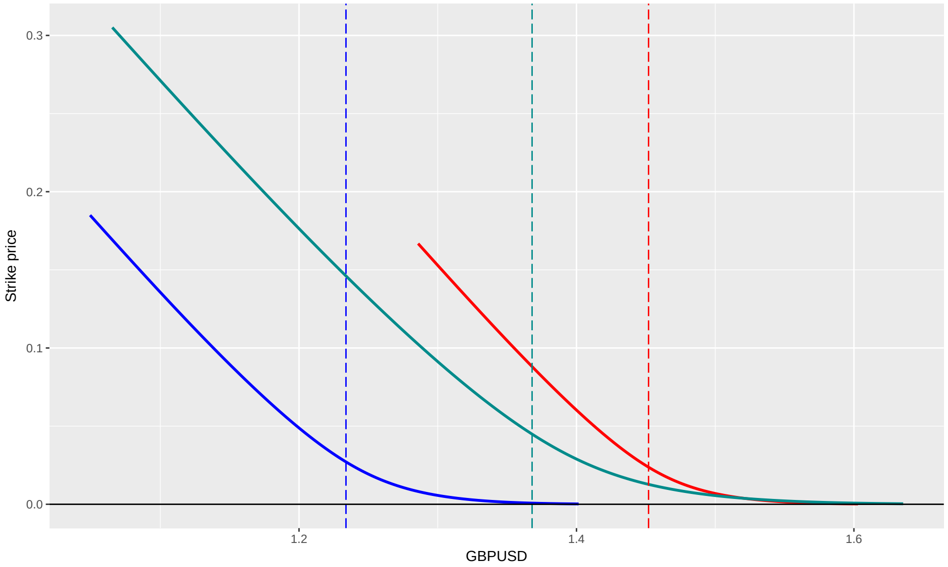
```
callstrike.df = data.frame(K_preBrexit, Call_preBrexit,
                           K_Brexit, Call_Brexit,
                           K_postBrexit, Call_postBrexit)
```

Constructing the market call function (6)

Plot the market call function

```r
ggplot(callstrike.df) +
  geom_line(aes(x=K_preBrexit,y=Call_preBrexit), col="red", size=1) +
  geom_line(aes(x=K_Brexit, y=Call_Brexit), col="darkcyan", size=1) +
  geom_line(aes(x=K_postBrexit, y=Call_postBrexit), col="blue", size=1) +
  labs(x="GBPUSD", y="Strike price") +
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +
  geom_hline(yintercept = 0, col="black")
```

Constructing the market call function (7)



Constructing the market call function (8)

Question: the call premium-strike functions are downward sloping, i.e. the call premium is higher for lower strike prices. Does this make sense? Explain why.

Constructing the market put function (1)

```
Put_preBrexit = mapply(GKoption.premium, K_preBrexit,
                       smile.df$preBrexit,S=spot[1],Tenor=Tenor,
                       fwd=forward[1],rf=rf[1],option_type="p")

Put_Brexit = mapply(GKoption.premium, K_Brexit, smile.df$Brexit,
                        S=spot[2],Tenor=Tenor,fwd=forward[2],
                        rf=rf[2],option_type="p")

Put_postBrexit = mapply(GKoption.premium, K_postBrexit,
                        smile.df$postBrexit,S=spot[3],Tenor=Tenor,
                        fwd=forward[3],rf=rf[3],option_type="p")
```

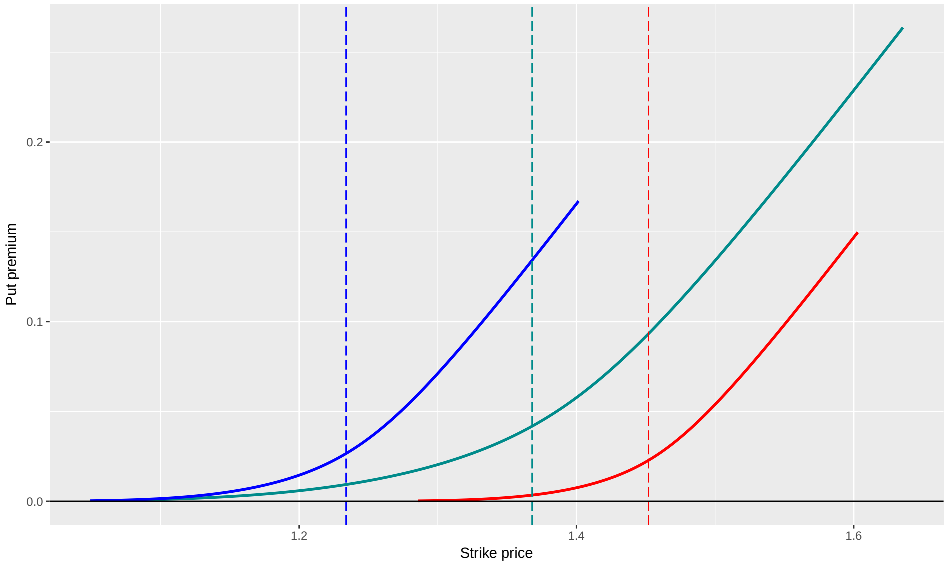
Constructing the market put function (2)

```
putstrike.df = data.frame(K_preBrexit, Put_preBrexit,
                          K_Brexit, Put_Brexit,
                          K_postBrexit, Put_postBrexit)
```

Constructing the market put function (3)

```r
ggplot(putstrike.df) +
  geom_line(aes(x=K_preBrexit,y=Put_preBrexit), col="red", size=1) +
  geom_line(aes(x=K_Brexit, y=Put_Brexit), col="darkcyan", size=1) +
  geom_line(aes(x=K_postBrexit, y=Put_postBrexit), col="blue", size=1) +
  labs(x="Strike price", y="Put premium") +
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +
  geom_hline(yintercept = 0, col="black")
```

Constructing the market put function (4)



Part F:

Extracting Risk-Neutral Densities

Generalized Beta Density

Generalized Beta Density (1)

- Parametric method
- Implemented in package `RND`
- Requires as inputs
  - call premia and strike range
  - put premia and strike range
  - spot rate
  - implied domestic interest rate
  - option tenor

Generalized Beta Density (2)

Let's start with the pre-Brexit period

```
r = imp_rd[1]                 # domestic interest rate
te= Tenor                     # tenor of the option
y = rf[1]                     # foreign interest rate
s0= spot[1]                   # spot exchange rate
call.premium = Call_preBrexit  # vector of call premium values
call.strikes = K_preBrexit     # vector of corresponding call strikes
put.premium = Put_preBrexit    # vector of put premium values
put.strikes = K_preBrexit      # vector of corresponding put strikes
```

Generalized Beta Density (3)

Extract the parameter distribution using the function `extract.gb.density()`:

```r
library(RND)
gb.preBrexit = extract.gb.density(initial.values=c(NA,NA,NA,NA),
                                  r=r, te=te, y=y, s0=s0,
                                  market.calls=call.premium,
                                  call.strikes = call.strikes,
                                  call.weights =1,
                                  market.puts = put.premium,
                                  put.strikes = put.strikes,
                                  put.weights = 1,
                                  lambda=1, hessian.flag=F)
```

Generalized Beta Density (4)

Obtain the risk neutral distribution by passing a range of strike prices

```
Krange = seq(0.9*min(K_preBrexit, K_Brexit, K_postBrexit),
             1.1*max(K_preBrexit, K_Brexit, K_postBrexit),
             0.01)
```

to the function `dgb()`:

```
gb = gb.preBrexit
gb.rnd.preBrexit = dgb(Krange,gb$a, gb$b, gb$v, gb$w)
```

Generalized Beta Density (5)

Do the same for the Brexit . . .

```
r = imp_rd[2]
y = rf[2]
s0= spot[2]
call.premium = Call_Brexit
call.strikes = K_Brexit
put.premium = Put_Brexit
put.strikes = K_Brexit
```

Generalized Beta Density (6)

```
gb.Brexit = extract.gb.density(initial.values=c(NA,NA,NA,NA),
                        r=r, te=te, y=y, s0=s0,
                        market.calls=call.premium,
                        call.strikes = call.strikes,
                        call.weights =1,
                        market.puts = put.premium,
                        put.strikes = put.strikes,
                        put.weights = 1,
                        lambda=1, hessian.flag=F)
gb = gb.Brexit
gb.rnd.Brexit = dgb(Krange,gb$a, gb$b, gb$v, gb$w)
```

Generalized Beta Density (7)

And more of the same for post-Brexit:

```
r = imp_rd[3]
y = rf[3]
s0= spot[3]
call.premium = Call_postBrexit
call.strikes = K_postBrexit
put.premium = Put_postBrexit
put.strikes = K_postBrexit
```

Generalized Beta Density (8)

```
gb.postBrexit = extract.gb.density(initial.values=c(NA,NA,NA,NA),
                           r=r, te=te, y=y, s0=s0,
                           market.calls=call.premium,
                           call.strikes = call.strikes,
                           call.weights =1,
                           market.puts = put.premium,
                           put.strikes = put.strikes,
                           put.weights = 1,
                           lambda=1, hessian.flag=F)
gb = gb.postBrexit
gb.rnd.postBrexit = dgb(Krange,gb$a, gb$b, gb$v, gb$w)
```

Generalized Beta Density (9)

```
gb.rnd.df = data.frame(Krange,gb.rnd.preBrexit,
                       gb.rnd.Brexit, gb.rnd.postBrexit)

ggplot(data=gb.rnd.df, aes(x=Krange)) +
  geom_line(aes(y=gb.rnd.preBrexit), col="red", size=1.25) +
  geom_line(aes(y=gb.rnd.Brexit), col="darkcyan", size=1.25) +
  geom_line(aes(y=gb.rnd.postBrexit), col="blue", size=1.25) +
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +
  geom_hline(yintercept=0, col="black", size=0.5) +
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

Generalized Beta Density (10)

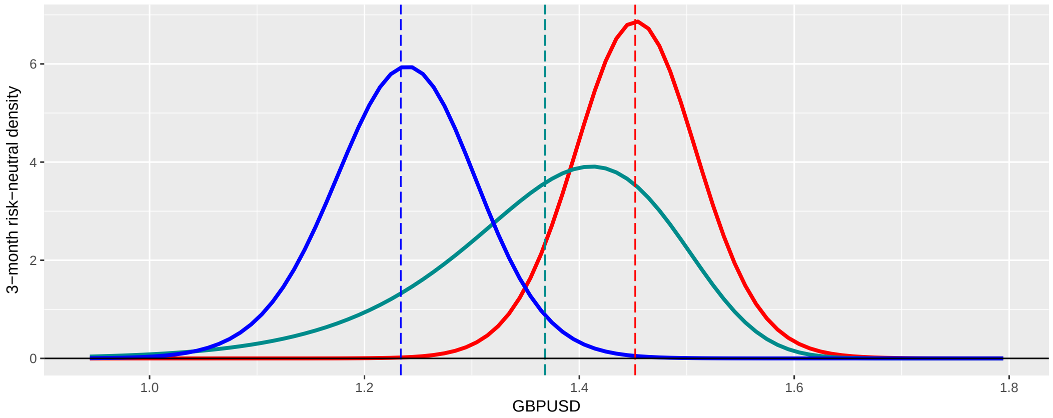


Figure 5: Generalized beta risk neutral distributions. Pre-Brexit: red; Brexit: cyan; post-Brexit: blue

Part F:
Extracting Risk-Neutral Densities
Edgeworth expansion

Edgeworth expansion (1)

- Semi-parametric method
- Implemented in package `RND`
- Requires as inputs
  - call premia and strike range
  - spot rate
  - implied domestic interest rate
  - option tenor

Edgeworth expansion (2)

Calculations in Brexit period

```
r = imp_rd[1]                          # domestic interest rate
te= Tenor                              # tenor of the option
y = rf[1]                              # foreign interest rate
s0= spot[1]                            # spot exchange rate
call.premium = Call_preBrexit          # vector of call premium values
call.strikes = K_preBrexit             # vector of corresponding call strikes
```

Edgeworth expansion (3)

The function `extract.ew.density()` calculates the parameters of the Edgeworth expansion:

```
ew.preBrexit = extract.ew.density(initial.values = rep(NA,2),
                      r=r, y=y, te=te, s0=s0,
                      market.calls=call.premium,
                      call.strikes = call.strikes,
                      call.weights =1, lambda=1, hessian.flag=F,
                      cl = list(maxit=10000))
```

which we then input into the function `dew()` to obtain the distribution.

```
ew = ew.preBrexit
ew.rnd.preBrexit  = dew(Krange,r,y,te,s0,
                        ew$sigma, ew$skew, ew$kurt)
```

Edgeworth expansion (4)

Repeat the recipe for Brexit . . .

```
r = imp_rd[2]                          # domestic interest rate
te= Tenor                              # tenor of the option
y = rf[2]                              # foreign interest rate
s0= spot[2]                            # spot exchange rate
call.premium = Call_Brexit             # vector of call premium values
call.strikes = K_Brexit                # vector of corresponding call strikes
```

Edgeworth expansion (5)

```
ew.Brexit = extract.ew.density(initial.values = rep(NA,2),
                r=r, y=y, te=te, s0=s0,
                market.calls=call.premium,
                call.strikes = call.strikes,
                call.weights =1, lambda=1, hessian.flag=F,
                cl = list(maxit=10000))
ew = ew.Brexit
ew.rnd.Brexit  = dew(Krange,r,y,te,s0,ew$sigma, ew$skew, ew$kurt)
```

Edgeworth expansion (6)

And for post-Brexit

```
r = imp_rd[3]                      # domestic interest rate
te= Tenor                          # tenor of the option
y = rf[3]                          # foreign interest rate
s0= spot[3]                        # spot exchange rate
call.premium = Call_postBrexit     # vector of call premium values
call.strikes = K_postBrexit        # vector of corresponding call strikes
```

Edgeworth expansion (7)

```r
ew.postBrexit = extract.ew.density(initial.values = rep(NA,2),
                    r=r, y=y, te=te, s0=s0,
                    market.calls=call.premium,
                    call.strikes = call.strikes,
                    call.weights =1, lambda=1, hessian.flag=F,
                    cl = list(maxit=10000))
ew = ew.postBrexit
ew.rnd.postBrexit  = dew(Krange,r,y,te,s0,ew$sigma, ew$skew, ew$kurt)
```

Edgeworth expansion (8)

Produce a pretty chart

```
ew.rnd.df = data.frame(Krange, ew.rnd.preBrexit,
                       ew.rnd.Brexit, ew.rnd.postBrexit)

ggplot(data=ew.rnd.df, aes(x=Krange)) +
  geom_line(aes(y=ew.rnd.preBrexit), col="red", size=1.25) +
  geom_line(aes(y=ew.rnd.Brexit), col="darkcyan", size=1.25) +
  geom_line(aes(y=ew.rnd.postBrexit), col="blue", size=1.25) +
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +
  geom_hline(yintercept=0, col="black", size=0.5) +
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

Edgeworth expansion (9)

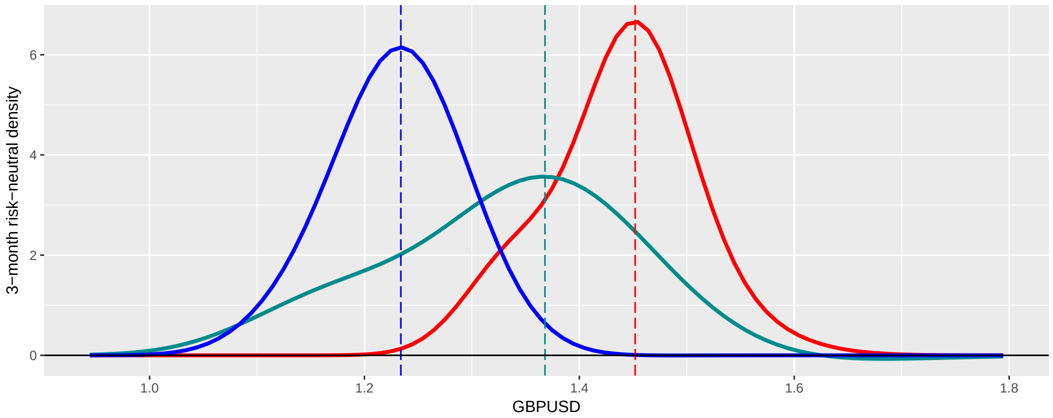


Figure 6: Edgeworth expansion risk neutral distributions. Pre-Brexit: red; Brexit: cyan; post-Brexit: blue

Part F:

Extracting Risk-Neutral Densities

Shimko method

Shimko method (1)

- Nonparametric method
- Implemented in package `RND`
- Method requires a volatility smile in the σ - K space

Shimko method (2)

Construct the volatility smile in the σ - K in the Brexit period

```
vol.preBrexit = vol.data[1:5,3]         # obtain implied volatility
delta.shimko = vol.data[1:5,1]          # obtain the deltas

# Calculate the strike prices corresponding to the observed deltas

Kshimko_preBrexit = mapply(get.strike,vol.preBrexit,
                           delta.shimko, S0=spot[1],
                           fwd=forward[1], rf=rf[1], Tenor=0.25)

# Calculate the call premium

shimko_preBrexit = mapply(GKoption.premium, Kshimko_preBrexit,
                          vol.preBrexit,S=spot[1],Tenor=Tenor,
                          fwd=forward[1],rf=rf[1],option_type="c")
```

Shimko method (3)

The function in the RND package that calculates the parameters of the Shimko's quadratic approximation to the volatility smile is extract.shimko.density():

```
# Inputs for extract.shimko.density()

r = imp_rd[1]                         # implied doomestic rate
te= Tenor                             # time to  maturity
y = rf[1]                             # foreign interest rate
s0= spot[1]                           # spot exchange rate
call.premium = shimko_preBrexit       # call premia values
call.strikes = Kshimko_preBrexit      # option strikes
b=r-y

shimko.preBrexit = extract.shimko.density(market.calls=call.premium,
                                         call.strikes = call.strikes,
                                         r=r, y=b, t=te, s0=s0,
                                         lower=0, upper= 30)
```

Shimko method (4)

The parameters are then fed to the function `dshimko()` to obtain the risk neutral distribution for a wider range of strikes, `Krange`:

```
shimko = shimko.preBrexit

shimko.rnd.preBrexit = dshimko(r=r, te=Tenor,
                               s0=s0, k=Krange, y=y,
                               a0=shimko[[1]]$a0, a1=shimko[[1]]$a1,
                               a2=shimko[[1]]$a2)
```

Shimko method (5)

Repeat for Brexit

```
vol.Brexit = vol.data[6:10,3]         # obtain implied volatility

Kshimko_Brexit = mapply(get.strike,vol.Brexit,
                        delta.shimko,S0=spot[2],
                        fwd=forward[2], rf=rf[2], Tenor=0.25)

shimko_Brexit = mapply(GKoption.premium, Kshimko_Brexit,
                       vol.Brexit,S=spot[2],Tenor=Tenor,
                       fwd=forward[2],rf=rf[2],option_type="c")
```

Shimko method (6)

```
r = imp_rd[2]
y = rf[2]
s0= spot[2]
call.premium = shimko_Brexit
call.strikes = Kshimko_Brexit
b=r-y

shimko.Brexit = extract.shimko.density(market.calls=call.premium,
                                       call.strikes = call.strikes,
                                       r=r, y=b, t=te, s0=s0,
                                       lower=0, upper= 30)

shimko = shimko.Brexit
shimko.rnd.Brexit = dshimko(r=r, te=Tenor, s0=s0, k=Krange, y=y,
                            a0=shimko[[1]]$a0, a1=shimko[[1]]$a1,
                            a2=shimko[[1]]$a2)
```

Shimko method (7)

Once more for post-Brexit

```
vol.postBrexit = vol.data[11:15,3]          # obtain implied volatility

Kshimko_postBrexit = mapply(get.strike,vol.postBrexit,
                        delta.shimko,S0=spot[3],
                        fwd=forward[3], rf=rf[3], Tenor=0.25)

shimko_postBrexit = mapply(GKoption.premium, Kshimko_postBrexit,
                       vol.postBrexit,S=spot[3],Tenor=Tenor,
                       fwd=forward[3],rf=rf[3],option_type="c")
```

Shimko method (8)

```
r = imp_rd[3]
y = rf[3]
s0= spot[3]
call.premium = shimko_postBrexit
call.strikes = Kshimko_postBrexit
b=r-y

shimko.postBrexit = extract.shimko.density(market.calls=call.premium,
                                            call.strikes = call.strikes,
                                            r=r, y=b, t=te, s0=s0,
                                            lower=0, upper= 30)

shimko = shimko.postBrexit
shimko.rnd.postBrexit = dshimko(r=r, te=Tenor, s0=s0, k=Krange, y=y,
                                a0=shimko[[1]]$a0, a1=shimko[[1]]$a1,
                                a2=shimko[[1]]$a2)
```

Shimko method (9)

Fun part: plots

```
shimko.rnd.df = data.frame(Krange,shimko.rnd.preBrexit,
                           shimko.rnd.Brexit, shimko.rnd.postBrexit)

ggplot(data=shimko.rnd.df, aes(x=Krange)) +
  geom_line(aes(y=shimko.rnd.preBrexit), col="red", size=1.25) +
  geom_line(aes(y=shimko.rnd.Brexit), col="darkcyan", size=1.5) +
  geom_line(aes(y=shimko.rnd.postBrexit), col="blue", size=1.25) +
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +
  geom_hline(yintercept=0, col="black", size=0.5) +
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

Shimko method (10)

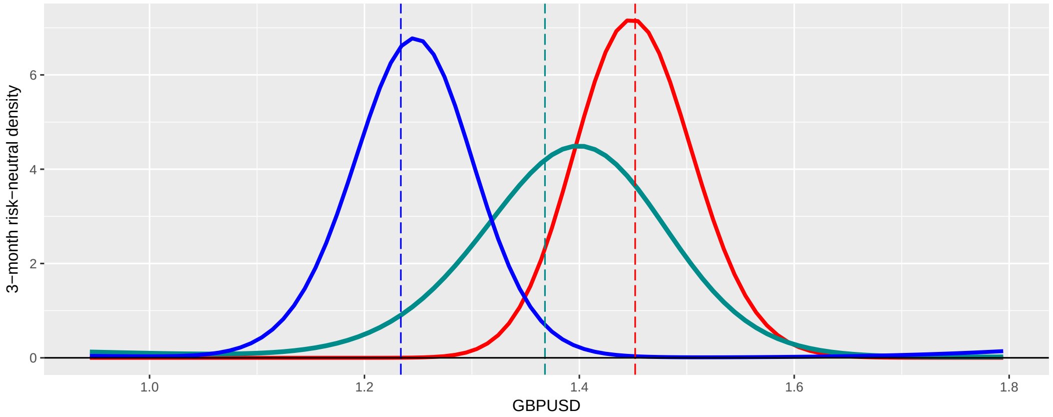


Figure 7: Shimko risk neutral distributions. Pre-Brexit: red; Brexit: cyan; post-Brexit: blue

Part G:
Comparing distributions

Pre-Brexit (1)

```r
ggplot(data=shimko.rnd.df, aes(x=Krange)) +
  geom_line(aes(y=shimko.rnd.preBrexit), col="red", size=1.25) +
  geom_line(data=gb.rnd.df, aes(x=Krange, y=gb.rnd.preBrexit),
            col="darkcyan", size=1.25) +
  geom_line(data=ew.rnd.df, aes(x=Krange, y=ew.rnd.preBrexit),
            col="blue", size=1.25) +
  geom_hline(yintercept=0, col="black", size=0.5) +
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

Pre-Brexit (2)

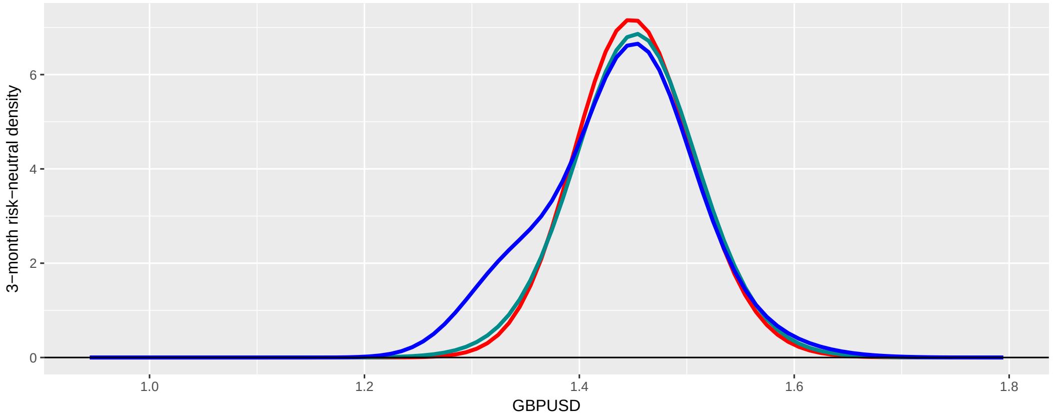


Figure 8: Pre-Brexit risk neutral distributions. Generalized beta: cyan; Edgeworth expansion: blue; Shimko: red.

Brexit (1)

```
ggplot(data=shimko.rnd.df, aes(x=Krange)) +
  geom_line(aes(y=shimko.rnd.Brexit), col="red", size=1.25) +
  geom_line(data=gb.rnd.df, aes(x=Krange, y=gb.rnd.Brexit),
            col="darkcyan", size=1.25) +
  geom_line(data=ew.rnd.df, aes(x=Krange, y=ew.rnd.Brexit),
            col="blue", size=1.25) +
  geom_hline(yintercept=0, col="black", size=0.5) +
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

Brexit (2)

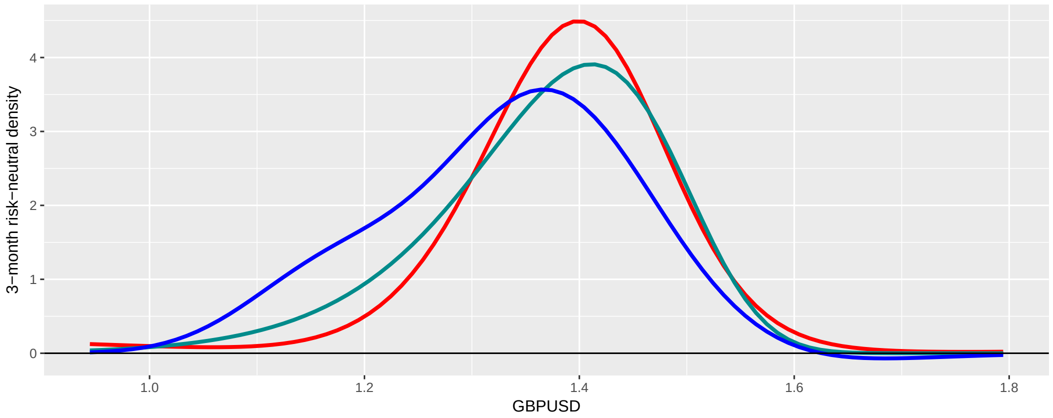


Figure 9: Brexit risk neutral distributions. Generalized beta: cyan; Edgeworth expansion: blue; Shimko: red.

Post-Brexit (1)

```r
ggplot(data=shimko.rnd.df, aes(x=Krange)) +
  geom_line(aes(y=shimko.rnd.postBrexit), col="red", size=1.25) +
  geom_line(data=gb.rnd.df, aes(x=Krange, y=gb.rnd.postBrexit),
            col="darkcyan", size=1.25) +
  geom_line(data=ew.rnd.df, aes(x=Krange, y=ew.rnd.postBrexit),
            col="blue", size=1.25) +
  geom_hline(yintercept=0, col="black", size=0.5) +
  labs(x="GBPUSD", y="3-month risk-neutral density")
```

Post-Brexit(2)

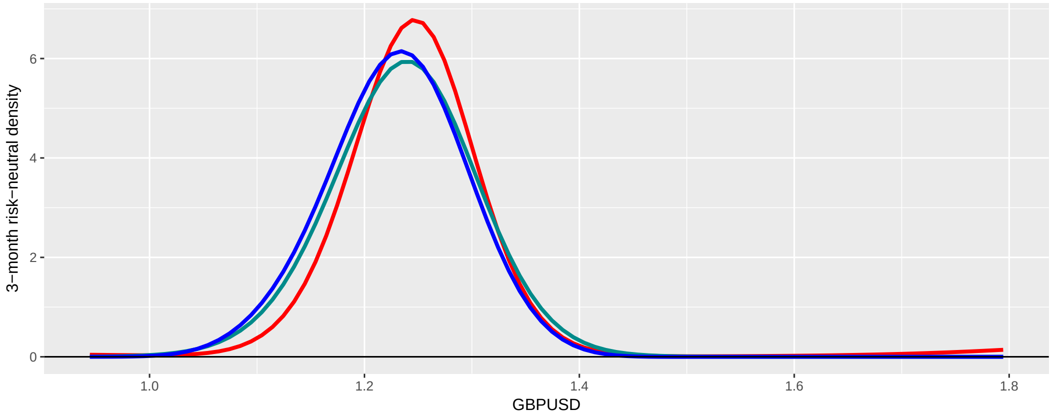


Figure 10: Post-Brexit risk neutral distributions. Generalized beta: cyan; Edgeworth expansion: blue.