### Extracting information from FX Options

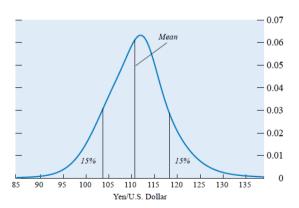
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February 7, 2018

### **Objectives**

▶ We want to build this

Figure 50. Distribution for Yen-Dollar Exchange Rate in Early September 1997 Implied by Options Prices on May 20, 1997



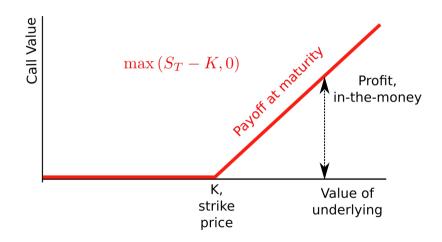
### What we will cover

- Learn about basic FX structures
- Develop some intuition about FX option prices
- Get comfortable with a variety of methods
- ► Side benefit: learn R and RStudio!
- Lecture notes available at https://jchanlauimf.github.io/IMF\_FXOptions

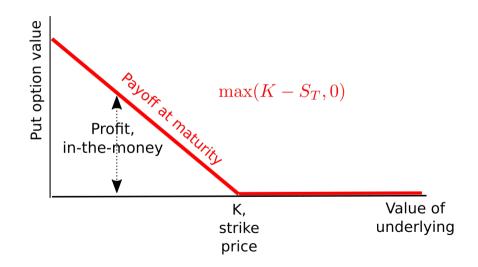
Part A:

The Very Basics

### Basics: a plain vanilla call option



# Basics: a plain vanilla put option



# Market reality

- ► Price quote available for
  - ATM plain options
- ▶ Price quotes available for **FX structures** 
  - risk reversals
  - butterfly spreads
- Main data sources
  - Bloomberg
  - Reuters
  - ▶ Investment banks' data portals

Part B:

Eyeballing the Data

# Getting the data in (1)

- 1. Start RStudio
- 2. Set the working directory to where we placed the data file
- 3. Issue this command in the console (replace with your directory)

```
my_wdir = "D:/IMF_FXCourse" # replace with your directory
setwd(my_wdir)
```

### Getting the data in (2)

Clean memory, set up the needed libraries

```
rm(list=ls())  # Clean up memory
library(ggplot2)  # Graphic library
library(lubridate)  # Data manipulation library
library(dplyr)  # Data manipulation library
source("auxFunctions.R")  # Auxiliary functions
```

# Getting the data in (3)

```
filename = "2018_IET_Options_data.csv"
data = read.csv(filename, header=TRUE)
data$Dates = mdy_hm(as.character(data$Dates))
```

2018_ChanLau_Brexit_Case_Study.Rmd ×			data ×								
♦ ♦ D Filter											
	Dates <sup>‡</sup>	Spot ‡	FWD3M <sup>‡</sup>	ATM <sup>‡</sup>	RR25D3M	BF25D3M	RR10D3M	BF10D3M	Rf ‡	Rd ≎	ImpliedRå
1	2012-12-21	1.61725	1.616769	5.9500	-0.516	0.21	-1.03	0.5750	0.490	0.340	0.4800
2	2012-12-28	1.61705	1.616364	6.4250	-0.539	0.23	-1.03	0.5750	0.530	0.340	0.4135
3	2013-01-04	1.60715	1.606236	6.1000	-0.475	0.21	-1.13	0.5750	0.490	0.330	0.4510
4	2013-01-11	1.61325	1.611847	6.0750	-0.544	0.21	-1.03	0.5690	0.570	0.320	0.4225
5	2013-01-18	1.58680	1.586463	7.0625	-0.680	0.23	-1.28	0.7750	0.560	0.310	0.4410
6	2013-01-25	1.58030	1.579088	6.7800	-0.715	0.23	-1.30	0.8130	0.480	0.405	0.3690
7	2013-02-01	1.56925	1.569613	7.1500	-0.724	0.21	-1.53	0.6000	0.480	0.310	0.3540

# The data (1)

#### FX related

- ▶ Spot: the GBPUSD spot exchange rate, i.e. USD per GBP
- ► FWD3M: the 3-month GBPUSD forward exchange rate
- ▶ Rf: the 3-month GBP money market deposit rate, annualized (in percent)
- ▶ Rd: the 3-month USD money market deposit rate, annualized (in percent)

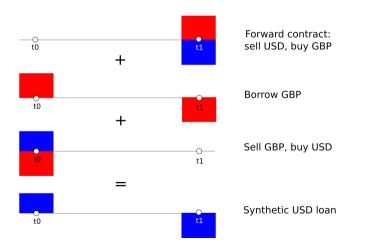
# The data (2)

### FX option related

- ► ATM: the at-the-money implied volatility of a GBPUSD option with strike price equal to ATM
- ▶ RR25D3M: the price of a  $25\Delta$  risk reversal, in annualized volatility units (in percent)
- ▶ BF25D3M: the price of a  $25\Delta$  butterfly spread, in annualized volatility units (in percent)
- ightharpoonup RR10D3M: the price of a  $10\Delta$  risk reversal, in annualized volatility units (in percent)
- ▶ BF10D3M: the price of a  $10\Delta$  butterfly spread, in annualized volatility units (in percent)

### The data (3)

Implied domestic rate from covered interest parity:  $F = S \frac{\exp(\text{Implied } R_d \times T)}{\exp(R_f \times T)}$ 

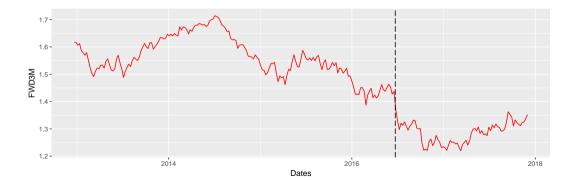


# What the data tell us: spot and forward FX rates (1)

p1

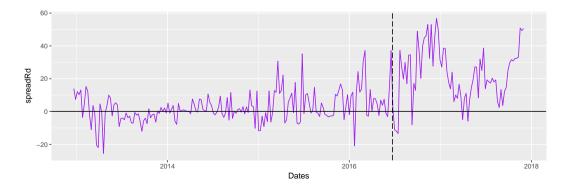


# What the data tell us: spot and forward FX rates (2)



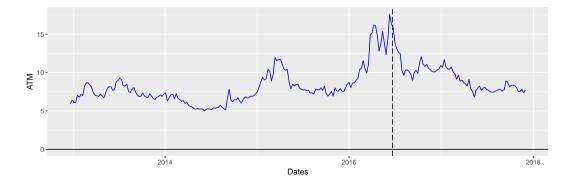
### What the data tell us: domestic rate differentials

рЗ



### What the data tell us: ATM volatility

p4



# What the data tell us: risk reversals (1)

#### Risk reversal (measure of vol-skew) [edit]

Risk reversal can refer to the manner in which similar out-of-the-money call and put options, usually foreign exchange options, are quoted by finance dealers. Instead of quoting these options' prices, dealers quote their volatility.

$$R_{25} = \sigma_{call,25} - \sigma_{put,25}$$

In other words, for a given maturity, the 25 risk reversal is the vol of the 25 delta call less the vol of the 25 delta put. The 25 delta put is the put whose strike has been chosen such that the delta is -25%.

Figure 3: Risk reversal definition (Wikipedia)

# What the data tell us: risk reversals (2)

- ightharpoonup Full understanding of the risk reversal requires knowing what  $\Delta$  is
- ▶ But without knowing, we still use risk reversal quotes to assess the prices market participants place on potential exchange rate movements.

$$RR_{25\Delta} = \sigma_{25\Delta}C - \sigma_{25\Delta}P$$

- ▶ Pay  $\sigma_{25\Delta C}$  for owning the call
- ▶ Offset cost somewhat by selling the put at  $\sigma_{25\Delta P}$

# What the data tell us: risk reversals (3)

▶ Risk reversal payoff, long a call and short a put, both of them OTM:

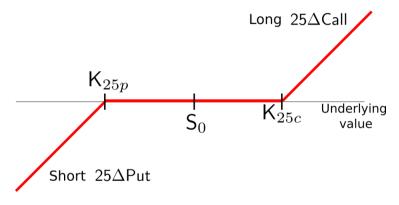


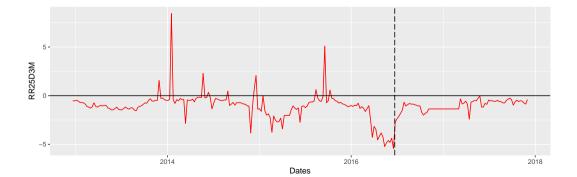
Figure 4: Risk reversal payoff at maturity

Explain why the price could be positive (or negative)?

# What the data tell us: risk reversals (4)

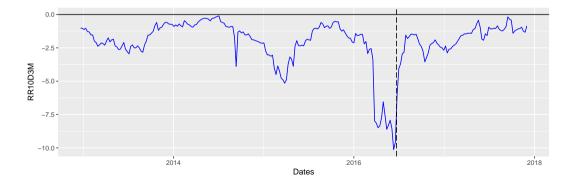
```
p5 = ggplot(data, aes(Dates, RR25D3M)) + geom_line(colour="red") +
        geom hline(vintercept=0)
p5 = p5 + geom_vline(xintercept=as.POSIXct(as.Date(("2016-06-22 UTC"))),
                     linetype="longdash")
```

р5



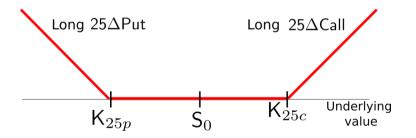
# What the data tell us: risk reversals (5)

р6



# What the data tells us: butterfly spreads (1)

- ▶ Suppose we want to profit from large movements of the exchange rate
- ▶ The strangle would make our day !!



# What the data tells us: butterfly spreads (2)

► The cost of the strangle is:

$$S_{25\Delta} = \sigma_{25\Delta C} + \sigma_{25\Delta P}$$

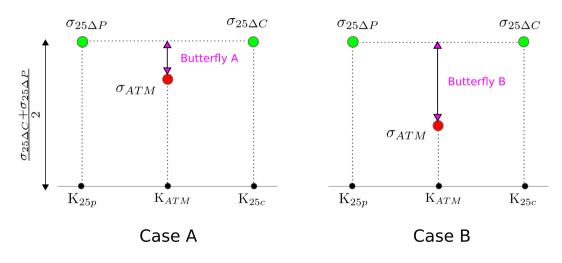
But markets don't quote strangles

► They quote **Butterfly Spreads** 

$$BF_{25\Delta} = \frac{\sigma_{25\Delta C} + \sigma_{25\Delta P}}{2} - \sigma_{ATM}$$

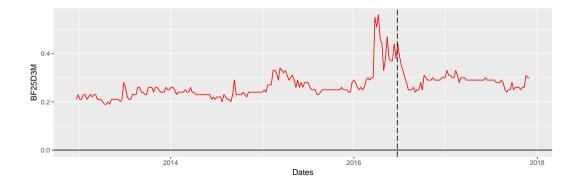
# What the data tells us: butterfly spreads (3)

▶ Butterfly spreads convey more information



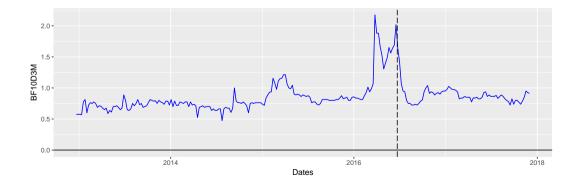
# What the data tells us: butterfly spreads (4)

p7



# What the data tells us: butterfly spreads (5)

p8



Part C:

A Little Bit of Option Pricing

# What is $\Delta$ (1)

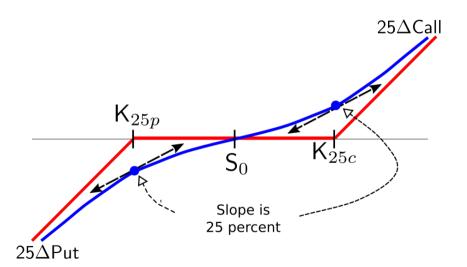
ightharpoonup The  $\Delta$  of a call

$$\Delta_{\mathcal{C}} = \frac{\partial \mathcal{C}}{\partial \mathcal{S}} \geq 0$$

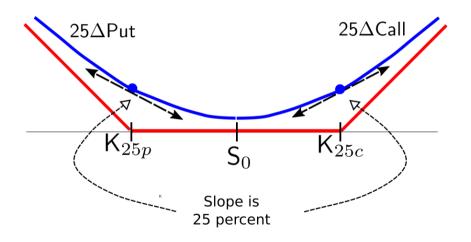
► The ∆ of a put

$$\Delta_P = \frac{\partial P}{\partial S} \le 0$$

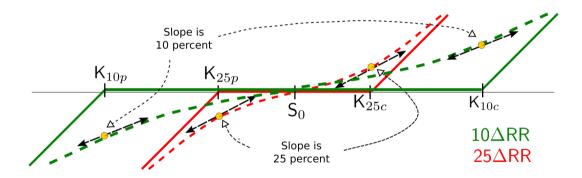
# What is $\Delta$ (2)



# What is $\Delta$ (3)



# What is $\Delta$ (4)



# Option prices = replication cost (1)

Dealer costs when buying call option from client

- ▶ Funding cost: borrow C at  $R_d$ :  $R_dC_t\delta t$
- Hedging cost (using delta-hedging)
  - ightharpoonup Borrow and sell  $\Delta$  units of foreign currency
  - Receive  $\Delta S$  and reinvest at  $R_d$
  - Pay accrued interest on borrowed amount of currency
  - Net gain:  $(R_d R_f)\Delta S_t \delta t$

# Option price = replication cost (2)

▶ Time decay of option: loses value as maturity approaches

$$\frac{\partial C_t}{\partial t} \delta t$$

Convexity gains since options are non-linear

$$\frac{\partial^2 C_t}{\partial S^2} (\delta S_t)^2$$

# Option price = replication cost (3)

▶ Gains must offset costs, yielding the pricing partial differential equation:

$$\frac{\partial C_t}{\partial t} \delta t + (R_d - R_f) \Delta S_t \delta t + \frac{\partial^2 C}{\partial S_t^2} (\delta S_t)^2 = R_d C_t \delta t$$

# Garman-Kohlhagen formula (1)

Assume FX follows a geometric brownian motion, the call option price is:

$$C(K, S_t, R_d, R_f, T, \sigma) = S_t \exp(-R_f \times (T - t))N(d_1) - K \exp(-R_d \times (T - t))N(d_2)$$

and the put option price is:

$$P(K, S_t, R_d, R_f, T, \sigma) = K \exp(-R_d \times (T - t))N(-d_2) - S_t \exp(-R_f \times (T - t))N(-d_1)$$

# Garman-Kohlhagen formula (2)

K is the strike price,

- $ightharpoonup S_t$  is the current spot exchange rate,
- $ightharpoonup R_d$  is the domestic interest rate,
- $ightharpoonup R_f$  is the foreign interest rate,
- ightharpoonup T-t is the remaining life of an option maturing at time T,
- lackrow  $\sigma$  is the implied volatility of the exchange rate used to price the option,

$$d_1 = rac{\ln(S_t/K) + (R_d - R_f + \sigma^2/2)(T - t)}{\sigma \times (T - t)}$$
 $d_2 = d_1 - \sigma \times (T - t)$ 

## Implied volatility $\sigma$ (1)

lacktriangle All GK formula inputs are observable except implied volatility  $\sigma$ 

▶ Option prices are quoted as **vols**, i.e. volatility

- ► To obtain price
  - ► Take quoted vol
  - ▶ Use reference values for all other variables
  - ► Plug into GK formula

## Implied volatility $\sigma$ (2)

#### Implied volatility

is different from historical or realized volatility

▶ is not a forecast of future volatility

- yields an GK option premium reflecting
  - profit margin
  - hedging costs
  - demand and supply in the FX market

Part D:

The Volatility Smile

### Finding vols for different $\Delta s$

- ightharpoonup For a given  $\Delta$  we have prices for:
  - ► Risk reversal (RR)
  - Butterfly spread (BF)
- ▶ The ATM vol,  $\sigma_{ATM}$  is also given
- From the definitions of the RR and the BF

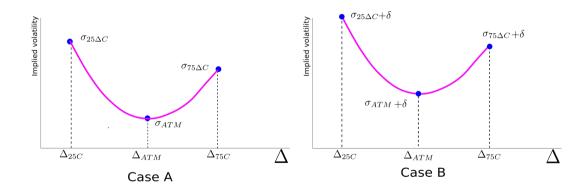
$$\sigma_{25\Delta C} = \sigma_{ATM} + BF_{25\Delta} + \frac{1}{2}RR_{25\Delta}$$

$$\sigma_{25\Delta P} = \sigma_{ATM} + BF_{25\Delta} - \frac{1}{2}RR_{25\Delta}$$

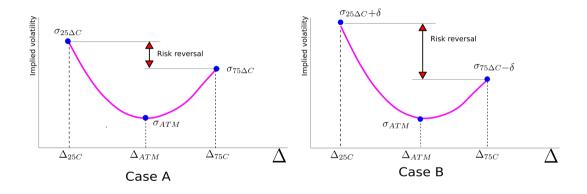
$$\sigma_{75\Delta C} = \sigma_{25\Delta P} \text{ by put-call parity}$$

▶ Plotting  $\sigma$  against  $\Delta$  yields the volatility smile

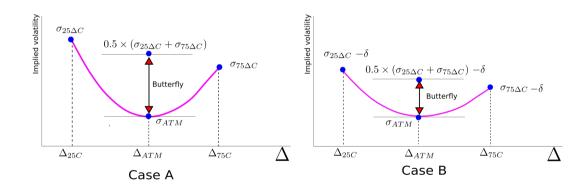
### The level of the smile: $\sigma_{ATM}$



#### The slope of the smile: the risk reversal



#### The slope of the smile: the butterfly spread



Part E:

Constructing the Volatility Smile: Brexit

#### Selecting the dates

We are interested in the behavior of the GBPUSD for three dates:

- ▶ January 8, 2016 (Pre-Brexit)
- ▶ June 24, 2016 (Brexit)
- ▶ December 1, 2017 (Post-Brexit)

#### Getting the data (1)

```
rm(list=ls())
filename = "2018 IET Options data.csv"
data = read.csv(filename, header=TRUE)
data$Dates = mdy hm(as.character(data$Dates))
rownames(data)=NULL
# Specify dates for analysis
date01 = as.Date("2016-01-08 UTC")
date02 = as.Date("2016-06-24 UTC")
date03 = as.Date("2016-12-30 UTC")
```

```
# Clean up memory
# Name of CSV data file
# Load datafile
# convert dates to Date
# remove row names
```

#### Getting the data (2)

```
# Create data frame this.data
this.data = rbind(
  data[which(data$Dates==date01),],
  data[which(data$Dates==date02),],
  data[which(data$Dates==date03).])
# Delete row names and change the names of the columns
rownames(this.data) = NULL
colnames(this.data) = c("Date", "spot", "forward", "atm", "rr25", "bf25",
                        "rr10"."bf10"."rf"."rd"."imp rd")
```

#### Get additional vols

In addition to the 25 $\Delta$  and 75 $\Delta$  vols, obtain the 10 $\Delta$  and 90 $\Delta$  vols:

$$\sigma_{10\Delta C} = \sigma_{ATM} + BF_{10\Delta} + \frac{1}{2}RR_{10\Delta}$$
$$\sigma_{10\Delta P} = \sigma_{ATM} + BF_{10\Delta} - \frac{1}{2}RR_{10\Delta}$$
$$\sigma_{90\Delta C} = \sigma_{10\Delta P}$$

## Calculating the implied vols (1)

```
# Vols are in percent, expressed them as simple numbers
this.data$atm = this.data$atm/100
this.data$rr25 = this.data$rr25/100
this.data$bf25 = this.data$bf25/100
this data rr10 = this data rr10/100
this data$bf10 = this data$bf10/100
this.data$rf = this.data$rf/100
this.data$rd = this.data$rd/100
this.data$imp rd=this.data$imp rd/100
# We will use this.data repeatedly
# Attach it to access its elements
attach(this.data)
```

## Calculating the implied vols (2)

```
# Recover vols for different deltas and put them in the data frame
```

```
this.data$sigma10c = atm + bf10 + 0.5*rr10
this.data$sigma25c = atm + bf25 + 0.5*rr25
this.data$sigma75c = this.data$sigma25c - rr25
this.data$sigma90c = this.data$sigma10c - rr10
this.data$sigmaatm = atm
```

Tenor = 3/12 # Maturity of options, 3 months, in years

#### What is the ATM strike?

Retail products

$$K_{ATM} = S$$
 
$$\Delta_{ATM} = N \left( \frac{\log(F/S) + \frac{1}{2}\sigma_{ATM}^2 T}{\sigma_{ATM}\sqrt{T}} \right)$$

EM currencies, maturities more than one year

$$K_{ATM} = F$$

$$\Delta_{ATM} = \exp(-Rf \times T)N(\frac{1}{2}\sigma_{ATM}\sqrt{T})$$

Major currencies, maturities of one year or less

$$K_{ATM} = F \times \exp\left(0.5\sigma_{ATM}^2 \times T\right)$$
  
 $\Delta_{ATM} = 0.5 \times \exp(-Rf \times T) \simeq 0.5$ 

#### Calculate the $\Delta_{ATM}$

```
# Calculate the strike of the ATM option
K_atm = forward*exp((0.5*(this.data$sigmaatm)^2)*Tenor)
deltaATM = 0.5*exp(-rf*Tenor)
deltaATM
```

## [1] 0.4989636 0.4993504 0.4994441

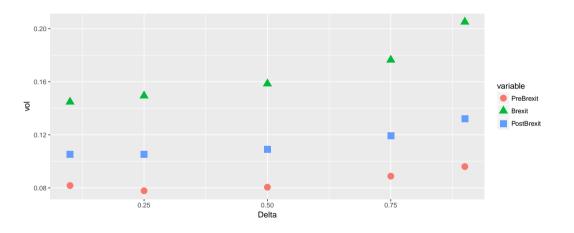
### Rough volatility smile (1)

```
# Select only the vols for each delta
list variables = c("sigma10c", "sigma25c", "sigmaatm",
                   "sigma75c". "sigma90c")
# Read the data as a matrix
vol data = t(as.matrix(subset(this.data. select=list variables)))
# Group the deltas in a vector, to be used in the x-axis
delta vector = c(0.10, 0.25, 0.5, 0.75, 0.9)
# Create the data frame for the chart
vol.smile = data.frame(delta vector, vol data)
rownames(vol.smile) = NULL
colnames(vol.smile) = c("Delta", "PreBrexit", "Brexit", "PostBrexit")
```

#### Rough volatility smile (2)

```
library(reshape2)
vol.data = melt(vol.smile, id="Delta")
ggplot(data=vol.data, aes(x=Delta, y=value, shape=variable)) +
   geom_point(aes(colour=variable), size=4) +
   labs(y="vol")
```

## Rough volatility smile (3)



# A more refined volatility smile (1)

```
# Fit second degree polynomial to smile
fit.Vol = function(data.vol, data.delta,delta.range)
 poly.fit = lm(data.vol ~ poly(data.delta, 2, raw=TRUE))
  # Use fitted polynomial to interpolate Delta-Vol Curve
  delta.square = delta.range*delta.range
 delta.interc = rep(1,length(delta.range))
  X = cbind(delta.interc, delta.range, delta.square)
  iVolInterpol = t(t(X)*poly.fit$coefficients)
  iVolInterpol = rowSums(iVolInterpol)
  return(iVolInterpol)
```

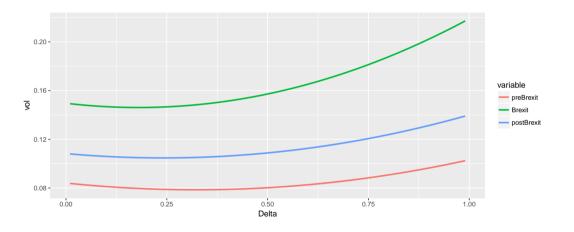
## A more refined volatility smile (2)

```
# Get the data points from the vol.smile data frame
data.delta = vol.smile$Delta
data.vol = vol.smile$PreBrexit
# Interpolation and extrapolation range
delta.range = seq(from=0.01, to = 0.99, by=0.005)
# Obtain the volatility smiles
smile.preBrexit =fit.Vol(vol.smile$PreBrexit, data.delta, delta.range)
                =fit.Vol(vol.smile$Brexit, data.delta, delta.range)
smile.Brexit
smile.postBrexit=fit.Vol(vol.smile$PostBrexit, data.delta, delta.range)
```

## A more refined volatility smile (3)

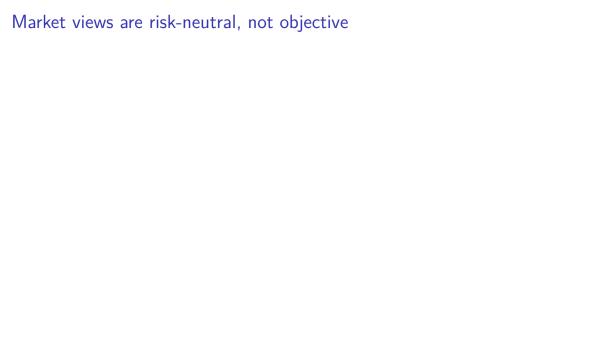
```
# Group the extended volatility smiles in a data frame
smile.df = data.frame(delta.range, smile.preBrexit,
                      smile.Brexit, smile.postBrexit)
rownames(smile.df) = NULL
colnames(smile.df) = c("Delta", "preBrexit", "Brexit", "postBrexit")
# Create the chart
library(reshape2)
smile.data = melt(smile.df, id="Delta")
ggplot(data = smile.data, aes(x=Delta, y=value, colour=variable)) +
 geom line(size=1) + labs(v="vol")
```

## A more refined volatility smile (4)



Part F:

Extracting Risk-Neutral Densities



#### Extraction methods

Methods fall under one of three categories:

- ► Parametric methods
- Semiparametric methods
- ► Non-parametric methods

All methods require constructing call and/or put functions

Option premia vs. strike price

#### Parametric methods

- ▶ Specify a parametric distribution function *F* for the exchange rate
- $\blacktriangleright$  For the distribution parameters  $\Lambda$ , the price of a call option is

$$C^F(K) = \exp(-R_d \times T) \int_K^\infty (S_T - K) dF(S_T; \Lambda),$$

The parameters that fit the market-implied distribution better solve

$$\arg\min_{\Lambda} \sum_{j=1}^{N} |C^F(K_j) - C^{Market}(K_j)|^2$$

#### Semi-parametric methods

- ► Start with a particular density
- Add expansion terms to distribution
- ► Each term helps to approximate market-implied distribution

#### Non-parametric methods

- ▶ Plot value of calls against different strike prices, i.e. market call function
- ▶ Breeden and Litzenberger: risk-neutral distribution proportional to second derivative of the call function:

$$\left. \frac{\partial^2 C}{\partial K^2} \right|_{K=S} = \exp(-R_d \times T) q(S).$$

▶ Use numerical methods to calculate derivative and find risk neutral distribution

Part F: Extracting Risk-Neutral Densities

Constructing option premium functions

### Constructing the market call function (1)

- ▶ We need to obtain the market call function
- ▶ In other words, move from  $\Delta$   $\sigma$  space to *Option premium Strike* space
- ▶ Two useful functions in auxFunctions.R
  - get.strike()
  - GKoption.premium()

## Constructing the market call function (2)

```
get.strike = function(vol,delta,S0,fwd,rf,Tenor)
{
  aux = qnorm(delta*exp(rf*Tenor))
  aux = aux*vol*sqrt(Tenor)
  aux = aux - 0.5*vol*vol*Tenor
  K = exp(-aux)*fwd
  return(K)
}
```

## Constructing the market call function (3)

```
GKoption.premium = function(K, sigma, S, Tenor, fwd, rf, option_type)
  if (option_type =="c") {w=1}
  if (option_type =="p") {w=-1}
 d1 = log(fwd/K)+0.5*sigma*sigma*Tenor
 d1 = d1/(sigma*sqrt(Tenor))
  d2 = d1 - sigma*sqrt(Tenor)
  rd = log(fwd/S)/Tenor + rf
 premium = exp(-rd*Tenor)*(w*fwd*pnorm(w*d1) - w*K*pnorm(w*d2))
  return(premium)
```

### Constructing the market call function (3)

Strike ranges for the selected dates:

#### Constructing the market call function (4)

Generate the call premia:

```
Call preBrexit = mapply(GKoption.premium, K preBrexit,
                        smile.df$preBrexit, S=spot[1],Tenor=Tenor,
                        fwd=forward[1],rf=rf[1], option type="c")
Call_Brexit = mapply(GKoption.premium, K_Brexit,
                     smile.df$Brexit, S=spot[2], Tenor=Tenor,
                     fwd=forward[2],rf=rf[2],option type="c")
Call postBrexit = mapply(GKoption.premium, K postBrexit,
                         smile.df$postBrexit,S=spot[3],Tenor=Tenor,
                         fwd=forward[3],rf=rf[3],option type="c")
```

## Constructing the market call function (5)

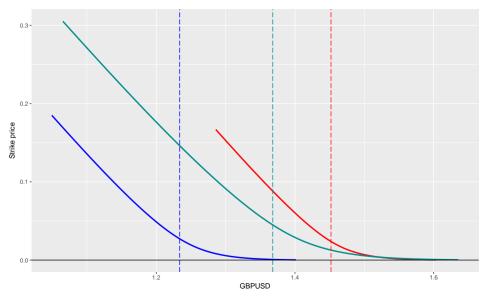
The data frame callstrike.df collects the call premium-strike functions for the selected dates:

#### Constructing the market call function (6)

Plot the market call function

```
ggplot(callstrike.df) +
  geom_line(aes(x=K_preBrexit,y=Call_preBrexit), col="red", size=1) +
  geom_line(aes(x=K_Brexit, y=Call_Brexit), col="darkcyan", size=1) +
  geom_line(aes(x=K_postBrexit, y=Call_postBrexit), col="blue", size=1) +
  labs(x="GBPUSD", y="Strike price") +
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +
  geom_hline(yintercept = 0, col="black")
```

# Constructing the market call function (7)





Question: the call premium-strike functions are downward sloping, i.e. the call premium is higher for lower strike prices. Does this make sense? Explain why.

### Constructing the market put function (1)

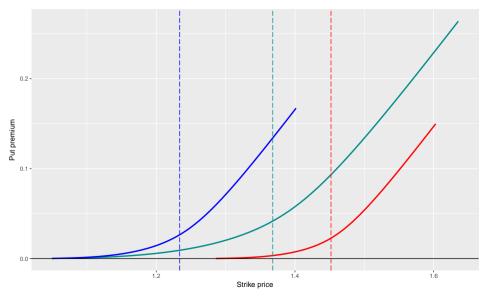
```
Put preBrexit = mapply(GKoption.premium, K preBrexit,
                       smile.df$preBrexit,S=spot[1],Tenor=Tenor,
                       fwd=forward[1],rf=rf[1],option type="p")
Put_Brexit = mapply(GKoption.premium, K_Brexit, smile.df$Brexit,
                        S=spot[2], Tenor=Tenor, fwd=forward[2],
                        rf=rf[2].option type="p")
Put postBrexit = mapply(GKoption.premium, K postBrexit,
                        smile.df$postBrexit,S=spot[3],Tenor=Tenor,
                        fwd=forward[3].rf=rf[3].option type="p")
```

## Constructing the market put function (2)

### Constructing the market put function (3)

```
ggplot(putstrike.df) +
  geom_line(aes(x=K_preBrexit,y=Put_preBrexit), col="red", size=1) +
  geom_line(aes(x=K_Brexit, y=Put_Brexit), col="darkcyan", size=1) +
  geom_line(aes(x=K_postBrexit, y=Put_postBrexit), col="blue", size=1) +
  labs(x="Strike price", y="Put premium") +
  geom_vline(xintercept = spot[1], col="red", linetype="longdash") +
  geom_vline(xintercept = spot[2], col="darkcyan", linetype="longdash") +
  geom_vline(xintercept = spot[3], col="blue", linetype="longdash") +
  geom_hline(yintercept = 0, col="black")
```

# Constructing the market put function (4)



Part F:

Extracting Risk-Neutral Densities
Using the RND package to generate densities